# Definitions

Nonlinear differential equs

$$\dot{x} = f(x, u)$$

$u$ ← vector of control inputs

$x$ ← vector of ~~stat~~ state

$\dot{x}$ ← $\frac{d}{dt}x$

For mechanical systems ($F = ma$)

$q$ – position variables (joint angles, etc)

$\dot{q}$ – velocities

$\ddot{q} = f(q, \dot{q}, u)$ second-order diff equ

$$\underbrace{\ddot{q}}_{\mathbb{R}^n} = f_1(q, \dot{q}) + \underbrace{f_2(q, \dot{q})}_{\mathbb{R}^{n \times m}} \underbrace{u}_{\mathbb{R}^m}$$ "control affine"

**Def** A system is fully-actuated in state $(q, \dot{q})$ if $f_2(q, \dot{q})$ is full row rank

**Def** underactuated in $(q, \dot{q})$ if $\text{rank}[f_2(q, \dot{q})] < n$

may be state dependent,

# Feedback Cancellation

Given $\ddot{q} = f_1(q, \dot{q}) + f_2(q, \dot{q})u$ $f_1, f_2$ are known

~~Consider the~~ ~~Consider the~~

Consider the control law

$$u = f_2^{-1}(q, \dot{q})\left(\ddot{q}^d - f_1(q, \dot{q})\right)$$

$\Rightarrow \quad \ddot{q} = \ddot{q}^d$ "feedback equivalent" to $\ddot{q} = u$

$f_2^{-1}(q, \dot{q})$ exists

| vs Feedback linearization

not the same

Defn

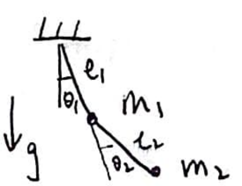
$$\ddot{q} = f(q, \dot{q}, u)$$

Input limits

State constraints

Model uncertainty / state estimation

Eqs of motion




Kinematics

$$P_1 = \begin{bmatrix} l_1 \sin\theta_1 \\ -l_1 \cos\theta_1 \end{bmatrix} = \begin{bmatrix} l_1 S_1 \\ -l_1 C_1 \end{bmatrix}$$

$x_1$ of $m_1$

$$P_2 = P_1 + \begin{bmatrix} l_2 \sin(\theta_1 + \theta_2) \\ -l_2 \cos(\theta_1 + \theta_2) \end{bmatrix}$$

Kinematic energy $T = \frac{1}{2} \dot{P}_1^T m_1 \dot{P}_1 + \frac{1}{2} \dot{P}_2^T m_2 \dot{P}_2$

Potential energy $U = m_1 g Y_1 + m_2 g Y_2$

$\Rightarrow$ Lagrangian

$$(m_1 + m_2) l_1^2 \ddot{q}_1 + m_2 l_2^2 (\ddot{q}_1 + \ddot{q}_2) + m_2 l_1 l_2 (2\ddot{q}_1 + \ddot{q}_2) C_2$$
$$- m_2 l_1 l_2 (2\dot{q}_1 + \dot{q}_2) \dot{q}_2 S_2 + (m_1 + m_2) l_1 g S_1$$
$$+ m_2 g l_2 S_{1+2} = \tau_1$$

$$m_2 l_2^2 (\ddot{q}_1 + m_2 l_1 l_2 \ddot{q}_1 c_2 + m_2 l_1 l_2 \dot{q}_1^2 \cancel{q_2} S_2 + m_2 g l_2 S_{1+2} = \tau_2$$

## The manipulator eqs

$$ma = \sum_i F_i$$

$$\underbrace{M(q)}_{\text{mass/inertia matrix}}\ddot{q} + C(q,\dot{q})\dot{q} = \tau_g(q) + B(q)u$$

$B(q)$ ← map from control inputs to $q$

$$T = \frac{1}{2}\dot{q}^T M(q)\dot{q}$$

$M(q)$: positive definite, invertible

$$\ddot{q} = M^{-1}(q)\left[\tau_g(q) + B(q)u - C(q,\dot{q})\dot{q}\right]$$

$$f_2(q,\dot{q}) = M^{-1}(q)\,B(q)$$

$B(q)$ full-row rank ⇒ fully-actuated

underactuated iff $\text{rank}[B] < \dim(q)$

dimension $B(q)$: $\dim(q) \times \dim(u)$

$\dim(q)$: degree of freedom; $\dim(u)$: control inputs

if $\dim(u) < \dim(q)$, ~~under articulated~~ under actuated

## Feedback Cancellation of Double Pendulum

target single pendulum dynamics:

$$\ddot{\theta}_1 = -\frac{g}{l}\sin\theta_1 - b\dot{\theta}_1$$

$$\ddot{\theta}_2 = 0$$

$$\bar{u} = B^{-1}\left[C\dot{q} - \tau_g + M\begin{bmatrix} -\frac{g}{l}s_1 - b\dot{q}_1 \\ 0 \end{bmatrix}\right]$$

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