BRAC University (Department of Computer Science and Engineering)

CSE 330 (Numerical Methods) for Summer 2024 Semester

Name: Time: 25 minutes

Student ID:

Section:

Quiz 2 [co1]

1. Let f(x) = x Cos(x). Where x ϵ {0, π/6, π/3}
   a) [1 Mark] What will be the degree of the interpolating polynomial for the above scenario?
   b) [2 Marks] Construct the Vandermonde Matrix, V.
   c) [3.5 Marks] Find out the interpolating polynomial using Lagrange Interpolation.
   d) [2 Marks] For x = π/2, what will be the corresponding value of P(x)?
   e) [1.5 Marks] State the scenario for which we will face problems with Lagrange Interpolation.

(a) 2

(b) $V = \begin{bmatrix} 1 & 0 & 0 \\ 1 & \pi/6 & (\pi/6)^2 \\ 1 & \pi/3 & (\pi/3)^2 \end{bmatrix}$

(c) $P_2(x) = l_0(x) f(x_0) + l_1(x) f(x_1) + l_2(x) f(x_2)$ ($f(x_0)$ multiplies 0)

$l_1(x) = \frac{x - x_0}{x_1 - x_0} \times \frac{x - x_2}{x_1 - x_0}$

$= \frac{x - 0}{\pi/6 - 0} \times \frac{x - \pi/3}{\pi/6 - \pi/3} = -\frac{x(x - \pi/3)}{\pi^2/36}$

$$l_2(x) = \frac{x - x_0}{x_2 - x_0} \times \frac{x - x_1}{x_2 - x_1} =$$

$$= \frac{x - 0}{\pi/3 - 0} \times \frac{x - \pi/6}{\pi/3 - \pi/6} = \frac{x(x - \pi/6)}{\pi^2/18}$$

$$P_2(x) = -\frac{x(x - \pi/3)}{\pi^2/36} \frac{\pi}{6} \cos\left(\frac{\pi}{6}\right) + \frac{x(x - \pi/6)}{\pi^2/18} \frac{\pi}{3} \cos\left(\frac{\pi}{3}\right)$$

d) $$P_2(\pi/2) = -\frac{\pi/2\left(\pi/2 - \pi/3\right)}{\pi^2/36} \pi/6 \cos \pi/6 + \frac{\pi/2\left(\pi/2 - \pi/6\right)}{\pi^2/18} \pi/3 \cos\left(\pi/3\right)$$

e) If we add extra nodes, we have to recalculate everything from the start.

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Quiz 2 [co1]

1. Let f(x) = x Sin(x). Where x ϵ {π/3, π/6, 0}
   a) [1 Mark] What will be the degree of the interpolating polynomial for the above scenario?
   b) [2 Marks] Construct the Vandermonde Matrix, V.
   c) [3.5 Marks] Find out the interpolating polynomial using Lagrange Interpolation.
   d) [2 Marks] For x = π, what will be the corresponding value of P(x)?
   e) [1.5 Marks] State the scenario for which we will face problems while interpolating with Vandermonde's matrix.

(a) 2

(b) $V = \begin{bmatrix} 1 & \pi/3 & (\pi/3)^2 \\ 1 & \pi/6 & (\pi/6)^2 \\ 1 & 0 & 0 \end{bmatrix}$

(c) $P_2(x) = l_0(x) f(x_0) + l_1(x) f(x_1) + l_2(x) f(x_2)$ → 0

$l_0(x) = \frac{x - x_1}{x_0 - x_1} \times \frac{x - x_2}{x_0 - x_2}$

$= \frac{x - \pi/6}{\pi/3 - \pi/6} \times \frac{x - 0}{\pi/3 - 0} = \frac{x(x - \pi/6)}{\pi^2/18}$

$$l_1(x) = \frac{x - x_0}{x_1 - x_0} \times \frac{x - x_2}{x_1 - x_2}$$

$$= \frac{x - \pi/3}{\pi/6 - \pi/3} \times \frac{x - 0}{\pi/6 - 0} = -\frac{x(x - \pi/3)}{\pi^2/36}$$

$$P_2(x) = \frac{x(x - \pi/6)}{\pi^2/18} \pi/3 \sin(\pi/3) - \frac{x(x - \pi/3)}{\pi^2/36} \pi/6 \sin(\pi/6)$$

d) $$P_2(\pi) = \frac{\pi(\pi - \pi/6)}{\pi^2/18} \times \pi/3 \sin(\pi/3) - \frac{\pi(\pi - \pi/3)}{\pi^2/36} \pi/6 \sin \pi/6$$

e) For large number of nodes the size of the V matrix becomes large. Thus computation becomes complex.