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Influence of the shape of the inner boundary on thermomagnetic convection in the annulus between horizontal cylinders: Heat transfer enhancement

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ABSTRACT

The paper studies convection in a horizontal annular gap filled with a magnetic fluid or gaseous oxygen under the influence of gravitational and magnetic forces. A current-carrying conductor and an external uniform magnetic field magnetizing an internal cylinder of a material with high magnetic permeability were considered as a source of a magnetic field. The influence of the shape of the inner cylinder and the magnitude of the magnetic field gradient on the heat transfer was studied. It is found that the strength of the magnetic field created by a current-carrying conductor is insufficient to compete with gravitational convection. In the absence of gravity, a variety of convective structures and the hysteretic nature of the transition between them were discovered. The shape of the conductor is an additional factor affected the convection.

The effect of a high gradient magnetic field of a magnetizing inner cylinder on convection is studied too. In the case of magnetic fluid, this can ensure the excess of magnetic forces over gravitational by hundreds of times. Heat transfer in an external field with strength of 150 kA/m from a circular magnetized cylinder can be increased 3–4 times, and in a field of 1000 kA/m 4–6 times compared with natural gravitational convection. Heat transfer can be further enhanced by 40–50% by choosing the shape of the inner cylinder.

The magnetic field can increase heat transfer through an annular gap filled with gaseous oxygen, 2 times.

1. Introduction

Natural convection in the horizontal annular gap between two circular cylinders has been studied since the 30s of the last century [1]. The use of natural convection heat transfer for cooling nuclear reactors, underground electric cables [2], airplane cabins, and electronic equipment caused an intensive continuation of these studies in 60–70s. The classical paper of Kuehn and Goldstein [3] provides a detailed list of papers in this field. In this paper, a detailed experimental and numerical study of all factors affecting the intensity of heat transfer was carried out and it was shown that the average coefficient of thermal conductivity is related to the Rayleigh number by the ratio $\mathrm{Nu} \sim \mathrm{Ra}^{m}$. The power of the Rayleigh number changed in the experiment ranged from 0.272 for air to 0.238 for water in the range of Rayleigh numbers from $2 \cdot 10^4$ to $9 \cdot 10^4$. In the numerical simulation, the exponent also changed from 0.25 for a gap equal to the radius of the inner cylinder to 0.2 for a gap with a width of 16 radii.

Attempts to find ways to intensify heat transfer in the annular gap led to the study of the eccentric arrangement of the cylinders or a change in their shape [4–8]. It was found that a change in the relative position of the cylinders and their shape noticeably affects the streamlines, but slightly changes the average Nusselt number. The intensity of heat transfer through the annular gap is still determined mainly by the Rayleigh number.

A different situation occurs if magnetic fluid is used as a heat carrier. Magnetic fluid is an artificial medium, which is a colloidal solution of magnetic nanoparticles, the average diameter of which is about 10 nm [9]. To prevent agglomeration of magnetic particles, a surfactant layer is deposited on their surface. Magnetic fluids are stable, keep their properties unchanged for decades (the authors have a sample of magnetic fluid produced at the Institute of Heat and Mass Transfer in Minsk, Belarus, in 1980, which has retained its properties to this day) and are widely used in many devices, technological processes and medicine (magnetofluid seals, bearings, ore separation, targeted drug delivery,

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hyperthermia, etc.) [9].

From the point of view of the influence on hydrodynamics and heat transfer, the main feature of magnetic fluids is the presence of an additional volume force $\mu_0 M\nabla H$, the value of which is determined by the magnetization of the fluid M and the magnetic field gradient ∇H. The magnetization of a fluid is the magnetic moment of a unit volume. When heated, the liquid expands, the volume increases and the magnetization, like density, decreases: $M = M_0(H)$ (1 - $\beta\Delta T$), (where β is the coefficient of thermal expansion of the fluid, the temperature dependence of the magnetic properties of magnetite particles in the range of existence of magnetic fluids can be neglected; this issue is discussed in detail in Ref. [9]), and the magnetic buoyancy force $\mu_0 M_0 \nabla H \beta \Delta T$ occurs. Since in the case of natural convection, the driving force is the gravity buoyancy force $\rho g \beta \Delta T$, the intensity of thermomagnetic convection can be estimated in comparison with natural gravitational convection by comparing ρg and $\mu_0 M_0 \nabla H$. Obviously, the value of ρg is given to us by nature and varies in a rather narrow range (differing, of course, for gases and liquids), while the value $\mu_0 M_0 \nabla H$ can vary in a wide range due to changes in the magnetization of the fluid and the gradient of the magnetic field.

The magnetization of a fluid is specified either by its physical natural properties (oxygen, air), or by the concentration of magnetic particles (magnetic fluids). The magnitude of the volume magnetic force is proportional to the magnetization and, accordingly, the greater the magnetization, the greater the intensity of thermomagnetic convection and heat transfer. The maximum magnetic fluid magnetization values are well known (no more than 100 kA/m) and cannot be significantly changed. Using instead of magnetite materials with a higher magnetization does not increase the magnetization of the magnetic fluid, since stabilization of such a material requires a larger thickness of the surfactant layer and its volume fraction, i.e. the magnetization of the fluid, falls [9]. Therefore, the main way to enhance heat transfer is to create the maximum possible gradient of the magnetic field in the volume of the magnetizable coolant. For this purpose, it can be possible to use either a current that creates a nonuniform magnetic field, or a uniform external magnetic field acting on objects that have a high magnetization (iron, permalloy). Such objects introduce significant distortions into the field structure, creating areas of non-uniformity, the magnitude of which is determined by the shape of these objects. Having a high magnetic permeability and being in an external uniform magnetic field, the inner circular cylinder of the heat exchanger can create a magnetic field with a significant gradient (dipole field) near its surface, which will cause a volume magnetic force that will enhance convection. In the case of an elliptical shape, the gradient of the induced magnetic field in the vicinity of the vertex near the semi-major axis should be rather great, that is, the effect of enhancing convective heat transfer should also be great.

In this connection, three situations are considered in this paper: 1) a magnetic field is created by a current-carrying conductor, convection in magnetic fluid is considered; 2) an external uniform magnetic field created by permanent magnets magnetizes an internal cylinder having high magnetic permeability, convection in magnetic fluid is considered; 3) an external uniform magnetic field created by permanent magnets magnetizes an internal cylinder having high magnetic permeability, convection in gaseous oxygen or air is considered.

The main goal of this study is to determine the possibility of a significant increase in heat flux through the horizontal annulus by a magnetic field due to the use of a magnetized medium as a heat carrier and a change in the shape of the inner cylinder.

2. Governing equations

2.1. Magnetic field

In the framework of this study, we assume that the non-uniform magnetic field is described by the equations.

$$\mathrm{div}\boldsymbol{B} = 0, \mathrm{rot}\boldsymbol{H} = 0, \boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M}(\boldsymbol{H}, T)), \boldsymbol{M}(\boldsymbol{H}, T) = M(H, T)\boldsymbol{H} / H \tag{1}$$

For paramagnetic fluids (oxygen, air), the dependence of magnetization on the external magnetic field is linear: $M(H) = \chi H$, where χ is the magnetic susceptibility. To describe the magnetization of magnetic fluids, the Langevin function $M(H) = M_S(\coth\varsigma - 1/\varsigma)$ is often used, where $\varsigma = \mu_0 mH/kT$, m is the magnetic moment of the particles, and M_S is the saturation magnetization of the fluid. However, this function describes well the dependence of magnetization on the magnetic field only for a monodisperse magnetic fluid, while in a real magnetic fluid there are particles of various sizes. There are more complex models for describing the dependence of magnetization on the field, for example [10], but for their application it is necessary to set the dipole interaction constant λ, which is unknown for a particular fluid. Therefore, in this paper we use the empirical dependence proposed in [11].

$$M(H) = M_S \frac{\chi_0 H / M_S}{1 + \chi_0 H / M_S} = M_S \frac{\chi_0 \widehat{H}}{1 + \chi_0 \widehat{H}} = M_S f(\widehat{H}) \tag{2}$$

where χ_0 is the initial magnetic susceptibility, $\widehat{H} = H/M_S$ is the dimensionless magnetic field strength. The variables χ_0 and M_S in this dependence are measured experimentally and expression (2) describes the dependence of the magnetization of a real magnetic fluid on a magnetic field with an error below 3–5%.

Generally speaking, when calculating the magnetic field in an annular gap filled with a non-uniformly heated magnetic fluid, the temperature dependence of magnetization should be taken into account. This is especially significant if the magnetic field is uniform. We assume that the magnetic field in the gap is substantially non-uniform. In this case, if the non-uniformity of the magnetic field is greater than the non-uniformity caused by the heating of the fluids (i.e., the non-uniformity of the temperature), the latter can be neglected. This is possible if the condition is fulfilled:

$$|\nabla H| \gg M_S \beta |\nabla T| \tag{3}$$

Since the coefficient of thermal expansion is of the order of 10^{-3} K^{-1}, this condition, as a rule, is satisfied for all real sources of the magnetic field. For example, for a permanent magnet 5 cm in size with a magnetic field on the surface of 150 kA/m (weak ferrite-barium magnet) acting on a magnetic fluid with a saturation magnetization $M_S = 40$ kA/m in a volume with a linear size of 5 cm, we obtain the condition for the temperature difference at the boundaries of the volume $\Delta T \ll 3.3 \cdot 10^4$ K, which is certainly performed for any fluid.

Therefore, when calculating the magnetic field, we assume that in equation (1) the magnetization of the liquid medium does not depend on temperature. When formulating the equations of convective motion of a liquid, the fact that the buoyancy force is determined by the temperature dependence of the magnetization is taken into account, and this dependence has the form (analog of the Boussinesq approximation for density).

$$M(H, T) = M_S f(H)(1 - \beta \Delta T) \tag{4}$$

Since $\nabla \times \boldsymbol{H} = 0$, we can introduce the potential of the magnetic field F so that $\boldsymbol{H} = \nabla F$. Then finally the magnetic field is determined by the equation

$$\nabla(\mu(H)\nabla F) = 0, \quad \mu(H) = 1 + f(\widehat{H}) / \widehat{H} \tag{5}$$

with boundary conditions for the equality of the normal components of the induction and the tangential components of the strength of magnetic field:

$$\left\{\mu \frac{\partial F}{\partial n}\right\} = 0, \left\{\frac{\partial F}{\partial \tau}\right\} = 0 \tag{6}$$

where braces mean $\{a\} = a_1 - a_2$, and indices 1 and 2 refer to media on both sides of the interface.

2.2. Equations of motion

To describe convective motion, the Boussinesq approximation is used, in which the dependence of density and magnetization on temperature is taken into account only when formulating the buoyancy force:

$$\rho = \rho_0[1 - \beta_\rho(T - T_0)], \quad M = M_0[1 - \beta_m(T - T_0)]$$

where T_0 is the average temperature, $\beta_\rho = -(1/\rho)\partial\rho/\partial T$ is the coefficient of thermal expansion, $\beta_m = -(1/M)\partial M/\partial T$ is the pyromagnetic coefficient. Considering that for materials of magnetic particles used in magnetic fluids, in most cases the dependence of magnetization on temperature in the temperature range of the existence of liquid media is much smaller than the change in magnetization due to thermal expansion, we assume below that $\beta_m = \beta_\rho$ (a detailed conclusion on equality can be found in Ref. [9]). Within the framework of these approximations, the two-dimensional dimensionless system of equations describing convective motion in variables of the stream function - vorticity has the form (a detailed derivation can be found in Refs. [12]):

$$\frac{\partial\omega}{\partial t} + \frac{1}{\mathrm{Pr}}(\boldsymbol{u}\cdot\nabla)\omega = \Delta\omega + \mathrm{Ra}\frac{\partial\theta}{\partial x} + \mathrm{Ra}_m f(H)\left[\left(\frac{\partial H}{\partial x}\right)\left(\frac{\partial\theta}{\partial y}\right) - \left(\frac{\partial H}{\partial y}\right)\left(\frac{\partial\theta}{\partial x}\right)\right]$$

$$\Delta\psi = -\omega \tag{7}$$

The governing equations are made dimensionless with the following reference quantities: $[t] = D^2/\nu$, $[x, y] = D$, $[T] = \Delta T$, $[u] = \kappa/D$, $[\psi] = \kappa$, $[\omega] = \kappa/D^2$, $[H] = M_S$, where D is the characteristic size of the system, κ is the thermal diffusivity, θ is the dimensionless temperature, and dimensionless numbers (the Rayleigh number, magnetic Rayleigh number and the Prandtl number) are defined as follows

$$\mathrm{Ra} = \frac{g\beta\Delta T D^3}{\nu\kappa}, \mathrm{Ra}_m = \frac{\mu_0 M_S^2\beta\Delta T D^2}{\rho\nu\kappa}, \mathrm{Pr} = \frac{\nu}{\kappa} \tag{8}$$

The stream function ψ and vorticity $\boldsymbol{\omega} = [0, 0, \omega]$ are related to the fluid velocity $\boldsymbol{u}$ by the expressions

$$u_x = \frac{\partial\psi}{\partial y}, \quad u_y = -\frac{\partial\psi}{\partial x}, \quad \boldsymbol{\omega} = \mathrm{curl}\boldsymbol{u}$$

2.3. Energy equation

The convective heat transfer equation has the standard form

$$\mathrm{Pr}\frac{\partial\theta}{\partial t} + (\boldsymbol{u}\cdot\nabla)\theta = \Delta\theta \tag{9}$$

3. Geometry of the problem and mesh

The convective flow of magnetic fluid is considered in the annular gap between two cylinders (Fig. 1a). The outer cylinder of radius $R_e = R + L$ is circular, and the inner cylinder is either circular with radius R or elliptical. In order to be able to compare the heat transfer intensities of circular and elliptical cylinders, we assume that their areas are equal, i.e. the semi-axes of the elliptical cylinder and the radius of the circular are connected by the relation $ab = R^2$. The magnetic field is calculated in accordance with equation (5) and the boundary conditions (6) in the entire space. In this case, the external radius of the calculation domain corresponding to "infinity" was assumed to be equal to $R_\infty = 16R$. This value guaranteed that the dependence of the numerical results on R_∞ was negligible. A typical triangular mesh on which numerical simulation was performed is shown in Fig. 1b.

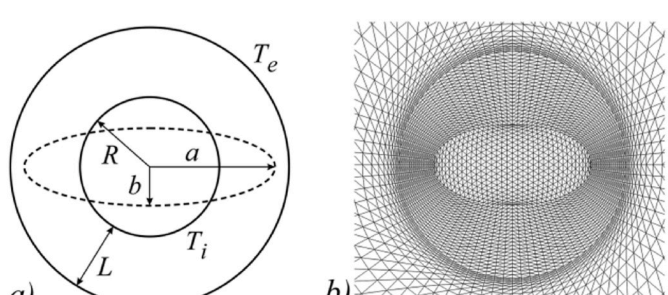

Fig. 1. Sketch of the physical domain (a) and typical mesh (b).

Details of the numerical method and validation of the code are discussed in the Appendix.

4. Nusselt number

The aim of this study is to investigate the effect of the magnitude of the magnetic field and the shape of the inner cylinder on the intensity of heat transfer. As a value characterizing the heat transfer of the cylinder, we use the integral value of the heat flux. In dimensionless form, this is the Nusselt number, defined as

$$\mathrm{Nu} = \oint_\Omega \frac{d\theta}{dn} d\Omega$$

where Ω is the contour coinciding with the surface of the inner cylinder, $\boldsymbol{n}$ is the normal vector to this surface. In the absence of convective motion, equation (7) for a circular inner cylinder has an exact solution that gives the value of the Nusselt number in the heat conduction mode (see, for example, the text-book [20])

$$\mathrm{Nu}_0 = \frac{2\pi}{\ln(R_e/R)}$$

5. Results and discussion

In this section, we consider the results of a study of three different physical situations: a) convection in magnetic fluid in the field of a current-carrying conductor; b) convection in magnetic fluid in the field of a magnetizing cylinder; c) convection in oxygen or air in the field of a magnetizing cylinder.

5.1. Convection in magnetic fluid under the magnetic field of current-carrying conductor

First of all, let us estimate the ratio of gravitational and magnetic forces in a magnetic fluid in the field of a conductor with current. Gravitational forces are represented in equation (7) by the Rayleigh number Ra, and magnetic forces are the product of the magnetic Rayleigh number and quantities related to the magnetic field $\mathrm{Ra_m} f(H)\partial H/\partial r$. Since the structure of the magnetic field is determined by the radius of the conductor, we use the radius R as the reference size of the system. The Rayleigh number in this case has the form $\mathrm{Ra} = g(\beta R^3\Delta T/\nu\kappa)$. To estimate the magnetic forces, it is necessary to know the quantity $f(H)\partial H/\partial r$. The dimensionless field of the conductor (reference value is M_S) with current I has the form $H = I/(2\pi R M_S) = jR/2M_S$ and depends on the current density in the conductor $j = I/\pi R^2$. Then the magnitude of the magnetic forces can be estimated as $\mathrm{Ra_m} f(H)\partial H/\partial r = (\mu_0 j M_S/2\rho) f(H)\partial H/\partial r(\beta R^3\Delta T/\nu\kappa)$. The ratio of magnetic F_m and gravitational F_g forces in this case is equal to $F_m/F_g = (\mu_0 j M_S/2\rho g) f(H)\partial H/\partial r$. As it is known, the current density in conductor j is limited due to its possible overheating and destruction (Table 1 [15]). The data in this table are approximated by the dependence $j_{max} = 10.1R^{-0.7}$ (dimension of j is A/mm^2, dimension of R is mm).

Data showing the ratio of magnetic and gravitational forces in the annulus near the current-carrying conductor for a magnetic fluid with the common characteristics ($M_S = 40$ kA/m, $\chi_0 = 2$, $\rho = 1300\ kg/m^3$) are presented in Table 2 (taking into account the expressions for the

Table 1
The dependence of the maximum current density on the conductor cross-section [15].

Cross-section, mm^2	0.75	1.0	2.5	4.0	6.0	10	16	25	35	50	70	150	240	400
I, A	13	15	27	36	46	68	92	123	152	192	245	392	532	737
j_{max}, A/mm^2	17.4	15	10.8	9.0	7.6	6.8	5.7	4.9	4.3	3.8	3.5	2.6	2.2	1.8

dimensionless magnetic field H and the function $f(H)$).

Since for reasonable values of the current in the conductor ($j = j_{max}/2$), the magnetic buoyancy force is an order of magnitude lower than the gravitational force even for very thin conductors, compared with gravity, the influence of the magnetic field of the current-carrying conductor on natural convection will be insignificant. However, in the absence of gravity, thermomagnetic convection can be used to cool hot conductors. Therefore, below we consider thermomagnetic convection in the absence of gravity.

5.1.1. Circular conductor

The width of the annular gap in this study is assumed to be equal to the radius of the conductor: $L = R$. In the case of a circular conductor, the magnetic field is axisymmetric, as is the temperature distribution. This means that the condition $\nabla \boldsymbol{H} \times \nabla \boldsymbol{T} = 0$ is fulfilled, which is the condition for the existence of the equilibrium state $\boldsymbol{u} = 0$. In this case, convection occurs when the magnetic Rayleigh number exceeds a certain critical value. A similar problem was studied in Refs. [16], however, comparison with the results of this investigation is rather complicated, since it uses the linear law of magnetization of a magnetic fluid and its magnetic susceptibility is included in the Rayleigh number. However, the qualitative results we obtained for a circular cylinder correspond to the results of [16], with the exception of the hysteresis and non-continuity of the curve $\mathrm{Nu}(\mathrm{Ra_m})$ that we found, which is discussed below.

Since the buoyancy force arising in the non-uniform magnetic field of the conductor is determined by the current value and the radius of the conductor, further consideration of convection in the annulus was performed for parameters simply realized in the experiment: $I = 25$ A, $R = 2$ mm, $L = R$, for magnetic fluid with the abovementioned properties. Steady convection is considered. Two versions of the initial conditions in the iterative process of solving convection equations are investigated. In the first version, the initial values of the stream function and vorticity were set equal to 0, and the exact solution of the heat equation was used for temperature. The obtained dependence of the relative Nusselt number ($\mathrm{Nu}/\mathrm{Nu_0}$, $\mathrm{Nu_0} = 9.0647$) is shown in Fig. 2 by a solid line. It can be seen that this curve is not continuous, which is associated with a change in convective structures with increasing magnetic Rayleigh number.

The convective flow arises above the critical value of the magnetic Rayleigh number $\mathrm{Ra_m} = 77{,}970$ in the form of a structure consisting of 10 convective cells (Fig. 3, shown for $\mathrm{Ra_m} = 112{,}000$). Under zero initial conditions, this structure exists in the range of the magnetic Rayleigh number from 77,970 to 83,500. In the range from 83,500 to 111,500, the convective structure consists of 8 cells, and over 112,000 and up to 144,000 again a structure of 10 cells is realized. Over $\mathrm{Ra_m} = 144{,}000$, there are 12 cells in the annulus.

In the second variant, the solution obtained for the previous value of the magnetic Rayleigh number is used as the initial condition, and then the Rayleigh number changes step by step (either downward or upward from the initial one). In this case, the range of existence of these

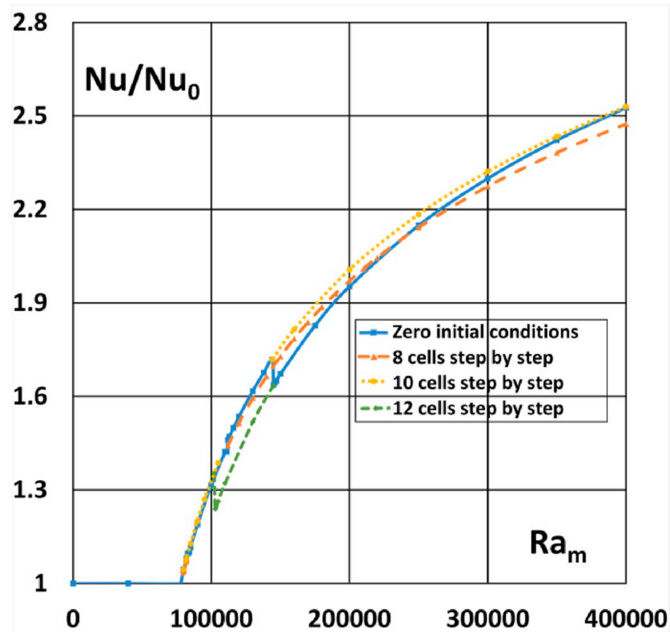

Fig. 2. Nusselt number dependence on magnetic Rayleigh number.

structures expands significantly: a structure of 10 cells exists in the entire studied range, 8 cells are implemented in the entire range of over 80,000, and 12 cells are possible at $\mathrm{Ra_m} > 103{,}000$ (Fig. 2). It should be noted that the Nusselt numbers for different structures do not differ very much, therefore, below, when comparing with convection around an elliptical cylinder, we will only show results obtained under zero initial conditions.

It should be noted that for a magnetic fluid with the above properties ($M_S = 40$ kA/m, $\rho = 1300$ kg/m^3) and $\beta = 10^{-3}$ 1/K, $\eta = 0.015$ Pa s, $\kappa = 7 \cdot 10^{-8}$ m^2/s, $R = 2$ MM, $I = 25$ A we obtain the following value of the magnetic Rayleigh number $\mathrm{Ra_m} = 7650\Delta T$. This means that for such a fluid its critical value is achieved at a temperature difference of 11 K. In Refs. [16] the critical temperature has the same order of magnitude, but the maximum values of the magnetic Rayleigh number for which the calculation was performed correspond to the current in the wire $I \sim 3000$ A, which is difficult to realize in reality. Unfortunately, it is not possible to compare the obtained results with known experimental data, since convection in the vertical gap between the cylinders was mainly studied [22,23]. In a few studies of the horizontal cylindrical gap, either a non-axisymmetric magnetic field is used [24], or insufficient data on the properties of the magnetic fluid and the magnetic field presented [25].

5.1.2. Elliptical current-carrying conductor

If the shape of the conductor is different from the circular, then the situation changes fundamentally. In this case, the isolines $H = \mathrm{const}$ do not coincide with the isolines $T = \mathrm{const}$ (Fig. 4). If the lines $H = \mathrm{const}$ have a shape close to ellipsoids, then the isolines $T = \mathrm{const}$ turn into circles as they approach the outer cylinder, since a constant temperature is set at the boundaries. This means that the equilibrium condition $\nabla H \times \nabla T = 0$ is not satisfied for any values of the magnetic field, i.e. convection occurs at any nonzero values of the magnetic Rayleigh

Table 2
The possible ratio of magnetic and gravitational buoyancy forces depending of conductor radius.

R, mm	0.01	0.05	0.1	0.49	1.13	4.7	11.3
F_m/F_g, $j = j_{max}$	0.90	0.74	0.68	0.54	0.47	0.37	0.31
F_m/F_g, $j = j_{max}/2$	0.11	0.097	0.089	0.074	0.066	0.054	0.047

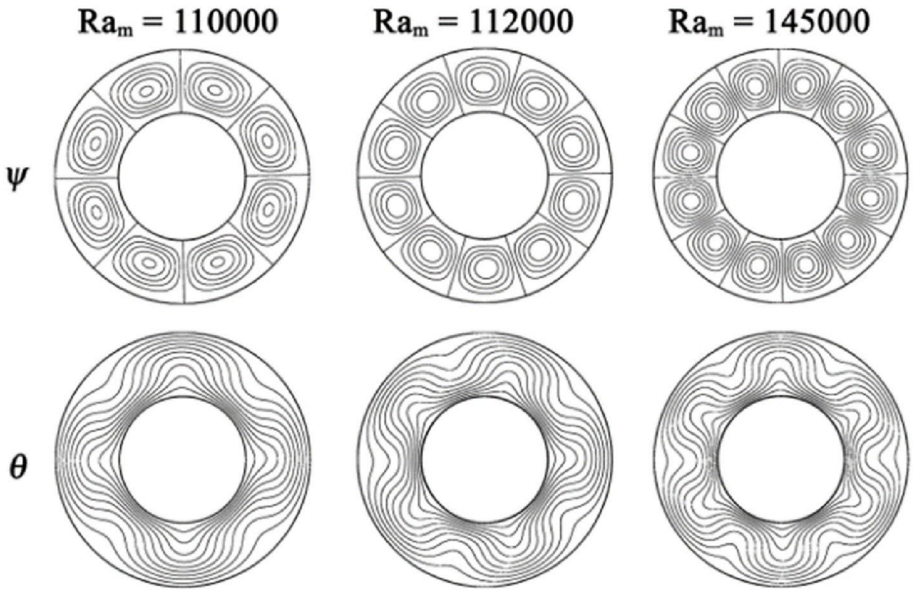

Fig. 3. Streamlines and isotherms for circular inner cylinder. $a = 1$, $L = 1$.

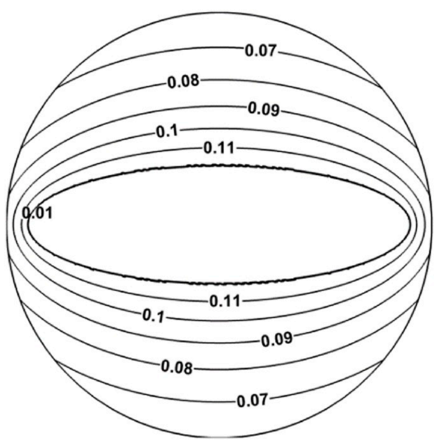

Fig. 4. Isolines of magnetic field strength $H = \text{const.}$ $a = 1.8$, $M_S = 30$ kA/m, $I = 25$ A, $R = 2$ mm.

number. The structure of convective flow in this case depends both on the shape of the inner cylinder and on the magnitude of the magnetic Rayleigh number. The flow structure and temperature distribution in the gap for various values of the magnetic Rayleigh number are shown in Fig. 5 for $a = 1.3$. The flow occurs in the form of four symmetric cells, while the isotherms repeat the shape of the boundaries of the domain and differ little from the isotherms in the heat conduction mode. This mode exists in the range 0 <$\mathrm{Ra_m}$ <70,000. The maximum value of the stream function for $\mathrm{Ra_m} = 50{,}000$ is $|\psi_{\max}| = 0.30$. In the range of 70,000 < $\mathrm{Ra_m}$ <96,000, additional weak convective cells are formed near the top of the ellipse, a weak plume appears, directed from the inner cylinder to the outer one, the structure consists of 12 cells. The maximum value of the stream function for $\mathrm{Ra_m} = 80{,}000$ is $|\psi_{\max}| = 1.94$. At $\mathrm{Ra_m} > 96{,}000$, the convective flow changes: weak cells near the vertices of the ellipse merge, 8 cells remain, while the direction of the flow in the main cells and, accordingly, the direction of the plume change. The maximum value of the stream function for $\mathrm{Ra_m} = 124{,}000$ is $|\psi_{\max}| = 3.85$. In the range of $130{,}000 < \mathrm{Ra_m} < 176{,}000$, additional cells again appear, but now there are 10 cells in the structure, and in the main cells in the wide part of the annulus the rotation is opposite, and therefore the pattern of isotherms is asymmetric with respect to the

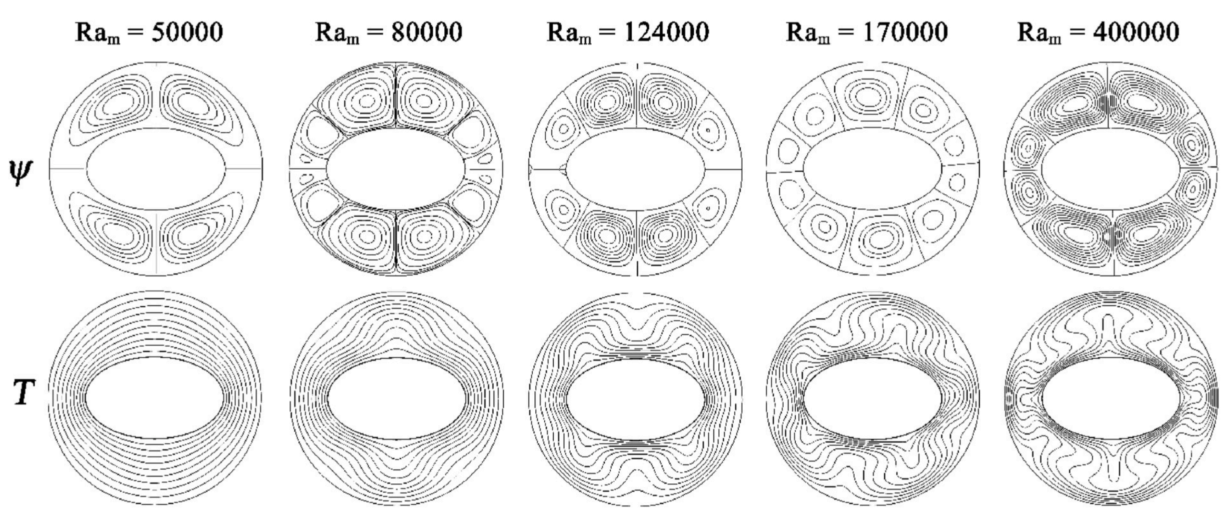

Fig. 5. Streamlines and isotherms for inner elliptic cylinder with semi-axes a = 1.3, L = 1.

vertical axis of the ellipse. The maximum value of the stream function for $\mathrm{Ra_m} = 170{,}000$ is $|\psi_{\max}| = 5.87$. Finally, at $\mathrm{Ra_m} > 176{,}000$ the flow again consists of 8 cells, but their rotation is opposite: in the wide part of the annulus, the plume is directed towards the outer cylinder. The maximum value of the current function for $\mathrm{Ra_m} = 400{,}000$ is $|\psi_{\max}| = 9.84$. Similarly, the structure of the flow changes in the case of $a = 1.6$ (Fig. 6).

The only difference is that even at small $\mathrm{Ra_m}$ values, weak cells are formed near the vertices of the ellipse, which do not disappear even at $\mathrm{Ra_m} = 400{,}000$. For the values of the magnetic Rayleigh number given in Fig. 6, the maximum value of the current function is respectively equal to: $|\psi_{\max}| = 0.71$, 3.6, 6.14, 8.37, 10.70.

In the case $a = 1.8$, the development of the convective flow with increasing $\mathrm{Ra_m}$ is more simple: for small $\mathrm{Ra_m}$ values, 4 symmetric cells are formed, then small weak convective cells appear near the vertex (Fig. 7, $\mathrm{Ra_m} = 210{,}000$), which gradually, with increasing $\mathrm{Ra_m}$, expand cross the annulus (Fig. 7, $\mathrm{Ra_m} = 300{,}000$), and for $\mathrm{Ra_m} > 355{,}000$ the flow is simplified, the number of cells decreases, and the main cells in the wide part of the annulus change the direction of rotation (Fig. 7, $\mathrm{Ra_m} = 360{,}000$). Since the main question of this paper is to study the effect of the shape of the inner cylinder on thermomagnetic convection, then Fig. 8 shows the dependence of $\mathrm{Nu/Nu_0}$ on the magnetic Rayleigh number. Here the Nusselt value in the heat conduction mode for a circular cylinder is used as $\mathrm{Nu_0}$ for all curves. Dependencies are obtained for zero initial conditions. It can be seen that, despite the fact that there is no threshold for convection, at small values of the magnetic Rayleigh number, the Nusselt number remains almost unchanged with increasing $\mathrm{Ra_m}$. Only at $\mathrm{Ra_m} \approx 30{,}000$–$50{,}000$ (this depends on the semi-major axis a) heat transfer from the inner cylinder begin to grow. The larger the semi-axis, the earlier the growth of heat transfer begins. However, with a slight elongation of the ellipse (Fig. 8, $a = 1.3$), the heat transfer at $\mathrm{Ra_m} > 100{,}000$ becomes even worse than from a circular cylinder. With large elongations (Fig. 8, $a = 1.6$ and $a = 1.8$) up to $\mathrm{Ra_m} \approx 250{,}000$, the Nusselt number is higher than in the case of a circular cylinder.

Thus, summing up the study of convective flow around a current-carrying conductor, we can say that the magnetic force generated by the current is weaker than the gravitational one and, as a result, thermomagnetic convection in this case cannot compete with natural gravitational convection. However, it should be noted that this study revealed the fact that a change in the shape of the inner cylinder, which changes the distribution of the magnetic field, can significantly increase the heat transfer from the inner cylinder through the annulus.

5.2. Magnetizable inner cylinder. Convection in magnetic fluid

If the inner cylinder is made of a material that can be magnetized in an external magnetic field and has strong magnetic properties (iron, permalloy, etc.), then the non-uniformity of the magnetic field near it can be much larger than near a current-carrying conductor. Indeed, if a magnetizable circular cylinder with relative magnetic permeability μ is in a transverse magnetic field H_0, then the magnetic field around it is described by the expression [21].

$$H_r = H_0\left(1 + \frac{\mu - 1}{\mu + 1}\frac{R^2}{r^2}\right)\cos\alpha; \quad H_\varphi = H_0\left(1 - \frac{\mu - 1}{\mu + 1}\frac{R^2}{r^2}\right)\sin\alpha$$

where the angle α is counted from the direction of the magnetic field. If the magnetic permeability μ is sufficiently large, then the magnetic field on the surface of the cylinder at the point $\alpha = 0$ doubles, and at $\alpha = \pi/2$ it is 0. Fig. 9 shows the pattern of isolines of the magnetic field strength in the annulus near the circular cylinder for $H_0 = 5$. It can be seen that the magnetic field strength near the upper and lower points of the circular cylinder (external field is vertical) is 10, i.e. twice as much as H_0. In the case of an ellipse elongated along the field, this value can be even larger. Thus, Fig. 10 shows the distribution of the magnetic field strength for an ellipse with $a = 1.8$. In this case, the maximum value of the magnetic field is 21.24. Since the minimum value of the magnetic field on the side surface of the cylinder is 0, a magnetic field gradient is created along the surface of the magnetized cylinder, almost perpendicular to the temperature gradient. It should be expected that such a magnetic field will induce intense thresholdless thermomagnetic convection in a non-isothermal fluid, the intensity of which is higher, the higher the magnitude of the external magnetic field. Since the external magnetic field can be significant (for example, the magnetic field strength of permanent magnets can reach 0.5–$1.5 \cdot 10^6$ A/m), and the field of a magnetized cylinder is more than twice as large at its surface, the magnitude of the magnetic force can be orders of magnitude exceed the value of gravitational volume force ρg. Indeed, the ratio of the magnetic buoyancy force and the gravitational one is $F_m/F_g = \mathrm{Ra}_m f(\widehat{H})\partial\widehat{H}/\partial r/\mathrm{Ra} = \mu_0 M_S^2 f(\widehat{H})\widehat{H}/\rho g R$. If the magnetic field is large enough (more than 200 kA/m), then the magnetic fluid is close to saturation and we can assume that $f(\widehat{H}) = 1$. Assuming an external field strength of 10^6 kA/m, for a fluid with magnetization $M_S = 40$ kA/m we get $\widehat{H} = 2H_0/M_S = 50$ and the ratio F_m/F_g for a fluid with a density $\rho = 1300\ \mathrm{kg/m^3}$ and a cylinder with a radius of 1 cm is 740. Even in the field of a cheap barium-ferrite magnet $H_0 = 160$ kA/m $f(\widehat{H}) = 0.89$, $\widehat{H} = 4$, the ratio of these forces is $F_m/F_g = 55$.

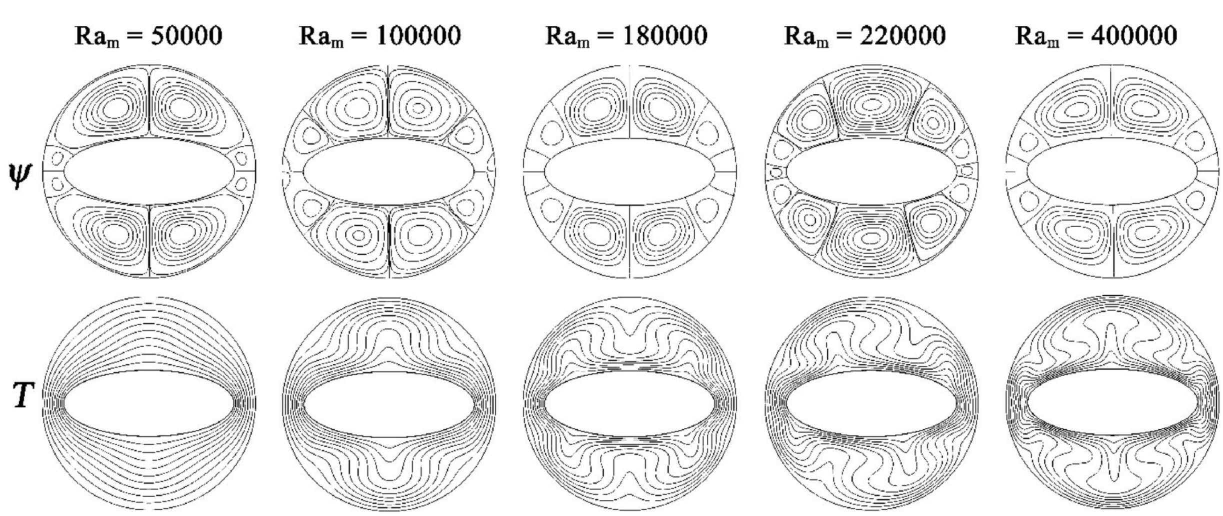

Fig. 6. Streamlines and isotherms for inner elliptic cylinder with semi-axes a = 1.6, $L = 1$.

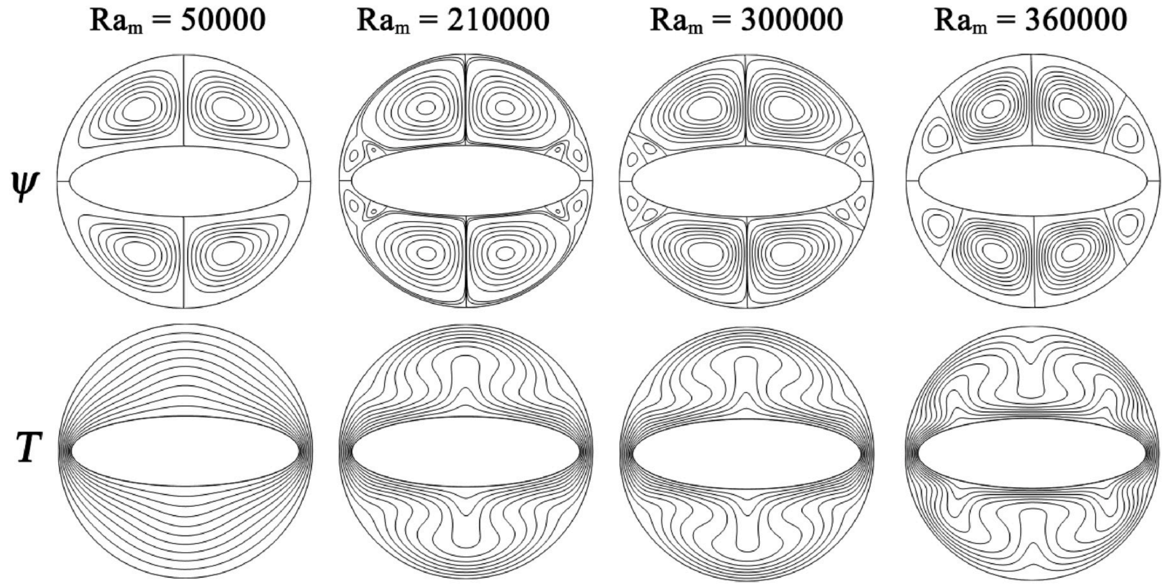

Fig. 7. Streamlines and isotherms for inner elliptic cylinder with semi-axes a = 1.8, $L = 1$.

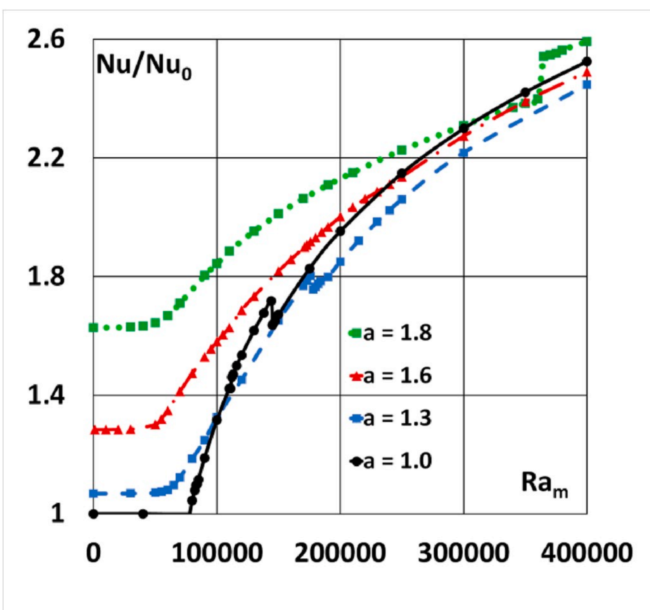

Fig. 8. Nusselt number dependence on Ra$_m$. Pr = 700.

Thus, in the case of a cylinder magnetized in an external field made of a material with high magnetic permeability, thermomagnetic convection can far exceed natural gravitational convection. This means that we can expect significant enhancement of heat transfer from the inner cylinder in a magnetic fluid due to an external magnetic field. Moreover, the degree of intensification should depend on the shape of the cylinder, since the non-uniformity of the magnetic field (i.e. and intensity of thermomagnetic convection) depends on the shape of the cylinder.

To test this hypothesis, the problem posed was investigated numerically. In the absence of a magnetic field, the streamlines of the convective flow in the annulus for a circular cylinder and an elliptical cylinder oriented horizontally and vertically, and isotherms are shown in Figs. 11 and 12. It is seen that with an increase in the Rayleigh number, the center of the convective cell in all cases shifts upward and the maximum value of the current function increases. However, in the case of an elliptical cylinder, the flow structure and heat transfer

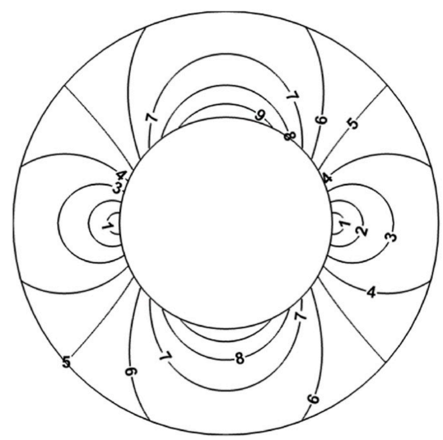

Fig. 9. The pattern of magnetic field strength value distribution. $a = 1$, $\mu = 1000$, $H_0 = 5$, $H_{max} = 10$.

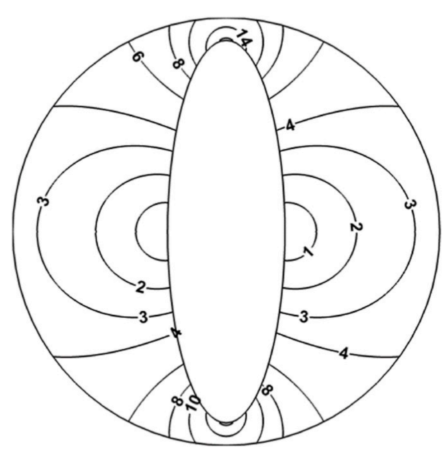

Fig. 10. The pattern of magnetic field strength value distribution. $a = 1.8$, $\mu = 1000$, $H_0 = 5$, $H_{max} = 21.24$.

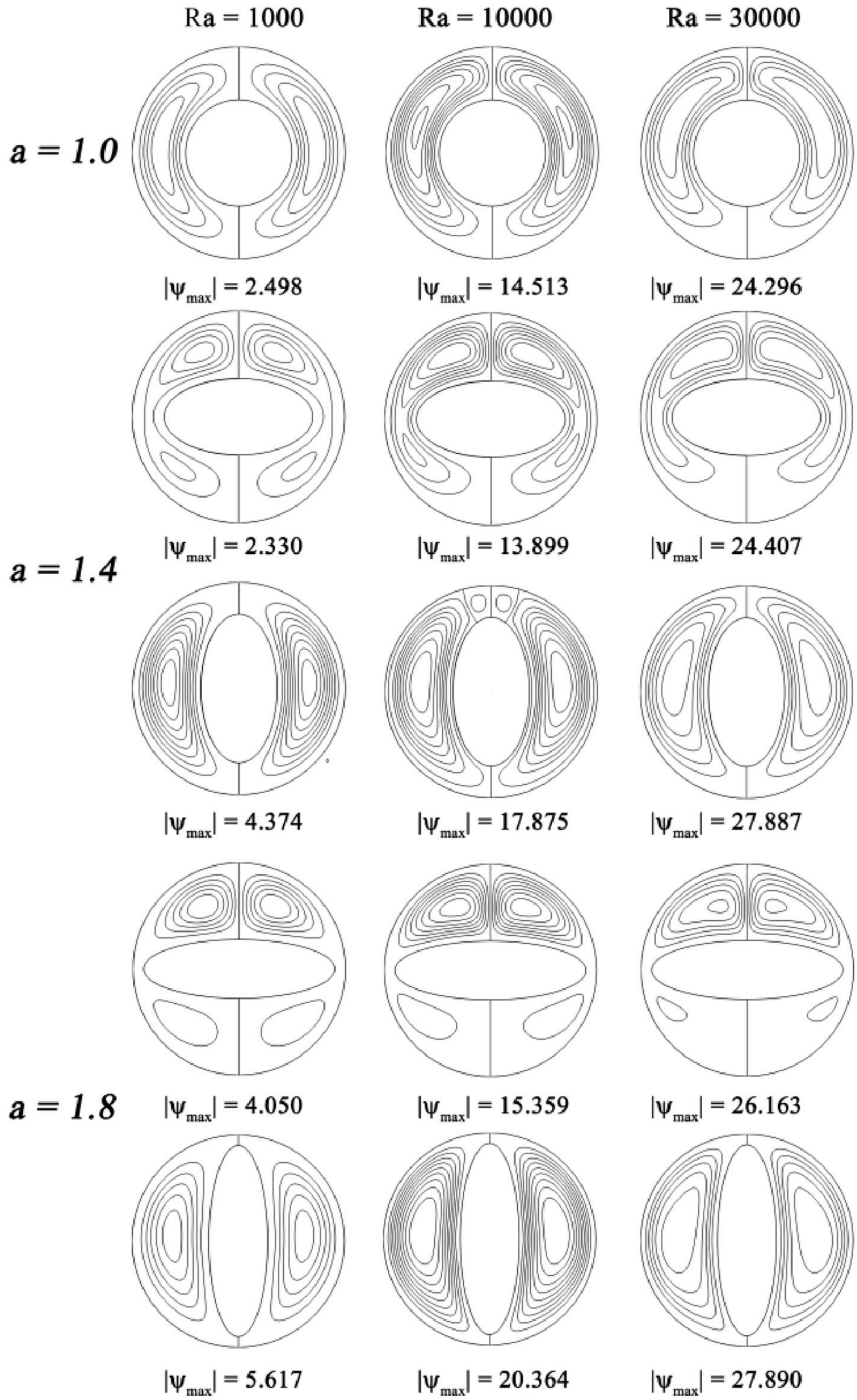

Fig. 11. Typical streamlines for $Ra_m = 0$, $R = 1$, $R_e = 2$ and semi-major axis of the elliptical cylinder $a = 1$, 1.4, 1.8. The elliptical cylinder is oriented horizontally or vertically.

intensity depend on both its elongation and orientation. With moderate elongation ($a = 1.4$) of the cylinder and its horizontal orientation, the flow does not differ very much from the flow around a circular cylinder, but the isotherms in the lower part of the annulus get closer to the inner cylinder. As a result, although the upper plume increases slightly, the Nusselt number, i.e. the heat transfer intensity, with moderate values of the Rayleigh number, even decreases. With the vertical orientation of the elliptical cylinder, the flow is significantly intensified; at Ra = 10,000, two additional cells appear in the upper part of the gap, which leads to the appearance of additional plumes and an increase in the Nusselt number.

With significant elongation ($a = 1.8$), in the case of horizontal orientation of the elliptical cylinder, the flow is almost absent in the lower part of the gap, since it is almost separated from the upper half of the gap and heating from above is realized in it. Therefore, although the intensity of the plume in the upper part of the gap increases with

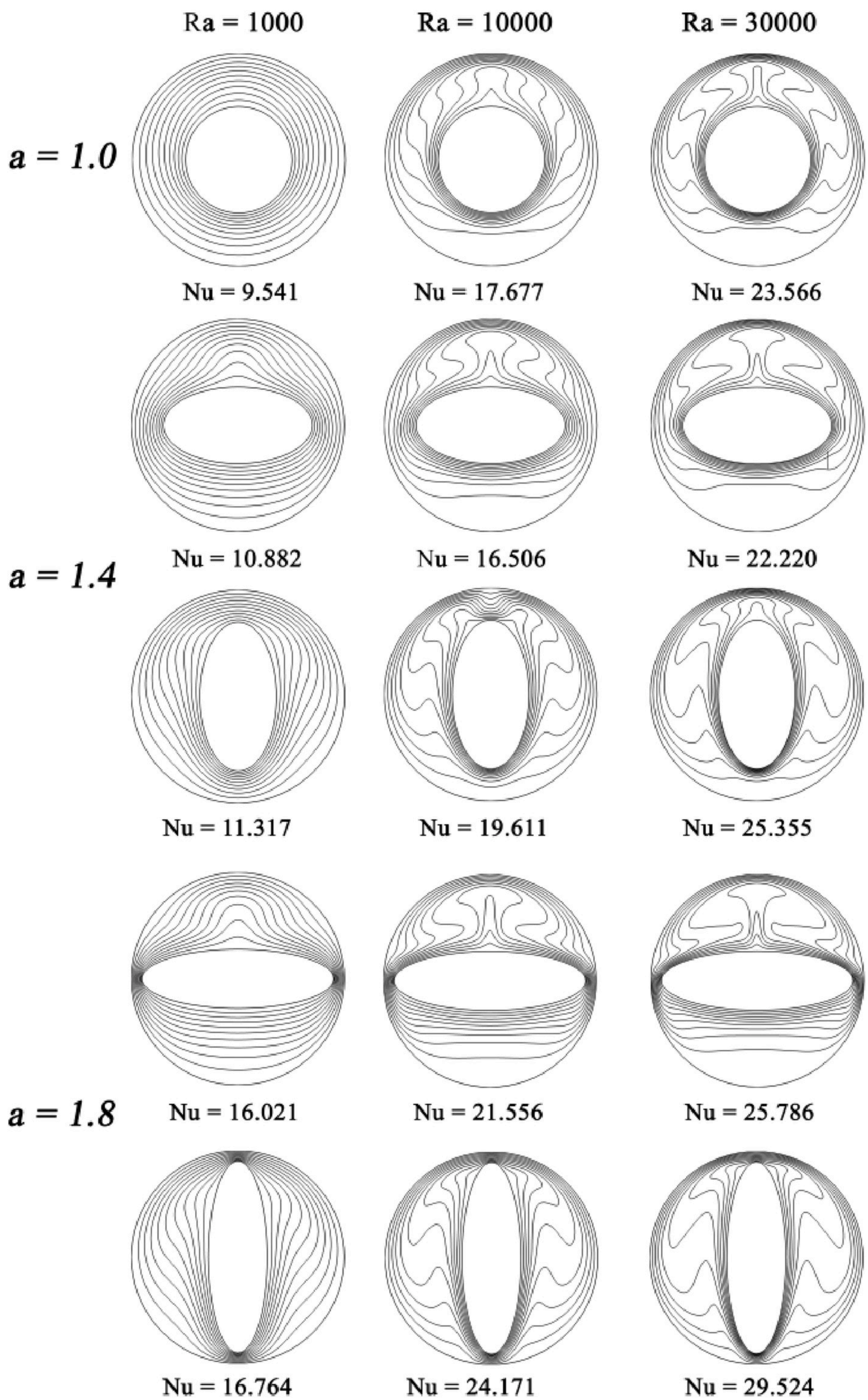

Fig. 12. Typical isotherms for $\mathrm{Ra_m} = 0$, $R = 1$, $R_e = 2$ and semi-major axis of the elliptical cylinder $a = 1$, 1.4, 1.8. The elliptical cylinder is oriented horizontally or vertically.

elongation, the total growth of the Nusselt number is small. With a vertical orientation, the enhancement of heat transfer is much more noticeable.

In total, the described development of the natural convection qualitatively corresponds to the results given in Ref. [8] for the annulus with the ratio $R_e/R = 2.6$. We can say that in the absence of a magnetic field, the influence of the shape of the inner cylinder on the heat transfer rate is noticeable only at small values of the Rayleigh number (Ra $\sim$1000), at moderate values (Ra $> 10^4$) the influence of the shape on the Nusselt number is not very large. Since the horizontal orientation of the elliptical cylinder from the point of view of heat transfer enhancement is not of interest, then we will focus on the cylinder with a vertical orientation. The dependence of the heat transfer intensity on the Rayleigh number for a vertically oriented elliptical cylinder is shown in Fig. 13. For the values of the Rayleigh number Ra $\geq 10^4$, this dependence has the form

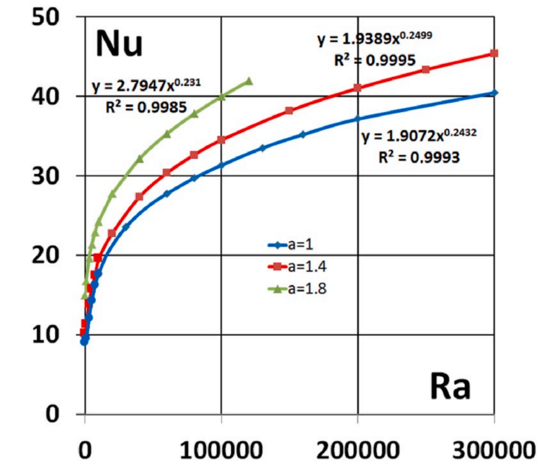

Fig. 13. Nusselt vs Ra for the natural convection around vertically oriented elliptical cylinder. $Ra_m = 0$, $R = 1$, $R_e = 2$, $Pr = 700$, $H = 5$.

$$
\mathrm{Nu} = \begin{cases} 1.91\mathrm{Ra}^{0.24}, & a = 1.0 \\ 1.94\mathrm{Ra}^{0.25}, & a = 1.4 \\ 2.79\mathrm{Ra}^{0.23}, & a = 1.8 \end{cases} \tag{10}
$$

which shows that the shape of the inner cylinder does not significantly affect the intensity of natural convection in the annulus between horizontal cylinders. Note that for $a = 1.8$ a stationary solution symmetric with respect to the vertical axis of the ellipse was obtained only for the values of the Rayleigh number $Ra \leq 1.2 \cdot 10^5$.

In the field of a magnetized cylinder in the absence of gravity, convective flow occurs in a magnetic fluid. Since the magnetic field gradient is directed along the surface of the inner cylinder, and the temperature gradient is directed across the annular gap, they are not parallel. This means that there is no threshold for convection and convection occurs for any non-zero values of the magnetic Rayleigh number. Since the distribution of the magnetic field is symmetric with respect to both the vertical and horizontal axis of the ellipse (Fig. 9), the flow pattern is also symmetrical with respect to them. As can be seen from Fig. 14, the flow at all values of the magnetic Rayleigh number consists of four cells, its intensity increases significantly with increasing magnetic Rayleigh number, but weakly depends on the shape of the inner cylinder. Moreover, at $Ra_m = 10^5$, the maximum value of the stream function even decreases with increasing elongation of the elliptical cylinder. Due to the pronounced symmetry and non-uniformity of the magnetic field created by the magnetizable cylinder, the flow structure practically does not change with increasing cylinder elongation or increasing the Rayleigh magnetic number, only the shape of convective cells changes. In such a situation, it would seem that one does not have to expect a significant change in heat transfer with elongation of the cylinder. However, as can be seen from the examples presented in Fig. 15, with increasing elongation of the ellipse, the Nusselt number increases by about 30%.

The dependences $Nu(Ra_m)$ for several values of the elongation of the inner cylinder are shown in Fig. 16 (for $H = 5$). For the values of the magnetic Rayleigh number $Ra_m > 10^4$, these dependences are, as in the case of natural convection, a power-law character. The exponent in this case is also close to 0.25:

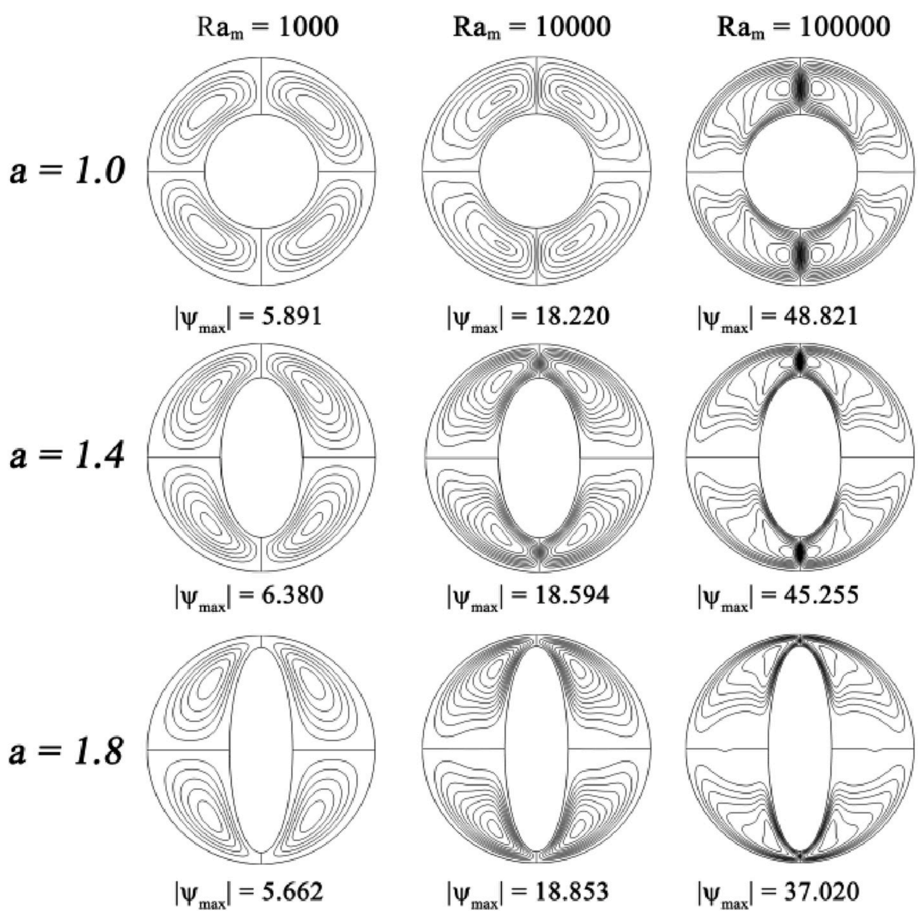

Fig. 14. Typical streamlines for $Ra = 0$, $R = 1$, $R_e = 2$ and semi-axes of the elliptical cylinder $a = 1$, 1.4, 1.8, $H = 5$.

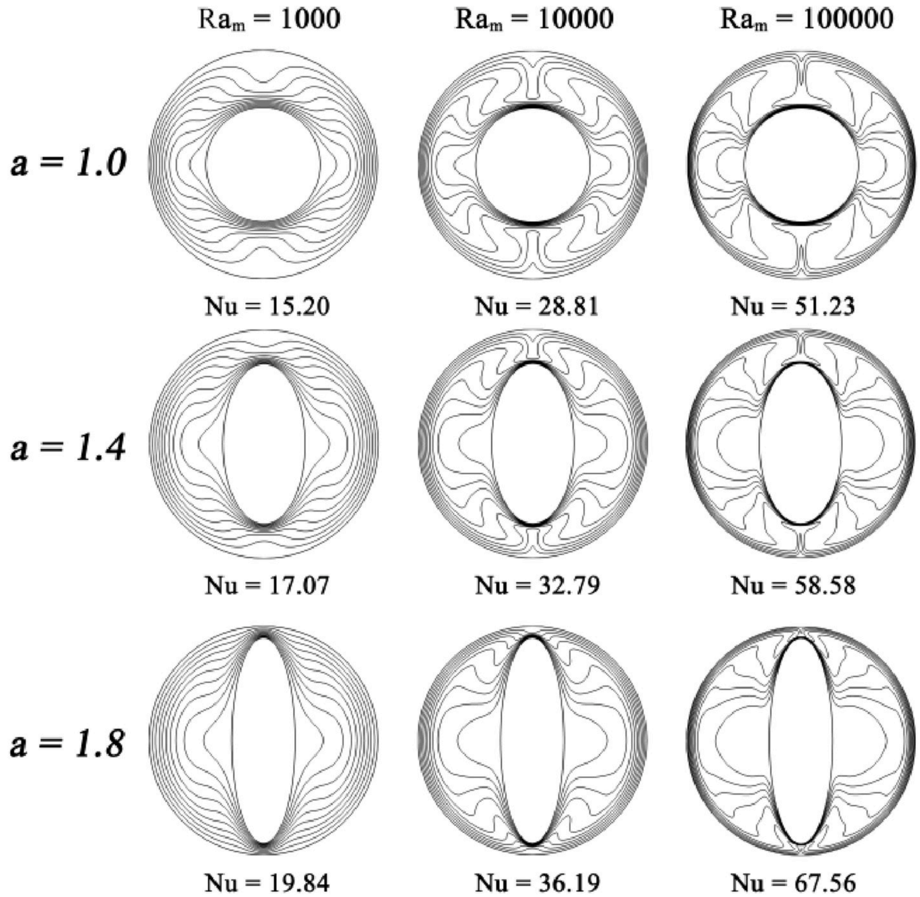

Fig. 15. Typical isotherms for $\mathrm{Ra} = 0$, $R = 1$, $R_e = 2$ and semi-axes of the elliptical cylinder $a = 1$, 1.4, 1.8, $H = 5$.

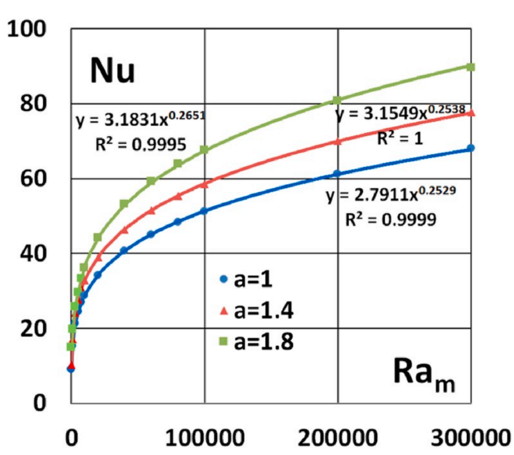

Fig. 16. Heat transfer intensity vs $\mathrm{Ra_m}$ in the case of gravity absence: $\mathrm{Ra} = 0$. $R = 1$, $R_e = 2$, $a = 1.0$, 1.4, 1.8, $H = 5$.

$$\mathrm{Nu} = \begin{cases} 2.79\mathrm{Ra_m^{0.253}}, & a = 1.0 \\ 3.15\mathrm{Ra_m^{0.254}}, & a = 1.4 \\ 3.18\mathrm{Ra_m^{0.265}}, & a = 1.8 \end{cases} \tag{11}$$

It can be seen that although the exponent in expressions (10) and (11) are close, the coefficient before the power dependence (11) is noticeably larger than for gravitational convection. This suggests that, with other characteristics unchanged, with increasing cylinder temperature, thermomagnetic convection will increase much faster than gravitational convection and, ultimately, as we will see later, it will determine the heat transfer intensity.

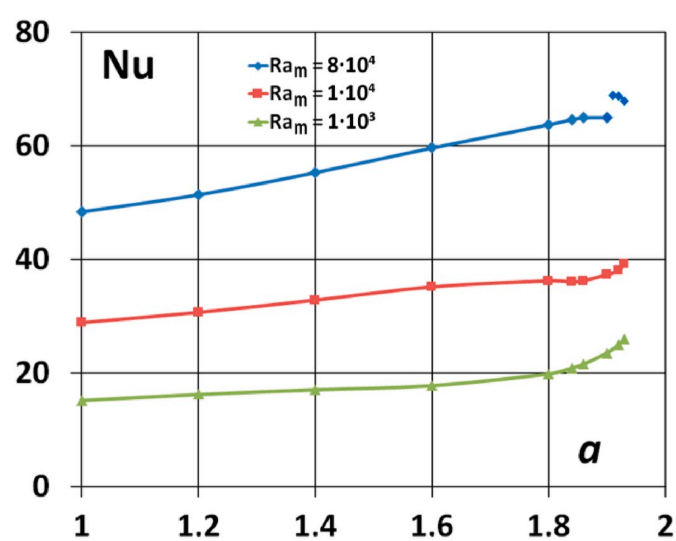

Fig. 17. Heat transfer intensity vs elongation of the ellipse in the case of gravity absence: $\mathrm{Ra} = 0$. $R = 1$, $R_e = 2$, $\mathrm{Ra_m} = 10^3$, 10^4, $8 \cdot 10^4$, $H = 5$.

In explicit form, the influence of the shape of the magnetizable cylinder on the heat transfer intensity is shown in Fig. 17. Up to $a = 1.8$, the Nusselt number increases almost linearly, increasing by about 30% compared with a circular cylinder ($\mathrm{Ra_m} = 10^3$: $a = 1$, $\mathrm{Nu} = 15.2$; $a = 1.8$, $\mathrm{Nu} = 19.8$; $\mathrm{Ra_m} = 10^4$: $a = 1$, $\mathrm{Nu} = 28.8$; $a = 1.8$, $\mathrm{Nu} = 36.2$; $\mathrm{Ra_m} = 8 \cdot 10^4$: $a = 1$, $\mathrm{Nu} = 48.4$; $a = 1.8$, $\mathrm{Nu} = 63.8$). With an elongation greater than 1.8, the Nu(Ra$_m$) dependence behaves differently depending on the magnitude of the magnetic Rayleigh number. At Ra$_m$ = 1000, the Nusselt number increases more rapidly, reaching a value of 25.9 at $a = 1.93$, which is 70% higher than for a circular cylinder. With Ra$_m$ = 10,000, the growth is not so noticeable, but still Nu = 39.2, which is 36% more than for a circular cylinder. And at Ra$_m$ = 80,000, the Nu (Ra$_m$) curve breaks at $a = 1.9$ and the Nusselt number jumps from 65.03 at $a = 1.90$ to 69.0 at $a = 1.91$. This break is due to the fact that with decreasing distance between the vertex of the ellipse and the outer circular cylinder, the flow structure changes. Two additional small vortices arise in a narrow gap near the vertex in which heat transfer was previously carried out mainly due to heat conduction, and the heat transfer increases significantly. Compared with a circular cylinder (Nu = 48.4), the increase in heat transfer is 43%.

It is obvious that the intensity of thermomagnetic convection should depend on the magnitude of the external uniform magnetic field magnetizing the cylinder. This dependence is shown in Fig. 18 for an ellipse with a semi-major axis $a = 1.8$, when the influence of the magnetic field is close to the maximum. For small values of the magnetic field, the magnetization law (2) is linear and the magnetic force is proportional to the square of the magnetic field strength $F_m \sim H^2$. Therefore, the dependence approximating those presented in Fig. 18 data was selected as

$$\mathrm{Nu} = \mathrm{Nu}|_{H=0}\left(1 + \frac{AH^2}{1 + BH^m}\right) \tag{12}$$

where $\mathrm{Nu}|_{H=0} = 14.94$ is the Nusselt number for heat conduction mode, and the values of the coefficients A, B, and m, as well as standard deviation σ for the fitting curve, are presented in Table 3.

We limited ourselves in constructing the dependencies shown in Fig. 18 with a maximum value of $H = 15$, which for a magnetic fluid with magnetization $M_S = 30$ kA/m corresponds to a magnetic field of 450 kA/m. It is easy to create such a field with Nd-Fe-B or Sm–Co permanent magnets. All other simulations in this study were performed for $H = 5$, which corresponds to a magnetic field of 150 kA/m, which can be created using cheap ferrite-barium magnets. Thus, the values of the enhancement of heat transfer due to magnetic fluid given in this paper should be considered not as the maximum possible, but as very moderate. Another factor that can affect the intensity of heat transfer is the angle between the direction of the magnetic field and the semi-major axis of the ellipse. The data presented above related to the case $\alpha = 0$, i.e. an external magnetic field was considered parallel to the semi-major

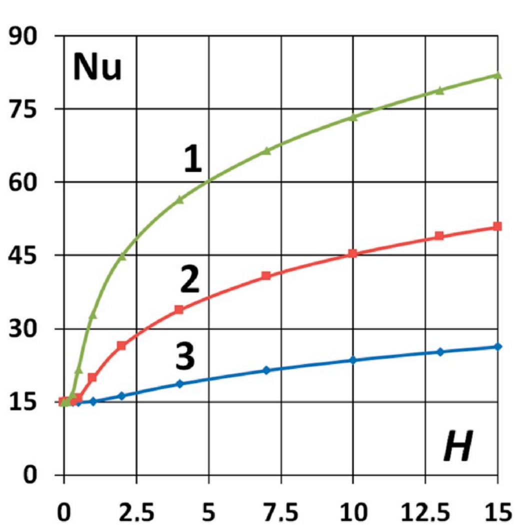

Fig. 18. The dependence of the Nusselt number on the magnetic field. Ra = 0. $R = 1$, $R_e = 2$, $a = 1.8$. 1 – Ra$_m$ = 60,000, 2 – Ra$_m$ = 10,000, 1 – Ra$_m$ = 1000. Markers are the result of the numerical simulation.

Table 3
Coefficients for the expression (12).

	Ra$_m$ = 10^3	Ra$_m$ = 10^4	Ra$_m$ = $6 \cdot 10^4$
A	0.03033	0.4929	2.7801
B	0.1118	0.5356	1.4292
m	1.5775	1.6383	1.6895
σ	0.0051	0.0224	0.0383

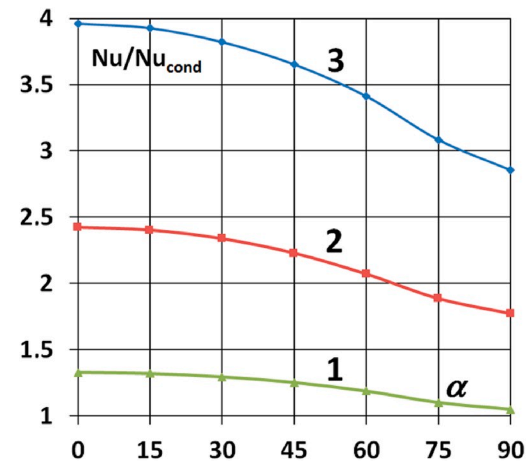

Fig. 19. Dependence of relative intensity of the heat transfer on the orientation of magnetic field. Ra = 0. $R = 1$, $R_e = 2$, $a = 1.8$. 1 – Ra$_m$ = 1000, 2 – Ra$_m$ = 10,000, 1 – Ra$_m$ = 60,000, $H = 5$. Nu$_{cond}$ = 14.94.

axis. As can be seen from Fig. 19, when the direction of the magnetic field deviates from the vertical, the heat transfer intensity decreases and is minimal for the horizontal orientation of the magnetic field. This drop is significant: from 3.96 to 2.86 for Ra$_m$ = $6 \cdot 10^4$, i.e. by 28%. For Ra$_m$ = 10^4, and Ra$_m$ = 10^3, this decrease is 27% and 21%, respectively. Since we are interested in the possibility of enhancing heat transfer due to a magnetic field, all the other data presented in the paper relate to the case of a magnetic field oriented along the semi-major axis of the ellipse.

Of particular interest from the point of view of the possibility of experimental verification of the results obtained is convection in the

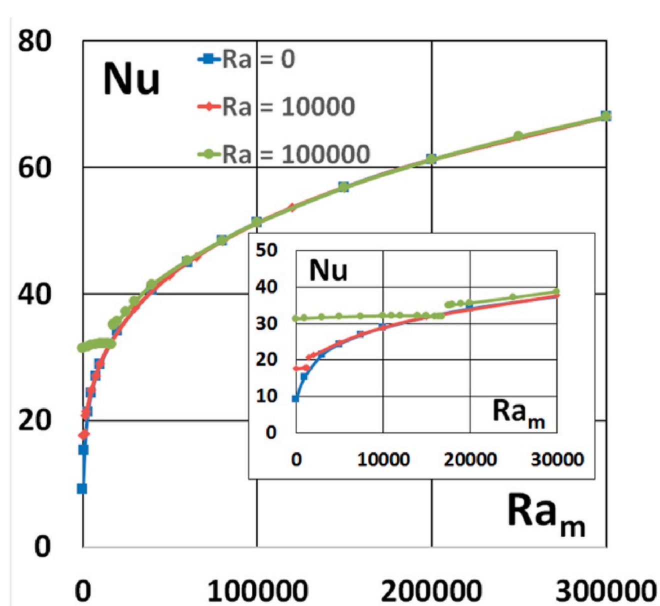

Fig. 20. Dependence of the Nusselt number on the magnetic Rayleigh number for Ra = 0, 10,000, and 100,000. $R = 1$, $R_e = 2$, $a = 1.0$. $H = 5$. α = 0.

presence of both gravity and a magnetic field. The rivalry between these two forces, causing convective flow, is of considerable interest from the practical point of view. Fig. 20 shows the change in the intensity of heat transfer depending on the magnetic Rayleigh number for various values of the gravitational Rayleigh number in the case of a circular inner cylinder. In the absence of gravity (Ra = 0), the Nu(Ra_m) dependence is continuous and monotonically increasing with increasing magnetic Rayleigh number. Moreover, the Nusselt number increases most rapidly at low Ra_m values. If Ra ≠ 0, then the competition between gravitational and magnetic forces leads to the fact that, at low Ra_m values, the heat transfer intensity almost does not increase, and then, when magnetic forces exceed gravitational ones, it increases jump-wise. This jump corresponds to a change in the flow structure (Fig. 21), which passes from a flow consisting of two convective cells, typical of gravitational convection in an annular gap (Fig. 11), to a flow of four cells, typical of convection in a magnetic dipole field (Fig. 14). In this case, instead of one thermal plume, which was in the upper part of the annulus, three plumes arise, two directed from the inner cylinder and one from the external. As a result, the Nusselt number sharply increases with a change in the flow structure. We also note that at $Ra_m > 20{,}000$ the heat transfer is completely determined by magnetic forces, all three curves practically coincide. In the case of a large elongation of the inner cylinder ($a = 1.8$, Fig. 22) in the narrow part of the annulus, the magnetic forces are very large and even for small values of the magnetic Rayleigh number ($Ra_m \sim 1000$) additional convective cells arise, the intensity of which increases with increasing Ra_m and the curve Nu(Ra_m) is continuous. It is of interest to compare the rivalry of magnetic and gravitational forces for real magnetic fluids. In an experimental study [17], a magnetic fluid with the following properties was used: density $\rho = 1370$ kg/m^3, dynamic viscosity $\eta = 0.069$ Pa s, thermal diffusivity $\kappa = 6.9 \cdot 10^{-8}$ m^2/s, magnetization saturation $M_S = 44.9$ kA/m. For this fluid, which is in the annulus 1 cm wide (radius of the inner circular cylinder 1 cm), the Rayleigh number is $Ra = 1.76 \cdot 10^3 \Delta T$ and for a temperature difference between the cylinders 23 K $Ra \approx 40{,}000$. In the absence of a magnetic field, the Nusselt number is $Nu = 1.908 Ra^{0.243} \approx 25.1$. The magnetic Rayleigh number in this case is equal to $Ra_m = 1.48 \cdot 10^4 \Delta T \approx 340{,}000$.

The Nusselt number in this case is $Nu = 2.963 Ra_m^{0.248} \approx 69.7$. Thus, in the described situation, when using cheap ferrite-barium magnets (magnetic field strength is 150 kA/m, $H = 5$), an increase in the intensity of heat transfer by almost three times is possible. Note that in a strong magnetic field ($\sim 1.5 \cdot 10^6$ kA/m), the factor $f(H)H$ will increase by 10 times and the heat transfer gain will also increase.

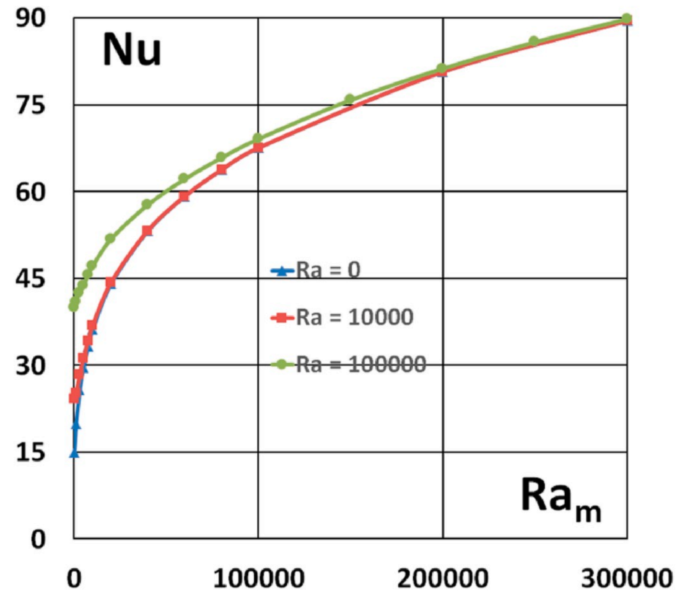

Fig. 22. Dependence of the Nusselt number on the magnetic Rayleigh number for Ra = 0, 10,000, and 100,000. Ra = 0. $R = 1$, $R_e = 2$, $a = 1.8$. $H = 5$. $\alpha = 0$.

For an elliptical cylinder with a semi-major axis length $a = 1.8$ without a magnetic field, $Nu = 2.796 Ra^{0.231} \approx 32.1$, and in a vertical magnetic field ($H = 5$) $Nu = 5.434 Ra_m^{0.222} \approx 91.5$. Thus, the intensity of heat transfer in the annular gap between two cylinders by thermomagnetic convection in this case is almost 3 times greater than by gravitational one.

It should be noted that $Ra \sim L^3$, and $Ra_m \sim L^2$, i.e. the ratio of magnetic to gravitational forces is inversely proportional to the gap width: $F_m/F_g \sim 1/L$. This means that with a decrease in the width of the gap, the efficiency of thermomagnetic convection in comparison with gravitational convection will increase, and vice versa.

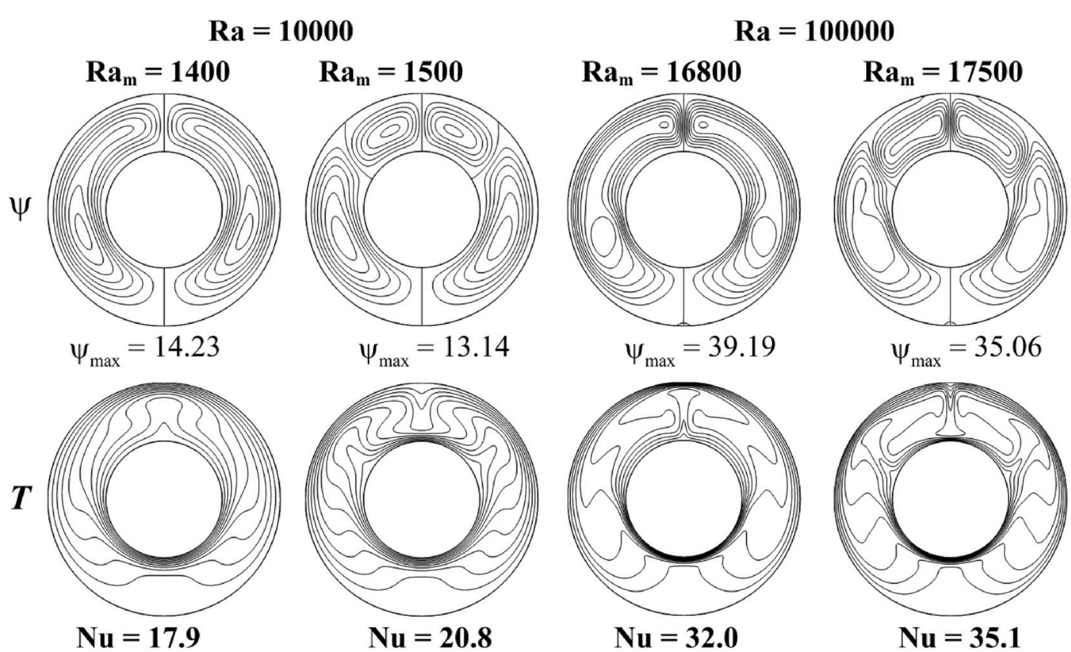

Fig. 21. Streamlines and isotherms near the gap in the curve Nu(Ra_m). $a = 1$, $H = 5$, $L = 1$.

5.3. Magnetizable inner cylinder. Convection in oxygen

From a practical point of view, it is of interest to evaluate the possibility of enhancing convective heat transfer using a magnetic field not only in such an artificial medium as a magnetic fluid, but also in natural fluids. One of the strongest paramagnets is gaseous oxygen. Convection in oxygen and in air was studied in a number of works [18,19], where it was shown that it can exceed gravitational convection in intensity. Since the oxygen magnetization is linearly related to the magnetic field strength ($M = \chi H$) and there is no saturation magnetization in practically achievable magnetic fields, in this case, as a reference value for the magnetic field, it is necessary to use not the saturation magnetization, as before, but the external magnetic field strength H_0. Then the equation of motion (7) takes the form

$$\frac{\partial\omega}{\partial t} + \frac{1}{\Pr}(\vec{u}\cdot\nabla)\omega = \Delta\omega + \mathrm{Ra}\frac{\partial\theta}{\partial x} + \mathrm{Ra}_m\left[\left(\frac{\partial H}{\partial x}\right)\left(\frac{\partial\theta}{\partial y}\right) - \left(\frac{\partial H}{\partial y}\right)\left(\frac{\partial\theta}{\partial x}\right)\right]$$

where the external magnetic field H_0 is used as the reference value for the magnetic field, and the magnetic Rayleigh number is

$$\mathrm{Ra}_m = \frac{\mu_0\beta\chi H_0^2\Delta TL^2}{\rho\nu\kappa}$$

It is interesting to estimate the value of the numbers Ra and $\mathrm{Ra_m}$ achievable in real technical devices. Since the physical properties of oxygen are known (presented in Table 4 for a temperature of 300 K), a change in the values of these numbers is possible only by changing the annular gap width L, the temperature difference ΔT, and the magnitude of the magnetic field H_0.

Then $\mathrm{Ra} = 7.15\cdot10^7L^3\Delta T$, $\mathrm{Ra_m} = 1.59\cdot10^{-5}H_0^2L^2\Delta T$. For an annular gap of 2 mm wide, the values of these parameters are $\mathrm{Ra} = 0.745\Delta T$ and $\mathrm{Ra_m} = 6.37\cdot10^{-11}H_0^2\Delta T$. If the magnetic field is $H_0 = 3\cdot10^5$ A/m, then $\mathrm{Ra_m} = 5.73\Delta T$, and in the case of a magnetic field $H_0 = 10^6$ A/m, $\mathrm{Ra_m} = 63.7\Delta T$. It can be seen that the magnetic Rayleigh number significantly exceeds the gravitational Rayleigh number even with a small magnetic field strength, but the absolute values of these numbers are small. With a temperature difference at the boundaries of the gap $\Delta T = 40$ K $\mathrm{Ra} = 30$, $\mathrm{Ra_m} = 230$ (for $H_0 = 3\cdot10^5$ A/m) and 2550 (for $H_0 = 10^6$ A/m). For such values of the Rayleigh numbers, the contribution of convection to heat transfer is not the main one, and although thermomagnetic convection is much stronger than gravitational, both of these methods of heat transfer are small compared to thermal conductivity. Indeed, in the numerical simulation of convection for an ellipse with a semi-major axis $a = 1.8$ with parameter values $\mathrm{Ra} = 30$, $\mathrm{Ra_m} = 2500$, $\Pr = 0.717$, the Nusselt number $\mathrm{Nu} = 17.68$ was obtained, which is not very different from the Nusselt number for the thermal conductivity $\mathrm{Nu_{cond}} = 14.95$.

With an increase in the gap width, both Rayleigh numbers also increase. For L = 2 cM we obtain $\mathrm{Ra} = 745\Delta T$, $\mathrm{Ra_m} = 573\Delta T$ (for $H_0 = 3\cdot10^5$ A/m), and $6370\Delta T$ (for $H_0 = 10^6$ A/m). Although the magnetic Rayleigh number grows more slowly with increasing annular gap width than the gravitational one, at high magnetic fields it remains much larger than the gravitational Rayleigh number and, at reasonable temperature differences, can significantly affect heat transfer in an annular gap filled with oxygen or air (for which magnetic susceptibility is 5 times less than that of oxygen). Fig. 23 shows the dependence of the Nusselt number on the temperature difference between the boundaries of the annular gap in a magnetic field $H_0 = 10^6$ A/m. As the temperature difference increases, both the gravitational and magnetic Rayleigh numbers increase. Combined convection leads to an increase in the Nusselt number from 14.95 in the heat conduction mode to 59, i.e. about four times. The temperature dependence for $\Delta T > 8$ K is well described by the dependence $\mathrm{Nu} = 20.56\,(\Delta T)^{0.2866}$. As can be seen from Fig. 24, in the absence of a magnetic field $\mathrm{Nu} = 29.7$ and for $H > 1.5\cdot10^5$ A/m, the Nusselt number begins to increase noticeably. For $H = 10^6$ A/m, the heat transfer intensity in gaseous oxygen doubles in the studied geometry.

Table 4
Physical properties of the oxygen at 300 K, normal pressure.

Density at 300 K (ρ)	1.301 kg/m3
Kinematic viscosity (ν)	$1.588\cdot10^{-5}$ m²/s
Viscosity (μ)	$2.065\cdot10^{-5}$ Pa s
Thermal conductivity (λ)	$2.65\cdot10^{-2}$ W/(m·K)
Thermal diffusivity (κ)	$2.214\cdot10^{-5}$ m²/s
Volumetric magnetic susceptibility (χ)	$2.261\cdot10^{-6}$
Volumetric thermal expansion coefficient (β)	1/300.0 (1/K)
Prandtl number (Pr)	0.717

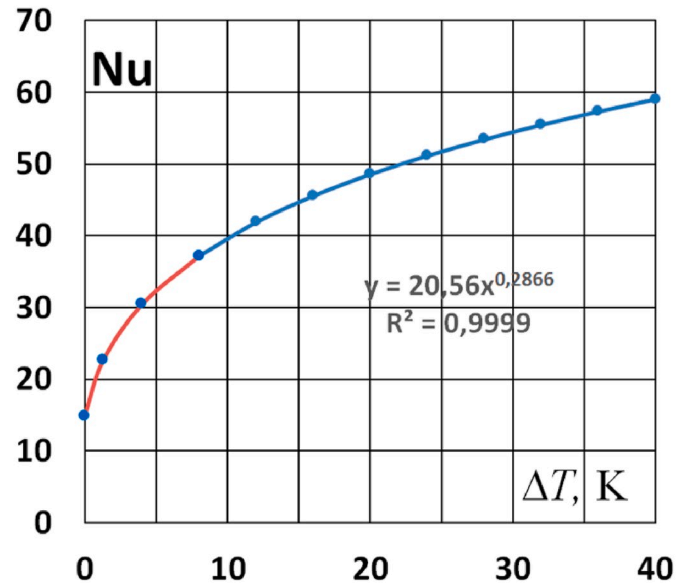

Fig. 23. Dependence of the heat transfer on the temperature difference for oxygen. $R = L = 2$ cm, $a = 1.8$, $H_0 = 10^6$ A/m, $\Pr = 0.717$, markers are the results of numerical simulation; the formula describes the blue part of the fitting line.

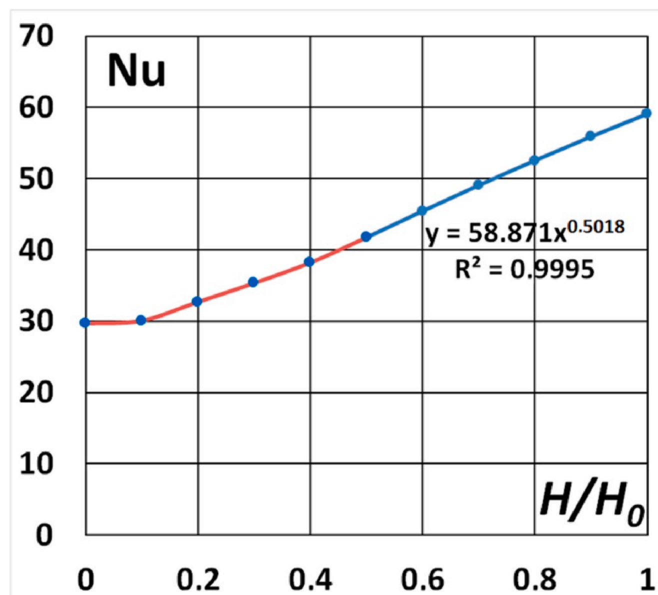

Fig. 24. Dependence of the heat transfer on the magnetic field for oxygen. $R = L = 2$ cm, $\Delta T = 40$ K ($\mathrm{Ra} = 31{,}000$), $a = 1.8$, $H_0 = 10^6$ A/m, $\Pr = 0.717$, markers are the results of numerical simulation; the formula describes the blue part of the fitting line.

Note that, as can be seen from Fig. 24, $\mathrm{Nu} \sim H^{0.5}$. Indeed, from the data presented in Fig. 23 it follows that $\mathrm{Nu} \sim \mathrm{Ra}_m^{0.28}$, but since $\mathrm{Ra}_m \sim H^2$, it is natural that $\mathrm{Nu} \sim H^{0.5}$.

The magnetic susceptibility of air is 5 times less than that of oxygen. This means that the corresponding magnetic Rayleigh number will also be 5 times less and the Nusselt number will be $5^{0.2866} \approx 1.6$ times less than for oxygen. That is, if for oxygen the Nusselt number will increase by 2 times, then for air this growth will be 25%, ceteris paribus.

6. Conclusions

This paper studies convection in a horizontal annular gap filled with a magnetizing medium under the influence of gravitational and magnetic forces. Two magnetizing media were considered: an artificial magnetic fluid, which is a colloidal solution of nanoparticles stabilized by a surfactant, and gaseous oxygen. A current-carrying conductor and an external uniform magnetic field magnetizing an internal cylinder of a material with high magnetic permeability were considered as a source of a non-uniform magnetic field. The influence of the shape of the inner cylinder and the magnitude of the external magnetic field on the intensity of heat transfer was studied.

It is found that the strength of the magnetic field created by a current-carrying conductor is insufficient to compete with gravitational convection. In the absence of gravity, a change in the shape of the conductor significantly changes the nature of thermomagnetic convection: in the case of an elliptical conductor, there is no threshold value of the magnetic field above which convection occurs. A variety of convective structures and the hysteretic nature of the transition between them were discovered. The influence of the shape of the conductor on heat transfer intensity (in the range of 30%) turned out to be significant only in a small range of the magnetic Rayleigh number.

In the case of a magnetizable inner cylinder, it is possible to create a magnetic field with a high gradient. In the case of magnetic fluid, this can ensure the excess of magnetic forces over gravitational by hundreds of times. Heat transfer in an external field with a strength of 150 kA/m from a circular magnetized cylinder can be increased 3–4 times, and in a field of 10^6 A/m 4–6 times compared with gravitational convection. Heat transfer can be further enhanced by 40–50% by choosing the shape of the inner cylinder.

The magnetic field can increase heat transfer through an annular gap filled with gaseous oxygen, 2 times.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix

A1. Numerical method

The problem was solved numerically using the control volume method on a triangular grid. Linear interpolation functions are used for the scalar potential of the magnetic field and the stream function

$$F = A_F x + B_F y + C_F, \quad \psi = A_\psi x + B_\psi y + C_\psi,$$

and exponential functions [13] for the temperature and the vorticity are used:

$$\omega = A_\omega \exp\left[\frac{U_{\mathrm{el}}}{\mathrm{Pr}}(X - X_{\max})\right] + B_\omega Y + C_\omega$$

$$T = A_T \exp[U_{\mathrm{el}}(X - X_{\max})] + B_T Y + C_T$$

where the coefficients A, B, C are calculated from the values of the functions at the nodes of the elements of the triangle, the local coordinate system X, Y is oriented on each element so that the X axis is directed along the local velocity $\boldsymbol{U}_{el}$ within this element.

The vorticity at the solid boundary of the cavity is calculated taking into account the curvature of the boundary by the method proposed in Ref. [14]. Details of the numerical method can also be found in Ref. [14].

A2. Grid-independence of the code and its validation

A2.1. Grid-independence study

The grid-independence of numerical results is studied in the case of $\mathrm{Ra}_m = 0$, $\mathrm{Ra} = 20{,}000$. For the circular cylinder the gap of the annulus is $L = R$ ($\varepsilon = L/2R = 0.5$), and for vertically oriented elliptical cylinder L is the same and the semi-major axis is $a/L = 1.8$. Since transformer oil is usually used as a heat carrier, the value of the Prandtl number $\mathrm{Pr} = 700$ is used in the simulations for magnetic fluid based on it. The numerical results obtained on several grids are presented in Table A1.

Table A1
Grid-independence study for the case $\mathrm{Ra}_m = 0$, $\mathrm{Ra} = 20{,}000$, $\mathrm{Pr} = 700$.

Circular cylinder				Elliptic cylinder, $a = 1.8$			
Mesh size	$\psi_{\max}$	Nu_0	Nu	Mesh size	$\psi_{\max}$	Nu_0	Nu
31 × 161	20.9105	8.9348	21.4382	41 × 201	26.7389	15.2974	29.0583
71 × 281	20.6369	9.0009	21.3523	81 × 401	25.7872	15.1289	27.5950
106 × 421	20.5243	9.0219	21.2866	121 × 601	25.8376	14.9483	27.7046
141 × 661	20.4521	9.0323	21.2300	161 × 801	25.4203	14.9377	27.1772
				201 × 1001	25.3725	14.9371	27.0746

The first number in the Mesh size column indicates the number of nodes across the annulus, and the second the number of nodes along the cylinder surface. The exact value of the Nusselt number in the thermal conductivity mode for $\varepsilon = 0.5$ is $Nu_0 = 9.0647$. We can conclude that the 106×421 and 161×801 grids generate fairly accurate numerical results, so most of our simulations were performed on these grids. Grid-independent solution depends on the complexity of the flow and temperature fields. In order to obtain correct numerical results, different mesh sizes are used in a particular cases, depending of the values of Ra, ε, and a.

A2.2. Validation of present code

The numerical results generated by present code are compared with those available in the literature at different cases. Comparison of present results for steady convection in the annulus between circular cylinders with results of [3,4] for the case $R_e/R = 2.6$ ($\varepsilon = 0.8$) and $Pr = 0.71$ is presented in Table A2. The results of present simulation are in good agreement with known results of [3,4]. It can be seen from Table A3 that the present results generally also agree well with results of [4,5] for the case of $R_e/R = 2.36$ ($\varepsilon = 0.68$) and $Pr = 0.71$.

Table A2
Comparison of heat transfer intensity, $\varepsilon = 0.8$, $Pr = 0.71$, $Ra_m = 0$.

	[3]	[4]	Present
Ra	Nu/Nu_0	Nu/Nu_0	Nu/Nu_0
1000	1.081	1.0822	1.0800
3000	1.404	1.3973	1.4054
6000	1.736	1.7161	1.7298
10000	2.010	1.9801	1.9990
20000	2.405	2.3763	2.4046
30000	2.661	2.6283	2.6618
50000	3.024	2.9647	3.0070
70000	3.308	3.1870	3.2520

Table A3
Comparison of numerical results, $\varepsilon = 0.68$, $Pr = 0.71$, $Ra_m = 0$.

	[5]	[4]	Present
Ra	Nu/Nu_0	Nu/Nu_0	Nu/Nu_0
5300	2.422	2.5560	2.578
45900	4.449	4.5355	4.609
82700	5.140	5.2004	5.300

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