Shape Fragments

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ABSTRACT

In constraint languages for RDF graphs, such as ShEx and SHACL, constraints on nodes and their properties in RDF graphs are known as "shapes". Schemas in these languages list the various shapes that certain targeted nodes must satisfy for the graph to conform to the schema. Using SHACL, we propose in this paper a novel use of shapes, by which a set of shapes is used to extract a subgraph from an RDF graph, the so-called shape fragment. Our proposed mechanism fits in the framework of Linked Data Fragments. In this paper, (i) we define our extraction mechanism formally, building on recently proposed SHACL formalizations; (ii) we establish correctness properties, which relate shape fragments to notions of provenance for database queries; (iii) we compare shape fragments with SPARQL queries; (iv) we discuss implementation options; and (v) we present initial experiments demonstrating that shape fragments are a feasible new idea.

CCS CONCEPTS

• **Information systems** → **Semantic web description languages**; **Query languages**; • **Theory of computation** → **Data provenance**.

KEYWORDS

data on the Web, Linked Data Fragments, SHACL, provenance

PVLDB Reference Format:
Thomas Delva, Anastasia Dimou, Maxime Jakubowski, and Jan Van den Bussche. Shape Fragments. PVLDB, 15(X): XXX-XXX, 2021.
doi:XX.XX/XXX.XX

PVLDB Artifact Availability:
The source code, data, and/or other artifacts have been made available at https://github.com/shape-fragments.

1 INTRODUCTION

This paper proposes and investigates the use of *shapes* to retrieve subgraphs from RDF graphs. The Resource Description Framework (RDF) [51] is a data model, often used on the Web, to represent data as sets of subject–property–object triples. Viewing properties as edge labels, such data is naturally interpreted as a labeled graph.

Shapes are constraints on nodes (IRIs or literals) and their properties in the context of an RDF graph. These constraints can be complex and can traverse the graph, checking properties of properties, and so on. Shapes are specified in schemas, used to validate RDF graphs; a popular language for writing such schemas is the W3C recommended language SHACL [54].[1] In SHACL, schemas are called "shapes graphs", but we will mostly refer to them simply as *shape schemas*.

Specifically, a shape schema specifies a set of shapes, each associated with a *target*, which is a simple type of node-returning query. Each such target–shape pair states that all nodes returned by the target satisfy the shape. When these inclusions hold in an RDF graph, the graph is said to *conform* to the schema.

Example 1.1. A node on which a shape is evaluated is referred to as a *focus node*. We give two simple examples of shapes (for now just expressed in English) about nodes in a publication graph:

Shape s_1**:** "The focus node has at least one author property, and at least one journal or at least one conference property."
Shape s_2**:** "The focus node has at least one editor property."

Targets are simple forms of queries such as:

Target t_1**:** "All nodes of type paper."
Target t_2**:** "All nodes that are the conference property of some node."

For example, a shape schema may specify the two shapes s_1 and s_2, and associate target t_1 to s_1 and t_2 to s_2. A graph then conforms to this shape graph if every node returned by t_j satisfies shape s_j, for $j = 1, 2$. □

Shape schemas play the same role for RDF graphs as database schemas and integrity constraints do for databases. By specifying the intended structural constraints on the data, they help maintaining data quality [24]. The shape schema can also be used by the query optimizer in processing SPARQL queries [2, 50]. Knowledge of a schema helps data consumers to effectively formulate their SPARQL queries in the first place. Moreover, a shape schema may be specified at the data consumer's side, expressing the structural constraints required for the data to be usable by local applications (e.g., [38, 44]).

[1] The other popular schema language for RDF data is ShEx [13], which we will discuss later in this paper.

Proceedings of the VLDB Endowment, Vol. 15, No. X ISSN 2150-8097.
doi:XX.XX/XXX.XX

SHACL is a rather powerful language, allowing the expression of quite complex shapes. However, the task of checking whether an input RDF graph conforms to a shape schema can, at least in principle, also be done by evaluating a, typically complex, SPARQL query.[2] That SHACL was still invented and proposed within the Web community strongly indicates that this language allows for easier expression of typical constraints on the structure of the data. This suggests that there may be an opportunity to leverage SHACL beyond conformance checking, and use it also to retrieve data.

In this paper, we explore this opportunity. We certainly do not consider SHACL to be a substitute for a general-purpose RDF query language. Instead, we specifically focus on *subgraph-returning* queries. While SPARQL can define such queries too, we propose that shapes and shape schemas can offer an attractive alternative.

Concretely, we propose the following mechanism.

Shape fragment: Let S be a set of shapes, which we may refer to as *request shapes*. It is natural to consider the query that simply returns all nodes in the input graph G that satisfy at least one of the request shapes. However, we can go beyond that, and retrieve the subgraph of G formed by tracing out, for each node v satisfying some request shape, the *neighborhood* of v in G, as prescribed by that shape. We call the subgraph thus formed the *shape fragment* of G for S.

Schema fragment: A special case of shape fragment that we expect to be common is when the request shapes are taken from a shape schema H to which G is known to conform. Specifically, for the request shapes, we take the shapes from the schema, conjoined with their targets. The resulting shape fragment of G with respect to H, will then consist of the neighborhoods of all target nodes, as prescribed by the shape associated to the target.

To specify the above mechanism precisely, we need to give a formal definition of the neighborhood of a node in a graph with respect to some shape. Finding the "right" definition has been one of the main goals of the present work. This is not easy, since shapes can amount to powerful logical expressions, involving universal quantifiers and negation. This will be discussed in detail later in the paper; here, we just continue our simple example.

Example 1.2. For the shape schema from Example 1.1, the shape fragment of a graph would consist of all nodes of type paper, together with their outgoing edges labeled author, journal or conference; these outgoing edges comprise the neighborhood for target–shape pair (t_1, s_1). Furthermore, the shape fragment will contain all nodes that are the conference of some node, together with their incoming edges labeled conference, and outgoing edges labeled editor; these edges comprise the neighborhood for target–shape pair (t_2, s_2). □

This paper is organized as follows. We begin in Section 2 with an informal introduction to shape fragments, and explain how they fit the framework of Linked Data Fragments (LDF [8, 30, 36, 59]). Section 3 gives a self-contained formal definition of shape fragments, revolving around our definition of neighborhood. We work from a formalization of SHACL proposed by Corman, Reutter and Savkovic [17], which is gaining traction [4, 37, 45]. Crucially, we prove that our definition of neighborhoods is *correct* in the following sense:

Correctness for shape fragments: We show that a node v satisfies a request shape in the shape fragment, if and only if v satisfies that shape in the original graph. Thus, neighborhoods consist of exactly the right information to satisfy shapes.

Correctness for schema fragments: If a graph G conforms to a schema H, then we show that the shape fragment of G with respect to H still conforms to H, as one would expect.

In Section 4, we explore how shape fragments can be implemented, either by translation to SPARQL, or by instrumenting an existing SHACL validator. We present initial experiments showing that computation of shape fragments is feasible. In this paper we mainly introduce the idea of shape fragments; a detailed investigation on processing strategies for shape fragments is an obvious direction for further research.

Section 5 relates shape fragments to data provenance, and compares the expressive power of shape fragments to Triple Pattern Fragments [59]. We also compare to a recent proposal, similar to shape fragments, made by Labra Gayo [34], which appeared independently of our own work. We believe that two independent researchers or research groups proposing a similar idea can underline that the idea is indeed natural. Section 6 concludes the paper by discussing topics for further research.

2 SHAPE FRAGMENTS

In this Section we give an introduction to shape fragments for readers having already some familiarity with RDF and SHACL. A self-contained formal development is given in the next Section. Moreover, we have defined a complete specification of shape fragments which closely follows the existing W3C SHACL recommendation, and explains in detail how each construct of core SHACL contributes to the formation of the shape fragment [56].

As a first example of a shape, consider data for a student-oriented workshop, where we require that every workshop paper has at least one author of type student. This constraint is expressed by the following shape–target pair in SHACL:

```
:WorkshopShape sh:targetClass :WorkshopPaper;
  sh:property [
    sh:path :author; sh:qualifiedMinCount 1 ;
    sh:qualifiedValueShape [ sh:class :Student ] ] .
```

The first triple shown above specifies the target, in this case, all nodes of type :WorkshopPaper. The remaining triples specify the shape itself. Shapes express constraints on individual nodes, called focus nodes. In this example, the shape expresses that the focus node should be related, through property :author, to at least one node of type :Student.

An RDF graph conforms to a shape–target pair if every node specified by the target conforms to the shape. For example, consider the following RDF graph in Turtle syntax [52]:

```
:p1 a :WorkshopPaper ; :author :Anne, :Bob, :Alice .
```

[2] SPARQL is the W3C recommended query language for RDF data. Our statement is taking exception of *recursive* shape schemas [4, 12], but note that also non-recursive SHACL is already quite expressive. Also, recursive SHACL is not yet standardized.

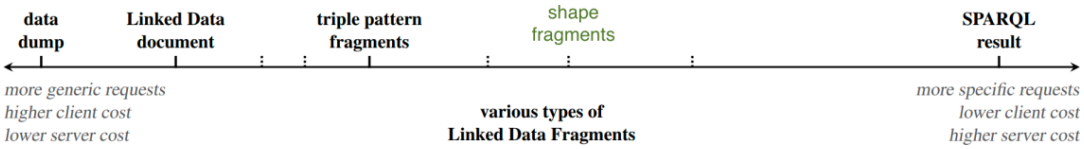

Figure 1: Positioning shape fragments in the LDF Framework (adapted from [59]).

```
:p2 a :WorkshopPaper ; :author :Anne, :Bob .
:Anne a :Professor . :Bob a :Professor .
:Alice a :Student .
```

Paper :p1 has author :Alice, who is a student, so :p1 conforms to the shape. However, :p2 clearly violates the shape. Hence, the graph does not conform to the shape–target pair.

In general, a *shapes graph* in SHACL is a collection of shape–target pairs. The task of validating an RDF graph, with respect to a shapes graph, produces a validation report. The report lists, for each shape–target pair, the target nodes that violate the shape. When the report is empty, the RDF graph conforms to the shapes graph.

In the present paper, we refer to shapes graphs as shape schemas, and propose that they can also be used to *retrieve* information. Roughly speaking, we retrieve, for each shape–target pair, all target nodes conforming to the shape, including all triples "tracing out the shape". We call these triples the *neighborhood* of the conforming node, and we refer to the union of all neighborhoods as the *shape fragment* for the schema.

For our example :WorkshopShape, the neighborhood of a conforming node v would consist of all triples (v :author x) from the graph where x is of type :Student, implying that, for each such x, also the triple (x a :Student) is included in the neighborhood. In SPARQL terms, the shape fragment for our simple example schema corresponds to the result of the query

```
CONSTRUCT WHERE
{ ?v a :WorkshopPaper . ?v :author ?x . ?x a :Student }
```

If we start from a data graph that conforms to the schema, the shape fragment will also conform to this schema. For example, consider the following shape schema:

```
:AddressShape a sh:NodeShape ;
  sh:property [
    sh:path :postalCode ;
    sh:datatype xsd:string ;
    sh:maxCount 1 ; ] .

:PersonShape a sh:NodeShape ;
  sh:targetClass :Person ;
  sh:property [
    sh:path :address ;
    sh:minCount 1 ;
    sh:node :AddressShape ;  ] .
```

On a conforming data graph, the shape fragment will consist of the neighborhoods of all nodes of target type :Person according to :PersonShape. This shape expresses that the focus node must be the subject of at least one :address triple; moreover, all objects of such triples must conform to :AddressShape. The latter shape (which is only auxiliary; it has no target of its own) expresses that the focus node can be the subject of at most one :postalCode triple; moreover, if this triple is present, the object must be a literal of type xsd:string. Thus, the shape fragment will consist of all triples of the form (v a :Person) in the graph, plus, for each such v, all triples (v :address x) in the graph, plus, for each such x, a triple (x :postalCode c) if present in the graph.

SHACL is a rather powerful language; shapes can be negated, can test for disjointness, etc. For example, consider:

```
:HappyAtWork a sh:NodeShape ;
  sh:not [
    sh:path :friend ;
    sh:disjoint :colleague ; ] .
```

This shape expresses that the focus node has at least one friend who is also a colleague. The shape fragment would retrieve, for every conforming node v, the union of all pairs of triples (v :friend x) and (v :colleague x) for each common x that exists in the data graph.

Shape fragments fit the framework of Linked Data Fragments [8, 30, 36, 59] (LDF) for publication interfaces to retrieve RDF (sub)graphs. At one end of the spectrum the complete RDF graph is retrieved; at the other end, the results of arbitrary SPARQL queries. Triple Pattern Fragments (TPF) [59], for example, represent an intermediate point where all triples from the graph that match a given SPARQL triple pattern are returned.

On this spectrum, shape fragments lie between TPF and arbitrary SPARQL, as shown in Fig. 1, taking advantage of the merits of both approaches. On the one hand, shape fragments may reduce the server cost, similarly to TPF, but they can also perform fewer requests as multiple TPFs can be expressed as a single shape fragment. On the other hand, shape fragments may also perform quite powerful requests, similarly to SPARQL endpoints, but without reaching the full expressivity of SPARQL. In fact, as opposed to TPF and SPARQL endpoints that predetermine where the cost will be, our proposed shape fragments mechanism may be implemented both on client- or server-side, adjusting and balancing the costs.

By returning a subgraph rather than a set of variable bindings (as in SPARQL select-queries), results can be better compressed [8]. Returning subgraphs also simplifies keeping track of provenance [26] or footprint [58]. Trading off efficiency against expressive power, processing shape fragments may be more efficient than processing arbitrary SPARQL (and with fewer requests than in the case of TPF) although this needs to be confirmed by further research. A final advantage of shape fragments is software reuse; shapes now serve the double purpose of describing the available data, and of accessing the data.

3 FORMAL DEFINITION AND SUFFICIENCY

In this section, we first present a formalization of SHACL, following the work by Corman, Reutter and Savkovic [17]. We extend their formalization to cover all features of SHACL, such as disjointness, zero-or-one property paths, closedness, language tags, node tests, and literals. We then proceed to the formal definition of neighborhoods and shape fragments, and establish an important correctness result, called the *sufficiency property.*

3.1 SHACL formalization

We assume three pairwise disjoint infinite sets I, L, and B of *IRIs*, *literals*, and *blank nodes*, respectively. We use N to denote the union $I \cup B \cup L$; all elements of N are referred to as *nodes*. Literals may have a "language tag" [51]. We abstract this by assuming an equivalence relation $\sim$ on L, where $l \sim l'$ represents that l and l' have the same language tag. Moreover, we assume a strict partial order $<$ on L that abstracts comparisons between numeric values, strings, dateTime values, etc.

An *RDF triple* (s, p, o) is an element of $(I \cup B) \times I \times N$. We refer to the elements of the triple as the subject s, the property p, and the object o. An *RDF graph* G is a finite set of RDF triples. It is natural to think of an RDF graph as an edge-labeled, directed graph, viewing a triple (s, p, o) as a p-labeled edge from node s to node o.

We formalize SHACL property paths as *path expressions* E. Their syntax is given by the following grammar, where p ranges over I:

$$E ::= p \mid E^- \mid E_1/E_2 \mid E_1 \cup E_2 \mid E^* \mid E?$$

SHACL can do many tests on individual nodes, such as testing whether a node is a literal, or testing whether an IRI matches some regular expression. We abstract this by assuming a set Ω of *node tests*; for any node test t and node a, we assume it is well-defined whether or not *a satisfies t*.

The formal syntax of *shapes* ϕ is now given by the following grammar.

$$\begin{aligned}
F &::= E \mid \mathrm{id} \\
\phi &::= \top \mid \bot \mid \mathit{hasShape}(s) \mid \mathit{test}(t) \mid \mathit{hasValue}(c) \\
&\quad \mid \mathit{eq}(F, p) \mid \mathit{disj}(F, p) \mid \mathit{closed}(P) \\
&\quad \mid \mathit{lessThan}(E, p) \mid \mathit{lessThanEq}(E, p) \mid \mathit{uniqueLang}(E) \\
&\quad \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \\
&\quad \mid \geq_n E.\phi \mid \leq_n E.\phi \mid \forall E.\phi
\end{aligned}$$

with E a path expression; $s \in I \cup B$; $t \in \Omega$; $c \in N$; $p \in I$; $P \subseteq I$ finite; and n a natural number.

Remark 3.1. Note that in shapes of the form $\mathit{eq}(F, p)$ or $\mathit{disj}(F, p)$, the argument expression F can be either a path expression E or the keyword 'id'. We will see soon that 'id' stands for the focus node. We need to include these id-variants in our formalisation, to reflect the distinction made in the SHACL recommendation between "node shapes" (expressing constraints on the focus node itself) and "property shapes" (expressing constraints on nodes reachable from the focus node by a path expression). □

We formalize SHACL shapes graphs as *schemas*. We first define the notion of *shape definition*, as a triple (s, ϕ, τ) where $s \in I \cup B$, and ϕ and τ are shapes. The elements of the triple are referred to as the *shape name*, the *shape expression*, and the *target expression*, respectively.[3]

Table 1: Conditions for conformance of a node to a shape.

ϕ	$H, G, a \models \phi$ if:
$\mathit{hasValue}(c)$	$a = c$
$\mathit{test}(t)$	a satisfies t
$\mathit{hasShape}(s)$	$H, G, a \models \mathit{def}(s, H)$
$\geq_n E.\psi$	$\sharp\{b \in [\![E]\!]^G(a) \mid H, G, b \models \psi\} \geq n$
$\leq_n E.\psi$	$\sharp\{b \in [\![E]\!]^G(a) \mid H, G, b \models \psi\} \leq n$
$\forall E.\psi$	every $b \in [\![E]\!]^G(a)$ satisfies $H, G, b \models \psi$
$\mathit{eq}(F, p)$	the sets $[\![F]\!]^G(a)$ and $[\![p]\!]^G(a)$ are equal
$\mathit{disj}(F, p)$	the sets $[\![F]\!]^G(a)$ and $[\![p]\!]^G(a)$ are disjoint
$\mathit{closed}(P)$	for all triples $(a, p, b) \in G$ we have $p \in P$
$\mathit{lessThan}(E, p)$	$b < c$ for all $b \in [\![E]\!]^G(a)$ and $c \in [\![p]\!]^G(a)$
$\mathit{lessThanEq}(E, p)$	$b \leq c$ for all $b \in [\![E]\!]^G(a)$ and $c \in [\![p]\!]^G(a)$
$\mathit{uniqueLang}(E)$	$b \nsim c$ for all $b \neq c \in [\![E]\!]^G(a)$.

Now a *schema* is a finite set H of shape definitions such that no two shape definitions have the same shape name. Moreover, as in the current SHACL recommendation, in this paper we consider only *nonrecursive* schemas. Here, H is said to be recursive if there is a directed cycle in the directed graph formed by the shape names, with an edge $s_1 \longrightarrow s_2$ if $\mathit{hasShape}(s_2)$ occurs in the shape expression defining s_1.

In order to define the semantics of shapes and shape schemas, we first recall that a path expression E evaluates on an RDF graph G to a binary relation on N, denoted by $[\![E]\!]^G$ and defined as follows. $[\![p]\!]^G = \{(a, b) \mid (a, p, b) \in G\}$; $[\![E^-]\!]^G = \{(b, a) \mid (a, b) \in [\![E]\!]^G\}$; $[\![E?]\!]^G = \{(a, a) \mid a \in N\} \cup [\![E]\!]^G$; $[\![E_1 \cup E_2]\!]^G = [\![E_1]\!]^G \cup [\![E_2]\!]^G$; $[\![E_1/E_2]\!]^G = \{(a, c) \mid \exists b : (a, b) \in [\![E_1]\!]^G \;\&\; (b, c) \in [\![E_2]\!]^G\}$; and $[\![E^*]\!]^G$ = the reflexive-transitive closure of $[\![E]\!]^G$. Finally, we also define $[\![\mathrm{id}]\!]^G$, for any G, to be simply the identity relation on N.

We are now ready to define when a focus node *a conforms* to a shape ϕ in a graph G, in the context of a schema H, denoted by $H, G, a \models \phi$. For the boolean operators $\top$ (true), $\bot$ (false), $\neg$ (negation), $\wedge$ (conjunction), $\vee$ (disjunction), the definition is obvious. For the other constructs, the definition is shown in Table 1. In this table, we employ the following notations:

- In the definition for $\mathit{hasShape}(s)$ we use the notation $\mathit{def}(s, H)$ to denote the shape expression defining shape name s in H. If s does not have a definition in H, we let $\mathit{def}(s, H)$ be $\top$ (this is the behavior in real SHACL).
- We use the notation $R(x)$, for a binary relation R, to denote the set $\{y \mid (x, y) \in R\}$. We apply this notation to the case where R is of the form $[\![E]\!]^G$ and x is a node. For example, $[\![\mathrm{id}]\!]^G(a)$ equals the singleton $\{a\}$.
- We also use the notion $\sharp X$ for the cardinality of a set X.

Note that the conditions for $\mathit{lessThan}(E, p)$ and $\mathit{lessThanEq}(E, p)$ imply that b and c must be literals.

In the Appendix, we show that our formalization fully covers real SHACL.

[3] Real SHACL only supports specific shapes for targets, but our development works equally well when allowing any shape for a target.

Example 3.2.

- The shape :WorkshopShape from Section 2 can be expressed in the formalisation as follows:

$$\geq_1 \text{:author}.\geq_1 \text{rdf:type/rdf:subclassOf}^*.\mathit{hasValue}(\text{:Student})$$

- The shape :HappyAtWork from Section 2 is expressed as $\neg \mathit{disj}(\text{:friend}, \text{:colleague})$.
- For an IRI p, the shape $\neg \mathit{disj}(\mathrm{id}, p)$ expresses that the focus node has a p-labeled self-loop, and the shape $\mathit{eq}(\mathrm{id}, p)$ expresses that its *only* outgoing p-edge is a self-loop. □

Finally, we can define conformance of a graph to a schema as follows. RDF graph G *conforms* to schema H if for every shape definition $(s, \phi, \tau) \in H$ and for every $a \in N$ such that $H, G, a \models \tau$, we have $H, G, a \models \phi$.

Remark 3.3. Curiously, SHACL provides shapes *lessThan* and *lessThanEq* but not their variants *moreThan* and *moreThanEq* (with the obvious meaning). Note that $\mathit{moreThan}(E, p)$ is not equivalent to $\neg \mathit{lessThanEq}(E, p)$. In this paper we stay with the SHACL standard, but our treatment is easily extended to *moreThan* and *moreThanEq*.

3.2 Neighborhoods

The fundamental notion to be defined is that of the *neighborhood* of a node v for a shape ϕ in a graph G. The intuition is that this neighborhood consists of those triples in G that show that v conforms to ϕ; if v does not conform to ϕ, the neighborhood is set to be empty. We want a generic, tractable, deterministic definition that formalizes this intuition. Our definition should also omit unnecessary triples; for otherwise, one could simply define the neighborhood to be G itself!

Before developing the definition formally, we discuss the salient features of our approach.

Negation Following the work by Grädel and Tannen on supporting where-provenance in the presence of negation [27], we assume shapes are in *negation normal form*, i.e., negation is only applied to atomic shapes. This is no restriction, since every shape can be put in negation normal form, preserving the overall syntactic structure, simply by pushing negations down. We push negation through conjunction and disjunction using De Morgan's laws. We push negation through quantifiers as follows:

$$\neg \geq_{n+1} E.\psi \equiv \leq_n E.\psi \quad \neg \leq_n E.\psi \equiv \geq_{n+1} E.\psi \quad \neg \forall E.\psi \equiv \geq_1 E.\neg\psi$$

The negation of $\geq_0 E.\psi$ is simply false.

Node tests We leave the neighborhood for *hasValue* and *test* shapes empty, as these involve no properties, i.e., no triples.

Closedness We also define the neighborhood for $\mathit{closed}(P)$ to be empty, as this is a minimal subgraph in which the shape is indeed satisfied. A reasonable alternative approach would be to return all properties of the node, as "evidence" that these indeed involve only IRIs in P. Indeed, we will show in Section 3.3 that our definitions, while minimalistic, are taken such that they can be relaxed without sacrificing the sufficiency property.

Disjointness Still according to our minimal approach, the neighborhood for disjointness shapes is empty. Analogously, the same holds for *lessThan* and *uniqueLang* shapes.

Equality The neighborhood for a shape $\mathit{eq}(E, p)$ consists of the subgraph traced out by the E-paths and p-properties of the node under consideration, evidencing that the sets of end-nodes are indeed equal. Here, we can no longer afford to return the empty neighborhood, although equality would hold trivially there. Indeed, this would destroy the relaxation property promised above. For example, relaxing by adversely adding just one E-path and one p-property with distinct end-nodes, would no longer satisfy equality.

Nonclosure The neighborhood for a shape $\neg \mathit{closed}(P)$ consists of those triples from the node under consideration that involve properties outside P, as expected.

Nonequality For $\neg \mathit{eq}(E, p)$ we return the subgraph traced out by the E-paths from the node v under consideration that end in a node that is *not* a p-property of v, and vice versa. A similar approach is taken for nondisjointness and negated *lessThan* shapes.

Quantifiers The neighborhood for $\forall E.\psi$ consists, as expected, of the subgraph traced out by all E-paths from the node under consideration to nodes x, plus the ψ-neighborhoods of these nodes x. For $\geq_n E.\psi$ we do something similar, but we take only those x that conform to ψ. Given the semantics of the $\geq_n$ quantifier, it seems tempting to instead just take a selection of n of such nodes x. However, we want a deterministic definition of neighborhood, so we take all x. Dually, for $\leq_n E.\psi$, we return the subgraph traced out by E-paths from the current node to nodes *not* conforming to ψ, plus their $\neg\psi$-neighborhoods.

Towards a formalization of the above ideas, we first make precise the intuitive notion of a path in an RDF graph, and of the subgraph traced out by a path. Paths are finite sequences of adjacent steps. Each step either moves forward from the subject to the object of a triple, or moves backward from the object to the subject. We make backward steps precise by introducing, for each property $p \in I$, its *reverse*, denoted by p^-. The set of reverse IRIs is denoted by I^-. We assume I and I^- are disjoint, and moreover, we also define $(p^-)^-$ to be p for every $p \in I$.

For any RDF triple $t = (s, p, o)$, the triple $t^- := (o, p^-, s)$ is called a *reverse triple*. As for IRIs, we define $(t^-)^-$ to be t. A *step* is an RDF triple (a forward step) or a reverse triple (a backward step). For any step $t = (x, r, y)$, we refer to x as the *tail*, denoted by $\mathit{tail}(t)$, and to y as the *head*, denoted by $\mathit{head}(t)$. A *path* is a nonempty finite sequence π of steps so that $\mathit{head}(t_1) = \mathit{tail}(t_2)$ for any two subsequent steps t_1 and t_2 in π. The *tail* of π is the tail of its first step; the *head* of π is the head of its last step. Any two paths π and π' where $\mathit{head}(\pi) = \mathit{tail}(\pi')$ can be concatenated; we denote this by $\pi \cdot \pi'$.

The *graph* traced out by a path π, denoted by $\mathit{graph}(\pi)$, is simply the set of RDF triples underlying the steps of the path. Thus, backward steps must be reversed. Formally,

$$\mathit{graph}(\pi) = \{t \mid t \text{ forward step in } \pi\} \cup \{t^- \mid t \text{ backward step in } \pi\}.$$

For a set Π of paths, we define $\mathit{graph}(\Pi) = \bigcup\{\mathit{graph}(\pi) \mid \pi \in \Pi\}$.

We are not interested in arbitrary sets of paths, but in the set of paths generated by a path expression E in an RDF graph G, denoted by $\mathit{paths}(E, G)$ and defined in a standard manner as follows. $\mathit{paths}(p, G) = \{(a, r, b) \in G \mid r = p\}$; $\mathit{paths}(E/E', G) = \{\pi \cdot \pi' \mid \pi \in$

Table 2: Neighborhood in the context of a schema H, when $G, v \models \phi$ and ϕ is in negation normal form. In particular, in rules 2 and 6, we assume that $\neg def(s, H)$ and $\neg\psi$ are put in negation normal form. In the omitted cases, and when $G, v \not\models \phi$, the neighborhood is defined to be empty.

ϕ	$B(v, G, \phi)$
$hasShape(s)$	$B(v, G, def(s, H))$
$\neg hasShape(s)$	$B(v, G, \neg def(s, H))$
$\phi_1 \wedge \phi_2$	$B(v, G, \phi_1) \cup B(v, G, \phi_2)$
$\phi_1 \vee \phi_2$	$B(v, G, \phi_1) \cup B(v, G, \phi_2)$
$\geq_n E.\psi$	$\bigcup\{graph(paths(E, G, v, x)) \cup B(x, G, \psi) \mid (v, x) \in [\![E]\!]^G \;\&\; G, x \models \psi\}$
$\leq_n E.\psi$	$\bigcup\{graph(paths(E, G, v, x)) \cup B(x, G, \neg\psi) \mid (v, x) \in [\![E]\!]^G \;\&\; G, x \models \neg\psi\}$
$\forall E.\psi$	$\bigcup\{graph(paths(E, G, v, x)) \cup B(x, G, \psi) \mid (v, x) \in [\![E]\!]^G\}$
$eq(E, p)$	$graph(pathsfrom(E \cup p, G, v))$
$eq(\mathrm{id}, p)$	$\{(v, p, v)\}$
$\neg eq(E, p)$	$graph(\{\pi \in pathsfrom(E, G, v) \mid (v, p, head(\pi)) \notin G\}) \cup \{(v, p, x) \in G \mid (v, x) \notin [\![E]\!]^G\}$
$\neg eq(\mathrm{id}, p)$	$\{(v, p, x) \in G \mid x \neq v\}$
$\neg disj(E, p)$	$\bigcup\{graph(paths(E, G, v, x)) \cup \{(v, p, x)\} \mid (v, x) \in [\![E]\!]^G \;\&\; (v, p, x) \in G\}$
$\neg disj(\mathrm{id}, p)$	$\{(v, p, v)\}$
$\neg lessThan(E, p)$	$\bigcup\{graph(paths(E, G, v, x)) \cup \{(v, p, y)\} \mid (v, x) \in [\![E]\!]^G \;\&\; (v, p, y) \in G \;\&\; x \not< y\}$
$\neg lessThanEq(E, p)$	$\bigcup\{graph(paths(E, G, v, x)) \cup \{(v, p, y)\} \mid (v, x) \in [\![E]\!]^G \;\&\; (v, p, y) \in G \;\&\; x \not\leq y\}$
$\neg uniqueLang(E)$	$graph(\{\pi \in pathsfrom(E, G, v) \mid \exists x \in [\![E]\!]^G(v) : x \neq head(\pi) \;\&\; x \sim head(\pi)\})$
$\neg closed(P)$	$\{(v, p, x) \in G \mid p \notin P\}$

$paths(E, G)$ & $\pi' \in paths(E', G)$ & $tail(\pi) = head(\pi')\}$; $paths(E \cup E', G) = paths(E, G) \cup paths(E', G)$; $paths(E?, G) = paths(E, G)$; $paths(E^*, G) = \bigcup_{i=1}^{\infty} paths(E^i, G)$; and $paths(E^-, G) = \{\pi^- \mid \pi \in paths(E, G)\}$. Here, E^i abbreviates $E/\cdots/E$ (i times), and $\pi^- = t_l^-, \ldots, t_1^-$ for $\pi = t_1, \ldots, t_l$. Note that $paths(p, G)$ is a set of length-one paths.

In order to link E-paths to the evaluation of shapes below, we introduce some more notation, for any two nodes a and b:

$$pathsfrom(E, G, a) := \{\pi \in paths(E, G) \mid tail(\pi) = a\}$$
$$paths(E, G, a, b) := \{\pi \in pathsfrom(E, G, a) \mid head(\pi) = b\}$$

Note that $graph(\pi)$, for every $\pi \in paths(E, G)$, is a subgraph of G. This will ensure that neighborhoods and shape fragments are always subgraphs of the original graph. Moreover, the following observation ensures that path expressions will have the same semantics in the fragment as in the original graph:

Proposition 3.4. *Let $F = graph(paths(E, G, a, b))$. Then $(a, b) \in [\![E]\!]^G$ if and only if $(a, b) \in [\![E]\!]^F$.*

Note that $paths(E, G)$ may be infinite, due to the use of Kleene star in E and cycles in G. However $graph(paths(E, G))$ is always finite, because G is finite.

We are now ready to define neighborhoods in the context of an arbitrary but fixed schema H. To avoid clutter we will omit H from the notation. Let v be a node, G be a graph, and ϕ be a shape. We define the *ϕ-neighborhood of v in G*, denoted by $B(v, G, \phi)$, as the empty RDF graph whenever v does not conform to ϕ in G. When v does conform, the definition is given in Table 2. As already discussed above, by pushing negations down, we can and do assume that ϕ is put in *negation normal form*, meaning that negation is only applied to atomic shapes. (Atomic shapes are those from the first three lines in the production for ϕ, in the grammar for shapes given in Section 3.1.)

3.3 Shape fragments and sufficiency

The *shape fragment* of an RDF graph G, for a finite set S of shapes, is the subgraph of G formed by the neighborhoods of all nodes in G for the shapes in S. Formally:

$$Frag(G, S) = \bigcup\{B(v, G, \phi) \mid v \in N \;\&\; \phi \in S\}.$$

Here, v ranges over the universe N of all nodes, but since neighborhoods are always subgraphs of G, it is equivalent to let v range over all subjects and objects of triples in G.

The shapes in S can be interpreted as arbitrary "request shapes". An interesting special case, however, is when S is derived from a shape schema H. Formally, we define the shape fragment of G for H as $Frag(G, H) := Frag(G, S)$, where $S = \{\phi \wedge \tau \mid \exists s : (s, \phi, \tau) \in H\}$. Thus, the shape fragment for a schema requests the conjunction of each shape in the schema with its associated target.

In order to state our main correctness result, we need to revisit the definition of schema. Recall that a schema is a set of shape definitions, where a shape definition is of the form (s, ϕ, τ). Until now, we allowed both the shape expression ϕ and the target τ to be arbitrary shapes. In real SHACL, however, only shapes of the following specific forms can be used as targets:

- $hasValue(c)$ (node targets);
- $\geq_1 p/r^*.hasValue(c)$ (class-based targets: p and r stand for type and subclass from the RDF Schema vocabulary [51], and c is the class name);
- $\geq_1 p.\top$ (subjects-of targets); and
- $\geq_1 p^-.\top$ (objects-of targets).

For our purposes, however, what counts is that real SHACL targets τ are *monotone*, in the sense that if $G, v \models \tau$ and $G \subseteq G'$, then also $G', v \models \tau$.

We establish:

Theorem 3.5 (Conformance). *Assume schema H has monotone targets, and assume RDF graph G conforms to H. Then $Frag(G, H)$ also conforms to H.*

In the Appendix, we prove the Conformance Theorem using the following result:

Lemma 3.6 (Sufficiency). *If $G, v \models \phi$ then also $G', v \models \phi$ for any RDF graph G' such that $B(v, G, \phi) \subseteq G' \subseteq G$.*

We call this the Sufficiency Lemma, because of the following corollary:

Corollary 3.7. *Let G be an RDF graph, let S be a finite set of shapes, let ϕ be a shape in S, and let v be a node. If $G, v \models \phi$, then also $Frag(G, S), v \models \phi$.*

The above corollary shows that the shape fragment $Frag(G, S)$ is sufficient in the sense of providing provenance for any shape in S evaluated in G. Thinking of a shape as a unary query, returning all nodes that conform to it, this is exactly the "sufficiency property" that has been articulated in the theory of data provenance [26].

The Sufficiency Lemma is stated not just for the neighborhood, but more strongly for all subgraphs that encompass the neighborhood. This formulation serves both a technical and a conceptual purpose. The technical purpose is that it helps to prove the lemma by induction: in order to deal with universal and maxcount quantifiers ($\forall$ and $\leq_n$), a stronger hypothesis is needed to carry the induction through.

Moreover, the conceptual contribution of the Sufficiency Lemma is that our definition of neighborhood is not "take it or leave it". Indeed, we allow for a shape fragment processor to return *larger* neighborhoods than the ones we strictly define. Theorem 3.5 and Corollary 3.7 will continue to hold. As discussed in Section 3.2, this "relaxation property" allows the server to provide more information, and justifies our minimalistic approach.

Example 3.8. Recall the example about the student-oriented workshop from Section 2. As a variation, suppose each paper must have at least one author, but can have at most one author who is *not* of type student. These two constraints are captured by a schema H with two shape definitions. One has the shape expression $\geq_1$ author.$\top$, and the other has the shape expression

$$\leq_1 \text{author}.\neg \geq_1 \text{type.student},$$

which in negation normal form becomes $\leq_1$ author. $\leq_0$ type.student. Both shape definitions have target $\geq_1$ type.paper. We denote the two shape expressions by ϕ_1 and ϕ_2, and the target by τ.

Consider the simple graph G consisting of a single paper, say p1. This paper has two authors: Anne, who is a professor, and Bob, who is a student. Formally, G consists of the five triples (p1, type, paper), (p1, auth, Anne), (p1, auth, Bob), (Anne, type, prof) and (Bob, type, student).

The neighborhood of p1 for $\phi_1 \wedge \tau$ consists of the three triples (p1, type, paper), (p1, auth, Anne) and (p1, auth, Bob). The neighborhood of p1 for $\phi_2 \wedge \tau$ consists of the three triples (p1, type, paper), (p1, auth, Bob) and (Bob, type, student). Thus, $Frag(G, H)$, being the union of the two neighborhoods, consists of the four triples (p1, type, paper), (p1, auth, Anne), (p1, auth, Bob) and (Bob, type, student).

Note that the triple (Bob, type, student) is essential in the neighborhood for $\phi_2 \wedge \tau$; omitting it from the shape fragment would break conformance to H. On the other hand, we are free to add the triple (Anne, type, prof) to the fragment without breaking conformance.

Finally, note that we could add to G various other triples unrelated to the shapes in H. The shape fragment would omit all this information, as desired. □

Example 3.9. For monotone shapes, the converse of Corollary 3.7 clearly holds as well. In general, however, the converse does not always hold. For example, consider the shape $\phi = \leq_0 p.\top$ ("the node has no property p"), and the graph $G = \{(a, p, b)\}$. Then the fragment $Frag(G, \{\phi\})$ is empty, so a trivially conforms to ϕ in the fragment. However, a clearly does not conform to ϕ in G.

4 IMPLEMENTATION AND EXPERIMENTAL VALIDATION

In this section we show that shape fragments can be effectively computed, and report on initial experiments.

4.1 Translation to SPARQL

Our first approach to computing shape fragments is by translation into SPARQL, the recommended query language for RDF graphs [29]. SPARQL select-queries return sets of *solution mappings*, which are maps μ from finite sets of variables to N. Variables are marked using question marks. Different mappings in the result may have different domains [7, 47].

Shape fragments are unions of neighborhoods, and neighborhoods in an RDF graph G are unions of subgraphs of the form $graph(paths(E, G, a, b))$, for path expressions E mentioned in the shapes, and selected nodes a and b. Hence, the following lemma is important. For any RDF graph G, we denote by $N(G)$ the set of all subjects and objects of triples in G.

Lemma 4.1. *For every path expression E, there exists a SPARQL select-query $Q_E(?t, ?s, ?p, ?o, ?h)$ such that for every RDF graph G:*

1. *The binary relation $\{(\mu(?t), \mu(?h)) \mid \mu \in Q_E(G)\}$ equals $[\![E]\!]^G$, restricted to $N(G)$.*
2. *For all $a, b \in N(G)$, the RDF graph*

$$\{(\mu(?s), \mu(?p), \mu(?o)) \mid \mu \in Q_E(G) \;\&\; (\mu(?t), \mu(?h)) = (a, b) \;\&\; \mu \text{ is defined on } ?s, ?p \text{ and } ?o\}$$

equals $graph(paths(E, G, a, b))$.

We emphasize that the above Lemma is not obvious. While SPARQL queries, through property paths, can readily test if $(a, b) \in [\![E]\!]^G$, it is not obvious one can actually return $graph(paths(E, G, a, b))$. The detailed proof is in the Appendix; the following example gives an idea of the proof on a simpler case.

Example 4.2. For IRIs a, b, q and r, the following SPARQL query, applied to any graph G, returns $graph(paths((q/r)^*, G, a, b))$:

```
SELECT ?s ?p ?o
WHERE { a (q/r)* ?t . ?h (q/r)* b . {
```

```
{ SELECT ?t (?t AS ?s) (q AS ?p) ?o ?h
  WHERE { ?t q ?o . ?o r ?h } }
UNION
{ SELECT ?t ?s (r AS ?p) (?h AS ?o) ?h
  WHERE { ?t q ?s . ?s r ?h } } }
```

□

Using Lemma 4.1, and expressing the definitions from Table 2 in SPARQL, we obtain a further result as follows. (Proof in the Appendix.)

Lemma 4.3. *For every shape ϕ, there exists a SPARQL select-query $Q_\phi(?v, ?s, ?p, ?o)$ such that for every RDF graph G,*

$$\{(\mu(?v), \mu(?s), \mu(?p), \mu(?o)) \mid \mu \in Q_\phi(G)\} = \{(v, s, p, o) \in N^4 \mid (s, p, o) \in B(v, G, \phi)\}$$

Remark 4.4. The above Lemma should not be confused with the known result [16, Proposition 3] that SPARQL can compute the set of nodes that *conform* to a given shape. Our result states that also the neighborhoods can be computed.

The above two Lemmas, combined with the definition of shape fragments, now imply the announced result:

Proposition 4.5. *For every finite set S of shapes, there exists a SPARQL select-query $Q_S(?s, ?p, ?o)$ such that for every RDF graph G,*

$$\{(\mu(?s), \mu(?p), \mu(?o)) \mid \mu \in Q_S(G)\} = Frag(G, S).$$

Query expressions for shapes can quickly become quite complex, even for just retrieving the nodes that satisfy a shape. For the simple example shape :PersonShape from Section 2, such a query needs to retrieve persons with at least one address, which is just a semijoin, but must also test that all addresses have at most one postal code, which requires at least a not-exists subquery involving a non-equality join. Shapes involving equality constraints require nested not-exists subqueries in SPARQL, and would benefit from specific operators for set joins, e.g., [31, 40]. Shapes of the form $\leq_5 p.\top$ requires grouping the p-properties and applying a condition count ≤ 5, plus a union with an outer join to retrieve the nodes without any p-property. Such shapes would benefit from specific operators for group join [20, 41].

Obviously, queries that actually retrieve the shape fragment are no simpler. Our Proposition 4.5 only states that SPARQL is sufficient in principle, and leaves query optimization for future work.

Example 4.6. For IRIs p, q and c, consider the request shape $\forall p.\geq_1 q.hasValue(c)$. The corresponding shape fragment is retrieved by the following SPARQL query:

```
SELECT ?s ?p ?o WHERE {
{ SELECT ?v WHERE
  { ?v p ?x MINUS { ?v p ?y OPTIONAL { ?y q c . ?v p ?z }
                                FILTER (!bound(?z)) } } } .
{ { SELECT (?v AS ?s) (p AS ?p) (?x as ?o)
    WHERE { ?v p ?x . ?x q c } }
  UNION
  { SELECT (?x AS ?s) (q AS ?p) (c as ?o)
    WHERE { ?v p ?x . ?x q c } } } }
```

The first subselect retrieves nodes $?v$ conforming to the shape; the UNION of the next two subselects then retrieves the neighborhoods. □

One may wonder about the converse to Proposition 4.5: is every SPARQL select-query expressible as a shape fragment? This does not hold, if only because shape fragments always consist of triples from the input graph, while select-queries can return arbitrary variable bindings. However, also more fundamentally, SHACL is strictly weaker than SPARQL; we give two representative examples.

4-clique Let $p \in I$. There does not exist a shape ϕ such that, on any RDF graph G, the nodes that conform to ϕ are exactly the nodes belonging to a 4-clique of p-triples in G. We can show that if 4-clique would be expressible by a shape, then the corresponding 4-clique query about a binary relation P would be expressible in 3-variable counting infinitary logic $C^3_{\infty\omega}$. The latter is known not to be the case, however [42]. (Infinitary logic is needed here to express path expressions, and counting is needed for the $\geq_n$ quantifier, since we have only 3 variables.)

Majority Let $p, q \in I$. There does not exist a shape ϕ such that, on any RDF graph G, the nodes that conform to ϕ are exactly the nodes v such that $\sharp\{x \mid (v, p, x) \in G\} \geq \sharp\{x \mid (v, q, x) \in G\}$ (think of departments with at least as many employees as projects). We can show that if Majority would be expressible by a shape, then the classical Majority query about two unary relations P and Q would be expressible in first-order logic. Again, the latter is not the case [33]. (Infinitary logic is not needed here, since for this query, we can restrict to a class of structures where all paths have length one.)

4.2 Adapting a validation engine

A shape fragments processor may also be obtained by adjusting a SHACL validator to return the validated RDF terms and their neighborhood, instead of a validation report.

A SHACL validation engine checks whether a given RDF graph conforms to a given schema, and produces a validation report detailing eventual violations. A validation engine needs to inspect the neighborhoods of nodes anyway. Hence, it requires only reasonably lightweight adaptations to produce, in addition to the validation report, also the nodes and their neighborhoods that validate the shapes graph, without introducing significant overheads for tracing out and returning these neighborhoods, compared to doing validation alone.

To test this hypothesis, we extended the open-source, free-license engine pySHACL [49]. This is a main-memory engine and it achieves high coverage for the core fragment of SHACL [23]. Written in Python, we found it easy to make local changes to the code [46]; starting out with 4501 lines of code, 482 lines were changed, added or deleted. Our current implementation covers most of SHACL core with the exception of complex path expressions. Our software, called **pySHACL-fragments**, is available open-source [56].

4.3 Experiments

We validated our approach by (i) assessing the correctness and practical applicability of shape fragments; (ii) measuring the overhead of shape fragment extraction, compared to mere validation, using our pySHACL-fragments implementation; and (iii) testing the viability of computing shape fragments by translation to SPARQL. All RDF

graphs, shapes and queries mentioned below, along with scripts to download the datasets and run the experiments, are publicly available [56].

4.3.1 Applicability of shape fragments. We *simulated* a range of SPARQL queries by shape fragments. Queries were taken from the SPARQL benchmarks BSBM [11] and WatDiv [3]. Unlike a shape fragment, a SPARQL select-query does not return a subgraph but a set of variable bindings. SPARQL construct-queries do return RDF graphs directly, but not necessarily subgraphs. Hence, we followed the methodology of modifying SPARQL select-queries to construct-queries that return all *images* of the pattern specified in the where-clause.

For tree-shaped basic graph patterns, with given IRIs in the predicate position of triple patterns, we can always simulate the corresponding subgraph query by a shape fragment. Indeed, a typical query from the benchmarks retrieves nodes with some specified properties, some properties of these properties, and so on. For example, a slightly simplified WatDiv query, modified into a subgraph query, would be the following. (To avoid clutter, we forgo the rules of standard IRI syntax.)

```
CONSTRUCT WHERE {
?v0 caption ?v1 . ?v0 hasReview ?v2 . ?v2 title ?v3 .
?v2 reviewer ?v6 . ?v7 actor ?v6 }
```

(Here, CONSTRUCT WHERE is the SPARQL notation for returning all images of a basic graph pattern.) We can express the above query as the fragment for the following request shape:

$$\geq_1 \mathsf{caption}.\top \wedge \geq_1 \mathsf{hasReview}.(\geq_1 \mathsf{title}.\top \wedge \geq_1 \mathsf{reviewer}.\geq_1 \mathsf{actor}^-.\top)$$

Of course, patterns can involve various SPARQL operators, going beyond basic graph patterns. Filter conditions on property values can be expressed as node tests in shapes; optional matching can be expressed using $\geq_0$ quantifiers. For example, consider a simplified version of the pattern of a typical BSBM query:

```
?v text ?t . FILTER langMatches(lang(?t),"EN")
OPTIONAL { ?v rating ?r }
```

The images of this pattern can be retrieved using the shape

$$\geq_1 \mathsf{title}.\mathit{test}(\mathsf{lang} = \text{“EN”}) \wedge \geq_0 \mathsf{rating}.\top.$$

Interestingly, the BSBM workload includes a pattern involving a combination of optional matching and a negated bound-condition to express absence of a certain property (a well-known trick [5, 6]). Simplified, this pattern looks as follows:

```
?prod label ?lab . ?prod feature 870
OPTIONAL { ?prod feature 59 . ?prod label ?var }
FILTER (!bound(?var))
```

The images of this pattern can be retrieved using the shape

$$\geq_1 \mathsf{label}.\top \wedge \geq_1 \mathsf{feature}.\mathit{hasValue}(870) \wedge \leq_0 \mathsf{feature}.\mathit{hasValue}(59).$$

A total of 39 out of 46 benchmark queries, modified to return subgraphs, could be simulated by shape fragments in this manner. The remaining seven queries involved features not supported by SHACL, notably, variables in the property position, or arithmetic.

We have verified equality between the 39 SPARQL subgraph queries executed on the benchmark data, and the corresponding shape fragments. This experiment served as a correctness test of our system pySHACL-fragments.

4.3.2 Extraction overhead. To measure the overhead of extracting shape fragments, compared to doing validation alone, we compared execution times of retrieving shape fragments using pySHACL-fragments, with producing the corresponding validation report using pySHACL. Here, we used the SHACL performance benchmark [53] which consists of a 30-million triple dataset known as the “Tyrolean Knowledge Graph”, accompanied by 58 shapes. For this experiment we have only worked with the five segments given by the first N million triples of the dataset, for $N = 1, \ldots, 5$.

We executed each of the shapes five times with each engine. Timers were placed around the validator.run() function, so *data loading and shape parsing time is not included.* In this experiment, the average overhead turns out to be below 10%, as illustrated in Figure 2. We used a 2x 6core Intel Xeon E5-2620 v3s processor with 128GB DDR4 RAM and a 1TB hard disk.

Going in more detail, we identified three different types of behaviours. Figure 2 shows a representative plot for each behaviour. The first type of behavior shows a clear, linear, increase in execution time for larger input sizes, going up to thousands of seconds for size 5M. This behavior occurs for three benchmark shapes, among which the shape PostalAddressShape. We show this, and following benchmark shapes, here in abridged form by simplifying IRI notation and omitting specific node tests. Below we use ‘τ’ to indicate the presence of a node test; we also use $=_n E.\phi$ to abbreviate $\geq_n E.\phi \wedge \leq_n E.\phi$.

PostalAddressShape:

$$\geq_1 \mathsf{type}.\mathit{hasValue}(\mathsf{PostalAddress}) \wedge =_1 \mathsf{addressCountry}.\top \wedge \forall \mathsf{addressCountry}.\tau \wedge =_1 \mathsf{addressLocality}.\top \wedge \forall \mathsf{addressCountry}.\tau \wedge \forall \mathsf{addressRegion}.\tau \wedge \geq_1 \mathsf{postalCode}.\top \wedge \forall \mathsf{postalCode}.\tau \wedge =_1 \mathsf{streetAddress}.\top \wedge \forall \mathsf{streetAddress}.\tau$$

The second type of behavior shows only a modest increase in execution time, increasing 10–20% between sizes 1M and 5M. This occurs for five benchmark shapes, among which the shape OpeningHourSpecificationShape, shown below. That the slope of the linear increase is smaller here than in the previous type can be explained by the distribution of nodes of type Opening Hour Specification in the data segments, which occur less densely than, e.g., Postal Addresses.

OpeningHourSpecificationShape:

$$\geq_1 \mathsf{type}.\mathit{hasValue}(\mathsf{OpeningHourSpecification}) \wedge \forall \mathsf{dayOfWeek}.\tau \wedge \forall \mathsf{closes}.\tau \wedge \forall \mathsf{opens}.\tau \wedge \forall \mathsf{validFrom}.\tau \wedge \forall \mathsf{validThrough}.\tau \wedge \leq_1 \mathsf{description}.\top \wedge \forall \mathsf{description}.\tau$$

The third and last type of behavior we observed shows execution times that remain constant over the five data segments. This behavior actually occurs for the majority of the benchmark shapes; we give OfferShape as an example. The explanation for this behavior is that all relevant triples for these shapes already occur in the first segment of 1M triples (recall that we do not measure data loading time). This first segment is indeed intended to be also used as a self-contained benchmark dataset.

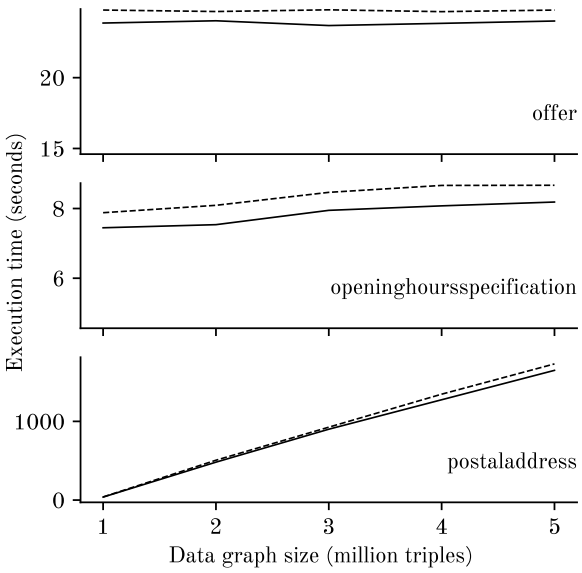

Figure 2: Adding shape extraction (dashed line) to pySHACL (full line) did not have large impact on the execution time, shown here for three representative shapes from the Tyrolean benchmark.

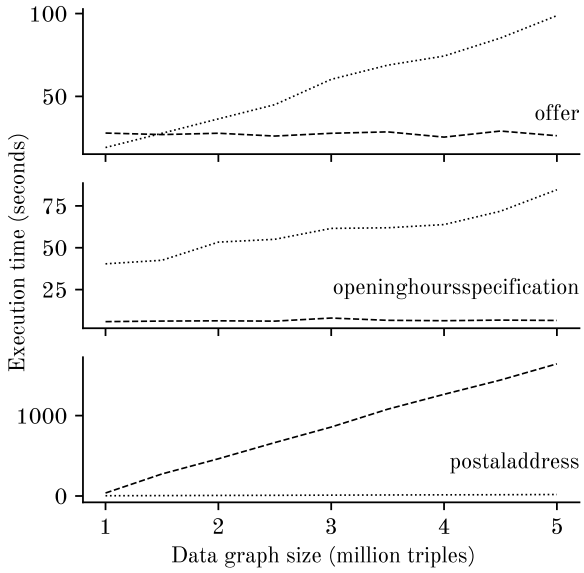

Figure 3: Jena ARQ in-memory SPARQL execution time (dotted) and pySHACL-fragments execution time (dashed).

Independently of how execution times vary over the five data segments, our measurements consistently report an average 10% overhead of extracting shape fragments.

OfferShape:

$$\geq_1 \mathsf{type}.\mathit{hasValue}(\mathsf{Offer}) \wedge =_1 \mathsf{name}.\top \wedge \forall \mathsf{name}.\tau \wedge \leq_1 \mathsf{description}.\top \wedge \forall \mathsf{description}.\tau \wedge =_1 \mathsf{availability}.\top \wedge \forall \mathsf{availability}.\tau \wedge \geq_1 \mathsf{itemOffered}.\top \wedge \forall \mathsf{itemOffered}.(\geq_1 \mathsf{type}.\mathit{hasValue}(\mathsf{Service}) \vee \geq_1 \mathsf{type}.\mathit{hasValue}(\mathsf{Product}) \vee \geq_1 \mathsf{type}.\mathit{hasValue}(\mathsf{Apartment})) \wedge =_1 \mathsf{price}.\top \wedge \forall \mathsf{price}.\tau \wedge =_1 \mathsf{priceCurrency}.\top \wedge \forall \mathsf{priceCurrency}.\tau \wedge =_1 \mathsf{url}.\top \wedge \forall \mathsf{url}.\tau \wedge \forall \mathsf{validFrom}.\tau \wedge \forall \mathsf{validThrough}.\tau$$

4.3.3 Computing shape fragments in SPARQL. As already discussed in Section 4.1, shapes give rise to complex SPARQL queries which pose quite a challenge to SPARQL query processors. It is outside the scope of the present paper to do a performance study of SPARQL query processors; our goal rather is to obtain an indication of the practical feasibility of computing shape fragments in SPARQL. Initial work by Corman et al. has reported satisfying results on doing *validation* for nonrecursive schemas by a single, complex SPARQL query [16]. The question is whether we can observe a similar situation when computing shape fragments, where the queries become even more complex.

We have obtained a mixed picture. We used the main-memory SPARQL engine Apache Jena ARQ. Implementing the constructive proof of Proposition 4.5, we translated the shape fragment queries for the benchmark shapes from the previous Section 4.3.2 into large SPARQL queries. The generated expressions can be thousands of lines long, as our translation procedure is not yet optimized to generate "efficient" SPARQL expressions. However, we can reduce the shapes by substituting $\top$ for node tests, and simplify the resulting expressions. For the three example shapes PostalAddressShape, OpeningHourSpecificationShape and OfferShape shown above, this amounts to substituting $\top$ for τ. This reduction preserves the graph-navigational nature of the queries. Note also that, while a constraint like $\forall p.\top$ is voidlessly true, it causes (as desired) the inclusion of p-triples in the shape fragment.

Execution times for the three SPARQL expressions, thus simplified, are shown in Figure 3, where they are compared, on the same test data as before, with the pySHACL-fragments implementation. We realize this is an apples-to-oranges comparison, but we can still draw some tentatively positive conclusions. Two SPARQL queries execute slower than pySHACL-fragments, but not so much slower that a log-scale y-axis would be needed to get them on the same picture. The SPARQL query for PostalAddressShape is even much faster. This is explained by the absence of $\leq_n$ constraints, which have a complex neighborhood definition. The generated query has only joins and counts, but no negated subqueries, which appears to run well on the ARQ processor. Reported timings are averages over five runs. We used a 2x 8core Intel Xeon E5-2650 v2 processor with 48GB DDR3 RAM and a 250GB hard disk.

Finally, to test the extraction of paths in SPARQL, we used the DBLP database [19], and computed the shape fragment for shape $\geq_1 a^-/a/a^-/a/a^-/a.\mathit{hasValue}(\mathsf{MYV})$, where a stands for the property dblp:authoredBy, and MYV stands for the DBLP IRI for Moshe Y. Vardi. This fragment extracts all authors at co-author distance three or less from this famous computer scientist, plus all a-triples

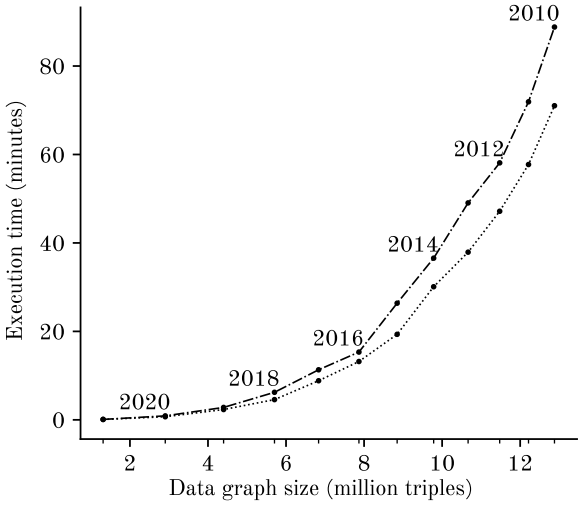

Figure 4: Jena ARQ store-based SPARQL execution time (dotted) and store-based GraphDB execution time (dashed-dotted) for the Vardi-distance-3 shape fragment.

involved. The generated SPARQL query is similar to the query from Example 4.2.

We ran this heavy analytical query on the two secondary-memory engines Apache Jena ARQ on TDB2 store, and GraphDB. The execution times over increasing slices of DBLP, going backwards in time from 2021 until 2010, are comparable between the two engines (see Figure 4). Vardi is a prolific and central author and co-author; just from 2016 until 2021, almost 7% of all DBLP authors are at distance three or less, or almost 145 943 authors. The resulting shape fragment contains almost 3% of all dblpl:authoredBy triples, or 219 085 unique triples.

5 RELATED WORK

5.1 Provenance

Shapes may be viewed as queries on RDF graphs, returning the nodes that conform to the shape. A close affinity is then apparent between neighborhoods as defined in this paper, and notions of *provenance* for database queries [15, 26].

A seminal work in the area of data provenance is that on *lineage* by Cui, Widom and Wiener [18]. Like shape fragments, the lineage of a tuple returned by a query on a database D is a subdatabase of D. Lineage was defined for queries expressed in the relational algebra. In principle, we can express shapes in relational algebra. So, instead of defining our own notion of neighborhood, should we have simply used lineage instead? The answer is no; the following example shows that Theorem 3.5 and Corollary 3.7 would fail.

Example 5.1. Recalling Example 3.8, consider a relational database schema with three relation schemes Paper(P), Author(P, A), and Student(A), and the integrity constraint that every paper should have at least one author, but no paper should have a non-student author. Consider the database D given by

$$D(\text{Paper}) = \{\text{p1}\};\ D(\text{Author}) = \{(\text{p1}, \text{Bob})\};\ D(\text{Student}) = \{\text{Bob}\}.$$

Note that D satisfies the integrity constraint. The papers with at least one author but without non-student authors are retrieved by the relational algebra expression $Q = \text{Paper} \bowtie (\pi_P(\text{Author}) - V)$ with $V = \pi_P(\text{Author} - (\text{Author} \bowtie \text{Student}))$. Since V is empty on D, the lineage of p1 for Q in D is the database D' where

$$D'(\text{Paper}) = \{\text{p1}\};\ D'(\text{Author}) = \{(\text{p1}, \text{Bob})\};\ D'(\text{Student}) = \emptyset.$$

This database no longer satisfies the integrity constraint. □

An alternative approach to lineage is *why-provenance* [14]. This approach is non-deterministic in that it reflects that there may be several "explanations" for why a tuple is returned by a query (for example, queries involving existential quantification). Accordingly, why-provenance does not yield a single neighborhood (called witness), but a set of them. While logical, this approach is at odds with our aim of providing a *deterministic* retrieval mechanism through shapes. Of course, one could take the union of all witnesses, but this runs into similar problems as illustrated in the above example. Indeed, why-provenance was not developed for queries involving negation or universal quantification.

A recent approach to provenance for negation is that by Grädel and Tannen [27, 57] based on the successful framework of provenance semirings [28]. There, provenance is produced in the form of provenance polynomials which give a compact representation of the several possible proof trees showing that the tuple satisfies the query. Thus, like why-provenance, this approach is inherently non-deterministic. Still, we were influenced by Grädel and Tannen's use of negation normal form, which we have followed in this work.

5.2 Triple pattern fragments

Figure 1 is not meant to reflect expressiveness comparisons. That figure places shape fragments to the right of triple pattern fragments (TPF [59]). However, while shape fragments are in general obviously much more powerful than TPFs, not all TPFs are actually expressible by shape fragments.

A TPF may be viewed as a query that, on an input graph G, returns the subset of G consisting of all images of some fixed triple pattern in G. For example, TPFs of the form $(?x, p, ?y)$, $(?x, p, c)$, $(c, p, ?x)$, or (c, p, d), for IRIs p, c, and d, are easily expressed as shape fragments using the request shapes $\geq_1 p.\top$, $\geq_1 p.hasValue(c)$, $\geq_1 p^-.hasValue(c)$, or $hasValue(c) \wedge \geq_1 p.hasValue(d)$, respectively.

The TPF $(?x, p, ?x)$, asking for all p-self-loops in the graph, corresponds to the shape fragment for $\neg disj(\text{id}, p)$. A TPF of the form (c, p, d), for IRI d, asking for a single triple on condition that it is present in the graph, corresponds to the shape fragment for

Furthermore, the TPFs $(?x, ?y, ?z)$ (requesting a full download) and $(c, ?y, ?z)$ are expressible using the request shapes $\neg closed(\emptyset)$ and $hasValue(c) \wedge \neg closed(\emptyset)$. Here, the need to use a "trick" via negation of closedness constraints exposes a weakness of shapes: properties are not treated on equal footing as subjects and objects. Indeed, other TPFs involving variable properties, such as $(?x, ?y, c)$, $(?x, ?y, ?x)$, or $(c, ?x, d)$, are not expressible as shape fragments.

The above discussion can be summarized as follows. The proof is in the Appendix.

Proposition 5.2. *The TPFs expressible as a shape fragment (uniformly over all input graphs) are precisely the TPFs of the following forms:*

(1) $(?x, p, ?y)$;
(2) $(?x, p, c)$;
(3) $(c, p, ?x)$;
(4) (c, p, d);
(5) $(?x, p, ?x)$;
(6) $(?x, ?y, ?z)$;
(7) $(c, ?y, ?z)$.

Remark 5.3. SHACL does not allow *negated properties* in path expressions, while these are supported in SPARQL property paths. Extending SHACL with negated properties would readily allow the expression of *all* TPFs as shape fragments. For example, the TPF $(?x, ?y, c)$, for IRI c, would become expressible by requesting the shape

$$\geq_1 p.hasValue(c) \vee \geq_1 !p.hasValue(c),$$

with p an arbitrary IRI. Here, the negated property $!p$ matches any property different from p.

5.3 Knowledge graph subsets

Very recently, the idea of defining subgraphs (or fragments as we call them) using shapes was independently proposed by Labra Gayo [34]. An important difference with our SHACL-based approach is that his approach is based on ShEx, the other shape language besides SHACL that is popular in practice [13, 24]. Shapes in ShEx are quite different from those in SHACL, being based on bag-regular expressions over the bag of properties of the focus node. As a result, the technical developments of our work and Labra Gayo's are quite different. Still, the intuitive and natural idea of forming a subgraph by collecting all triples encountered during conformance checking, is clearly the same in both approaches. This idea, which Labra Gayo calls "slurping", is implemented in our pyshacl-fragments implementation, as well as a "slurp" option in the shex.js implementation of ShEx [55]. Labra Gayo also gives a formal definition of ShEx + slurp, extending the formal definition of ShEx [13].

In our work we make several additional contributions compared to the development by Labra Gayo:

- We consider the important special case of shape fragments based on schemas with targets.
- We support path expressions directly, which in ShEx need to be expressed through recursion.
- We support negation, universal quantification, and other non-monotone quantifiers and shapes, such as $\leq_n$, equality, disjointness, lessThan.
- We make the connection to database provenance and to Linked Data Fragments.
- We establish formal correctness properties (the Conformance Theorem and the Sufficiency Lemma).
- We investigate the translation of shape fragments into SPARQL. On the other hand, Labra Gayo discusses Pregel-based implementations of his query mechanism.

5.4 Path-returning queries on graph databases

Our definition of neighborhood of a node v for a shape involving a path expression E returns E-paths from v to relevant nodes x (see Table 2). Notably, these paths are returned as a subgraph, using the *graph* constructor applied to a set of paths. Thus, shape fragments are loosely related to path-returning queries on graph databases, introduced as a theoretical concept by Barceló et al. [9] and found in the languages Cypher [22] and G-CORE [35].

However, to our knowledge, a mechanism to return a set of paths in the form of a subgraph is not yet implemented by these languages. We have showed in Section 4.1 that, at least in principle, this is actually possible in any standard query language supporting path expressions, such as SPARQL. Barceló et al. consider a richer output structure whereby an infinite set of paths (or even set of tuples of paths) resulting from an extended regular path query can be finitely and losslessly represented. In contrast, our *graph* constructor is lossy in that two different sets S_1 and S_2 of paths may have $graph(S_1) = graph(S_2)$. However, our Sufficiency Lemma shows that our representation is sufficient for the purpose of validating shapes.

6 CONCLUSION

The idea that shapes can serve not only for data validation, but also for data access, has been floating largely informally within the community (e.g., [10, 58]). Our work is a step towards putting this idea on a solid formal footing. Many questions open up for further research; we mention a few very briefly.

SHACL is a quite powerful language, so an obvious direction is to investigate efficient processing and optimization strategies for SHACL, both just for validation, and for computing shape fragments. Recent work on validation optimization was done by Figuera et al. [21]. Yet we believe many more insights from database query optimization can be beneficial and specialized to shape processing. (A related direction is to use shapes to inform SPARQL query optimization [2, 50].)

We have seen that shape fragments are strictly less expressive than SPARQL subgraph queries. Is the complexity of evaluation lower? For those SPARQL subgraph queries that *are* expressible as shape fragments, are queries in practice often easier to write in SHACL? Can we precisely characterise the expressive power of SHACL?

Our approach to defining neighborhoods has been somehow *minimal* and *deterministic*. However, we miss postulates stating in what precise sense our definitions (or improved ones) are really minimal.

The SHACL recommendation only defines the semantics for nonrecursive shape schemas, and we have seen in this paper that shape fragments are already nontrivial for this case. Nevertheless, there is current interest in *recursive* shape schemas [4, 12, 13, 16, 24]. Extending shape fragments with recursion is indeed another interesting direction for further research.

Finally, it would be desirable to extend shapes so that properties are treated on equal footing as subjects and objects, as is indeed the spirit of RDF [39, 48].

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APPENDIX

A TRANSLATING REAL SHACL TO FORMAL SHACL

In this section we define the function t which maps a SHACL shapes graph $\mathcal{S}$ to a schema H.

Assumptions about the shapes graph:

- All shapes of interest must be explicitly declared to be a `sh:NodeShape` or `sh:PropertyShape`
- The shapes graph is well-formed

Let the sets $\mathcal{S}_n$ and $\mathcal{S}_p$ be the sets of all node shape shape names, respectively property shape shape names defined in the shapes graph $\mathcal{S}$. Let d_x denote the set of RDF triples in $\mathcal{S}$ with x as the subject. We define $t(\mathcal{S})$ as follows:

$$t(\mathcal{S}) = \{(x, t_{nodeshape}(d_x), t_{target}(d_x)) \mid x \in \mathcal{S}_n\} \cup \{(x, t_{propertyshape}(d_x), t_{target}(d_x)) \mid x \in \mathcal{S}_p\}$$

where we define $t_{nodeshape}(d_x)$ in Section A.1, $t_{propertyshape}(d_x)$ in Section A.3 and $t_{target}(d_x)$ in Section A.4.

Remark A.1. We treat node shapes and property shapes separately. In particular, minCount, maxCount, qualified minCount, qualified maxCount, and uniqueLang constraints are only treated below under property shapes. Strictly speaking, however, these constraints may also be used in node shapes, where they are redundant, as the count equals one in this case. For simplicity, we assume the shapes graph does not contain such redundancies.

A.1 Defining $t_{nodeshape}(d_x)$

This function translates SHACL node shapes to shapes in the formalization. We define $t_{nodeshape}(d_x)$ to be the following conjunction:

$$t_{shape}(d_x) \wedge t_{logic}(d_x) \wedge t_{tests}(d_x) \wedge t_{value}(d_x) \wedge t_{in}(d_x) \wedge t_{closed}(d_x) \wedge t_{pair}(\mathrm{id}, d_x) \wedge t_{languagein}(d_x)$$

where we define $t_{shape}(d_x), t_{logic}(d_x), t_{tests}(d_x), t_{value}(d_x), t_{in}(d_x), t_{closed}(d_x), t_{langin}(d_x)$ and $t_{pair}(\mathrm{id}, d_x)$ in the following subsections.

A.1.1 Defining $t_{shape}(d_x)$. This function translates the Shape-based Constraint Components from d_x to shapes from the formalization. This function covers the SHACL keywords: `sh:node` and `sh:property`.

We define $t_{shape}(d_x)$ to be the conjunction:

$$\bigwedge_{(x,\mathsf{sh:node},y)\in d_x} hasShape(y) \wedge \bigwedge_{(x,\mathsf{sh:property},y)\in d_x} hasShape(y)$$

A.1.2 Defining $t_{logic}(d_x)$. This function translates the Logical Constraint Components from d_x to shapes from the formalization. This function covers the SHACL keywords: `sh:and`, `sh:or`, `sh:not`, `sh:xone`.

We define $t_{logic}(d_x)$ as follows:

$$\begin{aligned}
&\bigwedge_{(x,\mathsf{sh:not},y)\in d_x} (\neg hasShape(y)) \wedge \\
&\bigwedge_{(x,\mathsf{sh:and},y)\in d_x} (\bigwedge_{z\in y} hasShape(z)) \wedge \\
&\bigwedge_{(x,\mathsf{sh:or},y)\in d_x} (\bigvee_{z\in y} hasShape(z)) \wedge \\
&\bigwedge_{(x,\mathsf{sh:xone},y)\in d_x} (\bigvee_{a\in y} (a \wedge \bigwedge_{b\in y-\{a\}} \neg hasShape(b)))
\end{aligned}$$

where we note that the object y of the triples with the predicate sh:and, sh:or, or sh:xone is a SHACL list.

A.1.3 Defining $t_{tests}(d_x)$. This function translates the Value Type Constraint Components, Value Range Constraint Components, and String-based Constraint Components, with exception to the `sh:languageIn` keyword which is handled in Section A.1.5, from d_x to shapes from the formalization. This function covers the SHACL keywords:

> `sh:class`, `sh:datatype`, `sh:nodeKind`, `sh:minExclusive`, `sh:maxExclusive`, `sh:minLength`, `sh:maxLength`, `sh:pattern`.

We define $t_{tests}(d_x)$ as follows:

$$t_{tests'}(d_x) \wedge \bigwedge_{(x,\texttt{sh:class},y)\in d_x} \geq_1 \texttt{rdf:type/rdf:subclassOf}^*.hasValue(y)$$

where $t_{tests'}(d_x)$ is defined next. Let Γ denote the set of keywords just mentioned above, except for `sh:class`.

$$t_{tests'}(d_x) = \bigwedge_{c\in\Gamma} \bigwedge_{(x,c,y)\in d_x} test(\omega_{c,y})$$

where $\omega_{c,y}$ is the node test in Ω corresponding to the SHACL constraint component corresponding to c with parameter y. For simplicity, we omit the `sh:flags` for `sh:pattern`.

A.1.4 Defining $t_{pair}(\mathrm{id}, d_x)$. This function translates the Property Pair Constraint Components when applied to a node shape from d_x to shapes from the formalization. This function covers the SHACL keywords: `sh:equals`, `sh:disjoint`, `sh:lessThan`. `sh:lessThanOrEquals`.

We define the function $t_{pair}(\mathrm{id}, d_x)$ as follows:

- If $\exists p : (x, \texttt{sh:lessThan}, p) \in d_x$ or $(x, \texttt{sh:lessThanEq}, p) \in d_x$, then we define $t_{pair}(\mathrm{id}, d_x)$ as $\bot$.
- Otherwise, we define $t_{pair}(\mathrm{id}, d_x)$ as

$$\bigwedge_{(x,\texttt{sh:equals},p)\in d_x} eq(\mathrm{id}, p) \wedge \bigwedge_{(x,\texttt{sh:disjoint},p)\in d_x} disj(\mathrm{id}, p)$$

A.1.5 Defining $t_{languagein}(d_x)$. This function translates the constraint component Language In Constraint Component from d_x to shapes from the formalization. This function covers the SHACL keyword: `sh:languageIn`.

The function $t_{languagein}(E, d_x)$ is defined as follows:

$$t_{languagein}(E, d_x) = \bigwedge_{(x,\texttt{sh:languageIn},y)\in d_x} \forall E. \bigvee_{lang\in y} test(\omega_{lang})$$

where y is a SHACL list and ω_{lang} is the element from Ω that corresponds to the test that checks if the node is annotated with the language tag *lang*.

A.1.6 Defining other constraint components. These functions translate the Other Constraint Components from d_x to shapes from the formalization. This function covers the SHACL keywords: `sh:closed`, `sh:ignoredProperties`, `sh:hasValue`, `sh:in`.

We define the following functions:

$$t_{value}(d_x) = \bigwedge_{(x,\texttt{sh:hasValue},y)\in d_x} hasValue(y)$$

$$t_{in}(d_x) = \bigwedge_{(x,\texttt{sh:in},y)\in d_x} (\bigvee_{a\in y} hasValue(a))$$

Let P be the set of all properties $p \in I$ such that $(y, \texttt{sh:path}, p) \in \mathcal{S}$ where y is a property shape such that $(x, \texttt{sh:property}, y) \in d_x$ union the set given by the SHACL list specified by the

sh:ignoredProperties parameter. Then, we define the function $t_{closed}(d_x)$ as follows:

$$t_{closed}(d_x) = \begin{cases} \top & \text{if } (x, \texttt{sh:closed}, \mathit{true}) \notin d_x \\ \mathit{closed}(P) & \text{otherwise} \end{cases}$$

A.2 Defining $t_{path}(pp)$

In preparation of the next Subsection, this function translates the Property Paths to path expressions. This part of the translation deals with the SHACL keywords:

sh:inversePath, sh:alternativePath, sh:zeroOrMorePath,
sh:oneOrMorePath, sh:zeroOrOnePath, sh:alternativePath.

For an IRI or blank node pp representing a property path, we define $t_{path}(pp)$ as follows:

$$t_{path}(pp) = \begin{cases} pp & \text{if } pp \text{ is an IRI} \\ t_{path}(y)^- & \text{if } \exists y : (pp, \texttt{sh:inversePath}, y) \in \mathcal{S} \\ t_{path}(y)^* & \text{if } \exists y : (pp, \texttt{sh:zeroOrMorePath}, y) \in \mathcal{S} \\ t_{path}(y)/t_{path}(y)^* & \text{if } \exists y : (pp, \texttt{sh:oneOrMorePath}, y) \in \mathcal{S} \\ t_{path}(y)? & \text{if } \exists y : (pp, \texttt{sh:zeroOrOnePath}, y) \in \mathcal{S} \\ \bigcup_{a \in y} t_{path}(a) & \text{if } \exists y : (pp, \texttt{sh:alternativePath}, y) \in \mathcal{S} \text{ and } y \text{ is a SHACL list} \\ t_{path}(a_1)/\ldots/t_{path}(a_n) & \text{if } pp \text{ represents the SHACL list } [a_1, \ldots, a_n] \end{cases}$$

A.3 Defining $t_{propertyshape}(d_x)$

This function translates SHACL property shapes to shapes in the formalization. Let pp be the property path associated with d_x. Let E be $t_{path}(pp)$. We define $t_{propertyshape}(d_x)$ as the following conjunction:

$$t_{card}(E, d_x) \wedge t_{pair}(E, d_x) \wedge t_{qual}(E, d_x) \wedge t_{all}(E, d_x) \wedge t_{uniquelang}(E, d_x)$$

where we define t_{card}, t_{pair}, t_{qual}, t_{all}, and $t_{uniquelang}$ in the following subsections.

A.3.1 Defining $t_{card}(E, d_x)$. This function translates the Cardinality Constraint Components. from d_x to shapes from the formalization. This function covers the SHACL keywords: sh:minCount, sh:maxCount.

We define the function $t_{card}(E, d_x)$ as follows:

$$\bigwedge_{(x,\texttt{sh:minCount},n) \in d_x} \geq_n E.\top \wedge \bigwedge_{(x,\texttt{sh:maxCount},n) \in d_x} \leq_n E.\top$$

A.3.2 Defining $t_{pair}(E, d_x)$. This function translates the Property Pair Constraint Components when applied to a property shape from d_x to shapes from the formalization. This function covers the SHACL keywords: sh:equals, sh:disjoint, sh:lessThan. sh:lessThanOrEquals.

We define the function $t_{pair}(E, d_x)$ as follows:

$$\bigwedge_{(x,\texttt{sh:equals},p) \in d_x} \mathit{eq}(E, p) \wedge \bigwedge_{(x,\texttt{sh:disjoint},p) \in d_x} \mathit{disj}(E, p) \wedge \bigwedge_{(x,\texttt{sh:lessThan},p) \in d_x} \mathit{lessThan}(E, p) \wedge \bigwedge_{(x,\texttt{sh:lessThanOrEquals},p) \in d_x} \mathit{lessThanEq}(E, p)$$

A.3.3 Defining $t_{qual}(E, d_x)$. This function translates the (Qualified) Shape-based Constraint Components from d_x to shapes from the formalization. This function covers the SHACL keywords:

`sh:qualifiedValueShape`, `sh:qualifiedMinCount`, `sh:qualifiedMaxCount`, `sh:qualifiedValueShapesDisjoint`.

We distinguish between the case where the parameter `sh:qualifiedValueShapesDisjoint` is set to *true*, and the case where it is not:

$$t_{qual}(E, d_x) = \begin{cases} t_{sibl}(E, d_x) & \text{if } (x, \texttt{sh:qualifiedValueShapesDisjoint}, \mathit{true}) \in d_x \\ t_{nosibl}(E, d_x) & \text{otherwise} \end{cases}$$

where we define $t_{sibl}(E, d_x)$ and $t_{nosibl}(E, d_x)$ next. Let $ps = \{v \mid (v, \texttt{sh:property}, x) \in \mathcal{S}\}$. We define the set of sibling shapes

$$sibl = \{w \mid \exists v \in ps\, \exists y (v, \texttt{sh:property}, y) : (y, \texttt{sh:qualifiedValueShape}, w) \in \mathcal{S}\}.$$

We also define:

$$\begin{aligned} Q &= \{y \mid (x, \texttt{sh:qualifiedValueShape}, y) \in d_x\} \\ Qmin &= \{z \mid (x, \texttt{sh:qualifiedMinCount}, z) \in d_x\} \\ Qmax &= \{z \mid (x, \texttt{sh:qualifiedMaxCount}, z) \in d_x\} \end{aligned}$$

We now define

$$t_{sibl}(E, d_x) = \bigwedge_{y \in Q} \bigwedge_{z \in Qmin} \geq_z E.(hasShape(y) \wedge \bigwedge_{s \in sibl} \neg hasShape(s)) \\ \wedge \bigwedge_{y \in Q} \bigwedge_{z \in Qmax} \leq_z E.(hasShape(y) \wedge \bigwedge_{s \in sibl} \neg hasShape(s))$$

and

$$t_{nosibl}(E, d_x) = \bigwedge_{y \in Q} \bigwedge_{z \in Qmin} \geq_z E.hasShape(y) \wedge \bigwedge_{y \in Q} \bigwedge_{z \in Qmax} \leq_z E.hasShape(y).$$

A.3.4 Defining $t_{all}(E, d_x)$. This function translates the constraint components that are not specific to property shapes, but which are applied on property shapes.

We define the function $t_{all}(E, d_x)$ to be:

$$\forall E.(t_{shape}(d_x) \wedge t_{logic}(d_x) \wedge t_{tests}(d_x) \wedge t_{in}(d_x) \wedge t_{closed}(d_x) \wedge t_{languagein}(d_x)) \wedge t_{allvalue}(E, d_x)$$

where

$$t_{allvalue}(E, d_x) = \begin{cases} \top & \text{if } \nexists v : (x, \texttt{sh:hasValue}, v) \in d_x \\ \geq_1 E.t_{value}(d_x) & \text{otherwise} \end{cases}$$

and t_{shape}, t_{logic}, t_{tests}, t_{value}, $t_{languagein}$, and t_{closed} are as defined earlier. Note the treatment of the `sh:hasValue` parameter when used in a property shape. Unlike the other definitions, it is not universally quantified over the value nodes given by E.

A.3.5 Defining $t_{uniquelang}(E, d_x)$. This function translates the constraint component Unique Lang Constraint Component from d_x to shapes from the formalization. This function covers the SHACL keyword: `sh:uniqueLang`.

The function $t_{uniquelang}(E, d_x)$ is defined as follows:

$$t_{uniquelang}(E, d_x) = \begin{cases} uniqueLang(E) & \text{if } (x, \texttt{sh:uniqueLang}, \mathit{true}) \in d_x \\ \top & \text{otherwise} \end{cases}$$

A.4 Defining $t_{target}(d_x)$

This function translates the Target declarations to shapes from the formalization. This function covers the SHACL keywords:

`sh:targetNode`, `sh:targetClass`, `sh:targetSubjectsOf`, `sh:targetObjectsOf`.

We define the function as follows:

$$
\begin{aligned}
t_{target}(d_x) = & \bigvee_{(x,\texttt{sh:targetNode},y)\in d_x} hasValue(y) \vee \\
& \bigvee_{(x,\texttt{sh:targetClass},y)\in d_x} \geq_1 \texttt{rdf:type/rdf:subclassOf}^*.hasValue(y) \vee \\
& \bigvee_{(x,\texttt{sh:targetSubjectsOf},y)\in d_x} \geq_1 y.\top \vee \\
& \bigvee_{(x,\texttt{sh:targetObjectsOf},y)\in d_x} \geq_1 y^-.\top
\end{aligned}
$$

If none of these triples are in d_x we define $t_{target}(d_x) = \bot$

B PROOF OF CONFORMANCE THEOREM

Assuming the Sufficiency Lemma, the proof of the Conformance Theorem straightforwardly goes as follows. Let $F = Frag(G, H)$; we must show that F conforms to H. Thereto, consider a shape definition $(s, \phi, \tau) \in H$, and let v be a node such that $F, v \models \tau$. Since $F \subseteq G$ and τ is monotone, also $G, v \models \tau$, whence $G, v \models \phi$ since G conforms to H. Since by definition, F contains $B(v, G, \phi)$, the Sufficiency Lemma yields $F, v \models \phi$ as desired.

Toward a proof of the Sufficiency Lemma, we first prove:

Proof of Proposition 3.4. That $[\![E]\!]^F \subseteq [\![E]\!]^G$ is immediate from $F \subseteq G$ and the monotonicity of path expressions. For the reverse inclusion, we proceed by induction on the structure of E. The base case, where E is a property p, is immediate from the definitions. The inductive cases where E is one of $E_1 \cup E_2$, E_1^-, or E_1/E_2, are clear. Two cases remain:

- E is $E_1?$. If $a = b$, then $(a, b) \in [\![E]\!]^F$ by definition. Otherwise, (a, b) must be in $[\![E_1]\!]^G$. Therefore, by induction, $(a, b) \in [\![E_1]\!]^F \subseteq [\![E]\!]^F$.
- E is E_1^*. If $a = b$, then $(a, b) \in [\![E]\!]^F$ by definition. Otherwise, there exist $i \geq 1$ nodes $c_0, \ldots, c_i$ such that $a = c_0$ and $b = c_i$, and $(c_j, c_{j+1}) \in [\![E_1]\!]^G$ for $0 \leq j < i$. By induction, each $(c_j, c_{j+1}) \in [\![E_1]\!]^F$, whence (a, b) belongs to the transitive closure of $[\![E_1]\!]^F$ as desired.

□

We now give the

Proof of the Sufficiency Lemma. For any shape ϕ, we consider its expansion with relation to the schema H, obtained by repeatedly replacing subshapes of the form $hasShape(s)$ by $def(s, H)$, until we obtain an equivalent shape that no longer contains any subshapes of the form $hasShape(s)$. The proof proceeds by induction on the height of the expansion of ϕ in negation normal form, where the height of negated atomic shapes is defined to be zero. When ϕ is $\top$, $test(t)$, or $hasValue(c)$, and v conforms to ϕ in G, then v clearly also conforms to ϕ in G', as the conformance of the node is independent of the graph. We consider the following inductive cases:

- ϕ is $\phi_1 \wedge \phi_2$. By induction, we know v conforms to ϕ_1 in G' and conforms to ϕ_2 in G'. Therefore, v conforms to $\phi_1 \wedge \phi_2$ in G'.
- ϕ is $\phi_1 \vee \phi_2$. We know v conforms to at least one of ϕ_i for $i \in \{1, 2\}$ in G. Assume w.l.o.g. that v conforms to ϕ_1 in G. Then, our claim follows by induction.

- ϕ is $\geq_n E.\psi$. Here, and in the following cases, we denote $B(v, G, \phi)$ by B. As $G, v \models \phi$, we know there are at least n nodes $x_1, \ldots, x_n$ in G such that $x_i \in [\![E]\!]^G(v)$ and $G, x_i \models \psi$ for all $1 \leq i \leq n$. Let $F = \mathit{graph}(\mathit{paths}(E, G, v, x))$. By Proposition 3.4, $x_i \in [\![E]\!]^F(v)$. By definition of ϕ-neighborhood $F \subseteq B$, and we know $B \subseteq G'$. Therefore, because E is monotone, $x_i \in [\![E]\!]^{G'}(v)$. Furthermore, since $B(x_i, G, \psi) \subseteq B \subseteq G'$, by induction, $G', x_i \models \psi$ as desired.
- ϕ is $\leq_n E.\psi$. First we show that every $x \in [\![E]\!]^{G'}(v)$ that conforms to ψ in G', must also conform to ψ in G.
  Proof by contradiction. Suppose there exists a node $x \in [\![E]\!]^{G'}(v)$ that conforms to ψ in G', but conforms to $\neg\psi$ in G. By definition of ϕ-neighborhood, $B(x, G, \neg\psi) \subseteq B$, and we know $B \subseteq G'$. Therefore, by induction, x conforms to $\neg\psi$ in G', which is a contradiction.
  Because of the claim above, the number of nodes reachable from v through E that satisfy ψ in G' must be smaller or equal to the number of nodes reachable from v through E that satisfy ψ in G. Because we know $G, v \models \phi$, the lemma follows.
- ϕ is $\forall E.\psi$. For all nodes x such that $x \in [\![E]\!]^{G'}(v)$, as E is monotone, $x \in [\![E]\!]^G(v)$. As $G, v \models \phi$, $G, x \models \psi$. By definition of ϕ-neighborhood, $B(x, G, \psi) \subseteq B$. We know $B \subseteq G'$. Thus, by induction, $G', x \models \psi$ as desired.
- ϕ is $\mathit{eq}(E, p)$. We must show that $[\![E]\!]^{G'}(v) = [\![p]\!]^{G'}(v)$. For the containment from left to right, let $x \in [\![E]\!]^{G'}(v)$. Since E is monotone, $x \in [\![E]\!]^G(v)$. Since $G, v \models \phi$, $x \in [\![p]\!]^G(v)$. Let $F = \mathit{graph}(\mathit{paths}(p, G, v, x))$. By Proposition 3.4, $x \in [\![p]\!]^F(v)$. By definition of ϕ-neighborhood, $F \subseteq B$, and we know $B \subseteq G'$. Therefore, because path expressions are monotone, we also have $x \in [\![p]\!]^{G'}(v)$ as desired. The containment from right to left is analogous.
- ϕ is $\mathit{eq}(\mathrm{id}, p)$. We must show that $[\![id]\!]^{G'}(v) = [\![p]\!]^{G'}(v)$, or equivalently we must show that $\{v\} = [\![p]\!]^{G'}(v)$. We know that $G, v \models \phi$, therefore $[\![p]\!]^G(v) = \{v\}$. Now we only need to show that $(v, p, v) \in G'$ as $G' \subseteq G$ (and therefore G' does not contain more p-edges than G). Because by definition of neighborhood $B = \{(v, p, v)\}$, and because $B \subseteq G'$, the claim follows.
- ϕ is $\mathit{disj}(E, p)$. Let $x \in [\![E]\!]^{G'}(v)$. Since E is monotone, $x \in [\![E]\!]^G(v)$. Since $G, v \models \phi$, $x \notin [\![p]\!]^G(v)$. Therefore, as p is monotone, $x \notin [\![p]\!]^{G'}(v)$. The case where $x \in [\![p]\!]^{G'}(v)$ is analogous.
- ϕ is $\mathit{disj}(\mathrm{id}, p)$. We must show that $(v, p, v) \notin G'$. Because $G, v \models \phi$, we know that $(v, p, v) \notin G$. As $G' \subseteq G$, the claim follows.
- ϕ is $\mathit{lessThan}(E, p)$. Let $x \in [\![E]\!]^{G'}(v)$. Let $(v, p, y) \in G'$. We must show that $x < y$. Since E is monotone, $x \in [\![E]\!]^G(v)$ and since $G' \subseteq G$, $(v, p, y) \in G$. As $G, v \models \phi$, we know that $x < y$ in G and thus also in G'.
- ϕ is $\mathit{lessThanEq}(E, p)$. Analogous to the case where ϕ is $\mathit{lessThan}(E, p)$.
- ϕ is $\mathit{uniqueLang}(E)$. Let $x \in [\![E]\!]^{G'}(v)$. Let $y \in [\![E]\!]^{G'}(v)$ such that $y \neq x$. As E is monotone, $x \in [\![E]\!]^G(v)$ and $y \in [\![E]\!]^G(v)$. As $G, v \models \phi$, we know $y \nsim x$ in G and thus also in G'.
- ϕ is $\mathit{closed}(P)$. Let $(v, p, x) \in G'$. Then, $(v, p, x) \in G$. Therefore, as $G, v \models \phi$, $p \in P$ as desired.
- ϕ is $\neg\mathit{eq}(E, p)$. Since $G, v \models \phi$, there are two cases. First, there exists a node $x \in [\![E]\!]^G(v)$ such that $x \notin [\![p]\!]^G(v)$. Let $F = \mathit{graph}(\mathit{paths}(E, G, v, x))$. By Proposition 3.4, $x \in [\![E]\!]^F(v)$. By definition of ϕ-neighborhood $F \subseteq B$, and we know $B \subseteq G'$. Therefore, since E is monotone, $x \in [\![E]\!]^{G'}(v)$. Next, since $(v, p, x) \notin G$, we know $(v, p, x) \notin G'$. Thus, $[\![E]\!]^G(v) \neq [\![p]\!]^G(v)$ as desired. For the other case, there exists a node x such that $(v, p, x) \in G$ and $x \notin [\![E]\!]^G(v)$. By definition of ϕ-neighborhood, $(v, p, x) \in B \subseteq G'$. However, because E is monotone $x \notin [\![E]\!]^{G'}(v)$. Therefore $[\![p]\!]^G(v) \neq [\![E]\!]^G(v)$ as desired.
- ϕ is $\neg\mathit{eq}(\mathrm{id}, p)$. Since $G, v \models \phi$, there are two cases. First, $(v, p, v) \notin G$. We know $G' \subseteq G$, therefore if $(v, p, v) \notin G$, then $(v, p, v) \notin G'$ as desired. Second, $(v, p, v) \in G$ and there exists a node x such that $(v, p, x) \in G$ and $x \neq v$. From the definition of neighborhood, we know that this also holds for B and therefore also in G' as $B \subseteq G'$.

- ϕ is $\neg disj(E, p)$. Since $G, c \models \phi$, we know that there exists a node $x \in [\![E]\!]^G(v)$ such that $(v, p, x) \in G$. Let $F = graph(paths(E, G, v, x))$. By Proposition 3.4, $x \in [\![E]\!]^F(v)$. By definition of ϕ-neighborhood $F \subseteq B$, and we know $B \subseteq G'$. Then, since E is monotone, $x \in [\![E]\!]^{G'}(v)$. Next, by definition of ϕ-neighborhood, also $(v, p, x) \in B \subseteq G'$. Thus, $x \in [\![E]\!]^{G'}(v) \cap [\![p]\!]^{G'}(v)$ as desired.
- ϕ is $\neg disj(\mathrm{id}, p)$. We need to show that $(v, p, v) \in G'$. By definition of neighborhood, $(v, p, v) \in B$. As $B \subseteq G'$, $(v, p, v) \in G'$ as desired.
- ϕ is $\neg lessThan(E, p)$. Since $G, v \models \phi$, there exists a node $x \in [\![E]\!]^G(v)$ and a node $y \in [\![p]\!]^G(v)$ with $x \not< y$. If we can show that $x \in [\![E]\!]^{G'}(v)$ and $x \in [\![p]\!]^{G'}(v)$, it will follow that $G', v \models \phi$ as desired. Let $F = graph(paths(E, G, v, x))$. By Proposition 3.4, $x \in [\![E]\!]^F(v)$. By definition of ϕ-neighborhood, $F \subseteq B$, and we know $B \subseteq G'$. Then, since E is monotone, $x \in [\![E]\!]^{G'}(v)$. Next, by definition of ϕ-neighborhood, $(v, p, x) \in B$. Since $B \subseteq G'$, also $x \in [\![p]\!]^{G'}(v)$ as desired.
- ϕ is $\neg lessThanEq(E, p)$. Analogous to the case where ϕ is $\neg lessThan(E, p)$.
- ϕ is $\neg uniqueLang(E)$. Since $G, v \models \phi$, there exists $x_1 \neq x_2 \in [\![E]\!]^G(v)$ such that $x_1 \sim x_2$. As in the previous case, we must show that x_1 and x_2 are in $[\![E]\!]^{G'}(v)$. By Proposition 3.4, for both $i = 1, 2$, we have $x_i \in [\![E]\!]^{F_i}(v)$ with $F_i = graph(paths(E, G, v, x_i))$. By definition of ϕ-neighborhood $F_i \subseteq B \subseteq G'$. Therefore $x_i \in [\![E]\!]^{G'}(v)$ as desired.
- ϕ is $\neg closed(P)$. As $G, v \models \phi$, there exists a property $p \notin P$ and a node x such that $(v, p, x) \in G$. By definition $(v, p, x) \in B(v, G, \phi) \subseteq G'$. Hence, $G', v \models \phi$ as desired.

□

C SHAPE FRAGMENTS IN SPARQL

C.1 Proof of Lemma 4.1

Proceeding by induction on the structure of E, we describe Q_E in each case.

- E is a property name p.

```
SELECT (?s AS ?t) ?s (p AS ?p) ?o (?o AS ?h)
WHERE { ?s p ?o. }
```

- E is $E_1?$.

```
SELECT ?t ?s ?p ?o ?h
WHERE {
  { Q_E1 }
  UNION
  {
    SELECT (?v AS ?t) (?v AS ?h)
    WHERE { { ?v ?_p1 ?_o1 } UNION { ?_s2 ?_p2 ?v } }
  }
}
```

- E is E_1^-.

```
SELECT (?h AS ?t) ?s ?p ?o (?t as ?h)
WHERE {
  Q_E1
}
```

- E is $E_1 \cup E_2$.

```
SELECT ?t ?s ?p ?o ?h
WHERE {
  { Q_E1 }
```

```
  UNION
  { Q_{E_2} }
}
```

- E is E_1/E_2.

```
SELECT ?t ?s ?p ?o ?h
WHERE {
  {
    {
      SELECT ?t ?s ?p ?o (?h AS ?h1)
      WHERE { Q_{E_1} }
    }.
    {
      SELECT (?t AS ?h1) ?h
      WHERE { ?t E_2 ?h }
    }
  }
  UNION
  {
    {
      SELECT ?t (?h AS ?h1)
      WHERE { ?t E_1 ?h }
    }.
    {
      SELECT (?t AS ?h1) ?s ?p ?o ?h
      WHERE { Q_{E_2} }
    }
  }
}
```

- E is E_1^*.

```
SELECT ?t ?s ?p ?o ?h
WHERE {
  {
    ?t E_1^* ?x1.
    ?x2 E_1^* ?h.
    {
      SELECT (?t AS ?x1) ?s ?p ?o (?h AS ?x2)
      WHERE { Q_{E_1} }
    }
  }
  UNION
  {
    SELECT (?v AS ?h) (?v AS ?t)
    WHERE { { ?v ?_p1 ?_o1 } UNION { ?_s2 ?_p2 ?v } }
  }
}
```

C.2 Proof of Lemma 4.3

As always we work in the context of a schema H. We assume ϕ is put in negation normal form and proceed by induction as in the proof of the Sufficiency Lemma.

Note that Q_ϕ should not merely check conformance of nodes to shapes, but actually must return the neighborhoods. Indeed, that conformance checking in itself is possible in SPARQL (for nonrecursive shapes) is well known; it was even considered for recursive shapes [16]. Hence, in the constructions below, we use an auxiliary SPARQL query $CQ_\phi(?v)$ (C for conformance) which returns, on every RDF graph G, the set of nodes $v \in N(G)$ such that $G, v \models \phi$.

We now describe Q_ϕ for all the cases in the following.

- ϕ is $\top$: empty
- ϕ is $hasValue(c)$: empty
- ϕ is $test(t)$: empty
- ϕ is $closed(P)$: empty
- ϕ is $hasShape(s)$: $Q_{def(s,H)}$
- ϕ is $\phi_1 \wedge \phi_2$ or $\phi_1 \vee \phi_2$:

```
SELECT ?v ?s ?p ?o
WHERE {
  { CQ_φ } .
  { Q_φ1 }
  UNION
  { Q_φ2 }
}
```

- ϕ is $\geq_n E.\phi_1$:

```
SELECT (?t AS ?v) ?s ?p ?o
WHERE {
  {
    { SELECT (?v AS ?t) WHERE { CQ_φ } } .
    { Q_E } .
    { SELECT (?v AS ?h) WHERE { CQ_φ1 } }
  } UNION
  {
    { SELECT (?v AS ?t) WHERE { CQ_φ } } .
    ?t E ?h .
    {
      SELECT (?v AS ?h) ?s ?p ?o
      WHERE { { Q_φ1 } . { CQ_φ1 }}
    }
  }.
}
```

- ϕ is $\leq_n E.\phi_1$:

```
SELECT (?t AS ?v) ?s ?p ?o
WHERE {
  {
    { SELECT (?v AS ?t) WHERE { CQ_φ } } .
    { Q_E } .
    { SELECT (?v AS ?h) WHERE { CQ_¬φ1 } }
  } UNION
  {
```

```
    { SELECT (?v AS ?t) WHERE { CQ_φ } } .
    ?t E ?h .
    {
      SELECT (?v AS ?h) ?s ?p ?o
      WHERE { { Q_¬φ1 } . { CQ_¬φ1 }}
    }
  }
}
```

- ϕ is $\forall E.\phi_1$:

```
SELECT (?t AS ?v) ?s ?p ?o
WHERE {
  {
    { SELECT (?v AS ?t) WHERE { CQ_φ } } .
    { Q_E }
  } UNION
  {
    { SELECT (?v AS ?t) WHERE { CQ_φ } } .
    ?t E ?h .
    {
      SELECT (?v AS ?h) ?s ?p ?o
      WHERE { Q_φ1 }
    }
  }
}
```

- ϕ is $eq(E, p)$:

```
SELECT (?t AS ?v) ?s ?p ?o
WHERE {
  { SELECT (?v AS ?t) WHERE { CQ_φ } } .
  {
    { Q_E } UNION { Q_p }
  }
}
```

- ϕ is $eq(\text{id}, p)$:

```
SELECT ?v (?s AS ?v) (p AS ?p) (?v AS ?o)
WHERE {
  { CQ_φ } .
  ?v p ?v
}
```

- ϕ is $disj(E, p)$: empty
- ϕ is $disj(\text{id}, p)$: empty
- ϕ is $lessThan(E, p)$: empty
- ϕ is $lessThanEq(E, p)$: empty
- ϕ is $uniqueLang(E)$: empty
- ϕ is $\neg\top$: empty
- ϕ is $\neg hasValue(c)$: empty
- ϕ is $\neg test(t)$: empty
- ϕ is $\neg hasShape(s)$: $Q_{\neg def(s,H)}$
- ϕ is $\neg closed(P)$):

```
SELECT ?v (?v AS ?s) ?p ?o
WHERE {
  { CQ_φ } .
  ?v ?p ?o.
  FILTER (?p NOT IN P)
}
```

- ϕ is $\neg eq(E, p)$:

```
SELECT (?t AS ?v) ?s ?p ?o
WHERE {
  { SELECT (?v AS ?t) WHERE { CQ_φ } } .
  {
    { { Q_E } MINUS { ?t p ?h } }
    UNION
    { { Q_p } MINUS { ?t E ?h } }
  }
}
```

- ϕ is $\neg eq(\mathrm{id}, p)$:

```
SELECT ?v (?v AS ?s) (p AS ?p) (?v AS ?o)
WHERE {
  { CQ_φ } .
  { ?v p ?o }
  FILTER (?o != ?v)
}
```

- ϕ is $\neg disj(E, p)$:

```
SELECT (?t AS ?v) ?s ?p ?o
WHERE {
  { SELECT (?v AS ?t) WHERE { CQ_φ } } .
  {
    { { Q_E } . { ?t p ?h } }
    UNION
    { { Q_p } . { ?t E ?h } }
  }
}
```

- ϕ is $\neg disj(\mathrm{id}, p)$:

```
SELECT ?v (?v AS ?s) (p AS ?p) (?v AS ?o)
WHERE {
  { CQ_φ } .
  ?v p ?v
}
```

- ϕ is $\neg lessThan(E, p)$:

```
SELECT (?t AS ?v) ?s ?p ?o
WHERE {
  { SELECT (?v AS ?t) WHERE { CQ_φ } } .
  {
    { { Q_E } . { ?t p ?h2 } FILTER (! ?h < ?h2) }
    UNION
    { { Q_p } . { ?t E ?h2 } FILTER (! ?h2 < ?h) }
```

```
  }
}
```

- ϕ is $\neg lessThanEq(E, p)$:

```
SELECT (?t AS ?v) ?s ?p ?o
WHERE {
  { SELECT (?v AS ?t) WHERE { CQ_φ } } .
  {
    { { Q_E } . { ?t p ?h2 } FILTER (! ?h <= ?h2) }
    UNION
    { { Q_p } . { ?t E ?h2 } FILTER (! ?h2 <= ?h) }
  }
}
```

- ϕ is $\neg uniqueLang(E)$:

```
SELECT (?t AS ?v) ?s ?p ?o
WHERE {
  { SELECT (?v AS ?t) WHERE { CQ_φ } } .
  { Q_E } . { ?t E ?h2 }
  FILTER (?h != ?h2 && lang(?h) = lang(?h2))
}
```

D PROOF OF PROPOSITION 5.2

That the seven forms of TPF mentioned in the proposition can be expressed as shape fragments was already shown in the main body of the paper. So it remains to show that all other forms of TPF are not expressible as shape fragments. Since, for any finite set S of shapes, we can form the disjunction $\bigvee S$ of all shapes in S, and $Frag(G, S) = Frag(G, \{\bigvee S\})$ for any graph G, it suffices to consider single shapes ϕ instead of finite sets of shapes. We abbreviate $Frag(G, \{\phi\})$ to $Frag(G, \phi)$.

Formally, let $Q = (u, v, w)$ be a triple pattern, i.e., u, v and w are variables or elements of N. Let V be the set of variables from $\{u, v, w\}$ to N. A solution mapping is a function $\mu : V \to N$. For any node a, we agree that $\mu(a) = a$. Then the TPF query Q maps any input graph G to the subset

$$Q(G) = \{(\mu(u), \mu(v), \mu(w)) \mid \mu : V \to N \;\&\; (\mu(u), \mu(v), \mu(w)) \in G\}.$$

We now say that a shape ϕ *expresses* a TPF query Q if $Frag(G, \phi) = Q(G)$ for every graph G.

We begin by showing:

Lemma D.1. *Let G be an RDF graph and let ϕ be a shape. Assume $Frag(G, \phi)$ contains a triple (s, p, o) where p is not mentioned in ϕ. Then $Frag(G, \phi)$ contains all triples in G of the form (s, p', o'), where p' is not mentioned in ϕ.*

Proof. Since shape fragments are unions of neighborhoods, it suffices to verify the statement for an arbitrary neighborhood $B(v, G, \phi)$. This is done by induction on the structure of the negation normal form of ϕ. In almost all cases of Table 2, triples from $B(v, G, \phi)$ come from E-paths, with E mentioned in ϕ; from $B(v, G, \psi)$, with ψ a subshape of ϕ or the negation thereof; or involve a property p clearly mentioned in ϕ. Triples of the first kind never have a property not mentioned in ϕ, and triples of the second kind satisfy the statement by induction.

The only remaining case is $\neg closed(P)$. Assume (v, p, x) is in the neighborhood, and let $(v, p', x') \in G$ be a triple such that p' is not mentioned in ϕ. Then certainly $p' \notin P$, so (v, p', x') also belongs to the neighborhoods, as desired. □

Using the above Lemma, we give:

Proof of Proposition 5.2. Consider the TPF $Q = (?x, ?x, ?y)$ and assume there exists a shape ϕ such that $Q(G) = Frag(G, \phi)$ for all G. Consider $G = \{(a, a, b), (a, c, b)\}$, where a and c are not mentioned in ϕ. We have $(a, a, b) \in Q(G)$ so $(a, a, b) \in Frag(G, \phi)$. Then by Lemma D.1, also $(a, c, b) \in Frag(G, \phi)$. However, $(a, c, b) \notin P(G)$, so we arrive at a contradiction, and ϕ cannot exist.

Similar reasoning can be used for all other forms of TPF not covered by the proposition. Below we give the table of these TPFs Q, where c and d are arbitrary IRIs, possibly equal, and $?x$ and $?y$ are distinct variables. The right column lists the counterexample graph G showing that $Q(G) \neq Frag(G, \phi)$. Importantly, the property (a or b) of the triples in G is always chosen so that it is not mentioned in ϕ, and moreover, a, b and e are distinct and also distinct from c and d.

Q	G
$(?x, ?y, ?x)$	$\{(a, b, a), (a, b, c)\}$
$(?x, ?y, ?y)$	$\{(a, b, b), (a, b, c)\}$
$(?x, ?x, ?x)$	$\{(a, a, a), (a, a, b)\}$
$(?x, ?y, c)$	$\{(a, b, c), (a, b, d)\}$
$(?x, ?x, c)$	$\{(a, a, c), (a, a, d)\}$
$(?x, ?y, ?y)$	$\{(a, b, b), (a, b, c)\}$
$(c, ?x, ?x)$	$\{(c, a, a), (c, a, b)\}$
$(c, ?x, d)$	$\{(c, a, d), (c, a, e)\}$

□