Predicate logic and Quantifiers
Understand enough predicate logic to make sense of quantified statements.
In predicate logic, propositions depend on variable x, y, z, so their truth value may change depending on which values th
Introduce quantifiers $\exists$ existential quantifier
$\exists x P(x)$
$\exists x P(x)$ is true if $P(x)$ is true or some value of x; it is false otherwise.
$\forall$ universal quantifier
$\forall x P(x)$
$\forall x P(x)$ is true if $P(x)$ is true for all allowable values of x. It is false otherwise.
$\exists!$ for one and only one: Uniqueness Quantifier
$\exists! x P(x) or \exists_1$
$\exists! x P(x)$ is true if $P(x)$ is true for exactly one value of x and false for all often values of x; otherwise, $\exists! x P(x)$ is false.
Alternation of Quantifiers: Nested Quantifiers $\forall x \exists y \forall z P(x, y, z)$
NB: The order <u>cannot</u> be exchanged as it might modify the truth values of the statement (think of examples with two c
domain: real numbers, P(x,y):= x + y = y + x
$\forall x \forall y P(x, y) \equiv \forall y \forall x P(x, y)$
Negation of Quantifiers $\begin{matrix} \neg(\exists x P(x)) \leftrightarrow \forall x \neg P(x) \\ \neg(\forall x P(x)) \leftrightarrow \exists x \neg P(x) \end{matrix}$ Precedence of Quantifiers The quantifiers $\forall$ and $\exists$ have higher preced
When the domain of a quantifier if finite, quantified statements can be expressed using propositional logic. Binding vari
A quantifier is used on a variable x, which we say x is **bound**
No quantifier or set bounds a variable, which we say x is **free**
The part of a logical expression a quantifier is applied, which we say the part is the **scope** of the quantifier
the same letter is often used of represent variables bound by different quantifiers with scopes that do **not overlap**.
Substitution: $\exists(P(x) \wedge Q(x)) \vee \forall y R(y) \xrightarrow{y \to x} \exists(P(x) \wedge Q(x)) \vee \forall x R(x)$ Logical Equavalences Involving Quantifiers

$\forall x(P(x) \wedge Q(x)) \equiv \forall x P(x) \wedge \forall x Q(x)$
$\exists x(P(x) \vee Q(x)) \equiv \exists x P(x) \vee \exists x Q(x)$