

# Machine Learning for Trading

Notes for *CS 7646*



George Kudrayvtsev

[george.k@gatech.edu](mailto:george.k@gatech.edu)

Last Updated: June 21, 2019

<b>I Manipulating Financial Data</b>	<b>4</b>
<b>1 Python for Finance</b>	<b>5</b>
<b>2 Python for Finance</b>	<b>6</b>
2.1 Global Statistics . . . . .	6
2.1.1 Bollinger Bands® . . . . .	7
2.1.2 Daily Returns . . . . .	7
2.1.3 Cumulative Returns . . . . .	8
2.2 Fixing Bad Data . . . . .	9
2.3 Graphing Financial Data . . . . .	10
2.3.1 Histograms . . . . .	10
2.3.2 Scatter Plots . . . . .	12
2.4 Portfolios . . . . .	14
2.5 Optimizers . . . . .	16
2.5.1 Using an Optimizer . . . . .	17
2.5.2 Building Parameterized Models . . . . .	17
Error Metrics . . . . .	18
$n$ Dimensions . . . . .	19
2.5.3 Portfolio Optimization . . . . .	19
<b>II Computational Investing</b>	<b>21</b>
<b>3 Hedge Funds</b>	<b>22</b>
3.1 Types of Managed Funds . . . . .	22
3.2 Compensation . . . . .	23
3.3 Attracting Investors . . . . .	25
3.3.1 Goals . . . . .	25
3.3.2 Metrics . . . . .	25
3.4 Hedge Fund Computing Architecture . . . . .	26

<b>4 The Order Book</b>	<b>27</b>
4.1 Making Orders . . . . .	27
4.1.1 The Order Book . . . . .	28
4.1.2 Filling Orders . . . . .	31
4.1.3 Exploiting the Order Book . . . . .	33
Exploit 1: Faster-Than-Light Travel . . . . .	33
Exploit 2: Geographic Arbitrage . . . . .	33
4.1.4 Selling Short . . . . .	34
What Can Go Wrong? . . . . .	35
<b>5 Evaluating a Company</b>	<b>36</b>
5.1 Metrics for Evaluation . . . . .	36
5.1.1 Intrinsic Value . . . . .	37
5.1.2 Book Value . . . . .	38
5.1.3 Market Capitalization . . . . .	39
5.1.4 Knowledge is Power . . . . .	39
5.2 Capital Asset Pricing Model . . . . .	40
5.2.1 Portfolio Management Under CAPM . . . . .	41
5.2.2 CAPM for Portfolios . . . . .	41
5.2.3 Arbitrage Pricing Theory . . . . .	42
5.2.4 CAPM for Hedge Funds . . . . .	42
5.3 Technical Analysis . . . . .	44
5.3.1 Indicators . . . . .	45
Momentum . . . . .	45
Simple Moving Average . . . . .	46
Bollinger Bands® . . . . .	47
Normalization . . . . .	48
5.4 Anomalous Price Changes . . . . .	48
<b>III Learning Algorithms for Trading</b>	<b>49</b>
<b>6 Supervised Regression Learning</b>	<b>50</b>
6.1 Regression . . . . .	51
6.1.1 Linear Regression . . . . .	51
6.1.2 $k$ -Nearest Neighbor . . . . .	52
6.1.3 Training vs. Testing . . . . .	53
6.1.4 Problems with Regression . . . . .	53
6.2 Decision Trees . . . . .	54
6.2.1 Representing a Decision Tree . . . . .	55
6.2.2 Learning a Decision Tree . . . . .	56
<b>7 Reinforcement Learning</b>	<b>57</b>
<b>Index of Terms</b>	<b>58</b>

# CODE SNIPPETS

2.1 Normalizing prices . . . . .	6
2.2 Calculating daily returns . . . . .	7
2.3 Calculating cumulative returns . . . . .	9
2.4 Calculating portfolio statistics . . . . .	15
2.5 2D model fitting via vertical error. . . . .	19
2.6 Polynomial function fitting via vertical error. . . . .	19
5.1 Calculating momentum. . . . .	46
5.2 Calculating the simple moving average. . . . .	46
5.3 Calculating Bollinger Bands®. . . . .	47

# PART I

## MANIPULATING FINANCIAL DATA

# PYTHON FOR FINANCE

# PYTHON FOR FINANCE

**P**YTHON has emerged as a dominant language in financial analysis in recent years due to its accessibility and the development powerful libraries such as `pandas` and `numpy` which enable highly-optimized manipulation of stock market data. Many of the statistics we'll talk about here have some accompanying snippets that describe how calculating that statistic would look in `pandas`. This is especially useful for various projects in *CS 7646* as there are many equally-correct ways to calculate certain values, but a particular calculation is expected.

**Snippet 2.1:** A convenient way to compare price data for stocks is to normalize them to all start at 1.0. This is trivial in `pandas`: just divide a dataframe by its first price.

```
def normalize(df):
    return df / df[0]
```

## 2.1 Global Statistics

Given a selection of stock market data, there are a number of global statistics we can calculate that can give us insights to trends in a particular stock over time and allow us to compare performance of various stocks in a portfolio.

These include things like the `rolling mean`, the `rolling standard deviation`, and more.

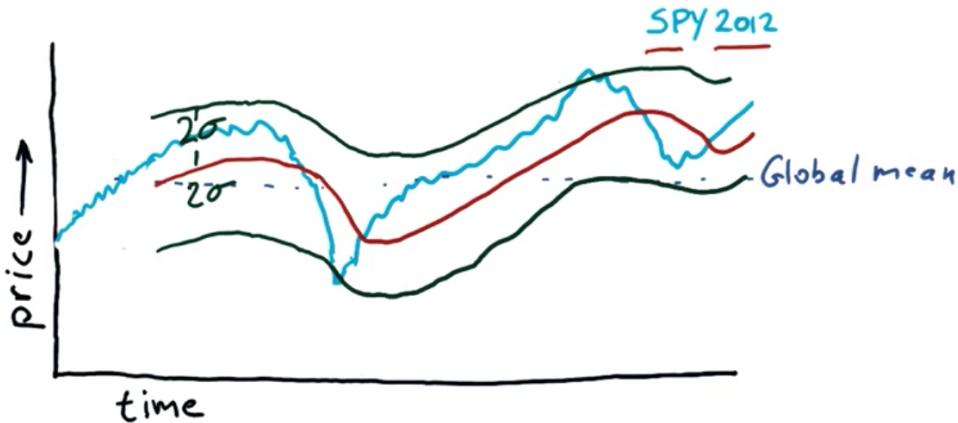
### ROLLING STATISTICS IN `PANDAS`

There are a number of functions in the `pandas` library that give us global statistics. These aren't members of the `DataFrame` class; rather, we pass in a `DataFrame` and some additional parameters that specify a few details about the computation we want.

- `stats.moments.rolling_mean` — notable parameters include the window size and the number of minimum necessary observations

### 2.1.1 Bollinger Bands®

Invented by John Bollinger in the 1980s, **Bollinger bands** are a measurement of a stock's volatility. By leveraging the rolling standard deviation—specifically,  $2\sigma$  above and below the global mean—we can get a sense of the volatility based on when the current stock intersects the bands.



**Figure 2.1:** A visual demonstration of Bollinger bands® applied to \$SPY. In this particular example, buying at the dip and selling at the peak would've rendered great profits, but there are plenty of examples in which this isn't an effective strategy.

A point of intersection, such as the dip in Figure 2.1, can indicate a trading opportunity. A double intersection (as in, it dips through the lower band and back up through it) with  $-2\sigma$  suggests a buy, whereas a double intersection with  $+2\sigma$  suggest a sell. We won't discuss the effectiveness of this method as an actual trading strategy until a later chapter, but it's an important demonstration of the power of these simple global statistics.

### 2.1.2 Daily Returns

One of the most important statistics used in financial analysis is **daily returns**. It's very straightforward: how much did the price go *up* or *down* on a particular day? The calculation is very simple: the daily return at time  $t$  is based on the price that day divided by the prices on the previous day.

$$\text{return}_t = \frac{\text{price}_t}{\text{price}_{t-1}} - 1 \quad (2.1)$$

**Snippet 2.2:** Given the portfolio value (either as a **DataFrame**, **Series**, or other **pandas** data structure) in `port_val`, we can find the daily returns (as a percentage) rather easily by dividing the portfolio value by itself, offset by one day.

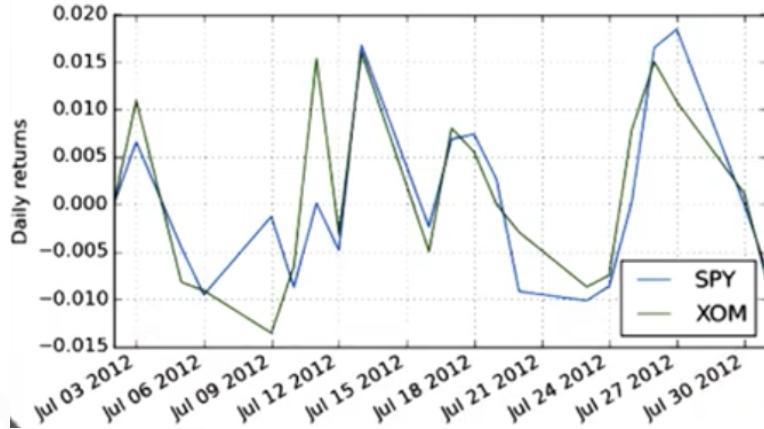
```
daily = (port_val[1:] / port_val[:-1].values) - 1
```

For example, if a stock was at \$100 on Monday and \$110 on Tuesday, the daily return would simply be:  $\frac{110}{100} - 1 = 10\%$ . We can plot the daily return for a stock over time to get a sense of its growth trend; it will generally zig-zag around zero, though its mean will be above zero if the stock price is increasing over time and below zero otherwise.



**Figure 2.2:** A potential daily return graph for an arbitrary stock.

Daily returns aren't very useful on their own relative to other global statistics like the rolling mean. Instead, they shine when *comparing* stocks: we can compare the daily returns of, for example, \$AAPL<sup>1</sup> against the S&P 500.



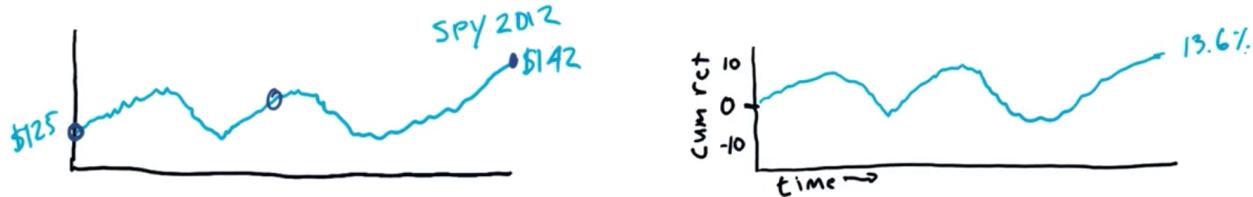
**Figure 2.3:** A comparison of the daily returns of Exxon Mobile (\$XOM) to the daily returns of the S&P 500 (\$SPY). We can see that it matches the ups and downs of the stock market in general quite closely.

### 2.1.3 Cumulative Returns

Another important statistic is **cumulative returns**. This is a measure of the stock price over a large period of time starting from some  $t_0$ . For example, you might say that the S&P has grown 10% in 2012; that would be its cumulative return (for  $t_0$  = January 1<sup>st</sup> 2012). The calculation is almost identical to (2.1), except it uses a fixed initial time:

$$\text{return}_t = \frac{\text{price}_t}{\text{price}_{t_0}} - 1 \quad (2.2)$$

<sup>1</sup> Generally speaking, we use the notation \$SYM to indicate a stock symbol.



**Figure 2.4:** An example of calculating and subsequently charting cumulative returns for \$SPY in 2012.

**Snippet 2.3:** As usual, we assume an existing portfolio value calculation in `port_val`. Note that the syntax `.iloc[idx]` is a way of directly accessing a particular row in a `pandas DataFrame` or `Series` object.

```
cumulative = (port_val.iloc[-1] / port_val_value.iloc[0]) - 1
```

## 2.2 Fixing Bad Data

We need to work around a number of (invalid) assumptions about financial data in order to manipulate it successfully. Many people assume stock ticker data is perfectly recorded minute-by-minute and has no gaps or missing data. The harsh reality is that the data is actually an amalgamation from multiple sources. A particular stock can trade at different (though similar) prices on different exchanges (like the NYSE or the NASDAQ).

Naturally, data can also be missing. Low-volume stocks that don't have a lot of trading activity might not have any data for a particular day; similarly, stocks come in and out of existence as the company (d)evolves.

The best way to work around gaps in a stock's price data is to **fill forward** then **fill backward**. To fill forward is to fill the price data from the last known price until trading activity resumes. This is far better than **interpolation** because interpolation is a “prediction” about the stock's price in the future which is a big no-no when it comes to analysis. After filling forward, there may still be an initial gap; for this, we fill backwards.

### FIXING BAD DATA IN PANDAS

The key to fixing bad data in `pandas` is `DataFrame.fillna`, typically called with the parameter `method='ffill'` (or `'bfill'` for backwards filling).

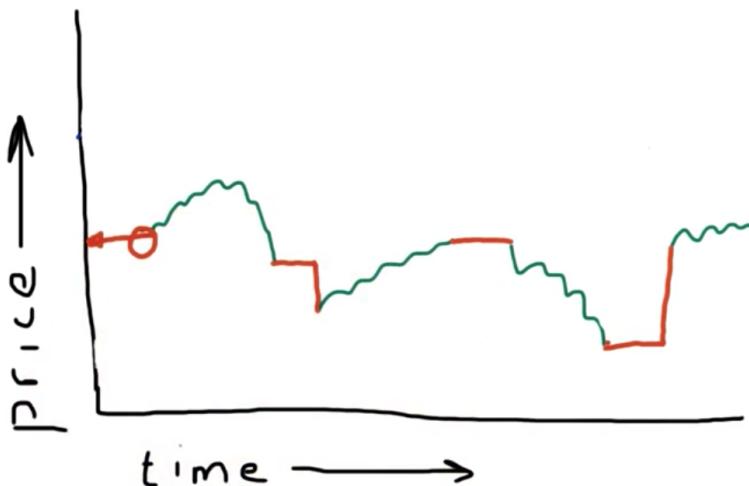


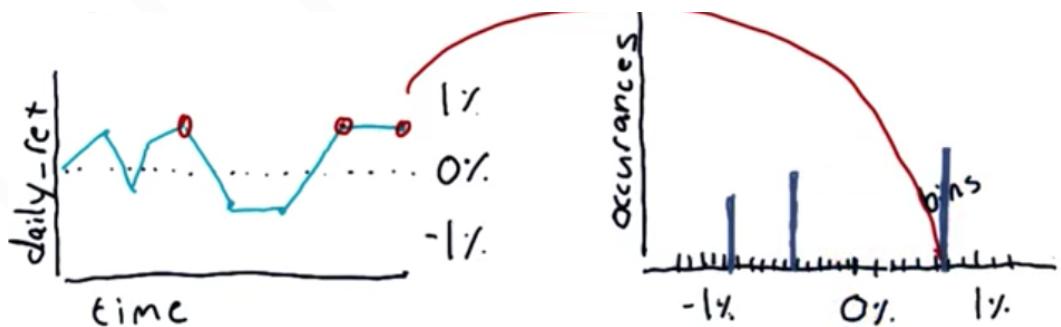
Figure 2.5

## 2.3 Graphing Financial Data

We'll be discussing **histograms** and **scatterplots** in this section, and we'll start by taking a closer look at **daily returns**.

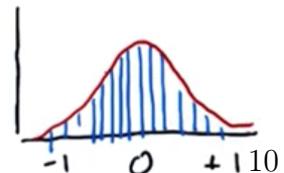
### 2.3.1 Histograms

On their own, daily return graphs don't offer us much. What we can do to get a better insight on a stock's behavior is to turn it into a histogram, which is essentially a bar graph that relates values to the frequency of their occurrences. For example, if there were 3 days in which you had 1% daily returns, you would have a 3-unit tall bar for  $x = 1\%$ . An important factor is choosing the granularity of the histogram (i.e. how many "buckets" of values we want to track).

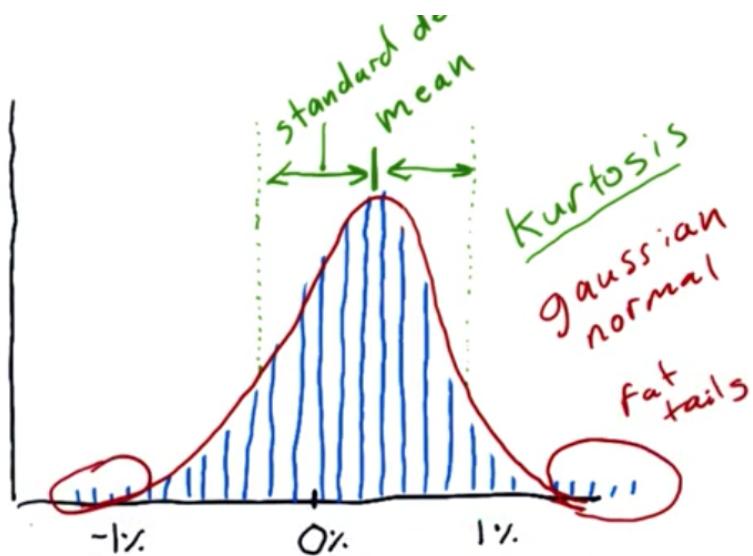


**Figure 2.6:** An example demonstrating how a daily return graph can be turned into a discretized histogram.

Suppose we looked at \$SPY over many years and created a histogram, normalizing it to  $[-1, 1]$ . What would it look like? Interestingly enough, it results in a **normal distribution** (also called a Gaussian distribution).



With our neat little histogram, we can now compute interesting values like the mean (in the case of a normal distribution, this is its peak) and the standard deviation (which gives us a sense of how often and how far things deviate from the mean). Another important thing to note is the measure of **kurtosis**. Kurtosis shows us how much our histogram would deviate from an actual perfect normal distribution.

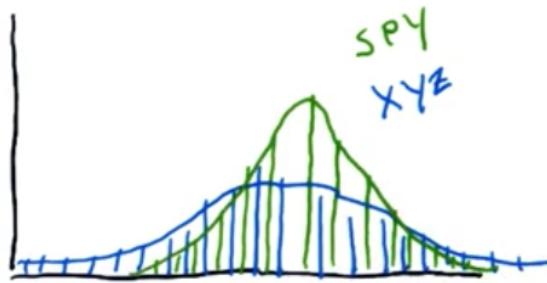


**Figure 2.7:** A visualization of kurtosis. Tails of the histogram that don't conform to the typical normal distribution (in the above example, the tails are too fat) speak to the deviation of outliers relative to the norm.

This is especially notable at the tails of the distribution; we can see “fat tails” in the histogram in [Figure 2.7](#). This tells us there are frequently large outliers relative to a perfect normal distribution, indicated by a positive kurtosis.

Kurtosis has real-world consequences. If we assume that our models are perfect normal distributions, we mathematically disregard inconsistencies in the data. This is a big mistake that is amplified with the amount of outliers actually present in the data. Kurtosis contributed to the Great Recession of 2008: banks built bonds based on mortgages whose returns they assumed were normally distributed. Thus, they claimed that the bonds had a very low probability of default. This proved to be false as massive numbers of homeowners defaulted on their loans, precipitating the economic collapse.

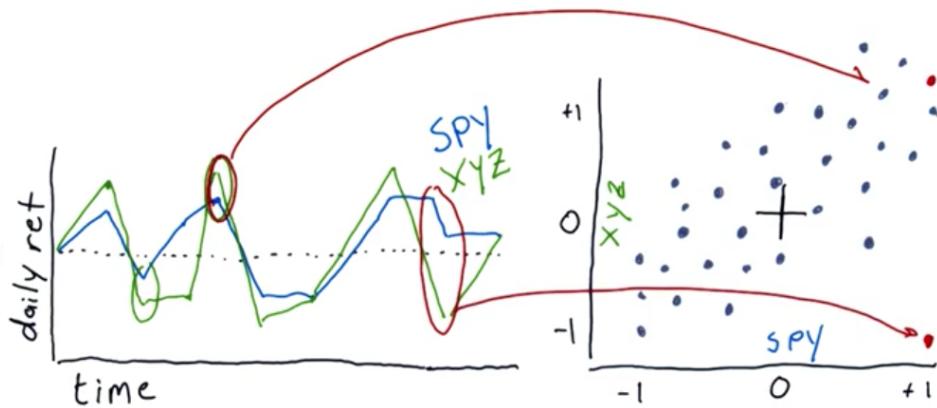
Comparing the histograms of stocks is an important part of financial analysis. For example, consider the two histograms for \$SPY and \$XYZ:



We can see that \$XYZ has a lower return and a higher volatility than \$SPY: its mean is lower and the standard deviation is larger. There is a higher spread of data, so there is more variation in daily returns, which is the definition of volatility.

### 2.3.2 Scatter Plots

A scatter plot lets us visualize differences between stocks at particular points in time by plotting them against each other. For example, in Figure 2.8 we can see the darkly circled area, which corresponds to both \$SPY and \$XYZ having a positive daily return, but \$XYZ's is higher.

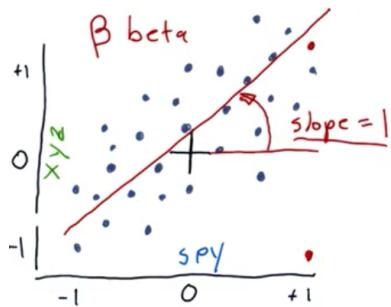


**Figure 2.8:** Turning the daily return plots of two stocks into a scatter plot.

Given enough data, we can typically see a trend. In Figure 2.8, we can see that there appears to be a linear relationship, however it's quite loose as the dots are relatively spread apart. We can use [linear regression<sup>2</sup>](#) to fit a line to the plot. The **slope** of that line—referred to in the financial world as  $\beta$  when comparing to a general market-tracking stock like \$SPY—describes how the stock reacts to the market. If  $\beta = 1$ , it means that when the market goes up 1%, that stock will also go up 1%.

Notice that the line of best fit in Figure 2.9 is *above* the origin. This value (the  $y$ -intercept

<sup>2</sup> We won't cover linear regression for now. It should be a familiar concept, but if not, you can think of it as the mathematical way to find the "line of best fit" for some points.



**Figure 2.9:** The slope of a line indicates a relationship between market trends and a specific stock (when comparing to \$SPY).

of the line) known as  $\alpha$  describes how much better a stock performs relative to the market on average. In this case, \$XYZ often outperforms the market slightly.

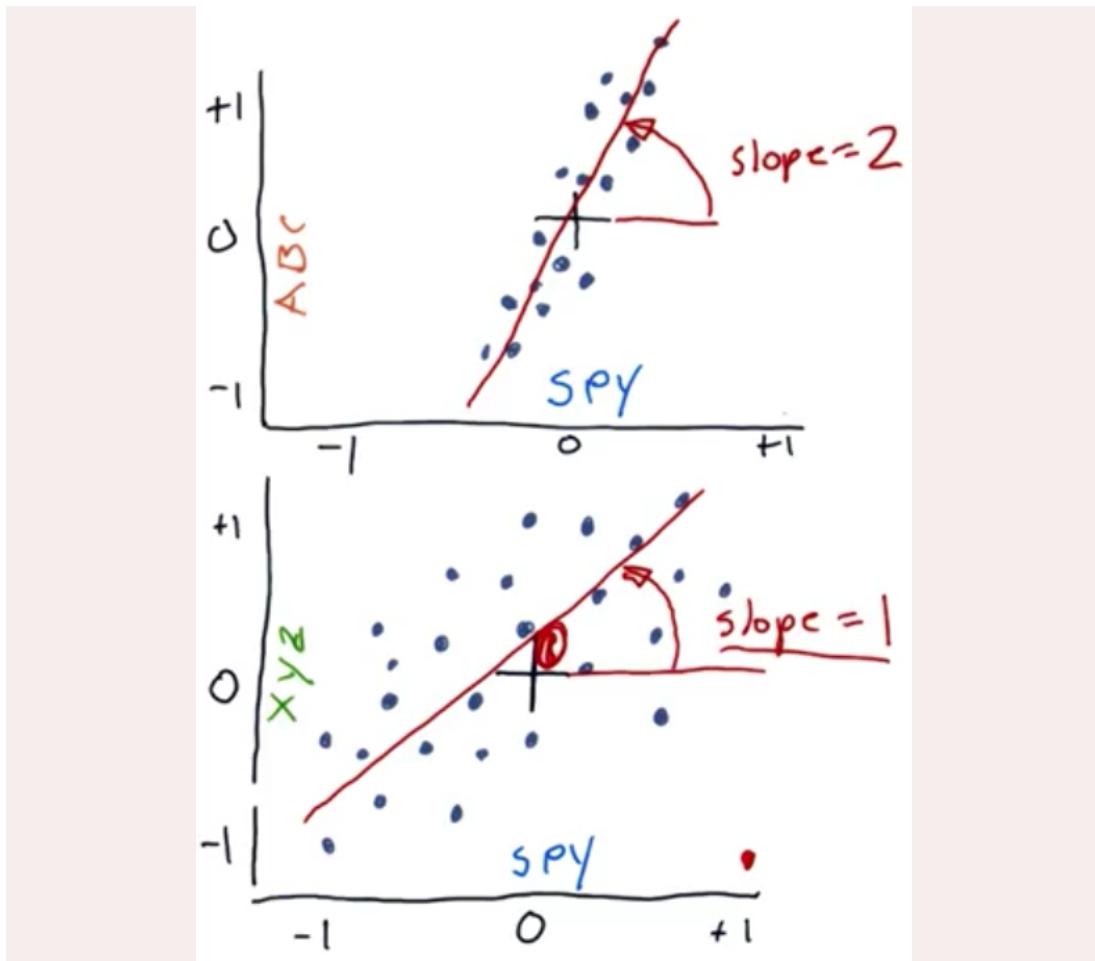
Another important metric on the scatterplot is **correlation**. This is a measure of how tight the points are to the line of best fit, in the range  $[0, 1]$ . In Figure 2.9, the dots are typically fairly far from the line,<sup>3</sup> which means there is a **low** correlation. It's critical to keep in mind that slope  $\neq$  correlation: it's just a measure of how tightly the dots fit a line of a particular slope.

### UNDERSTANDING CHECK: Correlation vs. Slope

Below are scatter plots for two stocks in relation to the S&P 500: \$ABC and \$XYZ. Which of the following is true about \$ABC?

- (a) \$ABC has a higher  $\beta$  (slope) and a lower correlation.
- (b) \$ABC has a lower  $\beta$  and a higher correlation.
- (c) \$ABC has a higher  $\beta$  and a higher correlation.

<sup>3</sup> Mathematically speaking, this would be a measure of the total error using something like the distance.



(c) Answer:

## 2.4 Portfolios

Let's expand beyond the realm of single-stock-analysis. We'll start out with a "buy-and-hold" strategy for our **portfolio** in which we have some initial principal investment spread out over a number of stocks; we will observe its behavior and performance over time. A portfolio is just a weighted set of assets. For example, your portfolio might be composed of \$AAPL, \$GOOG, and \$AMZN; the weight of each asset is the portion of total funds allocated to it.

Suppose our portfolio has the following setup:

- Principal starting value: \$1,000,000 *(I wish...)*
- Investment period: Jan. 1<sup>st</sup> 2009 to Dec. 31<sup>st</sup> 2011
- Stocks: \$SPY, \$XOM, \$GOOG, \$GLD

- Allocation ratios: [0.4, 0.4, 0.1, 0.1]

To calculate our daily portfolio value, we can use the following formula (assuming an initial data frame `prices` which has daily prices for our stocks):

$$\text{portfolio} = \underbrace{\sum_{\text{per-stock principal}} \text{ratio} * \text{start}}_{\text{daily position}} * \underbrace{\frac{\text{prices}}{\text{prices}[0]}}_{\text{normalized prices}} \quad (2.3)$$

Let's assume now we have a data frame `port_val` which is the list of portfolio values over time and the data frame `daily_rets` which is its daily returns. We can now calculate some statistics that everyone wants to know about a portfolio: the cumulative and average daily return, the standard deviation of the daily return, and the **Sharpe ratio**.

The cumulative return is simply the percentage of total portfolio growth, so its final value minus the initial value. The next two we have already discussed. The Sharpe ratio gives us an insight about our returns in the context of risk. Finance folk often associate risk with the standard deviation as that represents a stock's volatility (hence why cryptocurrency investment is risky: your coin could be worth \$100 tomorrow or \$1000 or \$3).

**Snippet 2.4:** In this snippet, we assume the existence of a `DataFrame` of stock data called `prices` that's formed as we've discussed: each row is labeled by a date and each column corresponds to a `$SYMBOL`. We'll also assume that `allocs` is our ratio of allocations and that `principal` is our initial (total) investment.

```
normalized = prices / prices.iloc[0]      # normalized prices
allocated = normalized_prices * allocs    # prices scaled by allocation ratio
positions = allocated * principal        # per-stock investment
portfolio = positions.sum(axis=1)          # portfolio value every day
```

Naturally, given two stocks with similar returns, we'd choose the one with less volatility; similarly, given similar volatility, we'd choose higher returns. What about a stock that is more volatile but has higher returns than another? This typically involves a bit of a “gut feeling,” but we want to do better than that.

That's where the **Sharpe ratio** comes in: it gives us a **risk-adjusted** return. All else being equal, it considers low levels of risk and high returns to be good. The Sharpe ratio additionally considers the risk-free rate of return which is critical when comparing financial strategies. A risk-free investment is something like an FDIC-insured bank account or a short-term treasury, which are (more-or-less) guaranteed to grow at a particular percentage. If your investment strategy doesn't outperform a risk-free return rate, you might want to reconsider it.

Given the portfolio return  $R_p$ , the risk-free return rate  $R_f$ , and the standard deviation of  $R_p$

(i.e. the volatility or risk), we can formulate the Sharpe ratio as such:

$$\frac{R_p - R_f}{\sigma_p}$$

Notice that as volatility goes up, the ratio shrinks. Similarly, as the return goes up (sans the risk-free return), the ratio grows. The actual formula is a little more complicated, but describes the same pattern (here  $E$  is the expectation):

$$S = \frac{E[R_p - R_f]}{\text{std}[R_p - R_f]} \quad (2.4)$$

The “risk-free rate” can be taken from LIBOR, the 3-month treasury bill, or just set as 0% which has been an accurate approximation lately. Instead of having to constantly check and update the aforementioned values every day, we can approximate the daily risk-free rate by taking the 252<sup>th</sup> root of the annual rate (remember, there are 252 trading days in a year):  $R_f = \sqrt[252]{1.0 + R_{f\text{annual}}} - 1$ .

The Sharpe ratio can vary wildly depending on how frequently its sampled. It was originally envisioned as an **annual** measure. If we want to sample more frequently, we need to add an adjustment factor  $k$  to make it all work out, where  $k$  is the square root of the number of samples per year. For an  $R_p$  that represents daily portfolio returns, then,  $k = \sqrt{252}$ . Similarly,  $k = \sqrt{52}$  for weekly samples, etc.

### EXAMPLE 2.1: Sharpe ratio

Suppose we've had our portfolio for 60 trading days. Our average daily return is 1/10% per day (0.001%, or 10 **basis points**<sup>a</sup>). Our daily risk-free return is 2 basis points, and our volatility ( $\sigma$ ) is 10 basis points.

What's the Sharpe ratio of this strategy?

Since we are operating in days, we know  $k = \sqrt{252}$ . Then, we just use (2.4) directly:

$$S = \sqrt{252} \cdot \frac{10 - 2}{10} = 12.7$$

---

<sup>a</sup> Basis points are often called “bps,” and their unit is bps.

## 2.5 Optimizers

**Optimization problems** are a class of mathematical problems that require us to solve for a value or values that maximize (or minimize) an equation. In this section, we'll be discussing optimizers in code that can solve these types of problems in a financial context.

An optimizer can:

- find the minimum values of a function,
- build parameterized models from data, and
- refine stock allocations in a portfolio

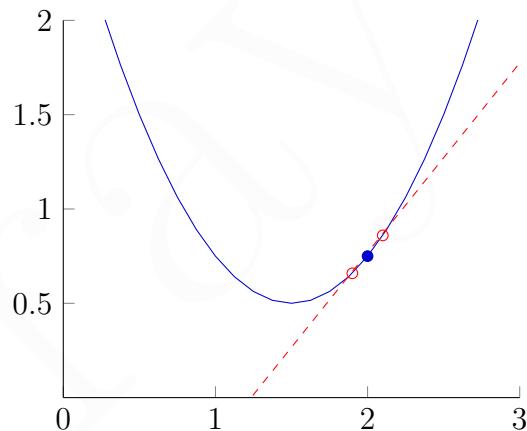
Using a pre-built optimizer is fairly simple. All we need to do is provide a function to minimize (such as  $f(x) = x^2 + 5$ ) and an initial guess.

### 2.5.1 Using an Optimizer

Let's walk through a simple minimization example. Suppose we're given the function (plotted below):

$$f(x) = (x - 1.5)^2 + 0.5$$

How does the minimizer work? Suppose our initial guess was  $x = 2$ . At that value, it uses the neighbors (like  $x = 1.9$  and  $x = 2.1$ ) to approximate a local slope and “marches downhill” (this is [gradient descent](#)). It repeats the process with another value that is down this slope, eventually converging to the minimum value  $x = 1.5$ .



Gradient descent is just one of a myriad of ways to minimize a function; it's an easy configurable parameter to [`scipy.optimize.minimize`](#).

Unfortunately, this isn't a bulletproof method. Functions with many **local** minima and flat areas are tough for minimizers to solve. Specifically, **convex functions** are the easiest for optimizers to solve. Formally-speaking, a convex function is one in which you can draw a line segment between *any* two points and that line would not intersect the function. A convex function must have just one minimum and not have any problematic flat areas.

### 2.5.2 Building Parameterized Models

Our goal in this section is to build a parameterized model from data.

Those of you who have taken a course on computer vision may have covered a [similar topic](#): instead of treating a line as an equation  $y = mx + b$  in which  $m$  and  $b$  are known, and we can plug in arbitrary  $x$  values to get their corresponding  $y$ s, we instead already *have* a set of  $\{(x_0, y_0), (x_1, y_1), \dots\}$  data points and wish to solve for  $m$  and  $b$ .

For convenience and generalization in code, we'll use  $c_i$  to refer to our parameters, so  $c_1 = m$  and  $c_2 = b$  in our aforementioned parameterized model of a line.

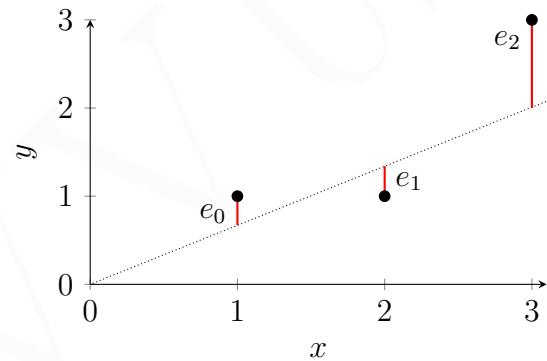
## Error Metrics

If the data fit our model perfectly, solving the parameters would be trivial. It's unlikely, though, that all of our points would be perfectly colinear, for example. Thus, we need a way to evaluate the quality of a "candidate model."

Given our understanding of minimizers, we want to reframe the "model of best fit" problem into something a minimizer can process. A common go-to error metric is simply the total vertical error from the points to the line.

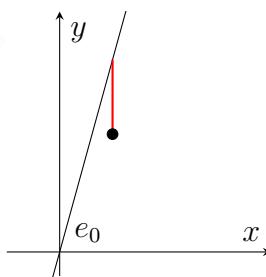
Our error function in this case is the vertical distance between point  $(x_i, y_i)$  and the line, for all points.  $mx_i + b$  is the "real  $y$ " at that  $x$  value via the true equation of the line:

$$E = \sum_{i=1}^n (y_i - (mx_i + b))^2$$



**Figure 2.10:** Fitting a line to a set of points, with the errors in red.

We can feed the minimizer the above error function applied to our simple line function:  $f(x) = c_1x + c_2$ , resulting in the parameters that best fit our data.



**Figure 2.11:** Measuring vertical error over-emphasizes errors in steep lines, but don't let that distract you from the fact that the Warriors blew a 3-1 lead in the 2016 Finals.

This error function isn't perfect: it over-emphasizes the error of steep lines because it only considers the  $y$  component, as seen in [Figure 2.11](#). A better function would be distance, or *perpendicular* error, but we won't get into that here because apparently it's too advanced of a topic for a graduate-level course on financial algorithms.

**Snippet 2.5:** We can find the best-fitting model to a set of observed data by feeding the model function into SciPy's optimization toolkit, specifically `scipy.optimize.minimize`.

```
import numpy as np

def error_function(line, data):
    """ Computes the error between a line (m, b) and observed data.

    line: a 2-tuple (c0, c1) where c0 is the slope and c1 is the y-intercept
    data: a 2D array where each row is an (x, y) point
    """
    return np.sum((data[:, 1] - (line[0] * data[:, 0] + line[1])) ** 2)
```

## *n* Dimensions

This process is extensible to  $n$ -dimensional data fairly simply. It does rely on the key assumption that the function can be modeled as an  $n^{\text{th}}$ -degree polynomial. That's okay for our current purposes, though, because we'll be applying this technique to optimize a portfolio which can be viewed as a simple function of allocation ratios applied to prices.

**Snippet 2.6:** We can find the best-fitting polynomial function to a set of observed data much like in [Snippet 2.5](#).

```
import numpy as np

def error_function(C, data):
    """ Computes the error between a line (m, b) and observed data.

    C: np.poly1d object (or 1D array) representing polynomial coefficients
    data: 2D array where each row is an (x, y) point
    """
    return np.sum((data[:, 1] - np.polyval(C, data[:, 0] + line[1])) ** 2)
```

### 2.5.3 Portfolio Optimization

Given a set of funds (i.e. stock symbols), assets (i.e. an initial amount of money to invest), and a time period (e.g. 1 year), find the ratio of assets to funds that maximizes performance. “Performance” here is the variable that depends on your investment strategy. If all you care about is growth, for example, you'd want to maximize cumulative returns (see [Snippet 2.3](#)).

In our running example, we'll optimize the portfolio's Sharpe ratio instead. The set of variables we wish to optimize is the allocation ratios across our chosen stock symbols. For example, we might want 20% of our funds to \$AAPL, 10% to \$GLD, and the rest to \$XOM for a nice, diversified portfolio. Of course, this might not be the optimal ratio. Framing this problem is a straightforward process:

- Provide a function for `minimize()` to work with.

We're trying to find the optimal allocations across a fixed set of stocks. Thus, in this function we'd need to calculate the Sharpe ratio for a given "best allocation" guess. Because we're using a *minimizer*, though, we need to multiply our result by -1.

- Provide an initial guess for our allocation ratios.

For this parameter, many values are likely to result in the same resulting optimal ratio. It might make sense to pick your true best guess, your ideal resulting ratio, or just a uniform distribution (like 33% across our above 3 stocks).

- Set up our ranges and constraints.

Obviously, we don't want to try any input values that result in allocations beyond 100% of our funds. In other words, our constraint must require that the sum of the ratios is 1. Similarly, we can restrict the possible inputs to our function to fall in [0, 1].

- Call the optimizer.

*(I don't think this bears more explanation.)*

# PART II

## COMPUTATIONAL INVESTING

# HEDGE FUNDS

**I**N this part of the course, we'll open up by discussing the different types of hedge funds. Following that, we'll look at how the market works, then talk about evaluating a company's worth and how that can be reflected in its stock price.

Let's cover a few vocabulary words here before we dive into details.

- **Liquidity** is a measurement of how easy it is to buy or sell shares in a fund.

As we'll see soon, ETFs, or exchange-traded funds are the most liquid of funds. They can be bought and sold easily and near-instantly during the trading day just like individual stocks; ETFs, though, represent some distribution of stocks. The **volume** of an ETF is just as important to its liquidity: because there are often millions of people trading it, it's easy to get your buy / sell order filled.

- A **large-cap stock** like Apple refers to a stock with a large **market capitalization**.

Market cap is a metric of a stock's total shares times its price. It's worth noting that the price of a stock has no relation to the value of a company; it only describes the cost of owning a single share *in* that company.

If you can afford the market cap of a company, you can afford to buy the company in its entirety and take over its ownership.

- A **bull market** or a **bullish** position on a stock is an optimistic viewpoint that implies that things will continue to grow. On the other hand, a **bear market** or a **bearish** position is pessimistic (or cautionary, or realistic, depending on how you see the glass) about the future of an asset.

## 3.1 Types of Managed Funds

With that in mind, managed funds can be broken down into three categories:

**ETFs** These funds function almost identically to stocks. Though they represent buckets of various ratios of stocks rather than shares in a single company, they can still be bought and sold just like regular stocks during the trading day. They are (usually) very liquid, especially if they have a high market capitalization. They often track either an entire

market (like \$SPY is an ETF tracking the S&P 500) or a specific sector. Furthermore, they're completely transparent: their structure, strategy, and distribution of allocations is publicly available.

For example, \$SMH, the VanEck Vectors semiconductor ETF, tracks various semiconductor producers. Their [holdings](#) are publicly available (with their highest allocation ratio falling onto Intel \$INTC at 12.3%) and are tradeable via most brokers and trading platforms.

**Mutual Funds** These funds are typically less transparent and less liquid than ETFs. Their allocations are disclosed quarterly, and they can only be traded at the end of a trading day. In line with that, their strategies are often much less transparent as well. Often, you have to sign an NDA or some other “hush-hush” agreement to buy shares in a mutual fund; generally-speaking, there's a much higher barrier to entry relative to an ETF.

**Hedge Funds** These funds, the biggest kahuna of them all, are highly secretive about their strategy and don't have to disclose their allocations or plans at all. They entice people to invest in them by demonstrating results—if they can show you a 30% gain in value over the last quarter, do you *really* care what their strategy is? Yeah, me neither.

Of course, there is evidence that hedge funds [generally don't outperform mutual funds](#), but that's beside the point here. ☺

**Table 3.1** summarizes the three categories. Now besides the difference in liquidity, composition, and transparency, why would you want to be the fund manager of one of these or another? Let's talk about the only thing that matters—**compensation**.

ETFs	Mutual Funds	Hedge Funds
Bought/sold like stocks	Bought/sold end of day	Bought/sold by agreement
Are backed by “baskets” of stocks	Disclosed quarterly	No disclosure
Transparent and liquid	Less transparent	Not transparent

**Table 3.1:** A brief summarization of the differences between the three types of managed funds.

## 3.2 Compensation

Let's discuss how the managers of each of these funds is compensated for their hard work of deciding how to spend other people's money. First let's introduce the idea of **assets under management** or AUM, which is the amount of other people's money the fund manager is responsible for.

In ETFs, managers are compensated by an **expense ratio** which is often simply a percentage of the AUM. They're generally pretty low: anything from 0.01% (1 “bip”) to 1% is typical (though the higher end is more unusual).

In mutual funds, the managers are also compensated by an expense ratio, though it's much higher than that of an ETF. They typically range from about 0.5% to 3%. If you ask a mutual fund manager why they get compensated more, they'll tell you it's because their job requires more skill than an ETF's manager. An ETF is typically designed to track an index; the manager of \$SPY, for example, just needs to make sure s/he allocates the stocks in the S&P 500 as they are represented. Because a mutual fund has more discretion about the ratios in its portfolio, it can incorporate research, deeper insights, etc. into its strategy to get better returns than an index.

Hedge funds follow an “old school” model known as **two and twenty**: they get 2% of the AUM as well as 20% of the profits. Now that's quite a big chunk of the profits; however, if you're an investor and you're making more money than you would be at a mutual fund (even after the manager's cut), why do you care?

### EXAMPLE 3.1: A Hedge Fund Manager's Salary

Imagine you're a hedge fund manager managing 100 million dollars. Why are you taking this class again? Oh right, we're just imagining... Okay, so suppose you had a 15% return this year. How much would you be compensated?

Well, the “two”—two percent of the AUM—comes out to  $\$100M \times 0.02 = \$2M$ , plus another “twenty”—twenty percent of the profits—or  $\$15M \times 0.20 = \$3M$ , meaning you made \$5M this year.

Now stop imagining and snap back to cold reality.

A common follow-up to this might be, “Is the ‘two’ calculated on the initial principal of \$100M or the end fund balance of \$115M?” Naturally, this depends on the hedge fund's compensation policy. In reality it'll probably be somewhere between the two as they take snapshots of the value of the fund throughout the year.

Modern-day hedge funds typically don't offer this classic 2/20 rate anymore. More common is things like 1/10 or somewhere inbetween, but the split compensation structure usually remains the same.

With the compensation model in mind, what motivates each respective fund manager? Well with just AUM factoring into the expense ratio, increasing that number is basically the only motivation for managers of expense-ratio-compensated funds—ETFs and mutual funds. Hedge funds, on the other hand, are additionally-motivated by profit, and so are consequently also motivated by risk-taking strategies.

Let's move forward assuming we want to be hedge fund managers since it seems like the most glamorous option and look deeper into what that entails.

### 3.3 Attracting Investors

A hedge fund with no investors is meaningless. How can we make money without risking someone else's? *Invest our own?* Unbelievable. So let's talk about how we can ~~eon~~ attract some people to give us their money to invest.

For whatever reason, hedge funds typically don't have more than 100 investors. As a result, they generally want each one to offer them a fairly large principal investment. So **who** are they targeting? Investors can be broken down into three categories: (very wealthy) **individuals**, (very large) **institutions**, and **funds of funds**.

Now **why** would anyone trust someone like you with their wealth? At the very least, you need to offer them a **track record** of success in the past, a convincing **simulation** and **story** for your investment strategy, and a **portfolio fit** that aligns with their investment goals. If you want to go all-in on the technology startup sector whereas Georgia Tech's foundation needs a stable place to keep their money, it might not be a good fit.

In addition to strategy, variety might be important to keep in mind. Diversity is key in a good portfolio that can endure market dips and general volatility. If an investor already has a particular area covered (like small-cap growth stocks, for instance), if you can only offer them a similar sector, things might not work out.

#### 3.3.1 Goals

Obviously your goal as a hedge fund manager is to make money. Things can be a little more nuanced than that, though. Some common goals that are helpful in attracting investors include:

- **Beating a benchmark:** if our portfolio can consistently beat the S&P 500 by choosing the best-performing stocks of the bunch, that can be a good and convincing benchmark.
- **Absolute return:** no matter what happens, we want a way to ensure a positive return overall. This is typically accomplished by “going long” (investing long-term) on good stocks and “selling short” (betting on declines on bad stocks in the short-term) on bad stocks. We'll discuss shorting later.

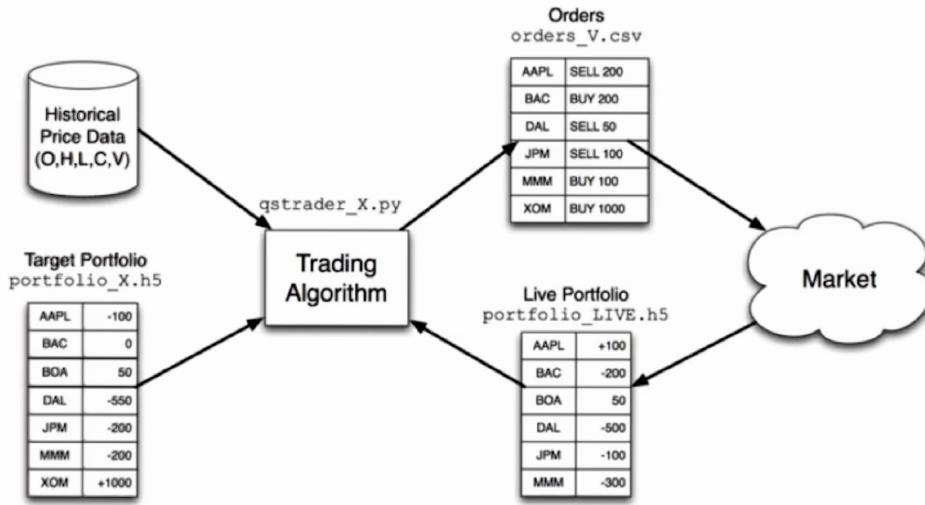
Your choice of goal as a hedge fund manager should depend on your experience and expertise. For example, if you're really good at picking stocks in emerging markets overseas, your best bet is targeting beating an index that tracks a similar investment strategy.

#### 3.3.2 Metrics

Given these goals, how can we measure our fund to see if our portfolio meets them? Well we already discussed quite a few metrics in the previous minicourse. **Cumulative returns**, **volatility**, and a measurement of the risk / reward (like the **Sharpe ratio**) are all reliable and convincing metrics.

## 3.4 Hedge Fund Computing Architecture

Hedge funds and high-frequency trading firms are some of the most computationally-demanding environments out there. There is a massive interconnected infrastructure that connects real-time market data, massive databases of historical pricing data, and state-of-the-art machine learning models to create trading algorithms. Figure 3.1 maps out a (very) high-level architecture for a typical firm.



**Figure 3.1:** The architectural structure of a hedge fund.

The current prices constitute a real portfolio. The trading algorithm uses that as well as real-time data about the market (pending orders, etc. which we'll get to shortly) to send out orders that make progress towards matching the *target* portfolio. There are complexities in this “progress-making” process: if your target portfolio wants 10,000 shares of \$AAPL, buying those 10,000 shares outright would be detrimental to our portfolio and probably make us change our target portfolio midway. This is because buying such a large quantity of shares will make the price rise as the market price goes up to meet demand. Hence there is a financial benefit to incrementally reaching the target portfolio with minimal market effects.

We'll work through this architecture backwards, starting from the market itself and eventually getting into the nitty-gritty inner workings of a machine learning model applied to market data.

# THE ORDER BOOK

**W**HAT happens when you click “buy” on a share of stock through an online broker?<sup>1</sup> Probably a lot more than you might imagine. Let’s dive into the details and discuss orders and the order book.

## 4.1 Making Orders

An **order** is a well-defined structure that contains very specific instructions about the stock to buy. It is made up of:

- **Action:** this is either a **BUY** or **SELL** instruction.
- **Symbol:** this is the unique stock symbol (like \$AAPL or \$SPY) to execute the order on.
- **Quantity:** this is the number of shares to purchase.
- **Order Type:** this is either a **market** or **limit** order. There are significant differences between these two order types.

A market order does not specify a price for the order. It generally indicates that you are willing to accept a “good price” for a stock. In reality, though, it’s filled at whatever price someone is willing to accept the order.<sup>2</sup> This can vary wildly during volatile market times. For example, if \$TSLA stock is plummeting because of Elon’s latest Twitter rant, your market sell order might get filled at far below the “current” price you saw in your broker’s stock ticker.

A limit order, on the other hand, *does* specify a price for the order. It guarantees that when the order is executed, it will be executed at that price. For buy orders, it can also

<sup>1</sup> By the way, if this course has motivated you to start investing, RobinHood is a great way to get started. It has no trading fees (!) and is extremely mobile-friendly. If you use my invite link, we’ll each get a free stock! <https://share.robinhood.com/georgek84>

<sup>2</sup> This is typically where high-frequency trading firms will take advantage of their literal physical proximity to the exchanges’ order books to fill market orders for slightly more than the current price and make a few pennies in profit. Well, a few pennies per share for a few million or billion orders a day... Check out [Is the stock market rigged?](#) for details on this surprisingly-legal phenomenon.

be filled *below* that price; similarly, for sell orders, it can be filled *above* your specified sale price. The outcome is guaranteed to be what you specify or better, however, it's entirely possible that your order will *not be filled* in the reasonable time frame you expect. During the same aforementioned period of volatility, if you ask for price  $x$  on your stock, but the market has fallen far below that point, it may take months or years for the share price to recover to  $x$  and thus for your order to be filled.

- **Price:** for limit orders, this specifies the precise minimum value you're willing to pay or receive for your order.

Let's take a look at how a typical market order will be filled.

#### FUN FACT: Other Order Types

Your online broker may offer you some other order types such as **stop loss** or **stop limit**. These are actually not directly supported by the market but are special orders that your broker automatically converts to one of the above types when the condition is fulfilled.

For example, a stop-loss sell order specifies that once a stock drops below a particular user-defined price, it should be executed as a market sell. It is generally used to, as the name suggests, *stop losses* as a stock drops in value. You might, for example, never want to risk losing money on an investment, so you could set a stop-loss order to be at (or slightly below) the price you purchased the stock at; your broker would convert and execute the sale any (!) time the stock dipped that low.

The most impactful and well-known of these is the ability to **sell short**, which we'll discuss in further detail [soon](#).

### 4.1.1 The Order Book

Let's suppose we executed the following order on the exchange, "Buy 100 shares of \$IBM at a limit price of \$99.95":

BUY, IBM, 100, LIMIT, 99.95

Suppose this is actually the first order of the day. This action adds a **BID** entry to the *public* order book for the exchange:

Type	Price	Quantity
BID	99.95	100

The order itself is anonymized, but it becomes public knowledge that there is interest in 100 shares of \$IBM. It's important to note that identical orders are lumped together. If another person comes along asking for another 200 shares at that same price, the order book would look like this:

Type	Price	Quantity
BID	99.95	300

**NOT** like this:

Type	Price	Quantity
BID	99.95	200
BID	99.95	100

Only your broker maintains the relationship between identities and orders in the exchange's order book.

Let's see what happens when a sale, or ASK, entry comes in. Suppose you submit an order to buy 1000 shares of \$IBM with a limit price of \$99.95. There's nobody selling \$IBM at that price, so it just gets added to the order book.

Type	Price	Quantity
ASK	100.00	200
BID	99.95	1000

Suppose that after some time, the order book looks like this:

Type	Price	Quantity
ASK	100.10	100
ASK	100.05	500
ASK	100.00	1000
BID	99.95	1000
BID	99.90	50
BID	99.85	50

Then a market order arrives to buy 100 shares of \$IBM. Remember, market orders don't specify a price, so they can always be filled (well, provided that there *are* sellers). The order is filled by the cheapest "ask" order, resulting in the client getting their shares for \$100 each.

Type	Price	Quantity
ASK	100.10	100
ASK	100.05	500
ASK	100.00	1000
BID	99.95	1000
BID	99.90	50
BID	99.85	50

The gap in price between the asks and bids is called the **bid-ask spread**, representing the supply and demand for the asset. The "current value" of a stock is somewhere in this spread.

#### EXAMPLE 4.1: How Does the Spread Reflect Stock Value?

Now that you know how an order book works, let's consider how we can use information about it to learn about the state of the market. Given the following order book, do you think the price of the stock is going to go *up* or *down*?

Type	Price	Quantity
ASK	100.10	100
ASK	100.05	500
ASK	100.00	1000
BID	99.95	100
BID	99.90	50
BID	99.85	50

---

The stock is much more likely to go down in price because of the selling pressure. Consider the difference between someone placing a market-buy vs. a market-sell order for 500 shares. For the buy, you would immediately get rid of all of the bids in the order book, meaning the price goes down. On the other hand, for the sell, the price wouldn't budge at all; you would only get rid of 500 shares in the cheapest ask.

Let's walk through the changes that occur to the order book through a handful of order examples. We will start with the order book from the [previous example](#).

Type	Price	Quantity
ASK	100.10	100
ASK	100.05	500
ASK	100.00	1000
BID	99.95	100
BID	99.90	50
BID	99.85	50

The following orders come in (oldest first):

BUY, 100, Market  
BUY, 100, Limit, 100.02  
SELL, 175, Market

The resulting order book is as follows:

Type	Price	Quantity
ASK	100.10	100
ASK	100.05	500
ASK	100.00	800
BID	99.85	25

Do you follow the flow? The first order is filled by the cheapest ask, dropping its quantity to 900. Then, the second order is likewise filled by the cheapest ask—remember, a limit-sell

guarantees the specified price *or better*—dropping it to 800. Finally, the third order fills out the first bid completely, then the second bid completely, and even 25 shares in the third bid. All together, the client would probably have their market purchase “cost” be the average across the three orders it used:

$$\text{cost} = \frac{(99.95 \times 100) + (99.90 \times 50) + (99.85 \times 25)}{175} \\ \approx 99.92$$

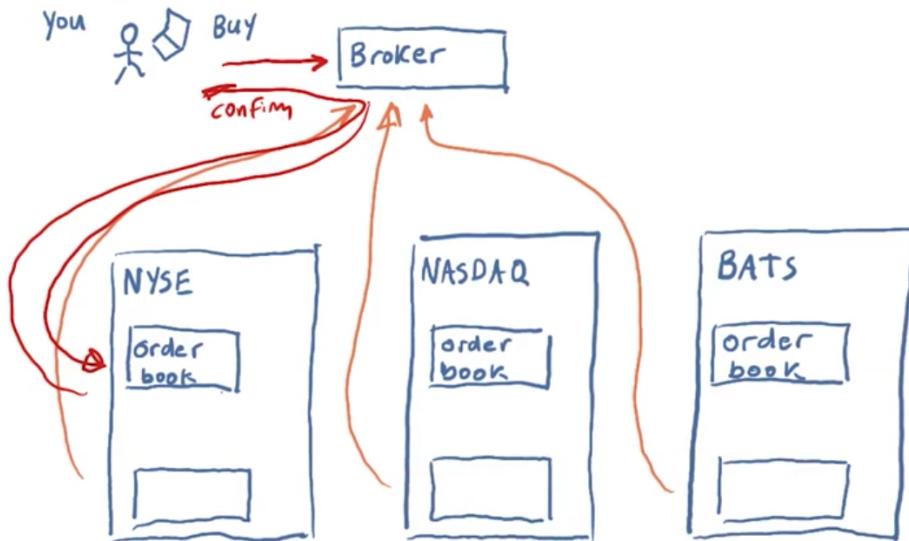
We can concretely see why the stock was more likely to decrease in price prior to this transaction, as explained in the [previous example](#). At the end of these orders, the price is somewhere between \$100 and \$99.85 rather than between \$100 and \$99.95 as it was previously.



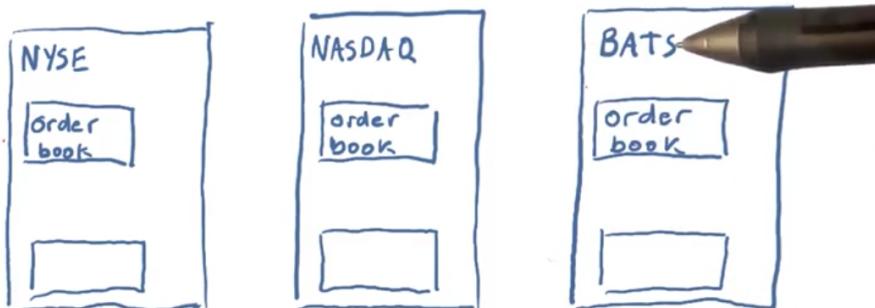
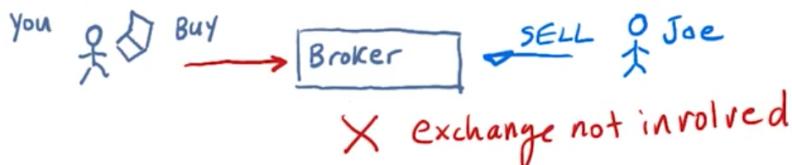
**Figure 4.1:** The order book of a real stock using a real trading platform.

### 4.1.2 Filling Orders

The path your order takes varies wildly depending on a number of factors. There is always an intermediary between you and a filled order: your broker. Your broker is connected to a number of exchanges (like the NASDAQ or NYSE). Certain stocks might be listed at one or more exchanges. Your broker has devices at each exchange that query the order book and report back results to the broker’s “home base” where it gathers and analyses that data to give you the best price. It executes your order on the best exchange and gives you a confirmation.



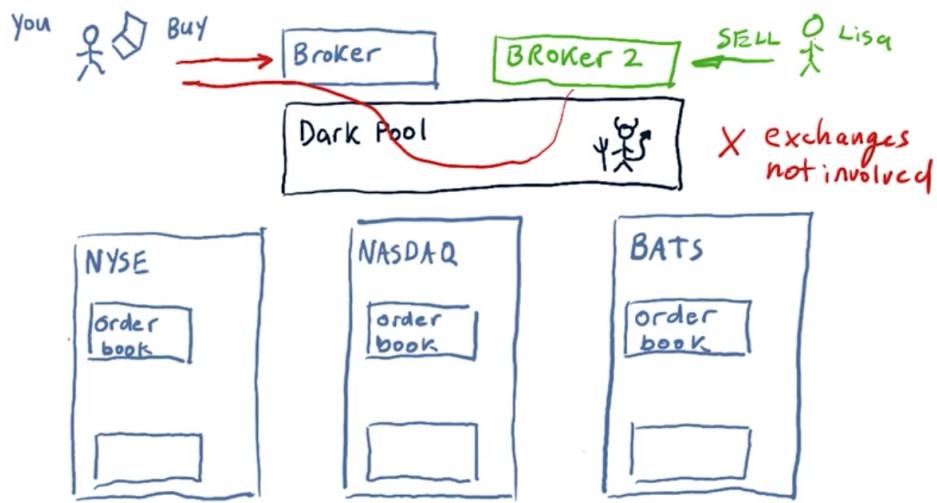
This is the simplest, clearest scenario. What if there are multiple clients using the same broker to make orders, and you want to buy the same stock that Joe Shmoe is selling? The trade can be made within the broker without having to contact (or pay fees to) the exchanges for their order information.<sup>3</sup>



Can we extrapolate this scenario across brokers? Turns out there's an entity out there called the **dark pool**, which is an intermediary between brokerages and exchanges. The dark pool often makes predictions about price movement; they pay the brokerages for the privilege of seeing orders before they're executed, and will actually execute advantageous ones without involving exchanges. Both of these entities—the brokerages and the dark pool members—claim this is kosher, legally-speaking, because they are still guaranteeing at least as good

<sup>3</sup> By law, the broker *must* give you a price as good as you would get on the exchange.

of a price as is available on the exchanges themselves.<sup>4</sup> Apparently around 80-90% of the trades made by “retail traders” never actually make it to the exchanges.



#### 4.1.3 Exploiting the Order Book

There are far more avenues for exploitation and (legal) skimming available than the simplistic happenings between the dark pool and brokerages we talked about in [Filling Orders](#). Let’s talk about how *hedge funds* can exploit inefficiencies in this system.

##### Exploit 1: Faster-Than-Light Travel

We’re located in Seattle and use e-Trade as our broker. That means our view of the order book, as well as any orders we make, are out-of-date by the amount of time it takes for that information to get to (or from) us. If we traveled at the speed of light, this would be almost 13ms one way, but since we have to deal with traveling through thousands of routers, switches, NSA taps, etc. through inefficient copper or even fiber optic cables, it’s more like 76ms on average. [\[1, 2\]](#)

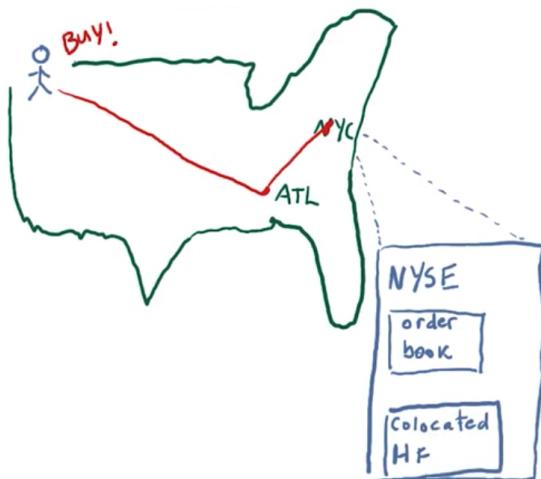
Hedge funds exploit their geographical advantage by having a device at the exchange itself, giving them instant access to order book changes.

With this massive advantage in reaction time, the hedge fund can make predictions about the price of a stock and place orders ahead of *your* prediction. For example, it might buy the stock as you’re buying it, then sell it to you (12ms later) for a few pennies more.

##### Exploit 2: Geographic Arbitrage

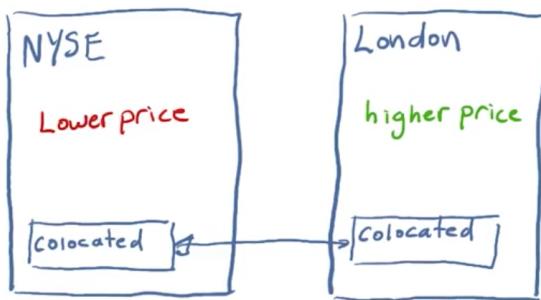
Suppose there is a stock listed in two exchanges: the NYSE and the London Stock Exchange. They operate independently, so there may occasionally be a small difference in the price of a

<sup>4</sup> Of course, by pooling together orders they get a better view of the market as a whole than the exchanges or any independent trader ever would, but I guess they bribe lobby the government enough to turn a blind eye to that competitive, monopolistic advantage.



**Figure 4.2:** Demonstrating the geographical advantage that hedge funds have. Your actions take at least 12ms to reach the market as they travel from Seattle to New York (even ignoring the pit stop at your broker in Atlanta), but the machine(s) that hedge funds have co-located on the exchange can beat you to any action every time, making a small profit every few milliseconds.

stock from one exchange to another. A hedge fund can have servers at both exchanges wired with an ultra-high-speed, dedicated connection. When any of these differences arise (such as NYSE's price going below London's), it immediately buys shares on the NYSE exchange and sells shares on the London exchange (either the same existing ones, depending on whether or not it already owns some).



**Figure 4.3:** Demonstrating geographic arbitrage. A hedge fund uses its dedicated connection to take advantage of price differences between exchanges.

Because this process occurs and is well-understood, such price differentials rarely arise, but when they do, exchanges are there to pick the pennies up off the ground.

#### 4.1.4 Selling Short

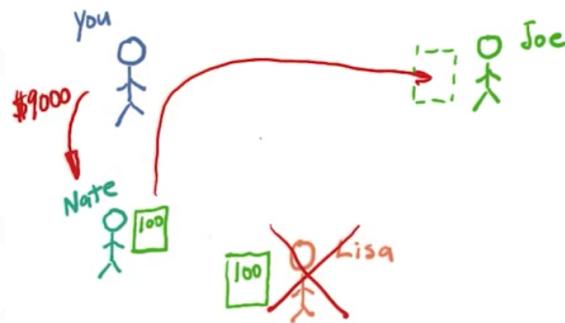
The most important and impactful order type “added” to the stock market by brokers (in addition to those we described previously) enables **selling short**—the bane of [/r/wallstreetbets](#). Shorting a stock involves taking a *negative* position; you bet that its price will go down rather than up.

Suppose \$IBM is selling at \$100. You think \$IBM is overhyped and want to short it. Lisa, on the other hand, thinks its still got plenty of growth ahead and wants to buy in. Joe holds 100 shares of \$IBM, and Joe is a great guy. Even though he wants to hold onto his shares, he's willing to lend them to you (well, his broker is).

You borrow Joe's shares and sell them to Lisa; she gives you \$10,000 in exchange. At the end of the day, you now have \$10,000 in your account and owe Joe 100 shares of \$IBM.



Joe is eventually going to want his shares back. Suppose \$IBM is now selling at \$90 and you want to cash out. Lisa doesn't want to get rid of her 100 shares; she's still **bullish** on \$IBM. Thankfully, there are plenty of people out there buying or selling shares at any given time; Nate is willing to sell his 100 shares. You give those 100 shares back to Joe, fulfilling your obligation. But you only paid \$9,000 for those shares, so you're left with a nice \$1,000 profit in your account!



### What Can Go Wrong?

What if the price didn't drop down to \$90? If you bought them from Nate at \$150 you'd be out \$5,000. This is worse than the situation with regular trades. If your (owned) stock goes down in value after you buy and you decide to sell, you still have money, just less of it. In this case, you actually owe your brokerage money once you exit your short position.

# EVALUATING A COMPANY

**I**NVESTING in a company can be a gamble. To properly evaluate a stock and make an informed decision about whether it should be bought, held, or sold, we need to evaluate the company that issues it. There are a number of different factors worth taking into consideration that we'll cover in this chapter.

## 5.1 Metrics for Evaluation

Why does company's value matter in the first place? We want to make money by purchasing a company's stock when it's undervalued and sell it when it's overvalued, right? Generally-speaking, a company has a "true value" that increases monotonically over time; its actual stock price, though, varies from that wildly. If we can identify deviations correctly, we can make good trades and steady profits.



**Figure 5.1:** Determining a company's value helps you identify times at which a trade would be appropriate.

Suppose *Acme Corporation* can output \$1 every year. How much would you say that company is worth? There are a few potential answers. Is it worth:

- \$1,
- \$70, because after that you'll be dead,
- $\$ \infty$  because it will output \$1 forever, or
- between \$10 and \$25 depending on interest rates?

Turns out all of these are valid choices depending on your valuation approach. The last one is probably the “most correct” given a traditional point of view of valuation.

A company is often evaluated on one or more of the following metrics:

- **Intrinsic value:** this is a measurement based on the **dividends** that a company puts out. A dividend is a cash payment to owners of that stock every quarter. If you owned one share of \$AAPL, for example, it would pay out around \$1 per year. Note that this payout is in addition to (or in spite of) changes to the share price itself. Intrinsic value assigns a valuation to a company based on this payout.
- **Book value:** this is a valuation based on the assets that a company owns. Assets include things like factories, stock (in the “this item is out of stock” sense, not a financial sense), property, etc.
- **Market cap:** this assigns the value of the company to the value of a share times the number of shares there are, as previously defined.

### 5.1.1 Intrinsic Value

If someone told you, “Hey, I can offer you a guarantee that I’ll give you a dollar a year from now.” How much would you be willing to pay for that guarantee? Would you rather have:

- that guarantee from the U.S. government (this is called a **bond**),
- that guarantee from your professor, or
- \$1 right now?

Obviously, the best thing to have is a dollar right now. There is no risk or need to wait. Furthermore, you probably trust the government more than your professor (well, maybe not...), so you’d take that guarantee over the other.

It should be obvious that the value of a dollar in the future is worth less than a dollar right now. There’s a risk that that dollar will never be delivered. Furthermore, the value of a dollar often decreases over time due to inflation. How do we estimate the value of these promises to deliver money in the future? The calculation of this **intrinsic value** comes

down to the **interest rate**:

$$\text{present value} = \frac{\text{future value}}{(1 + \text{interest rate})^i}$$

Here,  $i$  is the amount of time<sup>1</sup> before you receive your future value. If the U.S. government offered you a 1% annual interest rate on your \$1, the present value comes out to:

$$\frac{1.00}{1 + 0.01} \approx \$0.99$$

In other words, it's worth it to pay  $\leq \$0.99$  today for a government bond of \$1 a year from now. What if your professor is offering you a 5% interest rate in order to entice you and offset the fact that he's less trustworthy? You should be willing to pay even less for that:

$$\frac{1.00}{1 + 0.05} \approx \$0.95$$

There is an obvious exponential decay as the amount of time you need to wait grows. The higher the interest rate, the lower you should be willing to pay and the faster the value drops.

This problem is analogous to dividends in real companies. If \$AAPL pays out \$1/share in dividends every year, what is it worth paying for a single share? The “interest rate” in this case is now a metric of risk; we instead refer to it as a **discount rate**. Given that value, the value of the company to you would be the sum of the value of all of its dividends, where  $d$  is the discount rate. This infinite sum converges to a simple calculation:

$$\sum_{i=1}^{\infty} \frac{1}{(1+d)^i} = \frac{\text{future value}}{\text{discount rate}}$$

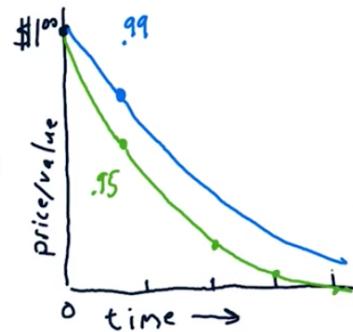
If we assume a \$1 annual dividend as above and a discount rate of 5%, the intrinsic value of that company would be  $1/0.05 = \$20$ .

### 5.1.2 Book Value

The **book value** of a company is defined by the total assets that company owns excluding its intangible assets and minus its liabilities.

---

<sup>1</sup>  $i$  operates on the same time scale as the interest rate. For example, if it's an annual interest rate,  $i$  is a measure of years.



**Figure 5.2:** The exponential decay of the worth of future promises.

Suppose a company owns four factories valued at \$10M each, three patents at \$5M each, and a \$10M loan. The book value of this company would be:

$$(4 \times 10) - 10 = \$30\text{M}$$

Of course, this is a overly-complicated way to think about this. The intangible assets are just irrelevant to the calculation, so it's just tangible assets minus liabilities.

### 5.1.3 Market Capitalization

We can let the market determine the value of our company for us. We've already defined the concept of [market capitalization](#), but for completeness sake here it is again:

$$\text{market capitalization} = \text{share price} \times \text{total shares}$$

### 5.1.4 Knowledge is Power

Why does information about a company like news reports, leaks, and new products affect its stock price? The way investors express their opinions on the value of a company is through its stock price (and consequently its market cap). If they believe the news devalues the company, they sell their shares and drive the price down. Let's look at how information can influence the other two metrics—**intrinsic** and **book** value.

If news comes out that a CEO is ill, the probability of investors getting the same dividends from that company as before decreases. This is “company” news and it directly lowers the intrinsic value.

If news comes out that the soil in Silicon Valley releases a chemical that increases brain capacity, the value of the assets of all companies in that area increases. This is “sector” news and directly increases the book value. We could similar have “market” news that affects all sectors.

#### EXAMPLE 5.1: Stock Evaluation

Let's bring it all together and check our understanding with an example.

The company owns 10 airplanes valued at \$10M each, its brand name is worth \$10M, and it has a \$20M loan. It pays out \$1M in dividends annually across 1 million shares and has a 5% discount rate. Its stock price is \$75.

What is the company's **book value**, **intrinsic value**, and **market capitalization**? Finally, **would you buy this stock?**

Give yourself a moment to work it out before you continue reading.

Its book value is \$80M:  $(10 \times \$10) - \$20$ .

Its intrinsic value is \$20M:  $\$1M/0.05$ .

Its market capitalization is \$75M:  $1M \text{ shares} \times \$75$ .

You should **absolutely buy** this entire company right now. For \$75M, you can turn around and immediately sell all of its assets and pay off its loans for a cool \$5M profit. The “low” intrinsic value here is completely irrelevant, and this is why the market cap for a company *very* rarely drops below its book value.

## 5.2 Capital Asset Pricing Model

Developed independently by researchers in the 1960’s, the **capital asset pricing model** (CAPM for short) explains how the market impacts individual stock prices and provides a mathematical framework for hedge fund investing.

Let’s briefly recall how **portfolios** work. The total return of a portfolio is the sum of each asset’s returns weighed by their ratio  $w_i$ :

$$r_p(t) = \sum_i w_i r_i(t) \quad (5.1)$$

### EXAMPLE 5.2: Portfolio Return

As a quick example, assume Stock A goes up +1% and Stock B goes down -2%. 75% of your portfolio is made up of \$A and you have a short position on \$B so its weight is -25%.

**What is the return on your portfolio?**

Applying our formula from (5.1) is trivial:

$$r_p(t) = (0.75 \cdot 0.01) + (-0.25 \cdot -0.02) = 1.25\%$$

This leads us to the concept of a **market portfolio**, which is effectively an index of a particular market. For example, the S&P 500 (tracked by \$SPY) represents the 500 largest companies that are trading on U.S. exchanges; it changes each day based on the value of those companies. There are similar indices in other countries like \$FTA in the United Kingdom or \$TOPIX in Japan. The market portfolio is a combination of these stocks, often weighed by their **market cap** (called **cap-weighted**):

$$w_i = \frac{\text{cap}_i}{\sum_j \text{cap}_j}$$

Each of these markets are broken into sectors like energy, technology, manufacturing, finance, etc... As we’ve discussed, news affects sectors and individual companies more often than it

affects an entire market. Some stocks have large weights and impact the market portfolio more significantly than others, for example, Apple (\$AAPL) and Exxon (\$XON) are each about 5% of the S&P 500.

This leads us to the **CAPM equation**:

$$r_i(t) = \underbrace{\beta_i r_m(t)}_{\text{market}} + \underbrace{\alpha_i(t)}_{\text{residual}} \quad (5.2)$$

You might recognize this as the equation of a line:  $y = mx + b$ ; that's no coincidence. We've seen  $\alpha$  and  $\beta$  before in [Python for Finance](#) when we discussed scatter plots:  $\beta$  was the slope and  $\alpha$  was the  $y$ -intercept of a stock plotted against an index.

The CAPM equation states that the return for an individual stock  $i$  on a particular day  $t$  is the market return  $r_m$  scaled by some  $\beta$  plus some residual  $\alpha$ . It asserts something important: a significant part of the return of a stock is affected *just* by the market at large. Most stocks have a  $\beta \approx 1$ : when the market goes up, the stock goes up approximately the same amount. The CAPM further asserts that the residual  $\alpha_i(t)$  is essentially a random variable; as  $t \rightarrow \infty$ , the factor becomes irrelevant because it continually cancels itself out. In other words, its expectation is 0:  $E_\alpha = 0$ .

### 5.2.1 Portfolio Management Under CAPM

The inception of CAPM led to a debate about whether **passive** or **active** investment is better. Passive investing involves simply buying an index and holding it, letting the growth of the market bring in profits. Active investing is (obviously) more involved: you pick and choose the individual stocks (or change the weights of an index) in hopes you can beat the market.

Let's examine the CAPM equation under these two philosophies:

$$r_i(t) = \beta_i r_m(t) + \alpha_i(t)$$

Both camps agree that the  $\beta$  term is correct: the return is most significantly affected by the market. The contention comes from the  $\alpha$  term: CAPM says its unpredictable and 0 on average. Active managers, though, claim they can predict  $\alpha$  better than a coin flip.

### 5.2.2 CAPM for Portfolios

To compute the return for an entire portfolio under the CAPM, we just slap the weighted summation on (5.2):

$$r_p(t) = \sum_i w_i \beta_i r_m(t) + \alpha_i(t) \quad (5.3)$$

We can pull out the  $\beta$  term from the summation:  $\beta_p = \sum_i w_i \beta_i$ . Here's where the management strategies we just discussed differ, though. CAPM says  $E_{\alpha_i} = 0$  so we can lump them all into a single term:

$$r_p(t) = \beta_p r_m(t) + \alpha_p(t)$$

Active managers, though, claim they can adjust the weights to outperform randomness, so the summation is still necessary:

$$r_p(t) = \beta_p r_m(t) + \sum_i w_i \alpha_i(t)$$

The CAPM implies some things about a robust portfolio. In a downward market, we want  $\beta \leq 1$ , so we don't lose as much money as the market in general. Contrarily, we want  $\beta \geq 1$  in an upward market to outperform just tracking the market.

### 5.2.3 Arbitrage Pricing Theory

The **arbitrage pricing theory**, developed in 1976 by Stephen Ross, digs deeper into the CAPM equation and notes that the  $\beta_i$  for a stock  $i$  is not representative of the sectors its composed of. For example, RobinHood could swing fairly significantly with both the financial and technology sectors, but less so with manufacturing. To properly model that, we'd want to break up its  $\beta$  into  $\beta_F r_F$  and  $\beta_T r_T$ :

$$r_i = \beta_{iF} r_F + \beta_{iT} r_T + \beta_{im} r_m + \dots + \alpha_i$$

This would, in theory, give us a more accurate pricing model.

### 5.2.4 CAPM for Hedge Funds

By picking stocks with appropriate  $\beta$  values, the CAPM pricing model virtually guarantees a positive return. The key, of course, still lies in picking the stocks.

Suppose we're geniuses (or time travelers) and make perfect predictions about two stocks: Stock A has a  $\beta = 1.0$  and will be  $+1\%$  over market, and Stock B has a  $\beta = 2.0$  and will be  $-1\%$  below market. The behaviour of the stocks is plotted in [Figure 5.3](#) below; if we enter our positions at the dashed line below—long \$50 on Stock A and short  $-\$50$  on Stock B—how much money will we make?

Well, the market (magically) had no change during our investment period, so  $\beta_A r_m = \beta_B r_m = 0$ . Our return, then, is:

$$\begin{aligned} r &= r_A + r_B \\ &= \beta_A r_m + \alpha_A + \beta_B r_m + \alpha_B \\ &= \alpha_A + \alpha_B \\ &= (0.01 \cdot \$50) + (-0.01 \cdot -\$50) \\ &= \$0.50 + \$0.50 = \$1 \end{aligned}$$

What if the market went up  $10\%$  during our investment time frame? In that case, the

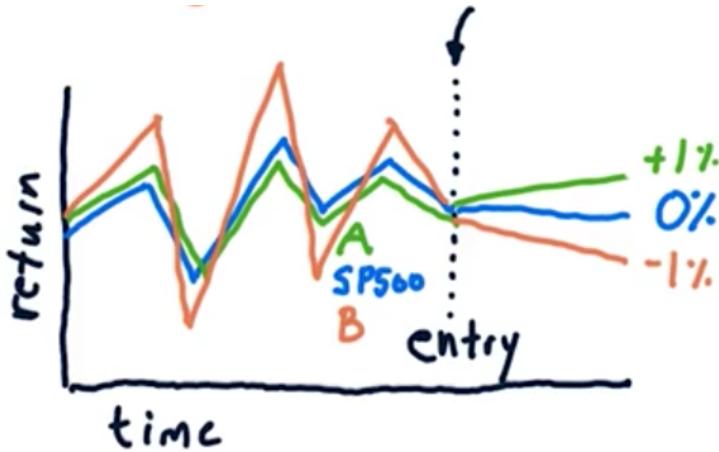


Figure 5.3

equation gets a little more complicated:

$$\begin{aligned}
 r &= \beta_A r_m + \alpha_A + \beta_B r_m + \alpha_B \\
 &= (1 \cdot 0.10 + 0.01) \cdot \$50 + (2 \cdot 0.10 - 0.01) \cdot -\$50 \\
 &= \$5.50 - \$9.50 = -\$4
 \end{aligned}$$

Contrarily, what if the market went down 10%? Our short position on a  $\beta = 2$  stock pulls us into the black:

$$\begin{aligned}
 r &= \beta_A r_m + \alpha_A + \beta_B r_m + \alpha_B \\
 &= (1 \cdot -0.10 + 0.01) \cdot \$50 + (2 \cdot -0.10 - 0.01) \cdot -\$50 \\
 &= -\$4.50 + \$10.50 = \$6
 \end{aligned}$$

If we're not careful about how we allocate money, perfect  $\alpha$  and  $\beta$  knowledge won't save us. Lets plug our assumptions into the CAPM and see how our stock choices affect our portfolio outcome, keeping the market return as our variable. From (5.3), we know that:  $r_p = \sum_i w_i \beta_i r_m(t) + \alpha_i(t)$ .

Plugging in our terms gives us:

$$\begin{aligned}
 r_p &= \sum_i w_i \beta_i r_m(t) + \alpha_i(t) \\
 &= (w_A \beta_A + w_B \beta_B) r_m + w_A \alpha_A + w_B \alpha_B \\
 &= (0.5 \cdot 1 - 0.5 \cdot 2) r_m \\
 &= -0.5 \cdot r_m + 0.01
 \end{aligned}$$

This corresponds to our calculations above. Though we have perfect predictions and information about our  $\alpha$ s, we have no knowledge or control over the market returns,  $r_m$ . Is there a way we can remove that component? In other words, can we make  $-0.5 \cdot r_m = 0$ ?

$$\beta_p = -0.5 = w_A \beta_A + w_B \beta_B$$

The question is, then: how can we choose weights for our portfolio such that they evaluate to zero? For what  $w_A, w_B$  is the following true, under the constraints that  $w_A > 0$ ,  $w_B < 0$  (since we're shorting Stock B), and the absolute value of their sum is 1.

$$w_A + 2w_B = 0$$

We can arrange these into a system of equations:

$$\begin{cases} w_A = -2w_B \\ |w_A| + |w_B| = 1 \end{cases}$$

We can easily solve this system to get  $w_B = -1/3$ ,  $w_A = 2/3$ . Plugging this back into the CAPM portfolio equation gives us:

$$\begin{aligned} r_p &= \underbrace{(w_A\beta_A + w_B\beta_B)}_{=0} r_m + w_A\alpha_A + w_B\alpha_B \\ &= w_A\alpha_A + w_B\alpha_B \\ &= 0.67 \cdot 1.0 + -0.33 \cdot -1.0 \\ &= 0.01 \end{aligned}$$

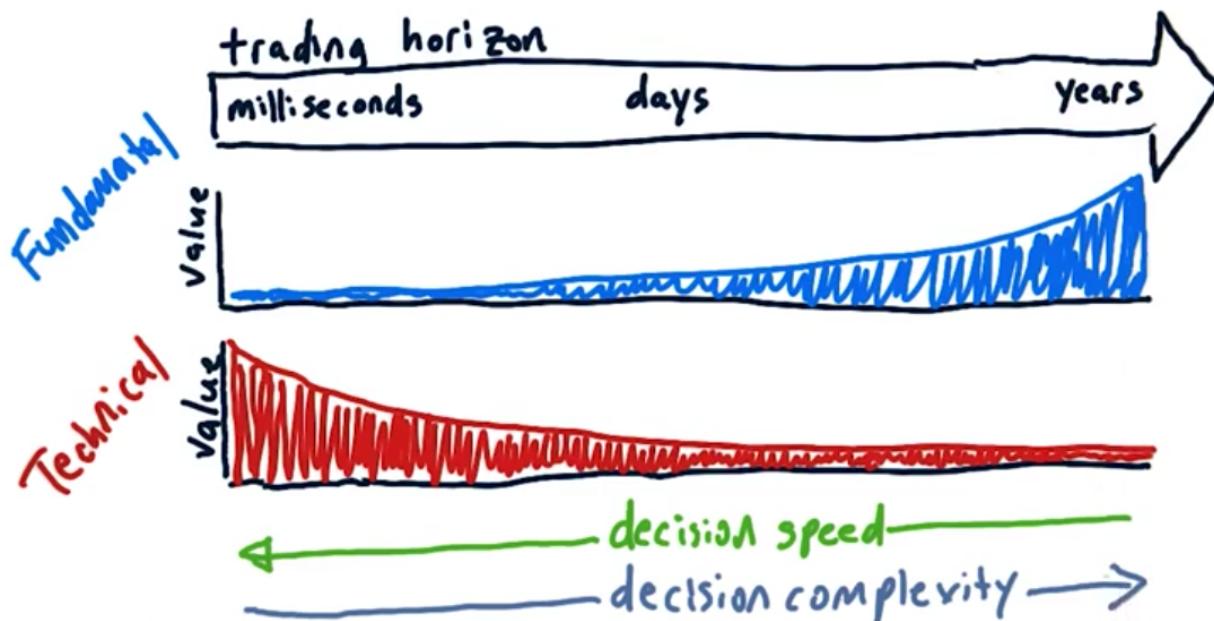
This means that regardless of the way the market moves, we will get a 1% return. Of course, this heavily relies on the fact that the  $\alpha$ s we claimed to know are in fact true, and the  $\beta$ s accurately represent the stocks' reactions to market changes. The main purpose of this was to demonstrate that we can minimize the effects of the market on our portfolio with  $\beta_p = 0$ .

## 5.3 Technical Analysis

There are two broad categories of choosing stocks to buy or sell: **fundamental analysis** and **technical analysis**. In the first section of this chapter, we discussed Metrics for Evaluation that fall in the former category; fundamental investors analyze the aspects of a company that estimate its value and look for opportunities to trade when the stock price goes above or below that value. On the other hand, technical analysis disregards the true value of a company, instead looking for and leveraging trends in a stocks price.

Technical analysis looks at **only** at historical price and volume. It involves computing statistics called **indicators** that are heuristics that hint at a buy or sell opportunity. Since it doesn't consider the value of a company, it might be more correct to call this a *trading* strategy rather than an *investment* strategy.

Technical analysis is often effective in the short term: Buffet invests in the long-term after doing thorough due diligence about the true value of a company, while high-frequency traders use indicators to buy and sell stocks within seconds. Since there are so many traders looking at individual indicators, it's harder to leverage them as an individual. Consequently, well-crafted combinations of indicators can be much more unique and effective. Finding stocks that contrast heavily to the market can also be beneficial; if every stock follows the market, you simply can't do better than the market.



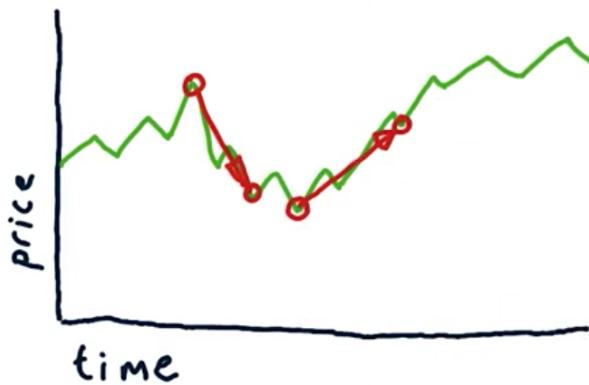
**Figure 5.4:** The contrast between fundamental and technical analysis. In the short term, technical analysis has a lot of value that quickly peters out; fundamental analysis, though, is most effective long-term as the true value of the company can shine. There are similar differences in decision speed vs. complexity, showing us how computers can shine in short-term trading, though they're less useful in long-term value-based investing.

### 5.3.1 Indicators

There are thousands of technical indicators out there, but we'll cover a few solid, common ones you may see: **momentum**, simple moving average, and Bollinger bands.

#### Momentum

Momentum is one of the simplest indicators; it's the rate of change of a stock. Over a certain number of days, how much has the price changed?



The first red line in the price graph above has steep negative momentum, whereas the second

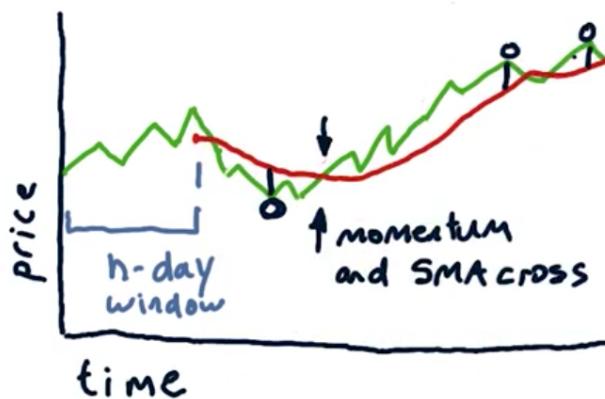
red line has steady positive momentum. The common way some traders will use momentum as an individual indicator is by assuming that a stock's momentum will continue.

**Snippet 5.1:** Momentum is a simple calculation measured over a time window of  $n$  days. It's a percentage, so it's often  $\in [-0.50, 0.50]$ .

```
momentum[t] = (price[t] / price[t - n]) - 1
```

## Simple Moving Average

Much like momentum, the **simple moving average** (or SMA) operates on an  $n$ -day window of time. The SMA is then just the average over those  $n$  days; it looks like a smoothed value of the price chart that lags behind.



The points of intersection between the simple moving average and the price graph tend to be important when using SMA as an indicator. Combined with momentum, this can indicate a trading opportunity. For example, in the above graph, the downward momentum and SMA intersection suggests that the stock may turn around its trend soon, indicating a buying opportunity.

Furthermore, gaps between a stock price and its moving average can be interpreted as a proxy for its true underlying value. The far-left downward excursion suggests a buying opportunity since the stock price has dipped significantly below its “true value” estimated by the SMA, and vice-versa for the two upward excursions on the right.

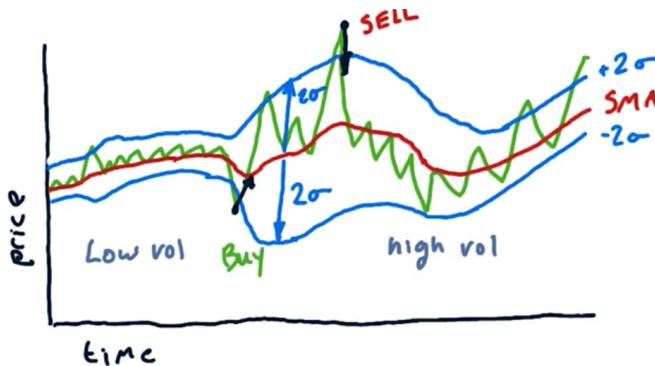
**Snippet 5.2:** The simple moving average is a ratio between the current price and the average price over a time window of  $n$  days. The SMA is a percentage, so it does not often deviate from being  $\in [-0.50, 0.50]$ .

```
sma[t] = (price[t] / price[t - n : t].mean()) - 1
```

## Bollinger Bands®

We've seen Bollinger bands before: it's a  $\pm 2\sigma$  deviation from the simple moving average. The bands are good thresholds for acting on price excursions, allowing short spikes and deviations in the stock price.

The reason why  $2\sigma$  is a good value is because it naturally varies with the volatility of the stock. In periods of low volatility, we would want to act on larger spikes, whereas in periods of high volatility we would want to let those slide a little more.



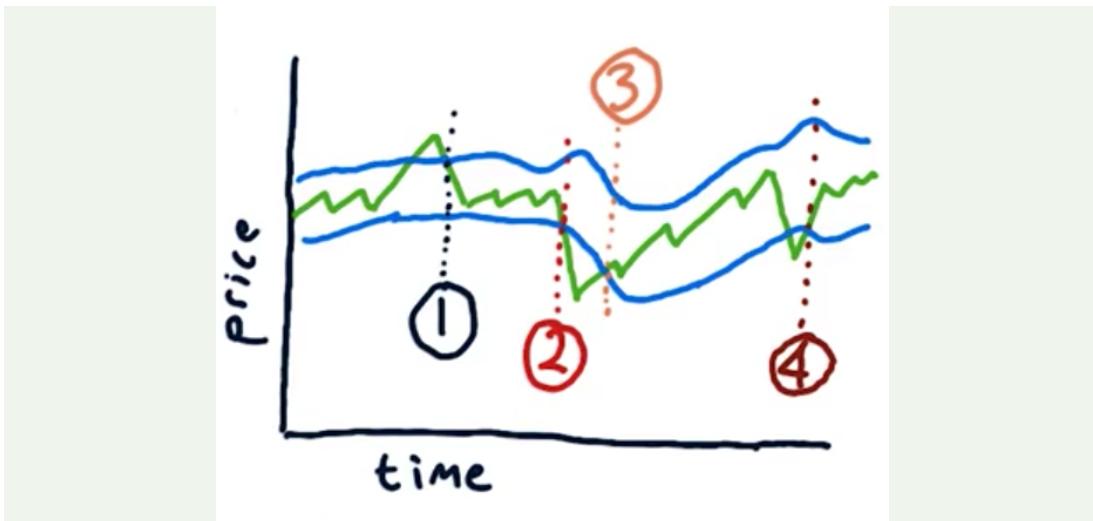
Points at which a stock crosses to the outside of its Bollinger bands *then re-enters* are often considered to be trading opportunities. For example, in the above graph when the stock spikes above the  $2\sigma$  band and dips back in, that is a selling opportunity.

**Snippet 5.3:** Bollinger bands are  $\mu \pm 2\sigma$ . We typically expect to see BB values  $\in [-1, 1]$ ; deviations from that often indicate trading opportunities.

```
bb[t] = (price[t] - sma[t]) / (2 * std[t])
```

### EXAMPLE 5.3: Bollinger Bands — Buy or Sell?

Given the following graph, do each of the circled points indicate a **buy** signal, a **sell** signal, or **neither**?



### Answers

In ①, we see a spike outside of the upper band, then back in. That is a signal to sell.

In ②, we see a spike outside of the lower band, but no re-entering. This **isn't an indication** of anything and prevents us from a premature buy when the stock could be plummeting for a while.

In ③, we re-enter through the lower band, indicating a **buy** signal.

In ④, we spike below the lower band and re-enter, again signaling a **buying** opportunity like in ③.

### Normalization

Notice that each of our indicators has a different range. This makes it difficult to plug them into plots or learning models, so we need to normalize them. This is straightforward and something we saw very early on in [chapter 2](#).

Given an array of values,  $\mathbf{v}$ , the normalized array in the range  $[-1, 1]$  is simply:

$$\hat{\mathbf{v}} = \frac{\mathbf{v} - \mu_{\mathbf{v}}}{\sigma_{\mathbf{v}}}$$

## 5.4 Anomalous Price Changes

*To be continued...*

# PART III

## LEARNING ALGORITHMS FOR TRADING

**T**HE third part of this topic will cover applying machine learning algorithms to financial data in order to create models that give us insights about stock behavior. We can use these insights to (try to) predict future price data and make trading decisions based on those predictions. Financial models have existed for a long time, but using machine learning lets us create “data-centric models” in which the evidence speaks for itself; there is no longer a need to subjectively interpret the data.

Let’s talk about machine learning in broad terms at first, then get into specific techniques and algorithms. What problem does machine learning solve? Well, given an observation input  $X$  (like the last year’s daily returns of a stock, for example), we can feed it into a **model** and get some output prediction  $Y$  (like the *next* set of daily returns). The model, of course, is created from massive amounts of data fed into a machine learning algorithm.

$$X \longrightarrow \boxed{\text{model}} \longrightarrow Y$$

# SUPERVISED REGRESSION LEARNING

**T**HE first part of this chapter will focus on **supervised regression learning**: **supervised** by a sequence of example predictions  $(\mathbf{x}_i, y_i)$ , we will **learn** a numerical prediction (the **regression**, a poor naming choice). Some examples that fall under this category of machine learning algorithms that we'll be discussing soon include linear regression,  $k$ -nearest neighbor (or  $k$ NN), decision trees, and decision forests.

In terms of stock data, the most pertinent “prediction” is a future price. This allows us to make decisions on top of that price data rather than restricting our model to making strategy-specific decisions (like “Buy!”) directly.

Remember, we can have many feature vectors  $\mathbf{x}_i$  for a particular stock. Typically, we have a **pandas DataFrame** for each feature in which each column is the data for a particular stock and each row is the data at a specific point in time. When we extend this to many features, we can think of it as a third dimension to our data in which the “depth” is the feature number. What is our  $\mathbf{y}$ , then? Well, as we've already established, it will be price. Specifically, we can create an association between a particular feature and a “future” price. We say “future” because we actually have historical price data. So we could look at the Sharpe ratio for \$IBM on January 1<sup>st</sup> (this is our  $x_i$ ) and correlate it with its price on January 5<sup>th</sup> (this is our  $y_i$ ).



**Figure 6.1:** Our machine learning model associates a 3-dimensional array of feature values for a variety of stocks with their (historically) “future” price data.

The “art” of model creation is picking these predictive factors—the more accurate your resulting predictions, the more someone is probably willing to pay to use your model.

**Backtesting** How accurate can these models *really* be? The stock market is a fickle beast and some argue it runs more on emotion than reason. We need a way to test the effectiveness of our model without losing tons of money in the process. This is where **backtesting** comes in: we apply our model on a different period of time than the one it was trained on and see how accurately its predictions match the actual stock behavior.

This is somewhat akin to splitting the training data into a “training” and “test” set that we’ll dive into further soon. Given a model based on a set of training data, we can place a set of orders for a future point in time. We then use the historical data to see how those orders would’ve performed, plotting the performance of our fake portfolio over time (in terms of any of the statistics we learned about in [Part I](#)). We can repeat this for as long as we have historical data for, seeing how our model would’ve “really” performed over that period of time.



**Figure 6.2:** Describing the process of backtesting our model over historical price data, plotting the results over time.

With a cursory overview out of the way, let’s dive into our first set of machine learning methods that will make us teh big bux.

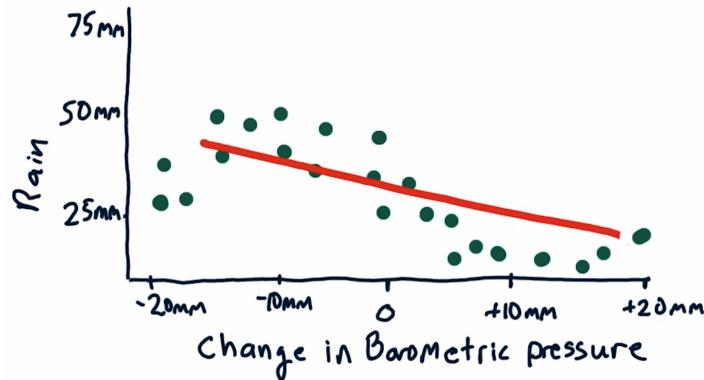
## 6.1 Regression

Don’t let the spooky term “regression” scare you: it’s just a numerical model. It outputs a number (its prediction) based on a bunch of input numbers.

### 6.1.1 Linear Regression

Let’s take a look at a *parametric* method first. We’ve seen the concept of a parametric model before: in [Building Parameterized Models](#), we chose parameters to our line ( $m$ , the slope, and  $b$ , the  $y$ -intercept from the familiar equation of a line,  $y = mx + b$ ) that best fit our data set. We defined “best fit” as minimizing the total vertical error between the line and the points. This is called **linear regression**.

Once we’ve found our best-fitting model parameters, calculating predictions is just a matter of plugging our new input values into the model. We can throw the initial data away. The problem, as you can see in [Figure 6.3](#), is that the real world seldom follows a linear model. Furthermore, it’s hard to say what degree of a polynomial you’d need to best fit a model. It might be better to let the data speak for itself, which would lead us to...



**Figure 6.3:** Using linear regression to fit a line to a data set mapping the correlation between changes in barometric pressure and rainfall. Notice that a higher-dimensional polynomial might have fit this data better, but this is the best we could do with our model (a line).

### 6.1.2 *k*-Nearest Neighbor

This data-centric approach (called an *instance* method, in contrast with the *parametric* method earlier) uses instances of the data points to create an approximate prediction based on the distance of a novel input to 1, 2, or  $k$  of its nearest neighbors in the data. In ***k*-nearest neighbor**, we just take the mean of the  $y$  values. This method actually works surprisingly well for  $k > 1$ , but isn't always ideal because it requires storing the data which might have many dimensions and take up a lot of space.

If we actually take the *weighted* mean of the  $y$  values based on the distance of the neighbors to the new input, that's actually called **kernel regression**.

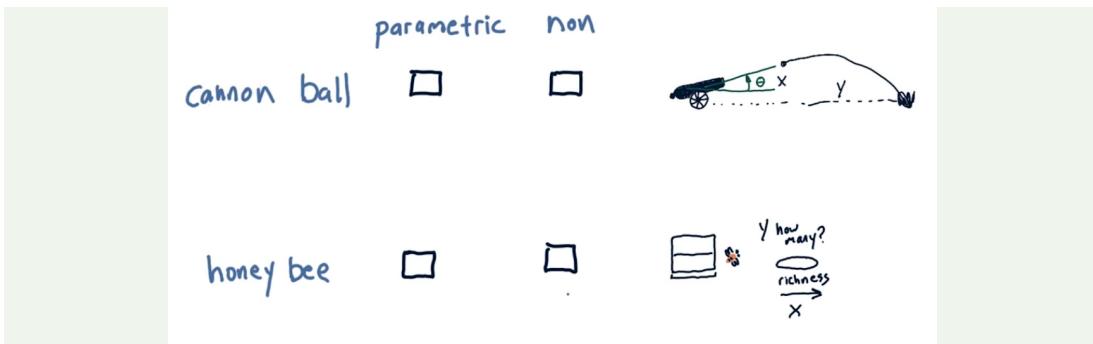
#### EXAMPLE 6.1: Choosing an Approach

How should we decide when a parametric approach like linear regression should be preferred to a situation over an instance or data-centric approach like *kNN*?

Suppose we have two situations we'd like to have a model for:

- A projectile is fired from a cannonball, and we'd like to predict where it lands. Our observable is  $\theta$ , the angle of the cannon, and its landing point.
- A hive of honey bees is attracted to a food source; we'd like to predict how many bees will go to a particular food source given its richness. Our observable data is  $n$ , the number of bees, and a food “richness” metric.

To which of these should we apply a parametric model? A non-parametric model?



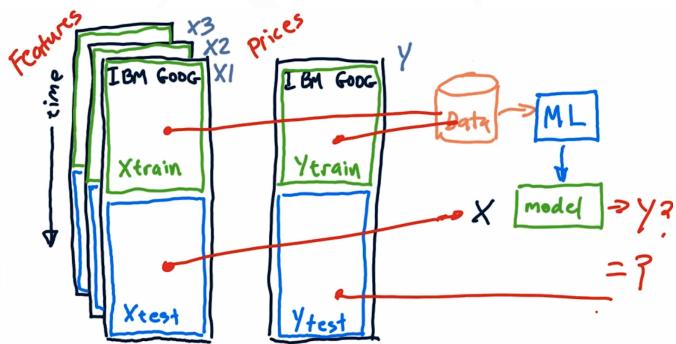
would be better modeled in an instance-based way.

**ANSWER:** The cannon would probably use a parametric model, whereas the bees

Generally speaking, the more well-defined a problem is, the more applicable a mathematical model might be, meaning a parametric approach is more suitable. We don't really have much "mathematical bee theory," so crafting a model based on the things we see is much more applicable.

### 6.1.3 Training vs. Testing

We have a limited amount of historical stock data to work with, right? But we need to evaluate our model without risking real money. Thus, we need to split our data into **training data**—which is fed into the algorithm to create the model—and **testing data**—which we use to evaluate the quality of our model before taking out a 2<sup>nd</sup> mortgage on our house.



An important caveat of train-test splitting in historical price data that doesn't necessarily apply to other applications of machine learning algorithms is that we want training data to always be earlier in time than testing data.

### 6.1.4 Problems with Regression

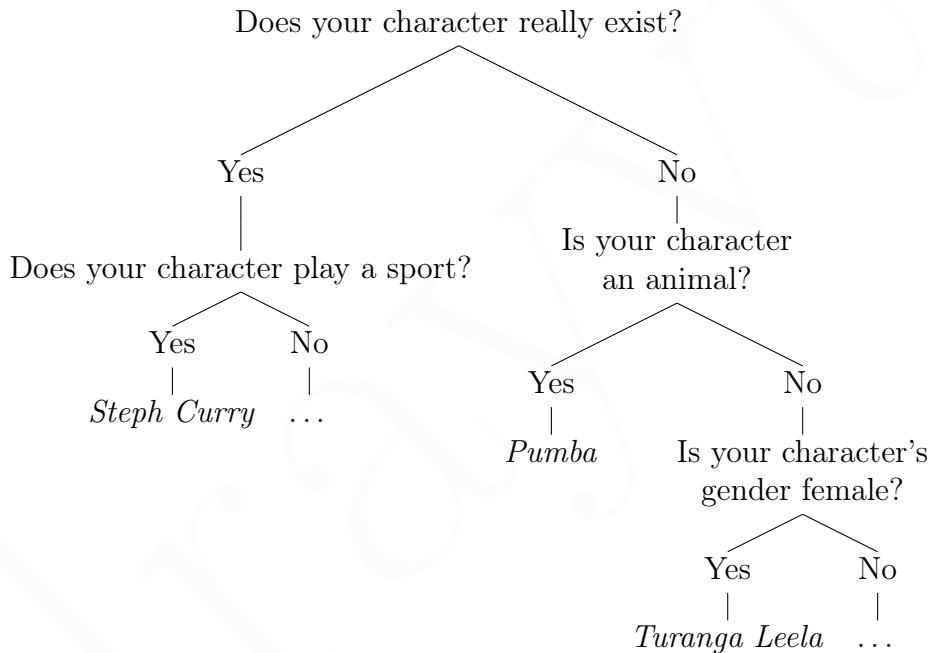
Obviously if this was a perfect solution we could call it here and become millionaires. Unfortunately, reality is often disappointing. Regression is a noisy and uncertain method. Furthermore, it's challenging to estimate confidence in a model and reflect that to the user in an understandable way (just showing the standard deviation is not very reliable nor user-friendly). There are also challenges nested in the problem that are specific to stocks: how

long do you hold for?<sup>1</sup> How do you optimize allocation ratios?

## 6.2 Decision Trees

We'll open our discussion of machine learning algorithms with **decision trees**.<sup>2</sup> What is a decision tree? It's fairly self-explanatory: it's a tree that maps various choices to diverging paths that end with some decision. For example, one can imagine the "intelligence" behind the famous "character guessing" AI [Akinator](#): for each yes-or-no question it asks, there are branches the answers that lead down a tree of further questions until it can make a confident guess.

One could imagine the following (incredibly oversimplified) tree in Akinator's "brain:"

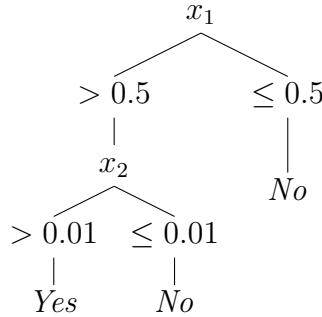


Except Akinator has played (and thus improved upon and expanded its decision tree) about 400 **million** games as of this writing.

In most applications of decision trees—especially with respect to financial algorithms—the branches are based on *numeric* boundaries in a particular **feature**. For example, we might decide to invest (or not invest) in a particular stock if its Sharpe ratio is above 0.5. Or, to branch further, we might only invest if  $S > 0.5$  *and* its daily returns are above 1%. We'd say that  $\mathbf{x}_1$  is the "feature vector" for Sharpe ratio (one element for each stock we're choosing from) and  $\mathbf{x}_2$  is the feature vector for its daily returns:

<sup>1</sup> Remember, you pay lower taxes on your capital gains (i.e. profits from stocks) if you hold them for over a year! This kind of calculation is important to keep in mind when analyzing a strategy or model.

<sup>2</sup> This section is based in-part on the [founding paper](#) by J.R. Quinlan on decision trees.



Note that a feature can (and likely will) appear more than once in a decision tree. Similarly, some features may be completely disregarded when making a decision for a particular input value depending on the path it takes. Consider the table:

	$x_1$	$x_2$
\$JPM	1	0.10
\$AAPL	0.2	0.07
\$SPY	0.7	-0.04

Which stocks would we invest in based on our decision tree above? Probably just \$JPM.

Coming to a conclusion given a decision tree seems pretty straightforward. The hard part is *creating* the tree. If we're *given* a set of data like the table above with an additional column describing what our choice *should* be, how do we find the best tree? How do we choose the feature to split our tree along? What values do we split a particular feature along? Etc.

### 6.2.1 Representing a Decision Tree

Before we dive into this topic, let's discuss the data structure itself. There are two main approaches: an object-oriented approach that many of you are probably familiar with in which there are **Node** objects that point to each other, and a matrix-based approach in which each row represents a node.

The matrix representation is more compact and more efficient to process, so let's go over it first. Let's continue with our running example of the buy/don't-buy decision tree and the feature vectors in the table above. Each row represents a node in the tree; the columns are the factor, the splitting value, and the left and right node that follow. For "special" (i.e. leaf) nodes, the splitting value is actually the decision value, and the factor is  $-1$  to indicate that it's a leaf. For the above, it'd look something like:

Index	Factor	Split Value	Left	Right
0	1	0.5	1	4
1	2	0.01	2	2
2	-1	Yes		
3	-1	No		
4	-1	No		

For a given input, say  $\mathbf{x} = [1, 0.10]$  (corresponding to \$JPM), we'd first check the root

(i.e.  $\text{index} = 0$ ) and see that it corresponds to the left value since we “pass” the split value<sup>3</sup>. The next node is at  $\text{index} = 1$ ; the input likewise “passes” and we move on to  $\text{index} = 2$ , which is a leaf node that says “Yes, buy!”

### 6.2.2 Learning a Decision Tree

Let’s talk about “learning” a decision tree now.

```
def build_tree(data):

    if data.shape[0] == 1:
        return [leaf, data.y, NA, NA]

    if all(data.ysame):
        return [leaf, data.y, NA, NA]

    determine best feature i to split
    onSplitVal= data[:,i].median()

    lefttree = build_tree(data[data[:,i] <= SplitVal])
    righttree = build_tree(data[data[:,i] > SplitVal])
    root = [i, SplitVal, 1, lefttree.shape[0] + 1]

    return append(root, lefttree, righttree)
```

---

<sup>3</sup> We arbitrarily decide that  $(x_i > \text{split})$  corresponds to the left index; this is just a matter of convention.

# REINFORCEMENT LEARNING

# INDEX OF TERMS

- k*-nearest neighbor, 50, 52  
arbitrage pricing theory, 42  
assets under management, 23  
backtesting, 51  
basis points, 16  
bear, 22  
bid-ask spread, 29  
Bollinger bands, 7, 45, 47  
bond, 37  
book value, 37, 38, 40  
bull, 22, 35  
cap-weighted, 40  
capital asset pricing model, 40  
convex functions, 17  
correlation, 13  
cumulative returns, 8, 25  
daily returns, 7, 10  
dark pool, 32  
decision forests, 50  
decision trees, 50, 54  
discount rate, 38  
dividends, 37  
expense ratio, 23  
feature, 54  
fundamental analysis, 44  
gradient descent, 17  
histogram, 10  
indicators, 44  
intrinsic value, 37, 37, 40  
kernel regression, 52  
kurtosis, 11  
large-cap stock, 22  
linear regression, 12, 50, 51  
liquid, 22  
market capitalization, 22, 37, 39, 40  
market portfolio, 40  
momentum, 45, 46  
normal distribution, 10  
order, 27  
order book, 27  
portfolio, 14, 40  
portfolio management, active, 41  
portfolio management, passive, 41  
ratio, 19  
rolling mean, 6  
rolling standard deviation, 6  
scatter plot, 12  
scatter plots, 41  
scatterplot, 10  
sell short, 28, 34  
Sharpe ratio, 15, 25  
short, 25  
simple moving average, 46, 47  
slope, 12, 41  
supervised regression learning, 50  
technical analysis, 44  
two and twenty, 24  
volume, 22