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Periodicity based Cruising Control of Passenger Cars for Optimized Fuel Consumption*

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Abstract—**Eco-driving technologies are able to largely reduce the fuel consumption of ground vehicles. This paper presents how to determine the fuel-optimized operating strategies of passenger cars under cruising process. The design naturally casts into an optimal control problem with the S-shaped engine fueling rate as the integrand of cost function. The solutions are numerically solved by the Legendre pseudospectral method, of which many are found to demonstrate periodic behaviors. In the periodic operation, the engine switches between the minimum brake specific fuel consumption (BSFC) point and the idling point, while the vehicle speed oscillates between its upper and lower bounds. The formation of periodic operation are analyzed and explained by the π-test theory and steady state analysis method.**

Keywords—autonomous vehicle; cruise control; fuel economy; optimal control

INTRODUCTION

The fuel consumption of ground vehicles is affected not only by how the vehicle is designed for energy efficiency, but also by how the vehicles are driven. Driving styles may result in large fluctuations in fuel efficiency. Following eco-driving program advices, aggressive drivers can improve fuel consumption by more than 15% [1]. The eco-driving programs attempt to change a driver's behavior through general advice (e.g., do not drive too fast; do not accelerate too quickly; shift gears sooner to keep engine speed lower; and maintain steady speeds. These programs are quite effective, but many drivers return to their before-training statues if not being trained regularly. Recent research on autonomous driving presents a high opportunity to implement eco-driving strategies in a more efficient way [2]. This kind of application requires the insightful understanding of fuel saving strategies. One approach is to learn the driving patterns of fuel-efficient drivers such as maintaining adequate headway, eliminating excessive idling, etc. [3][4]. Alternatively, model-based approaches are able to determine strategies from the characteristics of vehicle dynamics. Some known strategies include optimal shift-control of the transmission and smoothing the vehicle's acceleration [5].

Achieving the most fuel-efficient driving pattern often involves optimal control problems with nonlinear vehicle models. Monastyrsky and Golownykh [6] used a dynamic programming method to obtain numerical solutions for a number of short distance driving scenarios. Inspired by this work, Chang [7] and Froberg [8] determined that a constant speed is optimal on a flat road within certain bounds. For large gradients on downhill slopes, it is optimal to use the kinetic energy of the vehicle to accelerate in order to gain higher speed [9]. The two cases indicates the fact that the steady-state operation is commonly believed to be optimal in fuel economy. Using steady state operating strategies, a predictive cruise control (PCC) system enhances fuel economy by measuring the surface profile of the road ahead [10]. The PCC allows the vehicle speed to vary around the setting speed for better economy using elevation information from a three-dimensional (3-D) map [11]. A look-ahead real-time controller has been designed based on the dynamic programming algorithm to continuously feed the controller with reference speeds [12][13].

The aforementioned studies greatly rely on the assumption that there is a linear relationship between the engine consumption rate and engine power, which is not sufficiently accurate to describe the fueling characteristics of an engine. The fuel efficiency map of a spark-ignition engine is often in a contoured shape, with peak efficiency at a high-load zone and low efficiency at a part-load zone [14]. This characteristic of the efficiency map naturally leads to an S-shaped relationship between engine fueling rate and engine power, thus the associated fuel-optimized cruise control problem is not completely convex. In some cases, optimization usually leads to a periodic optimal solution, in which the control input alternates between distinctly different values [15]. Some periodic optimal control examples can be found in aerospace engineering and chemical engineering [16][17], but relatively less applications are found in automotive engineering.

In this paper we study how the engine fueling characteristics affect the fuel optimized operations for cruising passenger cars. The periodic operation is proved and demonstrated to be optimal in middle speed situations, which was commonly believed to be steady state operation before. The remainder of this paper is organized as follows. Section 2 introduces the model of vehicle longitudinal dynamics. In Section 3, we describe the formulation of an optimal control problem (OCP) for best fuel economy. Section 4 analyzes the periodicity of the OCP and introduce how to do the numerical computation. Section 5 presents the optimal solution and discusses the mechanism of periodicity formation. Section 6 concludes this paper.

VEHICLE LONGITUDINAL MODEL FOR CONTROL

The passenger car used is assumed to be equipped with cruise control (CC) system. The car has a 2-liter internal combustion engine (ICE) with a continuously variable transmission (CVT). The assumptions when modeling are: (1) the powertrain dynamics are lumped together to be a simplified transfer function; (2) the CVT ratio is selected to

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match the vehicle and engine speeds for the best fuel economy; and (3) no longitudinal slip on wheels. In typical pulse-and-gliding operations, the pulse acceleration is below 0.5 m/s^2, and the gliding deceleration is higher than -0.4m/s^2 [22]. The acceleration level is inside the range of road friction coefficient of dry asphalt road. Hence, even though no tyre slip is a strong assumption, but is actually reasonable here. The vehicle longitudinal dynamics are simplified to be:

$$
\begin{aligned}
M\dot{v} &= \frac{i_g i_0 \eta_T}{r_w} T_e - C_A v^2 - Mgf, \\
\tau_{eng}\dot{T}_e &= -T_e + T_{ecom},
\end{aligned}
\tag{1}
$$

where v is the vehicle speed, T_e is the engine torque, τ_{eng} is the time constant for lumped powertrain dynamics, T_{ecom} is the engine torque command, i_g is the CVT gear ratio, i_0 is the gear ratio of the final drive, η_T is the mechanical efficiency of the powertrain, r_w is the wheel radius, v is the vehicle speed, M is the vehicle mass, C_A is the aerodynamic drag coefficient, g is the gravity coefficient, and f is the coefficient of rolling resistance. In fuel saving research, an accurate fuel consumption model is critical. The engine fueling rate is modeled as a function of engine speed and torque as follows:

$$
\begin{aligned}
Q_{eng} &= \Psi(T_e, \omega_e) \cdot P_e, \\
P_e &= T_e \cdot \omega_e,
\end{aligned}
\tag{2}
$$

where Q_{eng} is the engine fueling rate, ω_e is the engine speed, P_e is the engine power, and $\Psi(\cdot,\cdot)$ is a function of engine torque and engine speed to describe the brake specific fuel consumption (BSFC) as shown in Fig. 1.

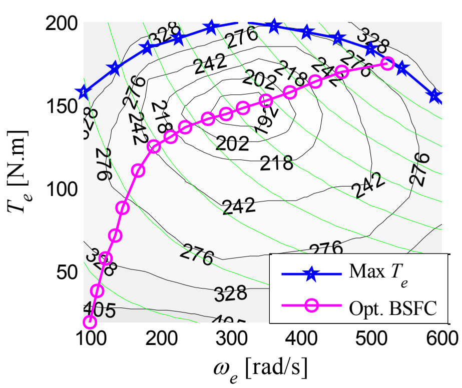


Fig. 1. The BSFC map of an engine.

In an internal combustion engine, the engine BSFC is mainly affected by heat losses, friction, and combustion efficiency. For an engine with a BSFC, as shown in Fig. 1, the minimum BSFC point is located around ω_e= 310 rad/s and T_e = 140 N·m. All other operating points have lower engine efficiency because of higher friction losses in a high speed region, richer fuel/air mixture in a high torque region, and higher heat losses in a low power region. When no engine power is required, the engine is idled at ω_e= 100 rad/s and T_e= 0 N·m instead of being shut down due to NVH and emission considerations. Thus, a small amount of fuel is consumed. The optimal BSFC line is shown by circles in Fig. 1, and is fitted by the follow function:

$$
\omega_{eopt} = b_{opt}\left(1 - k_{opt}T_e\right)^{-1},
\tag{3}
$$

where ω_{eopt} is the engine speed in the optimal BSFC line, and k_{opt}, b_{opt} are fitting parameters. The vehicle is equipped with a CVT, and the speed ratio is selected to achieve the best fuel economy:

$$
i_g = r_w \cdot \omega_{eopt}(i_0 v)^{-1}.
\tag{4}
$$

Since the CVT dynamics are neglected, the engine is assumed to follow the optimal BSFC line all the time by properly controlling the CVT. This assumption greatly simplifies the numerical optimization, and the results reflect the best performance achievable.

Optimal Cruising Problem for Best Fuel Economy

The cruising control system in a free driving scenario does not strictly track the speed reference set by drivers. A buffer for speed fluctuation around the reference is allowed to make the operation more flexible. The speed buffer is defined as

$$
v_{\min} \le v \le v_{\max},
\tag{5}
$$

where $v_{\min}$ and $v_{\max}$ are the bounds of speed fluctuation, and their average is the speed reference which is notated as v_{set}. The optimal control problem (OCP) is formulated to minimize the fuel consumption per kilometer over the time horizon $[0, T_f]$.

$$
\min J = \frac{\int_0^{T_f} Q_{eng}\, dt}{\int_0^{T_f} v\, dt},
\tag{6}
$$

while subjecting to the following constraints:

$$
\begin{aligned}
&\text{(a) Engine constraints:} \\
&\qquad T_{e\min} \le T_e \le T_{e\max}, \\
&\qquad \omega_{e\min} \le \omega_e \le \omega_{e\max}, \\
&\text{(b) Speed buffer constraint:} \\
&\qquad v_{\min} \le v \le v_{\max}.
\end{aligned}
\tag{7}
$$

In Eq. (7), constraints (a) are hard constraints due to physical limits of the engine, and constraint (b) is the constraint for speed fluctuation. When the CVT ratio is selected based on Eq. (4), the engine fueling rate Q_{eng} can be exclusively a function of the engine power P_e. For computational convenience, the engine fueling rate is fitted by a polynomial function of degree N (the thick solid line in Fig. 2):

$$
Q_{eng} = \varphi(P_e) = \sum_{i=0}^{N} \theta_i \cdot P_e^i,
\tag{8}
$$

where $\varphi(\cdot)$ is a polynomial function, θ_i are fitting coefficients, and N is the degree of the polynomial.

As shown in Fig. 2, the engine fueling rate has a nonlinear S-shaped relationship with respect to engine power. The curve between fueling rate and power is concave in a low power range and convex in a high power range. Two points are critical to analyze or determine the fuel-optimized operation: the saddle point and the tangent point. The saddle point divides the curve into a concave region and a convex region. The tangent point is the intersection of the curve and its lowest linear envelope, which is rather close to the minimum BSFC point. The engine power P_e at the saddle point is around $P_{e,sadd}$ =31 kW, corresponding to a speed of 36.5 m/s. The P_e at the tangent point is around $P_{e,tang}$ =45 kW, corresponding to a speed of 41.4 m/s. Fig. 3 shows the first and second derivatives of the fueling function. We know that the first derivative φ' is positive in the whole range, and the second derivative φ'' is negative for $P_e < P_{e,sadd}$ and becomes positive when $P_e > P_{e,sadd}$.

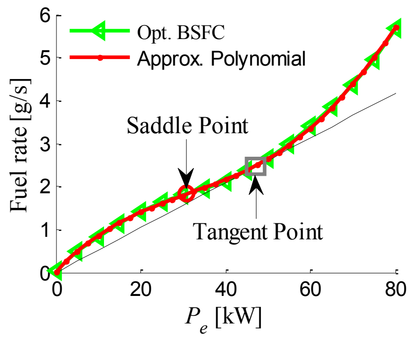


Fig. 2. Relationship between fuel rate and engine power.

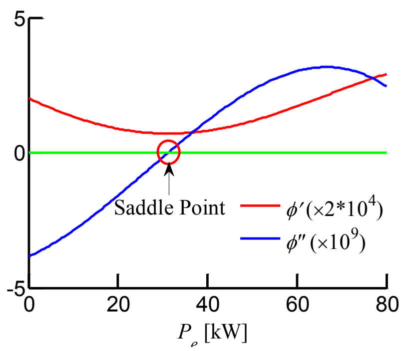


Fig. 3. Derivative analysis of engine fuel function.

PERIODICITY ANALYSIS AND NUMERICAL COMPUTATION

The S-shaped engine fueling characteristic leads to a possibility of achieving better fuel economy by time-dependent periodic operation. An important issue is to know when the periodic control is optimal. The π-test is a frequency domain criterion and works around the operating point [18]. It relies on Parseval's theorem to convert the second variation of the Hamiltonian into a frequency integral. The quadratic kernel of the integral must be negative-definite for some values of frequency if the periodic control is optimal [19].

Before using the π-test, we first formulate an optimal steady state (OSS) problem by fixing both state and control variables. For simplicity, the OSS was formulated to keep the main traits of the original OCP while reducing complexity by (a) using the average vehicle speed $\bar{v}$ in calculating the distance of travel and traction force; (b) neglecting the constant term gf in vehicle dynamics; (c) using engine torque T_{ecom} instead of engine power P_e as the control input; and (d) neglecting engine physical constraints because they are inactive in most cases. Thus, we have:

$$J_{ss} = \frac{1}{\bar{v}T_f}\int_0^{T_f} \varphi(x)\,dt, \tag{9}$$

Subject to

$$\dot{v} = f(x,u),$$
$$h(x) \le 0,$$

where the control input $u = P_e$, the state $x = v$, and

$$f(x,u) = \frac{\eta_T}{M\bar{v}}u - \frac{C_A}{M}x^2,$$
$$h^T(x) = [v_{min} - x \quad x - v_{max}].$$

The Hamiltonian of the OSS is given by:

$$H(x,u,\alpha,\lambda) = \varphi(u) + \alpha^T\begin{bmatrix} v_{min} - x \\ x - v_{max}\end{bmatrix} + \lambda\left(\frac{\eta_T}{M\bar{v}}u - \frac{C_A}{M}x^2\right), \tag{10}$$

where $\alpha^T = [\alpha_1 \quad \alpha_2]$ and λ are the Karush-Kuhn-Tucker (KKT) multipliers. The first-order necessary condition of the OSS for optimality gives:

$$\begin{gathered} \partial H/\partial x = 0, \partial H/\partial u = 0, \\ \alpha_i \cdot h_i = 0, \alpha_i \ge 0, \\ f(x,u) = 0, \lambda \ne 0. \end{gathered} \tag{11}$$

Solving Eq. (11), the optimal steady state solution is unique as follows:

$$\begin{gathered} \bar{x} = v_{min}, \qquad \bar{u} = 0.5\kappa\rho^{-1}v_{min}^2, \\ \bar{\lambda} = -\rho^{-1}\varphi'(\bar{u}), \qquad \bar{\alpha} = [\kappa\rho^{-1}\varphi'(\bar{u})v_{min} \quad 0], \end{gathered} \tag{12}$$

where κ and ρ are the intermittent variables

$$\kappa = \frac{2C_A}{M}, \rho = \frac{\eta_T}{M\bar{v}}. \tag{13}$$

Theorem [18][19]**:** If the pair $(\bar{x},\bar{u})$ is a local minimum of the OSS and the OCP is normal at $(\bar{x},\bar{u})$, and for steady state solutions $(\bar{x},\bar{u},\bar{\alpha},\bar{\lambda})$, there exists $\omega > 0$ such that the Hermitian matrix $\Pi(j\omega)$ is partially positive, and the optimal periodic solution exists for the OCP. The Hermitian matrix $\Pi(j\omega)$ is defined as:

$$\begin{gathered} \Pi(j\omega) = G^T(-j\omega)PG(j\omega) + Q^TG(j\omega) + \\ G(-j\omega)Q + R, \\ G(s) = (s-A)^{-1}B, \end{gathered} \tag{14}$$

where

$$\begin{gathered} A = \left.\frac{\partial f}{\partial x}\right|_{(\bar{x},\bar{u})}, B = \left.\frac{\partial f}{\partial u}\right|_{(\bar{x},\bar{u})}, \\ P = H_{xx}|_{(\bar{x},\bar{u},\bar{\alpha},\bar{\lambda})}, Q = H_{xu}|_{(\bar{x},\bar{u},\bar{\alpha},\bar{\lambda})}, \\ R = H_{uu}|_{(\bar{x},\bar{u},\bar{\alpha},\bar{\lambda})}. \end{gathered} \tag{15}$$

It is easy to validate that the pair $(\bar{x},\bar{u})$ in Eq. (12) is a local minimum of the OSS because no steady state is admissible when v is smaller than its lower bound v_{min}. The normality condition of the OCP also holds because A is nonsingular since the vehicle speed v is positive. In addition, the normality condition also means that the system is controllable, which definitely holds in the vehicle cruise control system. Then, substituting Eq. (12) into Eq. (14), we have the π-test criterion:

$$\Pi(j\omega) = \frac{\kappa\rho}{\kappa^2 {v_{min}}^2 + \omega^2}\varphi^{'}(\bar{u}) + \varphi^{''}(\bar{u}). \quad (16)$$

Here, $\kappa > 0$, $\rho > 0$, and $\varphi^{'}(\bar{u}) > 0$ in the admissible region. The sign of $\Pi(j\omega)$ depends on $\varphi^{''}(\bar{u})$ when $\omega \to +\infty$. When $\varphi^{''}(\bar{u}) < 0$ ($P_e < P_{e,sadd}$), $\Pi(j\omega)$ can be negative-definite at some values of frequency, so an optimal periodic operation exists. When $\varphi^{''}(\bar{u}) > 0$ ($P_e > P_{e,sadd}$), the steady state operation becomes optimal.

The computation of optimal solutions relies on the Legendre pseudospectral method. This approach is a type of direct method. The basis is to parameterize both control inputs and states using a finite set of Lagrange polynomials at orthogonal collocation points [20]. The associated nonlinear programming (NLP) has equally sparse structure, which allows the use of a sparse NLP solver for fast numerical computation. Another benefit is that the approximation will converge on the true function at an exponential rate as a function of the number of collocation points. In order to avoid infinite switching in solutions, we introduce a penalty term in the cost function by penalizing the derivative of the engine torque command:

$$\tilde{J} = \frac{1}{\int_0^{T_f} v\,dt}\int_0^{T_f}\left(Q_{eng} + \xi\left(\frac{dT_{ecom}}{dt}\right)^2\right)dt, \quad (17)$$

where $\xi > 0$ is the coefficient for the penalty. It should be noted that T_f is an important factor that affects the formation of periodic operation. If T_f is not well posed (for instance, not exactly the integer of a periodic period), a less perfect solution might be generated. Hence, we deliberately give a small flexibility to T_f for self-adjustment in the optimization.

RESULTS AND ANALYSIS

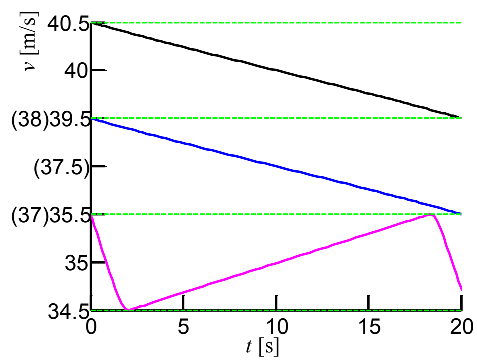
The parameters for the OCP are listed in Table 1. The speed buffer is set to be $\pm 0.5\mathrm{m/s}$ ($v_{max} - v_{min} = 1\mathrm{m/s}$).

Table 1 Parameters for the optimal control problem

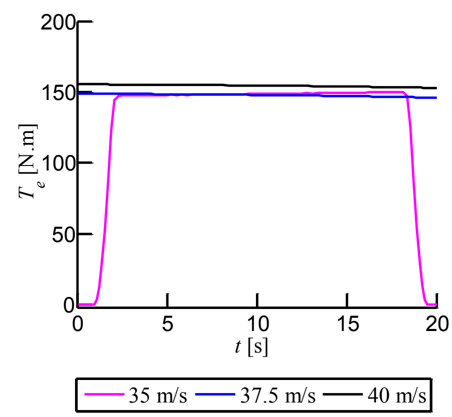
Parameter	Value	Parameter	Value
T_{emin}	0 N·m	T_{emax}	180 N·m
ω_{emin}	80 rad/s	ω_{emax}	600 rad/s
$v_{max} - v_{min}$	1 m/s	τ_{eng}	0.35 s
M	1845 kg	η_T	0.9
i_0	3.86	r_w	0.3 m
f	0.02	C_A	0.57 kg/m
ξ	1×10^{-5}		

Solutions for fuel-optimized operation

To investigate the relationship between periodicity and speed, v_{set} was selected to have different values, ranging from 5 to 40 m/s. The numerical solutions are shown in Fig. 4, Fig. 5 and Fig. 6. Note that Fig. 4(a), Fig. 5(a) and Fig. 6(a) use tick-label shared Y-axis. Three lines are drawn together in each figure, and the highest tick-label of every bottom sub-figure is overlapping with the lowest tick-label of above sub-figure. When v_{set} is 20, 25, and 30 m/s, the periodic operation is found to be optimal. The engine torque switches between 145 N·m (around the minimum BSFC point) and 0 N·m (around the idling point). The speed oscillates between its upper and lower bounds. This periodic operation is in agreement with the well-known pulse-and-glide (PnG) strategy, which has been proven to significantly reduce fuel consumption [21][22]. This strategy uses a periodic operation that first runs the engine at high power to accelerate, and then coasts down to a low speed. When v_{set} is 35, 37.5 and 40 m/s, the steady state operation is optimal. The engine torque is almost constant, and the vehicle speed drops from the upper bound to the lower bound of the speed buffer in order to maximally use kinetic energy. (Note that v_{set}=35 m/s is within the boundaries of periodic and steady state operation, which shows both features of the two kinds of operations). When v_{set} reduces to 5, 10, and 15 m/s, the periodic operation becomes less regular because the penalty term in the cost function counteracts the economic benefit of the switching operation. Therefore, a less regular PnG operation emerges with smoothing acceleration and deceleration. In such a strategy, the speed and range still fluctuate, but the engine slides in the half-load region, unable to reach the minimum BSFC point.

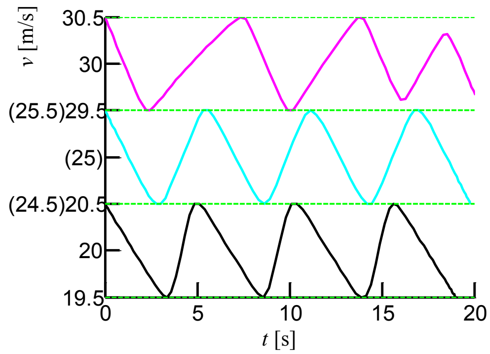


(a) vehicle speed

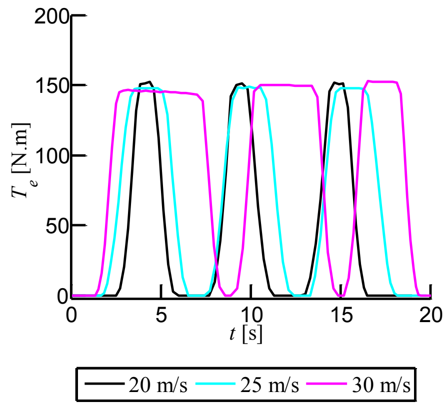


(b) engine torque

Fig. 4. Optimal solutions when v_{set} = 35, 37.5, and 40 m/s.

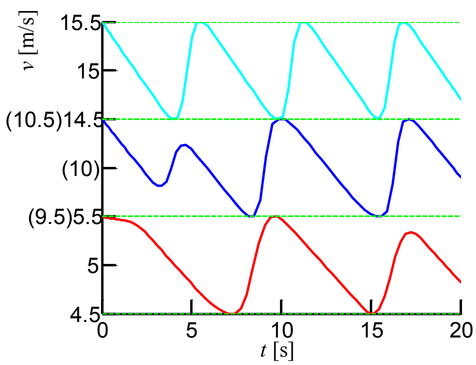


(a) vehicle speed

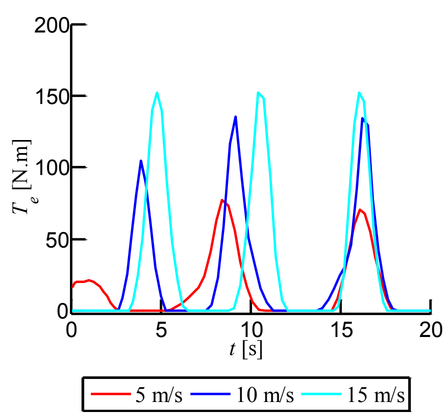


(b) engine torque

Fig. 5. Optimal solutions when v_{set} = 20, 25, and 30 m/s.



(a) vehicle speed



(b) engine torque

Fig. 6. Optimal solutions when v_{set} =5, 10, and 15 m/s.

Analysis of Periodic Operation

The fueling rate of the engine is the most critical factor for periodic operation. The aforementioned π-test predicts that the saddle point $P_{e,sadd}$ seems to be the threshold that separates the periodic region and non-periodic region. However, we note that the π-test relies on the linearization at the operating points. Due to the nonlinearity of the engine, the actual region with periodicity should be larger than the prediction of the π-test. Therefore, the tangent point $P_{e,tang}$, rather than the saddle point $P_{e,sadd}$, becomes the threshold of separation between the two regions. The reason is illustrated in Fig. 7, which adopts a graph-based method for steady state analysis.

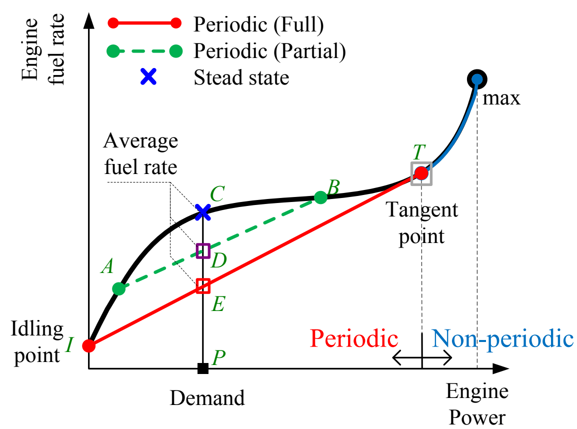


Fig. 7. Illustration of the formation of periodic operation.

Fig. 7 conceptually shows the S-shaped function of the engine fueling rate. Any periodic operation can be represented by a secant line in the S-shaped curve; e.g., line AB or line IT, wherein the two endpoints are the two engine operating points. Any steady state operation corresponds to a single point (e.g., point C). For a given power demand, the average fuel rate (e.g., Point C, D, and E) determines the overall fuel consumption. When $P_e < P_{e,tang}$, the secant line is always lower than a single point, which means that periodic operation is optimal. In this region, line IT gives the lowest average for any given power because it is the lowest linear envelope. This shows why the fuel optimized periodic operations fluctuate between the minimum BSFC point and the idling point (close to the two points, but not exactly; the following paragraph will explain the reason). When $P_e > P_{e,tang}$, a single point is always the lowest for a given power demand, which means that the steady state operation is optimal.

In addition, according to Fig. 7, the optimal periodic operation has no rate limitation, and any switching should be finished in infinite speed. However, the solutions of numerical computation do not follow this rule. In reality, besides the engine fueling rate, the formation of optimal operation is also affected by four other factors: (1) the aerodynamic drag; (2) the engine transient fuel consumption; and (3) the regulation of gear ratio in CVT; (4) the powertrain dynamics. The aerodynamic drag is a convex function of vehicle speed that tends to decrease the possibility of periodicity. The engine transient fuel consumption is not negligible in periodic control. It can increase engine fuel consumption by 4 to 5% in an accelerating situation. The transient fuel consumption caused by periodic operation partially counterbalances the fuel benefit, which has similar effects as the aerodynamic drag. The transmission and its control of the gear ratio also affect the formation of the optimal operation. A discrete gear ratio or inaccurate regulation of the gear ratio will largely change the shape of the engine fueling rate, thus altering the fuel-optimized operation. The powertrain dynamics can delay and smooth the switching timing, which will eliminate the possibility of avoiding the speed-infinite shifting from one point to the other.

CONCLUSION

This paper examined the fuel optimal control strategies of cruising passenger cars. Many ecological projects have used steady state operating strategies to improve the fuel economy of vehicles. It is not widely recognized that fuel-optimized operation can be periodic as opposed to steady state. We demonstrated by both the π-test and steady state analysis that optimal periodic operation do exists due to the S-shaped fueling rate of internal combustion engine. Our major findings include:

(1) The S-shaped engine fuel rate dominates the formation of fuel-optimized operation. The π-test results show that in the concave region the periodic control is optimal, and in the convex region the steady state operation becomes optimal;

(2) The optimal solution shows that as the vehicle speed increases, optimal driving operation changes from partial-periodic operation to full-periodic operation, and finally to steady speed operation. In the full-periodic operating strategy, the engine switches periodically between the minimum BSFC point and the idling point. The vehicle speed fluctuates between its upper and lower bounds.

It must be noted that the use of periodic operation is not completely free, but limited by other considerations in engineering practice. The hybridized vehicles largely reduce the dependence of fuel consumption on load and operation, which have weakened the fuel benefit of periodicity. Moreover, the periodic operation will be objectionable to the ride comfort of passengers, even to the smoothness of traffic flow. A balanced design should be achieved between fuel benefit and other requirements. One solution is to add a comfort penalty term to the cost function. The comfort penalty can be some convex functions of vehicle longitudinal acceleration, which will limit the fast transition of acceleration during switching. Another solution is to directly add the transient dynamics to the switching between pulse and gliding phases. The transient dynamics can also help reduce the maximum acceleration, thus improving ride comfort of passengers.

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