Today: Running

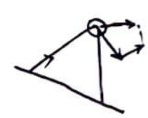
Defn of Running

1) Existence of aerial phase
2) Exchange of E_p and E_k

spring loaded (SLIP)
inverted pendulum

Why simple models?

1) Tractable
2) Mechanical Insights

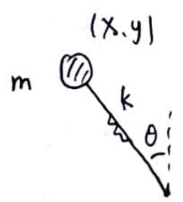


3) Comparative Biology

   Invariants in data $\Rightarrow$ fundamental principles?

4) As a "template" for high-dofs robots

Spring-loaded Inverted Pendulum



$$q = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \qquad u = \theta_{\text{touch down}}$$

Assumptions:

- massless leg

  $\Rightarrow$ command θ instantaneasly (in air)

- perfectly elastic collision

$\Rightarrow$ Energy is always conserved (threat to stability)

- when feet on ground, will have a pin joint
  "infinite stiding friction"

Poincare analysis

apex-to-apex map $(\dot{y} = 0)$

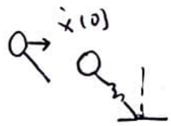
$$X = \begin{bmatrix} x \\ y \\ \theta \\ \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$

Given total E constant

$$y[n+1] = P_{u[n]}(y[n])$$

Flight phase

$$x = \begin{bmatrix} x \\ y \end{bmatrix}$$



$$m\ddot{x} = \begin{bmatrix} 0 \\ -mg \end{bmatrix}$$

$$y(t_{touchdown}) = \ell \cos\theta$$

Stance phase

$$X = \begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix}$$

$$m\ddot{r} - mr\dot{\theta}^2 + mg\cos\theta - k(r_0 - r) = 0$$

$$mr^2\ddot{\theta} + 2mr\dot{r}\dot{\theta} - mgr\sin\theta = 0$$

"minimal coordinates" no explicit constraints

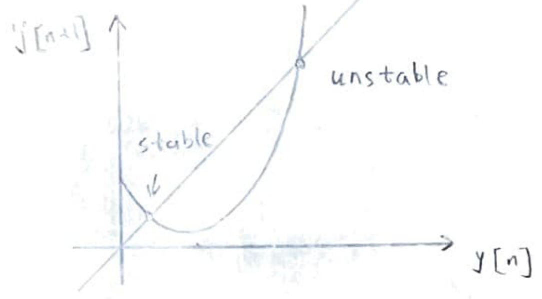
Harmut Geyer gave us linearized about small angles

Take-off to aerial

$x(0)$
$y(0)$ to $\dot{y}(t_{apex}) = 0$
$\dot{x}(0)$
$\dot{y}(0)$

+ Energy correction at apex

SLIP approximate apex-to-apex map



on the line stable points

failure

not enough $\dot{x}(0)$

SLIP Control

Choose $\theta_{touchdown}$ during aerial phase

$$y[n+1] = P(y[n], u[n])$$

Goal: design $u[n] = \pi(y[n])$

to stabilize y^d

Idea 1: find u^* s.t. $y^d = P(y^d, u^*)$

Linearize P around (y^d, u^*) and do (discrete time) LQR

$$x[n+1] = \cancel{A[n]x[}$$
$$Ax[n] + Bu[n]$$

LQR different form, but solvable

Idea 2: Deadbeat Control

if P is invertible

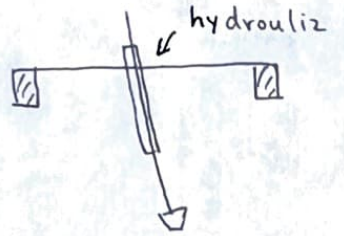
$$u[n] = P^{-1}(y^d, y[n])$$

push state to desired state in one step

this can be implemented in open-loop, with

no ~~knowleg~~ knowledge about $y[n]$ in contact

Continuous control (Raibert Hoppers)



Decomposed control

1) Hopping height (push at toe off)

2) Foot touchdown to regulate speed

3) Stabilize attitude during stance