

# Lecture 17: "Complete" Planning

AI / graph-search community

Def. complete if guaranteed to find a solution  
if one exists

"global optimality" if it finds the optimal plan

1) Decompose non-convexity in problem, then  
search all decompositions

2) Randomized motion planning



$x_{goal}$

Polygons are

$$P. \{x \mid Ax \leq b\}$$

$$x[n+1] = Ax[n] + Bu[n]$$

$$\min_u \sum (x[n] - x_{goal})^T Q (x[n] - x_{goal}) + \dots$$

$$s.t. \quad x[n+1] = \overset{Ax[n]}{\cancel{Ax[n]}} + Bu[n]$$

Exterior:

$$a_1 x \geq b_1 \quad \text{OR} \quad a_2 x \geq b_2 \quad \text{OR} \quad a_3 x \geq b_3$$

Disjunctive constraints

w/ integer variables (binary)

$$C[n] \in \{0, 1\}$$

$$\sum C_i[n] \leq m-1$$

$$a_1 x \geq b - C_1[n] M$$

↑ large number

at least one

$C_1[n] = 0$  constraint on

constraint on

$C_1[n] = 1$  constraint of +

nonlinear dynamics

$$x[n+1] = f(x[n], u[n])$$

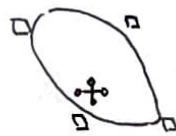
use linear dynamics to bound nonlinear constraints

$$x[n+1] \in \text{convex region around } f(x[n], u[n])$$

e.g., linear approximation bound

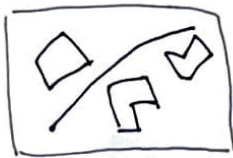
IRIS (Fast approximate convex segmentation)

Find convex region object is inside



ellipsoid approximation

Randomized motion planning

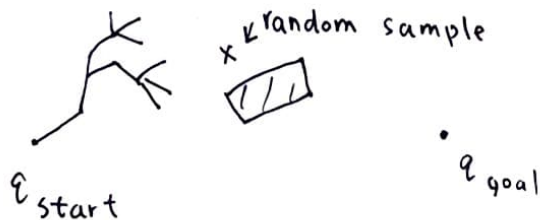


kinematics

collision checker

"sample-based planner"

RRT (Rapidly-exploring Random Tree)

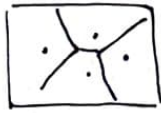


1) Random sample

2) find closest point on current tree

3) grow it towards sample point

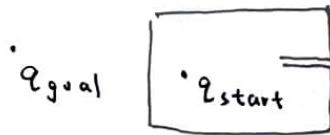
RRT has a Voronoi bias



boundary has equal distances to two points

prob  $\propto$  |region of voronoi|  
↑  
to select a point

compared to ~~na~~ grid search



narrow passage makes grid search not complete

reject collision config

→ uniform distribution in free space

→  $t \rightarrow \infty$  uniform distribution over free space

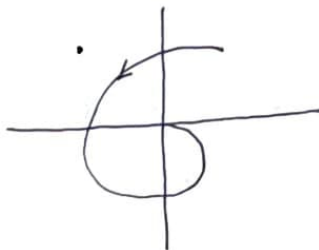
→ probabilistic complete

On humanoid James Kuffner sleek

reject not quasi-static configurations

pendulum dynamics example

sample ~~number~~ <sup>number</sup> very high



$$\dot{x} = f(x, u)$$

Euclidean distance does not work

Kinodynamic planners

(control-based planners

OMPL

- 1) Better sampling distributions
- 2) Better distance metric
- 3) Better extend (e.g., trajectory optimization?)

Approximate

"Distance" metrics

Min time to go (boundary value problem)

don't satisfies distance metric

cost to go

LQR RRT (quick cost calculation)

Unconstrained dynamics  $\rightarrow$  Value-iteration finds optimal policy

use cost to go to guide RRT search

Reachability - Guided RRT

Sample configs likely to be reachable

Build two trees

For every node in main tree



grow natural candidates

$\hookrightarrow$  reachability tree

closer to reachability tree  $\checkmark$

closer to original tree  $\times$

## RG-RRT

sample main RRT tree

extend each node to several reachable nodes

new sample

closer to reachable nodes - tree ✓

closer to main tree x reject

biased samples towards reachable regions

## RG-RRT

changed Voronoi bias

only from Voronoi regions for which differential constraints permit the expansion of the node towards sample

after the <sup>closest</sup> ~~closest~~ point is identified  $x_{new}^r$  ← reachable

it is added to main tree + <sup>reachable</sup> ~~reachable~~ points