COMP90038
Algorithms and Complexity

Lecture 21: Huffman Encoding for Data Compression

(with thanks to Harald Søndergaard & Michael Kirley)

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Recap

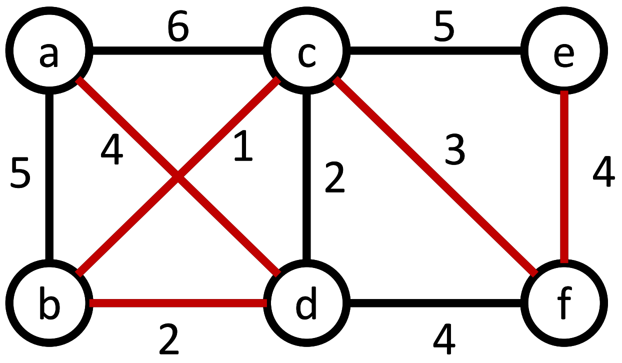
- We discussed **greedy algorithms**:
  - A problem solving strategy that takes the **locally best** choice among all feasible ones. Such choice is **irrevocable**.
  - Usually, **locally best** choices do not yield **global best** results.
  - In some exceptions a greedy algorithm is **correct and fast**.
  - Also, a greedy algorithm can provide good **approximations**.

- We applied this idea to two graph problems :
  - Prim's algorithm for finding **minimum spanning trees**
  - Dijkstra's algorithm for **single-source shortest path**

What is a Minimum Spanning Tree?

- A **minimum spanning tree** of a weighted graph $\langle V,E \rangle$ is a tree $\langle V,E' \rangle$ where E' is a subset of E, such that the connections have the lowest cost

- We use Prim's algorithm to find the minimum spanning tree.
  - It constructs a sequence of subtrees T, by **adding to the latest tree the closest node not currently on it**.



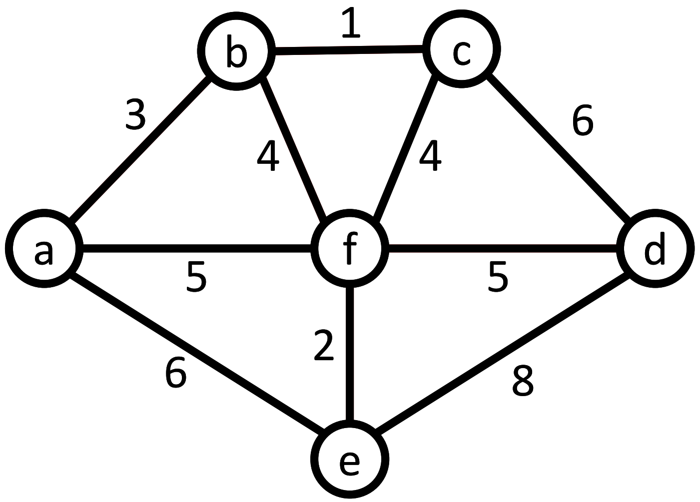
Prim's Algorithm

- We examined the complete algorithm, that uses priority queues:

function $\textsc{Prim}(\langle V, E\rangle)$
    for each $v \in V$ **do**
        $cost[v] \leftarrow \infty$
        $prev[v] \leftarrow nil$
    pick initial node v_0
    $cost[v_0] \leftarrow 0$
    $Q \leftarrow \textsc{InitPriorityQueue}(V)$ ▷ priorities are cost values
    while Q is non-empty **do**
        $u \leftarrow \textsc{EjectMin}(Q)$
        for each $(u, w) \in E$ **do**
            if $weight(u, w) < cost[w]$ **then**
                $\boxed{cost[w] \leftarrow weight(u, w)}$
                $prev[w] \leftarrow u$
                $\textsc{Update}(Q, w, cost[w])$ ▷ rearranges priority queue

Another example

- Let's work with the following graph:



a,b,c,f,e,d

Tree T		a	b	c	d	e	f
	cost	∞	∞	∞	∞	∞	∞
	prev	nil	nil	nil	nil	nil	nil
	cost	0					
	prev	nil					
	cost		3			6	5
	prev		a			a	a
	cost			1		6	4
	prev			b		a	b
	cost				6	6	4
	prev				c	a	b
	cost				5	2	
	prev				f	f	
	cost				5		
	prev				f		
	cost						
	prev						

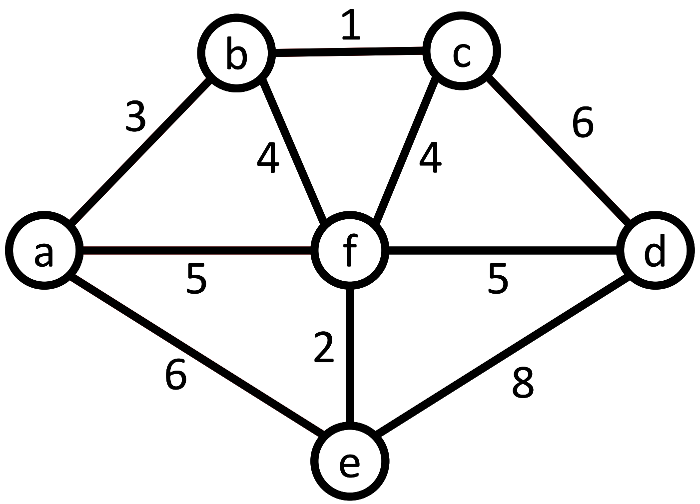
Dijkstra's Algorithm

- **Dijkstra's algorithm** finds all shortest paths **from a fixed start node**. Its complexity is the same as that of Prim's algorithm.

$$
\begin{array}{ll}
\textbf{function } \textsc{Dijkstra}(\langle V, E\rangle, v_0) & \\
\quad \textbf{for } \text{each } v \in V \textbf{ do} & \\
\quad\quad dist[v] \leftarrow \infty & \\
\quad\quad prev[v] \leftarrow nil & \\
\quad dist[v_0] \leftarrow 0 & \\
\quad Q \leftarrow \textsc{InitPriorityQueue}(V) & \triangleright \text{ priorities are distances} \\
\quad \textbf{while } Q \text{ is non-empty } \textbf{do} & \\
\quad\quad u \leftarrow \textsc{EjectMin}(Q) & \\
\quad\quad \textbf{for } \text{each } (u, w) \in E \textbf{ do} & \\
\quad\quad\quad \textbf{if } dist[u] + weight(u, w) < dist[w] \textbf{ then} & \\
\quad\quad\quad\quad \boxed{dist[w] \leftarrow dist[u] + weight(u, w)} & \\
\quad\quad\quad\quad prev[w] \leftarrow u & \\
\quad\quad\quad\quad \textsc{Update}(Q, w, dist[w]) & \triangleright \text{ rearranges priority queue}
\end{array}
$$

Another example

- Let's work with this graph again:



a,b,c,f,
e,d

Tree T		a	b	c	d	e	f
	cost	∞	∞	∞	∞	∞	∞
	prev	nil	nil	nil	nil	nil	nil
		0					
			3			6	5
			a			a	a
				4		6	5
				b		a	a
					10	6	5
					c	a	a
					10	6	
					10	6	

Data compression

- From an information-theoretic point of view, most computer files contain much redundancy.

- Compression is used to store files in less space.
  - For text files, savings up to 50 are common.
  - For binary files, savings up to 90 are common.

- Savings in space mean savings in time for file transmission.

Run-Length Encoding

- For a text with long runs of **repeated characters**, we could compress by counting the runs. For example:

AAAABBBAABBBBBCCCCCCCCDABCBAAABBBBCCCD

- can then be encoded as:

4A3BAA5B8CDABCB3A4B3CD

- This is not useful for normal text. However, for **binary files** it can be very effective.

Run-Length Encoding

```
000000000000000000000000000011111111111111000000000
000000000000000000000000001111111111111111110000000
000000000000000000000001111111111111111111111110000
000000000000000000000011111111111111111111111111000
000000000000000000001111111111111111111111111111110
000000000000000000011111110000000000000000001111111
000000000000000000011111000000000000000000000011111
000000000000000000011100000000000000000000000000111
000000000000000000011100000000000000000000000000111
000000000000000000011100000000000000000000000000111
000000000000000000011100000000000000000000000000111
000000000000000000001111000000000000000000000001110
000000000000000000000011100000000000000000000111000
011111111111111111111111111111111111111111111111111
011111111111111111111111111111111111111111111111111
011111111111111111111111111111111111111111111111111
011111111111111111111111111111111111111111111111111
011000000000000000000000000000000000000000000000011
```

Variable-Length Encoding

- Fixed-length encoding uses a static number of symbols (bits) to represent a character.
  - For example, the ASCII code uses 8 bits per character.

- Variable-Length encoding assigns shorter codes to common characters.
  - In English, the most common character is **E**, hence, we could assign **0** to it.
  - However, no other character code can start with **0**.

- That is, no character's code should be a prefix of some other character's code (unless we somehow put separators between characters, which would take up space).

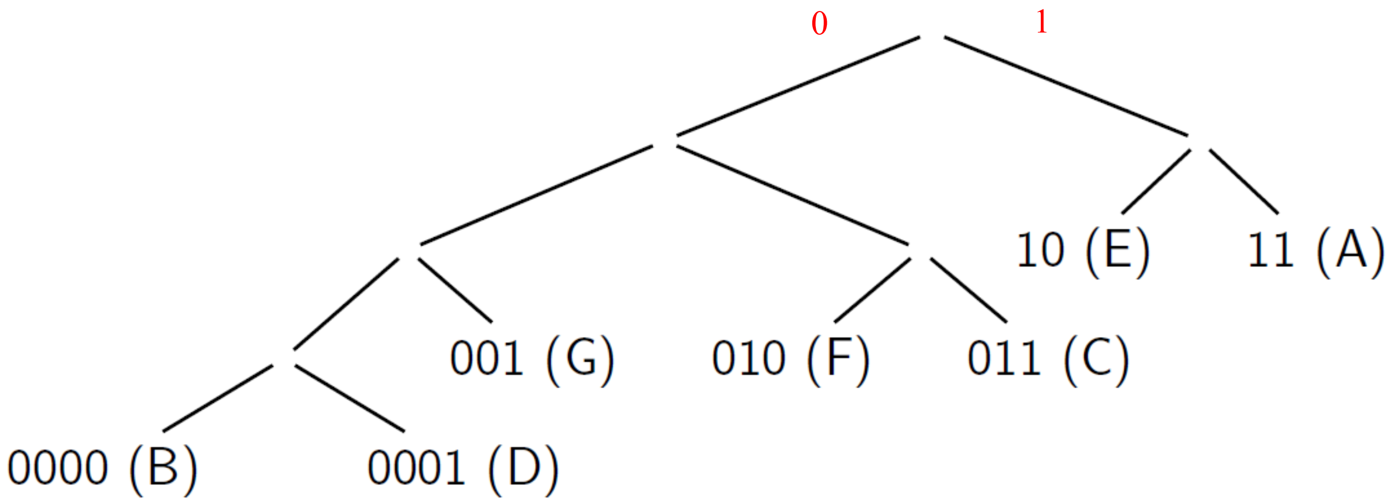
Variable-Length Encoding

- Suppose our alphabet is {A,B,C,D,E,F,G}
- We analyzed a text and found the following number of occurrences
- The last column shows some sensible codes that we may use for each symbol
  - Symbols with higher occurrence have shorter codes

SYMBOL	OCCURRENCE	CODE
A	28	11
B	4	0000
C	14	011
D	5	0001
E	27	10
F	12	010
G	10	001

Tries for Variable-Length Encoding

- A **trie** is a binary tree used on search applications
- To search for a key we look at individual **bits** of a key and descend to the **left** whenever a bit is **zero** and to the right whenever it is **one**
- Using a trie to determine codes means that no code will be the prefix of another



Encoding messages

- To encode a message, we just need to concatenate the codes. For example:

F	A	C	E
010	11	011	10

B	A	G	G	E	D
0000	11	001	001	10	0001

- If we were to assign three bits per character, FACE would use 12 bits instead of 10. For BAGGED there is no space savings

SYMBOL	CODE
A	11
B	0000
C	011
D	0001
E	10
F	010
G	001

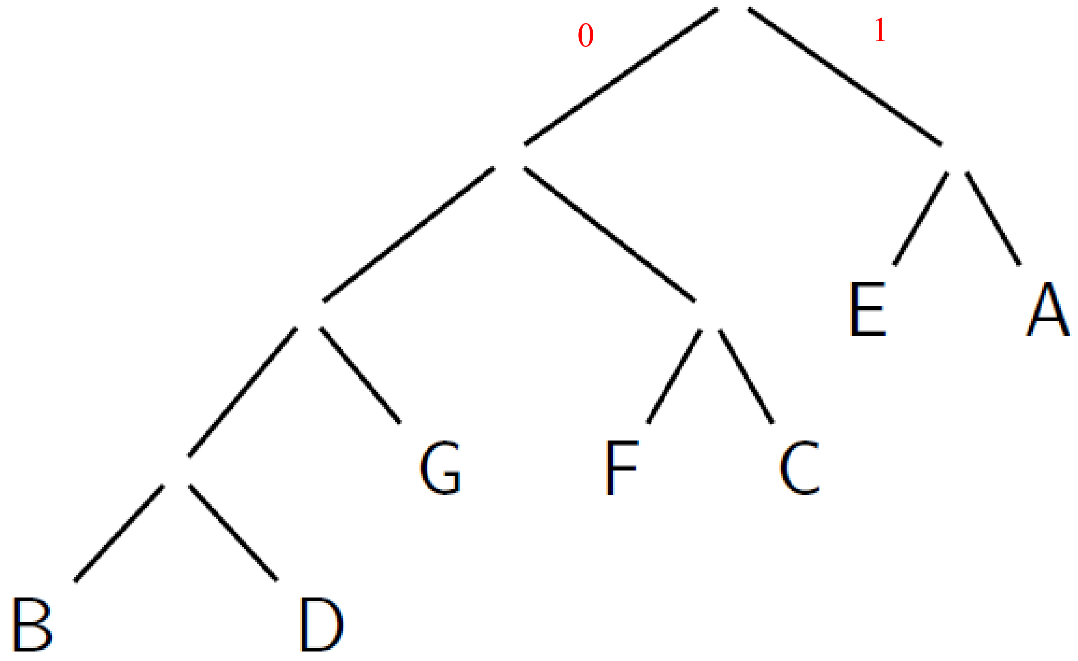
Decoding messages

- Try to decode **00011001111010** and **000011000100110** using the trie
  - Starting from the root, print each symbol found as a leaf
  - Repeat until the string is completed

DECAF

BADGE

- Remember the rules: Left branch is 0, right branch is 1



Huffman Encoding: Choosing the Codes

- Sometimes (for example for common English text) we may know the frequencies of letters fairly well.

- If we don't know about frequencies then we can still count all characters in the given text as a first step.

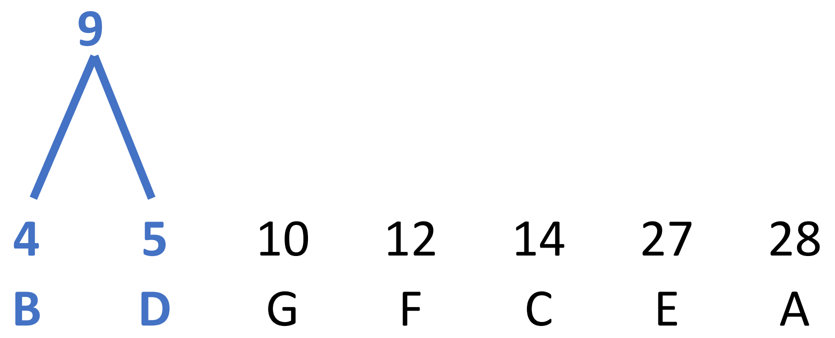
- But how do we assign codes to the characters once we know their frequencies?
  - By repeatedly selecting the two smallest weights and fusing them.

- This is **Huffman's algorithm** – another example of a **greedy method**.
  - The resulting tree is a **Huffman tree**.

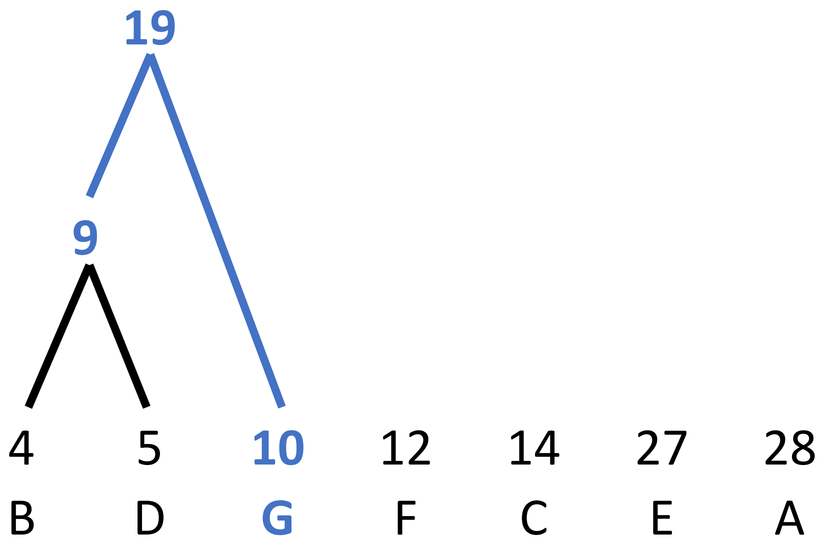
Huffman Trees (example)

4	5	10	12	14	27	28
B	D	G	F	C	E	A

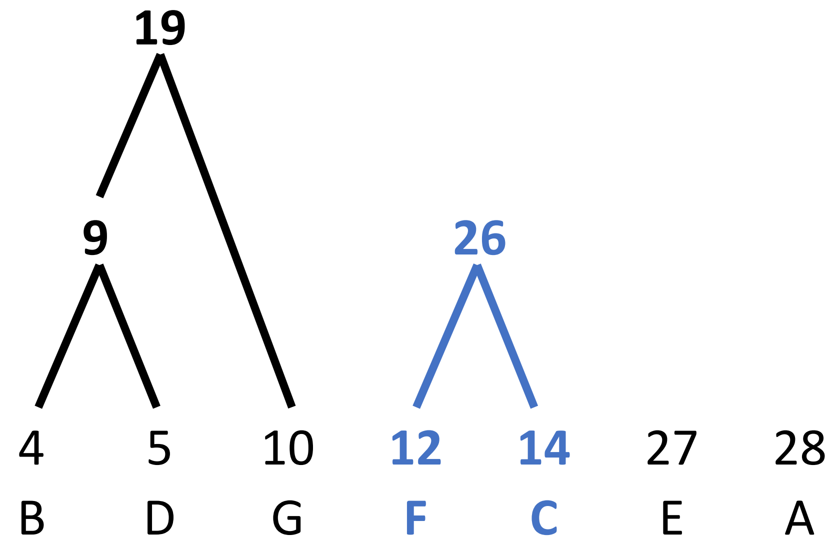
Huffman Trees (example)



Huffman Trees (example)



Huffman Trees (example)



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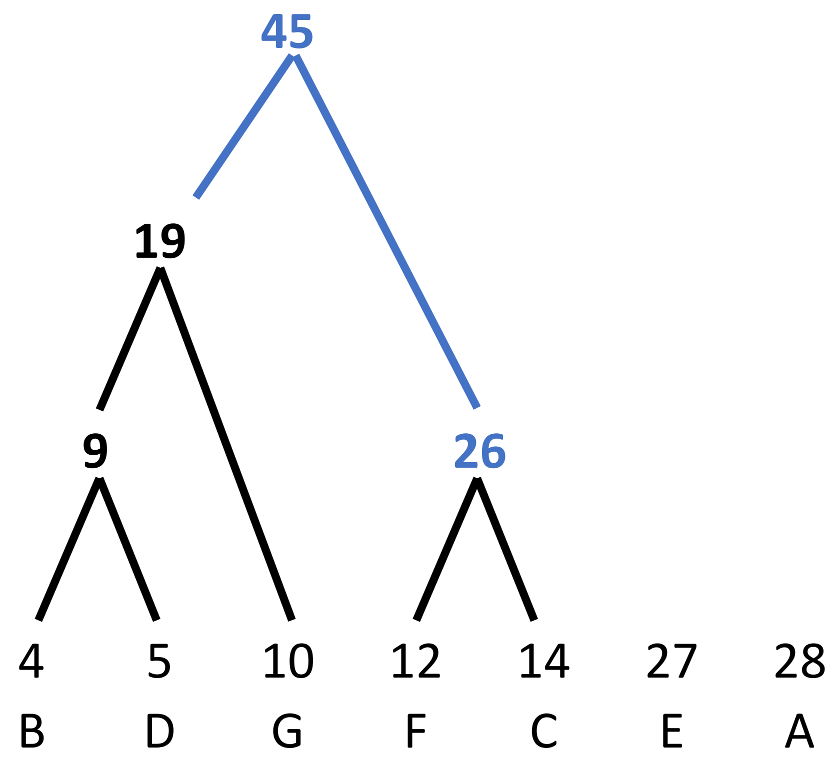
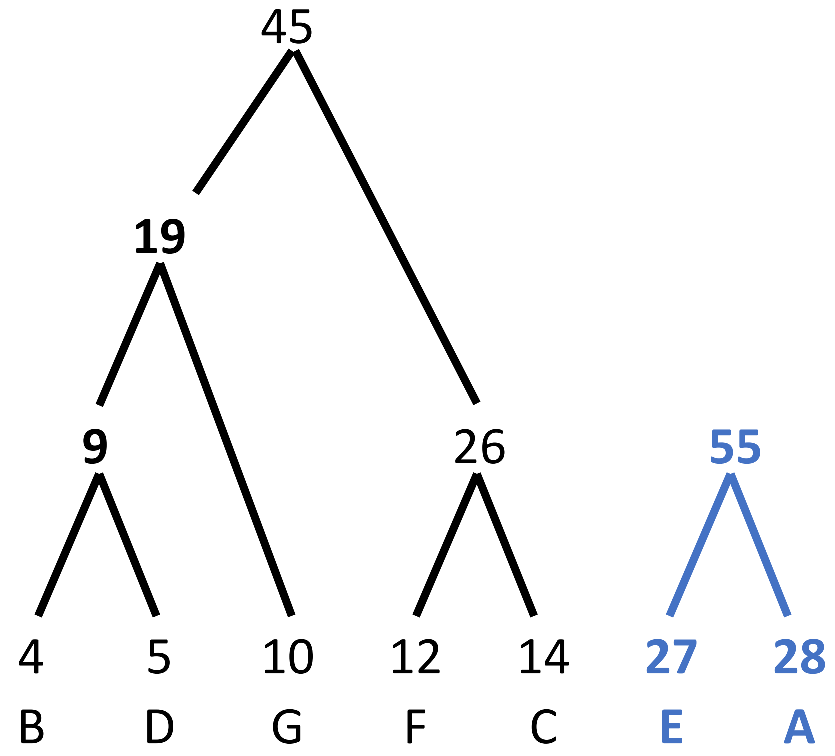
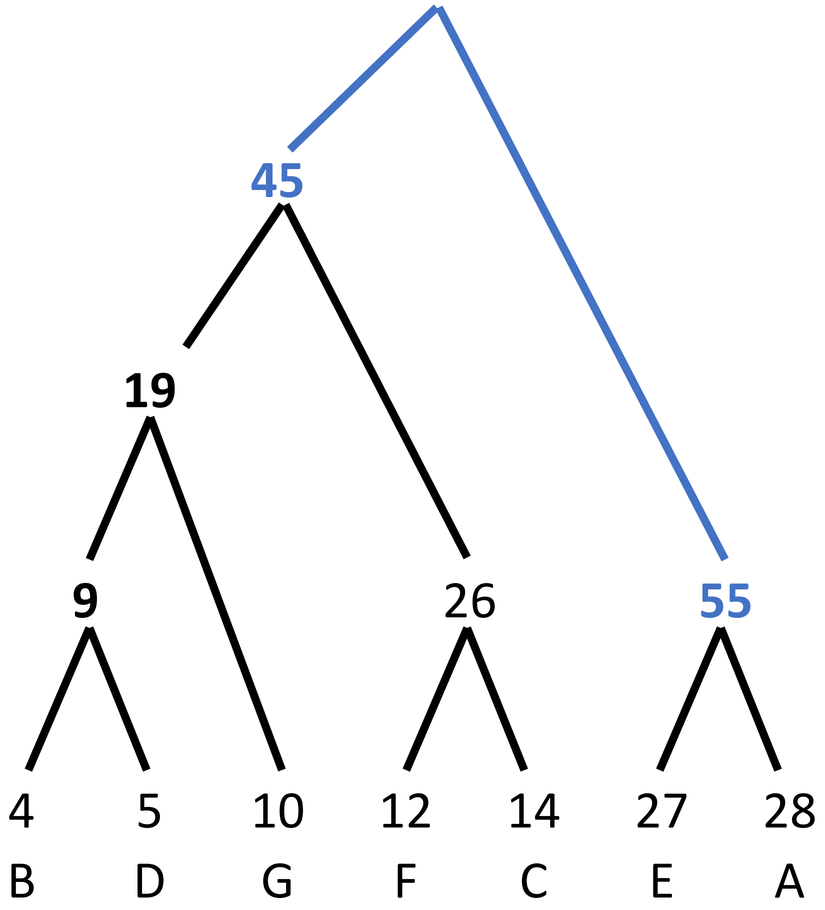


Huffman Trees (example)



Huffman Trees (example)



An exercise

- Construct the Huffman code for data in the table.
- Then, encode **ABACABAD** and decode **100010111001010**

ABACAD_

SYMBOL	FEQUENCY	CODE	
A	0.40		1
B	0.10		000
C	0.20		011
D	0.15		001
_	0.15		010

B D _ C A

Compressed Transmission

- If the compressed file is being sent from one party to another, the parties must agree about the codes used.
  - For example, the trie can be sent along with the message.

- For long files this extra cost is negligible.

- Modern variants of Huffman encoding, like **Lempel-Ziv compression**, assign codes not to individual symbols but to sequences of symbols.

Next lecture

- We briefly discuss complexity theory, NP-completeness and approximation algorithms
- On the final week we will devote time to review all the content