The Distribution of Innovations across Firms

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Abstract

This paper builds a macroeconomic framework with heterogeneous firms and heterogeneous inventors to quantitatively understand the allocation of innovations across firms and the implications of innovation tradability. The model characterizes the entire innovation process, including both firms hiring inventors to innovate in-house and trading innovations with each others. The model is calibrated to moments in the United States, for example, the probability of selling innovations. The model implies that inventors with more effort-sensitive ideas are hired by smaller firms. In a counterfactual scenario where firms cannot sell innovations, inventors move to larger firms (the share of innovations in firms with more than 100,000 employees increases by more than 10 percentage points), and growth drops by 0.166 percentage points.

JEL Classifications: D82, D86, E22, O31, O32, O40

Keywords: innovation, inventors, employee contracts, heterogeneous innovations, heterogeneous firms, innovation distribution, firms' boundaries, patent trading, endogenous growth.

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1 Introduction

Innovations are the fuel of growth and they are developed in many different types of firms. For example, AirPods were invented by Jason Giles at Apple, and both Facebook and Whatsapp were developed by inventors in start-ups. Also, firms resell innovations to other firms that complement them. For example, WhatsApp was sold to Facebook, which already owned a social network platform. Why are some innovations created in big firms and others in small firms? What are the forces pushing innovation inside or outside a firm's boundaries? Why are some innovations kept and some sold?

This paper studies how the option of selling innovations affects growth in an endogenous contracting setting. I model the entire process of producing an innovation, including both the primary and the secondary markets. In the primary market, an inventor with some innovative idea decides what firm to work for. In the secondary market, a firm decides whether to resell an innovation to another firm. I calibrate the model using US data moments, for example, the probability of selling innovations, and the distribution of innovations across firms. I run multiple counterfactual experiments on the innovations' tradability including reducing information asymmetry to improve tradability and imposing a transaction subsidy. The results indicate that the trade friction in secondary markets affects not only the innovation allocation and firm growth but also the aggregate growth rate. For example, in an extreme case, when firms were not allowed to trade innovations, inventors moved to larger firms, which contributed more to the growth than before. The share of innovations created in start-ups and firms with fewer than 500 employees would decrease from 8.0% to 1.3%. For firms with more than 100,000 employees, this share would increase by more than 10 percentage points. The overall aggregate growth rate in the economy would decrease by 0.166 percentage points, from 2.00% to 1.84%. The quantitative results heavily depends on the endogenous contracting environment, which mitigates the growth effect of shutting down secondary markets. If firms were not allowed to optimize contract terms, growth would drop 0.21 percentage points instead, and the innovation distribution would remain mostly unchanged. The paper shows that having an option to sell innovations is important for growth, and endogenous contracting matters for quantitatively evaluating the magnitude.

I model a population of firms of different and endogenously evolving sizes. They face a population of risk-averse, short-lived inventors with innovation projects that differ in effort-sensitivity, that is, the elasticity of the innovation rate with respect to the effort. The primary market is where firms offer compensations to attract inventors, and each inventor accepts the contract of the firm that gives the highest utility. Then the inventor works in the firm to develop the idea into an innovation.

In larger firms, an innovation is more likely to be more valuable since it has a higher probability of having complementarity with the firm (Figueroa and Serrano, 2019).[1] The complementarity includes that a firm can apply the technology to a larger production scale, the innovation is in line with the manager's expertise, or the firm has enough liquidity to meet any potential financial requirement in commercialization.[2] If there is complementarity, the firm is the most efficient user; otherwise, other firms are more efficient in using the technology.

A firm decides its compensation schedule to solve a principal-agent problem. The firm is risk neutral and improves its product quality using innovations produced by inventors. It offers a combination of equity and wage to provide incentives, share risk, and split surplus with the inventor; when the firm offers more equity, the inventor is better incentivized but also more exposed to the variance in equity. Since firms use equity to provide incentives, it exposes inventors both to the risks of their own innovations and all other risks the firm faces. The innovation-related risks contribute to a smaller portion of the equity variance in a larger firm. Specifically, in start-ups, the value of equity depends solely on the outcome of the inventor's innovation. Thus, start-ups can offer compensation in the form of equity and wages to maximize innovation outcomes merely restricted by the inventor's risk aversion. The inventor can be well-incentivized to put effort into innovating. In larger firms, by contrast, the inventor's innovation only makes up a small fraction of the equity value; much of the equity value depends on stochastic factors unrelated to the innovator's effort (e.g. success of other product lines). If big firms provided the same amount of incentive as start-ups, which means the big firms provided the same share of equity as start-ups, the inventors would be exposed to lots of unrelated risks and face a highly levered compensation package. Instead of receiving wages, the inventor would have to pay the big firm for the high amount of equity; this is not attractive to risk averse inventors. The equilibrium outcome is that both the equity and the incentive decrease with the firm size, and, in turn, an inventor works harder in a smaller firm.

One may think that the problem can be solved by having a bonus contingent on the delivery of an innovation. I assume that inventors can easily come up with a **fake** innovation, and whether an innovation is practical or not cannot be proved to a judge. Thus, the conflict of interest cannot be solved by assigning a bonus. Meanwhile, based on the same assumption, a firm cannot commit to paying a bonus contingent on the delivery of a useful innovation, because it is not credible. Potentially, a firm can use a detailed contract, together with a comprehensive consumer survey, to directly link the inventor's output to the firm's profit increase and use it as a bonus to incentivize inventors. However, as in the incomplete contract literature (Grossman

[1] This paper focuses on the quality-improvement innovations. In appendix, I consider when innovations are substitutes of existing technologies.

[2] In appendix, I show that under certain functional form assumptions, financial frictions shows up in the model in the same way as the complementarity.

and Hart, 1986), it is very hard to specify all innovation outcomes in contracts, and, in practice, innovations usually are not contractible (Acemoglu, 1996; Frésard et al., 2020); I assume that firms can only use equity and wages in the employee contracts, as commonly seen in the real world (Brickley and Hevert, 1991).[3]

Given contracts, an inventor chooses in which firm to work by trading off between chances and value. On the one hand, the smaller the firm is, the harder the inventor works, which leads to a higher probability of innovating. On the other hand, conditional on the delivery of an innovation, a larger firm can offer higher compensation to the inventor. The maximum compensation a firm can afford is determined by how much the innovation is worth to the firm. To sum up, an innovation, on average, is worth more in a larger firm. Therefore, the key trade-off is that working in a bigger firm means higher compensation when there is an innovation while working in a smaller firm leads to better incentives and a higher probability to get an innovation. The latter is more important for an inventor with a more effort-sensitive idea. As a result, inventors whose ideas are more effort-sensitive work for small firms or start-ups, whereas inventors with less effort-sensitive ideas work for big firms.

Once the innovation is created, the firm decides whether to resell it on a secondary market. Before making the decision, the firm observes the innovation step size,[4] which is random, and whether it has complementarity with the innovation. When the firm does not have complementarity with the innovation, it tries to resell the innovation on the secondary market. The buyers are the firms with complementarity. The secondary market serves as a reallocation to make efficient use of innovations.

A firm resells its innovation on a secondary market with asymmetric information. This is because the innovation step size is private information—a firm cannot prove it to others without telling them the technology details (Silveira and Wright, 2010; Chiu et al., 2017). This assumption of the unobservable innovation step size leads to a lemons market where only low-quality innovations are sold. This adverse selection feature in innovations is also true in Chatterjee and Rossi-Hansberg (2012), which analyzes the spin-off decisions under asymmetric information.

One special case is when the inventor works for a start-up. If the start-up keeps the innovation, either because of complementarity or failing to sell it, the start-up enters the market and begins to produce. On the other hand, if the start-up sells the innovation to another firm, it is the model counterpart to a start-up buyout. Namely, an incumbent acquires the start-up for its technology.

I calibrate the model to moments from US patenting firms between 1982 and 1997, for

[3]In the appendix, I discuss the case where firms can use stock options instead of equities.

[4]The step size is the quality of an innovation.

example, the aggregate growth rate, the probability of a firm selling its patents, and the distribution of patents by firm size. The patent distribution is calculated using data from the US Patent and Trademark Office (USPTO), the Center for Research in Security Prices (CRSP), the linked CRSP/USPTO data provided by Kogan et al. (2017), and CRSP/Compustat Merged Database. A public firm sample is first constructed using the datasets. Then, I build a statistical model to estimate the weight for each firm, as if being observed in the public firm sample were stratified based on firm sizes. The statistical model is used to estimate the distribution of innovation across all patenting firms. Only four data points in the distribution are targeted in the calibration; the rest are all untargeted. This model is able to match the majority part of the distribution of innovation across firms. Another untargeted moment is the growth rate gap between the 90th percentile and 50th percentile innovative firms, which my model can also match. Moreover, I use in-house R&D investment by firm sizes as the third set of untargeted moments. It confirms that the model can capture some of the trade-offs between in-house and outsourcing. Overall, the model is able to match untargeted moments, including the distribution of innovations across firms, the growth rate gap between the 90th-50th percentile, and the in-house R&D investment intensity.

Finally, I use the model to run counterfactuals, including adding signals to innovations for sale, providing subsidies on innovation trade, and shutting down the secondary market. Within counterfactuals, I examine the role of endogenous contracting and matching. The growth rate increases with efficiency. In the benchmark model, there is no signal about innovation quality. Due to the lemons market, only innovations with low step sizes are sold. When there is an additional signal related to the innovation step size, more innovations are sold. For example, if the R square of the signal is 0.6, which means that the signal can explain about 60% of the variance in an innovation step size, the probability of selling an innovation increases from 5.3% to 5.9%. Since firms can sell unwanted innovations more easily, smaller firms are more attractive. More innovations are created in start-ups and medium-small- to medium-large-sized firms—the share of innovations created in start-ups increases from 0.33% to 5.23%. The growth rate increases by 0.09 percentage points, from 2.00% to 2.09%.

When there is a subsidy on innovation transactions, it has a similar impact. Quantitatively, when the subsidy is 5% of the transaction value, the aggregate growth rate increases by 0.016 percentage points, and the share of innovations in start-ups increases from 0.33% to 1.2%. I also show that the endogenous contracting framework affects the results quantitatively. If, when facing a subsidy, inventors were not allowed to move to other firms, then both the aggregate effect and the firm-level changes would be smaller. For example, the share of innovations in start-ups would be about 0.34%, much smaller than 1.2%, as in the baseline counterfactual. Additionally, even if the inventors can move between firms, the endogenous contracts still matter. When firms could not change the contract terms, start-ups, small firms, and large firms would all become less attractive. Quantitatively, the share of innovations in firms with fewer than 500 employees would drop by 1.56 percentage points, higher than 1.23% in the baseline counterfactual.

In an extreme case where firms cannot trade innovations, the effects are stronger quantitatively. There is no innovation in start-ups and the share of innovations created in large firms (with more than 100,000 employees) increases by more than 10.25 percentage points—from 11.00% to 21.25%. The firm-level model implications are consistent with the empirical observations in Acikalin et al. (2022), which shows that when facing sudden patent invalidation, small firms lose disproportionately. The impact of shutting down the secondary market goes beyond firm-level distribution shift—the aggregate growth rate decreases by 0.166 percentage points, from 2.00% to 1.84%.

Related Literature My paper relates to a literature exploring the implications of trading knowledge on firm innovations, examples of which include Cassiman and Veugelers (2006), Higgins and Rodriguez (2006), Phillips and Zhdanov (2013), Bena and Li (2014), and Liu and Ma (2021), and the impact of the idea market on the economy growth (Eaton and Kortum, 1996; Silveira and Wright, 2010; Chatterjee and Rossi-Hansberg, 2012; Chiu et al., 2017; Cabral, 2018; Cunningham et al., 2021; Perla et al., 2021; Fons-Rosen et al., 2021). Recent papers studying the impact of the secondary market in a general equilibrium model include Akcigit et al. (2016) and Ma (2022). This work and my paper shares a common framework, inherited from the endogenous growth literature, for example, Romer (1986), Aghion and Howitt (1992), Aghion et al. (2001), Klette and Kortum (2004), Acemoglu et al. (2018), and Akcigit and Kerr (2018). My paper's firm production framework is closest to Acemoglu et al. (2018), which assumes that the intermediate goods producers use innovations to improve their qualities, and final goods producers assemble intermediate goods. My paper contributes to this literature by incorporating the endogenous contracting problem. I show that the endogenous firm boundaries are important, because the reallocation of inventors across firms is an important part of the counterfactual scenarios. The endogenous contracting setting mitigates the impact on the aggregate growth rate and amplifies the effect on the innovation distribution. For example, when innovations are not tradable, in my model, the growth rate drops by 0.166 percentage points; if inventors cannot relocate, the growth rate drops by 0.21 percentage points. For the distribution, the share of innovations in firms with more than 100,000 employees increases by more than 10 percentage points in my model, whereas it only increases by 0.01 percentage points without the endogenous contract and endogenous matching.

My model thinking about where inventors work relates to the firms' boundaries literature, important examples of which going back to Coase (1937), important examples include Grossman and Hart (1986), Hart and Moore (1990), and Hart and Moore (2008). Closest are Aghion and Tirole (1994) and Schmitz (2005) analyze the implications of the innovations' ownership. My paper studies the innovation ownership in a general economic setting with heterogeneous inventors and heterogeneous firms. My findings show that the endogenous firm boundaries matter quantitatively for both firm-level results and aggregate growth rate.

This paper is also connected to the discussion on IPR protection. Examples include Boldrin and Levine (2013), Galasso and Schankerman (2015), and Budish et al. (2015). I focus on the general equilibrium implications of tradability. My results are consistent with the empirical evidence derived from two natural experiments by Ma (2022) and Acikalin et al. (2022), namely, decreasing tradability means a larger portion of innovations are created in big firms and it undermines small firms disproportionately.

The rest of the paper is organized as follows: Section 2 describes the model and characterizes the equilibrium. Section 3 describes the calibration. Section 4 reports the quantitative results and counterfactuals, and Section 5 concludes.

2 A Theoretical Model

I have built a theoretical model where firms compete to attract inventors to innovate in-house. The goal is to study the allocation of innovations across firms and the implications of the innovation market on the firms' growth. I first introduce the modeling environment and then describe the equilibrium. There are two problems in the two markets for innovations. In the primary market, an inventor chooses what size of the firm she wants to work for. In the secondary market, firms buy and sell innovations with each other. I then characterize the equilibrium of this model. Within this framework, in section 4.5 I analyze how the tradability of innovations affects the innovation distribution across firms.

2.1 Environment

Here time is continuous. There are four types of agents: households, inventors, intermediate goods producers, and final goods producers. A representative household provides labor and consumes final goods. Inventors provide effort to produce innovations within intermediate firms and consume final goods. There is a continumm of intermediate firms. Each one produces one unique type of product of some quality and use innovations to improve its quality is improved using innovations. The final good producers use intermediate goods as input to produce final goods. I will describe the preference and production technologies in the following subsections.

2.1.1 Preference

The household is long-lived. Every household supplies one unit of labor inelastically. The household's utility function is

$$U_H = \int_0^{\infty} e^{-\rho t} ln(C_H(t))dt, \tag{1}$$

where $\rho > 0$ is the discount rate and $C_H(t)$ is the consumption of the household.

An inventor is short-lived and lives for dt time period. There is a continuum of inventors of measure 1 in every period. An inventor provides effort e_I to produce inventions. She is risk averse and has a mean-variance utility:

$$U_I(c_I, e_I) = \mathbb{E}(c_I) - \frac{var(c_I)}{2\bar{q}} - R(e_I)\bar{q}, \tag{2}$$

where c_I is the consumption, e_I is the effort level, and $R(e_I)\bar{q}$ is the associated cost. The cost is scaled by $\bar{q}$ to keep the problem stable over time. Denote the aggregate consumption of inventors using C_I.

2.1.2 Technology

Final goods producers are risk neutral. They produce final goods using a continuum of intermediate goods $j \in [0, N_F]$ with production technology which is similar to Akcigit and Kerr (2018)[5]:

$$Y(t) = \frac{1}{1-\beta} \int_0^{N_F} q_j^{\beta}(t) y_j^{1-\beta}(t) dj. \tag{3}$$

In this function, $q_j(t)$ is the quality of the intermediate good j, and $y_j(t)$ is its quantity. I normalize the price of the final good to one in every period. The final good producers are perfectly competitive, taking input prices as given. Henceforth, I will drop the time index t when it does not cause confusion. The final goods are consumed by the household and inventors. The resource constraint of the economy is:

$$Y = C_H + C_I. \tag{4}$$

The economy is composed of a continuum of measure N_F risk neutral firms producing intermediate goods. Each firm produces one kind of good: there is a one-to-one mapping

[5]The difference is that in my specification, the final good producers only use intermediate goods, not labor, to assemble the final good. My model yields similar results if the final good producers use labor as well.

between firms and intermediate goods. Thus, I use the same index j to denote both intermediate goods and firms. Each firm produces using a linear technology using only labor:

$$y_j = \bar{q} l_j, \tag{5}$$

where l_j is the labor input, $\bar{q} = \frac{1}{N_F}\int_0^{N_F} q_j dj$ is the average quality. The cost is linear in wage w, which firms take as given. In each period, the labor market satisfies the constraint:

$$\int_0^1 l_j dj \leq 1. \tag{6}$$

The production technologies, together with the market setting on innovation, ensure that a firm's value $V(q_j)$ is linear in quality q_j

$$V(q_j) = \nu q_j. \tag{7}$$

The proof is in Section 2.2.1.

This paper focuses on the balanced growth path. I normalize the variables using the average quality $\bar{q}$ so that

$$\tilde{q_j} \equiv \frac{q_j}{\bar{q}}, \tilde{Q} \equiv \frac{Q}{\bar{q}}, \tilde{V}\left(\tilde{q}\right) \equiv \frac{V\left(q_j\right)}{\bar{q}} = \nu \tilde{q_j}, \tilde{T} \equiv \frac{T}{\bar{q}}, \tilde{p_z} \equiv \frac{p_z}{\bar{q}}, \tilde{c_I} \equiv \frac{c_I}{\bar{q}}, \tag{8}$$

where $Q \equiv \int_0^{N_F} q_j dj$ is the total technology stock in the economy, T is the inventor's wage, and p_z is the price on the secondary market for innovations.

A firm's quality q is potentially affected by both business shocks and innovations. The business shock δ follows a Poisson arrival rate of 1. The shock is a random draw from a truncated normal distribution $F_\delta\left(\delta\right)\left(\delta \in [-1,1], \mathbb{E}\left(\delta\right) = 0, \mathrm{var}\left(\delta\right) = \sigma_\delta^2\right)$. Upon the realization of a business shock, the firm's quality becomes $q + \delta q$.

In intermediate firms, innovations are produced by inventors using effort as the input. Each inventor is born with one innovative idea type θ that measures the idea-specific effort sensitivity $\left(\theta \in (0,1), \theta \sim F_\theta(\theta)\right)$. She works in the firm she chooses, either an existing intermediate firm or a start-up. Hired by the firm, the inventor exerts unverifiable effort e_I to transform the idea into an innovation. Given the effort level e_I, an innovation arrives with the instantaneous Poisson flow rate:

$$\lambda_\theta(\theta, e_I) = \mu_\theta e_I^\theta. \tag{9}$$

The innovation production function is based on the growth theory literature (Romer, 1990; Klette and Kortum, 2004; Akcigit and Kerr, 2018). The literature usually treats θ as a parameter, whereas here, θ is heterogeneous across innovations. I distinguish different types of innovations to consider the mapping between innovations and firms. Additionally, in the literature, a unified firm produces and implements innovations; in my model, however, it is the inventor who creates innovations, and the firm only enjoys the outcome. It is costly to work, and the flow cost of choosing effort e_I is $R(e_I)\bar{q}$. Thus, without incentive, the inventor chooses $e_I = 0$. Meanwhile, she can always effortlessly make up a useless innovation that is not quality improved. The usefulness is not verifiable.

If the inventor creates a useful innovation, a patent is granted. The step size z is then drawn from a distribution $F_z(z)$. If a firm implements the innovation, then its quality increases by an increment, Δq, where

$$\Delta q = \begin{cases} \gamma_L z Q & \text{with } (1 - h(\tilde{q})) \\ \gamma_H z Q & \text{with } h(\tilde{q}), \gamma_H > \gamma_L \end{cases} . \tag{10}$$

and $h(\tilde{q})$ is the probability that a firm has complementarity with the innovation. I assume that $h'(\tilde{q}) > 0$, which means that larger firms are more likely to have complementarity with the innovation. The complementarity includes that a firm can apply the technology to a larger production scale, the innovation suits its business operation, or the firm has enough liquidity to meet any potential financial requirement in commercialization. γ_L and γ_H capture the different efficiency in applying the technology. When there is complementarity, the firm is the efficient user of the innovation. In this case, the quality increment is $\Delta q = \gamma_H z Q$. Otherwise, the firm is not an efficient user, and the quality improvement is lower. This reflects a possible technology mismatch in real life.[6]

Because the firm value is linear in its quality, given an innovation step size, its value only depends on the complementarity. If the firm is an efficient user, then there is no gain from trade. Otherwise, there is a positive gain from trade. All firms try to sell innovations without complementarity, and buyers are the firms who have complementarity with these innovations. For each innovation, assume that there are at least two firms that are efficient in implementing it.

2.1.3 Information Structure

After an inventor is born, the type θ is publicly known. Then the inventor chooses for which firm to work and the effort e_I. The effort e_I is unobservable and unverifiable. Hence, contracts cannot be contingent on the effort level.

[6] Akcigit et al. 2016 and Ma (2022) also consider the technology mismatch, but in a different setting. In their papers, if there is a mismatch, the firm cannot use the innovation while in my model, the firm can still apply it.

If the inventor created an innovation, the step size is not verifiable, so that the existence of a useful innovation is not verifiable. Therefore, contracts cannot be contingent on either. The firm that owns the innovation knows its step size, but cannot prove it to others. Hence, firms sell and buy innovations under asymmetric information. The complementarity is public information—all firms know which firms have complementarity with the innovation.

2.1.4 Employment Contracting Problem

On the primary market, a firm hires an inventor by offering an employment contract. I assume that the employment contract can only have two dimensions: a constant wage $T(q, \theta)$ and a share of the firm's equity $a(q, \theta) \in [0, 1]$. Firms can freely choose the combination of the two but cannot use other tools.

This setup yields a principal-agent framework. Firms are risk neutral, and inventors are risk averse. Firms enjoy the innovations produced by inventors, but it is costly for inventors to work and firms cannot monitor the effort. Thus, firms want to split surplus with the inventor by offering a constant wage; meanwhile, firms need to incentivize the inventor to exert effort by offering equities.

The incentive depends on the firm size. In start-ups, the value of equity depends solely on the outcome of the inventor's innovation. Thus, start-ups can offer compensation in the form of equity and wages to maximize innovation outcomes merely restricted by inventor's risk aversion. Inventor can be well-incentivized to put effort into innovating. In larger firms, by contrast, the inventor's innovation only makes up a small fraction of the equity value; much of the equity value depends on stochastic factors unrelated to the innovator's effort (e.g. success of other product lines). If larger firms were to offer the same incentive as start-ups for the same expected compensation, inventors would be exposed to many unrelated risks and face a highly levered compensation package, consisting of negative wages and a high proportion of firm equity; this is not attractive to risk averse inventors. Therefore, both the optimal equity level and the incentive decreases with firm sizes.

One may think that the problem can be solved by having a bonus contingent on the delivery of an innovation. However, inventors can earn bonuses effortlessly by providing useless innovations. Therefore, using a bonus cannot solve the conflict of interest. Potentially, a firm can use a detailed contract, together with a comprehensive consumer survey, to directly link the inventor's output to the firm's profit increase and use it as a bonus to incentivize inventors. However, as in the incomplete contract literature (Grossman and Hart, 1986), it is very hard to specify all innovation outcomes in contracts, and, in practice, it is usually not contractible (Acemoglu, 1996; Frésard et al., 2020); I assume that firms can only use equity and wages in the employee contracts, as commonly seen in the real world (Brickley and Hevert, 1991).

The timing is as follows. All firms simultaneously offer firm-inventor-specific contracts to each inventor. After viewing all contracts, the inventor chooses her favorite one and joins the firm. If the best contract for the inventor is offered by $q = 0$, we call that a start-up. Even in this case, the inventor does not bear all the risk by holding 100% equity—there is an endogenous $a\left(q = 0, \theta\right)$.

2.1.5 Secondary Market Setup

Firms can trade unwanted innovations on secondary markets. Sellers are the ones who don't have complementarity with their innovations. Each innovation is unique and not substitutable by others, which implies that there is one market for each innovation. Buyers are the firms that have complementarity with the innovation. Assume for each innovation for sale, a firm becomes a potential buyer with a probability proportional to the normalized quality $\tilde{q}$. Denote the probability with $\lambda_b\left(\tilde{q}\right) = \mu_b \tilde{q}$. For an innovation, assume that there are at least two potential buyers.

Firms sell and buy under incomplete information because the innovation step size is private information. Firms compete to buy innovations following Bertrand competition. Each buyer offers a price p_z. All buyers offer purchase contracts simultaneously, and the seller chooses whether to accept one. It is a one-shot game—neither sellers nor buyers keep track of past trades. The settings lead to a lemons market. As a result, only low-quality innovations are sold.

2.1.6 Entry and Exit

There are two types of entry. One is innovation-related, which happens when an inventor chooses to work in a firm with $q = 0$, successfully creates an innovation, and decides to keep it, denoted the amount of innovation-related entrants with λ_I. The other is the exogenous entry where firms enter the market due to reasons other than innovations. Denote the amount of exogenous entrants with λ_0. In both cases, upon entry, the firm draws a quality q from a distribution $F_{q0}(q)$ and pay for the quality at a fair price. This represents the spillover from incumbents to entrants. Assume that $f_{q0}(q) = f_q(q)/k_t$. When $k_t < 1$, it means that entrants are on average smaller than the average firm. When the new entrant owns an innovation, then its starting quality q is boosted by Δq because of the innovation.

Firms face an exogenous exit rate τ. I will focus on a lanced growth path such that entry equals exit

$$\tau N_f = \lambda_I + \lambda_0. \tag{11}$$

2.2 Equilibrium

I focus on the balanced growth path and solve the problem backward. I now characterize the equilibria of the economy in which aggregate variables $(Y, C, R, w, \bar{q})$ grow at the constant rate g.

2.2.1 Production

The final good producer chooses $\{y_j\}_j$ to maximize its profit using the technology described in Section 2.1.2:

$$\max_{\{y_j\}} \frac{1}{1-\beta} \int_0^{N_F} q_j^{\beta} y_j^{1-\beta} dj - \int_0^{N_F} y_j p_j dj. \tag{12}$$

The first-order condition (FOC) yields the demand function of intermediate firms $p_j = q_j^{\beta} y_j^{\beta}$. The intermediate goods are produced by corresponding firm $j \in [0, N_F]$ using only labor $y_j = \bar{q} l_j$, where $\bar{q} = \frac{1}{N_F} \int_0^{N_F} q_j dj$ is the average quality, and l_j is the labor input. Intermediate good producers are monopolistic competition and choose l_j, p_j, y_j to maximize its profit, given the wage level w:

$$\begin{aligned} &\max_{l_j, p_j, y_j} y_j p_j - w l_j. \\ &s.t.\ y_j = \bar{q} l_j \\ &\quad\ p_j = q_j^{\beta} y_j^{-\beta} \end{aligned} \tag{13}$$

Therefore, the FOC yields

$$y_j = q_j \left(\frac{\bar{q}(1-\beta)}{w} \right)^{\frac{1}{\beta}}, l_j = y_j/\bar{q}, p_j = \frac{w}{\bar{q}(1-\beta)}. \tag{14}$$

In each period, the labor market clearing satisfies $\int_0^{N_F} l_j dj = 1$, which gives that $\frac{\int_0^{N_F} q_j \left(\frac{\bar{q}(1-\beta)}{w}\right)^{\frac{1}{\beta}} dj}{\bar{q}} = 1$. The wage w can be solved

$$w = N_F^{\beta} (1-\beta) \bar{q}. \tag{15}$$

Plug it back into the intermediate firm's problem. Both production y_j and profit π_j are linear in quality

$$y_j = \frac{q_j}{N_F}, \pi_j = \frac{\beta q_j}{N_F^{1-\beta}}. \tag{16}$$

In this model, firm size is linear in firm quality.

I drop the subscript j from the firm-level variable when it does not cause confusion. Intermediate firms are the ones who hire inventors to innovate. Because of competition, any value from innovations is captured by inventors, and any value from acquisitions is captured by the seller. The discounted value of being a firm of quality q is the same as the net present value under the assumption that the firm never innovate. Thus, the value function of intermediate firm q at time t can be written as

$$V(q,t) = \int_t^\infty e^{-(r+\tau)(s-t)} \beta q / N_F^{1-\beta} ds. \tag{17}$$

$$V(q,t) = \nu q, \text{where } \nu = \frac{\beta}{(r+\tau) N_F^{1-\beta}}. \tag{18}$$

The value function is linear in q and does not depend on time. This result indicates that for each firm, the value of a same quality improve Δq is the same.

The aggregate production is linear in average quality $\bar{q}$. The resource constraint of the economy is $Y = C_H + C_I$, where R is the total R&D spending in each period. The Euler equation is:

$$g = \frac{\dot{Y}}{Y} = \frac{\dot{C}_H}{C_H} = \frac{\dot{\bar{q}}}{\bar{q}} = r - \rho. \tag{19}$$

2.2.2 Secondary market

The secondary innovation market is where firms buy and sell innovations. Buyers follow Betrand competition. They set the price $\tilde{p_z}$ according to the zero-profit condition

$$\int_0^{\hat{z}} \left(\gamma_H \nu z \tilde{Q} - \tilde{p_z} \right) f_z(z) dz = 0, \tag{20}$$

where $\gamma_H \nu z \tilde{Q}$ is the innovation value to the buyer and $\tilde{p_z}$ is the price of the innovation; $\hat{z}$ is the step size threshold. If $z \leq \hat{z}$, an innovation is sold. Because of the linearity of the firm value function, the price $\tilde{p_z}$ is uniform across all buyers and all innovations.

A seller accepts an offer only when it is profitable, which means

$$\gamma_L z \nu \tilde{Q} \leq \tilde{p_z}. \tag{21}$$

The left-hand side is the value of the innovation if the seller keeps it, whereas the right-hand side is the value if the seller sells it. The threshold $\hat{z}$ satisfies

$$\gamma_L \hat{z} \nu \tilde{Q} = \tilde{p_z}. \tag{22}$$

If a firm is not the efficient user of an innovation, it chooses to sell the innovation if its step size is lower than the threshold $\hat{z}$. Otherwise, the firm uses the innovation to improve its own technology.

For each innovation, the probability of being sold is

$$(1 - h(\tilde{q})) F_z(\hat{z}),$$

where $(1 - h(\tilde{q}))$ is the probability that the innovation does not have complementarity with the firm $\tilde{q}$ where the inventor works; $F_z(\hat{z})$ is the cumulative probability density of z at $\hat{z}$. It represents , conditional on no complementarity, the probability of an innovation being sold on the market with asymmetric information. The total amount of innovations sold is

$$F_z(\hat{z}) \int_0^1 \lambda_\theta^*(\theta)(1 - h(\tilde{q}^*(\theta))) f_\theta(\theta) d\theta, \tag{23}$$

where λ_θ^* is the equilibrium innovation arrival rate of type θ. $\tilde{q}^*$ denotes the firm for with inventor θ works in equilibrium. Recall that the probability of buying an innovation is $\lambda_b(\tilde{q}) = \mu_b \tilde{q}$. The secondary market clearing gives

$$N_F \int \lambda_b(\tilde{q}) f_q(\tilde{q}(\theta)) d\tilde{q} = F_z(\hat{z}) \int_0^1 \lambda_\theta^*(\theta)(1 - h(\tilde{q}^*(\theta))) f_\theta(\theta) d\theta, \tag{24}$$

which implies that

$$N_F \mu_b = F_z(\hat{z}) \int_0^1 \lambda_\theta^*(\theta)(1 - h(\tilde{q}^*(\theta))) f_\theta(\theta) d\theta. \tag{25}$$

It pins down the probability of buying an innovation.

The innovation value if the innovation is created in firm $\tilde{q}$ satisfies

$$x(z, \tilde{q}) = \begin{cases} \gamma_L \nu \hat{z} \tilde{Q} & \text{no complementarity and } z \leq \hat{z} \\ \gamma_L \nu z \tilde{Q} & \text{no complementarity and } z > \hat{z} \\ \gamma_H \nu z \tilde{Q} & \text{with complementarity.} \end{cases} \tag{26}$$

where $x(z, \tilde{q})$ depends on two things. One is the innovation step size z. The other is realization of the complementarity. I show in the next section that there is a threshold $\hat{z}$ in the secondary market. Only innovations with step size lower than $\hat{z}$ are sold.

When there is no complementarity, and the step size z is lower than the threshold $\hat{z}$, the firm sells the innovation on the innovation market at the price $\gamma_L \nu \hat{z} Q$. When there is no complementarity, and the step size z is higher than the threshold, the firm keeps it, and it is worth $\gamma_L \nu z Q$. When there is complementarity, the firm is the efficient user of the innovation.

Then the firm also implements it in-house, and its value is $\gamma_H \nu z Q$. Ideally, all innovations are sold to the efficient user. The asymmetric information on the secondary market, however, prevents this from happening.

The expected innovation value $\mathbb{E}\left(x\left(\tilde{q}\right)\right)$ can be written as

$$\begin{aligned} \mathbb{E}\left(x\left(\tilde{q}\right)\right) = & \gamma_L\left[F_z\left(\hat{z}\right)\hat{z} + \left(1-F_z\left(\hat{z}\right)\right)\mathbb{E}\left(z|z>\hat{z}\right)\right]\left(1-h(\tilde{q})\right)\nu Q \\ & + \gamma_H \mathbb{E}\left(z\right)h(\tilde{q})\nu Q. \end{aligned} \tag{27}$$

It increases with the firm size $\tilde{q}$. Therefore, conditional on an innovation being created, its value is higher if it is in a big firm.

The secondary market setting can be easily extended to consider changes in market features. For example, in Section 4.5, I run the counterfactual where there are signals about the invention step size and study how the innovation distribution is affected.

2.2.3 Primary market

The primary market is where firms compete to hire inventors using a combination of equity a and wage $\tilde{T}$. Because of the competition, the firm's problem is the same as to maximize the utility it offers to an inventor, subject to the zero profit condition. The problem can be written as follows

$$\begin{aligned} & max_{a,T,e_I} U_I\left(a,\tilde{T};\tilde{q},\theta,e_I\right) = \mathbb{E}\left(\tilde{c_I}\right) - \frac{1}{2}\text{var}\left(\tilde{c_I}\right) - R\left(e_I\right)dt. \\ & s.t. \lambda_\theta\left(\theta,e_I\right)\mathbb{E}\left(x\left(\tilde{q}\right)\right)dt - \mathbb{E}\left(\tilde{c_I}\right) \geq 0 \\ & e_I = \arg\max\left\{U_I\left(a,\tilde{T};\tilde{q},\theta,e_I\right)\right\} \\ & \mathbb{E}\left(\tilde{c_I}\right) = a\left(\mathbb{E}\left(\tilde{V_0}\left(\tilde{q}\right)\right) + \lambda_\theta\left(\theta,e_I\right)\mathbb{E}\left(x\left(\tilde{q}\right)\right)dt\right) + \tilde{T} \\ & \text{var}\left(\tilde{c_I}\right) = a^2\left(\lambda_\theta\left(\theta,e_I\right)\sigma_x^2(\tilde{q}) + \sigma_0^2(\tilde{q})\right)dt \end{aligned} \tag{28}$$

The optimal choice is $\{a^*\left(\theta,\tilde{q}\right), T^*\left(\theta,\tilde{q}\right)\}$. The first line is the objective function $U_I\left(a,\tilde{T};\tilde{q},\theta,e_I\right)$, which represents the inventor θ's utility when she works for the firm $\tilde{q}$ with effort level e_I; $\left(a,\tilde{T}\right)$ characterize the contract offered by the firm, and $\tilde{c_I}$ is the inventor's consumption level, while $R\left(e_I\right)dt$ is the cost of effort during the employment.

The second line is the firm's individual rationality constraint, where $x\left(\tilde{q}\right)$ is the outcome value of an innovation to a firm $\tilde{q}$. It multiplies by the probability of creating an innovation $\lambda_\theta\left(\theta,e_I\right)dt$, which gives the expected payoff of hiring an inventor. The cost of hiring is $\left(\tilde{c_I}\right)$. The firm will only participate if the expected gain is nonnegative.

The third line is the inventor's incentive compatibility constraint. The inventor chooses an effort level e_I to maximize the expected utility $U_I\left(a, \tilde{T}; \tilde{q}, \theta, e_I\right)$ given firm quality $\tilde{q}$ and the wage scheme $\left\{a, \tilde{T}\right\}$.

The fourth line describes the inventor's expected consumption. The first part, $a\left(\tilde{V_0}\left(\tilde{q}\right) + \lambda_\theta\left(\theta, e_I\right)\mathbb{E}\left(x\left(\tilde{q}\right)\right)dt\right)$, is expected value of the firm equity a, which is the sum of the value without innovations and the expected value of an innovation. The other component, $\tilde{T}$, is the constant transfer from the firm to the inventor.

The fifth line shows the variance of the inventor's consumption. The exposure is the equity a. The uncertainty comes from two sources: innovation-related part $\lambda_\theta\left(\theta, e_I\right)\sigma_x^2(\tilde{q})$ and the innovation-independent part $\sigma_0^2(\tilde{q})$. In start-ups, the second term is zero. I explain both in detail later.

Given contracts, the inventor with idea θ chooses the firm $\tilde{q}^*\left(\theta\right) \in [0, \infty)$ and exerts the corresponding effort level e_I to maximize her utility by solving the following problem:

$$\max_{\tilde{q}} U_I\left(a\left(\tilde{q}, \theta\right), \tilde{T}\left(\tilde{q}, \theta\right), e_I\left(\tilde{q}, \theta\right), \tilde{q}, \theta\right). \tag{29}$$

Here $a\left(\tilde{q}, \theta\right), \tilde{T}\left(\tilde{q}, \theta\right) and e_I\left(\tilde{q}, \theta\right)$ are the solutions to the firm's problem described by Equation (28). If $\tilde{q}^*\left(\theta\right) > 0$, then the inventor works in an incumbent firm; $\tilde{q}^*\left(\theta\right) = 0$ means that the inventor joins a new firm. The equilibrium choice of effort is $e_I^*\left(\theta\right)$ and the corresponding arrival rate is $\lambda_\theta^*\left(\theta\right) = \lambda_\theta\left(\theta, e_I^*\left(\theta\right)\right)$.

The firm value is affected by four factors: in-house innovation, purchased innovation, business shocks, and exogenous death. The firm value without in-house innovations $\tilde{V_0}\left(\tilde{q}\right)$ can be written as

$$\tilde{V_0}\left(\tilde{q}\right) = \nu\tilde{q} + \begin{cases} -\nu\tilde{q} & \tau dt \\ \delta\nu\tilde{q} & dt \\ \left(\gamma_H \nu z \tilde{Q} - \tilde{p}_z\right)|_{z \le \hat{z}} & \lambda_b(\tilde{q}) F_z\left(\hat{z}\right)dt \\ 0 & 1 - \left(\tau + 1 + \lambda_b\left(\tilde{q}\right)F_z\left(\hat{z}\right)\right)dt. \end{cases} \tag{30}$$

The first line is when the firm dies exogenous. The second line is when hit by a business shock δ that arrives with arrival rate 1. The shock is a random draw from a truncated normal distribution $F_\delta\left(\delta\right)\left(\delta \in [-1, 1], \mathbb{E}\left(\delta\right) = 0, \mathrm{var}\left(\delta\right) = \sigma_\delta^2\right)$. The third line is when the firm purchases an innovation. The quality improves by $\gamma_H z \tilde{Q}$, where z is a random draw from the $F_z\left(z\right)$ distribution truncated at $\hat{z}$. The cost of buying an innovation is $\nu\gamma_L \hat{z}\tilde{Q}$. The last line is the firm value otherwise.

The uncertainty in the firm equity leads to the variance in the inventor's consumption c_I.

Because the in-house innovation is independent with other activities, the firm level risk can be decomposed into two parts: in-house innovation-related risk and the rest. The former one is $\lambda_\theta(\theta, e_I)\,\sigma_x^2(\tilde{q})dt$, where $\sigma_x^2(\tilde{q}) = \mathbb{E}\left[(x(\tilde{q}))^2\right]$ is the second moment of the innovation value. The later one is $\sigma_0^2(\tilde{q})dt$ which contains all shocks unrelated to innovations, including the risk from death, business shocks, and unknown step size in innovation trade. It can be written as

$$\sigma_0^2(\tilde{q}) = \nu^2\left\{\tilde{q}^2\left(\tau + \sigma_\delta^2\right) + \lambda_b(\tilde{q})\,F_z(\hat{z})\,\mathbb{E}\left[(\gamma_H z - \gamma_L \hat{z})^2\,|z \le \hat{z}\right]\right\}. \tag{31}$$

The risk increases with the firm size $\tilde{q}$, which dilute the inventor's impact to the firm's equity. It restricts the attractiveness to offer stocks to the inventor.

The contracting problem between firms and inventors is described in above. A firm $\tilde{q}$ chooses the optimal contract $\{a^*(\theta, \tilde{q}), T^*(\theta, \tilde{q})\}$ by solving Equation (28) to offer to the inventor θ. After seeing all contracts, the inventor picks the contract offered by firm $\tilde{q}^*(\theta)$, which gives her the highest utility according to Equation (29). The arrival rate generated by this inventor in the firm $\tilde{q}$ is $\lambda_\theta^*(\theta)$. In firm $\tilde{q}$, the innovation arrival rate is affected by both the single innovation arrival rate and the inventor density within the firm. Denote the arrival rate in firm $\tilde{q}$ by $\lambda_q(\tilde{q})$ and the firm density function by $f_q(\tilde{q})$. The total amount of innovations in firms should be the same as the total innovations made by inventors. The inventor market clearing gives

$$T_F\lambda_q(\tilde{q}(\theta))\,f_q(\tilde{q}(\theta))\,d\tilde{q} = \lambda_\theta^*(\theta)\,f_\theta(\theta)d\theta. \tag{32}$$

2.2.4 Entry and Exit

Recall there are two types of entry in this model. One is innovation-related. It happens when an inventor chooses to work in a firm with $q = 0$, successfully gets an innovation, and decides to keep it. The number of firms that enter this way can be written as

$$\lambda_I = \int_{\tilde{q}^*(\theta)=0} \lambda_\theta^*(\theta)\left[(1 - F_z(\hat{z}))(1 - h(0)) + h(0)\right] f_\theta(\theta)\,d\theta. \tag{33}$$

The other type is the exogenous entry, which enters the market without an innovation. The number is denoted by a parameter λ_0.

On the exit front, the exit rate satisfies that the number of firms who exit equals the number that enters.

$$\tau N_F = \lambda_I + \lambda_0. \tag{34}$$

2.2.5 Growth rate

The growth rate of the aggregate quality $\bar{q}$ can be written as:

$$g = \frac{\mathbb{E}_t\left(\bar{q}(t+dt)\right) - \bar{q}(t)}{\bar{q}(t)dt}. \tag{35}$$

The expected average quality at $t + dt$ includes two parts: the quality of incumbents and the quality of new firms. $\mathbb{E}_t\left(\bar{q}(t+\Delta t)\right)$ can be written as

$$\begin{aligned}\mathbb{E}_t\left(\bar{q}(t+dt)\right) = & \overbrace{\int \mathbb{E}q\left(t+dt\right) f_q\left(q,t\right) dq}^{\text{Incuments}} + \overbrace{\tau k_t \bar{q} dt}^{\text{Entry}} \\ & + \overbrace{\left[\gamma_L \lambda_{I,L} \mathbb{E}\left(z|z>\hat{z}\right) + \gamma_H \lambda_{I,H} \mathbb{E}\left(z\right)\right]\bar{q}dt}^{\text{Innovative Entry}},\end{aligned} \tag{36}$$

where $\lambda_{I,L} = \frac{(1-h(0))(1-F_z(\hat{z}))}{(1-h(0))(1-F_z(\hat{z}))+h(0)}\lambda_I, \lambda_{I,H} = \frac{h(0))}{(1-h(0))(1-F_z(\hat{z}))+h(0)}\lambda_I,$.

Hence, the growth rate is

$$\begin{aligned} g = & -\tau\left(1-k_t\right) \\ & + \mu_b N_F\left[\gamma_L \frac{\mathbb{E}\left(z\right)}{F\left(\hat{z}\right)} + \gamma_H \mathbb{E}\left(z|z\leq \hat{z}\right)\right] \\ & + \gamma_H \mathbb{E}\left(z\right) \int h\left(q^*\left(\theta\right)\right) \lambda_\theta^*\left(\theta\right) f_\theta\left(\theta\right) d\theta. \end{aligned} \tag{37}$$

It includes two parts.The first row is creative destruction. The other is the second and third lines in Equation (37). It represents the growth rate due to innovation. When the innovation is worth more or the arrival rate increases, the growth rate goes up.

The in-house R&D expenditure C_I of the economy can be written as

$$C_I = \int \lambda_\theta\left(\theta\right) \mathbb{E}\left(x\left(q^*\left(\theta\right)\right)\right) f_\theta\left(\theta\right) d\theta. \tag{38}$$

It captures all transfers made to inventors. The total firm-level R&D expenditure is the sum of R and the spending on purchasing patents on the secondary market

$$X = C_I + \mu_b p_z. \tag{39}$$

Based on Equation 12, the equilibrium out put level Y is linear in $\bar{q}$

$$Y = \frac{1}{1-\beta}\frac{\bar{q}}{N_F^{1-\beta}}. \tag{40}$$

and the consumption level is

$$C_H = Y - C_I. \tag{41}$$

I end this section by summarizing the equilibrium.

Definition 1. A balanced growth path of this economy for any combination of t, q, is the mapping between q and θ, the allocation $\left(\left\{y_j^*\right\}_j, Y^*, X^*, C_I^*, C_H^*\right)$, the price levels $\left(w^*, \left\{p_j^*\right\}_j, \hat{z}\right)$, the aggregate growth rate g^*, the entry rates λ_{eI}^*, and the measure of firms T_F^*, such that (1) for any $j \in [0, 1]$, y_j^* and p_j^* satisfy Equation (14); (2) wage w^* satisfies Equation (15); (3) measure of the intermediate producers T_F^* satisfies Equation (34); (4) the mapping is the solution of Equation (29); (5) aggregate creative destruction τ_e satisfies Equation (34); (6) the price offer $\hat{z}$ for inventions satisfies Equation (22); (7) the entry rates λ_{eI}^* satisfy Equation (33); (8) in-house R&D spending C_I^* satisfies Equation (38); (9) total R&D spending X satisfies Equation (39); (10) aggregate output Y^* satisfies Equation (40); (11) aggregate consumption C_H^* satisfies Equation (41); and (12) steady-state growth rate g^* satisfies Equation (37).

3 Calibration

I calibrate the model to the United States. I construct the moments using firm-level information and use patents as a proxy for innovations. Section 3.1 describes the data. Section 3.2 explains how I calibrate the model. Appendix A outlines the algorithm I used to solve the model.

3.1 Data

This paper combines four datasets to calculate moments. The datasets are the US Patent and Trade Office (USPTO), the Center for Research in Security Prices (CRSP), the Merged CRSP-Compustat Database, and the linked CRSP-USPTO data provided by Kogan et al. (2017). In addition, I use the Survey on Business Strategies (ESEE) by Fundacion SEPI from Spain, which provides firm-level in-house R&D investment and outsourcing R&D expenditure.[7]. The sample period is from 1982 to 1997, as commonly used in the literature. To address the concern that the results are specific to the sample period, I also calibrate the model to 1998-2010 and the results are reported in the appendix.

3.1.1 US Data Development

I combine the four datasets to obtain firm-level in-house patenting features. I start with the linked CRSP/USPTO dataset. It covers all patents granted to public firms from 1926 to

[7]Spain has a similar employee invention policy as the US: firms can ask employees to give up ownership of all on-job innovation. Thus, the firm innovation decision in Spain is comparable with US.

2010. The dataset provides links between stocks and patents, and an estimated value for each patent. The value is estimated based on the stock market reaction in a three-day window after the announcement of a patent.(Kogan et al., 2017) I deflate the value of patents using the Consumer Price Index (CPI) for all urban consumers from Federal Reserve Economic Data.

Next, I match this dataset with patent information. I download the supplemental information, including application date and granted date, from USPTO.[8] I clean the data using the following steps. First, I only keep patents with nonmissing citation observations. Second, I omit any observations where the patent granted date is before the patent application date, or the patent is granted more than ten years after the initial application.[9] This gives me a dataset with all in-house patents owned by public firms.

Then, I link it with firm-level data. I download firm-level employment data from Compustat and stock data from CRSP. I deflate the market capitalization using CPI. I clean the firms using three criteria: (1) positive employment, (2) located in the United States, and (3) innovated at least one patent during the sample period. My final sample is the universe of patenting US public firms. There are 3,175 unique firms in the sample, compared with Akcigit and Kerr (2018), who use firm census and reports 23,927 firms in total. The discrepancy is because my sample includes only public firms, while their sample contains private firms as well.

3.1.2 A Statistical Model

The dataset described in the previous section only includes information for public firms. The problem with using public firm data is that the dataset includes big firms disproportionately, which means that the distribution is not representative—particularly it lacks input from small firms. Meanwhile, there is another concern that small public firms may not comparable with small private firms. I assume that being a public firm is a random sampling weighted by the firm size and have developed a statistical model to estimate the weights. Then I use the weights to estimate moments conditional on the firm size is not small, that is, the firm has at least 500 employees.

Because the firm size is linear in quality q in the model, I use q to denote the firm size. The public firm size follows distribution $g(q)$ while the population distribution of all innovative firms is $f_q(q)$. Assume that for a firm size q, the probability of being public is $p(q)$, which only depends on firm size q. I estimate $p(q)$ parametrically using generalized method of moments

[8]For patents granted after 1976, high-quality details are available at PatentsView, a patent data visualization and analysis platform supported by USPTO.

[9]On average, it takes 2.4 years (29 months) for a patent to be granted.

(GMM). The relationship between $g(q)$ and $f(q)$ is

$$f(q)\, p(q) = g(q)\, \mathbb{E}(p(q)), \tag{42}$$

where $\mathbb{E}(p(q))$ is the share of public firms among all patenting firms:

$$\mathbb{E}(p(q)) = \int f(q)\, p(q)\, dq, \tag{43}$$

which is 0.13 from the data, and I assume that $p(q)$ takes the form of the cumulative density function of a shifted gamma distribution: $p(q) = \Gamma(q + q_0, \Gamma_a, \Gamma_b)$. Then I use a generalized method of moments (GMM) to estimate the parameters $(q_0, \Gamma_a, \Gamma_b)$. There are five moments in the data. First, because $f(q)$ is probability density function, it satisfies

$$\int_0^\infty f(q)\, dq = 1, \tag{44}$$

which gives

$$\int_0^\infty f(q)\, dq = \int_0^\infty \frac{g(q)}{p(q)} \mathbb{E}(p(q))\, dq = 1. \tag{45}$$

This is one condition that $p(q)$ must meet. Second, the average employee size is 1,805 (Akcigit and Kerr, 2018). Set the unit of q to be 1,000:

$$\mathbb{E}(q) = 1.805, \tag{46}$$

It yields the moment below:

$$\int_0^\infty q \frac{g(q)}{p(q)} \mathbb{E}(p(q))\, dq = 1.805. \tag{47}$$

The rest of the moments are the three quartiles. The n^{th} quartile of the population q_n satisfies

$$\int_0^{q_n} f(q)\, dq = \frac{n}{4}, n = 1, 2, 3. \tag{48}$$

The quartiles of patenting firms are approximately 17, 70, and 370 employees (Akcigit and Kerr, 2018). Therefore, it gives three moment conditions:

$$\int_0^{q_n} \frac{g(q)}{p(q)} \mathbb{E}(p(q))\, dq = \frac{n}{100}, n = 1, 2, 3. \tag{49}$$

The five moments described in Equation (45), (49), and (47) overidentified the parameters.

Table 1: Parameter Values Used in the Statistical Model

q_0	Γ_a	Γ_b
6.8980×10^{-5}	0.7046	6.0645

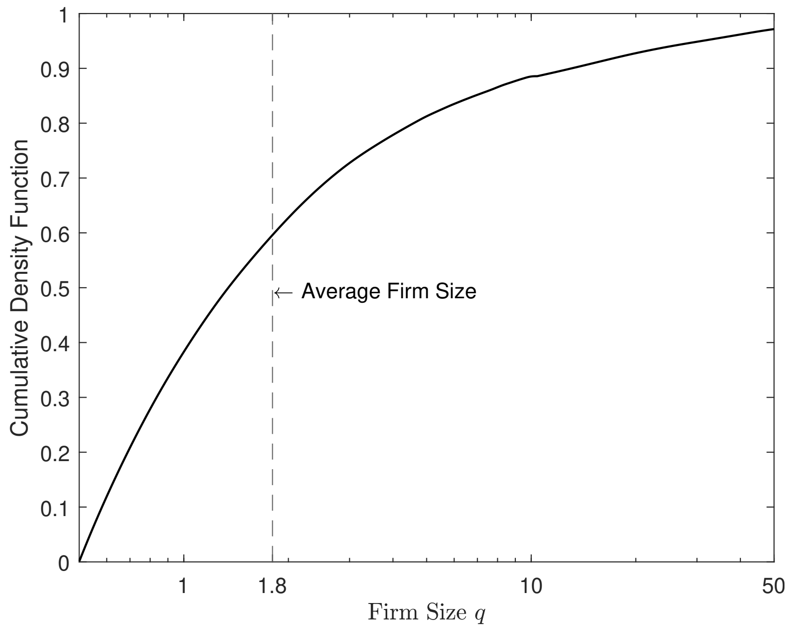

Figure 1: Estimated Firm Size Distribution for Innovative Firms with More Than 500 Employees

Notes: Unit of q: 1,000 employees. This is estimated using the public firm data and the statistical model.

I set up the estimation so that the parameters must satisfy the first moment and assign uniform weights to the other four moments. The estimated parameters are reported in Table 1.

Figure 1 reports the estimated distribution of firm sizes, conditional on more than 500 employees. The estimated moments of the firm size distribution are shown in Table 2.

I estimate the patent distribution across firms, which are used as moments in the quantitative analysis. Because the public firm data have only limited the observations of firms with a few employees, one concern is the sample of small public firms suffers from selection bias. That is, the reason why a small firm goes public is correlated with its innovations. Therefore, I only

Table 2: Model Fit for Key Moments of the Statistical Model

Moments	Data	Model
Share of firms with fewer than 17 employees	0.25	0.25
Share of firms with fewer than 70 employees	0.50	0.45
Share of firms with fewer than 370 employees	0.75	0.76
Average firm size	1,805	1,450

Table 3: Estimated Cumulative Share of Patents by Firm Sizes Using the Statistical Model

Firm Size (Thousand Employees)	Cumulative Share
0.5	0.10
5	0.30
10	0.36
20	0.44
50	0.59
100	0.76

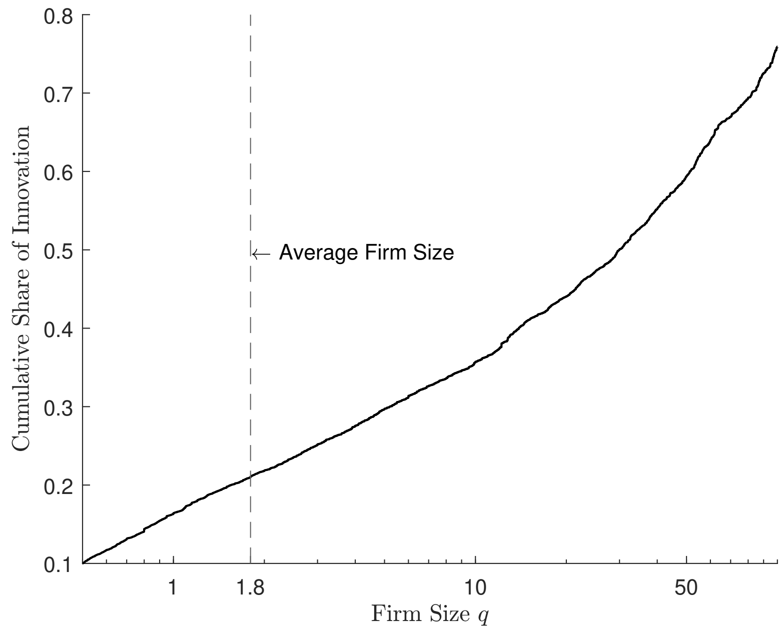

Figure 2: Estimated Innovation Distributions across Big Innovative Firms

Notes: Unit of q: 1,000 employees. The results are estimated using the public firm data and the statistical model. This figure shows the cumulative share for firms with more than 500 employees because the small public firms' observations are limited. For firms with fewer than 500 employees, the cumulative share of innovations is set to 10% according to Figueroa and Serrano (2019).

use the statistical model to estimate the share of patents held by firms with more than 500 employees and assume that the share of patents held by firms with fewer than 500 employees is 10%, as suggested by the literature (Figueroa and Serrano, 2019). The estimated cumulative share of patents is reported in Figure 2, and some moments are in Table 3. From the table, the 60th percentile is about 50,000 employees per firm.

3.1.3 ESEE

I compare the model implications on in-house versus purchased innovations with the Survey on Business Strategies (ESEE). ESEE is a panel survey of manufacturing firms in Spain conducted by Fundacion SEPI. The firms in the survey are selected through stratified, proportional, and systematic sampling with a random seed. It reports firms' R&D activities, including annual expenses on in-house innovations and external innovation-related activities. Although Spain is admittedly different from the United States, the employee invention law in the two countries are similar regarding who owns the rights of employee inventions. Therefore, the innovation activities in the two countries are comparable. I use this dataset as a qualitative reference for the in-house versus external expenses comparison.

The first wave of ESEE is from 1990. I use the surveys from 1990 and 1994 to estimate moments. I run four specifications for 1990 and 1994 according to:

$$y_{ft} = \text{constant}_t + \text{coeff}_t \times \ln\left(\text{Sales}_{ft}\right) + \epsilon_{ft}, t = 1990, 1994, \tag{50}$$

where y_{ft} is the dependent variable of firm f and time t. Four dependent variables are (1) total R&D expenditure to sales ratio, (2) internal R&D expenditure to sales ratio, (3) total R&D expenditure normalized by average sales, and (4) internal R&D expenditure normalized by average sales. The first two measure the innovation intensity, and the rest measure the absolute level of innovation expenditure. The results are reported in Table 4. It shows that both R&D and internal R&D expenditure intensity decreases with firm size and the absolute levels increase with firm size.

3.2 Calibration

In this section I calibrate the model. To solve the model, I use the following functional form assumptions. The cost function is quadratic:

$$R\left(e_I\right) = \frac{e_I^2}{2}. \tag{51}$$

The probability of having complementarity with an innovation is

$$h\left(\tilde{q}\right) = k_1 - k_2 \text{exp}\left(-\eta\tilde{q}\right), k_1 \geq k_2 > 0, \eta > 0. \tag{52}$$

The step size distribution of an innovation follows Pareto distribution:

$$F_z\left(z\right) = 1 - \frac{m^\alpha}{z^{\alpha+1}}. \tag{53}$$

The type distribution of inventors follows beta distribution:

$$F_\theta = \mathcal{B}\left(\theta; \beta_a, \beta_b\right). \tag{54}$$

Table 4: Firm R&D Investment Regressions[1]

	(1) $\frac{\text{R\&D}}{\text{Sales}}$	(2) $\frac{\text{Internal}}{\text{Sales}}$	(3) $\frac{\text{R\&D}}{\text{Avg Sales}}$	(4) $\frac{\text{Internal}}{\text{Avg Sales}}$
Panel A: Year = 1990				
ln(Sales)	-0.009^{***}	-0.006^{***}	0.016^{***}	0.012^{***}
	(-3.93)	(-4.75)	(8.24)	(8.00)
Constant	0.174^{***}	0.118^{***}	-0.246^{***}	-0.182^{***}
	(4.61)	(5.62)	(-7.63)	(-7.40)
Observations	869	869	869	869
R-Squared	0.017	0.025	0.073	0.069
Panel B: Year = 1994				
ln(Sales)	-0.001^{**}	-0.001^{*}	0.023^{***}	0.013^{***}
	(-2.02)	(-1.79)	(7.40)	(6.71)
Constant	0.042^{***}	0.030^{***}	-0.361^{***}	-0.203^{***}
	(3.53)	(3.14)	(-6.96)	(-6.28)
Observations	801	801	801	801
R-Squared	0.005	0.004	0.064	0.053

[1] The dependent variable used for each column is listed at the top of the column. The data are from Survey on Business Strategies (ESEE) of the SEPI Foundation. The t statistics are in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 5: List of Parameters Used in the Model

Parameter	Description	Identification
β	Quality share in final goods	External calibration
ρ	Discount rate	External calibration
τ	Exit rate	External calibration
σ_δ^2	Business shock arrival rate	External calibration
λ_0	Exogenous entry	Indirect+direct[1]
k_t	Avg quality of exogenous entrants	Indirect inference
μ_θ	R&D arrival rate multiplier	Indirect inference
η	Curvature of Pr(complementarity)	Indirect inference
k_1	Pr(complementarity|big firm[2])	Indirect inference
k_2	Pr(complementarity|big firm) -Pr(complementarity|start-up)	Indirect+direct
m	Scale para of step size dist	Indirect+direct
α	Shape para of step size dist	Indirect inference
β_a, β_b	Shape para of inventor dist	Indirect inference
γ_L	Scale of R&D without complementarity	Normalized to 1
γ_H	Scale of R&D value with complementarity	Indirect inference

[1] "Indirect+direct" means that parameters are estimated directly given the value of indirect estimated parameters.
[2] "Big firm" is defined as a firm with more than 500 employees, according to USPTO.

The model has 16 parameters, which are listed in Table 5. The parameters (γ_L, γ_H, m) cannot be identified separately. I normalize γ_L to 1.

I identify the parameters in three steps. First, I set parameters $(\rho, \beta, \tau, \sigma_\delta^2)$ to match the moments in the literature. The discount rate ρ is set to 2%, which roughly corresponds to an annual discount factor of 97%. I use β to target the profitability in the data, which is 10.9% for the sample period between 1982 to 1997 (Akcigit and Kerr, 2018). In the model, the profitability equals β. Thus, I set $\beta = 10.9\%$. τ is 6.5%, the average annual entry rate of innovative firms. The entry rate is estimated as follows. I obtain the annual entry rate for the whole economy from Decker et al. (2016). To estimate the entry rate for innovative firms, I use two other measures from the same paper: the share of employment at young firms for the whole economy and for the information industry. I calculate the ratio between the two and use this ratio as a proxy for the ratio between economy-wide entry rate, and the innovative firm entry rate is the same as the employment share because the information industry constitutes a large portion of innovation firms. Then combining the ratio and the economy-wide entry rate, I estimate the entry rate for innovative firms as 6.5%. Meanwhile, σ_ϵ^2 is related to the firm growth rate volatility. The median standard deviation of firm growth rates within 10 years is about 15% to 17% for all firms. Using public patenting firms, the results are similar. I set σ_ϵ^2 to 14%. The parameter value are listed in Table 6

Table 6: External Calibrated Parameters

ρ	β	τ	σ_δ^2
0.02	0.109	0.065	0.14

There are 11 remaining parameters to be estimated. I then jointly estimate $(k_t, \mu_\theta, \eta, k_1, \alpha, \beta_a, \beta_b, \gamma_H)$ using indirect inference. I identify these parameters using a simulated method of moments approach in the spirit of Lentz and Mortensen (2008), which is widely used in the growth literature to match firm-level evidence (Lentz and Mortensen, 2016; Acemoglu et al., 2018; Akcigit and Kerr, 2018). I locate eight moments that correspond to eight parameters, respectively. For each combination of $(k_t, \mu_\theta, \eta, k_1, \alpha, \beta_a, \beta_b, \gamma_H)$, I calculate the model implied moments based on the model and compare them to the data-generated moment to minimize

$$min \sum_{i=1}^{8} \left(\frac{model\,(i) - data\,(i)}{data\,(i)} \right)^2, \tag{55}$$

where each moment is indexed by i. Meanwhile, for each combination of the eight parameters, I estimate (m, λ_0, k_2) directly using external calibration:

m—This parameter is the size factor of the invention step size distribution. To simplify the calculation, I calibrate $m \times N_F$. I match this to the average value of an innovation in firms with more than 500 employees. In the model, it can be written as $\frac{\int_{q>500} \mathbb{E}(x|q) f_q(q) dq}{\int_{q>500} \nu q f_q(q) dq}$. In the data, I first keep all firms with more than 500 employees. Then I calculate the average patent value per firm. The average value across firms weighted by the statistical model developed before is about 3.6% of the average market capitalization.

λ_0—This parameter is the exogenous entry, and it is in terms of firm counts. Using Equation (34), it determines the measure of firms N_F and consequentially m, since $m \times N_F$ is constant. Hence, according to Equation (37), λ_0 affects aggregate growth rate. I set λ_0 so that the aggregate growth rate g is 2%.

k_2—When the firm is a start-up, $\tilde{q} = 0$, $h(\tilde{q}) = k_1 - k_2$. Then the start-up buyout rate is $(1 - k_1 + k_2) F_z(\hat{z})$. I use the start-up buyout rate to identify k_2. In the data, the probability is about 9.4% (Gao et al., 2013).

The calibrated parameters (m, λ_0, k_2) are in Table 7. I next describe how I identify $(k_t, \mu_\theta, \eta, k_1, \alpha, \beta_a, \beta_b, \gamma_H)$:

k_t—This parameter represents the average quality of entries. When k_t goes up, holding the aggregate growth rate constant, the measure of firms N_F needs to adjust. Therefore, k_t affects the innovation density per firm and consequentially firm-level growth rate. I match this to the average productivity growth rate. In the data, the average employment growth rate is 7.45%(Akcigit and Kerr, 2018). Because my model does not have population change or aggregate employment change, I first subtract the aggregate employment growth (2%, US Bureau of Labor Statistics) from the average growth rate. According to the Equation (14), the model implies that the growth rate of employment and the growth rate of productivity satisfy

$$\frac{\dot{l}_j}{l_j} = \frac{\dot{q}_j}{q_j} - \frac{\dot{\bar{q}}}{\bar{q}} = g_j - g, \tag{56}$$

if the firm still exists, and

$$\frac{\dot{l}_j}{l_j} = -1 = \frac{\dot{q}_j}{q_j} = g_j, \tag{57}$$

otherwise. Therefore, I can estimate the average productivity growth to be 7.3%.

μ_θ—This parameter affects both the value of innovating and the uncertainty of innovation linearly. When $\tilde{q}$ is small, the uncertainty of innovation dominates, and it has no impact on allocation; otherwise, it affects the value more than the uncertainty—μ_θ has a disproportional influence on the value-risk trade-off across firms. Numerically, in the relevant parameter space, μ_θ mainly affects the share of innovation held by small- to medium-sized firms. I match it to the share of innovations held by firms with fewer than 2,000 employees.

η—This parameter governs the curvature of the complementarity probability. When η increases, it acts as if all firms are larger by the same factor regarding the complementarity probability. It affects the firm size chosen by each inventor nearly uniformly. For example, if η increases by 50%, then the size each inventor chooses also increases by about 50%. So I match it to the 60th percentile of all patenting firms. Based on the distribution estimated in the previous section, the 60th percentile

k_1—When $\tilde{q} \to \infty$, $\lim h(\tilde{q}) = k_1$, and the probability of a large firm selling a patent is $(1 - k_1) F_z(\hat{z})$. I use the average probability of a big firm selling a patent to identify k_1. In the data, the probability is about 5.1% (Figueroa and Serrano, 2019).

α—This parameter is the shape factor of the invention step size distribution. It affects the volatility of the innovation value. If α is large, then the uncertainty in the inventor's income stream is dominated by operational reasons, including business shocks, exogenous death rate, and purchasing innovation from the secondary market. As α approaches 2, for more firms, the uncertainty mainly comes from the in-house innovation. In another word, it mainly affects the behavior of small firms. Thus, I adjust α to match the average probability of a firm selling a patent. In the data the probability is about 5.4% (Figueroa and Serrano, 2019).

β_a and β_b—Both are parameters of the idea type distribution $F_\theta(\theta)$. Given the mapping between firms and inventors, the share of innovation that happens in each firm depends on the density of the inventors. I use the cumulative share of innovations held by firms with fewer than 500 employees and 5,000 employees to match both β_a and β_b.

Table 7: Directly Calibrated Parameters Given Indirect Inference Results

λ_0	k_2	m
0.024	0.072	0.021

Table 8: Indirect Inference Calibrated Parameters

k_t	μ_θ	η	k_1	α	β_a	β_b	γ_H
0.013	2.28	0.13	0.97	2.0016	0.086	1.32	2.05

γ_H—This parameter measures the value of having complementarity with an invention. When γ goes up, having complementarity is more valuable. In the model, it influences the slope of the innovation allocation especially when θ is not small. Hence, it is related to the growth rate versus firm size regression (Akcigit and Kerr, 2018):

$$g_{ft} = \eta + \beta_g ln\left(size_{ft}\right) + \epsilon_{ft}.$$

The empirical coefficient from the literature is $\beta_g = -0.035$.

The results are shown in Table 8

4 Results

The calibration moments are reported in Table 9. Overall, my model matches closely the targeted moments. Next, I discuss model features in more detail.

4.1 Optimal Contracts

Figure 3 shows the share of innovation-related risk decreases across firms. As a result, it is more difficult for big firms to incentivize inventors without exposing them to unrelated risk. The is one optimal contract for each inventor in equilibrium. I plot the optimal contracts across firms in Figure 4. As shown in the first panel, in equilibrium, the stock share $a\left(\tilde{q}\right)$ decreases with the firm size. The second panel plots the resulting effort level for the inventor works for firm $\tilde{q}$. It also decreases with the firm sizes but with a less rapid speed—more effort-sensitive inventors work for smaller firms, receive more equity and choose a higher effort level.

4.2 The Distribution of Inventors across Firms

Figure 5 shows that an inventor with a more effort-sensitive idea chooses a smaller firm. Namely, inventors with low θ are less effort-sensitive and work for big firms; instead of working for an incumbent firm, inventors with θ higher than a threshold $\bar{\theta}$ join a start-up.

Table 9: Model Fit for Key Moments—Targeted Moments

Moment	Data	Model
Profitability	0.109	0.109
Discount rate	0.020	0.020
Entry rate	0.065	0.065
Firm growth volatility	0.17	0.18
Aggregate growth rate	0.02	0.020
Average patent value	0.036	0.036
start-up buyout rate	0.094	0.094
Pr(Sell|Big)[1]	0.051	0.049
Pr(Sell)	0.054	0.053
Growth-size relation β_g[2]	-0.035	-0.036
% Inno, emp$<$ 500[3]	0.10	0.080
% Inno, emp$<$ 2,000	0.22	0.24
% Inno, emp$<$ 5,000	0.30	0.34
60th pctl $\tilde{q}$ weighted by R&D	28.47	29.25
Average growth rate	0.073	0.071

[1] Pr(Sell|Big) measures the probability to sell an innovation given it is invented by a big firm. A "Big firm" is defined as a firm with more than 500 employees, according to USPTO.

[2] β_g is the coefficient of the growth-size regression.

[3] The % Inno is the cumulative density function of innovations created in firms with less than certain employment. For example, "% Inno, emp$<$ 500" means the share of innovations, among all innovations created in this period, that are invented in a firm with fewer than 500 employees.

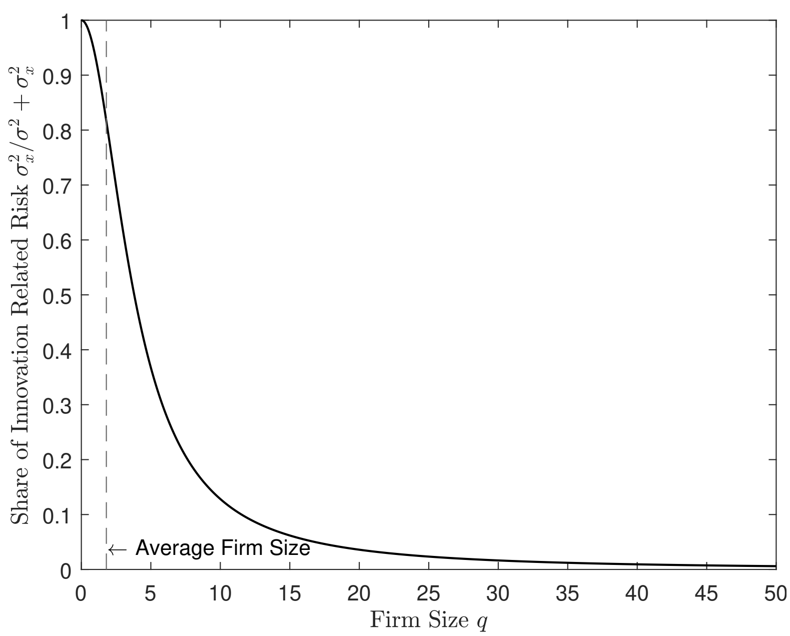

Figure 3: The Innovation-Related Risk across Firm Sizes

Notes: Unit of q: 1,000 employees.

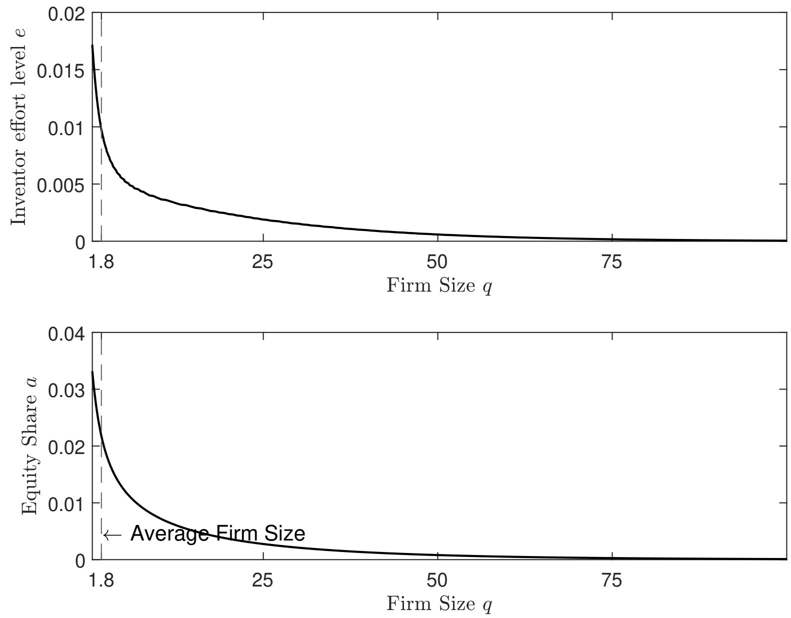

Figure 4: The Optimal Contract and the Corresponding Effort Level

Notes: Unit of q: 1,000 employees. The figure reports the equilibrium equity and effort across firms.

When choosing where to work, an inventor faces the trade-off between chances and value. The chance to successfully innovate decreases with firm sizes, because, as shown in Figure 4, smaller firms are better at incentivizing inventors to exert more effort. The value of an innovation increases with firm sizes, for larger firms are more likely to implement the innovation efficiently. When an inventor's idea is more effort sensitive, the incentive channel weighs more. Therefore, a more effort-sensitive inventor chooses to work for a smaller-size firm.

4.3 Growth Decomposition

I now use the model to document the sources of growth. The growth rate in Equation (37) can be written as

$$\begin{aligned} g = & \overbrace{-\tau\left(1-k_t\right)}^{\text{Destruction}} \\ & + \overbrace{\gamma_H F_z\left(\hat{z}\right)\mathbb{E}\left(z|z\leq\hat{z}\right)\int\left[1-h\left(q^*\left(\theta\right)\right)\right]\lambda^*_\theta\left(\theta\right)f_\theta\left(\theta\right)d\theta}^{\text{Secondary market}} \\ & + \overbrace{\mathbb{E}\left(z\right)\int\left\{\gamma_H h\left(q^*\left(\theta\right)\right)+\gamma_L\right\}\lambda^*_\theta\left(\theta\right)f_\theta\left(\theta\right)d\theta}^{\text{Primary market}}. \end{aligned} \tag{58}$$

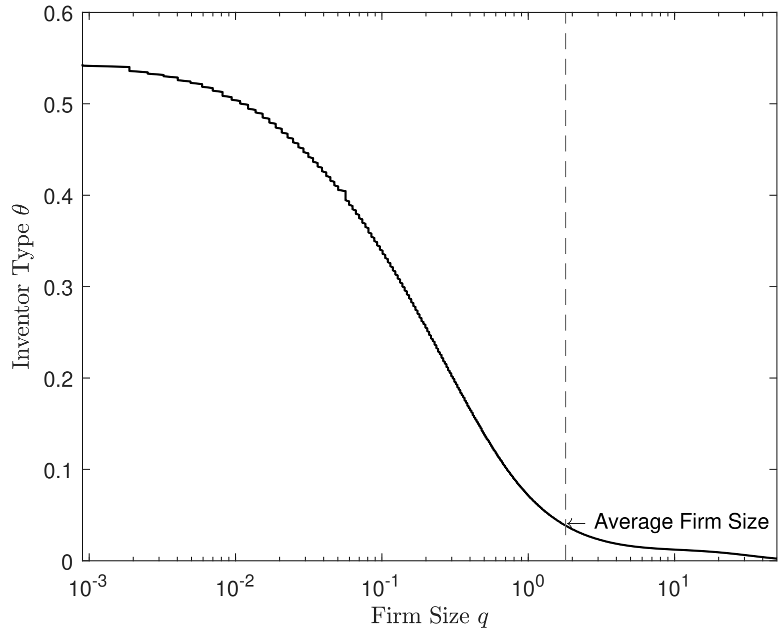

Figure 5: The Relationship between Inventors and Firms

Notes: Unit of q: 1,000 employees. Inventors with $\theta > 0.5$ work for start-ups with $q = 0$ not shown in this figure, because the x-axis is in log.

It depends on three forces: (1) destruction and replacement with an entrant subtracted the innovation effect, (2) firms purchasing innovation on the secondary market, and (3) firms hiring inventors from the primary market to innovate in-house.

My model estimates that the average quality decreases by 6.41 percentage points due to the destruction and replacement process. For exogenous entries, this is because new entrants start at a quality on average lower than the firms who exit due to exogenous destruction. For innovative start-ups, this destruction loss only captures the quality decreases because an incumbent left the market; the contribution of new innovation is estimated later together with the incumbent innovation.

Average quality increases by 8.25 percentage points because of innovation in-house. My model estimates that if we keep the innovation allocation constant, the secondary market adds 0.166 percentage points to the aggregate growth rate. The contribution of the secondary market is low, which is consistent with the fact that on average only about 5% of all innovations are sold (Figueroa and Serrano, 2019). In Section 4.5 I analyze the effects of changes on the secondary market.

4.4 Untargeted Moments

I next compare the model with untargeted features in the data.

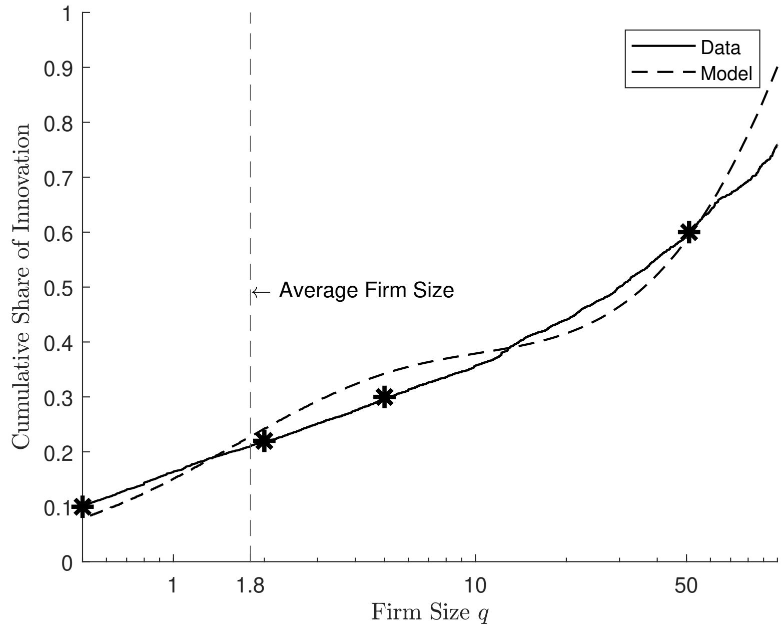

Figure 6: Cumulative Share of Innovations by Firm Sizes

Notes: Unit of q: 1,000 employees. The solid line plots the cumulative share for firms with more than 500 employees, the same as Figure 2. The stars represents the data points I used to calibrate the model.

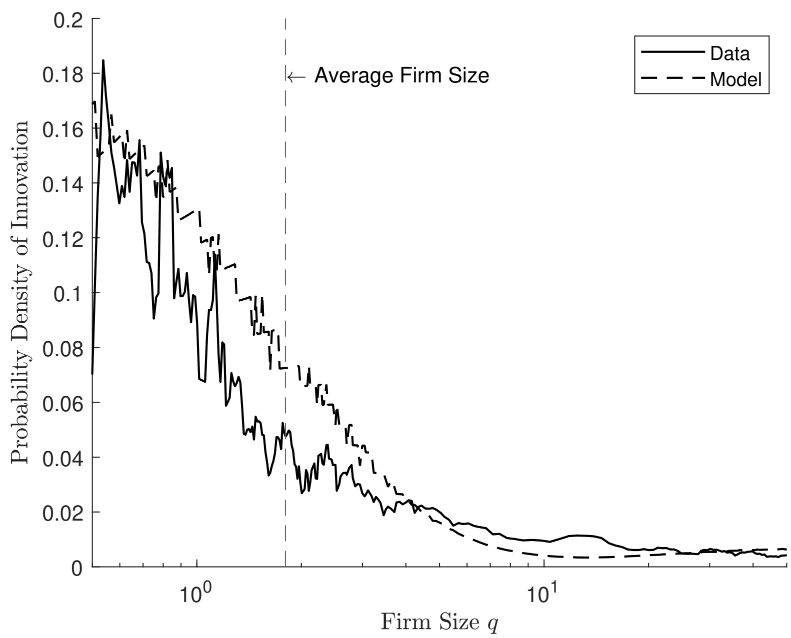

Figure 7: Probability Density of Innovation

Notes: Unit of q: 1,000 employees. This figure shows the probability density distribution of innovations for firms with more than 500 employees because the observations in small public firms are limited. The data are estimated using both public firm data and the statistical model. The model is estimated using the benchmark calibration.

Table 10: Model Fit for Untargeted Moments

Moment	Data	Model
Growth gap: 90th pctl to 50th pctl[1]	0.31	0.38
% Inno, emp$<$ 20	0.44	0.41
% Inno, emp$<$ 50	0.59	0.59
% Inno, emp$<$ 100	0.76	0.89

[1] The difference between the 90th percentile of the firm-level growth rate and the median growth rate.

Cumulative shares of inventions by firm sizes—The comparison is shown in Figure 6, and some moments are reported in Table 10. In general, the model is able to match the overall pattern of the cumulative shares. It overestimates when the firm size is medium-small to medium and large and underestimates when the size is around medium to medium-large. This is mainly because of the assumption that θ follows a beta distribution. Figure 7 shows the probability density of innovations by firm size q. The model follows a similar pattern as the data.

Growth rate gap between 90th percentile and 50th percentile—The growth rate difference is obtained from Decker et al. (2016), is about 31%. In the model, I simulate 5 million firms for one year to find the annual growth rate for each firm. Then I only keep firms who have innovations within this year. The growth rate of firms that die within this time interval is defined as -1, following the literature. I rank the firms according to their growth rate and find the growth rate gap between the 90th percentile and 50th percentile.

The comparison of untargeted moments are listed in Table 10. Overall we are able to match the data; specifically we match the 90th–50th growth rate gap and cumulative share held by medium- to medium-large-size firms closely.

In-house versus outsourcing choice—In additional, I report the comparison between in-house versus outsourcing choice both in the data and in the model in Table 12. Though the data are from Spain and the model is calibrated to the US the model can match at least the direction of the coefficients, which is reassuring because it captures some of the trade-offs between in-house and outsourcing.

4.5 Counterfactual

In this section, I consider counterfactuals about the innovation tradability on the secondary market. The analysis shows that tradability affects both firm-level innovation allocations and the aggregate growth rate. Section 4.5.1 analyzes the case if there is a signal about the invention step size. Section B.2 shows the results if there is an invention transaction tax. Section 4.5.3

Table 11: Model Fit for Untargeted Moments Using Spanish Data[1]

DV	$\frac{\text{R\&D}}{\text{Sales}}$	$\frac{\text{Internal}}{\text{Sales}}$	$\frac{\text{R\&D}}{\mathbb{E}(\text{Sales})}$	$\frac{\text{Internal}}{\mathbb{E}(\text{Sales})}$
1990	−0.009	−0.006	0.016	0.012
1994	−0.001	−0.001	0.023	0.013
Model	−0.024	−0.024	0.0026	0.0020

[1] The data are from ESEE of the SEPI Foundation, Spain. I use the Spanish dataset because it shares a similar employee invention law as the US, and provides detailed information on R&D investment.

Table 12: Model Fit for Untargeted Moments Using Spanish Data[1]

DV	$\frac{\text{R\&D}}{\text{Sales}}$	$\frac{\text{Internal}}{\text{Sales}}$	$\frac{\text{R\&D}}{\mathbb{E}(\text{Sales})}$	$\frac{\text{Internal}}{\mathbb{E}(\text{Sales})}$
1990	−0.009	−0.006	0.016	0.012
1994	−0.001	−0.001	0.023	0.013
Model	−0.024	−0.024	0.0026	0.0020

[1] The data are from ESEE of the SEPI Foundation, Spain. I use the Spanish dataset because it shares a similar employee invention law as the US, and provides detailed information on R&D investment.

considers the extreme case when the secondary market no longer exists.

4.5.1 Signal

Assume there is a publicly available signal χ associated with each innovation z. The signal satisfies

$$\chi = \varepsilon z, \tag{59}$$

where ε is a noise and it is always at least as large as 1. This is to reflect the fact that firms tend to brag about how good their innovations are. Assume that ε follows a Pareto distribution, where $f_{\varepsilon}(\varepsilon) = \frac{\alpha_{\varepsilon}}{\varepsilon^{\alpha_{\varepsilon}+1}}, \alpha_{\varepsilon} > 0$. Buyers use this information to update the innovation step size distribution, which yields:

$$f_{z|\chi}(z|\chi) = \begin{cases} \frac{-\tilde{\alpha} z^{-\tilde{\alpha}-1}}{\chi^{-\tilde{\alpha}} - m^{-\tilde{\alpha}}} & \text{if} z \leq \chi \\ 0 & \text{otherwise}, \end{cases} \tag{60}$$

where $\tilde{\alpha} = \alpha - \alpha_{\varepsilon}$.

Buyers now use this information to determine their bid price on the secondary market. As before, the buyer chooses a price $\tilde{p}_z(\chi)$ according to the zero profit condition.

$$\int_0^{\max(\hat{z},\chi)} \left[\gamma_H \nu z \tilde{Q} - \tilde{p}_z(\chi)\right] f_{z|\chi}(z|\chi)\, dz = 0. \tag{61}$$

The innovations, which are worth less than the price to the seller, are sold. Equivalently, all innovations with step size lower than the threshold $\hat{z}$ are sold, where

$$\gamma_L \hat{z} \nu \tilde{Q} = p_z(\chi). \tag{62}$$

The solution $\hat{z}$ satisfies

$$\begin{cases} \frac{\gamma_H}{\tilde{\alpha}-1}\left(m^{-\tilde{\alpha}+1} - \hat{z}^{-\tilde{\alpha}+1}\right) = \frac{\hat{z}}{\tilde{\alpha}}\left(m^{-\tilde{\alpha}} - \hat{z}^{-\tilde{\alpha}}\right) & \hat{z} \leq \chi \\ \hat{z} = \frac{\tilde{\alpha}(\gamma_H)\left[m\chi^{\tilde{\alpha}} - \chi m^{\tilde{\alpha}}\right]}{(\tilde{\alpha}-1)\left[\chi^{\tilde{\alpha}} - m^{\tilde{\alpha}}\right]} & \hat{z} > \chi. \end{cases} \tag{63}$$

The first line means that if the signal is good enough, then the bid price is independent of the signal. Using the updated expression for $\hat{z}$, the firm's problem and the inventor's problem are both the same as in the benchmark model.

To measure the informativeness of a signal, I regress $\ln z$ on the signal $\ln \chi$ and use R^2 as the information's accuracy. In the regression, the R^2 is

$$R^2 = \frac{\mathrm{Var}(\ln z)}{\mathrm{Var}(\ln \chi)} = \frac{\alpha_\varepsilon^2}{\alpha^2 + \alpha_\varepsilon^2}, \tag{64}$$

which increases with the shape parameter α_ε^2 of the signal distribution. It implies that when the signal is more accurate, R^2 is higher, and consequently, we can better infer z from the signal.

I consider two different signal levels: $\alpha_\varepsilon = 1, 2.5$, and the corresponding R^2 are 0.2 and 0.6, , which means the signal informative level is low and high, respectively. The results are reported in the second and third columns of Table 13. The benchmark results are shown in the first column for comparison.

The first row reports the percentage change in the aggregate growth rate. When $R^2 = 0.2$, the growth rate increases by 0.01 percentage points while it increases by 0.09 percentage points when $R^2 = 0.6$. Namely, when the R^2 of the signal is 0.6, the growth rate increases from 2.00% to 2.09%. This is because there are two opposite forces that determine the growth rate. On the one hand, more innovations are utilized efficiently, which increases the growth rate. On the other, there are more entries due to innovation, which drives up creative destruction as well. These two forces are of similar magnitude when the signal is weak, so the aggregate growth rate increases slightly. When the signal is more accurate, the first force is stronger, and the growth rate is higher.

The second to the sixth rows of Table 13 report the changes in the distribution. The signal has heterogeneous effects on inventors. The signal has a significant impact on the share in start-ups. Quantitatively, the share of innovations created in start-ups is more than double, from 0.33% to 0.72%, with a signal as noisy as $R^2 = 0.2$. When the signal is more informative, with $R^2 = 0.6$, the share of innovations in start-ups increases by 4.90 percentage points. Meanwhile, the share of innovations created in medium-small (with 500–20,000 employees) and medium-large firms (with 20,000–100,000 employees) also increases with the signal strength; the share of innovations in both small firms (with fewer than 500 employees) and large firms (with more than 100,000 employees) decreases with the signal strength.

The heterogeneity is because the signal effect is two-fold on the inventor's choice. Figure 8 reports the change in the mapping between inventors and firms. For inventors with the effort sensitivity level $\theta > 0.15$ and $\theta < 0.015$, more precise information leads to them working for smaller firms. On the contrary, for inventors with the effort sensitivity level $0.015 < \theta < 0.15$, the more informative the signal is, the bigger firm they choose. This is because the signal precision affects the inventor's decision in two ways. First, because firms can sell unmatched innovations more easily, the expected value of an innovation decreases more slowly with the firm size. Namely, for each innovation, there is a smaller value gap of being commercialized in different sizes. An innovation in a big firm, with a signal, is worth less more than in a small firm, compared with the benchmark case. This change makes small firms more attractive. Second, innovations contribute more to firm's equity. The innovative risk takes a larger share in a firm's total uncertainty profile, as shown in Figure 9. The increasing innovative uncertainty has two impacts: the firm equity variance is higher, and more uncertainty in the firm equity are related to the inventor's own choice. The former channel restricts the start-ups' and small firms' ability to offer equity since there is more demand for risk-sharing. The latter channel, on the contrary, enables medium- to large-sized firms to offer more equity by lowering the background noise level. Figure 10 confirms the mechanism. The share of equity offered by start-ups drops from 4% to 3.8% when there is a strong signal ($R^2 = 0.60$). And the slope is flatter in the signal's informativeness, especially at the left end. Therefore, the capacity to incentivize inventors drops more slowly with the firm size, especially for small firms.

The relative strength of the two forces determines the influence of the signal. For inventors with $\theta > 0.15$ and $\theta < 0.015$, the increasing innovation value is more important. Hence, inventors work for smaller firms. For the rest, the incentive improvement plays a more critical role, and inventors shift to bigger firms. As a result, more innovations are created in start-ups and medium-sized firms.

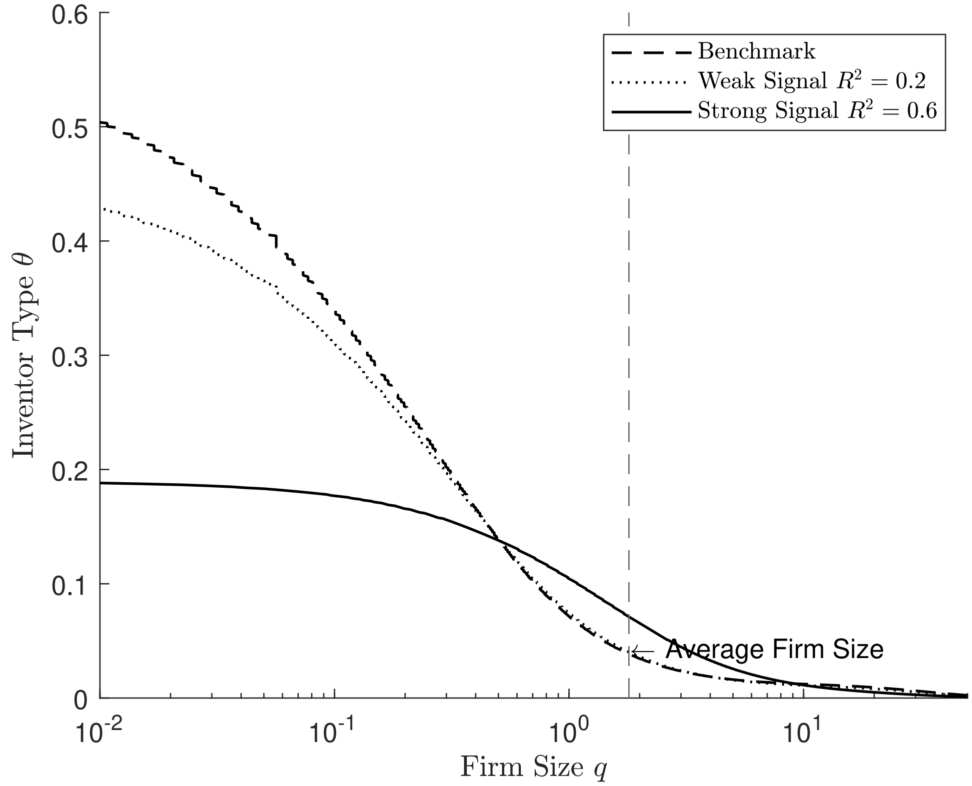

Figure 8: The Mapping between Inventors and Firms

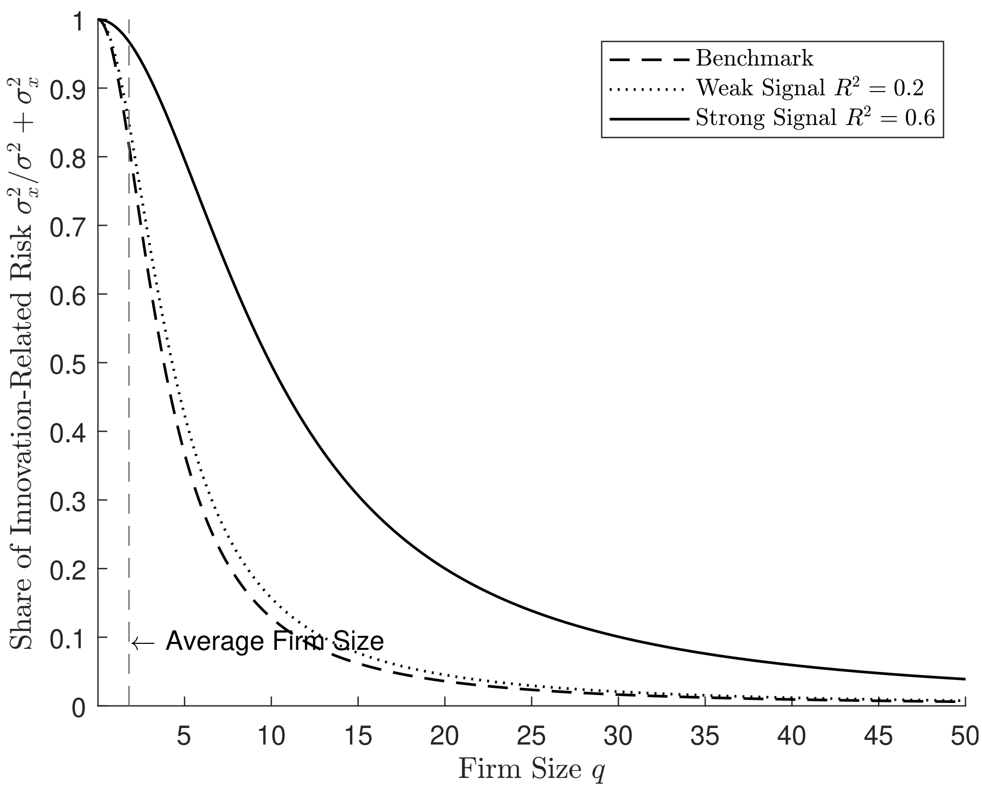

Figure 9: The Innovation-Related Risk Takes a Larger Share When the Signal Is Strong

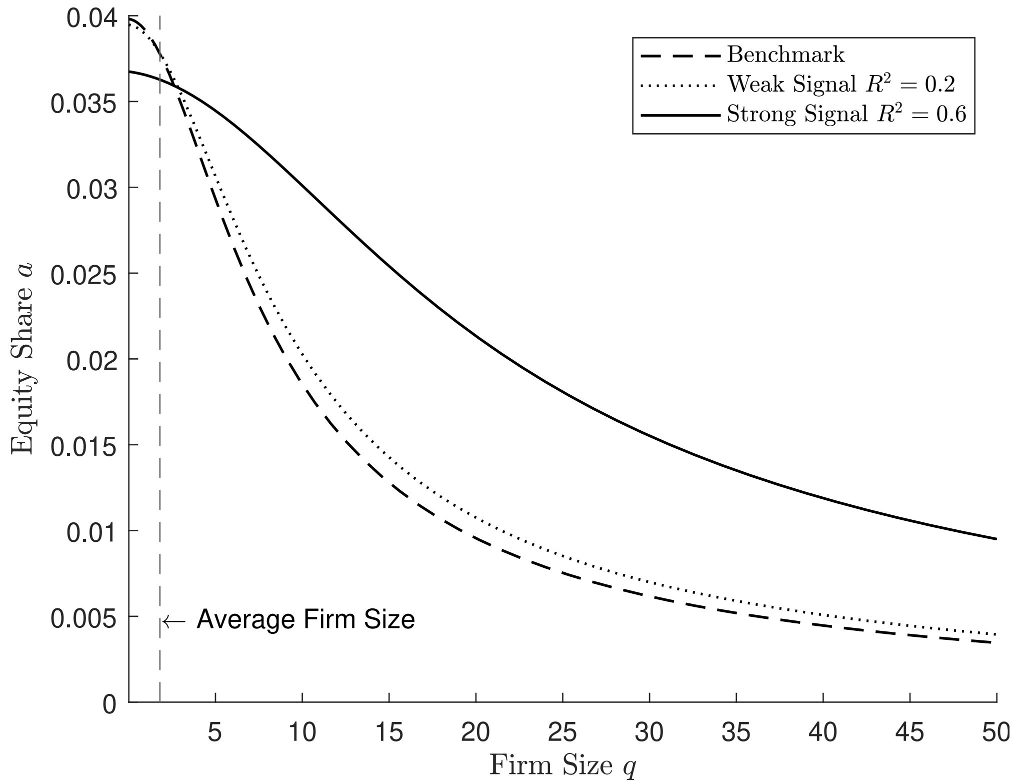

Figure 10: The Optimal Contract When the Signal Is Strong

Table 13: Counterfactuals[1]

	(1)	(2)	(3)	(4)	(5)	(6)
Moment	Benchmark	Signal	Signal	Subsidy	Subsidy	No Market
		$\alpha_\epsilon = 1$ $R^2 = 0.2$	$\alpha_\epsilon = 2.5$ $R^2 = 0.6$	1%	5%	
Growth rate	2.00	+0.013	+0.087	+0.013	+0.016	−0.166
% Inno in startups	0.33	+0.390	+4.900	+0.115	+0.897	−0.330
% Inno, emp$<$ 500	7.66	−0.430	−4.880	−0.112	−1.285	−6.350
% Inno, emp$\in [500, 20k)$	33.32	+1.974	+7.944	+0.811	+3.791	−8.585
% Inno, emp$\in [20k, 100k)$	47.70	−0.296	+3.034	+0.825	+7.594	+5.012
% Inno, emp$\geq$ 100k	11.00	−1.636	−10.996	−1.638	−10.996	+10.250
Pr(Sell)	5.28	+0.237	+0.617	+0.042	+0.168	−5.283

[1] The table reports the **percentage points difference** with respect to the benchmark. For example, the growth rate change is calculated according to $(g_{\text{counter}} - g) \times 100$. The first column reports the benchmark case in percentage points. The second and third columns analyze cases in which there is a noisy signal when firms trade innovations on the secondary market. The noise ϵ follows a Pareto distribution with scale parameter 1 and shape parameter α_ϵ. The fourth and fifth columns report the results when there is a one-time innovation transaction subsidy. The subsidy rates are 1% and 5% of the transaction price. The sixth column shows a case where firms cannot trade innovations at all.

4.5.2 Subsidy

This section considers when there is a subsidy to the secondary market. It is a fixed transfer Ψ per transaction, which means that the buyer now pays $(p_z - \Psi)$ to purchase an innovation. The rest of the settings are the same as in the benchmark model. The buyer's zero-profit condition is

$$\int_0^{\hat{z}} (\gamma_H \nu z Q - p_z + \Psi) f_z(z) dz = 0, \tag{65}$$

where $\hat{z} = \frac{p_z}{\gamma_L \nu Q}$ is the step size threshold; a firm agrees to sell an innovation when it does not have complementarity and the step size is lower than $\hat{z}$. Define $\psi \equiv \frac{p_z}{\gamma_L \nu Q}$. The zero-profit condition can be re-written as

$$\int_0^{\hat{z}} (\gamma_H z - \gamma_L \hat{z} + \gamma_L \psi) f_z(z) dz = 0. \tag{66}$$

With the subsidy, it is less costly to buy innovations. Therefore, $\hat{z}$ is higher than in the benchmark case, and a firm is more likely to sell an innovation. Adding a transaction tax has the opposite effect of a subsidy. The counterfactual results of a transaction tax are reported in Appendix B.2.

I evaluate two cases where the subsidy levels are 1% and 5% of the transaction price, respectively. The fourth and fifth column of Table 13 reports the results. The result is similar to the effect of adding a signal. It improves innovation tradability, which has two impacts on firm-level outcomes. It also has heterogeneous effect on inventors. There are more innovations in both start-ups and medium-small- to medium-large-sized firms and fewer innovations in small and big firms. The second to the sixth rows of Table 13 report the changes in the distribution. When there is a marginal subsidy, quantitatively, the share of innovations in start-ups increases by 0.115 percentage points. When the subsidy is 5%, the share in start-ups increases by 0.897 percentage points. Meanwhile, the share of innovations created in medium-small (with 500–20,000 employees) and medium-large firms (with 20,000–100,000 employees) also increases with the subsidy; the share of innovations in both small firms (with fewer than 500 employees) and large firms (with more than 100,000 employees) decreases with it. The seventh row confirms that it is more likely for a firm to sell its innovation.

Meanwhile, the quantitative results heavily depend on the endogenous contracting framework, especially the free movement of inventors. Table 14 explores the quantitative difference between settings when the subsidy is 5%. The first column is the benchmark model, and the second column reports the baseline case, where firms can adjust the contract terms and inventors can reallocate. The third column is when the firm cannot adjust contracts, while inventors can freely move to other firms. In this case, the growth rate increases by 0.007 percentage

Table 14: Counterfactuals on Subsidy—The Role of Endogenous Contracts and Matching[1]

	(1)	(2)	(3)	(4)	(5)
Moment	Benchmark	Baseline Model	Same Contract	Same Mapping	All Same
Subsidy$= 5\%$					
Growth rate	2.00	$+0.016$	$+0.007$	$+0.005$	$+0.005$
% Inno in startups	0.33	$+0.897$	$+0.909$	$+0.005$	$+0.005$
% Inno, emp< 500	7.66	-1.285	-1.579	$+0.006$	$+0.005$
% Inno, emp$\in [500, 20k)$	33.32	$+3.791$	$+4.683$	$+0.000$	$+0.000$
% Inno, emp$\in [20k, 100k)$	47.70	$+7.594$	$+6.984$	-0.009	-0.007
% Inno, emp$\geq 100k$	11.00	-10.996	-10.996	-0.002	-0.002
Pr(Sell)	5.28	$+0.168$	$+0.191$	$+0.074$	$+0.074$

[1] The table reports the **percentage points difference** with respect to the benchmark. For example, the growth rate change is calculated according to $(g_{\text{counter}} - g) \times 100$. The first column reports the benchmark case in percentage points. The second column analyzes the baseline case, where both the contracts and the firm-inventor mapping can adjust. The third and fourth columns study the cases in which the contracts and the firm-inventor mapping cannot adjust, respectively. The fifth column reports the results where neither the contracts nor the firm-inventor mapping can adjust.

points, lower than 0.016. The firm-level effect is more significant. For example, the share of innovations in small firms drops by 1.579 percentage points, compared with 1.285 percentage points in the baseline model. The firm-level changes are due to two reasons. First, since firms cannot adjust contracts, the contracts are not optimal–start-ups offer too much equity while large firms offer too little equity. Thus, inventors choose medium-sized firms, which leads to the rise in the shares of innovations in medium-small-sized and medium-large-sized firms. Second, inventors who still choose start-ups are over-incentivized. Therefore, they work harder than in the baseline counterfactual. As a result, there are fewer inventors in the start-ups, but everyone uses more effort. In equilibrium, the second channel dominates, and the share of innovations in start-ups is higher than in the baseline counterfactual. The fourth column is when the firm can adjust contracts, but inventors cannot move to other firms. The fifth column considers the extreme case where neither contracts nor the inventor-firm mapping can change. In both cases, the impacts on aggregate results and the firm-level changes are much smaller. The growth rates in the both cases increase by only 0.005 percentage points, whereas it is 0.016 percentage points in the baseline setting. Overall, both the free movement of inventors and the contract terms affect the firm-level outcomes and the aggregate growth rate.

4.5.3 Shut Down the Secondary Market

The tradability of an innovation increases with the protection of intellectual rights (Akcigit et al., 2016). An extreme case is when there is no protection for innovations, in which firms are not able to trade innovations, even though other firms are more efficient in implementing the technology. For example, before 1980, innovations in neither Genetic Engineering and Software are tradable, for they were not covered by the patent law. The results are reported in the fifth column of Table 13. The inventors shift to bigger firms. The share of innovations created in start-ups drop by 0.33 percentage points, which is 100%—there are no innovations in start-ups anymore. The share of innovations in small firms also decreases dramatically, by 6.35 percentage points. In the contrast to the well-known "small firms innovate" idea, without any intellectual rights protection, innovations are created in big firms. For example, quantitatively, the share of innovations created in large firms (with more than 100,000 employees) increases by more than 10.25 percentage points—from 11.00% to 21.25%. The firm-level model implications are consistent with the empirical observations in Acikalin et al. (2022), which shows that when facing sudden patent invalidation, small firms lose disproportionately.

The impact of shutting down the secondary market goes beyond firm-level distribution shift—it also has an aggregate implication. The aggregate growth rate decreases by about 0.166 percentage points. The growth rate drops from 2.00% to 1.84%. The magnitude of the growth rate drop depends on two groups of data moments: the probability of selling an innovation and the share of innovations in small firms. If firms are, in fact, more likely to sell an innovation than the data, the growth rate drop will be larger. For example, consider the case where the parameter $k_1 = 0.93$, which means instead of 5.3%, on average 8.5% of innovations are sold. Shutting down the secondary market will result in a drop that is 0.1 percentage points higher than before—the growth rate decreases by 0.26 percentage points, compared with the 0.166 percentage points in the baseline calibration. If in addition, more innovations are created in small firms than observed, then the results will be more salient. For example, consider a case in which on average 17% of innovations are sold, and no innovations are created in firms with more than 50,000 employees. Shutting down the secondary market leads to an additional 0.4 percentage points drop in the growth rate—now the growth rate would be 1.44% if there were no secondary market. This is because the impact of tradibility is mainly about innovation reallocation between firms. Both a high probability of selling and a small share of innovations in big firms means that the reallocation has a strong implication for efficiency improvement. Hence, shutting down the secondary market has a more salient effect.

Meanwhile, similar as before, the quantitative results heavily depends the endogenous contracting framework. Assume that firms cannot change the contract or the mapping, then the growth rate drops by 0.21 percentage points instead of 0.166 percentage points. For the firm-level outcome, the share of innovations almost stay unchanged. The share of innovations in firms with more than 100,000 employees only increases by 0.01 percentage points, whereas it increases by more than 10 percentage points in the endogenous contracting setting. Overall, the endogenous contracting setting mitigates the impact on the aggregate growth rate and amplifies the effect on the innovation distribution.

5 Conclusion

This paper explores why some inventions are invented inside a firm while others are outside a firm. The model characterizes the contractual relationship between two sets of heterogeneous agents: inventors and firms. Heterogeneous firms hire inventors to innovate in-house, and then trade non-complementary inventions on a secondary market. How much equity a firm offers depends on its size. Large firms find it difficult to give inventors high-power incentive without exposing them to unrelated risks. Therefore, both the incentive and the optimal equity level decreases with firm sizes. A key trade-off that an inventor faces when choosing firms is between value and opportunities. On average, innovation value is higher in bigger firms, since they are more likely to have synergy with the innovation; meanwhile, the chance to innovate is higher in small firms, for they are better at incentivizing inventors using equity. The model suggests that inventors work for small firms if their ideas are more sensitive to effort whereas inventors with ideas less sensitive to effort work for big firms.

This model offers a framework to think about the boundaries of firms quantitatively. The counterfactual exercise shows that, if we shut down the innovation market, the growth rate would decrease by 0.166 percentage points, and the share of innovations created in start-ups and firms with fewer than 500 employees will drop from 8.0% to 1.3%. It would be useful to extend and generalize the analysis in several directions. Adding the idea-generating process, such as inventors can have advantages in understanding the potential step size, would add richness to this model. The model would gain realism if breaking the linear relationship between the firm size and quality by introducing adjustment costs or adding financial frictions. Including the individual occupation choice between production and innovation can facilitate studies of how to motivate innovations. Quantitative models can gain greater insights into the nature of firms and policy implications on innovation by studying the inventor-firm game in a macroeconomic setting.

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A Algorithm

I use value function iteration to solve the model. First, I define a set of discrete points for normalized firm sizes $\{\tilde{q}_0, \tilde{q}_1, ..., \tilde{q}_N\}$ and for idea types $\{\theta_0, \theta_1, \theta_2, ...\theta_M\}$. The algorithm employs a computational loop with the following steps

Table 15: Counterfactuals on Signals—The Role of Endogenous Contracts and Matching[1]

	(1)	(2)	(3)	(4)	(5)
Moment	Benchmark	Baseline Model	Same Contract	Same Mapping	All Same
Signal= 20%					
Growth rate	2.00	+0.013	+0.014	+0.012	+0.012
% Inno in startups	0.33	+0.390	+0.392	+0.005	+0.005
% Inno, emp<500	7.66	−0.430	−0.485	+0.002	+0.002
% Inno, emp$\in[500, 20k)$	33.32	+1.974	+2.316	+0.000	+0.000
% Inno, emp$\in[20k, 100k)$	47.70	−0.296	−0.583	−0.005	−0.004
% Inno, emp$\geq 100k$	11.00	−1.636	−1.640	−0.001	−0.001
Pr(Sell)	5.28	+0.237	+0.244	+0.174	+0.174

[1] The table reports the **percentage points difference** with respect to the benchmark. For example, the growth rate change is calculated according to $(g_{\text{counter}} - g) \times 100$. The first column reports the benchmark case in percentage points. The second column analyzes the baseline case, where both the contracts and the firm-inventor mapping can adjust. The third and fourth columns study the cases in which the contracts and the firm-inventor mapping cannot adjust, respectively. The fifth column reports the results where neither the contracts nor the firm-inventor mapping can adjust.

1. Guess the measure of the firm, the innovation without complementarity, the death rate and the growth rate (N_F, μ_b, τ, g).

2. Based on the Equation (7), solve for the corresponding ν.

3. For each inventor firm pair $(\theta, \tilde{q})$, use Equation (28) to solve for the stock share offered a and the utility level.

4. For each inventor θ, find the firm $\tilde{q}$ that offers the highest utility level according to Equation (29).

5. Update the guess on the measure of the firm, the innovation without complementarity, the death rate, and the growth rate (N_F, μ_b, τ, g) using Equation 32, 25, 34, and 37.

6. Repeat steps 2 to 5 until convergence.

B Counterfactuals

B.1 Counterfactual: Endogenous Contracting

Table 15 and Table 16 compare the effects of adding a signal and shutting down the secondary market in four different cases: the baseline case, with exogenous contracts, exogenous matching, and both the exogenous contracts and exogenous matching. Similar to the counterfactual on subsidies in Section 4.5, the results are quantitatively different.

Table 16: Counterfactuals When Shut Down the Secondary Market—The Role of Endogenous Contracts and Matching[1]

	(1)	(2)	(3)	(4)	(5)
Moment	Benchmark	Baseline Model	Same Contract	Same Mapping	All Same
Signal= 20%					
Growth rate	2.00	−0.166	−0.162	−0.212	−0.205
% Inno in startups	0.33	−0.330	−0.330	−0.008	−0.010
% Inno, emp< 500	7.66	−6.350	−6.357	−0.072	−0.060
% Inno, emp$\in [500, 20k)$	33.32	−8.585	−8.556	+0.000	+0.004
% Inno, emp$\in [20k, 100k)$	47.70	+5.012	+5.006	+0.066	+0.054
% Inno, emp$\geq$ 100k	11.00	+10.250	+10.239	+0.015	+0.014
Pr(Sell)	5.28	−5.283	−5.283	−5.283	−5.283

[1] The table reports the **percentage points difference** with respect to the benchmark. For example, the growth rate change is calculated according to $(g_{\text{counter}} - g) \times 100$. The first column reports the benchmark case in percentage points. The second column analyzes the baseline case, where both the contracts and the firm-inventor mapping can adjust. The third and fourth columns study the cases in which the contracts and the firm-inventor mapping cannot adjust, respectively. The fifth column reports the results where neither the contracts nor the firm-inventor mapping can adjust.

B.2 Counterfactual: Tax

Now I add a sales tax to the secondary market. It is a fixed cost Ψ per transaction, which means that the buyer now pays $(p_z + \Psi)$ to purchase an innovation. The rest of the settings are the same as in the benchmark model. The buyer's zero-profit condition is

$$\int_0^{\hat{z}} \left(\gamma_H \nu z Q - p_z - \Psi\right) f_z(z) dz = 0, \tag{67}$$

where $\hat{z} = \frac{p_z}{\gamma_L \nu Q}$ is the step size threshold; a firm agrees to sell an innovation when it does not have complementarity and the step size is lower than $\hat{z}$. Define $\psi \equiv \frac{p_z}{\gamma_L \nu Q}$. The zero-profit condition can be re-written as

$$\int_0^{\hat{z}} \left(\gamma_H z - \gamma_L \hat{z} - \gamma_L \psi\right) f_z(z) dz = 0. \tag{68}$$

With the tax, it is more costly to buy innovations. Therefore, $\hat{z}$ is lower than in the benchmark case, and a firm is less likely to sell an innovation.

I evaluate a case where the tax level is 1% of the transaction price. The fourth column of Table B1 reports the results. The tax mainly affects the innovation distribution. The tax, opposite to a signal, adds friction to the secondary market. As a result, firms sell a smaller portion of innovations. There is a larger value gap between innovations implemented by a big firm and those implemented by a small firm, which makes big firms more attractive. Similar to the signal case, the tax also affects firms' risk composition—fewer risks come from innovations.

Table B1: Counterfactual on Secondary Market Tax: 1% of the Transaction Price[1]

Moment	Benchmark	Tax
Aggregate growth rate	2.00	−0.003
% Inno in startups	0.33	−0.084
% Inno, emp< 500	7.66	−0.067
% Inno, emp$\in [500, 20k)$	33.32	−0.378
% Inno, emp$\in [20k, 100k)$	47.70	+0.529
% Inno, emp$\geq 100k$	11.00	+0.001
Pr(Sell)	5.28	−0.045

[1] The table reports the **percentage points difference** with respect to the benchmark. For example, the growth rate change is calculated according to $(g_{\text{counter}} - g) \times 100$. The second column reports the results when there is a one-time innovation transaction tax. The tax rate is as low as 1% of the transaction price.

In this case, the second force is weaker than the first for all inventors. All innovations shift to larger firms. Quantitatively, the proportion of innovations in start-ups slides by 0.084 percentage points, which is a 25% drop. The share in small firms (with fewer than 500 employees) and medium-small firms (with 500–20,000 employees) drop by 0.067 percentage points and 0.38 percentage points, respectively. The medium-large firms take a share 0.53 percentage points higher than before. For firms with more than 100,000 employees, the share increases slightly, by about 0.001 percentage points. The tax has a mild impact on the aggregate growth rate. This is because the effect is mitigated by endogenous contracts. I will discuss this mechanism in the next counterfactual exercise.

C Extensions

C.1 A More Recent Sample Period

In this section, I calibrate the model to the sample period from 1998 to 2010 to cover more recent observations. The firm-level data are from the enter for Research in Security Prices (CRSP) and the Merged CRSP-Compustat Database. I apply the statistical model derived in Section 3.1 on the public firm data to estimate the economy-wide moments. I use patent data for innovations. The patent data are from Patent Examination Research Dataset and Patent Assignment Dataset (PAD), both provided by the US Patent and Trademark Office (USPTO).[10] I link firm-level data and the patent data using the linked CRSP-USPTO data

[10] I estimate the probability of selling an innovation using PAD for two sample periods. Then scale all probability related moments using the ratio. This is because, rather than using all patents observations, Figueroa and Serrano (2019) keeps only **corporate patents**, which take about 75% in the whole patent sample. But "corporate patents" are not marked in PAD. Hence, to ensure the samples are consistent, I scale all patent

Table C1: Directly Calibrated Parameters Given Indirect Inference Results (A More Recent Sample Period)

λ_0	k_2	m
0.021	0.119	0.007

Table C2: Indirect Inference Calibrated Parameters (A More Recent Sample Period)

k_t	μ_θ	η	k_1	α	β_a	β_b	γ_H
0.006	1.821	0.213	0.937	2.001	0.203	1.953	0.937

provided by Kogan et al. (2017).

I calibrate the model in the same way as explained in section 3. The external calibrated parameters are also the same as in Table 6. Other parameters are reported in Table C2 and Table C1. The targeted moments are reported in Table C3. Table C4 shows the counterfactual results. The patterns are the same as the baseline sample period.

C.2 Financial Friction

The model implicitly incorporates the idea of financial frictions in the complementarity. The financial friction is an additive part of the complementarity under certain assumptions. One example is described below.

When an inventor finishes working, the firm pays the wage and purchases the stock from the inventor. The total spending is $W(\tilde{q})$. Firms can borrow from outside at the interest rate r to finance the spending under a collateral constraint

$$W(\tilde{q}) \leq \varepsilon(\tilde{q}) V(\tilde{q}), \tag{69}$$

where $\varepsilon(\tilde{q})$ is a random variable ($\varepsilon > 0,\ f_\varepsilon(\varepsilon)$). If the constraint binds, the firm cannot fully develop the business potential of the innovation. The firm can sell the innovation on the secondary market, where the buyer can further commercialize the innovation. In this case, the effect of financial frictions shows up exactly as the complementarity in the benchmark model.

C.3 Innovations as Substitutions

The benchmark model assumes that each innovation can improve a firm's quality. Cunningham et al. (2021) suggests that some innovations may be substitutions of existing technology. The model can incorporate this by adding one assumption—firms are exposed to random random negative quality shocks, which are proportional to the aggregate new innovation arrival rate.

trade-related moments using the same ratio.

Table C3: Model Fit for Key Targeted Moments (A More Recent Sample Period)

Moment	Data	Model
Profitability	0.11	0.11
Discount rate	0.02	0.02
Entry rate	0.07	0.07
Firm growth volatility	0.17	0.17
Aggregate growth rate	0.02	0.02
Average patent value	0.03	0.03
start-up buyout rate	0.16	0.16
Pr(Sell|Big)[1]	0.09	0.09
Pr(Sell)	0.09	0.09
Growth-size relation β_g[2]	-0.04	-0.04
% Inno, emp<500[3]	0.07	0.07
% Inno, emp$<2,000$	0.24	0.24
% Inno, emp$<5,000$	0.34	0.35
60th pctl $\tilde{q}$ weighted by R&D	19.91	16.16
Average growth rate	0.07	0.05

[1] Pr(Sell|Big) measures the probability to sell an innovation given it is invented by a big firm. A "Big firm" is defined as a firm with more than 500 employees, according to USPTO.
[2] β_g is the coefficient of the growth-size regression.
[3] The % Inno is the cumulative density function of innovations created in firms with less than certain employment. For example, "% Inno, emp$<$ 500" means the share of innovations, among all innovations created in this period, that are invented in a firm with fewer than 500 employees.

Table C4: Counterfactuals (A More Recent Sample Period)

	(1)	(2)	(3)	(4)	(5)	(6)
Moment	Benchmark	Signal	Signal	Subsidy	Tax	No Market
		$\alpha_\epsilon = 1$ $R^2 = 0.2$	$\alpha_\epsilon = 2.5$ $R^2 = 0.6$	1%	1%	
Growth rate	2.003	$+0.031$	$+0.122$	$+0.003$	-0.004	-0.266
% Inno in startups	0.240	$+0.320$	$+4.270$	$+0.098$	-0.075	-0.240
% Inno, $q<0.5$	6.430	-0.410	-4.490	-0.10	-0.076	-5.680
% Inno, $q \in [0.5, 20)$	43.646	$+1.864$	$+7.484$	$+0.582$	-0.708	-11.185
% Inno, $q \in [20, 100)$	49.685	-1.775	-7.265	-0.585	$+0.858$	$+17.101$
% Inno, $q \geq 100$	0.000	$+0.000$	$+0.000$	$+0.000$	$+0.000$	$+0.000$
Pr(Sell)	2.451	$+0.390$	$+0.700$	$+0.069$	-0.079	-9.090

[1] The table reports the **percentage points difference** with respect to the benchmark. For example, the growth rate change is calculated according to $(g_{\text{counter}} - g) \times 100$. The first column reports the benchmark case in percentage points. The second and third columns analyze a case in which there is a noisy signal when firms trade innovations on the secondary market. The noise ϵ follows a Pareto distribution with scale parameter 1 and shape parameter α_ϵ. The fourth (fifth) column reports the results when there is a one-time innovation transaction subsidy(tax). The tax rate is as low as 1% transaction value. The sixth column shows a case where firms cannot trade innovations at all.

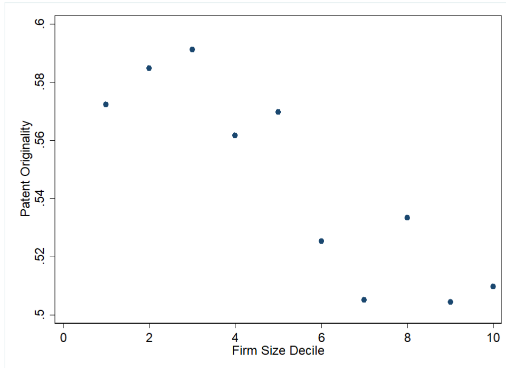

Figure 11: The Average Originality By Firm Size in 1997

C.4 Use Stock Options Instead of Equity

Using the stock options mainly affect the constant transfer of a contract; stock options capture the equity variance and protected by a lower bound. The same logic still applies: in a bigger firm, the innovation contributes to a smaller share of the equity value. Therefore, the only difference is the functional form of the variance associated with the incentive. Essentially, the mechanism behind the incentive problem does not change. The qualitative results still hold while the quantitative results may be different.

C.5 Patent Originality Distribution

One possible mapping of effort-sensitivity in the real life is the patent originality, as defined by Hall et al. (2001):

$$\text{Originality}_j = 1 - \sum_i^{n_j} s_{ij}^2$$

where s_{ij} denotes the percentage of citations made by patent j that belong to patent class i. Higher Originality_j means the patent j relies on technologies in many different fields, and hence it is more novel. Figure 11 plots the average originality by firm size in 1997. It decreases with firm size, which is consistent with the model implications.