# Assignment 04

1 (a) $f(x) = x^3 + x^2 - 4x - 4$

Solving $f(x)$ we get,

$$x_1 = -2$$

$$x_2 = 2$$

$$x_3 = -1$$

$f(x) = 0$

$\Rightarrow x^3 + x^2 - 4x - 4 = 0$

$\Rightarrow x^2 = -x^3 + 4x + 4$

$\Rightarrow x = \sqrt{-x^3 + 4x + 4}$

$\Rightarrow g_1(x) = \sqrt{-x^3 + 4x + 4}$

$f(x) = 0$

$\Rightarrow x^3 + x^2 - 4x - 4 = 0$

$\Rightarrow 4x = x^3 + x^2 - 4$

$\Rightarrow x = \dfrac{x^3 + x^2 - 4}{4}$

$\therefore g_2(x) = \dfrac{x^3 + x^2 - 4}{4}$

(b) $g_1(x) = \sqrt{-x^3+4x+4}$

$$g_1'(x) = \frac{1}{2}\left(-x^3+4x+4\right)^{-1/2}\left(-3x^2+4\right)$$

$$= \frac{4-3x^2}{2\sqrt{-x^3+4x+4}}$$

$$g_2(x) = \frac{x^3+x^2-4}{4}$$

$$g_2'(x) = \frac{3x^2+2x}{4}$$

for the roots $-2$, $2$ and $-1$,

$\lambda_1 = |g_1'(-1)| = 0.5$ [$\because$ linear convergence]

$\lambda_2 = |g_1'(-2)| = 2$ [$\because$ Divergence]

$\lambda_3 = |g_1'(2)| = 2$ [$\because$ Divergence]

$\lambda_4 = |g_2'(-1)| = 0.25$ [$\because$ linear convergence]

$\lambda_5 = |g_2'(-2)| = 2$ [$\because$ Divergence]

$\lambda_6 = |g_2'(2)| = 4$ [$\because$ Divergence]

2 a) $f(x) = xe^{x} - 1$

$f'(x) = e^{x} + xe^{x}$

$x_0 = 1.5$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1.5 - \frac{f(1.5)}{f'(1.5)} = 0.9891$$

$$x_2 = 0.9891 - \frac{f(0.9891)}{f'(0.9891)} = 0.6787$$

$$x_3 = 0.6787 - \frac{f(0.6787)}{f'(0.6787)} = 0.5766$$

$\downarrow$ continue iterations like this

b) $g(x) = \dfrac{2x+1}{\sqrt{x+1}}$

$g'(x) = \dfrac{2x+3}{2(x+1)^{3/2}}$

to be superlinearly convergent, $\lambda$ has to be 0.

$\therefore |g'(x)| = 0$

$\Rightarrow \dfrac{2x+3}{2(x+1)^{3/2}} = 0$

$\Rightarrow 2x+3 = 0$

$\Rightarrow x = -3/2$ [showed]