Quiz 1 for MATH 226, section 39559

Name: Date:

Problem 1: Show that the following equation represents a sphere in \mathbb{R}^3 , and find its center and radius:

$$x^2 + y^2 + z^2 - 10x + 2z - 3 = 0.$$

(Hint: $x^2 + 2ax = (x+a)^2 - a^2$.)

Problem 2: Consider the 2-dimensional vectors $\mathbf{a} = \langle 4, 1 \rangle$ and $\mathbf{b} = \langle 1, -2 \rangle$. Draw \mathbf{a} , \mathbf{b} , and $\mathbf{a} + \mathbf{b}$, illustrating the triangle law. Compute the length $|\mathbf{a} + \mathbf{b}|$.

Quiz 2 for MATH 226, section 39559

Date:

Name:

Problem 1: Consider the triangle ABC in \mathbb{R}^3 with vertices A = (1,3,2), B = (5,-1,0), and C = (0,2,-1). What is the angle of this triangle at the vertex A?

Problem 2: Compute the area of the triangle ABC defined in Problem 1, and find a vector orthogonal to the plane through A, B, and C.

Quiz 3 for MATH 226, section 39559

20 minutes; use back if necessary

Name:	Date:

Problem 1: Define L to be the line through P = (3, 1, 2) and Q = (-1, 0, 1). Express L in terms of either a vector equation or parametric equations. Where does L intersect the plane defined by x - y + z = 3?

Problem 2: Draw the surfaces defined by $x^2 + z^2 = 4$ and $y^2 + z^2 = 1$ in \mathbb{R}^3 . Draw their curve(s) of intersection, and find vector function(s) that represent these curve(s).

Quiz 4 for MATH 226, section 39559

15 minutes; use back if necessary

Date:

Problem 1: Consider $\mathbf{r}(t) = \langle e^t \cos t, e^t \sin t \rangle$. Compute the arclength function s(t) starting from t = 0. Reparametrize \mathbf{r} with respect to arclength, again starting from t = 0.

Problem 2: Consider $f(x,y) = \sqrt{x} + y$ and $g(x,y) = ye^x$. What are the domain and range of f and of g? Draw a contour plot for each function that shows a few level curves. For each level curve, provide the corresponding equation.

Quiz 5 for MATH 226, section 39559

15 minutes; use back if necessary

Name: Date:

Problem 1: If $\cos(xyz) = 1 + x^2y^2 + z^2$, find $\partial z/\partial x$ and $\partial z/\partial y$.

Problem 2: Compute the linearization of $f(x, y, z) = x^2 e^{xy+z^2}$ at (x, y, z) = (1, 0, 0). Use this to estimate f(0.9, -0.1, -0.2).

Quiz 6 for MATH 226, section 39559

15 minutes; use back if necessary. Due October 9, 2020.

Name: Date:

Problem 1: Using whichever method you like, find the point on the plane x - y + 3z = 1 closest to the origin.

Problem 2: Compute the volume under the surface z=x/y and above the rectangle $R=[0,2]\times[1,3]$ (i.e. $R=\{0\leq x\leq 2, 1\leq y\leq 3\}$) in the xy-plane.

Quiz 7 for MATH 226, section 39559

20 minutes. Due October 16, 2020.

Name: Date:

Problem 1: Calculate the iterated integral $I = \int_0^1 \int_{\sqrt{y}}^1 \frac{ye^{x^2}}{x^3} dx dy$ by first reversing the order of integration.

Problem 2: Evaluate the following two integrals:

- (a) $I = \iint_D (x^2 + y^2)^{3/2} dA$, where D is the region in the first quadrant bounded by y = 0, y = x, and $x^2 + y^2 = 9$. (b) $I = \int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} (x^3 + xy^2) dy dx$.

Quiz 8 for MATH 226, section 39553

15 minutes; use back if necessary

Name: Date:

Problem 1: Evaluate $I = \iiint_E e^{(x^2+y^2+z^2)^{3/2}} dV$, where E is the portion of the unit ball $x^2+y^2+z^2 \leq 1$ that lies in the first octant (i.e. the region with $x \geq 0, y \geq 0, z \geq 0$).

Problem 2: Evaluate $I = \iiint_E y \, dV$, where E is the solid that lies under $z = 1 - x^2 - y^2$, above the xy-plane, and in the half-space $y \ge 0$.

Quiz 9 for MATH 226, section 39559

15 minutes

Name: Date:

Problem 1: Using whichever technique you like, evaluate the integral

$$\int_C \sqrt{1+x^3} \, dx + 2xy \, dy,$$

where C is the triangle with vertices (0,0), (1,0), and (1,3).

Problem 2: Let **F** be a constant vector field $\mathbf{F}(x,y) = \langle a,b \rangle$, for a,b two real numbers. Compute the integral of **F** over the circle $x^2 + y^2 = 1$, traversed counterclockwise.

MATH 226 Final Exam SPRING 2018

Last Name:	First Name:	
Signature:	Student ID:	
D. Crombecque 9am G. Dreyer 11am	D. Crombecque 10am G. Dreyer 1pm	J-M. Leahy 12pm

Directions. Fill out your name, signature and student ID number on the lines above before starting the exam! Also, check the box next your professor's name.

- You must show all your work and justify your methods to obtain full credit. State any theorems that you use. Clearly indicate your final answers.
- Simplify your answers to a reasonable degree. Any fraction should be written in lowest terms. You need not evaluate expressions such as $\ln 5$, $e^{0.7}$ or $\sqrt{229}$.
- You may use the SINGLE sided HANDWRITTEN sheet of notes that you brought with you. This may be no more than one sheet of $8\frac{1}{2}'' \times 11''$ paper. You may have anything written on one side of it, but it must be written in your own handwriting. No other notes or books are allowed during the test.
- No calculators or other electronic devices are allowed. Turn off your cell phone.

• DO NOT WRITE OUTSIDE THE DESIGNATED BOXES.

• Remember, USC considers cheating to be a serious offense; the minimum penalty is failure for the course. Cheating includes "straying eyes" and failing to stop writing when told to do so at the end of the exam.

Problem	Points	Score	Problem	Points	Score	Problem	Points	Score
1	10		4	15		7	10	
2	15		5	20		8	25	
3	20		6	15		9	20	
Subtotal	45		Subtotal	50		Subtotal	55	

Total	
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Problem	1.	Consider	the	two	planes

$$P_1: x + y - z = 2$$
 and $P_2: 3x - 4y + 5z = 6$.

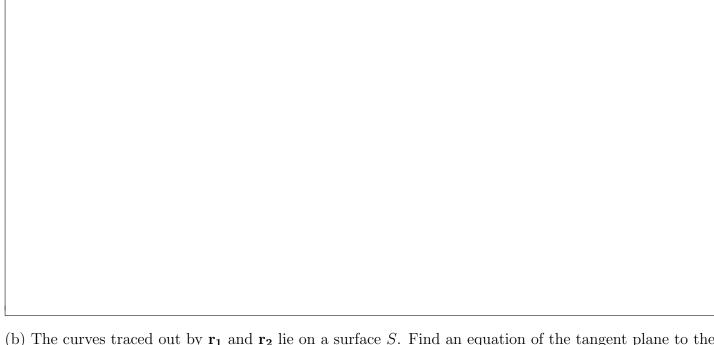
(a) Find the parametric equations of their line of intersection.					
) Find the angle between the two planes.					

Problem	2.	Consider	the	two	parametrized	curves
LIODICIII	⊿.	Consider	ULIC	UWU	parametrized	cur ves

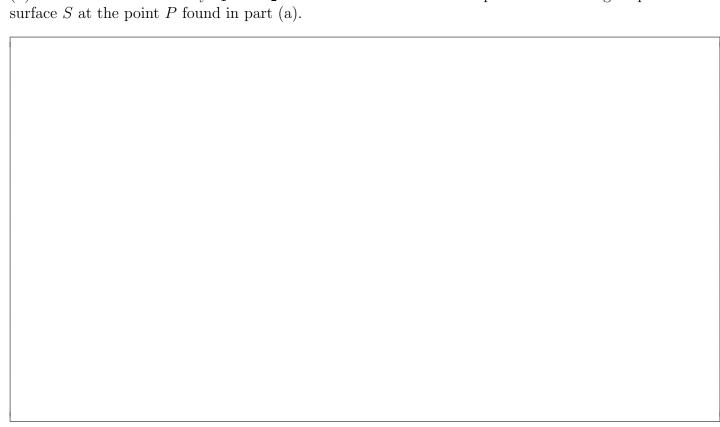
$$\mathbf{r_1}(t) = \langle 1 + t^2, 2 - t, t^4 + 3t^2 - 4t + 4 \rangle$$
 and $\mathbf{r_2}(u) = \langle u^2, 3 - u, u^4 + u^2 - 6u + 8 \rangle$,

where t and u are in \mathbb{R} .

((a)	Find	the	coordinates	of	the	point	of	intersection	P	of	the	two	curves.
١	(~)	11114	OIL	COOL GILLGOOD	01	OILO	POILE	01	11100100001011	-	01	OII	0110	Carton



(b) The curves traced out by $\mathbf{r_1}$ and $\mathbf{r_2}$ lie on a surface S. Find an equation of the tangent plane to the



Problem 3 (continued). b) Let E be the solid bounded from below by the cone $z = \sqrt{x^2 + y^2}$ and above by the sphere $x^2 + y^2 + (z - 2)^2 = 2$. Set up, BUT DO NOT EVALUATE, a triple integral in CYLINDRICAL COORDINATES that yields the volume of the solid E .						

$\sin(yz) - x^2z = 2.$
(a) Find the values of $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at the point $(1,0,-2)$ on S .

Problem 4. Consider the surface S defined by the equation

Proble	em 4 (continue	ed).					
(b) App	proximate the	value of z whe	en x = 1.1 and	1 y = -0.3.			
(c) Cor	nsider a path 1	$\mathbf{r}(t) = \langle x(t), y(t) \rangle$	$\langle t), z(t) \rangle$ lying	on the surface	e S such that \mathbf{r}	$c(0) = \langle 1, 0, -2 \rangle.$	Assume
that $\frac{dx}{dt}$	$\frac{d}{dt}(0) = -5 \text{ and }$	$\frac{dg}{dt}(0) = 5. \text{ Fi}$	ind the value	of $\frac{dz}{dt}(0)$.			

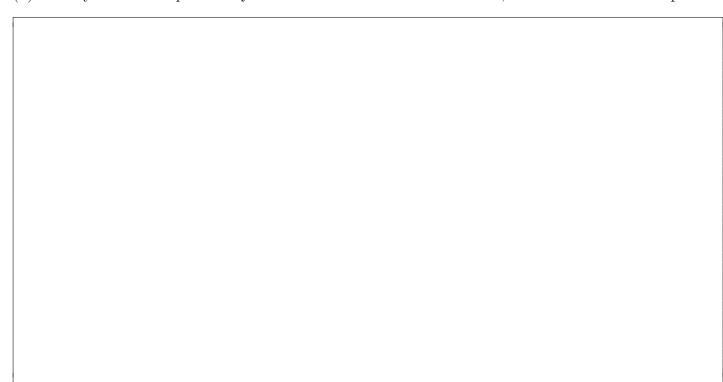
Problem 5. Consider the function

$$f(x,y) = \frac{xy^2}{2} + \frac{x^3}{3} - x.$$

on the domain $\mathcal{D} = \{(x, y) \in \mathbb{R}^2 : x^2 + \frac{y^2}{2} \le 3\}.$

(a) Find the critical points of f contained in the interior of \mathcal{D} .

(b) Classify the critical points of f	f in the interior of $\mathcal D$ as local m	axima, local minima or saddle points.



Problem 5 (continued).					
(c) Find the absolute minimum and maximum values of f on \mathcal{D} .					

	$\mathbf{F}(x,y,z) = \langle 2xyze^{x^2}$	$e^{2y}, z^2 + x^2 z e^{x^2 y}, e^{x^2 y} + 2yz - 2yz$	$-3z^2\rangle$.
For ALL possible sevaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.	mooth curves C initiating fr	om the point $(0, -1, 1)$ and e	ending at the point $(\sqrt{\ln(2)}, 1, 1)$

a) Let $G(x,y)$	z = -zi + 1	Ranetrize	the surface S	to evaluate f	f G. dS Th	ere is extra spa
rovided the ne	ext page.	. I diametize	one surface D	o cvarianc J.	$JS \hookrightarrow aS. 111$	ere is exira spa
	1.40					

Problem 8 (a) (extra	a space).		

Problem 8 (continued).					
Let $\mathbf{F}(x, y, z) =$ eld \mathbf{F} and the surfa	$x\mathbf{j} + xz\mathbf{k}$. Verify to S described in t	that $\operatorname{Curl} \mathbf{F} = \mathbf{G}$ the previous page	and then verif	y Stoke's Theore	em for the vec

Problem	Q	Consider	the	vector	field	
r romem	9.	Consider	ыне	vector	пеп	

$$\mathbf{F}(x, y, z) = (xy^2 + e^z)\mathbf{i} + (yz^2 + \tan(zx))\mathbf{j} + (\ln(y^2 + x^2) + zx^2)\mathbf{k}.$$

Use the method of your choice to evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where S is the closed boundary of the solid E bounded from below by the cone $z = \sqrt{x^2 + y^2}$ and above by the sphere $x^2 + y^2 + z^2 = 2z$. with outward orientation. There is extra space provided on the next page.

orientation. There is extra space provided on the next page.					

oblem 9 (extra space).					