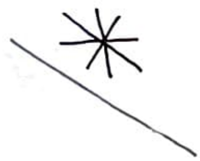
Today: Simple Models of Walking

Key ideas:

- Contact Mechanics (Non-smooth mechanics)
- Limit cycle stability

Simplest model:



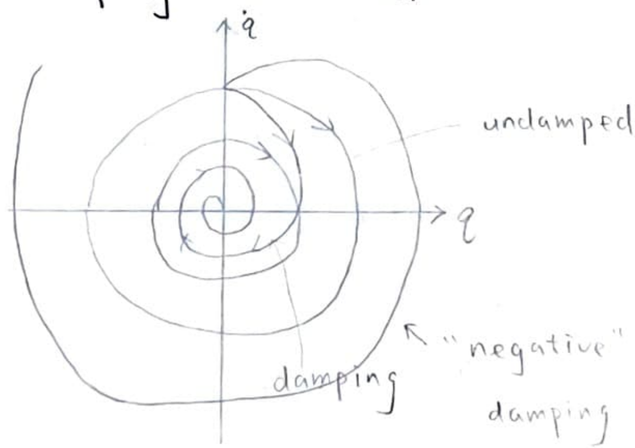
single stable state

↓ gravity, E↑ collision, E↓

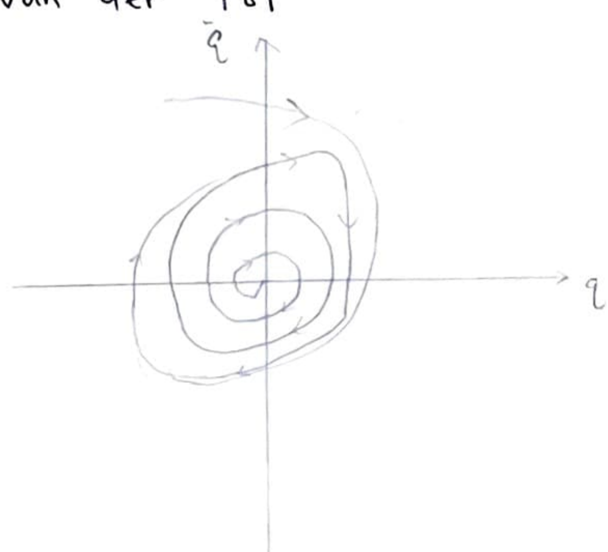
Example: (w/o impacts) van der Pol oscillator

$$\ddot{q} + \mu(q^2 - 1)\dot{q} + q = 0 \qquad \mu > 0$$

Linear Spring-mass model van der Pol



van der Pol



Stability of a trajectory

$\forall t > 0 \quad x^*(t)$

$$\lim_{t \to \infty} \| x(t) - x^*(t) \| = 0$$

Limit cycle stability

A periodic trajectory $\forall t > 0 \quad x^*(t) = x^*(t+T)$

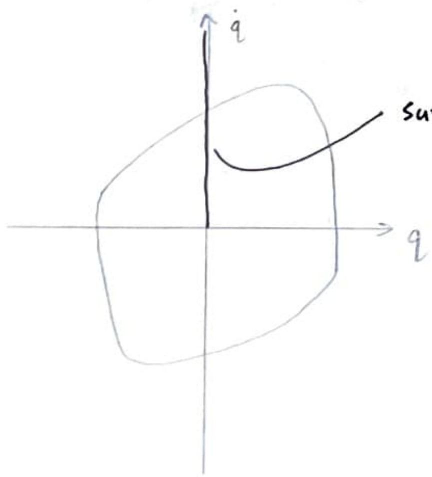
is limit cycle stable if

$$\min_{\tau} \| x(t) - x^*(\tau) \| \to 0$$

aka orbital stability

the minimum distance of arbitrary initial condition

solution to periodic solution



surface of section $\in \mathbb{R}^{n-1}$

$X \in \mathbb{R}^n$

section is transverse to the flow

all trajectories return to the section

$$x_p[n+1] = P(x_p[n])$$

$\mathbb{R}^{n-1}$

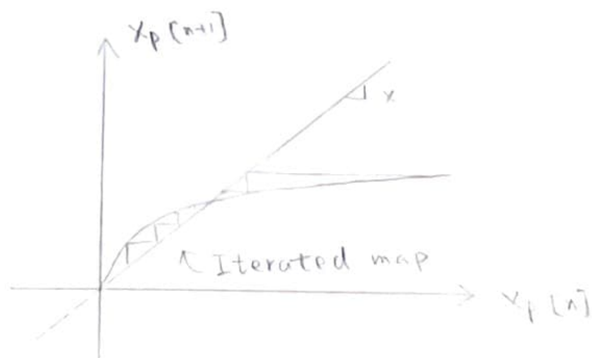
Poincare Map

in ~~Van~~ van der Pol ~~osicallator~~ oscillator

$S \quad q = 0, \ \dot{q} > 0$

Poincare Map

of vdP oscillator



find pt $x^* = P(x^*)$

local stability via

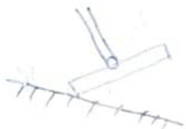
linearization

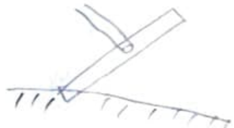
$\frac{\partial P}{\partial x}, \quad x[n+1] = A\,x[n]$

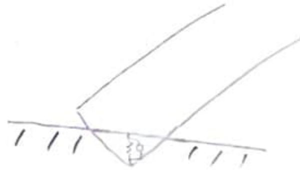
Contact Models

~~stiff~~ rigid vs soft contact

↑ e.g. spring (damper model)







$\lambda_{normal} = -k$ (penetration distance)

$+$ damping

k may be very large

bad for optimization and

closed form solution

strict non-penetration

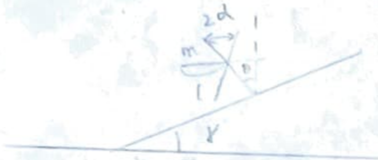
collisions are modelled as impulses

force :

better for optimization and

closed form solution

The Rimless Wheel



Assumptions:

- No slip (foot on ground as a pin joint)
- Collisions are inelastic impulsive
  $\Rightarrow$ no bouncing
- No double support

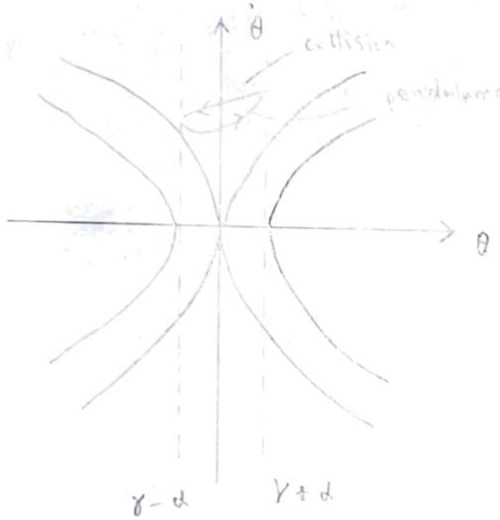
$\theta_1 = \gamma + (90^\circ - \alpha) - 90^\circ$

$= \gamma - \alpha$

$180^\circ - \theta_2 = 180^\circ - (\theta_1 + 2\alpha)$

$\theta_2 = \theta_1 + 2\alpha$

$= \gamma + \alpha$



Inelastic collision

All energy into ground is lost

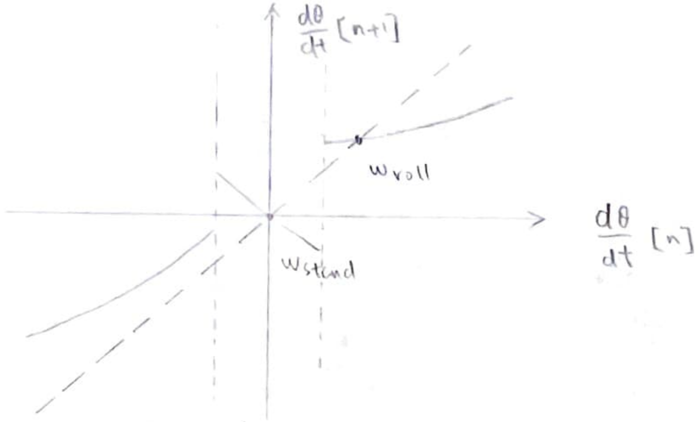
Angular momentum around pt of collision is conserved

$$L(t^-) = L^- = l \times m v^- = l\, m\, \dot\theta l \sin\left(\tfrac{\pi}{2} - 2\alpha\right) = m \dot\theta^- l^2 \cos 2\alpha$$

$$L^+ = l \times m v^+ = m l^2 \dot\theta^+$$

$$\dot\theta^+ = \dot\theta^- \cos 2\alpha$$

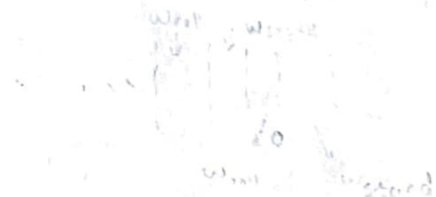
$\alpha > \frac{\pi}{2}$ collision once, stop



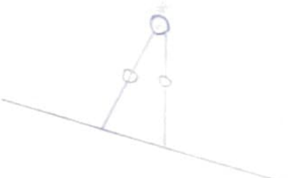
roll over
the top

roll back, not reach
the top

regions of attraction are not
simple ... always w_{roll}



Compass Gait



smaller region of attraction

limit cycle

impulse at right time

simple models
→ insights for physical robots
4 x more efficient w/ toe off