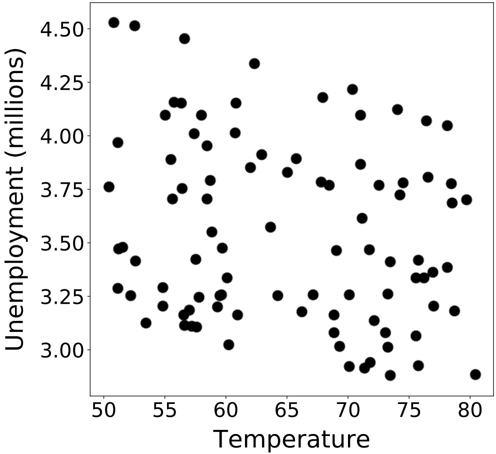
Correlation (Usually) Does Not Imply Causation

Probability and Statistics for Data Science

Carlos Fernandez-Granda

Unemployment and temperature in Spain (2015-2022)



Correlation coefficient: -0.21

Would an increase in temperature decrease unemployment?

Causal inference

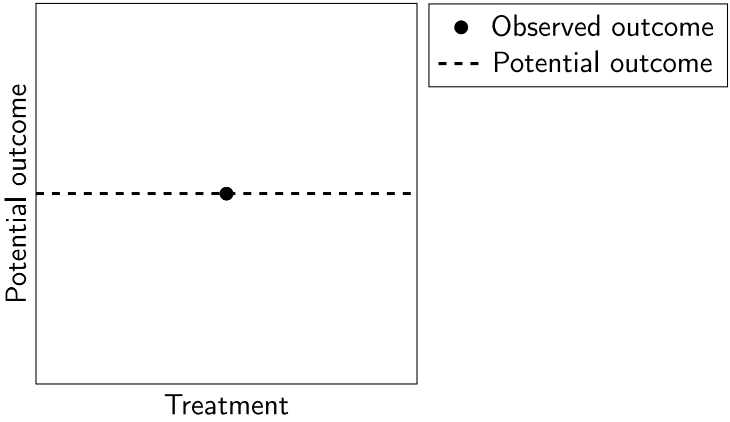
Key question: Does a treatment $\tilde{t}$ cause a certain outcome?

Potential outcome: $\widetilde{\text{po}}_t$

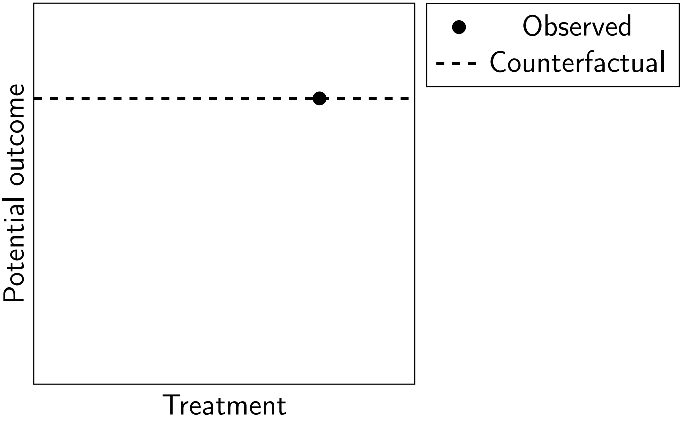
Observed data:

$$\tilde{y} := \widetilde{\text{po}}_t \qquad \text{if} \qquad \tilde{t} = t$$

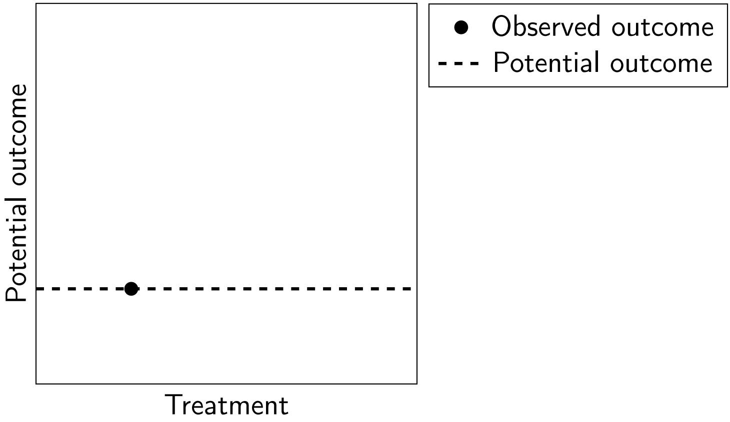
Potential outcomes



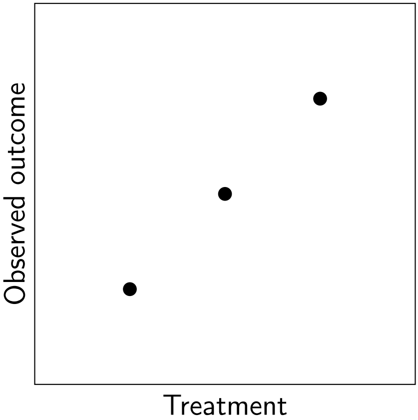
Potential outcomes



Potential outcomes



Observed data



Linear causal effect

For some constant $\beta \in \mathbb{R}$

$$\mathrm{E}\left[\widetilde{\mathsf{po}}_t\right] = \beta t$$

Key question: Can we estimate linear causal effects from data?

Idea

Use covariance between observed outcome $\tilde{y}$ and the treatment $\tilde{t}$

Necessary condition: $\widetilde{\mathrm{po}}_t$ and $\tilde{t}$ are independent for all t

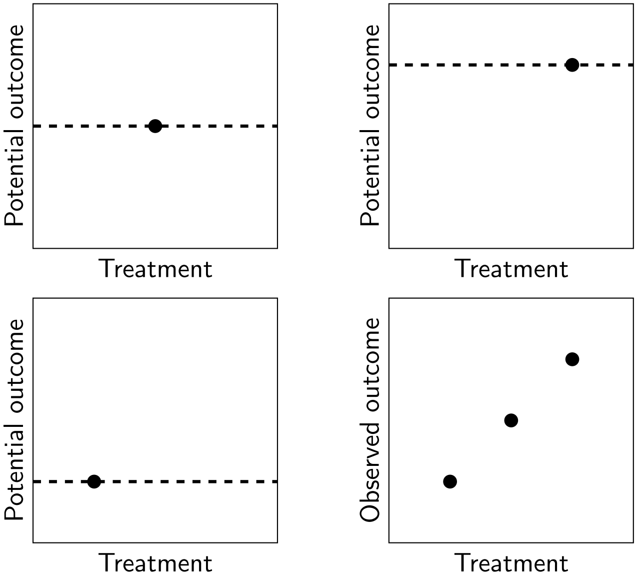
Iterated expectation

Assuming $\mathrm{E}[\tilde{t}] = 0$ and $\mathrm{E}[\tilde{t}^2] = 1$

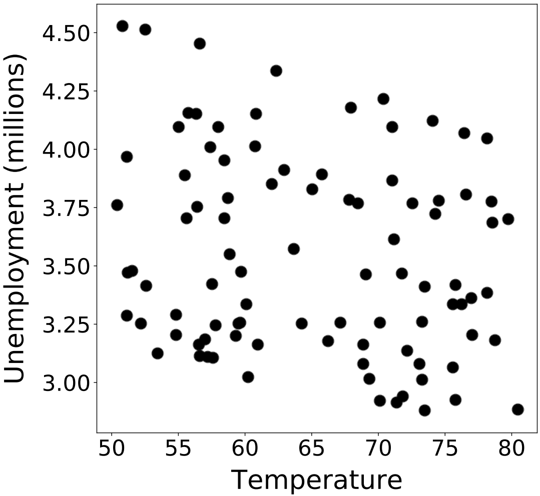
$$\begin{aligned}\mathrm{Cov}\left[\tilde{y}, \tilde{t}\,\right] = \mathrm{E}\left[\tilde{y}\tilde{t}\,\right] &= \mathrm{E}\left[\mu_{\tilde{y}\tilde{t}\,|\,\tilde{t}}(\tilde{t})\right] \\ &= \mathrm{E}\left[\beta\tilde{t}^{\,2}\right] \\ &= \beta\mathrm{E}\left[\tilde{t}^{\,2}\right] \; = \beta\end{aligned}$$

$$\begin{aligned}\mu_{\tilde{y}\tilde{t}\,|\,\tilde{t}}(t) &= \int_{y=-\infty}^{\infty} yt\, f_{\tilde{y}\,|\,\tilde{t}}(y \mid t)\,\mathrm{d}y \\ &= \int_{y=-\infty}^{\infty} yt\, f_{\widetilde{\mathrm{po}}_t\,|\,\tilde{t}}(y \mid t)\,\mathrm{d}y \\ &= t\int_{y=-\infty}^{\infty} y\, f_{\widetilde{\mathrm{po}}_t}(y)\,\mathrm{d}y \\ &= t\mathrm{E}\left[\widetilde{\mathrm{po}}_t\right] \\ &= \beta t^2\end{aligned}$$

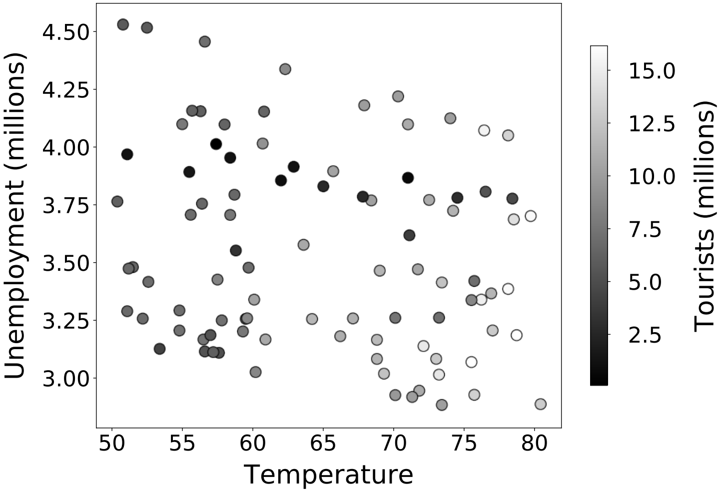
Why do we need independence?



Unemployment and temperature in Spain (2015-2022)



Unemployment and temperature in Spain (2015-2022)



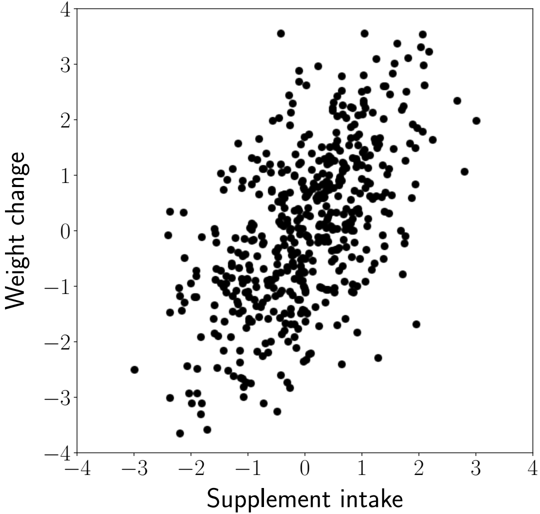
Guinea-pig rescue

Goal: Fatten the guinea pigs

Question: Does a nutritional supplement help?

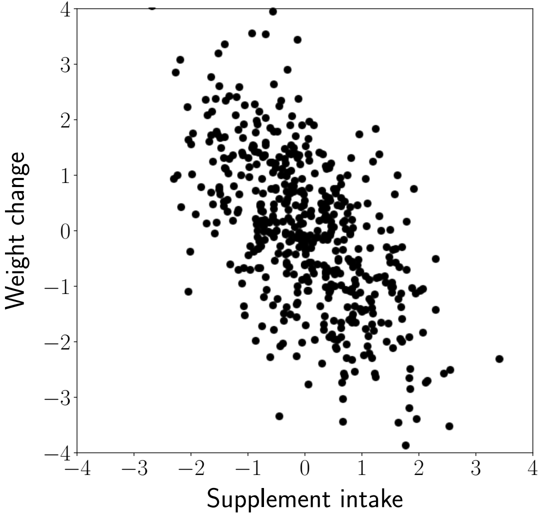
Supplement mixed with food

Covariance = 0.8



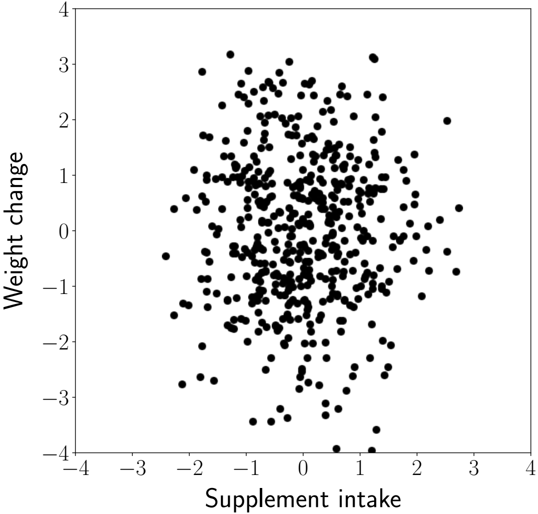
Supplement after the food

Covariance = -0.8



Randomized supplement

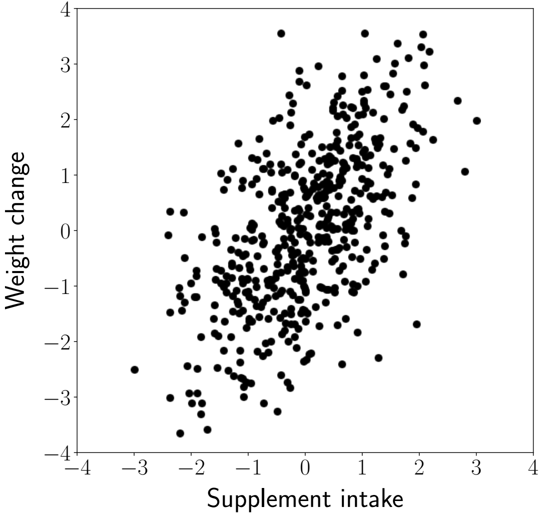
Covariance = 0



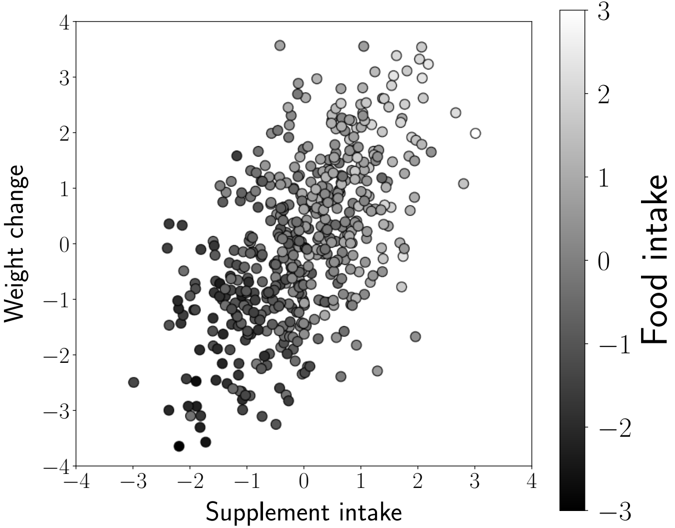
What's going on?

Weight change depends on food intake

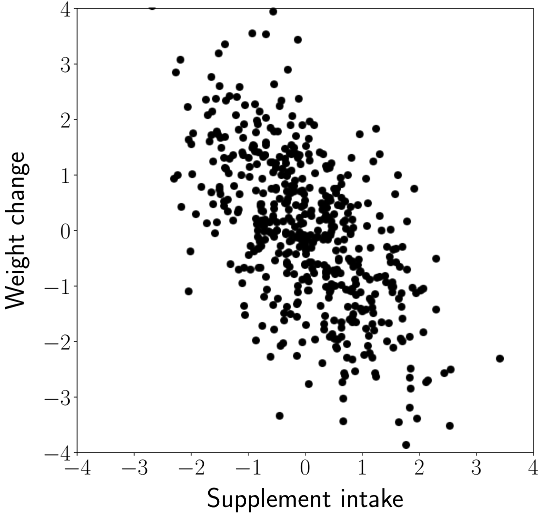
Supplement mixed with food



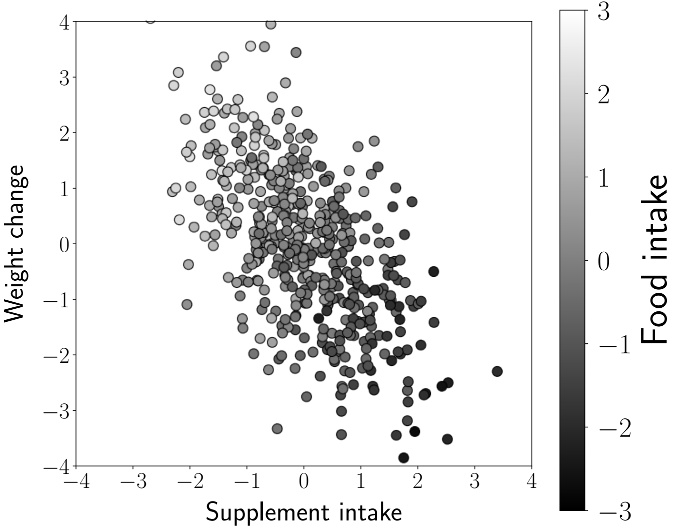
Supplement mixed with food



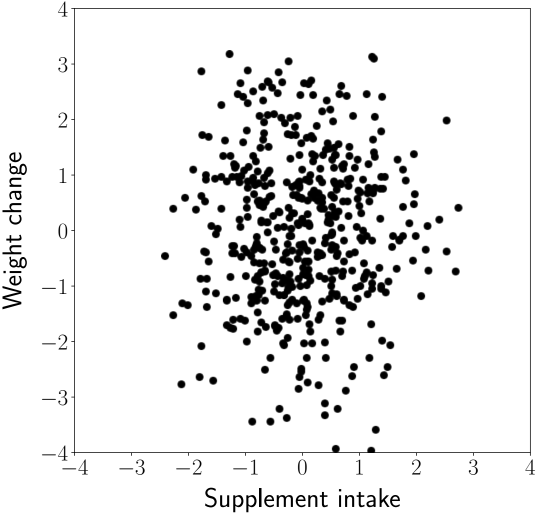
Supplement after the food



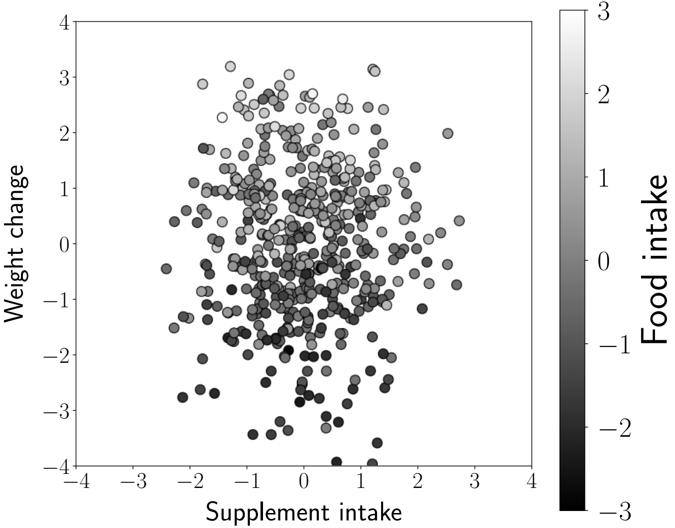
Supplement after the food



Randomized supplement



Randomized supplement



Unobserved confounder

Potential outcome $\widetilde{\mathrm{po}}_{t,c}$ depends on treatment $\tilde{t}$ and on confounder $\tilde{c}$

Observed data:

$$\tilde{y} := \widetilde{\mathrm{po}}_{t,c} \qquad \text{if} \qquad \tilde{t} = t, \tilde{c} = c$$

For some constants $\beta, \gamma \in \mathbb{R}$

$$\mathrm{E}\left[\widetilde{\mathrm{po}}_{t,c}\right] = \beta t + \gamma c$$

Can we still estimate β from covariance between $\tilde{t}$ and $\tilde{y}$?

Assumptions

$\tilde{t}$ and $\tilde{c}$ are standardized

No additional confounders: $\widetilde{\mathrm{po}}_{t,c}$ is independent from $(\tilde{t}, \tilde{c})$

Iterated expectation

$$\begin{aligned}
\mathrm{Cov}\left[\tilde{y},\tilde{t}\right] = \mathrm{E}\left[\tilde{y}\tilde{t}\right] &= \mathrm{E}\left[\mu_{\tilde{y}\tilde{t}\,|\,\tilde{t},\tilde{c}}(\tilde{t},\tilde{c})\right] \\
&= \mathrm{E}\left[\beta\tilde{t}^{\,2} + \gamma\tilde{t}\tilde{c}\right] \\
&= \beta\mathrm{E}\left[\tilde{t}^{\,2}\right] + \gamma\mathrm{E}\left[\tilde{t}\tilde{c}\right] \\
&= \beta + \gamma\rho_{\tilde{t},\tilde{c}}
\end{aligned}$$

$$\begin{aligned}
\mu_{\tilde{y}\tilde{t}\,|\,\tilde{t},\tilde{c}}(t,c) &= \int_{y=-\infty}^{\infty} yt\, f_{\tilde{y}\,|\,\tilde{t},\tilde{c}}(y\,|\,t,c)\,\mathrm{d}y \\
&= \int_{y=-\infty}^{\infty} yt\, f_{\widetilde{\mathrm{po}}_{t,c}\,|\,\tilde{t},\tilde{c}}(y\,|\,t,c)\,\mathrm{d}y \\
&= t\int_{y=-\infty}^{\infty} y\, f_{\widetilde{\mathrm{po}}_{t,c}}(y)\,\mathrm{d}y \\
&= t\mathrm{E}\left[\widetilde{\mathrm{po}}_{t,c}\right] \\
&= \beta t^2 + \gamma ct
\end{aligned}$$

Guinea pigs

Treatment $\tilde{t}$: Supplement intake

Confounder $\tilde{c}$: Food intake

Potential outcome $\widetilde{\text{po}}_{t,c}$: Weight change

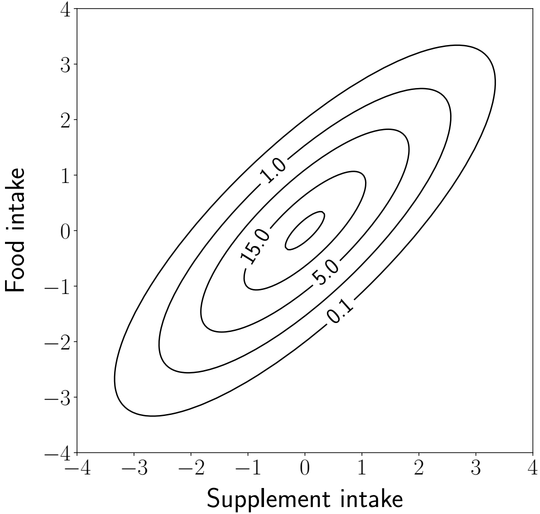
$$\mathrm{E}\left[\widetilde{\text{po}}_{t,c}\right] = c$$

Covariance between observed weight change and supplement?

$$\mathrm{Cov}\left[\tilde{y}, \tilde{t}\,\right] = \rho_{\tilde{t},\tilde{c}}$$

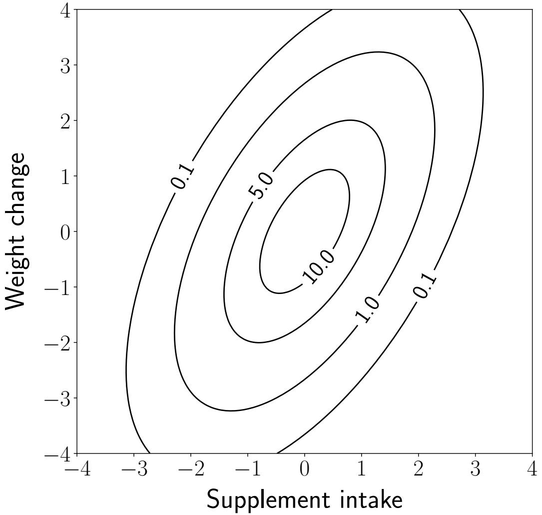
Supplement mixed with food: $\rho_{\tilde{t},\tilde{c}} := 0.8$

Assuming $\tilde{t}$ and $\tilde{c}$ are jointly Gaussian



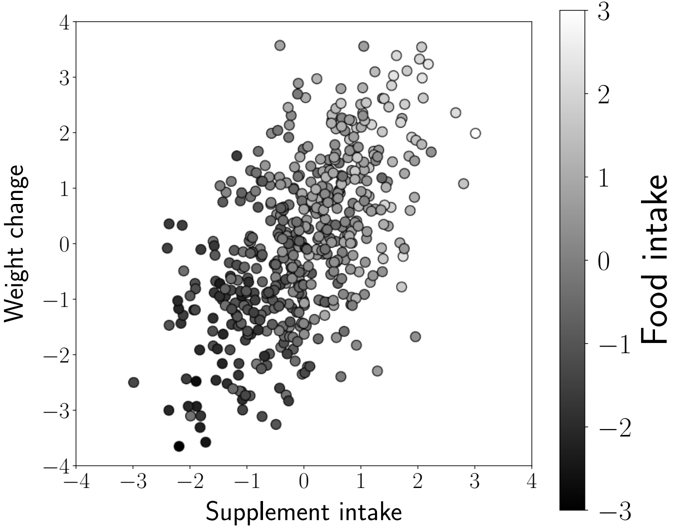
Supplement mixed with food: $\mathrm{Cov}\left[\tilde{y}, \tilde{t}\,\right] = 0.8$

Assuming $\widetilde{\mathrm{po}}_{t,c}$ is Gaussian with mean c and unit variance



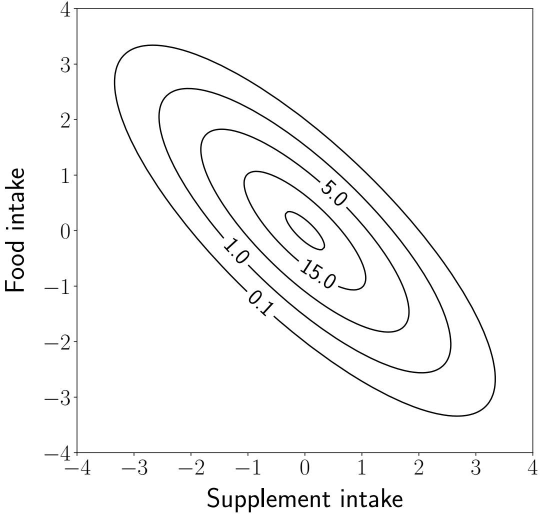
Supplement mixed with food: $\mathrm{Cov}\left[\tilde{y}, \tilde{t}\,\right] = 0.8$

Assuming $\widetilde{\mathrm{po}}_{t,c}$ is Gaussian with mean c and unit variance



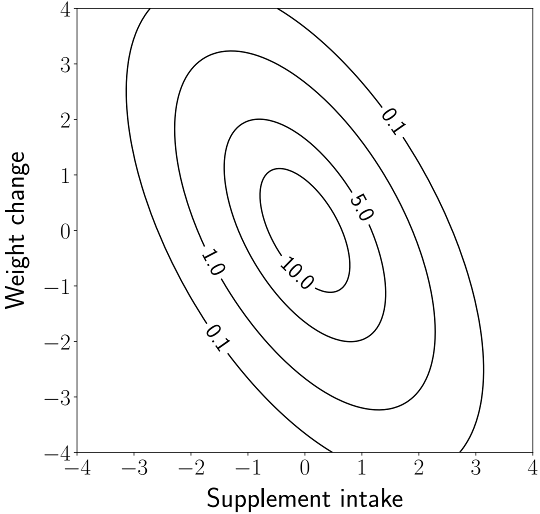
Supplement after the food: $\rho_{\tilde{t},\tilde{c}} := -0.8$

Assuming $\tilde{t}$ and $\tilde{c}$ are jointly Gaussian



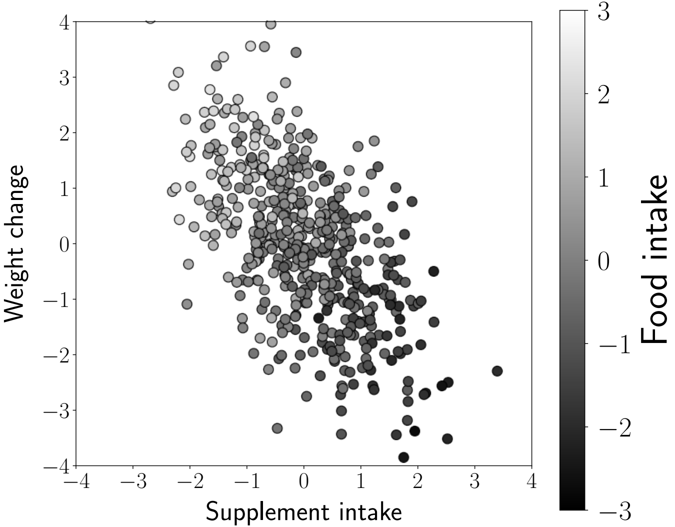
Supplement after the food: $\mathrm{Cov}\left[\tilde{y}, \tilde{t}\,\right] = -0.8$

Assuming $\widetilde{\text{po}}_{t,c}$ is Gaussian with mean c and unit variance



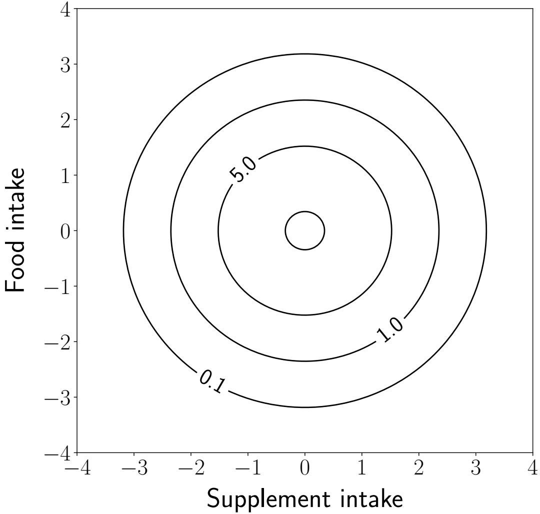
Supplement after the food: $\mathrm{Cov}[\tilde{y}, \tilde{t}\,] = -0.8$

Assuming $\widetilde{\mathrm{po}}_{t,c}$ is Gaussian with mean c and unit variance



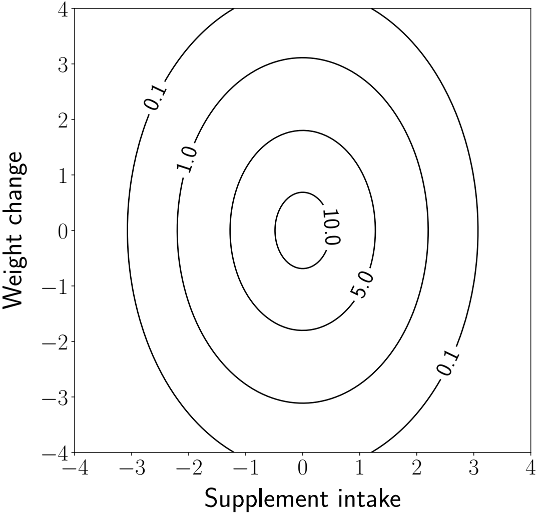
Randomized supplement: $\rho_{\tilde{t},\tilde{c}} := 0$

Assuming $\tilde{t}$ and $\tilde{c}$ are jointly Gaussian



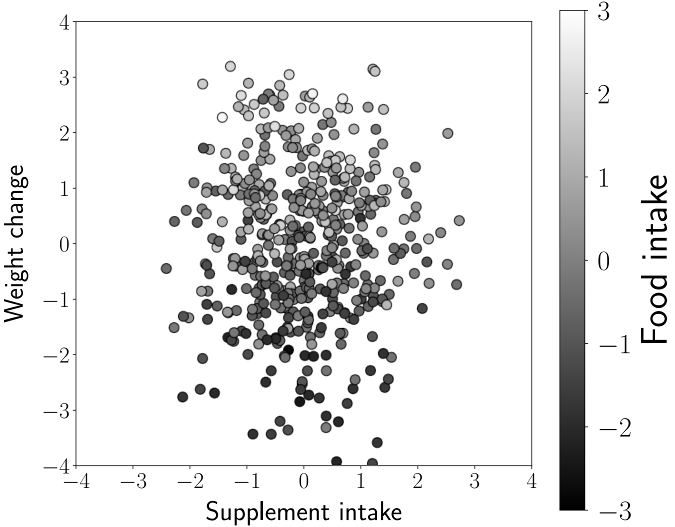
Randomized supplement: $\mathrm{Cov}\left[\tilde{y}, \tilde{t}\,\right] = 0$

Assuming $\widetilde{\text{po}}_{t,c}$ is Gaussian with mean c and unit variance



Randomized supplement: $\mathrm{Cov}\left[\tilde{y}, \tilde{t}\,\right] = 0$

Assuming $\widetilde{\mathrm{po}}_{t,c}$ is Gaussian with mean c and unit variance



What have we learned

Correlation does not imply causation

However, it does if the treatment is randomized

Otherwise, unobserved confounders produce spurious correlation