Lecture 18: Feedback Motion Planning

Last time:

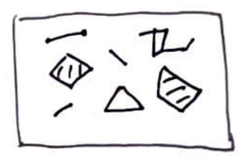
start state

goal state

will find a path if it exists

Getting to feedback

- Plan every timestep (MPC)
- Roadmaps

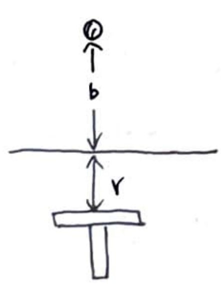


- RRT*

(Backwards trees)

Simplest Juggling Model

"linear Juggler"



$x \neq = \begin{bmatrix} b \\ r \\ \dot{b} \end{bmatrix}$, $u = [\dot{r}]$

Assumptions

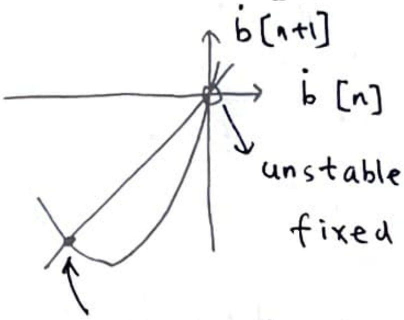
- instantaneous elastic collisions (w/ coefficient of restitution)

Mirror law

Impose $r(t) = -k \cdot b(t)$

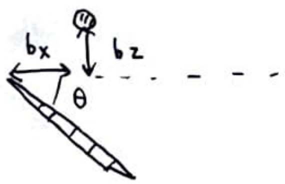
$\Rightarrow$ collisions at $r = 0, b = 0$

$\Rightarrow$ one-dimensional Poincare map collision velocity



"essentially globally stable"

2D juggling



Pre-image backchaining

LQR Tree

LQR + RRT sample

trajectory optimization to connect points

probabilistic complete

→ probabilistic feedback coverage

Lyapunov functions along trajectories

$$\dot{\bar{x}}(t) = A(t)\,\bar{x}(t) + B\,\bar{u}(t)$$

$$\Rightarrow \bar{u}(t) = -K\,\bar{x}(t) \qquad J(x,t) = \bar{x}^T S(t)\,\bar{x}$$

Trajectory $x_0(t)$ defined over finite $t \in [0, t_f]$

Find $\rho(t)$ s.t. $\dot{J}(x,t) \le \dot{\rho}(t)$ when $J(x,t) = \rho(t)$

$J(x, t_f) \le \rho(t_f)$ inside next controller

$x_0(t)$ numerical approx

ellipsoid constraint

$J(x,t) \le \rho(t)$ time varying funnel

$\uparrow$

scalar

"funnel" : finite-time invariant set

new node

x_{new}

connect to original trajectory

how to synthesze state machine to choose
controller in each step

$J(x,t) = 0$

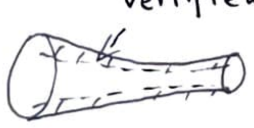
$J(x,t) = \rho(t)$

$= \bar{x}^T S(t) \bar{x}$

$\dot{J}(x,t) \le \dot{\rho}(t) \quad \forall x \quad J(x,t) \le \rho(t) \leftarrow$ stability

$\forall x \quad J(x,t) = \rho(t) \leftarrow$ finite time invariant

verified region



practical $\forall x \quad a\rho(t) \le J(x,t) \le \rho(t)$

$\uparrow$

$0 \le a \le 1$

$\dot{x} = f(x,u) = f_1(x) + f_2(x)\, u$

$u = -Kx$ or $u = -K\, m(x)$

polynomial of x

Given V $\quad \dot{V}(x) = \frac{\partial V}{\partial x}\left[f_1(x) + f_2(x)(-K m(x)) \right]$

$\dot{V}(x) \le 0$ for all x $\quad V(x) \le \rho$

fixed V, convex in K (controller)

cannot search K and V simultaneously

↳ Bilinear alternation

⇒ bigger and better funnels ?

These rely on full state feedback control

not work in manipulation now