Assignment 03

1a] hermite interpolation.

b] $f(x) = x \ln(x)$

x	$f(x)$	$f'(x)$
1	0	1
3	3.296	2.099

$$\therefore P_{2n+1} = h_0(x) f(x_0) + \hat{h}_0(x) f'(x_0) + h_1(x) f(x_1) + \hat{h}_1(x) f'(x_1)$$

$$= 0 + \hat{h}_0(x) + 3.296\, h_1(x) + 2.099 \hat{h}_1(x)$$

Now, $\hat{h}_0(x) = (x - x_0)\{l_0(x)\}^2$

$$= x - 1 \left(\frac{x-3}{-2}\right)^2$$

$$= \frac{(x-1)(x-3)^2}{4}$$

$l_0(x) = \frac{x - x_1}{x_0 - x_1}$

$= \frac{x-3}{1-3}$

$$h_1(x) = \{1 - 2(x - x_1) l_1'(x_1)\} \{l_1(x)\}^2$$

$$= \left\{1 - 2(x-3)\frac{1}{2}\right\}\left\{\frac{(x-1)}{2}\right\}^2$$

$$= \frac{(x-1)^2 (4-x)}{4}$$

$$\hat{h}_1(x) = (x - x_1)\{l_1(x)\}^2$$

$$= (x-3)\left(\frac{x-1}{2}\right)^2$$

$$= \frac{(x-3)(x-1)^2}{4}$$

$$l_1(x) = \frac{x - x_0}{x_1 - x_0}$$

$$= \frac{x-1}{3-1}$$

$$= \frac{x-1}{2}$$

$$l_1'(x) = 1/2$$

$$l_1'(x_1) = 1/2$$

c) $$P_3(x) = \frac{(x-1)(x-3)^2}{4} + 3.296 \times \frac{(x-1)^2(4-x)}{4} + 2.099 \frac{(x-3)(x-1)^2}{4}$$

2]

$$\theta_j = \frac{(2j+1)\pi}{2(n+1)}$$

$\theta_0 = \pi/10$, $\quad x_0 = 5\cos \pi/10$

$\theta_1 = 3\pi/10$, $\quad x_1 = 5\cos 3\pi/10$

$\theta_2 = 5\pi/10$, $\quad x_2 = 5\cos 5\pi/10$

$\theta_3 = 7\pi/10$, $\quad x_3 = 5\cos 7\pi/10$

$\theta_4 = 9\pi/10$, $\quad x_4 = 5\cos 9\pi/10$