Today: Feedback of Lyapunov Through Contact

Last time:

$$x^+ = \Delta(x^-)$$

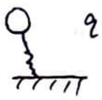
"mode 1" "mode 2"

$\phi(x,u) = 0$

"guard"

minimal coordinates

$$\dot{x}_1 = f_1(x_1, u) \qquad \dot{x}_2 = f_2(x, u)$$

 $q = \begin{bmatrix} x \\ y \end{bmatrix}$  $q = \begin{bmatrix} r \\ \theta \end{bmatrix}$

maximal coordinates (hybrid case)

mode 1

$$\dot{x} = f(x, u, \lambda)$$

contact constraint forces

$$g_1(x, u, \lambda) = 0$$

$\lambda = 0$

mode 2

$$\dot{x} = f(x, u, \lambda)$$

$$g_2(x, u, \lambda) = 0$$

foot(q) = ~~constraint~~ constant $\in \mathbb{R}^{2\times 1}$

hard contact: $\frac{d}{dt}\,\text{foot}(q, \dot{q}) = 0$

$$g_2(x, u, \lambda) \Rightarrow \frac{d^2}{dt^2}\,\text{foot}(q, \dot{q}, u, \lambda) = 0$$

soft contact: $\lambda = -k\,\text{foot}(q) + \cdots$

friction most cases fits this framework

maximal coordinates (complementarity)

$$\dot{x} = f(x, u, \lambda)$$

hard contact: $\phi_i(q) \geq 0$, $\lambda_i \geq 0$

$\phi_i(q) \cdot \lambda_i = 0$ [either $\phi_i(q) = 0$ or $\lambda_i = 0$]

soft contact: $\lambda = \begin{cases} 0 & \phi(q) = 0 \\ -k\phi(q) & \phi(q) \leq 0 \end{cases}$

traj optimization w/ mode sequence

works very well practical

w/o mode sequence

your mileage may vary

Sums-of-squares for Lyapunov analysis through contact

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} = \tau_g(q) + Bu + \sum_i J_i^T(q)\lambda_i$$

$\forall x, \lambda$ $V(q, \dot{q}) \geq 0$, $\dot{V}(q, \dot{q}) \leq 0$

S-procedure/ $V(q, \dot{q}) \leq \rho$, $\lambda \geq 0$

Lagrangian multiplier $\phi(q) \geq 0$

$\phi(q) \cdot \lambda = 0$ also polynomial

system with frictions, only stability in Lyapunov in most of the time, not asy or exp stability

passive systems works, with controller may not work

Stabilize a fixed pt. through contact

Ex. compass gait balancing

pin joint (minimal coordinates)

+ LQR works

($\equiv$ acrobat)

maximal flooting coordinate

$\dot{x} = f(x, u, \lambda)$

$g(x, u, \lambda) = 0$ foot(q)

can solve for λ, insert in

$\Rightarrow \dot{x} = f(x, u)$ $x \in \mathbb{R}^4$

$\approx Ax + Bu$

LQR (A, B, Q, R) will fail

(not controllable)

L.P. MPC will succeed from valid x

(w/ foot(q) = 0)

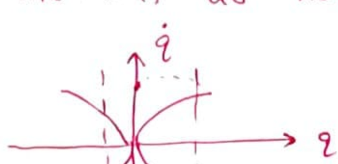
Equality Constrained LQR

$\dot{x} = Ax + Bu$

know additionally that $Gx = 0$

can project dynamics into nullspace of the (linearized) constraint

Note these methods do not stabilize across contacts



cannot stabilize when hit the wall

Local stabilize across modes as a mixed-integer optimization

$$\min_{x[\cdot], u[\cdot]} \sum x^T Q x + u^T R u$$

$$\text{s.t.} \quad x[n+1] = A x[n] + B u[n]$$

$$x[0] = x_0$$

$$x[N] = x_{GOAL}$$

now

$$x[n+1] = A^i x[n] + B^i u[n] + c^i \quad , \quad \text{when } D^i x + F^i u = 0$$

one way to write the problem

$$\min_{x[\cdot], u[\cdot], b[\cdot]}$$

$$x[n+1] = A x[n] + B u[n] + B_\lambda \lambda[n] + c \ldots$$

$$\lambda \geq 0 \qquad G x[n] \geq 0$$

$$\lambda_i[n] \leq b_i M$$

big positive number

"big M"

"mixed-integer optimization"

$$G_i x[n] \leq (1 - b_i) M$$

$$b_i \in \{0, 1\}$$

$$\min_x f(x)$$

$$\text{s.t.} \quad g(x) \leq 0$$

$$x_i \in \mathbb{Z}$$

Non-convex optimization always

"Mixed-integer convex" iff relaxation convex

Relaxation gives lower bounds

$\Rightarrow$ effective branch-and-bound search

A computational bottleneck

var : 2 x # of potential contacts x # timesteps

Tight formulations for PWA MPC

leverage results of disjunctive programming

Approximate Explicit MPC

still cannot achive real-time rates