ECE-GY 6123 Image and Video Processing, Fall 2023
Computer Assignment 3: Pyramid Transforms
Due 10/27/2023 at 11:59 PM

Please submit your solutions, including Python code and outputs (when relevant), **as a single PDF file to GradeScope**.

In this assignment, you will implement 2D Gaussian, Laplacian pyramids and a Wavelet transform in Python, and look at their reconstruction qualities and representation efficiency.

You'll need to have python packages for `OpenCV` and `PyWavelets` installed. Throughout the following problems, keep your input images on the `uint8` scale of [0,255].

Problem 1 (Gaussian and Laplacian pyramids)**.**

(a) Write functions `gaussian_pyramid` and `laplacian_pyramid` that decompose an input grayscale image into a J-level Gaussian and Laplacian pyramid, respectively, where J is an input to each function.

You can use `cv2.resize` for downsampling and upsampling with `INTER_LINEAR` and `INTER_CUBIC` filters, respectively.

(b) Write a function `reconstruct_laplacian` that reconstructs the original image from a J-level Laplacian pyramid. Verify it works correctly on a test image. Display the Gaussian and Laplacian pyramid images for $J = 3$.

(c) Write a function `quantize_pyramid` that takes in a Laplacian pyramid and quantizes the coefficients c with quantization step-size q as follows,

$$Q(c, q) = q \left\lfloor \frac{c - \mu}{q} + \frac{1}{2} \right\rfloor + \mu$$

where μ is the mean of the coefficient map, assumed to be $\mu = 0$ for residual (Laplacian) images and $\mu = 128$ otherwise (Gaussian images).

(d) For pyramid levels $J = 0, 1, 2, 3$ (where $J = 0$ is simply the original image) plot the reconstruction PSNR,

$$\text{PSNR} = 10 \log_{10} \left(\frac{255^2}{\text{MSE}} \right),$$

between the original and reconstructed image vs. the number of non-zeros in the representation for pyramids quantized at steps $q = 2^n$, $n = 0, 1, \ldots, 8$. Plot PSNR on the y-axis and NNZ on the x-axis.

What relationship do you observe between pyramid depth and representation efficiency for a desired reconstruction PSNR? Is this expected?

Such a curve helps us evaluate the representation efficiency of the Laplacian pyramid. Note that we're using the number of non-zeros (NNZ) as a surrogate for the number of bits needed to describe the image.

(e) For $J = 3$, determine qualitatively at what point the quantization level is unnoticeable. How do the number of non-zeros compare to the original image?

Help!: If your pyramid functions from problem 1 aren't working, you can use the skimage packages's functions.