The Distribution of Innovations across Firms

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Abstract

This paper builds a macroeconomic framework with heterogeneous firms and heterogeneous inventors to quantitatively understand the allocation of innovations across firms and the implications of innovation tradability. The model characterizes the entire innovation process, including both firms hiring inventors to innovate in-house and trading innovations with each other. The model is calibrated to moments in the United States, for example, the probability of selling innovations, and the distribution of innovations across firms. The model implies that projects whose outcomes are more effort-sensitive are developed by smaller firms. In a counterfactual scenario where firms cannot sell innovations, inventors move to larger firms (the share of innovations in firms with more than 100,000 employees increases by more than 10 percentage points), and growth drops by 0.16 percentage points.

JEL Classifications: D82, D86, E22, O31, O32, O40

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1 Introduction

Innovations are the fuel of growth and they are developed in many different types of firms. For example, AirPods were invented by Jason Giles at Apple, and both Facebook and Whatsapp were developed by inventors in start-ups. Also, firms resell innovations to other firms that complement them. For example, WhatsApp was sold to Facebook, which already owned a social network platform. Why are some innovations created in big firms and others in small firms? What are the forces pushing innovation inside or outside a firm's boundaries? Why are some innovations kept and some sold?

This paper studies how the option of selling innovations affects growth in an endogenous contracting setting. I model the entire process of producing an innovation, including both the primary and the secondary markets. In the primary market, an inventor with some innovative idea decides what firm to work for. In the secondary market, a firm decides whether to resell an innovation to another firm. I calibrate the model using US data moments, for example, the probability of selling innovations, and the distribution of innovations across firms. I run counterfactual experiments on the innovations' tradability. The results indicate that the trade friction in secondary markets affects not only the innovation allocation and firm growth but also the aggregate growth rate. For example, in an extreme case, when firms were not allowed to trade innovations, inventors would move to larger firms, which would contribute more to the growth than before. The share of innovations created in start-ups and firms with fewer than 500 employees would decrease from 8.0% to 1.3%. For firms with more than 100,000 employees, this share would increase by more than 10 percentage points.[1] The overall aggregate growth rate in the economy would decrease by 0.16 percentage points, from 2.00% to 1.84%. The quantitative results heavily depend on the endogenous contracting environment, which mitigates the growth effect of shutting down secondary markets. If firms were not allowed to optimize contract terms, growth would drop 0.21 percentage points instead, and the innovation distribution would remain mostly unchanged. The paper shows that having the option to sell innovations is important for growth, and endogenous contracting matters for quantitatively evaluating the magnitude.

I model a population of firms of different and endogenously evolving sizes. They face a population of risk-averse, short-lived inventors with innovation projects that differ in effort sensitivity, which measures the elasticity of the innovation rate with respect to the inventor's effort. The primary market is where firms offer compensations to attract inventors, and each inventor accepts the contract of the firm that gives the highest utility. Then the inventor works in the firm to develop the idea into an innovation.

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[1] The firm-level model implications provide one possible explanation for the empirical observations in Acikalin et al. (2022), which shows that when facing sudden patent invalidation, small firms lose disproportionately.

A firm decides its compensation schedule to solve a principal-agent problem. The firm is risk neutral and improves its product quality using innovations produced by inventors. It offers a combination of equity and wages to provide incentives, share risk, and split the surplus with the inventor; when the firm offers more equity, the inventor is better incentivized but also more exposed to the variance in equity. Since firms use equity to provide incentives, it exposes inventors both to the risks of their own innovations and all other risks the firm faces.[2] The innovation-related risks contribute to a smaller portion of the equity variance in a larger firm. Specifically, in start-ups, the value of equity depends solely on the outcome of the inventor's innovation. Thus, start-ups can offer compensation in the form of equity and wages to maximize innovation outcomes merely restricted by the inventor's risk aversion. The inventor can be well-incentivized to put effort. In larger firms, by contrast, much of the equity value depends on stochastic factors unrelated to the innovator's effort (e.g. success of other product lines). If large firms provided the same amount of incentive as start-ups, which means the large firms provided the same share of equity as start-ups, the inventors would be exposed to lots of unrelated risks and face a highly levered compensation package. The equilibrium outcome is that both the equity share and the incentive decrease with the firm size, and, in turn, an inventor works harder in a smaller firm.

In larger firms, an innovation is more likely to be more valuable since it has a higher probability of having complementarity with the firm (Figueroa and Serrano, 2019).[3] The complementarity includes that a firm can apply the technology to a larger production scope, the innovation is in line with the manager's expertise, or the firm has enough liquidity to meet any potential financial requirement in commercialization.[4] If there is complementarity, the firm is the most efficient user; otherwise, other firms are more efficient in using the technology.

Given contracts, an inventor chooses in which firm to work with by trading off between chances and value. On the one hand, the smaller the firm is, the harder the inventor works, which leads to a higher probability of innovating. On the other hand, conditional on the delivery of an innovation, a larger firm on average extract more value from the innovation. Therefore, the key trade-off is that working in a bigger firm means a higher surplus from using an innovation, while working in a smaller firm leads to better incentives and a higher probability to get an innovation. The latter is more important for an inventor with a more effort-sensitive idea. As a result, inventors whose ideas are more effort-sensitive work for small firms or start-ups, whereas

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[2]The problem cannot be solved by using a cash bonus contingent on the delivery of an innovation. This is based on the assumption that inventors can easily come up with a **fake** innovation, and whether an innovation is fake cannot be proved to a judge. See more in Section 2.1.4.

[3]This paper focuses on quality-improvement innovations. In the appendix, I consider when innovations are substitutes for existing technologies.

[4]In the appendix, I show that under certain functional form assumptions, financial frictions show up in the model in the same way as complementarity.

inventors with less effort-sensitive ideas work for large firms.

Once the innovation is created, the firm decides whether to resell it on a secondary market. Before making the decision, the firm observes the innovation step size,[5] which is random, and whether it has complementarity with the innovation. When the firm does not have complementarity with the innovation, it tries to resell the innovation on the secondary market. The buyers are the other firms that do have complementarity with the innovation. The secondary market serves as a reallocation mechanism to make efficient use of innovations.

A firm resells its innovation on a secondary market with asymmetric information. This is because the innovation step size is private information—a firm cannot prove it to others without telling them the technology details (Silveira and Wright, 2010; Chiu et al., 2017). This assumption of the unobservable innovation step size leads to a lemons market where only low-quality innovations are sold. Therefore, firms cannot fully capture the innovation's full first-best-use value, and this is why not everyone goes to startups. This adverse selection feature in innovations is also true in Chatterjee and Rossi-Hansberg (2012), which analyzes the spin-off decisions under asymmetric information.

One special case is when the inventor works for a start-up. If the start-up keeps the innovation, either because of complementarity or failing to sell it, the start-up enters the market and begins to produce. On the other hand, if the start-up sells the innovation to another firm, it is the model counterpart to a start-up buyout. Namely, an incumbent acquires the start-up for its technology.

I use patents as a real-world proxy for innovations. I calibrate the model to moments from US patenting firms between 1982 and 1997, matching the aggregate growth rate, the probability of a firm selling its patents, the start-up buyout rate, the distribution of patents by firm size, and the average value of an innovation. The patent distribution is calculated using data from the US Patent and Trademark Office (USPTO), the Center for Research in Security Prices (CRSP), the linked CRSP/USPTO data provided by Kogan et al. (2017), and CRSP/Compustat Merged Database. A public firm sample is first constructed using the datasets. Then, I build a statistical model to estimate the weight for each firm, as if being observed in the public firm sample were stratified based on firm sizes. The statistical model is used to estimate the distribution of innovations across all patenting firms. Only four data points in the distribution are targeted in the calibration; the rest are all untargeted. This model is able to match the majority part of the distribution of innovations across firms. Another untargeted moment is the growth rate gap between the 90th percentile and 50th percentile innovative firms, which my model can also match. Moreover, I use in-house R&D investment by firm sizes as the third set of untargeted moments. It confirms that the model can capture some of the trade-offs between in-house and

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[5]The step size is the change in firm quality resulting from an innovation.

outsourcing.

Finally, I use the model to run counterfactuals. In addition to the experiment where I shut down the secondary market, I simulate the cases in which there are signals for innovation step sizes and subsidies on innovation trade. The growth rate increases with efficiency. In the benchmark model, there is no signal about innovation quality. Due to the lemons market, only innovations with low step sizes are sold. When potential buyers can observe a signal about the innovation step size, more innovations are sold. For example, if the R square of the signal is 0.6, which means that the signal can explain about 60% of the variance in an innovation step size, the probability of selling an innovation increases from 5.3% to 5.9%. Since firms can sell unwanted innovations more easily, smaller firms are more attractive. More innovations are created in start-ups and medium-small- to medium-large-sized firms—the share of innovations created in start-ups increases from 0.33% to 5.23%. The growth rate increases by 0.09 percentage points, from 2.00% to 2.09%. When buyers receive a subsidy when purchasing an innovation, it has a similar impact. Quantitatively, when the subsidy is 5% of the transaction value, the aggregate growth rate increases by 0.016 percentage points, and the share of innovations in start-ups increases from 0.33% to 1.2%.

In the baseline counterfactuals, both the contract terms and the inventor-firm matching are endogenous. In each counterfactual, I also examine the role of endogenous contracting and matching on quantitative outcomes. If, in the counterfactual case where there is no secondary market, the contract terms are fixed and firm-inventor matching is flexible, the growth rate drops slightly less and the reallocation is less significant. This is because firms over-incentivize inventors by using a pre-fixed contract. In the second case, when the contracts are flexible and inventor-firm matching is fixed, the growth rate drops more, to more than 0.21 percentage points. Meanwhile, the innovation distribution almost remains un-changed. In the third case, when neither the contract nor the matching can adjust, the growth rate drops slightly less than in the second case. This is because the contracts cannot be adjusted and inventors are over-incentivized.

Related Literature My paper relates to a literature exploring the implications of trading knowledge on firm innovations, examples of which include Cassiman and Veugelers (2006), Higgins and Rodriguez (2006), Phillips and Zhdanov (2013), Bena and Li (2014), and Liu and Ma (2021), and the impact of the idea market on the economy growth (Eaton and Kortum, 1996; Silveira and Wright, 2010; Chatterjee and Rossi-Hansberg, 2012; Chiu et al., 2017; Cabral, 2018; Cunningham et al., 2021; Perla et al., 2021; Fons-Rosen et al., 2021). Recent papers studying the impact of the secondary market in a general equilibrium model include Akcigit et al. (2016) and Ma (2022). This work and my paper share a common framework, inherited from the endogenous growth literature, for example, Romer (1986), Aghion and Howitt (1992),

Aghion et al. (2001), Klette and Kortum (2004), Acemoglu et al. (2018), and Akcigit and Kerr (2018). My paper's firm production framework is closest to Acemoglu et al. (2018), which assumes that the intermediate goods producers use innovations to improve their qualities, and final goods producers assemble intermediate goods. My paper contributes to this literature by incorporating the endogenous contracting and matching problem. I show that the endogenous firm boundaries are important because the reallocation of inventors across firms is an important part of the counterfactual scenarios. The endogenous contracting and matching setting mitigates the impact on the aggregate growth rate and amplifies the effect on the innovation distribution.

My model thinking about where inventors work relates to the literature on the boundary of the firm, important examples of which going back to Coase (1937), important examples include Grossman and Hart (1986), Hart and Moore (1990), and Hart and Moore (2008). Closest are Aghion and Tirole (1994) and Schmitz (2005) who analyze the implications of the innovations' ownership. My paper casts these ideas in a quantitative general equilibrium economic setting with heterogeneous inventors and heterogeneous firms. My findings show that the endogenous firm boundaries matter quantitatively for both firm-level results and aggregate growth rate.

This paper is also connected to the discussion on intellectual property right protection. Examples include Boldrin and Levine (2013), Galasso and Schankerman (2015), and Budish et al. (2015). I focus on the general equilibrium implications of tradability. My results are consistent with the empirical evidence derived from two natural experiments by Ma (2022) and Acikalin et al. (2022), namely, decreasing tradability means a larger portion of innovations are created in large firms and it undermines small firms disproportionately.

The rest of the paper is organized as follows: Section 2 describes the model and characterizes the equilibrium. Section 3 describes the calibration. Section 4 reports the quantitative results and counterfactuals, and Section 5 concludes.

2   A Theoretical Model

I have built a theoretical model where firms compete to attract inventors to innovate in-house. The goal is to study the allocation of innovations across firms and the implications of the innovation market on the firms' growth. I first introduce the modeling environment and then describe the equilibrium. There are two problems in the two markets for innovations. In the primary market, an inventor chooses what size of the firm she wants to work for. In the secondary market, firms buy and sell innovations with each other. I then characterize the equilibrium of this model.

2.1 Environment

Here time is continuous. There are four types of agents: households, inventors, intermediate goods producers, and final goods producers. A representative household provides labor and consumes final goods. Inventors provide efforts to produce innovations within intermediate firms and consume final goods. There is a continuum of intermediate firms. Each one produces one unique type of product of some quality and uses innovations to improve its quality. The final good producers use intermediate goods as input to produce final goods. I will describe the preference and production technologies in the following subsections.

2.1.1 Preferences

The household is long-lived. Every household supplies one unit of labor inelastically. The household's utility function is

$$U_H = \int_0^\infty e^{-\rho t} ln(C_H(t)) dt, \tag{1}$$

where $\rho > 0$ is the discount rate and $C_H(t)$ is the consumption of the household.

An inventor is short-lived and lives for dt time periods. There is a continuum of inventors of measure 1 in every period. An inventor provides effort e_I to produce inventions. She is risk averse and has a mean-variance utility:

$$U_I(c_I, e_I) = \mathbb{E}(c_I) - \frac{var(c_I)}{2\bar{q}} - R(e_I)\bar{q}, \tag{2}$$

where c_I is the consumption, e_I is the effort level, and $R(e_I)\bar{q}$ is the associated cost. $\bar{q}$ (defined below) is the average quality in the economy. The cost is scaled by $\bar{q}$ to keep the problem stable over time. Denote the aggregate consumption of inventors using C_I.

2.1.2 Technology

Firms are owned by household. The final good producers produce final goods using a continuum of intermediate goods $j \in [0, N_F]$ with production technology which is similar to Akcigit and Kerr (2018)[6]:

$$Y(t) = \frac{1}{1-\beta} \int_0^{N_F} q_j^\beta(t) y_j^{1-\beta}(t) dj. \tag{3}$$

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[6]The difference is that in my specification, the final good producers only use intermediate goods, not labor, to assemble the final good. My model yields similar results if the final good producers use labor as well.

In this function, $q_j(t)$ is the quality of the intermediate good j, and $y_j(t)$ is its quantity. I normalize the price of the final good to one in every period. The final good producers are perfectly competitive, taking input prices as given. Henceforth, I will drop the time index t when it does not cause confusion. The final goods are consumed by the household and inventors. The resource constraint of the economy is:

$$Y = C_H + C_I. \tag{4}$$

There is a continuum of measure N_F risk neutral firms producing intermediate goods. Each firm produces one kind of good. Each firm produces using a linear technology using only labor:

$$y_j = \bar{q} l_j, \tag{5}$$

where l_j is the labor input, $\bar{q} = \frac{1}{N_F} \int_0^{N_F} q_j dj$ is the average quality. The cost is linear in wage w, which firms take as given. In each period, the labor market satisfies the constraint:

$$\int_0^1 l_j dj \leq 1. \tag{6}$$

The production technologies, together with the market setting on innovation, ensure that a firm's value $V(q_j)$ is linear in quality q_j

$$V(q_j) = \nu q_j, \tag{7}$$

where ν is endogenously determined by the general equilibrium. The proof is in Section 2.2.1.

This paper focuses on the balanced growth path. I normalize the variables using the average quality $\bar{q}$. I denote the normalized variables using tilde:

$$\tilde{q}_j \equiv \frac{q_j}{\bar{q}}, \tilde{Q} \equiv \frac{Q}{\bar{q}}, \tilde{V}(\tilde{q}) \equiv \frac{V(q_j)}{\bar{q}} = \nu \tilde{q}_j, \tag{8}$$

where $Q \equiv \int_0^{N_F} q_j dj$ is the total technology stock in the economy.

A firm's quality q is potentially affected by both business shocks and innovations. The business shock δ follows a Poisson arrival rate normalized to 1. The shock is a random draw from $\Xi(\delta)$. Upon the realization of a business shock, the firm's quality becomes $q + \delta q$.

In intermediate firms, innovations are produced by inventors using effort as the input. Each inventor is born with one innovative idea type θ that measures the idea-specific effort sensitivity $(\theta \in (0,1), \theta \sim \Psi(\theta))$. She works in the firm she chooses, either an existing intermediate firm or a start-up. Hired by the firm, the inventor exerts unverifiable effort e_I to transform the idea into an innovation. Given the effort level e_I, an innovation arrives with the instantaneous

Poisson flow rate:

$$\lambda_\theta(\theta, e_I) = \mu e_I^\theta. \tag{9}$$

The innovation production function is based on the growth theory literature (Romer, 1990; Klette and Kortum, 2004; Akcigit and Kerr, 2018). The literature usually treats θ as a parameter, whereas here, θ is heterogeneous across innovations. I distinguish different types of innovations to consider the mapping between innovations and firms. An larger θ means the production function is more effort-elastic and I refer to inventors with high θ as effort-sensitive inventors. Additionally, in the literature, a unified firm produces and implements innovations; in my model, however, it is the inventor who creates innovations, and the firm only enjoys the outcome. It is costly to work, and the flow cost of choosing effort e_I is $R(e_I)\bar{q}$. Thus, without incentive, the inventor chooses $e_I = 0$. Meanwhile, she can always effortlessly make up a useless innovation that does not improve quality. The usefulness is not verifiable.

If the inventor creates a useful innovation, a patent is granted. The step size z is then drawn from a distribution $\Phi(z)$. If a firm implements the innovation, then its quality increases by an increment, Δq, where

$$\Delta q = \begin{cases} \gamma_L z Q & \text{with probability } (1 - h(\tilde{q})) \\ \gamma_H z Q & \text{with probability } h(\tilde{q}), \gamma_H > \gamma_L. \end{cases} \tag{10}$$

$h(\tilde{q})$ is the probability that a firm has complementarity with the innovation. I assume that $h'(\tilde{q}) > 0$, which means that larger firms are more likely to have complementarity with the innovation. The complementarity includes that a firm can apply the technology to a larger production scope, the innovation suits its business operation, or the firm has enough liquidity to meet any potential financial requirement in commercialization. γ_L and γ_H capture the different efficiency in applying the technology. When there is complementarity, the firm is the efficient user of the innovation. In this case, the quality increment is $\Delta q = \gamma_H z Q$. Otherwise, the firm is not an efficient user, and the quality improvement is lower. In this case, there are always some firms taht have complementarity.[7]

Because the firm value is linear in its quality, given an innovation step size, its value to any given firm only depends on the complementarity. If the firm is an efficient user, then there is no gain from trade. Otherwise, there is a positive gain from trade. All firms try to sell innovations without complementarity, and buyers are the firms who have complementarity with these innovations. For each innovation, assume that there are at least two firms that are

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[7]Akcigit et al. 2016 and Ma (2022) also consider the technology mismatch, but in a different setting. In their papers, if there is a mismatch, the firm cannot use the innovation while in my model, the firm can still apply it.

efficient in implementing it.

2.1.3 Information Structure

After an inventor is born, the type θ is publicly known. Then the inventor chooses for which firm to work and the effort e_I. The effort e_I is unobservable and unverifiable. Hence, contracts cannot be contingent on the effort level.

If the inventor created an innovation, the step size is not verifiable, which means the existence of a useful innovation is not verifiable. Therefore, contracts cannot be contingent on either. The firm that owns the innovation knows its step size, but cannot prove it to others. Hence, firms sell and buy innovations under asymmetric information. The complementarity is public information—all firms know which firms have complementarity with the innovation.

2.1.4 Employment Contracting Problem

In the primary market, a firm hires an inventor by offering an employment contract. I assume that the employment contract can only have two dimensions: a constant wage T and a share of the firm's equity $a \in [0, 1]$. Firms can freely choose the combination of the two but cannot use other tools.

One may think that a better tool for providing incentives to the inventor would be to offer a bonus contingent on the delivery of an innovation, since this would avoid exposing inventors to business shocks that are unrelated to innovations. However, inventors can earn bonuses effortlessly by providing useless innovations. Therefore, using a bonus cannot solve the moral hazard problem. A firm could try to use a detailed contract, and try to directly link the inventor's innovation to the firm's profit increase and use this as a basis for a bonus. I will follow the incomplete contract literature (Grossman and Hart, 1986), who assumes that it is very hard to specify all innovation outcomes in contracts, and, in practice, it is usually not contractible (Acemoglu, 1996; Frésard et al., 2020); I assume that firms can only use equity and wages in the employee contracts, as commonly seen in the real world (Brickley and Hevert, 1991).

The timing is as follows. All firms simultaneously offer firm-inventor-specific contracts to each inventor. After viewing all contracts, the inventor chooses her favorite one and joins the firm. If the best contract for the inventor is offered by $q = 0$, we call that a start-up. Even in this case, the inventor does not bear all the risk by holding 100% equity. I assume that $q = 0$ firms (interpretable as venture capital funds) offer optimal contracts with endogenous a.

2.1.5 Secondary Market Setup

Firms can trade unwanted innovations on secondary markets. Sellers are the ones who don't have complementarity with their innovations. Each innovation is unique and not substitutable by others, which implies that there is one market for each innovation. Buyers are the firms that have complementarity with the innovation. For each innovation for sale, a firm becomes a potential buyer with a probability proportional to the normalized quality $\tilde{q}$. For an innovation, there are at least two potential buyers.

Firms sell and buy under incomplete information because the innovation step size is private information. Firms compete to buy innovations following Bertrand competition. Each buyer offers a price p_z. All buyers offer purchase contracts simultaneously, and the seller chooses whether to accept one. It is a one-shot game—neither sellers nor buyers keep track of past trades. The settings lead to a standard lemons market (Akerlof, 1978).

2.1.6 Entry and Exit

There are two types of entry. One is innovation-related, which happens when an inventor chooses to work in a firm with $\tilde{q} = 0$, successfully creates an innovation and decides to keep it. Denote the amount of innovation-related entrants with λ_I. The other is the exogenous entry where firms enter the market due to reasons other than innovations. Denote the amount of exogenous entrants with λ_0. In both cases, upon entry, the firm draws a quality $\tilde{q}$ from a distribution $\tilde{F}_{q0}(\tilde{q})$ and incurs a cost which equals the firm value. This represents the spillover from incumbents to entrants. Assume that $\tilde{f}_{q0}(\tilde{q}) = \tilde{f}_q(\tilde{q})/\kappa$. When $\kappa < 1$, it means that entrants are on average smaller than the average firm. When the new entrant owns an innovation, then its starting quality q is boosted by Δq because of the innovation.

Firms face an exogenous exit rate τ. I will focus on a balanced growth path such that entry equals exit

$$\tau N_f = \lambda_I + \lambda_0. \tag{11}$$

2.2 Equilibrium

I focus on the balanced growth path and solve the problem backward. I now characterize the equilibria of the economy in which aggregate variables $(Y, C, R, w, \bar{q})$ grow at the constant rate g.

2.2.1 Production

The final good producer chooses $\{y_j\}_j$ to maximize its profit using the technology described in Section 2.1.2:

$$\max_{\{y_j\}} \frac{1}{1-\beta} \int_0^{N_F} q_j^\beta y_j^{1-\beta} dj - \int_0^{N_F} y_j p_j dj. \tag{12}$$

The first-order condition yields the demand function of intermediate firms $p_j = q_j^\beta y_j^\beta$. The intermediate goods are produced by corresponding firm $j \in [0, N_F]$ using only labor $y_j = \bar{q} l_j$, where $\bar{q} = \frac{1}{N_F} \int_0^{N_F} q_j dj$ is the average quality, and l_j is the labor input. Intermediate good producers are in monopolistic competition and choose l_j, p_j, y_j to maximize their profit, given the wage level w:

$$\max_{l_j, p_j, y_j} y_j p_j - w l_j.$$
$$\text{s.t. } y_j = \bar{q} l_j \tag{13}$$
$$p_j = q_j^\beta y_j^{-\beta}$$

Therefore, the FOC yields

$$y_j = q_j \left(\frac{\bar{q}(1-\beta)}{w} \right)^{\frac{1}{\beta}}, l_j = y_j / \bar{q}, p_j = \frac{w}{\bar{q}(1-\beta)}. \tag{14}$$

In each period, the labor market clearing satisfies $\int_0^{N_F} l_j dj = 1$, which gives that $\frac{\int_0^{N_F} q_j \left(\frac{\bar{q}(1-\beta)}{w} \right)^{\frac{1}{\beta}} dj}{\bar{q}} = 1$. The wage w can be solved

$$w = N_F^\beta (1-\beta) \bar{q}. \tag{15}$$

Plug it back into the intermediate firm's problem. Both production y_j and profit π_j are linear in quality

$$y_j = \frac{q_j}{N_F}, \pi_j = \frac{\beta q_j}{N_F^{1-\beta}}. \tag{16}$$

I drop the subscript j from the firm-level variable when it does not cause confusion. In this model, the firm size is linear in its quality and I will use q to denote firm size when it does not cause confusion. Intermediate firms are the ones who hire inventors to innovate. Because of competition, any value from innovations is captured by inventors, and any value from acquisitions is captured by the seller. The discounted value of being a firm of quality q is the same as the net present value under the assumption that the firm never innovate. Thus,

the value function of intermediate firm q at time t can be written as

$$V(q, t) = \int_t^\infty e^{-(r+\tau)(s-t)} \beta q / N_F^{1-\beta} ds = \nu q. \tag{17}$$

where $\nu = \frac{\beta}{(r+\tau)N_F^{1-\beta}}$. The value function is linear in q and does not depend on time. This result implies that for any firm, the value of a same quality improvement Δq is the same.

The aggregate production is linear in average quality $\bar{q}$. The resource constraint of the economy is $Y = C_H + C_I$, where R is the total R&D spending in each period. The Euler equation is:

$$g = \frac{\dot{Y}}{Y} = \frac{\dot{C}_H}{C_H} = \frac{\dot{\bar{q}}}{\bar{q}} = r - \rho. \tag{18}$$

2.2.2 Secondary market

The secondary innovation market is where firms buy and sell innovations. For each innovation, there are at least two buyers. Buyers follow Betrand competition and offer prices to the seller. A seller accepts an offer only when it is profitable, which means

$$\gamma_L z \nu \tilde{Q} \le \tilde{p}_z. \tag{19}$$

The left-hand side is the value of the innovation if the seller keeps it, whereas the right-hand side is the value if the seller sells it. The threshold $\hat{z}$ satisfies

$$\gamma_L \hat{z} \nu \tilde{Q} = \tilde{p}_z. \tag{20}$$

They set the price $\tilde{p}_z$ according to the zero-profit condition

$$\int_0^{\hat{z}} \left(\gamma_H \nu z \tilde{Q} - \tilde{p}_z \right) \phi(z) dz = 0, \tag{21}$$

where $\gamma_H \nu z \tilde{Q}$ is the innovation value to any buyer with complementarity and $\tilde{p}_z$ is the price of the innovation; $\hat{z}$ is the step size threshold. If $z \le \hat{z}$, an innovation is sold.

If a firm is not the efficient user of an innovation, it chooses to sell the innovation if its step size is lower than the threshold $\hat{z}$. Otherwise, the firm uses the innovation to improve its own technology.

For each innovation, the probability of being sold is

$$(1 - h(\tilde{q})) \Phi(\hat{z}),$$

where $(1 - h(\tilde{q}))$ is the probability that the innovation does not have complementarity with the firm $\tilde{q}$ where the inventor works; $\Phi(\hat{z})$ is the cumulative distribution function of z at $\hat{z}$. The total amount of innovations sold per unit of time is

$$\Phi(\hat{z}) \int_0^1 \lambda_\theta^* (1 - h(\tilde{q}^*(\theta))) \psi(\theta) \, d\theta, \tag{22}$$

where λ_θ^* is the equilibrium innovation arrival rate of type θ. $\tilde{q}^*$ denotes the firm for which inventor θ works in equilibrium. Recall that the probability of buying an innovation is $\lambda_b \tilde{q}$. The secondary market clearing gives

$$N_F \int \lambda_b \tilde{q} f_q(\tilde{q}(\theta)) \, d\tilde{q} = \Phi(\hat{z}) \int_0^1 \lambda_\theta^* (1 - h(\tilde{q}^*(\theta))) \psi(\theta) \, d\theta, \tag{23}$$

which implies that

$$N_F \lambda_b = \Phi(\hat{z}) \int_0^1 \lambda_\theta^* (1 - h(\tilde{q}^*(\theta))) \psi(\theta) \, d\theta. \tag{24}$$

It pins down the endogenous variable λ_b, and the probability of buying an innovation.

The innovation value if the innovation is created in firm $\tilde{q}$ satisfies

$$x(z, \tilde{q}) = \begin{cases} \gamma_L \nu \hat{z} \tilde{Q} & \text{no complementarity and } z \leq \hat{z} \\ \gamma_L \nu z \tilde{Q} & \text{no complementarity and } z > \hat{z} \\ \gamma_H \nu z \tilde{Q} & \text{with complementarity.} \end{cases} \tag{25}$$

When there is no complementarity, and the step size z is lower than the threshold $\hat{z}$, the firm sells the innovation on the innovation market at the price $\gamma_L \nu \hat{z} Q$. When there is no complementarity, and the step size z is higher than the threshold, the firm keeps it, and it is worth $\gamma_L \nu z Q$. When there is complementarity, the firm is the efficient user of the innovation. Then the firm also implements it in-house, and its value is $\gamma_H \nu z Q$. In a first-best allocation, all innovations are sold to the efficient user. The asymmetric information on the secondary market, however, prevents this from happening.

The expected innovation value $\mathbb{E}(x(\tilde{q}))$ can be written as

$$\mathbb{E}(x(\tilde{q})) = \gamma_L \left[\Phi(\hat{z}) \hat{z} + (1 - \Phi(\hat{z})) \mathbb{E}(z | z > \hat{z}) \right] (1 - h(\tilde{q})) \nu Q \tag{26}$$
$$+ \gamma_H \mathbb{E}(z) h(\tilde{q}) \nu Q.$$

It increases with the firm size $\tilde{q}$ because $h\tilde{q}$ is increasing in $\tilde{q}$. Therefore, conditional on an innovation being created, its value is higher, if it is in a large firm.

2.2.3 Primary market

The primary market is where firms compete to hire inventors using a combination of equity a and wage $\tilde{T}$. The setup yields a principal-agent framework. Firms are risk neutral, and inventors are risk averse. Firms enjoy the innovations produced by inventors, but it is costly for inventors to work and firms cannot monitor the effort. Thus, firms want to split the surplus with the inventor by offering a constant wage; meanwhile, firms need to incentivize the inventor to exert effort by offering equity. Because of the competition, the firm's problem is the same as maximizing the utility it offers to an inventor, subject to the zero profit condition. The problem can be written as follows

$$max_{a,T,e_I} U_I\left(a, \tilde{T}; \tilde{q}, \theta, e_I\right) = \mathbb{E}\left(\tilde{c}_I\right) - \frac{1}{2}\text{var}\left(\tilde{c}_I\right) - R\left(e_I\right) dt.$$

$$s.t. \lambda_\theta\left(\theta, e_I\right) \mathbb{E}\left(x\left(\tilde{q}\right)\right) dt - \mathbb{E}\left(\tilde{c}_I\right) \geq 0$$

$$e_I = \arg\max\left\{U_I\left(a, \tilde{T}; \tilde{q}, \theta, e_I\right)\right\} \tag{27}$$

$$\mathbb{E}\left(\tilde{c}_I\right) = a\left(\mathbb{E}\left(\tilde{V}_0\left(\tilde{q}\right)\right) + \lambda_\theta\left(\theta, e_I\right) \mathbb{E}\left(x\left(\tilde{q}\right)\right) dt\right) + \tilde{T}$$

$$\text{var}\left(\tilde{c}_I\right) = a^2\left(\lambda_\theta\left(\theta, e_I\right)\sigma_x^2(\tilde{q}) + \sigma_0^2(\tilde{q})\right) dt$$

The optimal contract is denoted $\{a^*\left(\theta, \tilde{q}\right), T^*\left(\theta, \tilde{q}\right)\}$. The first line is the objective function $U_I\left(a, \tilde{T}; \tilde{q}, \theta, e_I\right)$, which represents the inventor θ's utility when she works for the firm $\tilde{q}$ with effort level e_I; $\left(a, \tilde{T}\right)$ characterize the contract offered by the firm, and $\tilde{c}_I$ is the inventor's consumption level, while $R\left(e_I\right) dt$ is the cost of effort during the employment.

The second line is the firm's individual rationality constraint, where $x\left(\tilde{q}\right)$ is the value of an innovation to a firm $\tilde{q}$. It multiplies by the probability of creating an innovation $\lambda_\theta\left(\theta, e_I\right) dt$, which gives the expected payoff of hiring an inventor. The cost of hiring is $\left(\tilde{c}_I\right)$. The firm will only participate if the expected gain is nonnegative.

The third line is the inventor's incentive compatibility constraint. The inventor chooses an effort level e_I to maximize the expected utility $U_I\left(a, \tilde{T}; \tilde{q}, \theta, e_I\right)$ given firm quality $\tilde{q}$ and the wage scheme $\left\{a, \tilde{T}\right\}$.

The fourth line describes the inventor's expected consumption. The first part, $a\left(\tilde{V}_0\left(\tilde{q}\right) + \lambda_\theta\left(\theta, e_I\right) \mathbb{E}\left(x\left(\tilde{q}\right)\right) dt\right)$, is expected value of the firm equity a, which is the sum of the value without innovations and the expected value of an innovation. The other component, $\tilde{T}$, is the constant transfer from the firm to the inventor.

The fifth line shows the variance of the inventor's consumption. The exposure is the equity a. The uncertainty comes from two sources: innovation-related part $\lambda_\theta\left(\theta, e_I\right)\sigma_x^2(\tilde{q})$ and the innovation-independent part $\sigma_0^2(\tilde{q})$. In start-ups, the second term is zero. I explain both in

detail later.

Given contracts, the inventor with idea θ chooses the firm $\tilde{q}^*(\theta) \in [0, \infty)$ and exerts the corresponding effort level e_I to maximize her utility by solving the following problem:

$$\max_{\tilde{q}} U_I\left(a(\tilde{q}, \theta), \tilde{T}(\tilde{q}, \theta), e_I(\tilde{q}, \theta), \tilde{q}, \theta\right). \tag{28}$$

Here $a(\tilde{q}, \theta), \tilde{T}(\tilde{q}, \theta)$ and $e_I(\tilde{q}, \theta)$ are the solutions to the firm's problem described by Equation (27). If $\tilde{q}^*(\theta) > 0$, then the inventor works in an incumbent firm; $\tilde{q}^*(\theta) = 0$ means that the inventor joins a new firm. The equilibrium choice of effort is $e_I^*(\theta)$ and the corresponding arrival rate is $\lambda_\theta^* = \lambda_\theta(\theta, e_I^*(\theta))$.

The firm value is affected by four factors: in-house innovation, purchased innovation, business shocks, and exogenous death. The firm value without in-house innovations $\tilde{V}_0(\tilde{q})$ can be written as

$$\tilde{V}_0(\tilde{q}) = \nu\tilde{q} + \begin{cases} -\nu\tilde{q} & \tau dt \\ \delta\nu\tilde{q} & dt \\ \left(\gamma_H \nu z \tilde{Q} - \tilde{p}_z\right)|_{z \le \hat{z}} & \lambda_b \tilde{q} \Phi(\hat{z}) dt \\ 0 & 1 - (\tau + 1 + \lambda_b \tilde{q} \Phi(\hat{z})) dt. \end{cases} \tag{29}$$

The first line is when the firm dies exogenous. The second line is when hit by a business shock δ that arrives with an arrival rate of 1. The shock is a random draw from a truncated normal distribution $\Xi(\delta)$. The third line is when the firm purchases an innovation. Though the expected gain from purchasing an innovation is zero, the realized gain usually is not zero—it is the gap between the price paid and the firm value improvement. The quality improves by $\gamma_H z \tilde{Q}$, where z is a random draw from the $\Phi(z)$ distribution truncated at $\hat{z}$. The cost of buying an innovation is $\nu\gamma_L \hat{z} \tilde{Q}$. The last line is the firm value otherwise.

The uncertainty in the firm equity leads to the variance in the inventor's consumption c_I. Because the in-house innovation is independent of other activities, the firm-level risk can be decomposed into two parts: in-house innovation-related risk and the rest. The former one is $\lambda_\theta(\theta, e_I) \sigma_x^2(\tilde{q}) dt$, where $\sigma_x^2(\tilde{q}) = \mathbb{E}\left[(x(\tilde{q}))^2\right]$ is the second moment of the innovation value. The latter one is $\sigma_0^2(\tilde{q}) dt$ which contains all shocks unrelated to innovations, including the risk from death, business shocks, and unknown step size in innovation trade. It can be written as

$$\sigma_0^2(\tilde{q}) = \nu^2 \left\{ \tilde{q}^2 \left(\tau + \sigma_\delta^2\right) + \lambda_b \tilde{q} \Phi(\hat{z}) \mathbb{E}\left[(\gamma_H z - \gamma_L \hat{z})^2 | z \le \hat{z}\right] \right\}. \tag{30}$$

The risk increases with the firm size $\tilde{q}$, and faster than σ_x^2. Therefore, in a larger firm, the innovation-related risk takes a smaller share in the total equity variance.

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The firm determines how much equity to offer to the inventor. The incentive problem depends on the firm size. In start-ups, the value of equity depends solely on the outcome of the inventor's innovation. Thus, start-ups can offer compensation in the form of equity and wages to maximize innovation outcomes merely restricted by the inventor's risk aversion. The inventor can be well-incentivized to put effort into innovating. In larger firms, by contrast, the inventor's innovation only makes up a small fraction of the equity value; much of the equity value depends on stochastic factors unrelated to the innovator's effort (e.g. success of other product lines). If larger firms were to offer the same incentive as start-ups for the same expected compensation, inventors would be exposed to many unrelated risks and face a highly levered compensation package, consisting of negative wages and a high proportion of firm equity; this is not attractive to risk averse inventors. Therefore, both the optimal equity level a and the resulting incentive decrease with firm sizes.

After seeing all contracts, the inventor picks the contract offered by firm $\tilde{q}^*(\theta)$, which gives her the highest utility. Equation (28) can be written as:

$$max_{\tilde{q},e_I} U_I(\tilde{q};\theta,e_I) = \lambda_\theta(\theta,e_I)\,\mathbb{E}(x(\tilde{q}))\,dt - \frac{1}{2}a^2(\tilde{q})\left[\lambda_\theta(\theta,e_I)\sigma_x^2(\tilde{q}) + \sigma_0^2(\tilde{q})\right]dt - R(e_I)\,dt.$$

$$s.t.e_I = \arg\max\{U_I(\tilde{q};\theta,e_I)\}.$$

(31)

The main trade-off an inventor facing is governed by the first term $\lambda_\theta(\theta,e_I)\,\mathbb{E}(x(\tilde{q}))$ (under relevant parameters). The two elements work in opposite directions. A firm with higher quality $\tilde{q}$ on average can implement an innovation more efficiently, which means that the value-added of an innovation $\mathbb{E}(x(\tilde{q}))\,dt$ is higher; meanwhile, a smaller firm can means better incentive and hence higher arrival rate $\lambda_\theta(\theta,e_I)$. The inventor's decision then depends on the relative strength of the two forces, which, in turn, is determined by the inventor type. For an effort-sensitive inventor with high θ, the incentive channel is more important. As a result, she chooses to work for a small firm. Similarly, when an inventor's idea has low effort-elasticity, the value channel is more crucial and she works for a large firm.

Next, I change the measure of the innovation arrival rate from per inventor to per firm. For each inventor, the equilibrium innovation arrival rate generated by herself in the firm $\tilde{q}$ is λ_θ^*. In firm $\tilde{q}$, the innovation arrival rate is affected by both the single innovation arrival rate and the number of inventors per firm. Denote the arrival rate in firm $\tilde{q}$ by $\lambda_q(\tilde{q})$ and the firm probability distribution function by $f_q(\tilde{q})$. The total amount of innovations in a firm should be the same as the total innovations made by inventors who work for the firm. The inventor market clearing gives

$$N_F\lambda_q(\tilde{q}(\theta))\,f_q(\tilde{q}(\theta))\,d\tilde{q} = \lambda_\theta^*\psi(\theta)d\theta,$$

(32)

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which pins down the innovation arrival rate per firm.

2.2.4 Entry and Exit

Recall there are two types of entries in this model. One is innovation-related. It happens when an inventor chooses to work in a firm with $q = 0$, successfully gets an innovation, and decides to keep it. The number of firms that enter this way can be written as

$$\lambda_I = \int_{\tilde{q}^*(\theta)=0} \lambda_\theta^* \left[\overbrace{\underbrace{(1 - \Phi(\hat{z}))(1 - h(0))}_{\text{Non-complementary \& big step size}} + \underbrace{h(0)}_{\text{Complementary}}}^{\text{Innovations kept}} \right] \psi(\theta) \, d\theta. \tag{33}$$

The other type is the exogenous entry, which enters the market without an innovation. The number is denoted by a parameter λ_0.

On the exit front, the exit rate satisfies that the number of firms who exit equals the number that enters.

$$\tau N_F = \lambda_I + \lambda_0. \tag{34}$$

I use (34) to solve for the measure of firms N_F.

2.2.5 Growth rate

The growth rate of the aggregate quality $\bar{q}$ can be written as:

$$g = \frac{\mathbb{E}_t(\bar{q}(t + dt)) - \bar{q}(t)}{\bar{q}(t)dt}. \tag{35}$$

The growth rate is

$$\begin{aligned}
g = &-\tau(1 - \kappa) \\
&+ [\gamma_L \mathbb{E}(z) + (\gamma_H - \gamma_L) \mathbb{E}(z | z \leq \hat{z}) F(\hat{z})] \int [1 - h(q^*(\theta))] \lambda_\theta^* \psi(\theta) \, d\theta \\
&+ \gamma_H \mathbb{E}(z) \int h(q^*(\theta)) \lambda_\theta^* \psi(\theta) \, d\theta.
\end{aligned} \tag{36}$$

It includes three parts. The first row is quality destruction if there were no innovations. For each unit of time, τ fraction of firms with an average quality $\bar{q}$ leave the economy, while the same fraction of firms enter the economy with an average quality κ. The second and third lines report the quality improvement due to the innovation. The second line measures the improvement that attributes to the innovations that do not complement the initial firm. If there were no secondary market, then the average quality is $\gamma_L \mathbb{E}(z)$. With the secondary

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market, some low-step-size innovations are sold to more efficient users, which increases the quality improvement by $(\gamma_H - \gamma_L)\,\mathbb{E}\,(z|z \le \hat{z})\,F\,(\hat{z})$. The third line is the quality improvement because of innovations that complement with the firms. The average quality is $\gamma_H \mathbb{E}\,(z)$. Based on (36), when the innovation is worth more or the arrival rate increases, the growth rate goes up.

2.2.6 Equilibrium

I end this section by summarizing the equilibrium. The in-house R&D expenditure C_I of the economy can be written as

$$C_I = \int \lambda_\theta\,(\theta)\,\mathbb{E}\,(x\,(q^*\,(\theta)))\,\psi\,(\theta)\,d\theta. \tag{37}$$

It captures all transfers made to inventors. The total firm-level R&D expenditure is the sum of R and the spending on purchasing patents on the secondary market

$$X = C_I + \lambda_b p_z. \tag{38}$$

Based on Equation (12), the equilibrium out put level Y is linear in $\bar{q}$

$$Y = \frac{1}{1-\beta}\frac{\bar{q}}{N_F^{1-\beta}}. \tag{39}$$

and the consumption level is

$$C_H = Y - C_I. \tag{40}$$

Definition 1. A balanced growth path of this economy for any combination of t, q, is the mapping between q and θ, the allocation $\left(\{y_j^*\}_j, Y^*, X^*, C_I^*, C_H^*\right)$, the price levels $\left(w^*, \{p_j^*\}_j, \hat{z}\right)$, the aggregate growth rate g^*, the entry rates λ_{eI}^*, and the measure of firms N_F^*, such that (1) for any $j \in [0, 1]$, y_j^* and p_j^* satisfy Equation (14); (2) wage w^* satisfies Equation (15); (3) measure of the intermediate producers N_F^* satisfies Equation (34); (4) the mapping is the solution of Equation (28); (5) aggregate creative destruction τ_e satisfies Equation (34); (6) the price offer $\hat{z}$ for inventions satisfies Equation (20); (7) the entry rates λ_{eI}^* satisfy Equation (33); (8) in-house R&D spending C_I^* satisfies Equation (37); (9) total R&D spending X satisfies Equation (38); (10) aggregate output Y^* satisfies Equation (39); (11) aggregate consumption C_H^* satisfies Equation (40); and (12) steady-state growth rate g^* satisfies Equation (36).

3 Calibration

I calibrate the model to the United States. I construct the moments using firm-level information and use patents as a proxy for innovations. Section 3.1 describes the data. Section 3.3 explains how I calibrate the model. Appendix A outlines the algorithm I used to solve the model.

3.1 Data

This paper combines four datasets to calculate moments. The datasets are the US Patent and Trade Office (USPTO), the Center for Research in Security Prices (CRSP), the Merged CRSP-Compustat Database, and the linked CRSP-USPTO data provided by Kogan et al. (2017). In addition, I use the Survey on Business Strategies (ESEE) by Fundacion SEPI from Spain, which provides firm-level in-house R&D investment and outsourcing R&D expenditure.[8]. The main sample period is from 1982 to 1997. I also calibrate the model to 1998-2010 and the results are reported in the appendix.

3.1.1 US Data Development

I combine the four datasets to obtain firm-level in-house patenting features. I start with the linked CRSP/USPTO dataset. It covers all patents granted to public firms from 1926 to 2010. The dataset provides links between stocks and patents, and an estimated value for each patent. The value is estimated based on the stock market reaction in a three-day window after the announcement of a patent.(Kogan et al., 2017) I deflate the value of patents using the Consumer Price Index (CPI) for all urban consumers from Federal Reserve Economic Data.

Next, I match this dataset with patent information. I download the supplemental information, including application date and granted date, from USPTO.[9] I clean the data using the following steps. First, I only keep patents with nonmissing citation observations. Second, I omit any observations where the patent granted date is before the patent application date, or the patent is granted more than ten years after the initial application.[10] This gives me a dataset with all in-house patents owned by public firms.

Then, I link it with firm-level data. I download firm-level employment data from Compustat and stock data from CRSP. I deflate the market capitalization using CPI. I clean the firms using three criteria: (1) positive employment, (2) located in the United States, and (3) obtained at least one patent during the sample period. My final sample is the universe of patenting US

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[8]Spain has a similar employee invention policy as the US: firms can ask employees to give up ownership of all on-job innovation. Thus, the firm innovation decision in Spain is comparable with US.

[9]For patents granted after 1976, high-quality details are available at PatentsView, a patent data visualization and analysis platform supported by USPTO.

[10]On average, it takes 2.4 years (29 months) for a patent to be granted.

public firms. There are 3,175 unique firms in the sample. For comparison, Akcigit and Kerr (2018) uses census data and reports 23,927 firms in total. The discrepancy is because my sample includes only public firms, while their sample contains private firms as well.

3.1.2 ESEE

I use Survey on Business Strategies (ESEE) to evaluate the model implications on untargeted moments (in-house versus purchased innovations). ESEE is a panel survey of manufacturing firms in Spain conducted by Fundacion SEPI. The firms in the survey are selected through stratified, proportional, and systematic sampling with a random seed. It reports firms' R&D activities, including annual expenses on in-house innovations and external innovation-related activities. Although Spain is admittedly different from the United States, the employee invention law in the two countries are similar regarding who owns the rights of employee inventions. Therefore, the innovation activities in the two countries are comparable. I use this dataset as a qualitative reference for the in-house versus external expenses comparison.

3.2 Moment Estimation

3.2.1 A Statistical Model

The dataset described in the previous section only includes information for public firms. The problem with using public firm data is that the dataset includes large firms disproportionately, which means that the distribution is not representative—particularly it lacks input from small firms. I assume that being a public firm is a random sampling weighted by the firm size.

Because firm size is linear in quality q in the model, I use q to denote the firm size. The public firm size follows distribution $g(q)$ (Figure 1) while the population distribution of all innovative firms is $f_q(q)$. I obtain $g(q)$ from the public firm data and need to estimate $f_q(q)$. Assume that for a firm size q, the probability of being public is $p(q)$, which only depends on firm size q. I estimate $p(q)$ parametrically using the generalized method of moments (GMM). The relationship between $g(q)$ and $f_q(q)$ is

$$f_q(q)\, p(q) = g(q)\, \mathbb{E}(p(q)),\tag{41}$$

where $\mathbb{E}(p(q))$ is the share of public firms among all patenting firms:

$$\mathbb{E}(p(q)) = \int f_q(q)\, p(q)\, dq.\tag{42}$$

There are 23,927 patenting firms in the economy among which 3,175 are public. Therefore,
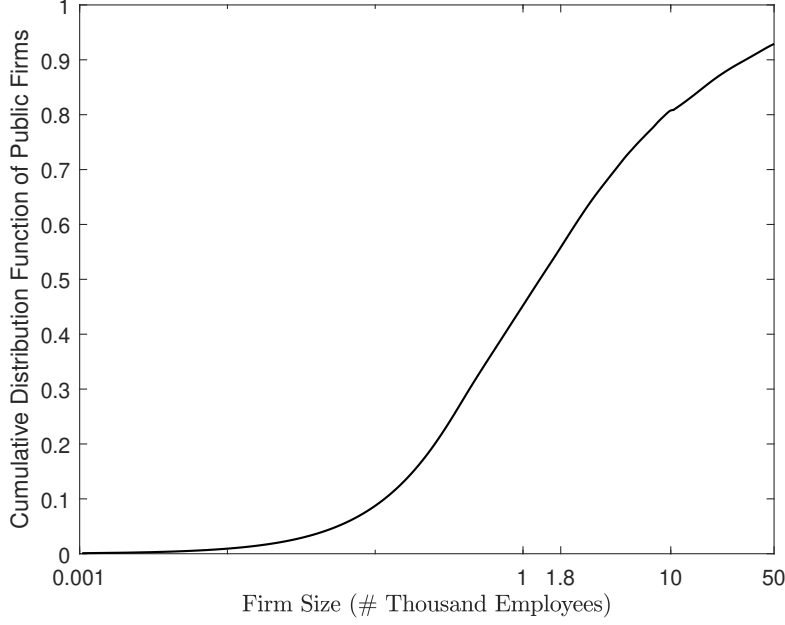
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Figure 1: Size Distribution of Public Innovative Firms

$\mathbb{E}\left(p\left(q\right)\right) = 0.13$. I assume that $p\left(q\right)$ takes the form of the cumulative distribution function of a shifted gamma distribution: $p\left(q\right) = \Gamma\left(q + q_0, \Gamma_a, \Gamma_b\right)$. Then I use a generalized method of moments (GMM) to estimate the parameters $\left(q_0, \Gamma_a, \Gamma_b\right)$.

There are five moments in the data. First, because $f_q\left(q\right)$ is probability distribution function, by definition, it satisfies

$$\int_0^\infty f_q\left(q\right) dq = 1, \tag{43}$$

which gives the first condition that $p\left(q\right)$ must meet:

$$\int_0^\infty \frac{g\left(q\right)}{p\left(q\right)} dq = \frac{1}{\mathbb{E}\left(p\left(q\right)\right)}. \tag{44}$$

Second, the average employee size of all patenting firms is 1,805 (Akcigit and Kerr, 2018). Set the unit of q to be 1,000:

$$\mathbb{E}\left(q\right) = 1.805, \tag{45}$$

It yields the moment below:

$$\int_0^\infty q \frac{g\left(q\right)}{p\left(q\right)} \mathbb{E}\left(p\left(q\right)\right) dq = 1.805. \tag{46}$$

Table 1: Parameter Values Used in the Statistical Model

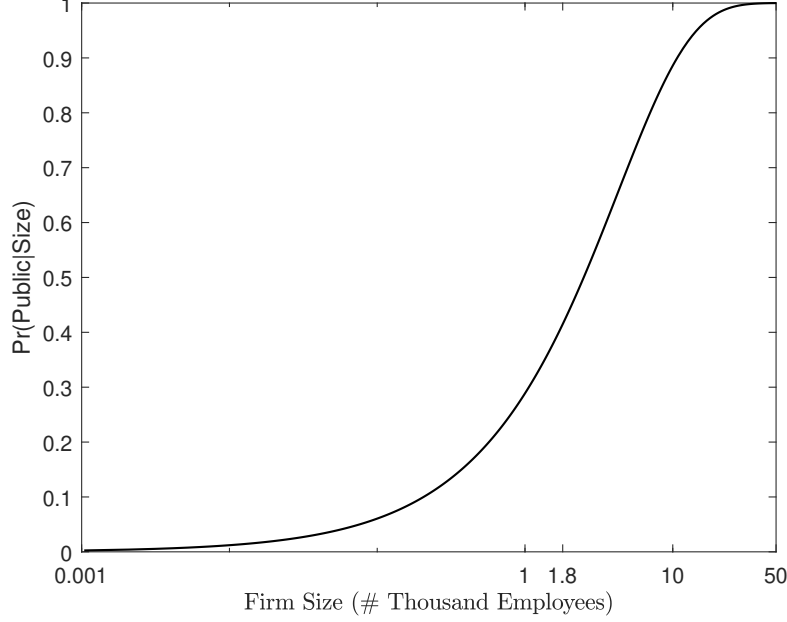| q_0	Γ_a	Γ_b
6.8980×10^{-5}	0.7046	6.0645

Figure 2: Probability of Being Public Conditional on Firm Size

Notes: This figure plots the estimated probability function $p(q)$.

The rest of the moments are the three quartiles. The nth quartile of the population q_n satisfies

$$\int_0^{q_n} f_q(q)\, dq = \frac{n}{4}, n = 1, 2, 3. \tag{47}$$

The quartiles of size of all patenting firms are approximately 17, 70, and 370 employees (Akcigit and Kerr, 2018). Therefore, it gives three moment conditions:

$$\int_0^{q_n} \frac{g(q)}{p(q)} \mathbb{E}(p(q))\, dq = \frac{n}{100}, n = 1, 2, 3. \tag{48}$$

The five moments described in Equation (44), (48), and (46) overidentified the parameters. I set up the estimation so that the parameters must satisfy the first moment and assign uniform weights to the other four moments. The estimated parameters are reported in Table 1 and the estimated probability of being public is in Figure 2

Combining the size distribution of public firms in Figure1 and the probability of being public in Figure 2, Figure 3 reports the estimated distribution of firm sizes. The estimated moments of the firm size distribution are shown in Table 2.
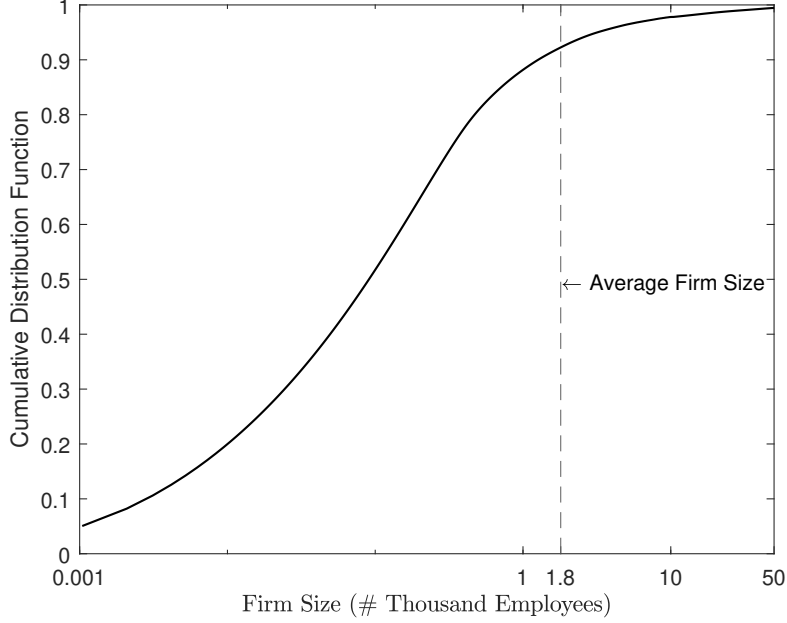
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Figure 3: Estimated Firm Size Distribution of Innovative Firms

Notes: This is estimated using the public firm data and the statistical model.

Table 2: Model Fit for Key Moments of the Statistical Model

Moments	Data	Model
Share of firms with fewer than 17 employees	0.25	0.25
Share of firms with fewer than 70 employees	0.50	0.45
Share of firms with fewer than 370 employees	0.75	0.76
Average firm size	1,805	1,450

3.2.2 Innovation Distribution

I estimate the patent distribution across firms as an empirical counterpart of the innovation distribution, which is used as moments in the quantitative analysis. Because the public firm data have only limited the observations of firms with a few employees, one concern is the sample of small public firms suffers from selection bias. That is, the reason why a small firm goes public is correlated with its innovations. Therefore, I only use the statistical model to estimate the share of patents held by firms with more than 500 employees and assume that the share of patents held by firms with fewer than 500 employees is 10%, as suggested by the literature (Figueroa and Serrano, 2019). The estimated cumulative share of patents is reported in Figure 4, and some moments are in Table 3. From the table, the 60th percentile is about 50,000 employees per firm.

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Table 3: Estimated Cumulative Share of Patents by Firm Sizes Using the Statistical Model

Firm Size (Thousand Employees)	Cumulative Share
0.5	0.10
5	0.30
10	0.36
20	0.44
50	0.59
100	0.76

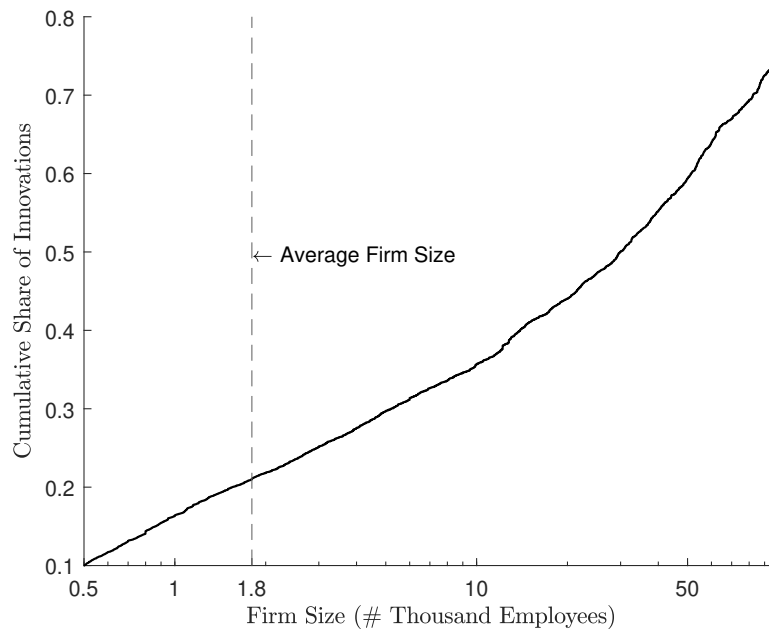Figure 4: Estimated Innovation Distributions across Innovative Firms (with more than 500 employees)

Notes: This figure shows the cumulative share of innovations for firms with more than 500 employees, estimated using the public firm data and the statistical model. The cumulative share of innovations in firms with fewer than 500 employees is set to 10% according to Figueroa and Serrano (2019).

3.3 Calibration

In this section I calibrate the model. To solve the model, I use the following functional form assumptions. The cost function is quadratic:

$$R\left(e_I\right) = \frac{e_I^2}{2}. \tag{49}$$

The probability of having complementarity with an innovation is

$$h\left(\tilde{q}\right) = k_1 - k_2 \exp\left(-\eta \tilde{q}\right), k_1 \geq k_2 > 0, \eta > 0. \tag{50}$$

The step size distribution of an innovation follows Pareto distribution:

$$\Phi\left(z\right) = 1 - \frac{m^\alpha}{z^{\alpha+1}}. \tag{51}$$

The type distribution of inventors follows beta distribution:

$$\Psi\left(\theta\right) = \mathcal{B}\left(\theta; \beta_a, \beta_b\right). \tag{52}$$

The business shock δ is a random draw from a truncated normal distribution:

$$\Xi\left(\delta\right)\left(\delta \in [-1, 1], \mathbb{E}\left(\delta\right) = 0, \text{var}\left(\delta\right) = \sigma_\delta^2\right) \tag{53}$$

The model has 16 parameters, which are listed in Table 4. The parameters (γ_L, γ_H, m) cannot be identified separately, so I normalize γ_L to 1.

The discount rate ρ is set to 2%. I use β to target the firm profitability, defined as $\frac{\pi_j}{y_j p_j}$, which is 10.9% for the sample period between 1982 to 1997 (Akcigit and Kerr, 2018). In the model, the profitability equals β. Thus, I set $\beta = 10.9\%$. τ is 6.5%, the average annual entry rate of innovative firms.

The entry rate is estimated as follows. I obtain the annual entry rate for the whole economy from Decker et al. (2016) which is about 11.6%. To estimate the entry rate for innovative firms, I use two other measures from the same paper: the share of employment at young firms for the whole economy (14.3%) and the information industry (8.0%). I calculate the ratio between the two and use this ratio as a proxy for the ratio between the economy-wide entry rate, and the innovative firm entry rate is the same as the employment share because the information industry constitutes a large portion of innovation firms. Then combining the ratio and the economy-wide entry rate, I estimate the entry rate for innovative firms as 6.5%. Meanwhile, σ_δ^2 is related to the firm growth rate volatility. The median standard deviation of firm growth

Table 4: List of Parameters Used in the Model

Parameter	Description	Identification
β	Quality share in final goods	External calibration
ρ	Discount rate	External calibration
τ	Exit rate	External calibration
σ_δ^2	Business shock arrival rate	External calibration
λ_0	Exogenous entry	Indirect inference
κ	Avg quality of exogenous entrants	Indirect inference
μ	R&D arrival rate multiplier	Indirect inference
η	Curvature of Pr(complementarity)	Indirect inference
k_1	Pr(complementarity|emp>500)	Indirect inference
k_2	Pr(complementarity|emp>500)	Indirect inference
	-Pr(complementarity|start-up)	
m	Scale para of step size dist	Indirect inference
α	Shape para of step size dist	Indirect inference
β_a, β_b	Shape para of inventor dist	Indirect inference
γ_L	Scale of R&D without complementarity	Normalized to 1
γ_H	Scale of R&D value with complementarity	Indirect inference

Table 5: External Calibrated Parameters

ρ	β	τ	σ_δ^2
0.02	0.109	0.065	0.14

rates within 10 years is about 16% to 20% for all firms (Comin and Philippon, 2005). Using public patenting firms, the results are similar. I set σ_δ to 14%. The parameter values are listed in Table 5

There are 11 remaining parameters to be estimated. I identify these parameters using a simulated method of moments approach in the spirit of Lentz and Mortensen (2008), which is widely used in the growth literature to match firm-level evidence (Lentz and Mortensen, 2016; Acemoglu et al., 2018; Akcigit and Kerr, 2018). I calculate the model implied moments based on the model and compare them to the data-generated moment to minimize

$$min \sum_{i=1}^{11} \left(\frac{model\,(i) - data\,(i)}{data\,(i)} \right)^2. \tag{54}$$

The moments and the corresponding parameters are described below.

The probability of selling an innovation—The secondary market is an essential feature of the model. To characterize the secondary market. I match three moments related to the probability of selling an innovation: the probability of a big firm[11] selling its innovation, the

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[11]Following USPTO, a big firm is defined as a firm with more than 500 employees.
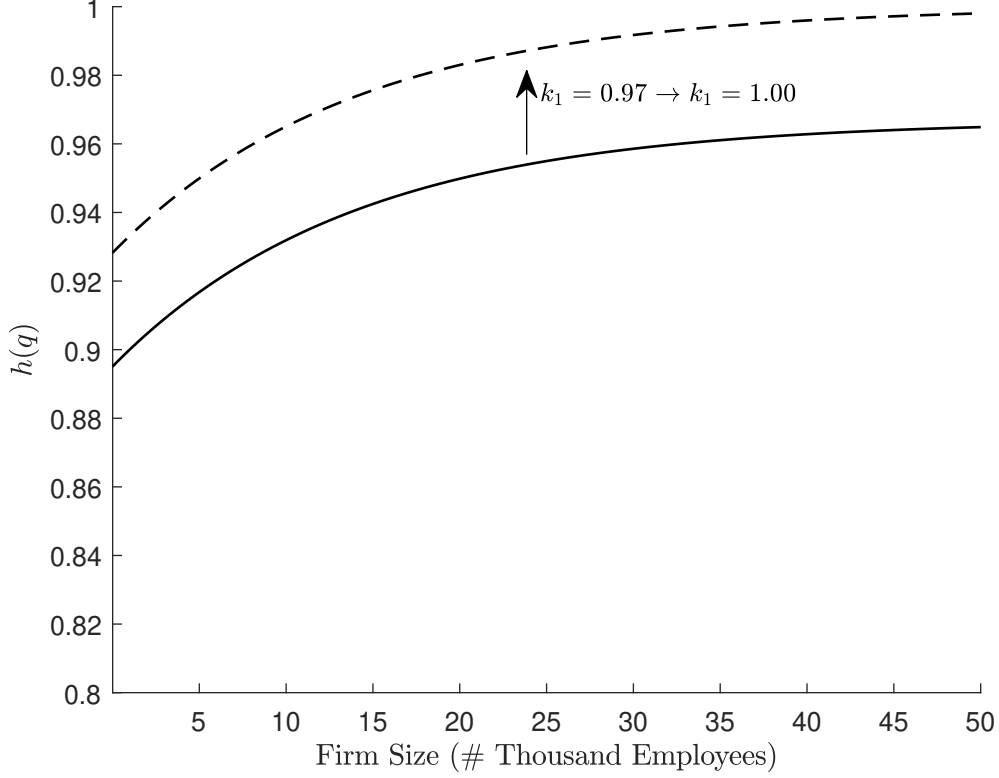
Figure 5: The Impact of k_1 on $h(q)$

start-up buyout rate, and the average probability of a firm selling its innovation. The probability of selling is directly affected by the probability of having complementarity. The first moment is 5.1% (Figueroa and Serrano, 2019) and it corresponds to k_1. As shown in Figure 5, when k_1 increases (decreases), it shifts up (down) the probability of having complementarity $h(\tilde{q})$. The second moment is corresponding to k_2. Figure 6 shows that $k2$ mainly affects the y-intercept of $h(q)$. Therefore, k_2 directly links to the startup-up buyout rate (9.4%, from Gao et al. (2013)), which in my model is the probability of a start-up selling its innovation. The third is matched to α, the shape factor of the invention step size distribution. When α is large, then the uncertainty in the inventor's income stream is dominated by operational reasons that are unrelated to her own innovating effort. As α approaches 2, the innovation-related uncertainty goes up, which affects small to medium-small-sized firms disproportionately. Thus, I adjust α to match the average probability of a firm selling a patent. In the data, the probability is about 5.4% (Figueroa and Serrano, 2019).

Innovation distribution across firms—I obtain the innovation distribution across firms using the public firm data, patent data from USPTO, and a statistical estimation in Section 3.2.2. I target four data points from the distribution to pin down four parameters: η, μ, β_a, and β_b. First, η governs the curvature of the complementarity probability. For firms large enough,
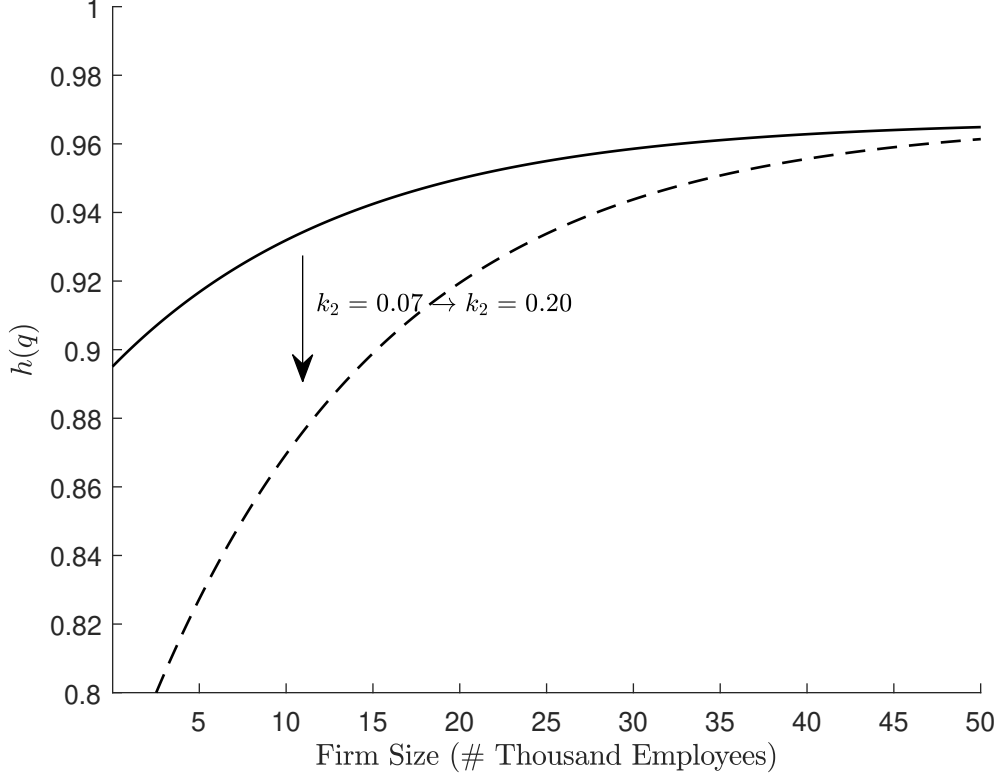
Figure 6: The Impact of k_2 on $h(q)$

because they all offer sufficiently low equity shares, *eta* governs the firm-inventor matching. When η increases, it acts as if those firms are larger by the same factor regarding the complementarity probability. The impact is shown in Figure 7. *eta* mainly affects the slope of the cumulative share of innovations by firm size. For example, if η increases by 100%, then the 60th percentile of the size of all patenting firms shrinks by half. So I match it to the 60th percentile of size (employment size=51,250, which is $28.47\bar{q}$). The second parameter μ disciplines the share of innovations held by firms with fewer than 2,000 employees. μ affects both the value of innovating and the innovation-related uncertainty linearly. It affects the innovation value in all firms uniformly; it acts as a normalization. However, it has a disproportional impact on the equity risk across firms, since it only affects the innovation-related risk. As a result, in the relevant parameter space, μ mainly governs the share of innovation held by small- to medium-sized firms. I match it to the share of innovations held by firms with fewer than 2,000 employees (22%). The last two parameters β_a and β_b determine the inventor type distribution $\Psi(\theta)$. β_a affects the amount of low effort-sensitive inventors whereas β_b affects the amount of high effort-sensitive inventors. Therefore, I use the cumulative share of innovations held by firms with fewer than 5,000 employees (10%) and 500 employees (30%) to match β_a and β_b, respectively.
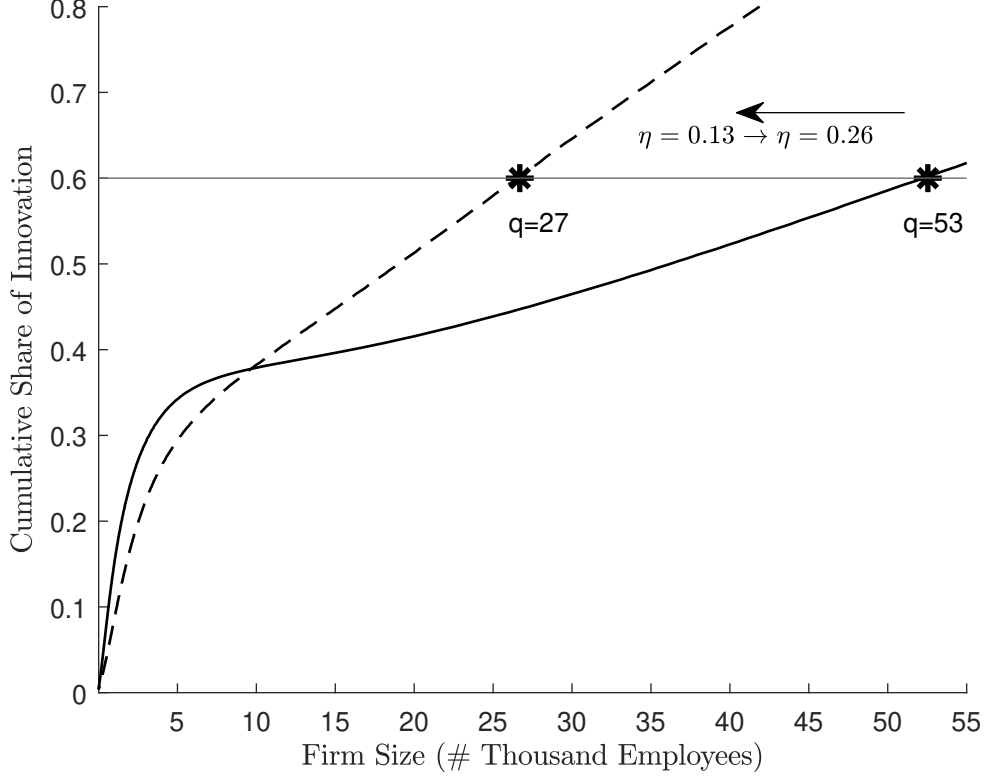
Figure 7: The Impact of η on Innovation Distribution

Average value of an innovation in firms with more than 500 employees—In the data, I use
the average value of a patent in firms with more than 500 employees as a proxy. The patent
value is estimated using the stock market reaction by Kogan et al. (2017). I first keep all firms
with more than 500 employees. Then I calculate the average patent value per firm and weigh
it by the statistical model developed in Section 3.2.2. The average value per patent is about
3.6% of the average market capitalization. In the model, given the measure of firm N_F, the
size factor m of the step size distribution determines the average value per innovation. It pins
down the value of m.

The aggregate growth rate—The amount of firm entry exogenously λ_0, from Equation (34),
determines the measure of firms N_F. Conditional on the innovation value parameter m, λ_0
impacts the aggregate growth rate through N_F, for innovation value is scaled by the total
technology stock $(Q = \bar{q}N_F)$. Hence, I set λ_0 so that the aggregate growth rate g is 2%.

The average growth rate—In the model, it is mainly affected by κ. When κ goes up, holding
the aggregate growth rate constant, the measure of firms N_F needs to adjust. Therefore,
κ affects the innovations per firm and consequentially firm-level growth rate. The average

Table 6: Directly Calibrated Parameters Given Indirect Inference Results

λ_0	k_2	m
0.024	0.072	0.021

Table 7: Indirect Inference Calibrated Parameters

κ	μ	η	k_1	α	β_a	β_b	γ_H
0.013	2.28	0.13	0.97	2.0016	0.086	1.32	2.05

productivity growth rate, based on Equation (14), can be written as:

$$\mathbb{E}\left(\frac{\dot{q}_j}{q_j}\right) = \mathbb{E}\left(\frac{\dot{l}_j}{l_j}\right) + g, \tag{55}$$

where $\mathbb{E}\left(\frac{\dot{l}_j}{l_j}\right)$ is the average employment growth controlling the working population constant and g is the aggregate growth rate. The average employment growth is 7.4%(Akcigit and Kerr, 2018) and the total employment growth is 2.1% (US Bureau of Labor Statistics). Therefore, the average employment growth controlling the working population is about 5.3%, which gives the average productivity growth rate equal to 7.3%.

Firm growth versus firm size regression—γ_H measures the value of having complementarity with an invention. When γ_H goes up, having complementarity is more valuable, which affects the inventor's trade-off. It is related to the growth rate versus firm size regression (Akcigit and Kerr, 2018):

$$g_{ft} = \eta + \beta_g ln\left(qft\right) + \epsilon_{ft}.$$

The empirical coefficient from the literature is $\beta_g = -0.035$.

The results are shown in Table 7

4 Results

The calibration moments are reported in Table 8. Overall, my model matches closely the targeted moments. Next, I discuss model features in more detail.

4.1 Optimal Contracts

Figure 8 shows what share of total risk is the innovation-related risk for firms of different sizes. As a result, it is more difficult for large firms to incentivize inventors without exposing them to unrelated risks. The is one optimal contract for the inventor each firm actually hires in equilibrium. I plot the contracts across firms in Figure 9. The top panel plots the resulting

Table 8: Model Fit for Key Moments—Targeted Moments

Moment	Data	Model	Parameter it informs
Profitability	0.109	0.109	β
Discount rate	0.020	0.020	ρ
Entry rate	0.065	0.065	τ
Firm growth volatility	0.17	0.18	σ_δ^2
Aggregate growth rate	0.02	0.020	λ_0
Average patent value	0.036	0.036	m
start-up buyout rate	0.094	0.094	k_2
Pr(Sell|emp>500)[1]	0.051	0.049	k_1
Pr(Sell)	0.054	0.053	α
Growth-size relation β_g[2]	-0.035	-0.036	γ_H
% Inno, emp< 500[3]	0.10	0.080	β_b
% Inno, emp< 2,000	0.22	0.24	μ
% Inno, emp< 5,000	0.30	0.34	β_a
60th pctl $\tilde{q}$ weighted by R&D	28.47	29.25	η
Average growth rate	0.073	0.071	κ

[1] A firm with more than 500 employees is defined as A "Big firm", according to USPTO.

[2] β_g is the coefficient of the growth-size regression.

[3] The % Inno is the cumulative density function of innovations created in firms with less than certain employment. For example, "% Inno, emp< 500" means the share of innovations, among all innovations created in this period, that are invented in a firm with fewer than 500 employees.

effort level e for the inventor works for a firm $\tilde{q}$. The bottom panel shows that, in equilibrium, the equity $a(\tilde{q})$ decreases with the firm size. More effort-sensitive inventors work for smaller firms, receive more equity and choose a higher effort level.

4.2 The Distribution of Inventors across Firms

Figure 10 shows that an inventor with a more effort-sensitive idea chooses a smaller firm. Namely, inventors with low θ are less effort-sensitive and work for big firms; instead of working for an incumbent firm, inventors with θ higher than a threshold $\bar{\theta}$ join a start-up.

When choosing where to work, an inventor faces the trade-off between chances and value. The chance to successfully innovate decreases with firm sizes, because, as shown in Figure 9, smaller firms are better at incentivizing inventors to exert more effort. The value of an innovation increases with firm size, for larger firms are more likely to implement the innovation efficiently. When an inventor's idea is more effort sensitive, the incentive channel weighs more. Therefore, a more effort-sensitive inventor chooses to work for a smaller-size firm.
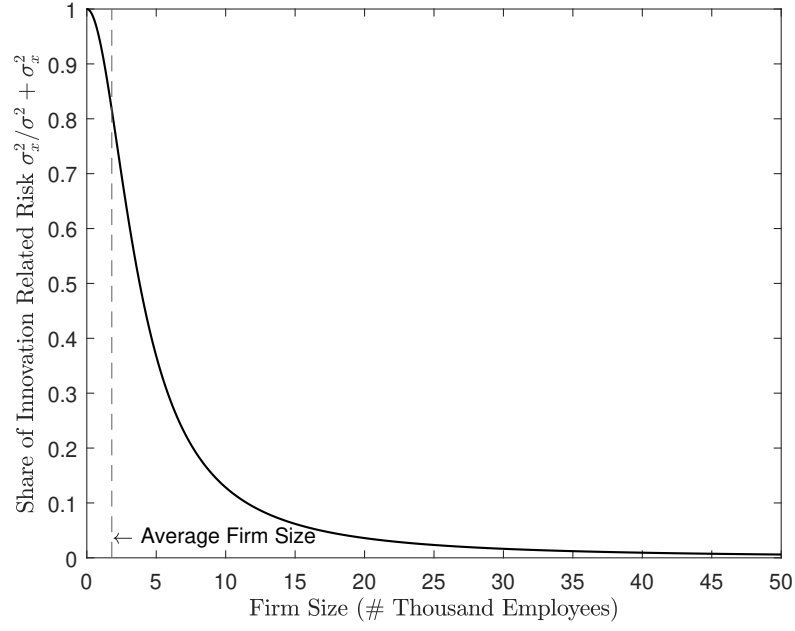
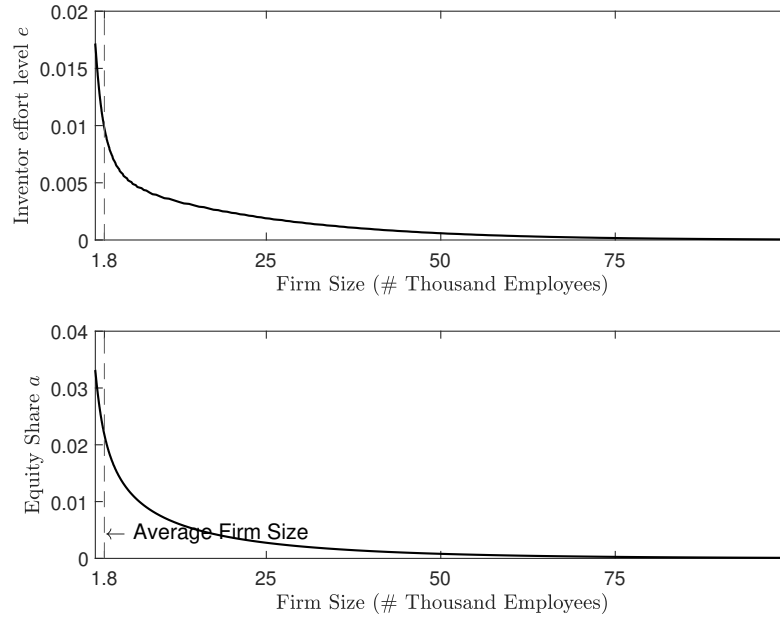Figure 8: The Innovation-Related Risk across Firm Sizes

Figure 9: The Equilibrium Effort and the Optimal Contract

Notes: The figures report the actual effort level and the optimal contract for each equilibrium firm-inventor matching.
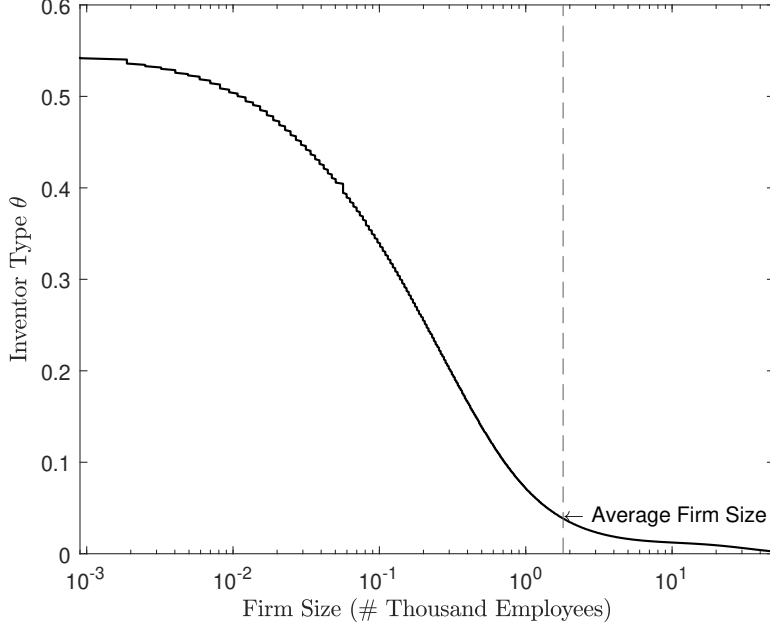
Figure 10: The Relationship between Inventors and Firms

Notes: Inventors with $\theta > 0.5$ work for start-ups with $q = 0$ are not shown in this figure, because the x-axis is in log.

4.3 Untargeted Moments

I next compare the model with untargeted features in the data.

Cumulative shares of inventions by firm sizes—I use four moments from the innovation distribution to calibrate the model; the rest is used as untargeted moments. The comparison is shown in Figure 11, and some moments are reported in Table 10. In general, the model is able to match the overall pattern of the cumulative shares. It overestimates when the firm size is medium-small to medium and large and underestimates when the size is around medium to medium-large. This is mainly because of the assumption that θ follows a beta distribution. Figure 12 shows the probability distribution function of Figure 11. The model follows a similar pattern as the data.

Growth rate gap between 90th percentile and 50th percentile—Decker et al. (2016) shows that in high-tech industries (defined by Heckler (2005)) the growth rate difference between 90th percentile and 50th percentile is about 31%. In the model, I simulate 5 million firms for one year to find the annual growth rate for each firm. Then I only keep firms that have innovations within this year. The growth rate of firms that die within this time interval is defined as -1, following the literature. I rank the firms according to their growth rate and find the growth rate gap between the 90th percentile and 50th percentile.

The comparison of untargeted moments is listed in Table 10. Overall I am able to match the data; specifically, we match the 90th–50th growth rate gap and cumulative share held by

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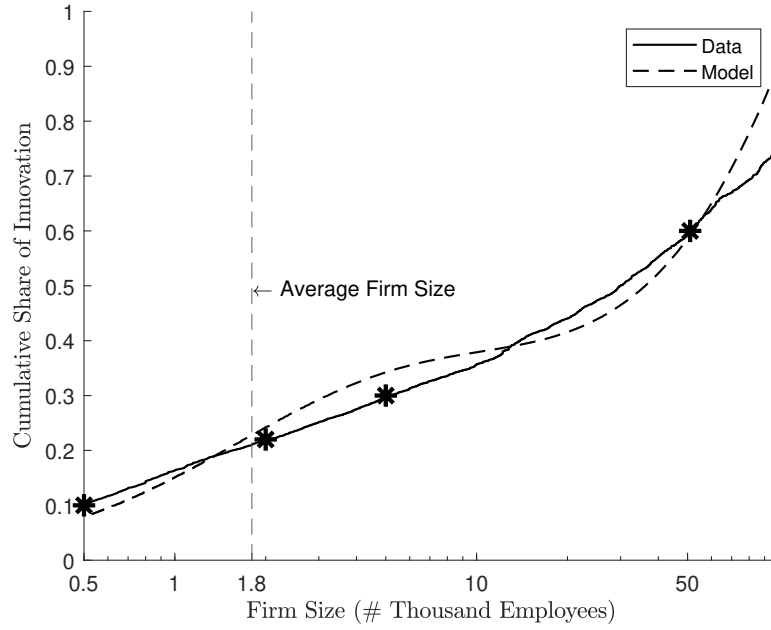
Figure 11: Cumulative Share of Innovations by Firm Sizes

Notes: The solid line plots the cumulative share for firms with more than 500 employees, the same as Figure 4. The stars represents the data points I used to calibrate the model.
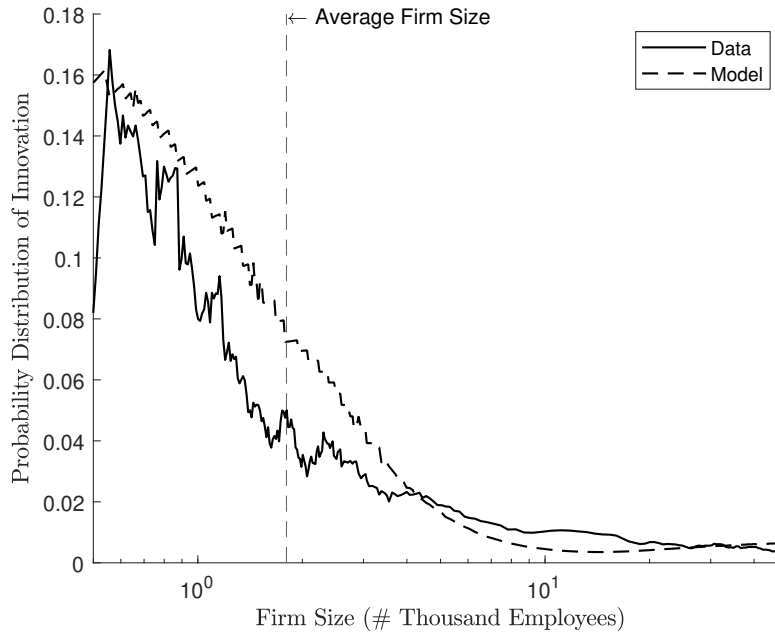
Figure 12: Probability Density of Innovation

Notes: This figure shows the probability distribution function of innovations for firms with more than 500 employees both in data and in model. The data are estimated using both public firm data and the statistical model. The model is estimated using the benchmark calibration.

medium- to medium-large-size firms closely.

In-house versus outsourcing choice—In addition, I report the comparison between in-house versus outsourcing choice both in the data and in the model in Table 11. The data moments are obtained from ESEE in 1990 and 1994. The first wave of ESEE is in 1990. I run four specifications for 1990 and 1994 according to:

$$y_{ft} = \text{constant}_t + \text{coeff}_t \times \ln\left(\text{Sales}_{ft}\right) + \epsilon_{ft}, t = 1990, 1994, \tag{56}$$

where y_{ft} is the dependent variable of firm f and time t. Four dependent variables are (1) total R&D expenditure to sales ratio, (2) internal R&D expenditure to sales ratio, (3) total R&D expenditure normalized by average sales, and (4) internal R&D expenditure normalized by average sales. The first two measure the innovation intensity, and the rest measure the absolute level of innovation expenditure. The results are reported in Table 9. It shows that both R&D and internal R&D expenditure intensity decreases with firm size and the absolute levels increase with firm size. Though the data are from Spain and the model is calibrated to the US the model can match at least the direction of the coefficients, which is reassuring because it captures some of the trade-offs between in-house and outsourcing.

4.4   Growth Decomposition

I now use the model to document the sources of growth. The growth rate in Equation (36) can be written as

$$
g = \overbrace{-\tau\left(1-\kappa\right)}^{\text{Destruction}}
$$
$$
+ \gamma_H \Phi\left(\hat{z}\right) \mathbb{E}\left(z | z \leq \hat{z}\right) \overbrace{\int \left[1 - h\left(q^*\left(\theta\right)\right)\right] \lambda_\theta^* \psi\left(\theta\right) d\theta}^{\text{Secondary market}}
$$
$$
+ \mathbb{E}\left(z\right) \overbrace{\int \left\{\gamma_H h\left(q^*\left(\theta\right)\right) + \gamma_L\right\} \lambda_\theta^* \psi\left(\theta\right) d\theta}^{\text{Primary market}}. \tag{57}
$$

It depends on three forces: (1) destruction and replacement with an entrant subtracted the innovation effect, (2) firms purchasing innovation on the secondary market, and (3) firms hiring inventors from the primary market to innovate in-house.

My model estimates that the average quality decreases by 6.41 percentage points due to the destruction and replacement process. For exogenous entries, this is because new entrants start at a quality on average lower than the firms who exit due to exogenous destruction.

Average quality increases by 8.25 percentage points because of innovation in-house. On

Table 9: Firm R&D Investment Regressions[1]

	(1) $\frac{\text{R\&D}}{\text{Sales}}$	(2) $\frac{\text{Internal}}{\text{Sales}}$	(3) $\frac{\text{R\&D}}{\text{Avg Sales}}$	(4) $\frac{\text{Internal}}{\text{Avg Sales}}$
Panel A: Year = 1990				
ln(Sales)	−0.009***	−0.006***	0.016***	0.012***
	(−3.93)	(−4.75)	(8.24)	(8.00)
Constant	0.174***	0.118***	−0.246***	−0.182***
	(4.61)	(5.62)	(−7.63)	(−7.40)
Observations	869	869	869	869
R-Squared	0.017	0.025	0.073	0.069
Panel B: Year = 1994				
ln(Sales)	−0.001**	−0.001*	0.023***	0.013***
	(−2.02)	(−1.79)	(7.40)	(6.71)
Constant	0.042***	0.030***	−0.361***	−0.203***
	(3.53)	(3.14)	(−6.96)	(−6.28)
Observations	801	801	801	801
R-Squared	0.005	0.004	0.064	0.053

[1] The dependent variable used for each column is listed at the top of the column. The data are from Survey on Business Strategies (ESEE) of the SEPI Foundation. The t statistics are in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 10: Model Fit for Untargeted Moments

Moment	Data	Model
Growth gap: 90th pctl to 50th pctl[1]	0.31	0.38
% Inno, emp< 20	0.44	0.41
% Inno, emp< 50	0.59	0.59
% Inno, emp< 100	0.76	0.89

[1] The difference between the 90th percentile of the firm-level growth rate and the median growth rate.

Table 11: Model Fit for Untargeted Moments Using Spanish Data[1]

DV	$\frac{\text{R\&D}}{\text{Sales}}$	$\frac{\text{Internal}}{\text{Sales}}$	$\frac{\text{R\&D}}{\mathbb{E}(\text{Sales})}$	$\frac{\text{Internal}}{\mathbb{E}(\text{Sales})}$
1990	−0.009	−0.006	0.016	0.012
1994	−0.001	−0.001	0.023	0.013
Model	−0.024	−0.024	0.0026	0.0020

[1] The data are from ESEE of the SEPI Foundation, Spain. I use the Spanish dataset because it shares a similar employee invention law as the US, and provides detailed information on R&D investment.

Table 12: Counterfactuals[1]

Moment	(1) Benchmark	(2) No Market	(3) Signal	(4) Signal	(5) Subsidy	(6) Subsidy
			$\alpha_\epsilon = 1$ $R^2 = 0.2$	$\alpha_\epsilon = 2.5$ $R^2 = 0.6$	1%	5%
Growth rate	2.00	-0.166	$+0.013$	$+0.087$	$+0.013$	$+0.016$
% Inno in startups	0.33	-0.330	$+0.390$	$+4.900$	$+0.115$	$+0.897$
% Inno, emp< 500	7.66	-6.350	-0.430	-4.880	-0.112	-1.285
% Inno, emp$\in [500, 20k)$	33.32	-8.585	$+1.974$	$+7.944$	$+0.811$	$+3.791$
% Inno, emp$\in [20k, 100k)$	47.70	$+5.012$	-0.296	$+3.034$	$+0.825$	$+7.594$
% Inno, emp$\geq 100k$	11.00	$+10.250$	-1.636	-10.996	-1.638	-10.996
Pr(Sell)	5.28	-5.283	$+0.237$	$+0.617$	$+0.042$	$+0.168$

[1] The table reports the **percentage points difference** with respect to the benchmark. For example, the growth rate change is calculated according to $(g_{\text{counter}} - g) \times 100$. The first column reports the benchmark case in percentage points. The second column shows a case where firms cannot trade innovations at all. The third and fourth columns analyze cases in which there is a noisy signal when firms buy and sell innovations. The noise ϵ follows a Pareto distribution with scale parameter 1 and shape parameter α_ϵ. The fifth and sixth columns report the results when there is a one-time innovation transaction subsidy. The subsidy rates are 1% and 5% of the transaction price.

average about 5% of all innovations are sold (Figueroa and Serrano, 2019), and the secondary market adds 0.16 percentage points to the aggregate growth rate. In Section 4.5 I analyze the effects of changes on the secondary market.

4.5 Counterfactual

In this section, I consider counterfactuals about the innovation tradability on the secondary market. The analysis shows that tradability affects both firm-level innovation allocations and the aggregate growth rate. I start with an extreme case when the secondary market no longer exists in Section 4.5.1. Section 4.5.2 analyzes the case if there is a signal about the invention step size. Section 4.5.3 studies the case when innovation transactions are subsidized. Section C.1 shows the results if there is an invention transaction tax.

4.5.1 Shut Down the Secondary Market

Consider a case where there is no secondary market and firms cannot buy or sell innovations. It hurts small firms disproportionately because they are the ones who rely more on the secondary market. The results are reported in the second column of Table 12. The benchmark results are shown in the first column for comparison. Inventors shift to bigger firms. The share of innovations created in start-ups drops by 0.33 percentage points, which is 100%—there are no innovations in start-ups anymore; start-ups cannot attract any inventors. The share of innovations in small firms also decreases dramatically, by 6.35 percentage points. This exercise

speaks to one of the effects of intellectual property rights protection: without protection, firms cannot trade patents and the secondary market does not exist. The model implications are consistent with the empirical observations in Acikalin et al. (2022), which shows that when facing sudden patent invalidation, small firms lose disproportionately. In the contrast to the well-known "small firms innovate" idea, without the secondary market, innovations are created in big firms. For example, quantitatively, the share of innovations created in large firms (with more than 100,000 employees) increases by more than 10.25 percentage points—from 11.00% to 21.25%. The impact of shutting down the secondary market goes beyond firm-level distribution shift—it also has an aggregate implication. The aggregate growth rate decreases by about 0.16 percentage points. The growth rate drops from 2.00% to 1.84%.

The magnitude of the growth rate drop depends on two groups of data moments: the probability of selling an innovation and the share of innovations in small firms. The results are shown in Table 13. If firms are, in fact, more likely to sell an innovation than the data, the growth rate drop will be larger. For example, consider the case where the parameter $k_1 = 0.93$, which means instead of 5.3%, on average 8.5% of innovations are sold if there is a secondary market. Shutting down the secondary market will result in a drop that is 0.1 percentage points higher than before—the growth rate decreases by 0.26 percentage points, compared with the 0.16 percentage points in the baseline calibration. If in addition, more innovations are created in small firms than observed, then the results will be more salient. For example, consider a case in which on average 17% of innovations are sold, and no innovations are created in firms with more than 50,000 employees. Shutting down the secondary market leads to an additional 0.4 percentage points drop in the growth rate—now the growth rate would be 1.44% if there were no secondary market. This is because the impact of tradibility is mainly about innovation reallocation between firms. Both a high probability of selling and a small share of innovations in big firms means that the reallocation has a strong implication for efficiency improvement in which case shutting down the secondary market has a more salient effect.

4.5.2 Signal

Assume there is a publicly available signal χ associated with each innovation z. The signal satisfies

$$\chi = \varepsilon z, \tag{58}$$

Table 13: The Secondary Market's Impact Depends on the Probability of Selling an Innovation
[1]

		(1) Baseline	(2) More Sell	(3) Fewer In Large Firms
Parameters	k_1	0.97	0.93	0.84
	η	0.13	0.13	0.29
Moments	Pr(sell)	5.3%	8.4%	16.5%
	% Inno, emp$\geq 50k$	58.7%	58.8%	100.0%
Counterfactual	Δ g(No Market)	0.16%	0.26%	0.56%

[1] The table reports the counterfactual aggregate growth rate drop when shutting down the secondary market. The first column reports the baseline counterfactual as in Table 12. The second column shows when holding other moments constant, and the probability of selling increases. In the third column, there are fewer innovations in big firms and more trade between firms.

where ε is noise. Assume that ε follows a Pareto distribution, where $f_\varepsilon(\varepsilon) = \frac{\alpha_\varepsilon}{\varepsilon^{\alpha_\varepsilon+1}}, \alpha_\varepsilon > 0$. Buyers use this information to update the innovation step size distribution, which yields:

$$f_{z|\chi}(z|\chi) = \begin{cases} \frac{-\tilde{\alpha}z^{-\tilde{\alpha}-1}}{\chi^{-\tilde{\alpha}}-m^{-\tilde{\alpha}}} & \text{if} z \leq \chi \\ 0 & \text{otherwise,} \end{cases} \tag{59}$$

where $\tilde{\alpha} = \alpha - \alpha_\varepsilon$.

Buyers now use this information to determine their bid price on the secondary market. As before, the buyer chooses a price $\tilde{p}_z(\chi)$ according to the zero profit condition.

$$\int_0^{\max(\hat{z},\chi)} \left[\gamma_H \nu z \tilde{Q} - \tilde{p}_z(\chi) \right] f_{z|\chi}(z|\chi) \, dz = 0. \tag{60}$$

The innovations are sold if they are worth less than the price to the seller. Equivalently, all innovations with step size lower than the threshold $\hat{z}$ are sold, where

$$\gamma_L \hat{z} \nu \tilde{Q} = p_z(\chi). \tag{61}$$

The solution $\hat{z}$ satisfies

$$\frac{\gamma_H}{\tilde{\alpha}-1} \left(m^{-\tilde{\alpha}+1} - \hat{z}^{-\tilde{\alpha}+1} \right) = \frac{\hat{z}}{\tilde{\alpha}} \left(m^{-\tilde{\alpha}} - \hat{z}^{-\tilde{\alpha}} \right), \tag{62}$$

if $\hat{z} \leq \chi$. In this case, the bid price is independent of the signal. Otherwise, it means that the

signal shows the step size is low enough, and the solution $\hat{z}$ satisfies

$$\hat{z} = \frac{\tilde{\alpha}\left(\gamma_H\right)\left[m\chi^{\tilde{\alpha}} - \chi m^{\tilde{\alpha}}\right]}{\left(\tilde{\alpha} - 1\right)\left[\chi^{\tilde{\alpha}} - m^{\tilde{\alpha}}\right]}. \tag{63}$$

Using the updated expression for $\hat{z}$, the firm's problem and the inventor's problem are both the same as in the benchmark model.

To measure the informativeness of a signal, I regress $\ln z$ on the signal $\ln \chi$ and use R^2 as the information's accuracy. In the regression, the R^2 is

$$R^2 = \frac{\text{Var}\left(\ln z\right)}{\text{Var}\left(\ln \chi\right)} = \frac{\alpha_\varepsilon^2}{\alpha^2 + \alpha_\varepsilon^2}, \tag{64}$$

which increases with the shape parameter α_ε^2 of the signal distribution. It implies that when the signal is more accurate, R^2 is higher, and consequentially, we can better infer z from the signal.

I consider two different signal levels: $\alpha_\varepsilon = 1, 2.5$, and the corresponding R^2 are 0.2 and 0.6, , which means the signal informativeness level is low and high, respectively. The results are reported in the third and forth columns of Table 12.

The first row reports the percentage change in the aggregate growth rate. When $R^2 = 0.2$, the growth rate increases by 0.01 percentage points while it increases by 0.09 percentage points when $R^2 = 0.6$. Namely, when the R^2 of the signal is 0.6, the growth rate increases from 2.00% to 2.09%. This is because that more information means less severe adverse selection and consequentially it is easier for firms to sell their innovations. Therefore, the economy is more efficient and the growth rate is higher.

The second to the sixth rows of Table 12 report the changes in the innovation distribution across firms. The signal has heterogeneous effects on inventors. The signal has a significant impact on the share in start-ups. Quantitatively, the share of innovations created in start-ups is more than double, from 0.33% to 0.72%, with a signal as noisy as $R^2 = 0.2$. When the signal is more informative, with $R^2 = 0.6$, the share of innovations in start-ups increases by 4.90 percentage points. Meanwhile, the share of innovations created in medium-small (with 500–20,000 employees) and medium-large firms (with 20,000–100,000 employees) also increases with the signal strength; the share of innovations in both small firms (with fewer than 500 employees) and large firms (with more than 100,000 employees) decreases with the signal strength.

The heterogeneity is because the signal effect is two-fold on the inventor's choice. Figure 13 reports the change in the mapping between inventors and firms. For inventors with the effort sensitivity level $\theta > 0.15$ and $\theta < 0.015$, more precise information leads to them working for smaller firms. On the contrary, for inventors with the effort sensitivity level $0.015 < \theta < 0.15$,

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the more informative the signal is, the bigger firm they choose. This is because the signal precision affects the inventor's decision in two ways. First, because firms can sell unmatched innovations more easily, the expected value of an innovation decreases more slowly with the firm size. Namely, for each innovation, there is a smaller value gap of being commercialized in different sizes. Second, innovations contribute more to firm's equity. The innovative risk takes a larger share in a firm's total uncertainty profile, as shown in Figure 14. The increasing innovative uncertainty has two impacts: the firm equity variance is higher, and more uncertainty in the firm equity are related to the inventor's own choice. The former channel restricts the start-ups' and small firms' ability to offer equity since there is more demand for risk-sharing. The latter channel, on the contrary, enables medium- to large-sized firms to offer more equity by lowering the background noise level. Figure 15 confirms the mechanism. The share of equity offered by start-ups drops from 4% to 3.8% when there is a strong signal ($R^2 = 0.60$). And the slope is flatter in the signal's informativeness, especially at the left end. Therefore, the capacity to incentivize inventors drops more slowly with the firm size, especially for small firms.

The relative strength of the two forces determines the influence of the signal. For inventors with $\theta > 0.15$ and $\theta < 0.015$, the increasing innovation value is more important. Hence, inventors work for smaller firms. For the rest, the incentive improvement plays a more critical role, and inventors shift to bigger firms. As a result, more innovations are created in start-ups and medium-sized firms.

4.5.3 Subsidy

This section considers when the transactions on the secondary market are subsidized. It is a fixed transfer Γ per transaction, which means that the buyer now pays $(p_z - \Gamma)$ to purchase an innovation. The rest of the settings are the same as in the benchmark model. The buyer's zero-profit condition is

$$\int_0^{\hat{z}} \left(\gamma_H \nu z Q - p_z + \Gamma \right) \phi(z) dz = 0, \tag{65}$$

where $\hat{z} = \frac{p_z}{\gamma_L \nu Q}$ is the step size threshold; a firm agrees to sell an innovation when it does not have complementarity and the step size is lower than $\hat{z}$. Define $\psi \equiv \frac{p_z}{\gamma_L \nu Q}$. The zero-profit condition can be re-written as

$$\int_0^{\hat{z}} \left(\gamma_H z - \gamma_L \hat{z} + \gamma_L \psi \right) \phi(z) dz = 0. \tag{66}$$

With the subsidy, it is less costly to buy innovations. Therefore, $\hat{z}$ is higher than in the benchmark case, and a firm is more likely to sell an innovation. Adding a transaction tax has the opposite effect of a subsidy. The counterfactual results of a transaction tax are reported in
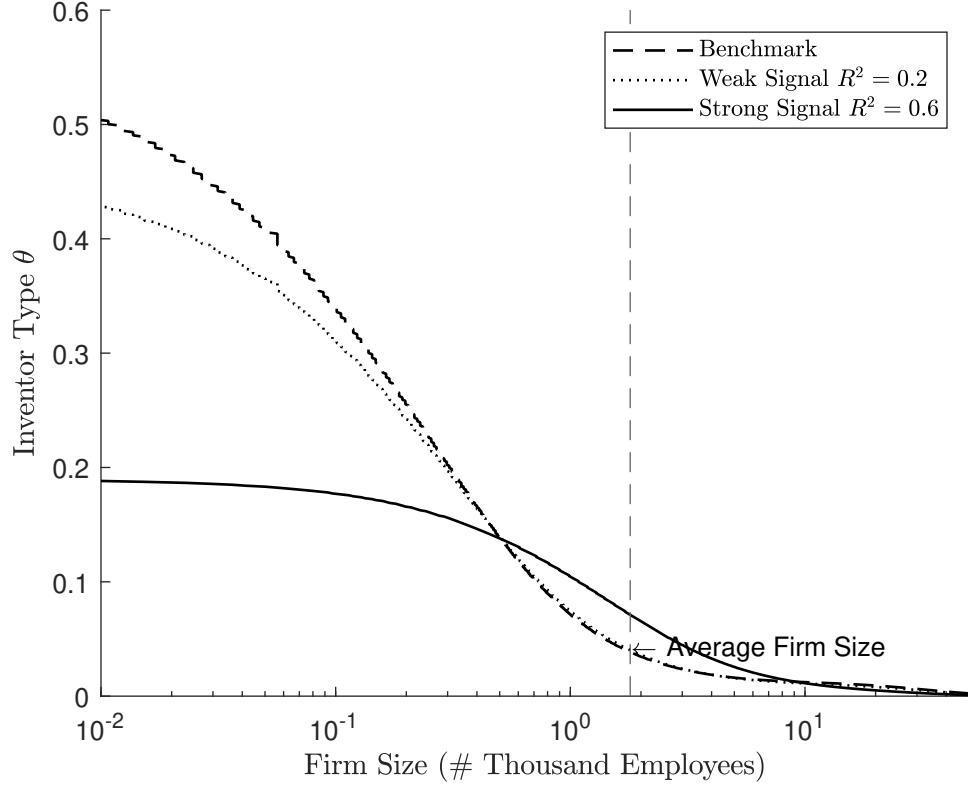
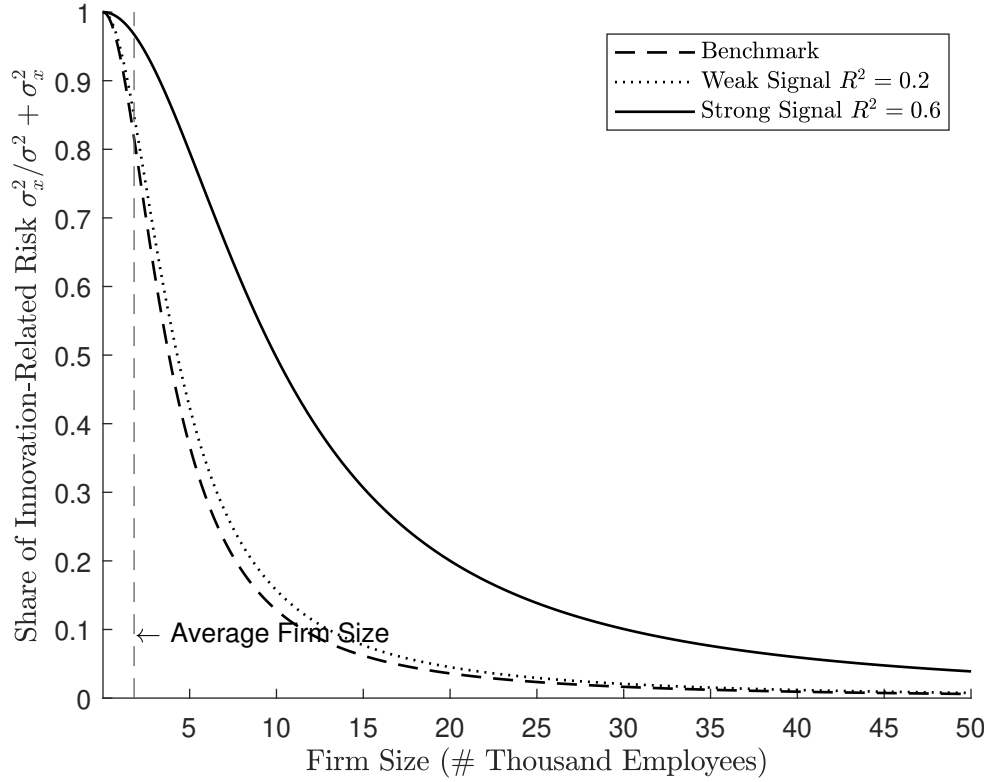Figure 13: The Mapping between Inventors and Firms

Figure 14: The Innovation-Related Risk Takes a Larger Share When the Signal Is Strong
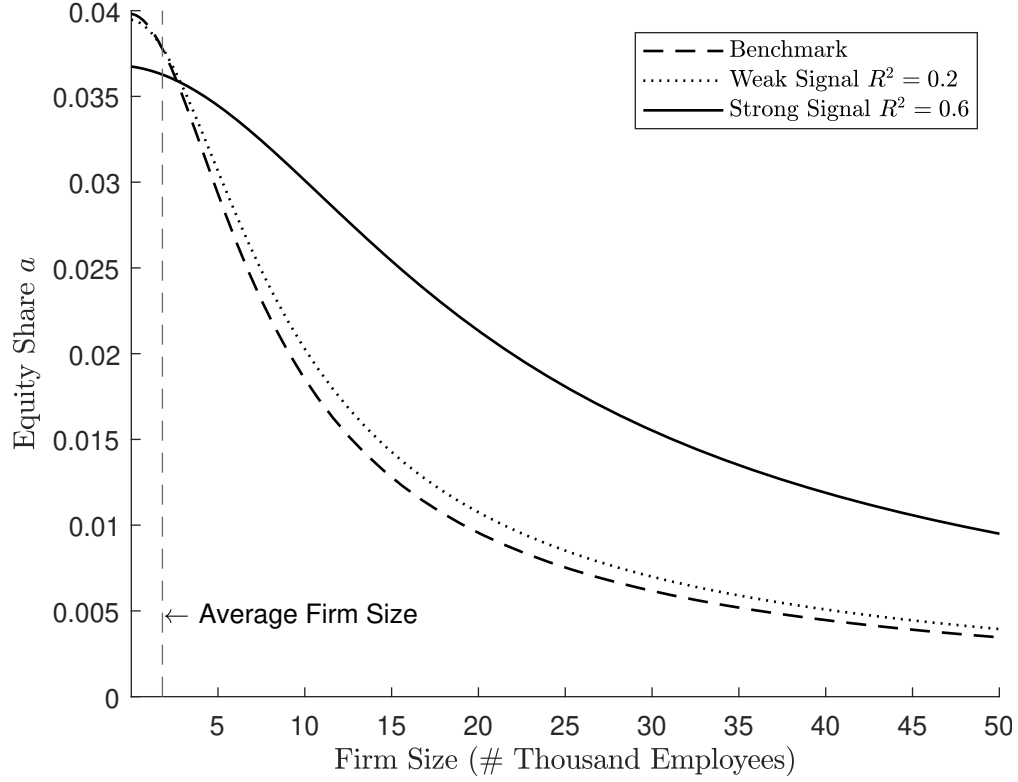
Figure 15: The Optimal Contract When the Signal Is Strong

Appendix C.1.

I evaluate two cases where the subsidy levels are 1% and 5% of the transaction price, respectively. The fourth and fifth column of Table 12 reports the results. The result is similar to the effect of adding a signal. It improves innovation tradability, which has two impacts on firm-level outcomes. It also has heterogeneous effect on inventors. There are more innovations in both start-ups and medium-small- to medium-large-sized firms and fewer innovations in small and big firms. The second to the sixth rows of Table 12 report the changes in the distribution. When there is a 1% subsidy, quantitatively, the share of innovations in start-ups increases by 0.115 percentage points. When the subsidy is 5%, the share in start-ups increases by 0.897 percentage points. Meanwhile, the share of innovations created in medium-small (with 500–20,000 employees) and medium-large firms (with 20,000–100,000 employees) also increases with the subsidy; the share of innovations in both small firms (with fewer than 500 employees) and large firms (with more than 100,000 employees) decreases with it. The seventh row confirms that it is more likely for a firm to sell its innovation.

Table 14: Counterfactuals When Shut Down the Secondary Market—The Role of Endogenous Contracts and Matching[1]

Moment	(1) Benchmark	(2) Baseline Model	(3) Same Contract	(4) Same Mapping	(5) All Same
Growth rate	2.00	-0.166	-0.162	-0.212	-0.205
% Inno in startups	0.33	-0.330	-0.330	-0.008	-0.010
% Inno, emp< 500	7.66	-6.350	-6.357	-0.072	-0.060
% Inno, emp$\in [500, 20k)$	33.32	-8.585	-8.556	$+0.000$	$+0.004$
% Inno, emp$\in [20k, 100k)$	47.70	$+5.012$	$+5.006$	$+0.066$	$+0.054$
% Inno, emp$\geq 100k$	11.00	$+10.250$	$+10.239$	$+0.015$	$+0.014$
Pr(Sell)	5.28	-5.283	-5.283	-5.283	-5.283

[1] The table reports the **percentage points difference** with respect to the benchmark. For example, the growth rate change is calculated according to $(g_{\text{counter}} - g) \times 100$. The first column reports the benchmark case in percentage points. The second column analyzes the baseline case, where both the contracts and the firm-inventor mapping can adjust. The third and fourth columns study the cases in which the contracts and the firm-inventor mapping cannot adjust, respectively. The fifth column reports the results where neither the contracts nor the firm-inventor mapping can adjust.

4.6 Endogenous Contracting and Firm-Investor Matching

This section explores how endogenous contracting and matching affect the quantitative results. For each counterfactual exercise, in addition to the baseline counterfactual in Section 4.5, I consider three cases: when firms cannot change contract terms but inventors can move freely, when firms can modify contracts but inventors cannot move to another firm, and when neither the contract terms nor the firm-inventor matching can adjust.

Table 14 reports the decomposition for the first counterfactual exercise: when there is no secondary market. If firms cannot change the contract or the mapping, then the growth rate drops by 0.21 percentage points instead of 0.166 percentage points. For the firm-level outcome, the share of innovations almost stays unchanged. The share of innovations in firms with more than 100,000 employees only increases by 0.01 percentage points, whereas it increases by more than 10 percentage points in the endogenous contracting setting. In this counterfactual exercise, allowing firm-inventor matching to change can capture most of the effects. This is because, in the extreme case, large firms are benefited a lot, which makes it very difficult for firms (especially small firms) to adjust contracts to attract inventors. The effect of the endogenous contract itself is more significant in the counterfactual exercises I consider below. Overall, the endogenous contracting setting mitigates the impact on the aggregate growth rate and amplifies the effect on the innovation distribution.

Table 15 and Table 16 compare the effects of adding a signal and introducing a subsidy. The quantitative implications of endogenous contracting and matching have similar implications in these two cases.

In both tables, the first column is the benchmark model when there is no signal or subsidies.

Table 15: Counterfactuals on Signals—The Role of Endogenous Contracts and Matching[1]

Moment	(1) Benchmark	(2) Baseline Model	(3) Same Contract	(4) Same Mapping	(5) All Same
Signal$= 20\%$					
Growth rate	2.00	+0.013	+0.014	+0.012	+0.012
% Inno in startups	0.33	+0.390	+0.392	+0.005	+0.005
% Inno, emp< 500	7.66	−0.430	−0.485	+0.002	+0.002
% Inno, emp$\in [500, 20k)$	33.32	+1.974	+2.316	+0.000	+0.000
% Inno, emp$\in [20k, 100k)$	47.70	−0.296	−0.583	−0.005	−0.004
% Inno, emp$\geq 100k$	11.00	−1.636	−1.640	−0.001	−0.001
Pr(Sell)	5.28	+0.237	+0.244	+0.174	+0.174

[1] The table reports the **percentage points difference** with respect to the benchmark. For example, the growth rate change is calculated according to $(g_{\text{counter}} - g) \times 100$. The first column reports the benchmark case in percentage points. The second column analyzes the baseline case, where both the contracts and the firm-inventor mapping can adjust. The third and fourth columns study the cases in which the contracts and the firm-inventor mapping cannot adjust, respectively. The fifth column reports the results where neither the contracts nor the firm-inventor mapping can adjust.

The second column reports the baseline counterfactual, where firms can adjust the contract terms and inventors can reallocate. The third column is, in the same counterfactual setting, when the firm cannot adjust contracts, but inventors can freely move to other firms. For example, in the subsidy case, when firms cannot adjust the contract terms, the growth rate increases by 0.007 percentage points, lower than 0.016. The firm-level effect is more significant. The share of innovations in small firms drops by 1.579 percentage points, compared with 1.285 percentage points in the baseline counterfactual. The firm-level changes are due to two reasons. First, since firms cannot adjust contracts, the contracts are not optimal–start-ups offer too much equity while large firms offer too little equity. Thus, inventors choose medium-sized firms, which leads to a rise in the shares of innovations in medium-small-sized and medium-large-sized firms. Second, inventors who still choose start-ups are over-incentivized. Therefore, they work harder than in the baseline counterfactual. As a result, there are fewer inventors in the start-ups, but everyone uses more effort. In equilibrium, the second channel dominates, and the share of innovations in start-ups is higher than in the baseline counterfactual. The fourth column is when the firm can adjust contracts, but inventors cannot move to other firms. The fifth column considers the extreme case where neither contracts nor the inventor-firm mapping can change. In both scenarios, the impacts on aggregate results and the firm-level changes are much smaller. In the subsidy case, the growth rates in both cases increase by only 0.005 percentage points, whereas it is 0.016 percentage points in the baseline setting. Overall, both the free movement of inventors and the contract terms affect the firm-level outcomes and the aggregate growth rate.

Table 16: Counterfactuals on Subsidy—The Role of Endogenous Contracts and Matching[1]

	(1)	(2)	(3)	(4)	(5)
Moment	Benchmark	Baseline Model	Same Contract	Same Mapping	All Same
Subsidy= 5%					
Growth rate	2.00	+0.016	+0.007	+0.005	+0.005
% Inno in startups	0.33	+0.897	+0.909	+0.005	+0.005
% Inno, emp< 500	7.66	−1.285	−1.579	+0.006	+0.005
% Inno, emp∈ [500, 20k)	33.32	+3.791	+4.683	+0.000	+0.000
% Inno, emp∈ [20k, 100k)	47.70	+7.594	+6.984	−0.009	−0.007
% Inno, emp≥ 100k	11.00	−10.996	−10.996	−0.002	−0.002
Pr(Sell)	5.28	+0.168	+0.191	+0.074	+0.074

[1] The table reports the **percentage points difference** with respect to the benchmark. For example, the growth rate change is calculated according to $(g_{\text{counter}} - g) \times 100$. The first column reports the benchmark case in percentage points. The second column analyzes the baseline case, where both the contracts and the firm-inventor mapping can adjust. The third and fourth columns study the cases in which the contracts and the firm-inventor mapping cannot adjust, respectively. The fifth column reports the results where neither the contracts nor the firm-inventor mapping can adjust.

5 Conclusion

This paper explores why some inventions are invented inside a large firm while others are in a small firm or start-up. The model characterizes the contractual relationship between two sets of heterogeneous agents: inventors and firms. Heterogeneous firms hire inventors to innovate in-house, and then trade non-complementary inventions on a secondary market. How much equity a firm offers depends on its size. Large firms find it difficult to give inventors high-power incentives without exposing them to unrelated risks. Therefore, both the incentive to exert effort and the optimal equity share decrease with firm size. A key trade-off that an inventor faces when choosing firms is between value and opportunities. On average, innovation value is higher in bigger firms, since they are more likely to have synergy with the innovation; meanwhile, the chance to innovate is higher in small firms, for they are better at incentivizing inventors using equity. The model suggests that inventors work for small firms if their ideas are more sensitive to effort whereas inventors with ideas less sensitive to effort work for big firms.

This model offers a framework to think about the boundaries of firms quantitatively. The counterfactual exercise shows that, if we shut down the innovation market, the growth rate would decrease by 0.166 percentage points, and the share of innovations created in start-ups and firms with fewer than 500 employees will drop from 8.0% to 1.3%.

It would be useful to extend and generalize the analysis in several directions. One possible direction is to consider how individuals can decide whether to become an inventor or a worker. Another direction is to incorporate financial frictions in the model and to think about when an inventor becomes an entrepreneur.

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Appendix A   Algorithm

I use value function iteration to solve the model. First, I define a set of discrete points for normalized firm sizes $\{\tilde{q}_0, \tilde{q}_1, ..., \tilde{q}_N\}$ and for idea types $\{\theta_0, \theta_1, \theta_2, ...\theta_M\}$. The algorithm employs a computational loop with the following steps

1. Guess the measure of the firm, the innovation without complementarity, the death rate and the growth rate $(N_F, \lambda_b, \tau, g)$.

2. Based on the Equation (7), solve for the corresponding ν.

3. For each inventor firm pair $(\theta, \tilde{q})$, use Equation (27) to solve for the stock share offered a and the utility level.

4. For each inventor θ, find the firm $\tilde{q}$ that offers the highest utility level according to Equation (28).

5. Update the guess on the measure of the firm, the innovation without complementarity, the death rate, and the growth rate $(N_F, \lambda_b, \tau, g)$ using Equation 32, 24, 34, and 36.

6. Repeat steps 2 to 5 until convergence.

Appendix B   Growth Rate

The growth rate of the aggregate quality $\bar{q}$ by definition is:

$$g = \frac{\mathbb{E}_t\left(\bar{q}(t + dt)\right) - \bar{q}(t)}{\bar{q}(t)dt}. \tag{67}$$

The expected average quality at $t + dt$ includes two parts: the quality of incumbents and the quality of new firms. $\mathbb{E}_t\left(\bar{q}(t + \Delta t)\right)$ can be written as

$$\mathbb{E}_t\left(\bar{q}(t+dt)\right) = \overbrace{\int \mathbb{E}q\left(t+dt\right)f_q\left(q,t\right)dq}^{\text{Incuments}} + \overbrace{\tau\kappa\bar{q}dt}^{\text{Entry}} \tag{68}$$

$$+ \overbrace{\left[\gamma_L\lambda_{I,L}\mathbb{E}\left(z|z>\hat{z}\right) + \gamma_H\lambda_{I,H}\mathbb{E}\left(z\right)\right]\bar{q}dt}^{\text{Innovative Entry}},$$

where $\lambda_{I,L} = \frac{(1-h(0))(1-\Phi(\hat{z}))}{(1-h(0))(1-\Phi(\hat{z}))+h(0)}\lambda_I$, $\lambda_{I,H} = \frac{h(0)}{(1-h(0))(1-\Phi(\hat{z}))+h(0)}\lambda_I$,.

Rearrange the equation to get the growth rate:

$$\begin{aligned} g &= -\tau\left(1-\kappa\right) \\ &+ \lambda_b N_F \left[\gamma_L \frac{\mathbb{E}\left(z\right)}{F\left(\hat{z}\right)} + \gamma_H \mathbb{E}\left(z|z\leq\hat{z}\right)\right] \\ &+ \gamma_H \mathbb{E}\left(z\right) \int h\left(q^*\left(\theta\right)\right)\lambda_\theta^*\psi\left(\theta\right)d\theta. \end{aligned} \tag{69}$$

Appendix C   Counterfactuals

C.1   Counterfactual: Tax

Now I add a sales tax to the secondary market. It is a fixed cost Γ per transaction, which means that the buyer now pays $(p_z + \Gamma)$ to purchase an innovation. The rest of the settings are the same as in the benchmark model. The buyer's zero-profit condition is

$$\int_0^{\hat{z}}\left(\gamma_H\nu zQ - p_z - \Gamma\right)\phi(z)dz = 0, \tag{70}$$

where $\hat{z} = \frac{p_z}{\gamma_L\nu Q}$ is the step size threshold; a firm agrees to sell an innovation when it does not have complementarity and the step size is lower than $\hat{z}$. Define $\psi \equiv \frac{p_z}{\gamma_L\nu Q}$. The zero-profit condition can be re-written as

$$\int_0^{\hat{z}}\left(\gamma_H z - \gamma_L\hat{z} - \gamma_L\psi\right)\phi(z)dz = 0. \tag{71}$$

With the tax, it is more costly to buy innovations. Therefore, $\hat{z}$ is lower than in the benchmark case, and a firm is less likely to sell an innovation.

I evaluate a case where the tax level is 1% of the transaction price. The fourth column of Table C1 reports the results. The tax mainly affects the innovation distribution. The tax, opposite to a signal, adds friction to the secondary market. As a result, firms sell a smaller portion of innovations. There is a larger value gap between innovations implemented by a big firm and those implemented by a small firm, which makes big firms more attractive. Similar to

Table C1: Counterfactual on Secondary Market Tax: 1% of the Transaction Price[1]

Moment	Benchmark	Tax
Aggregate growth rate	2.00	−0.003
% Inno in startups	0.33	−0.084
% Inno, emp< 500	7.66	−0.067
% Inno, emp∈ [500, 20k)	33.32	−0.378
% Inno, emp∈ [20k, 100k)	47.70	+0.529
% Inno, emp≥ 100k	11.00	+0.001
Pr(Sell)	5.28	−0.045

[1] The table reports the **percentage points difference** with respect to the benchmark. For example, the growth rate change is calculated according to $(g_{counter} - g) \times 100$. The second column reports the results when there is a one-time innovation transaction tax. The tax rate is as low as 1% of the transaction price.

the signal case, the tax also affects firms' risk composition—fewer risks come from innovations. In this case, the second force is weaker than the first for all inventors. All innovations shift to larger firms. Quantitatively, the proportion of innovations in start-ups slides by 0.084 percentage points, which is a 25% drop. The share in small firms (with fewer than 500 employees) and medium-small firms (with 500–20,000 employees) drop by 0.067 percentage points and 0.38 percentage points, respectively. The medium-large firms take a share 0.53 percentage points higher than before. For firms with more than 100,000 employees, the share increases slightly, by about 0.001 percentage points. The tax has a mild impact on the aggregate growth rate. This is because the effect is mitigated by endogenous contracts. I will discuss this mechanism in the next counterfactual exercise.

Appendix D   Extensions

D.1   A More Recent Sample Period

In this section, I calibrate the model to the sample period from 1998 to 2010 to cover more recent observations. The firm-level data are from the enter for Research in Security Prices (CRSP) and the Merged CRSP-Compustat Database. I apply the statistical model derived in Section 3.1 on the public firm data to estimate the economy-wide moments. I use patent data for innovations. The patent data are from Patent Examination Research Dataset and Patent Assignment Dataset (PAD), both provided by the US Patent and Trademark Office (USPTO).[12] I link firm-level data and the patent data using the linked CRSP-USPTO data

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[12]I estimate the probability of selling an innovation using PAD for two sample periods. Then scale all probability related moments using the ratio. This is because, rather than using all patents observations, Figueroa and Serrano (2019) keeps only **corporate patents**, which take about 75% in the whole patent sample. But

Table C2: Directly Calibrated Parameters Given Indirect Inference Results (A More Recent Sample Period)

λ_0	k_2	m
0.021	0.119	0.007

Table C3: Indirect Inference Calibrated Parameters (A More Recent Sample Period)

κ	μ	η	k_1	α	β_a	β_b	γ_H
0.006	1.821	0.213	0.937	2.001	0.203	1.953	0.937

provided by Kogan et al. (2017).

I calibrate the model in the same way as explained in section 3. The external calibrated parameters are also the same as in Table 5. Other parameters are reported in Table C3 and Table C2. The targeted moments are reported in Table C4. Table C5 shows the counterfactual results. The patterns are the same as the baseline sample period.

D.2   Financial Friction

The model implicitly incorporates the idea of financial frictions in the complementarity. The financial friction is an additive part of the complementarity under certain assumptions. One example is described below.

When an inventor finishes working, the firm pays the wage and purchases the stock from the inventor. The total spending is $W(\tilde{q})$. Firms can borrow from outside at the interest rate r to finance the spending under a collateral constraint

$$W(\tilde{q}) \leq \varepsilon(\tilde{q}) V(\tilde{q}), \tag{72}$$

where $\varepsilon(\tilde{q})$ is a random variable ($\varepsilon > 0$, $f_\varepsilon(\varepsilon)$). If the constraint binds, the firm cannot fully develop the business potential of the innovation. The firm can sell the innovation on the secondary market, where the buyer can further commercialize the innovation. In this case, the effect of financial frictions shows up exactly as the complementarity in the benchmark model.

D.3   Innovations as Substitutions

The benchmark model assumes that each innovation can improve a firm's quality. Cunningham et al. (2021) suggests that some innovations may be substitutions of existing technology. The model can incorporate this by adding one assumption—firms are exposed to random random

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"corporate patents" are not marked in PAD. Hence, to ensure the samples are consistent, I scale all patent trade-related moments using the same ratio.

Table C4: Model Fit for Key Targeted Moments (A More Recent Sample Period)

Moment	Data	Model
Profitability	0.11	0.11
Discount rate	0.02	0.02
Entry rate	0.07	0.07
Firm growth volatility	0.17	0.17
Aggregate growth rate	0.02	0.02
Average patent value	0.03	0.03
start-up buyout rate	0.16	0.16
Pr(Sell|Big)[1]	0.09	0.09
Pr(Sell)	0.09	0.09
Growth-size relation β_g[2]	−0.04	−0.04
% Inno, emp< 500[3]	0.07	0.07
% Inno, emp< $2,000$	0.24	0.24
% Inno, emp< $5,000$	0.34	0.35
60th pctl $\tilde{q}$ weighted by R&D	19.91	16.16
Average growth rate	0.07	0.05

[1] Pr(Sell\|Big) measures the probability to sell an innovation given it is invented by a big firm. A "Big firm" is defined as a firm with more than 500 employees, according to USPTO.

[2] β_g is the coefficient of the growth-size regression.

[3] The % Inno is the cumulative density function of innovations created in firms with less than certain employment. For example, "% Inno, emp< 500" means the share of innovations, among all innovations created in this period, that are invented in a firm with fewer than 500 employees.

Table C5: Counterfactuals (A More Recent Sample Period)

Moment	(1) Benchmark	(2) No Market	(3) Signal $\alpha_\epsilon = 1$ $R^2 = 0.2$	(4) Signal $\alpha_\epsilon = 2.5$ $R^2 = 0.6$	(5) Subsidy 1%	(6) Tax 1%
Growth rate	2.003	−0.266	0.031	0.122	0.003	−0.004
% Inno in startups	0.24	−0.240	0.32	4.27	0.098	−0.075
% Inno, $q < 0.5$	6.43	−5.680	−0.410	−4.490	−0.10	−0.076
% Inno, $q \in [0.5, 20)$	43.646	−11.185	1.864	7.484	0.582	−0.708
% Inno, $q \in [20, 100)$	49.685	17.101	−1.775	−7.265	−0.585	0.858
% Inno, $q \geq 100$	0	0	0	0	0	0
Pr(Sell)	2.451	−9.090	0.39	0.7	0.069	−0.079

[1] The table reports the **percentage points difference** with respect to the benchmark. For example, the growth rate change is calculated according to $(g_{\text{counter}} - g) \times 100$. The first column reports the benchmark case in percentage points. The second and third columns analyze a case in which there is a noisy signal when firms trade innovations on the secondary market. The noise ϵ follows a Pareto distribution with scale parameter 1 and shape parameter α_ϵ. The fourth (fifth) column reports the results when there is a one-time innovation transaction subsidy(tax). The tax rate is as low as 1% transaction value. The sixth column shows a case where firms cannot trade innovations at all.

negative quality shocks, which are proportional to the aggregate new innovation arrival rate.

D.4 Use Stock Options Instead of Equity

Using the stock options mainly affect the constant transfer of a contract; stock options capture the equity variance and protected by a lower bound. The same logic still applies: in a bigger firm, the innovation contributes to a smaller share of the equity value. Therefore, the only difference is the functional form of the variance associated with the incentive. Essentially, the mechanism behind the incentive problem does not change. The qualitative results still hold while the quantitative results may be different.

D.5 Patent Originality Distribution

One possible mapping of effort-sensitivity in the real life is the patent originality, as defined by Hall et al. (2001):

$$\text{Originality}_j = 1 - \sum_{i}^{n_j} s_{ij}^2$$

where s_{ij} denotes the percentage of citations made by patent j that belong to patent class i. Higher Originality$_j$ means the patent j relies on technologies in many different fields, and hence it is more novel. Figure D1 plots the average originality by firm size in 1997. It decreases with firm size, which is consistent with the model implications.
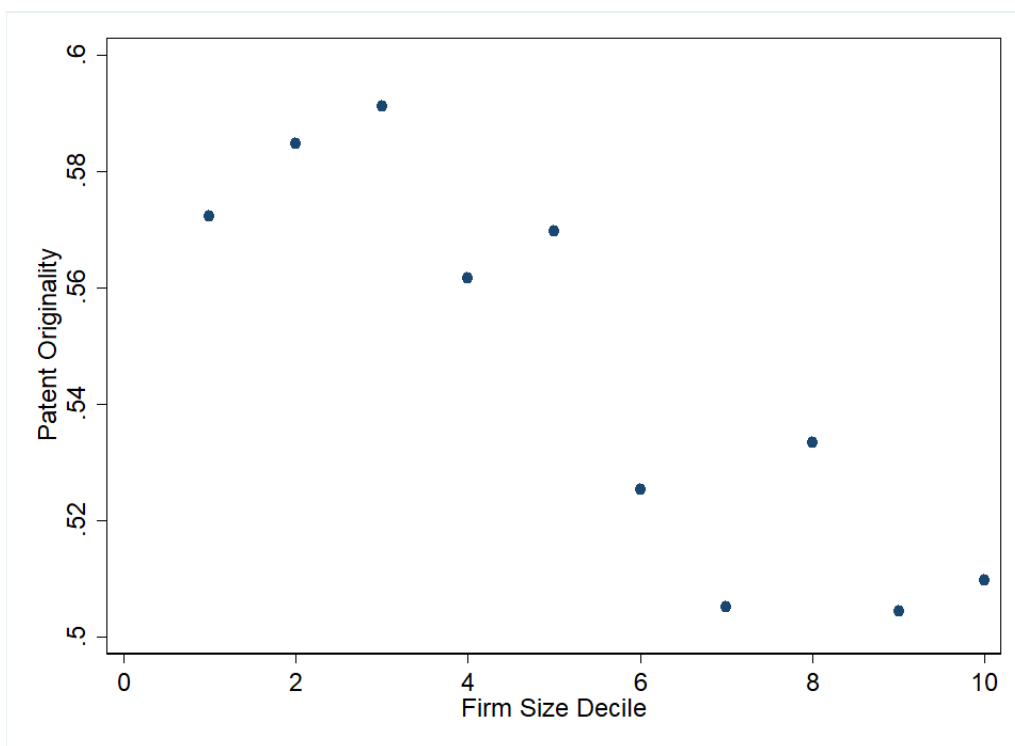
Figure D1: The Average Originality By Firm Size in 1997