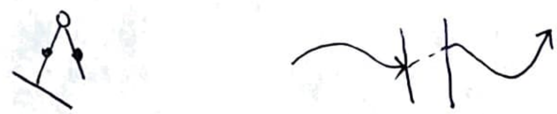
Lecture 16: Humanoids

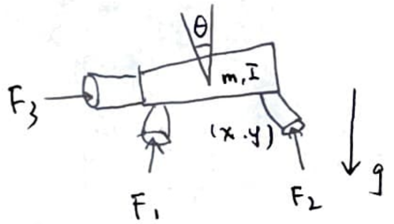
(more generally "highly-actuated legged robots")

Legs



traj opt, LQR, SoS ...

Spacecraft



Inputs $F_1, \dots, F_N \in \mathbb{R}^2$

Dynamics
$$m\ddot{x} = \sum_i F_{i,x}$$
$$m\ddot{z} = \sum_i F_{i,z} - mg$$
$$I\ddot{\theta} = \sum_i [r_i \times F_i]_y$$
$$= \sum_i (r_{ix} F_{iz} - r_{iz} F_{ix})$$

equations are affine

Constraints

subject to $|F| < F_{max}$

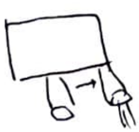
$|F_{ix}|$ $|F_{iz}|$ small

moving thrusters

$|\dot{r}| <$ small (velocity limit)

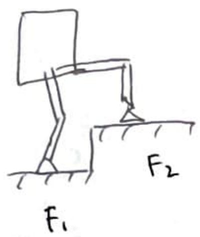
$\dot{r}\,|F| = 0$ suddenly much harder

↑ cannot move and force same time



only allow force in a few locations

view can explain humanoids



$F_{1,z} \geq 0 \qquad |F_{1,x}| \leq \mu F_{1,z}$

not compass gait, but yes for Asimo / ATLAS ...

Doesn't
~~Doesnt~~ have to be massless legs ...

center of mass $\qquad x_{com}(q)$

+ total angular momentum around com

Ignoring impacts $\qquad z_{com} = \text{constant} \quad + \quad \ddot{\theta} = 0$

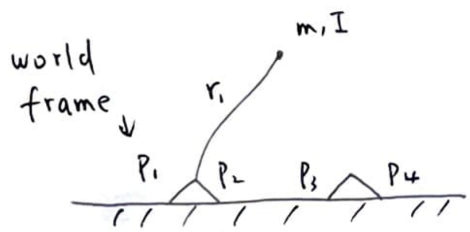
↑

sounds freak

but makes things easier

$\Rightarrow \dot{z}_{com} = 0 \qquad \ddot{z}_{com} = 0$

Flat terrian



$F_{i,z} \geq 0 \quad \Rightarrow \quad \ddot{z} \geq -g$

$|F_{i,x}| \leq \mu F_{i,z} \quad \Rightarrow \quad |\ddot{x}| \leq \mu(\ddot{z} + g)$

$\cancel{\ddot{x} \leq} \quad |\ddot{x}| = \left|\frac{1}{m}\sum_i F_{i,x}\right| \leq \frac{1}{m}\sum_i F_{i,z} = \frac{1}{m}(m\ddot{z} + mg)$

$$I\ddot{\theta} = \sum_i \left(r_{i,z} F_{i,x} - r_{i,x} F_{i,z} \right)$$

$$r_i = P_i - \begin{bmatrix} x \\ z \end{bmatrix}$$

$$X_{com} = \frac{\sum_i x_i m_i}{\sum_i m_i} \in \mathbb{R}^2$$

Def:
center of preassure

$$X_{cop} = \frac{\sum_i P_{i,x} F_{i,z}}{\sum_i F_{i,z}}$$

$$(m\ddot{z} + mg)(x_{cop} - x)$$

$$= (z_{cop} - z)\, m\ddot{x} - I\ddot{\theta}$$

proof.

~~Ix~~ $I\ddot{\theta} = (z_{cop} - z) \sum_i F_{i,x} - (x_{cop} - x) \sum_i F_{i,z}$

$$= (z_{cop} - z)\, m\ddot{x} - (x_{cop} - x)(m\ddot{z} + mg)$$

$$\Downarrow$$

$$0 = -h\, m\ddot{x} - (x_{cop} - x)\, mg$$

$$\ddot{x} = \frac{g}{h}(x - x_{cop})$$

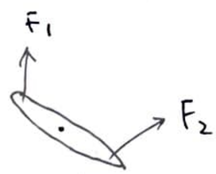
Assume:
~~Assume:~~

$\ddot{\theta} = 0$

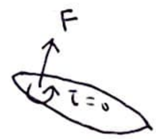
$\ddot{z} = 0 \quad \dot{z} = 0 \Rightarrow z \neq z_{cop} = h$

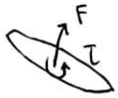
↑ height

Zero-moment Point ("ZMP")



special points





total force

total torque = 0 w.r.t. ZMP

in flat ground walking

X_{cop} ~~X_{com}~~ is a zero-moment point

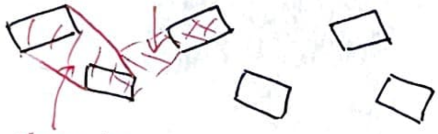
Three stage walking plan/control

1) Footstep planning

2) Plan CoM/CoP

3) Fill in the details $(q, \dot{q})$

after changed feet



x_{cop} in convex hull

$F_{i,z} \geq 0$

$\Rightarrow x_{cop} \in$ convex hull (p)

"support polygon"

cop traj

com traj according to dynamics

faster robots closer to center line

Footstep Planning

kinematic feasible regions

another region → Mix-integer convex opt

convex approximation

select ~~each~~ which region to put the foot

other ~~approaches~~ approaches: sampling-based

nonlinear opt

MI - convex opt

} simple models

simplified kinematics

almost no dynamics

Planning w/ more dynamics

$\tau = r \times f$ bilinear terms, non convex

angular momentum

Little dog planning

three
~~three~~ contact regions

divide free space into convex regions

note com/cop trajectory
may failed to generate joint contrals
for full nonlinear optimization
but most succeed ?

2019 receipe for reliable

constant com height → no impact events
linear com dynamics → com planning as convex opt

conter of mass controls ground reaction forces