Today Computational Methods for Legged Roberts

Simple models Rimless wheel

Compass gait

SLIP

Limit cycles

Hybrid dynamics of contact

Smooth

Fixed points / local stability

Local stabilization. (e.g. LQR)

Lyapunov analysis

Traj optimization

Smooth systems

$\dot{x} = f(x)$

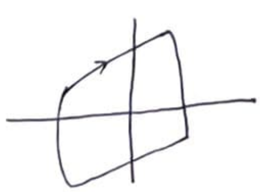
fixed pt is $f(x^*) = 0$

$\underset{x}{\text{find}}$ s.t. $f(x^*) = 0$ search problem

Periodic solutions

Find by : Traj optimization

Ex van der Pol oscillator



$\underset{x(\cdot),\ t(\cdot)}{\text{find}}$ s.t. $\dot{x}(t) = f(x)$

e.g. direct collocation

knots in spline rep ~~xx~~ $x(\cdot)$

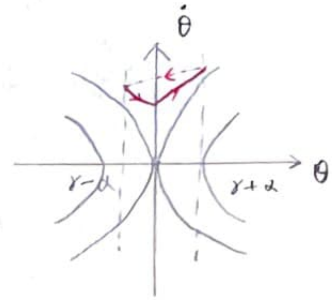
$x(0) = x(t_f)$

knock out origin point $q(0) = 0$

$\dot{q}(0) \geq .1$

Code demo to find periodic solution

Ex. RW cycle w/ dircol



$$ml^2\theta - mgl\sin\theta = 0$$

$$\dot\theta^+ = \cos(2\alpha)\,\dot\theta^-$$

find $x(\cdot), t(\cdot)$ s.t. $\dot x = f(x)$ via dircol

$$\theta(0) = \gamma - \alpha$$

$$\theta(t_f) = \gamma + \alpha$$

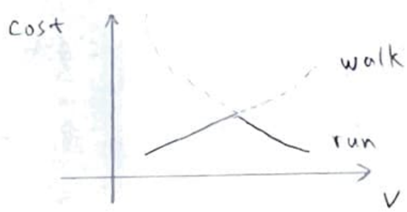
$$\dot\theta(0) = \cos(2\alpha)\,\dot\theta(t_f)$$

in RW, a unique solution found

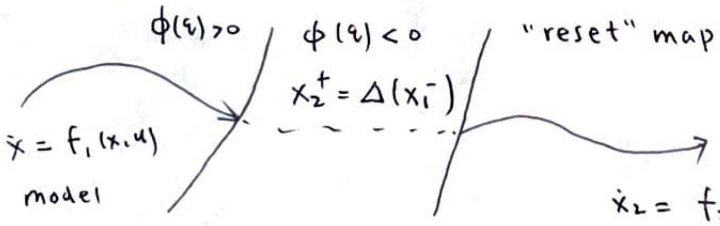
in Compass Gait, solution changes as system param changes

may be multiple solutions (walk & run)

speed ↗ running better than walking



More general (autonomous hybrid systems)



$\phi(q) = 0$ mode 2

"guard"

"witness function"

modes

mode 1 foot in air

mode 2 heel on ground

mode 3 heel + toe on ground

mode 4 toe on ground

If you prescribe a "mode schedule"

$$x[0] \; \dots \; x[k] \qquad x[n+1] = f_1(x[n], u[n])$$

$$x[k+1] \; \dots \; x[N] \qquad x[n+1] = f_2(x[n], u[n])$$

$$\forall i < k \qquad \phi(x[i]) > 0$$

$$\phi(x[k]) = 0$$

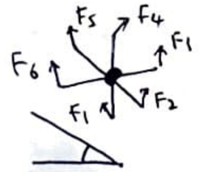
$$x[k+1] = \Delta(x[k])$$

$$\forall i > k \qquad \phi(x[i]) < 0 \quad ?$$

problem:

write "minimal coordinate" difficult in

"maximal coordinates"



$$q = \begin{bmatrix} x \\ z \\ \theta \end{bmatrix}$$

$\text{foot}_i(q) \geq \text{ramp}$

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} = \tau_g(q) + Bu + \sum_i \underbrace{J_i^T(q)}_{\text{Contact Jacobian}} \underbrace{F_i}_{\text{Contact forces}}$$

$$J_i = \frac{\partial \phi(q)}{\partial q}$$

if $\phi_i(q) > 0$, $F_i = 0$

if $\phi(q) = 0$

$\dot{\phi}(q) = 0 \quad \ddot{\phi}(q) = 0 \Rightarrow$ solve for F to impose constrain

$$\underset{x(\cdot),\, u(\cdot),\, F(\cdot)}{\text{find}} \quad \text{s.t.} \quad \dot{x} = f(x, u, F)$$

$$\phi(q) = 0$$

$$\rightarrow \dot{\phi}(q) = 0$$

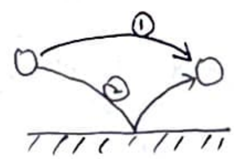
but hand designed contact sequence became impossible for more complicated systems

Idea #1: Soft contact

$$F_i = \text{soft-spring-model}(x)$$

$$= -k\,\phi_i(x) - \text{damping} \cdots$$

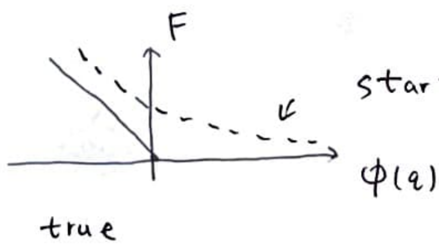
problem: contact modes are stiff, different modes



cannot discover ② traj

start initial ① traj

method



start with relaxation

Idea #2: Time-stepping approximation

$$x[n+1] = f(x[n], u[n], \lambda[n])$$

forces

backward Euler update

$$x[n+1] = x[n] + dt\, f(x[n+1], u[n+1], \lambda[n+1])$$

$$\phi(q) \geq 0, \quad \lambda[n] \geq 0$$

Complementary constraint

$$\phi(q[n]) \cdot \lambda[n] = 0 \quad \leq \varepsilon$$

$$\phi(q) = 0 \quad \text{OR} \quad \lambda = 0$$

Surprisingly effective solns via LCP