Modeling and Simulation for Six-Axis Industrial Robot ABB-IRB-2400

1. Introduction:

Since industrial robots are widely used in not only industry but other areas, modeling, and simulation for a manipulator would be a good initial practice in my study. This project will focus on modeling and simulation for a six-axis industrial robot, ABB-IRB-2400. ABB-IRB-2400 is popular six axes manipulator that mainly used on the production line. This project is separated into two parts, mathematical modeling, and SimScape simulation. At the end of the project, the mathematical model will be compared to SimScape simulation as a check of the result.

2. Assumptions:

During modeling, there are some assumptions in the following list.

1. No frictions in manipulator's joints.

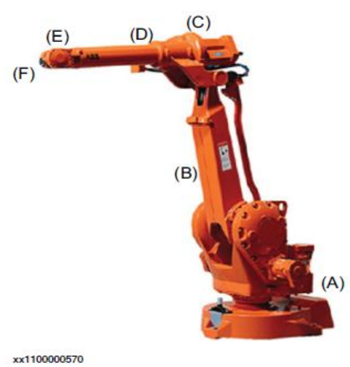
2. The complicated geometry of each component can be simplified to similar regular geometry such as cylinder or cuboid. In this case, the computation of component's inertial will be easier.

3. Every component exposes in gravitational field, and the gravitational acceleration is 9.8 m/s^2.

3. Mathematical Modeling:

3.1 Kinematic Analysis

3.1.1 D-H table of this manipulator



Pos	Description	Pos	Description
A	Axis 1	B	Axis 2
C	Axis 3	D	Axis 4
E	Axis 5	F	Axis 6

Figure 1: IRB 2400 Manipulator with Joint Denotation and Axis Description [1]

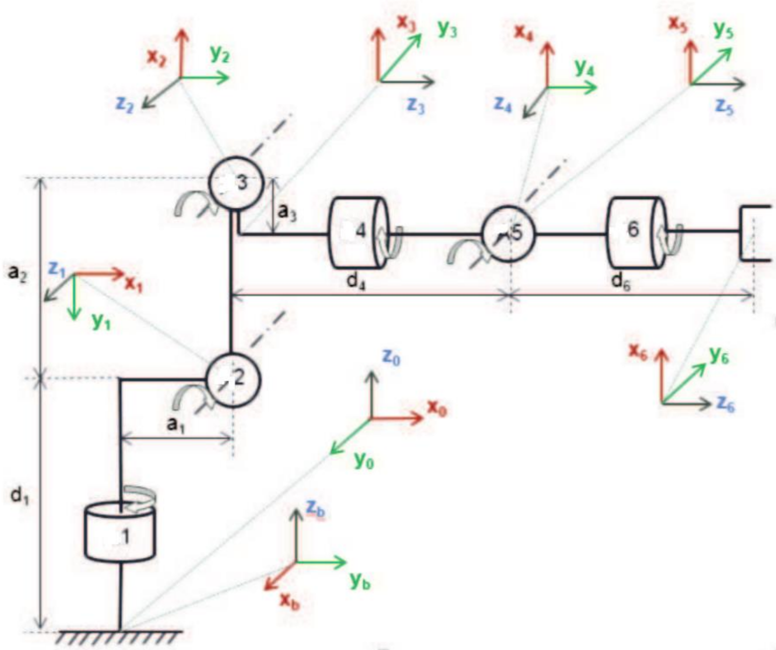


Figure 2: The joints of the robot with coordinate systems following the DH-convention [2]

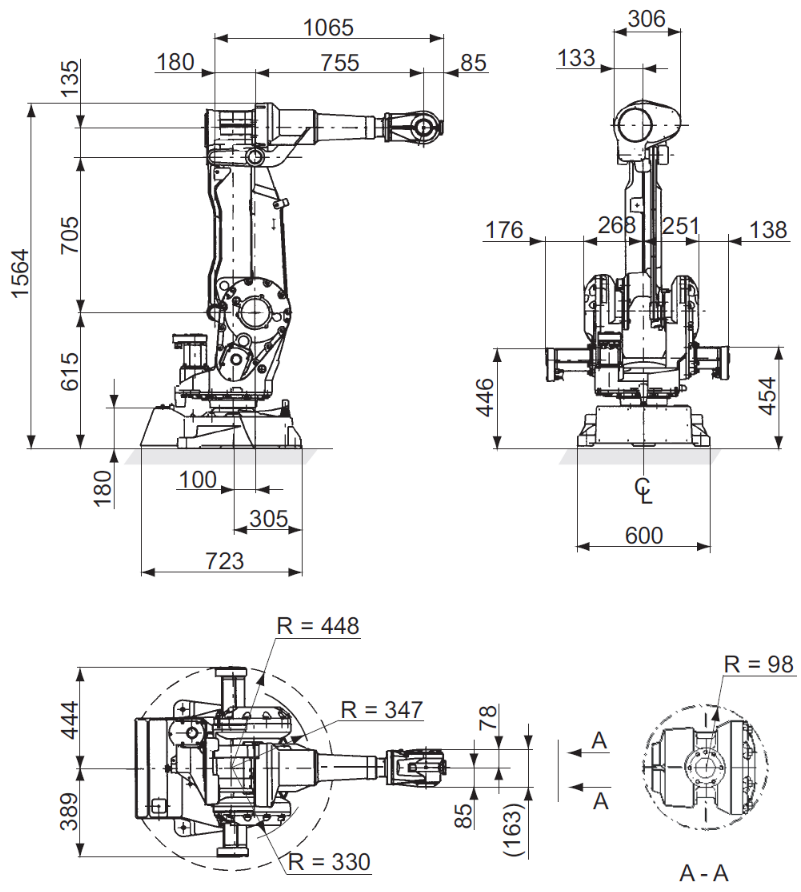
Axis	Type of motion	Range of movement
1	Rotation Motion	+ 180° to - 180°
2	Arm motion	+ 110° to - 100°
3	Arm motion	+ 65° to - 60°
4	Rotation Motion	+ 200° to - 200° (Unlimited as optional)
5	Bend motion	+ 120° to - 120°
6	Turn motion	+ 400° to - 400° + 250 rev.[i] to - 250 rev. Max.[ii]

Figure 3: Range of Movement in each Axis [1]

Based on Figure 2, Figure 3 and Figure 4, we can obtain the DH-table (*Table 1*) for IRB 2400 manipulator. Here we set up the origin of global coordinate is $x_0y_0z_0$ at joint 1.

Table 1: DH-table with range of movement

i	$\theta(°)$	*d(mm)*	a(mm)	α (°)	$\theta_{min}(°)$	$\theta_{max}(°)$
1	q_1	$d_1 = 435$	$a_1 = 205$	-90	-180	-180
2	q_2	0	$a_2 = 705$	0	-100	+110
3	$q_3 - 90$	0	$a_3 = 135$	-90	-60	+65
4	q_4	$d_4 = 755$	0	90	-200	+200
5	q_5	0	0	-90	-120	+120
6	q_6	$d_6 = 85$	0	0	-400	+400



xx1100000546

Figure 4: Dimensions of IRB 2400 in 3-views Engineering Graph

3.1.2 Homogenous Matrix

According the Equation 3.10 in textbook, we can compute the homogenous matrix using the D-H table.

$$
\begin{aligned}
A_i &= Rot_{z,\theta_i}\mathrm{Trans}_{z,d_i}\mathrm{Trans}_{x,a_i}Rot_{x,\alpha_i} \qquad (3.10)\\
&= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&\qquad \times \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

$$A_1 = \begin{bmatrix} c_1 & 0 & -s_1 & a_1c_1 \\ s_1 & 0 & c_1 & a_1s_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} s_3 & 0 & c_3 & a_3c_3 \\ -c_3 & 0 & s_3 & a_3s_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} c_4 & 0 & s_4 & 0 \\ s_4 & 0 & -c_4 & 0 \\ 0 & -1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_5 = \begin{bmatrix} c_5 & 0 & -s_5 & 0 \\ s_5 & 0 & c_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Based on the Equation 3.24 and 3.25 in the textbook, we have

$$T_6^0 = A_1 \cdots A_6 \tag{3.24}$$

$$= \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.25}$$

Where, $r_{11} = -c_6(c_1c_{23}s_5 - c_1c_4c_5s_{23}) - c_1s_4s_6s_{23}$

$$r_{21} = -c_6(s_1c_{23}s_5 - s_1c_4c_5s_{23}) - s_1s_4s_6s_{23}$$

$$r_{31} = c_6(s_5s_{23} + c_4c_5c_{23}) - c_{23}s_4s_6$$

$$r_{12} = s_6(c_1c_{23}s_5 - c_1c_4c_5s_{23}) - c_1c_6s_4s_{23}$$

$$r_{22} = s_6(s_1c_{23}s_5 - s_1c_4c_5s_{23}) - s_1c_6s_4s_{23}$$

$$r_{23} = -s_6(s_5s_{23} + c_4c_5c_{23}) - c_{23}s_4c_6$$

$$r_{13} = -c_1c_5c_{23} - c_1c_4s_5s_{23}$$

$$r_{23} = -s_1c_5c_{23} - s_1c_4s_5s_{23}$$

$$r_{33} = c_5s_{23} - c_4s_5c_{23}$$

$$d_x = a_1c_1 + d_6r_{13} + a_2c_1c_2 + (d_4 + a_3)c_1c_{23}$$

$$d_y = a_1s_1 + d_6r_{23} + a_2s_1c_2 + (d_4 + a_3)s_1c_{23}$$

$$d_z = d_6r_{33} - d_4s_{23} - a_2s_2 - a_3s_{23}$$

$$s_i = sin(q_i), \quad c_i = cos(q_i), \quad s_{ij} = sin(q_i + q_j), \quad c_{ij} = cos(q_i + q_j), \quad i, j = 1, 2, 3, 4, 5, 6$$

2.1.3 Jacobian Matrix

Based on the Equation 4.77 and 4.79, we can compute the Jacobian matrix using previous homogeneous matrix.

$$J_{v_i} = \begin{cases} z_{i-1} \times (o_n - o_{i-1}) & \text{for revolute joint } i \\ z_{i-1} & \text{for prismatic joint } i \end{cases} \tag{4.77}$$

$$J_{\omega_i} = \begin{cases} z_{i-1} & \text{for revolute joint } i \\ 0 & \text{for prismatic joint } i \end{cases} \tag{4.79}$$

At first, we need to have z_{i-1} *and* o_i, *i* = 1, 2, 3, 4, 5, 6

z_{i-1} can be observed from *Figure 2*.

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad z_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad z_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad z_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad z_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad z_5 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Next, we need to have o_i. Using the Equation 4.40 from textbook, we can extract o_i from homogenous matrix.

$$T_n^0(q) = \begin{bmatrix} R_n^0(q) & o_n^0(q) \\ 0 & 1 \end{bmatrix} \tag{4.40}$$

$$o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad o_1 = \begin{bmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix} \quad o_2 = \begin{bmatrix} a_1 c_1 + a_2 c_1 c_2 \\ a_1 c_1 + a_2 s_1 c_2 \\ -a_2 s_2 \end{bmatrix} \quad o_3 = \begin{bmatrix} a_1 c_1 + a_2 c_1 c_2 + a_3 c_1 c_{23} \\ a_1 s_1 + a_2 s_1 c_2 + a_3 s_1 c_{23} \\ -a_2 s_2 - a_3 s_{23} \end{bmatrix}$$

$$o_4 = \begin{bmatrix} a_1 c_1 + a_2 c_1 c_2 + a_3 c_1 c_{23} + d_4 c_1 c_{23} \\ a_1 s_1 + a_2 s_1 c_2 + a_3 c_1 s_{23} + d_4 s_1 c_{23} \\ -a_2 s_2 - (a_3 + d_4) s_{23} \end{bmatrix} \quad o_5 = \begin{bmatrix} a_1 c_1 + a_2 c_1 c_2 + a_3 c_1 c_{23} + d_4 c_1 c_{23} \\ a_1 s_1 + a_2 s_1 c_2 + a_3 c_1 s_{23} + d_4 s_1 c_{23} \\ -a_2 s_2 - (a_3 + d_4) s_{23} \end{bmatrix}$$

$$o_6 = \begin{bmatrix} a_1 c_1 + a_2 c_1 c_2 + a_3 c_1 c_{23} + d_4 c_1 c_{23} + d_6 r_{13} \\ a_1 s_1 + a_2 s_1 c_2 + a_3 c_1 s_{23} + d_4 s_1 c_{23} + d_6 r_{23} \\ d_6 r_{33} - a_2 s_2 - (a_3 + d_4) s_{23} \end{bmatrix}$$

Right now, we can use Equation 4.77 to compute the Jacobian matrix

$$J_v = \begin{bmatrix} J_{v_1} & J_{v_2} & J_{v_3} & J_{v_4} & J_{v_5} & J_{v_6} \\ J_{\omega_1} & J_{\omega_2} & J_{\omega_3} & J_{\omega_4} & J_{\omega_5} & J_{\omega_6} \end{bmatrix}$$

$$J_{v_1} = z_0 \times (o_6 - o_0) = \begin{bmatrix} s_1 - (a_1 s_1 + a_2 s_1 c_2 + a_3 c_1 s_{23} + d_4 s_1 c_{23} + d_6 r_{23}) \\ c_1 + a_1 s_1 + a_2 s_1 c_2 + a_3 c_1 s_{23} + d_4 s_1 c_{23} + d_6 r_{23} \\ 0 \end{bmatrix}$$

$$J_{v_2} = z_1 \times (o_6 - o_1) = \begin{bmatrix} 0 \\ a_2 s_2 + d_4 s_{23} - d_6 r_{33} + a_3 s_{23} \\ a_2 s_1 c_2 + a_3 c_1 s_{23} + d_4 s_1 c_{23} + d_6 r_{23} \end{bmatrix}$$

$$J_{v_3} = z_2 \times (o_6 - o_2) = \begin{bmatrix} 0 \\ d_4 s_{23} - d_6 r_{33} + a_3 s_{23} \\ a_3 c_1 s_{23} + d_4 s_1 c_{23} + d_6 r_{23} \end{bmatrix}$$

$$J_{v_4} = z_3 \times (o_6 - o_3) = \begin{bmatrix} -d_6 r_{23} - d_4 s_1 c_{23} \\ d_6 r_{13} + d_4 c_1 c_{23} \\ 0 \end{bmatrix}$$

$$J_{v_5} = z_4 \times (o_6 - o_4) = \begin{bmatrix} -d_6 r_{23} \\ d_6 r_{13} \\ 0 \end{bmatrix}$$

$$J_{v_6} = z_5 \times (o_6 - o_5) = \begin{bmatrix} -d_6 r_{23} \\ d_6 r_{13} \\ 0 \end{bmatrix}$$

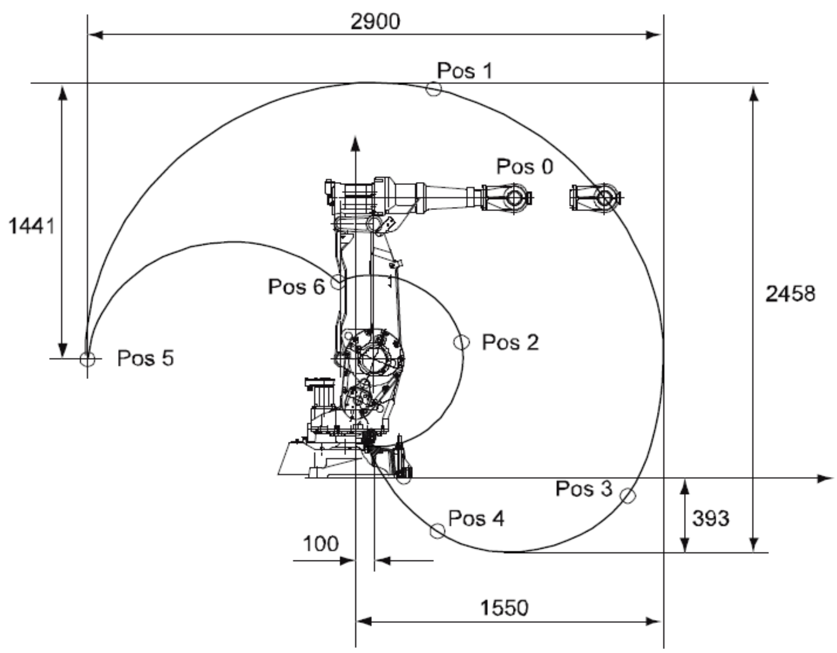
Since every joint is revolute joint, the Angular Velocity Jacobians are as following

$$J_{\omega_1} = z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad J_{\omega_2} = z_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad J_{\omega_3} = z_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$J_{\omega_4} = z_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad J_{\omega_5} = z_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad J_{\omega_6} = z_5 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

3.1.4 Workspace Area

This section is to define the workspace of IRB 2400 manipulator.



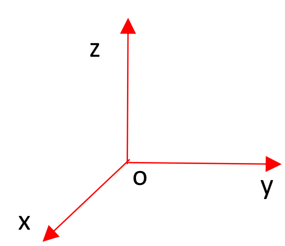


Figure 5: Workspace of IRB 2400 Manipulator [1]

The boundary of workspace can be expressed by following equations. Since joint 1 can rotate in 360°, the 2-D equation in x-z plane can be easily extended to 3-D equation in x-y-z plane.

Define R_1 is the distance between joint 2 to end effector when every point is in a straight line. Using the Figure 4, we can have $R_1 = 705 + 135 + 755 + 85 = 1680$

In this case,
$$z < \sin(q_2)\, R_1 + 615 \qquad -120° \le q_2 \le 120°$$
$$x < \sin(q_1)\, R_1 + 100 \qquad -180° \le q_1 \le 180° \qquad (1)$$
$$y < \cos(q_1)\, R_1 \qquad -180° \le q_1 \le 180°$$

The circle of Pos 2 is the most inner boundary, the trajectory of Pos 2 on positive y-direction. The radius $R_2 = \sqrt{(541-100)^2 + (693-435)^2} = 510$

In this case,
$$z > \sin(q_2)\, R_2 + 615 \qquad -120° \le q_2 \le 120°$$
$$x > \sin(q_1)\, R_2 + 100 \qquad -180° \le q_1 \le 180° \qquad (2)$$
$$y > \cos(q_1)\, R_2 \qquad -180° \le q_1 \le 180°$$

The arc between Pos 3 and Pos 4. When the end effector is on the Pos 3, the coordinate of joint 3 is $x_3 = 827, z_3 = 195, radius\ of\ joint\ 3\ rotation\ R_3 = 840, angle\ between\ x-axis\ and\ Pos\ 3, \beta = \operatorname{atan}(1351-727, 118+435) = -40°$ =

Thus,
$$z > -118 - R_3\cos(-40° + q_3) \qquad -60° \le q_3 \le 120° \qquad (3)$$
$$x > 1351 - R_3\sin(-40° + q_3) \qquad -120° \le q_3 \le 120°$$

Finally, we need to compute the arc between Pos4 and base. Since the boundary between Pos 4 and base is very close to straight line, we can linearize this arc to straight line. Thus,

$$400z + 302x > 0 \qquad (4)$$

Thus, every point satisfying the boundary equation 1, 2,3,4 is within the workspace of this manipulator.

3.1.5 Singularity

The singularity configuration of a manipulator is important to know for every user. Once the robot stacks on the singularity, it always needs a lot of time to move it out gradually. The most efficient way to measure the singularity configuration is to check whether the Jacobian Matrix loses any rank. Since 6-DOF manipulators have been used in industry for a long time, its singularity configurations have been well-known. This section will show singularity configurations usually happen in the industry.

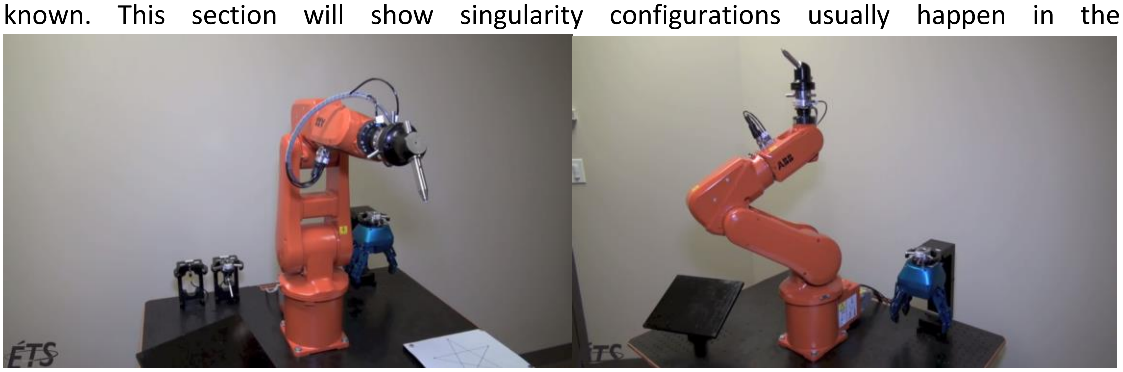


Figure 6: Wrist Singularity (Left) and Should Singularity (Right) [3]

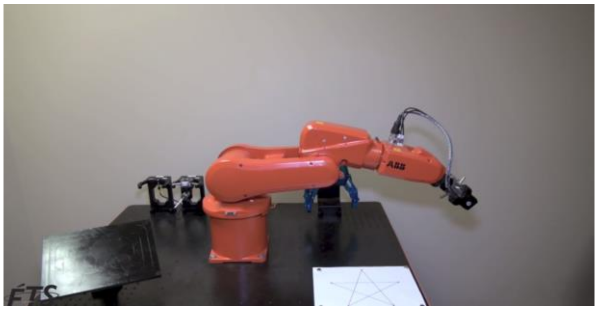


Figure 7: Elbow Singularity [3]

3.2 Dynamic Analysis

This section will discuss the dynamic of this manipulator. Since the 3D dynamics of this 6-dof robot arm is difficult to compute by hand, I simplify this 3D dynamics into 2D dynamics. In this case, 6-dof is reduced to 3-dof; only joint 2, 4, 5 have rotation and joint 1, 3, 6 are assumed to be fixed. Thus, this manipulator has been reduced to a three-link revolute joint arm.

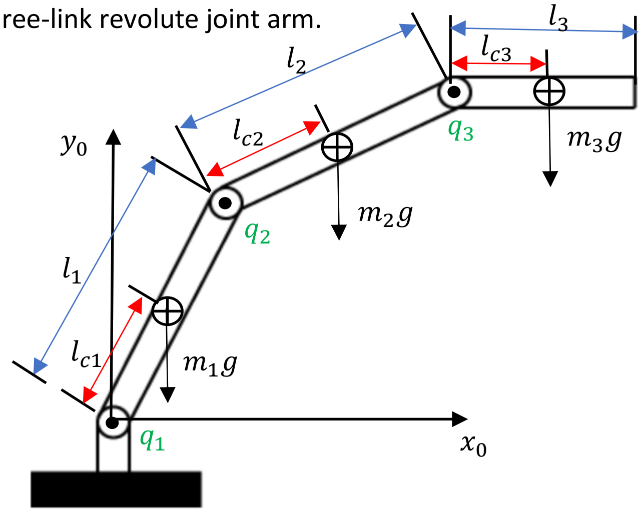


Figure 8: 3 Link Planar Arm. The circles indicate joints

The following table is to explain the parameter in simplified model *Figure 8.*

Table 2: Comparison of Parameters in the Simplified Model and the Actual Model

Simplified 3 Link Planar Arm	IRB2400 6-dof Arm	Value
q_1	q_2	variable
q_2	q_4	variable
q_3	q_5	variable
m_1	m_2	16.04 kg
m_2	$m_3 + m_4$	10.51 kg
m_3	$m_5 + m_6$	0.21 kg
l_1	a_2	0.705 m
l_2	d_4	0.755 m
l_3	d_6	0.085 m
l_{c1}	$a_2/2$	0.3525 m
l_{c2}	$d_4/2$	0.3775 m

l_{c3}	$d_6/2$	0.0425 m
I_1	I_2	$0.0883\ kg \cdot m^2$
I_2	I_3	$0.3359\ kg \cdot m^2$
I_3	I_5	$3.43 \times 10^{-4} kg \cdot m^2$

The length between joint to CoM of link l_{c1}, l_{c2}, l_{c3} equal to the half of link length $\frac{1}{2}l_1, \frac{1}{2}l_2, \frac{1}{2}l_3$, respectively.

The mass and inertia matrix can be found in the **Appendix 1**.

The Planar Elbow Manipulator example in Textbook can be a good reference for this dynamic analysis.

3.2.1 Inertia Tensor

The angular velocity for each joint is

$$\omega_1 = \dot{q}_1 k, \quad \omega_2 = (\dot{q}_1 + \dot{q}_2)k, \quad \omega_3 = (\dot{q}_1 + \dot{q}_2 + \dot{q}_3) \tag{5}$$

when expressed in the base inertial frame.

In general, the inertia tensor can be expressed as

$$I = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \tag{6}$$

Since ω_i is aligned with *k,* the triple product $\omega_i^T I_i \omega_i$ reduces simply to $(I_{zz})_i$ times the square of the magnitude of the angular velocity. To be simply, we denote $(I_{zz})_i$ as I_i. Thus, the rotational kinematic energy of this system is

$$\frac{1}{2}\dot{q}^T \left\{ I_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + I_2 \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + I_3 \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \right\} \dot{q}^T \tag{7}$$

3.2.2. Kinetic Energy

Here we can extend the Equation 6.76 in textbook to this model by adding one link.

Thus, the Jacobin Matrix for each joint in Figure 8 are as following.

<u>For Link 1,</u>

$$v_{c1} = J_{v_{c1}} \dot{q}$$

Where,

$$J_{v_{c1}} = \begin{bmatrix} -l_{c1} sinq_1 & 0 & 0 \\ l_{c1} cosq_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{8}$$

<u>For Link 2,</u>

$$v_{c2} = J_{v_{c2}} \dot{q} \tag{9}$$

Where, $$J_{v_{c2}} = \begin{bmatrix} -l_1 sinq_1 - l_{c2}\sin(q_1+q_2) & -l_{c2}\sin(q_1+q_2) & 0 \\ l_1 cosq_1 + l_{c2}\cos(q_1+q_2) & l_{c2}\cos(q_1+q_2) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

<u>For Link 3,</u>

$$v_{c3} = J_{v_{c3}}\dot{q}$$

Where,

$$J_{v_{c3}} = \begin{bmatrix} -l_1 sinq_1 - l_2\sin(q_1+q_2) - l_{c3}\sin(q_1+q_2+q_3) & -l_2\sin(q_1+q_2) - l_{c3}\sin(q_1+q_2+q_3) & -l_{c3}\sin(q_1+q_2+q_3) \\ l_1 cosq_1 + l_2\cos(q_1+q_2) + l_{c3}\cos(q_1+q_2+q_3) & l_2\cos(q_1+q_2) + l_{c3}\cos(q_1+q_2+q_3) & l_{c3}\cos(q_1+q_2+q_3) \\ 0 & 0 & 0 \end{bmatrix} \tag{10}$$

Using above Equations, we can form the inertia matrix *D(q)*.

$$D(q) = m_1 J_{v_{c1}}^T J_{v_{c1}} + m_2 J_{v_{c2}}^T J_{v_{c2}} + m_3 J_{v_{c3}}^T J_{v_{c3}} + \begin{bmatrix} I_1+I_2+I_3 & I_2 & I_3 \\ I_2 & I_2 & 0 \\ I_3 & 0 & I_3 \end{bmatrix} \tag{11}$$

$$D = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix} \tag{12}$$

The inetria matrix for *Link 1*,

$$m_1 J_{v_{c1}}^T J_{v_{c1}1} = \begin{bmatrix} m_1 l_{c1}^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{13}$$

The inertia matrix for *Link 2*,

$$m_2 J_{v_{c2}}^T J_{v_{c2}} = \begin{bmatrix} m_2(l_1^2 + l_{c2}^2 + 2l_1 l_{c2}^2 + 2l_1 l_{c2} c_2) & m_2(l_{c2}^2 + l_1 l_{c2} c_2) & 0 \\ m_2(l_{c2}^2 + l_1 l_{c2} c_2) & m_2 l_{c2}^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{14}$$

The inertia matrix for *Link 3,*

$$m_3 J_{v_{c3}}^T J_{v_{c3}} = \begin{bmatrix} m_3(l_1^2 + l_2^2 + l_{c3}^2 + 2l_1 l_2 c_2 + 2l_1 l_{c3} c_{23} + 2l_2 l_{c3} c_3) & m_3(l_1 l_2 c_2 + l_1 l_{c3} c_{23} + l_2^2 + 2l_2 l_{c3} c_3 + l_{c3}^2) & m_3(l_1 l_{c3} c_{23} + l_2 l_{c3} c_3 + l_{c3}^2) \\ m_3(l_1 l_2 c_2 + l_1 l_{c3} c_{23} + l_2^2 + 2l_2 l_{c3} c_3 + l_{c3}^2) & m_3(l_2^2 + l_{c3}^2 + 2l_2 l_{c3} c_3) & m_3(l_2 l_{c3} c_3 + l_{c3}^2) \\ m_3(l_1 l_{c3} c_{23} + l_2 l_{c3} c_3 + l_{c3}^2) & m_3(l_2 l_{c3} c_3 + l_{c3}^2) & m_3 c_3^2 \end{bmatrix} \tag{15}$$

The total inertia matrix $D = D_1 + D_2 + D_3$.Using Equation 12,

(16)

$$D = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix}$$

$$d_{11} = m_1 l_{c1}^2 + m_2(l_1^2 + l_{c2}^2 + 2l_1 l_{c2}^2 + 2l_1 l_{c2} c_2) \\ + m_3(l_1^2 + l_2^2 + l_{c3}^2 + 2l_1 l_2 c_2 + 2l_1 l_{c3} c_{23} + 2l_2 l_{c3} c_3) + I_1 + I_2 + I_3$$

$$d_{12} = d_{21} = m_2(l_{c2}^2 + l_1 l_{c2} c_2) + m_3(l_1 l_2 c_2 + l_1 l_{c3} c_{23} + l_2^2 + 2l_2 l_{c3} c_3 + l_{c3}^2) + I_2$$

$$d_{13} = d_{31} = m_3(l_1 l_{c3} c_{23} + l_2 l_{c3} c_3 + l_{c3}^2) + I_3$$

$$d_{22} = m_2 l_{c2}^2 + m_3(l_2^2 + l_{c3}^2 + 2l_2 l_{c3} c_3) + I_2$$

$$d_{23} = d_{32} = m_3(l_2 l_{c3} c_3 + l_{c3}^2)$$

$$d_{33} = m_3 c_3^2 + I_3$$

Next, we will compute the Christoffel symbols using the definition 6.58 in textbook

$$c_{111} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_1} = 0$$

$$c_{121} = c_{211} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = -m_2 l_1 l_{c2}\sin(q_2) - m_3 l_1 l_2 \sin(q_2) - m_3 l_1 l_2 \sin(q_2 + q_3)$$

$$c_{131} = c_{311} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_3} = -m_3 l_2\, l_{c3}\, sin(q_2 + q_3) - m_3 l_2 l_{c3} \sin(q_3)$$

$$c_{221} = \frac{\partial d_{12}}{\partial q_2} - \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = -m_2 l_1 l_{c2} \sin(q_2) - m_3 l_1 l_2 \sin(q_2) - m_3 l_1 l_{c3} \sin(q_2 + q_3)$$

$$c_{231} = c_{321} = \frac{1}{2}\left(\frac{\partial d_{13}}{\partial q_2} + \frac{\partial d_{12}}{\partial q_3} - \frac{\partial d_{23}}{\partial q_1}\right) \\ = \frac{1}{2}\,[-m_3 l_1 l_{c3} \sin(q_3 + q_2) \;+\; (-2m_3 l_1 l_{c3} \sin(q_3 + q_2) - 2m_3 l_2 l_{c3} \sin(q_3)) \\ + \;0] = -\frac{3}{2} m_3 l_1 l_{c3} \sin(q_3 + q_2) - m_3 l_2 l_{c3} \sin(q_3)$$

$$c_{331} = \frac{\partial d_{13}}{\partial q_3} - \frac{1}{2}\frac{\partial d_{33}}{\partial q_1} = -m_3 l_1 l_{c3} \sin(q_2 + q_3) - m_3 l_2 l_{c3} \sin(q_3)$$

$$c_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = m_2 l_1 l_{c2} \sin(q_2) + m_3 l_1 l_2 \sin(q_2) + l_1 l_{c3} \sin(q_2 + q_3)$$

$$c_{122} = c_{212} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = 0$$

$$c_{132} = c_{312} = \frac{1}{2}\left(\frac{\partial d_{23}}{\partial q_1} + \frac{\partial d_{21}}{\partial q_3} - \frac{\partial d_{13}}{\partial q_2}\right) = -\frac{1}{2} m_3 l_1 l_{c3} \sin(q_3 + q_2) - m_3 l_2 l_{c3} \sin(q_3) \tag{17}$$

$$c_{222} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_2} = 0$$

$$c_{223} = \frac{\partial d_{32}}{\partial q_2} - \frac{1}{2}\frac{\partial d_{22}}{\partial q_3} = m_3 l_2 l_{c3} \sin(q_3)$$

$$c_{232} = c_{322} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_2} = 0$$

$$c_{332} = \frac{\partial d_{23}}{\partial q_3} - \frac{1}{2}\frac{\partial d_{33}}{\partial q_2} = -m_3 l_2 l_{c3} \sin(q_3)$$

$$c_{113} = \frac{\partial d_{31}}{\partial q_1} - \frac{1}{2}\frac{\partial d_{11}}{\partial q_3} = m_3 l_1 l_{c3} \sin(q_2 + q_3) + m_3 l_2 l_{c3} \sin(q_2)$$

$$c_{123} = c_{213} = \frac{1}{2}\left(\frac{\partial d_{32}}{\partial q_1} + \frac{\partial d_{31}}{\partial q_2} - \frac{\partial d_{12}}{\partial q_3}\right) = m_3 l_2 l_{c3} \sin(q_3)$$

$$c_{133} = c_{313} = \frac{1}{2}\frac{\partial d_{33}}{\partial q_1} = 0$$

$$c_{233} = c_{323} = \frac{1}{2}\frac{\partial d_{33}}{\partial q_2} = 0$$

$$c_{333} = \frac{\partial d_{33}}{\partial q_3} = -m_3 \sin(2q_3)$$

Using above computation, we can have the matrix $C(q, \dot{q})$

$$C(q, \dot{q}) = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \tag{18}$$

Here,

$c_{11} = (c_{111}\dot{q}_1 + c_{211}\dot{q}_2 + c_{311}\dot{q}_3)$

$c_{12} = (c_{121}\dot{q}_1 + c_{211}\dot{q}_2 + c_{321}\dot{q}_3)$

$c_{13} = (c_{131}\dot{q}_1 + c_{231}\dot{q}_2 + c_{311}\dot{q}_3)$

$c_{21} = (c_{112}\dot{q}_1 + c_{212}\dot{q}_2 + c_{312}\dot{q}_3)$

$c_{22} = (c_{122}\dot{q}_1 + c_{222}\dot{q}_2 + c_{322}\dot{q}_3)$

$c_{23} = (c_{132}\dot{q}_1 + c_{232}\dot{q}_2 + c_{332}\dot{q}_3)$

$c_{31} = (c_{113}\dot{q}_1 + c_{213}\dot{q}_2 + c_{313}\dot{q}_3)$

$c_{32} = (c_{123}\dot{q}_1 + c_{223}\dot{q}_2 + c_{323}\dot{q}_3)$

$c_{33} = (c_{133}\dot{q}_1 + c_{233}\dot{q}_2 + c_{333}\dot{q}_3)$

3.2.3 Potential Energy

For Link 1:

$$P_1 = m_1 g l_{c1} \sin(q_1)$$

For Link 2:

$$P_2 = m_2 g(l_1 \sin(q_1) + l_{c2} \sin(q_1 + q_2))$$

For Link 3:

$$P_3 = m_3 g(l_1 \sin(q_1) + l_2 \sin(q_2) + l_{c3} \sin(q_1 + q_2 + q_3))$$

In sum:

$$\begin{aligned} P = P_1 + P_2 + P_3 & \\ &= (m_1 l_{1c} + m_2 l_1 + m_3 l_1) g \sin(q_1) + (m_2 l_{c2} + m_3 l_2) g \sin(q_1 + q_2) \\ &+ m_3 l_{c3} g sin(q_1 + q_2 + q_3) \end{aligned} \tag{19}$$

Therefore, the function g_k defined in Equation 7.61 become

$$g_1 = \frac{\partial P}{\partial q_1} \tag{21}$$

$$= (m_1 l_{c1} + m_2 l_1 + m_3 l_1) g \cos(q_1) + (m_2 l_{c2} + m_3 l_2) g \cos(q_1 + q_2) + m_3 l_{c3} g cos(q_1 + q_2 + q_3) \tag{20}$$

$$g_2 = \frac{\partial P}{\partial q_2} = (m_2 l_{c2} + m_3 l_2) g \cos(q_1 + q_2) + m_3 l_{c3} g cos(q_1 + q_2 + q_3) \tag{22}$$

$$g_3 = \frac{\partial P}{\partial q_3} = m_3 l_{c3} g cos(q_1 + q_2 + q_3)$$

3.2.4 Torque Analysis in each joint using Euler-Lagrange equation

Recall the Equation (7.63) from textbook,

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau \tag{7.63}$$

Thus, in this case, the torque τ in each is

$$\begin{aligned} d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + d_{13}\ddot{q}_3 + c_{11}\dot{q}_1 + c_{12}\dot{q}_2 + c_{13}\dot{q}_3 + g_1 &= \tau_1 \\ d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + d_{23}\ddot{q}_3 + c_{21}\dot{q}_1 + c_{22}\dot{q}_2 + c_{23}\dot{q}_3 + g_2 &= \tau_2 \\ d_{31}\ddot{q}_1 + d_{32}\ddot{q}_2 + d_{33}\ddot{q}_3 + c_{31}\dot{q}_1 + c_{32}\dot{q}_2 + c_{33}\dot{q}_3 + g_3 &= \tau_3 \end{aligned} \tag{23}$$

3.2.5 Torque Analysis Example Computation by Matlab

This section will show an example of computing the torque at each joint assuming we know the joint variable**. Please see 'IRB2400_Modeling_Simulation/Mathematic Modelling/ Dynamics/Dynamics.m'**

Here I set the joint variable $\ddot{q} = [0.1 \quad 0.1 \quad 0.1]$

The variation of torque on each joint is shown on the figure below.

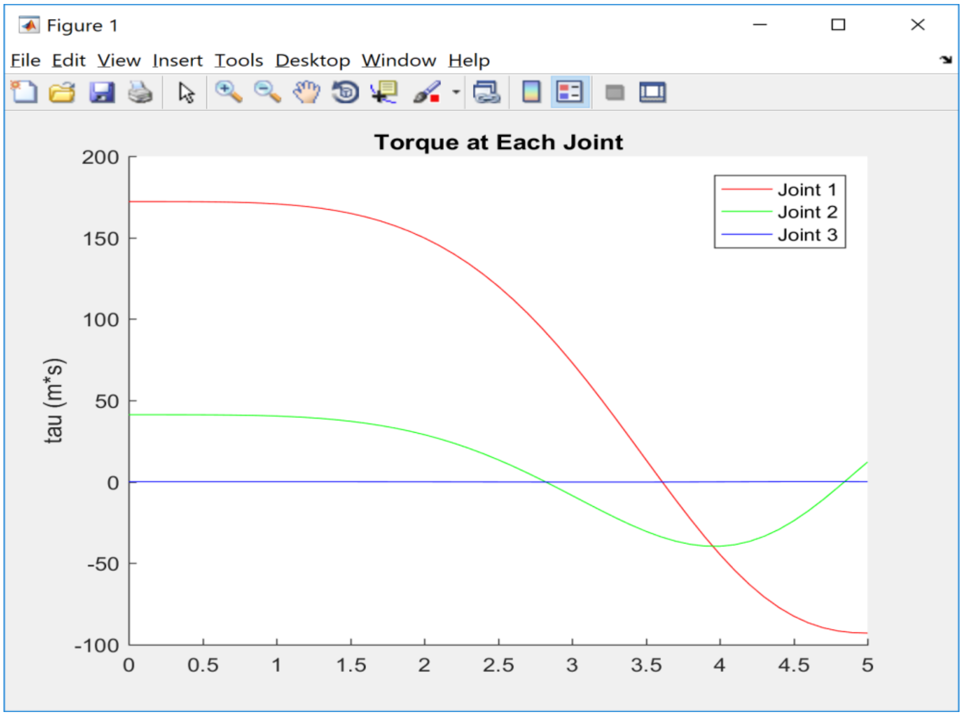


Figure 9: Example of Torque Analysis using MatLab

4. SimScape Modelling and Simulation

This section will compare the computed results from mathematical models and results from SimScape.

There are two hints to run the simscape model:

1. Please open the Simulink file in the folder. When you open the Simulink file, there may be warning for incorrect path of source data file. In this case, open the Model Explorer of Simulink Window (View -> Model Explorer or Ctrl + H). And then, select Model Workspace underneath the model. Change the data source to correct path of Assembly_DataFile.m and reinitialize it.

See 'IRB2400_Modeling_Simulation/SimScape Simulation/Data_file_Configuartion.png'.

2. Please keep the current folder to the directory of simscape mode; otherwise, Matlab will not animation.

See 'IRB2400_Modeling_Simulation/SimScape Simulation/Forward_Kinematics/ Position_in_Simluation.png'.

4.1 Comparison in Forward Kinematics

In this case, I set up the joint variable $q = \begin{bmatrix} \frac{\pi}{3} & -\frac{\pi}{4} & \frac{\pi}{4} & \frac{\pi}{4} & \frac{\pi}{4} & \frac{\pi}{4} \end{bmatrix}^T$

Mathematical Modelling: Please go to directory 'IRB2400_Modeling_Simulation/Mathematic Modelling/ Forward_Kinematics' and run 'transform_matrix.m'.

SimScape Simulation: Please go to directory 'IRB2400_Modeling_Simulation/SimScape Simulation/ Forward_Kinematics' and run 'Assembly.slx' by Simulink.

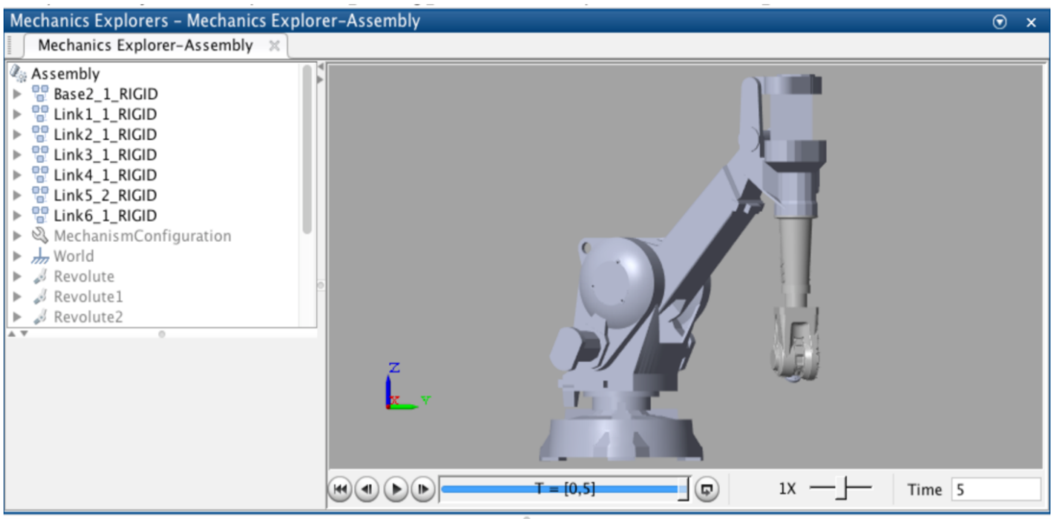


Figure 10: Manipulator Position Setup Joint Variable

In this section, four variables are comparison, x, y, z in translation from base frame to the frame of end effector and angle in rotation from base frame to frame of end effector. The results are shown in following figures.

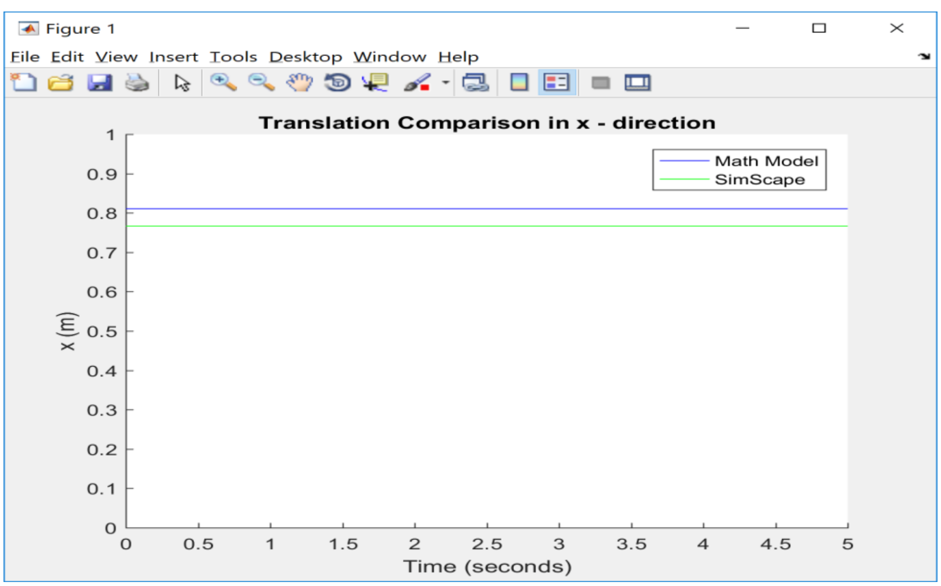


Figure 11: Translation Comparison in x -direction between Mathematical Model and SimScape

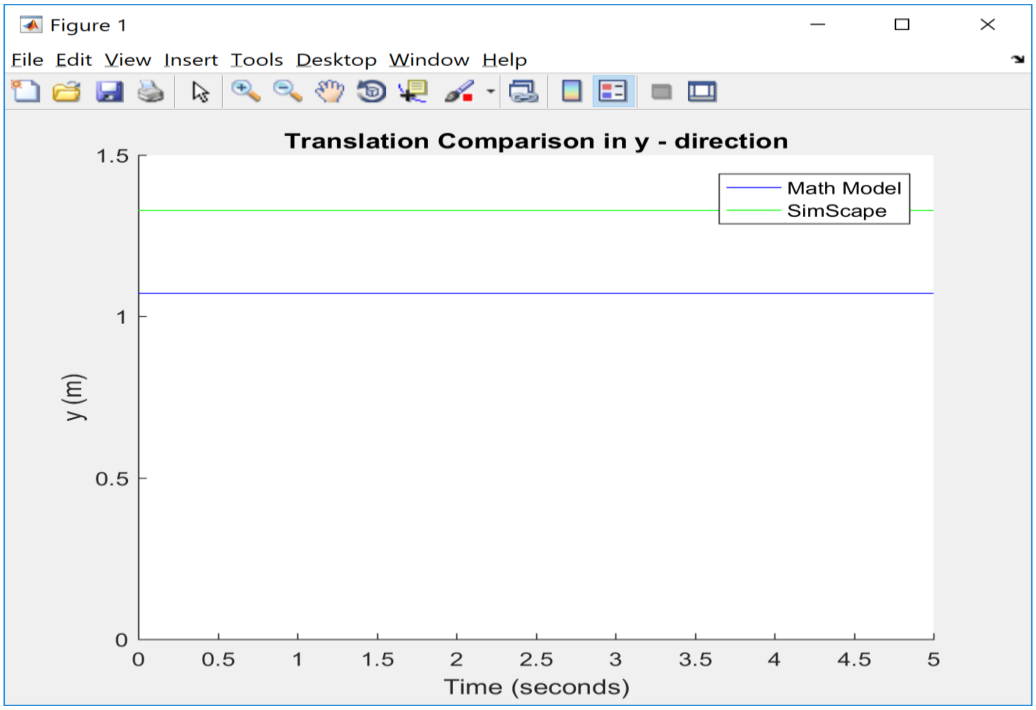


Figure 12: Translation Comparison in y -direction between Mathematical Model and SimScape

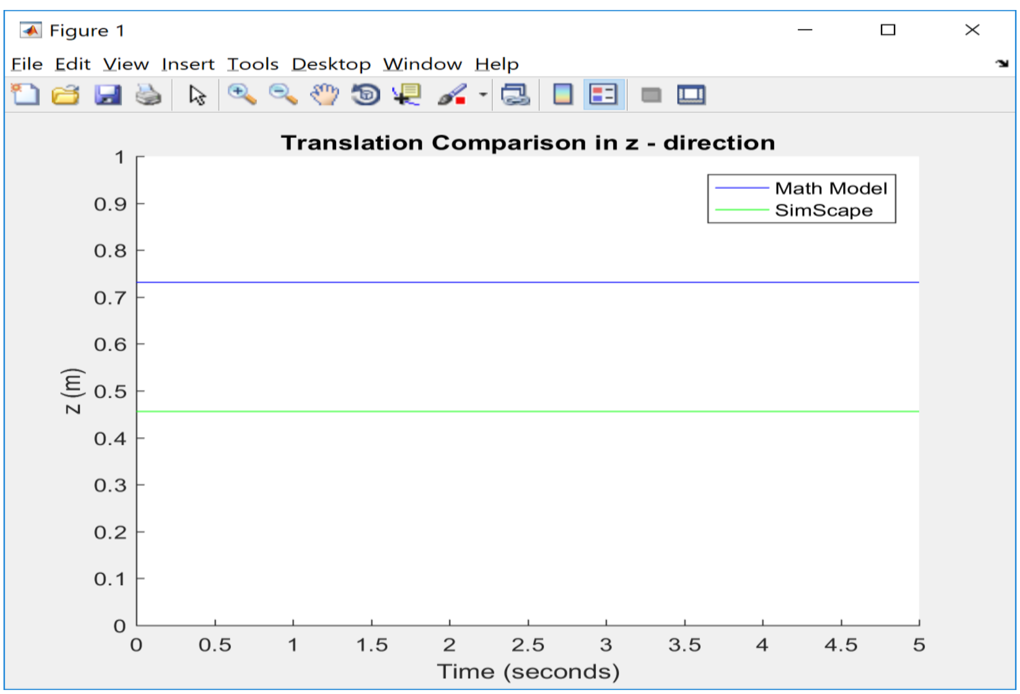


Figure 13: Translation Comparison in z -direction between Mathematical Model and SimScape

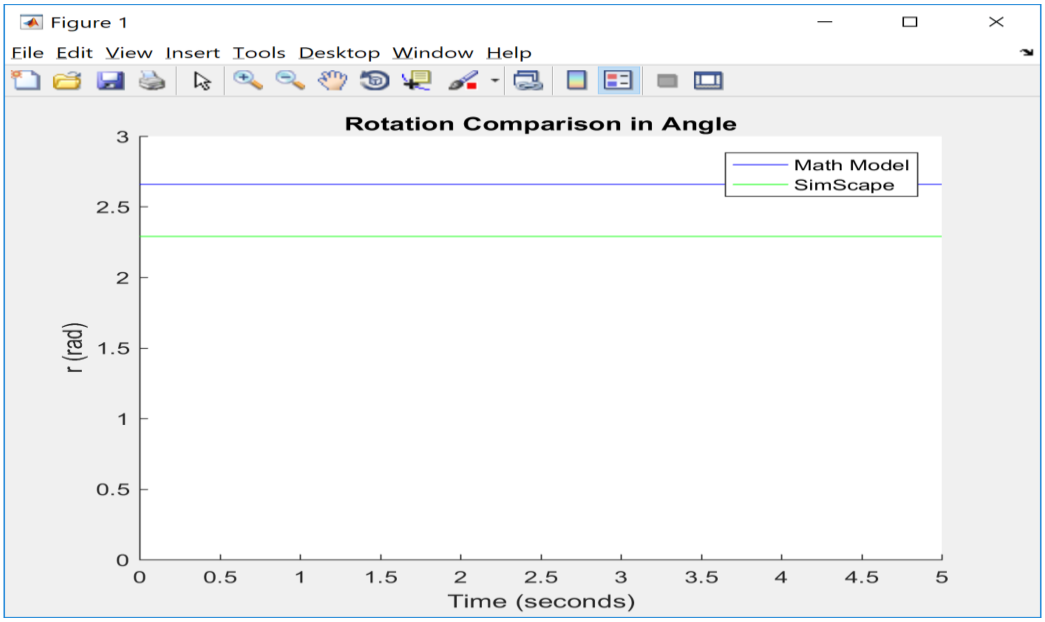


Figure 14: Rotation Transformation Comparison between Mathematical Model and SimScape

4.2 Comparison in Velocity Kinematics

In this simulation, I set $\dot{q} = [0.5 \quad -0.3 \quad -0.2 \quad 0.2 \quad 0.1 \quad 0.1]$

Mathematical Modelling: Please go to directory 'IRB2400_Modeling_Simulation/Mathematic Modelling/ Velocity_Kinematics' and run 'Jacbobian_Matrix.m'.

SimScape Simulation: Please go to directory 'IRB2400_Modeling_Simulation/SimScape Simulation/ Velcoity_Kinematics' and run 'Assembly.slx' by Simulink.

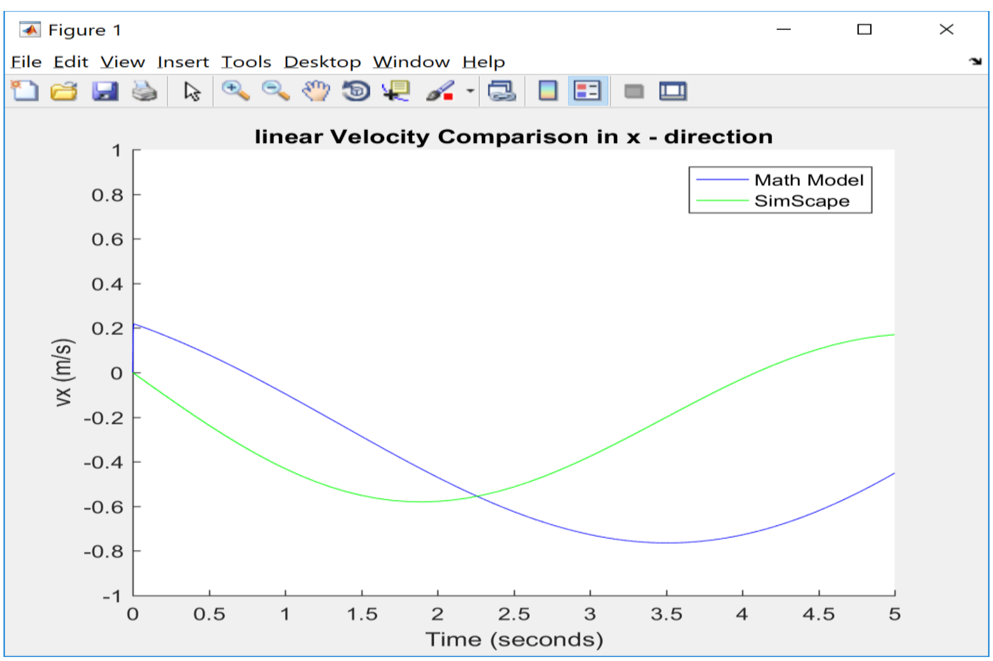


Figure 15: Linear Velocity Comparison in x -direction

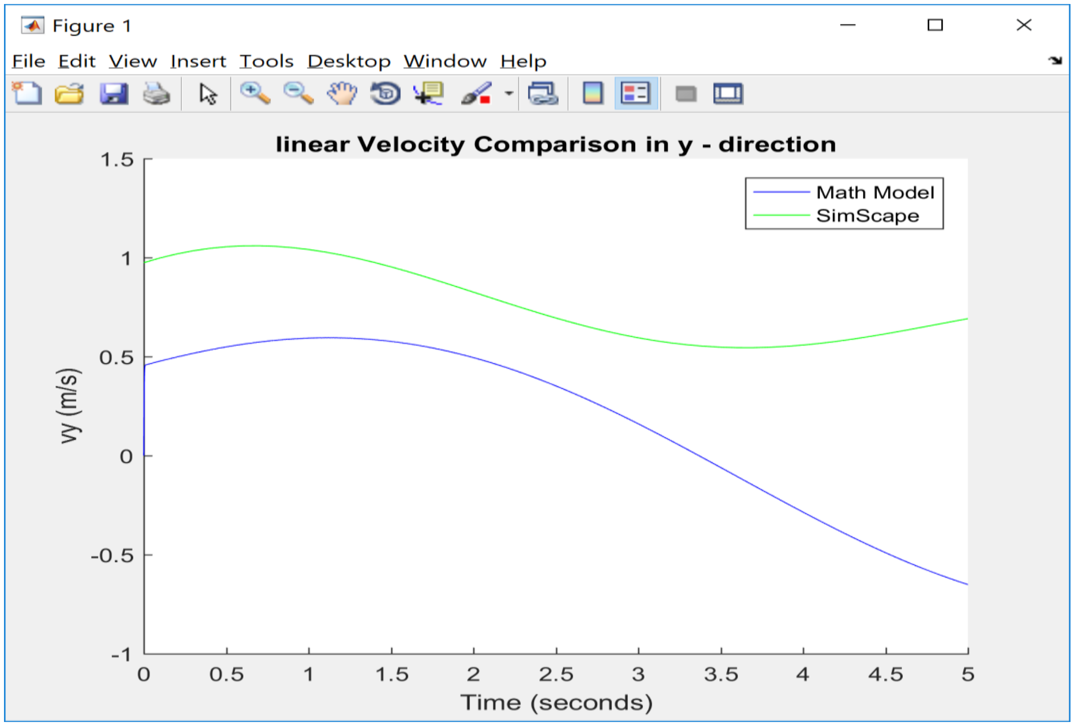


Figure 16: Linear Velocity Comparison in y – direction

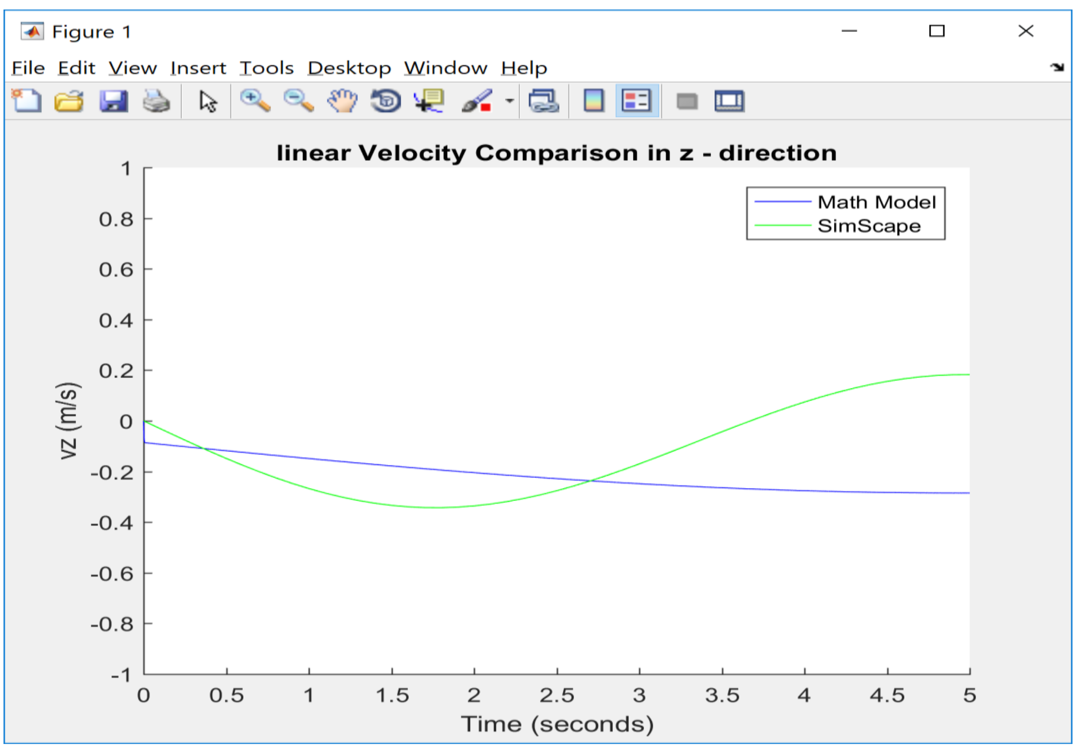


Figure 17: Linear Velocity Comparison in z – direction

Conclusion:

This project has modeled the ABB-IRB2400 Manipulator and simulated it in Matlab Simscape. In modeling, the forward kinematics and velocity kinematics have been modeled based on the original manipulator; the dynamics has been modeled based on a simplified model, 3 – revolute joint planar arm. Every step of equation derivations is shown clearly, and there are also Matlab codes to help computation. In the simulation, the position and velocity of the end effector are tracked in Simscape. The results from Simscape are compared to the results from the forward kinematics and velocity kinematics in modeling. The comparison in section 4 shows the results from modeling are different to the results from Simscape. The difference may come from the loss of information when building mathematical models. And it needs further check to make sure the source of these difference. *In conclusion, I believe that this project would be a good reference material for someone would like to learn 6 – axis manipulator.*

Reference:

1. IRB 2400 Manual. http://new.abb.com/products/robotics/industrial-robots/irb-2400

2. Piotrowski, N; Barylski, A. [2014] *Modelling A 6-DOF Manipulator using Matlab Software*, Archives of Mechanical Technology and Automation Vol.34 no. 3

3. Online Video (Singularity of six-axis robot) https://www.youtube.com/watch?v=zlGCurgsqg8

4. Mark W. Spong, Seth Hutchinson, M. Vidyasagar [2005] *Robot Modeling and Control*

Appendix 1:

Mass properties of Link1
Configuration: Default
Coordinate system: -- default --

Density = 1000.00000 kilograms per cubic meter

Mass = 52.62347 kilograms

Volume = 0.05262 cubic meters

Surface area = 1481836.49805 square millimeters

Center of mass: (millimeters)
X = 31.06709
Y = 19.67036
Z = 446.83042

Principal axes of inertia and principal moments of inertia: (kilograms * square millimeters)
Taken at the center of mass.
Ix = (-0.24499, 0.89506, -0.37263) Px = 1757300.55829
Iy = (0.33474, 0.43880, 0.83391) Py = 1963436.76818
Iz = (0.90991, 0.07957, -0.40711) Pz = 2918159.71212

Moments of inertia: (kilograms * square millimeters)
Taken at the center of mass and aligned with the output coordinate system.
Lxx = 2741508.66782 Lxy = -114320.53321 Lxz = 372478.73184
Lyx = -114320.53321 Lyy = 1804339.33880 Lyz = -37826.21296
Lzx = 372478.73184 Lzy = -37826.21296 Lzz = 2093049.03197

Moments of inertia: (kilograms * square millimeters)
Taken at the output coordinate system.
Ixx = 13268536.00935 Ixy = -82162.27878 Ixz = 1102983.12184
Iyx = -82162.27878 Iyy = 12361795.73553 Iyz = 424698.10094
Izx = 1102983.12184 Izy = 424698.10094 Izz = 2164200.56543

Mass properties of Link2
Configuration: Default
Coordinate system: -- default --

Density = 1000.00000 kilograms per cubic meter

Mass = 16.04354 kilograms

Volume = 0.01604 cubic meters

Surface area = 658600.11102 square millimeters

Center of mass: (millimeters)
X = 149.77346
Y = -0.28595
Z = 934.36927

Principal axes of inertia and principal moments of inertia: (kilograms * square millimeters)
Taken at the center of mass.

Ix = (-0.03433, 0.00362, 0.99940)	Px = 87655.39483
Iy = (-0.02703, -0.99963, 0.00270)	Py = 640735.19271
Iz = (0.99904, -0.02692, 0.03442)	Pz = 651852.33384

Moments of inertia: (kilograms * square millimeters)
Taken at the center of mass and aligned with the output coordinate system.

Lxx = 651179.17776	Lxy = 230.19633	Lxz = -19359.59041
Lyx = 230.19633	Lyy = 640735.98601	Lyz = 2013.58081
Lzx = -19359.59041	Lzy = 2013.58081	Lzz = 88327.75761

Moments of inertia: (kilograms * square millimeters)
Taken at the output coordinate system.

Ixx = 14657925.34911	Ixy = -456.91437	Ixz = 2225832.63597
Iyx = -456.91437	Iyy = 15007370.89734	Iyz = -2272.99392
Izx = 2225832.63597	Izy = -2272.99392	Izz = 448219.12129

Mass properties of Link3
Configuration: Default
Coordinate system: -- default --

Density = 1000.00000 kilograms per cubic meter

Mass = 14.57655 kilograms

Volume = 0.01458 cubic meters

Surface area = 680845.22676 square millimeters

Center of mass: (millimeters)
X = 98.59058
Y = -10.24809
Z = 1436.17927

Principal axes of inertia and principal moments of inertia: (kilograms * square millimeters)
Taken at the center of mass.

Ix = (0.99623, 0.01724, 0.08502)	Px = 113022.92362
Iy = (0.01100, 0.94704, -0.32092)	Py = 325884.35461
Iz = (-0.08605, 0.32064, 0.94328)	Pz = 338904.03951

Moments of inertia: (kilograms * square millimeters)
Taken at the center of mass and aligned with the output coordinate system.

Lxx = 114721.34694	Lxy = 4014.38904	Lxz = 19086.80837
Lyx = 4014.38904	Lyy = 327159.69771	Lyz = -3625.95293
Lzx = 19086.80837	Lzy = -3625.95293	Lzz = 335930.27309

Moments of inertia: (kilograms * square millimeters)
Taken at the output coordinate system.

Ixx = 30182010.72388	Ixy = -10713.25085	Ixz = 2083035.62164
Iyx = -10713.25085	Iyy = 30534603.78553	Iyz = -218165.02537
Izx = 2083035.62164	Izy = -218165.02537	Izz = 479146.74034

Mass properties of Link4
    Configuration: Default
    Coordinate system: -- default --

Density = 1000.00000 kilograms per cubic meter

Mass = 6.44343 kilograms

Volume = 0.00644 cubic meters

Surface area = 364240.76009 square millimeters

Center of mass: (millimeters)
    X = 653.53685
    Y = 1.14432
    Z = 1455.31020

Principal axes of inertia and principal moments of inertia: (kilograms * square millimeters)
Taken at the center of mass.

Ix = (0.99996, 0.00920, 0.00077)	Px = 15748.90179
Iy = (-0.00920, 0.99995, -0.00478)	Py = 182003.23346
Iz = (-0.00082, 0.00477, 0.99999)	Pz = 185622.01283

Moments of inertia: (kilograms * square millimeters)
Taken at the center of mass and aligned with the output coordinate system.

Lxx = 15763.07342	Lxy = 1529.39233	Lxz = 131.38970
Lyx = 1529.39233	Lyy = 181989.24588	Lyz = -16.08697
Lzx = 131.38970	Lzy = -16.08697	Lzz = 185621.82879

Moments of inertia: (kilograms * square millimeters)
Taken at the output coordinate system.

Ixx = 13662482.68548	Ixy = 6348.14970	Ixz = 6128466.49012
Iyx = 6348.14970	Iyy = 16580754.78891	Iyz = 10714.42777
Izx = 6128466.49012	Izy = 10714.42777	Izz = 2937684.63474

Mass properties of Link5
Configuration: Default
Coordinate system: -- default --

Density = 1000.00000 kilograms per cubic meter

Mass = 0.37055 kilograms

Volume = 0.00037 cubic meters

Surface area = 40256.72821 square millimeters

Center of mass: (millimeters)
X = 855.24882
Y = -0.02265
Z = 1455.89289

Principal axes of inertia and principal moments of inertia: (kilograms * square millimeters)
Taken at the center of mass.

Ix = (0.98192, -0.00156, -0.18931)	Px = 311.48917
Iy = (-0.18931, -0.00103, -0.98192)	Py = 344.20208
Iz = (0.00134, 1.00000, -0.00131)	Pz = 544.83147

Moments of inertia: (kilograms * square millimeters)
Taken at the center of mass and aligned with the output coordinate system.

Lxx = 312.66193	Lxy = -0.31835	Lxz = -6.08041
Lyx = -0.31835	Lyy = 544.83069	Lyz = 0.27281
Lzx = -6.08041	Lzy = 0.27281	Lzz = 343.03010

Moments of inertia: (kilograms * square millimeters)
Taken at the output coordinate system.

Ixx = 785735.86126	Ixy = -7.49641	Ixz = 461382.43491
Iyx = -7.49641	Iyy = 1057005.81215	Iyz = -11.94643
Izx = 461382.43491	Izy = -11.94643	Izz = 271380.81260

Mass properties of Link6
Configuration: Default
Coordinate system: -- default --

Density = 1000.00000 kilograms per cubic meter

Mass = 0.04890 kilograms

Volume = 0.00005 cubic meters

Surface area = 12443.19956 square millimeters

Center of mass: (millimeters)
X = 927.27756
Y = 0.03525
Z = 1454.79232

Principal axes of inertia and principal moments of inertia: (kilograms * square millimeters)
Taken at the center of mass.

Ix = (0.00301, 0.99670, 0.08111)	Px = 13.86440
Iy = (-0.01369, -0.08107, 0.99661)	Py = 14.07692
Iz = (0.99990, -0.00411, 0.01340)	Pz = 19.28762

Moments of inertia: (kilograms * square millimeters)
Taken at the center of mass and aligned with the output coordinate system.

Lxx = 19.28659	Lxy = 0.02207	Lxz = -0.06976
Lyx = 0.02207	Lyy = 13.86588	Lyz = 0.01747
Lzx = -0.06976	Lzy = 0.01747	Lzz = 14.07646

Moments of inertia: (kilograms * square millimeters)
Taken at the output coordinate system.

Ixx = 103509.45870	Ixy = 1.62029	Ixz = 65964.06360
Iyx = 1.62029	Iyy = 145549.25637	Iyz = 2.52489
Izx = 65964.06360	Izy = 2.52489	Izz = 42059.29496