Indian Institute of Information Technology Ranchi
Jharkhand-834010

Department of Mathematics

Assignment-III

Subject Name: Mathematics-I (Calculus and Differential Equations) **Subject Code:** MA1001

Instructions: Solve all the questions systematically and submit by **5:00PM** on **9th Feb 2023**.

1. Find the velocity and acceleration of a moving particle whose position is given by $x = 4t - \frac{t^2}{2}$, $y = 3 + 6t - \frac{t^3}{6}$ at $t = 2$.
2. Find the constants *m* and *n* such that the surface $mx^2 - 2nyz = (m+4)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point (1, -1, 2).
3. Find the directional derivative of the surface $\emptyset(x, y, z) = 4e^{2x-y+z}$ at (1, 1, -1) in the direction towards the point (-3, 5, 6).
4. Show that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$, where $r = |\vec{r}|$.
5. If $r = |\vec{r}|$, evaluate (i) Div $\left(\frac{\vec{r}}{r^2}\right)$ (ii) Div (Grad r^n) (iii) Curl $\left(\frac{\vec{r}}{r^3}\right)$
6. The temperature of the points in space is given by $T(x, y, z) = x^2 + y^2 - z$. A mosquito located at (1, 1, 2) desires to fly in such a direction that it will get warm as soon as possible. In what direction should it move?
7. Find the directional derivative of the scalar point function $\emptyset(x, y, z) = xyz$ in the direction of the outer normal to the surface $z = xy$ at (3, 1, 3).
8. If $\rho,\ \emptyset,\ z$ are the cylindrical coordinates, show that grad $(\log \rho)$ and grad $(\emptyset)$ are solenoidal vectors.
9. If $\vec{a}$ is a constant vector, show that $\vec{a} \times (\nabla \times \vec{r}) = \nabla(\vec{a}.\vec{r}) - (\vec{a}.\nabla)\vec{r}$.
10. Determine the constants *a*, *b*, and *c* so that $\vec{F} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is an irrotational vector. Hence find the scalar potential $\emptyset$ such that $\vec{F} = \nabla\emptyset$.
11. Find the work done when a force $\vec{F} = (x^2 - y^2 + x)\hat{\imath} - (2xy + y)\hat{\jmath}$ moves a particle from origin to (1,1) along a parabola $y^2 = x$.
12. Show that $\int_c (2xy + 3)dx + (x^2 - 4z)dy - 4ydz$, where *c* is any path joining (0, 0, 0) to (1, -1, 3) does not depend on the path *c*. Hence, evaluate the line integral.
13. Evaluate $\iint_s \vec{F}.\hat{n}\ ds$, where $\vec{F} = 2xy\hat{\imath} - yz\hat{\jmath} + x^2\hat{k}$ over the surface *s* of the cube bounded by the coordinate planes and planes $x = a, y = a, z = a$.
14. Verify Green's theorem to evaluate $\int_c 2y^2dx + 3xdy$, where *c* is the boundary of the closed region bounded by $y = x$ and $y = x^2$.
15. Apply Green's theorem to evaluate the area enclosed by the curves $y = x, y = \frac{x}{4}$ and $y = \frac{4}{x}$ in the first quadrant.
16. If $\vec{F} = (y^2 + z^2 - x^2)\hat{\imath} + (z^2 + x^2 - y^2)\hat{\jmath} + (x^2 + y^2 - z^2)\hat{k}$, evaluate $\iint_s (\nabla \times \vec{F}).\hat{n}\ ds$ integrated over the portion of the surface $x^2 + y^2 - 2ax + az = 0$ above the plane $z = 0$. Hence, verify Stoke's theorem.

17. Evaluate $\int_c y^2dx + z^2dy + x^2dz$, where c is the triangular closed path joining the points *(0, 0, 0)*, *(0, a, 0)* and *(0, 0, a)* using the Stoke's theorem.
18. Verify the divergence theorem for $\vec{F} = (x + y^2)\hat{\imath} - 2x\hat{\jmath} + 2yz\hat{k}$ and the volume of tetrahedron bounded by the coordinate planes and the plane $2x + y + 2z = 6$.
19. Evaluate $\iint_s \frac{ds}{\sqrt{a^2x^2+b^2y^2+c^2z^2}}$ over the closed surface s of the ellipsoid $ax^2 + by^2 + cz^2 = 1$ using divergence theorem.
20. Evaluate $\iint_s (y^2z^2\hat{\imath} + z^2x^2\hat{\jmath} + x^2y^2\hat{k}) \,.\, \overrightarrow{ds}$, where s is the upper part of the sphere $x^2 + y^2 + z^2 = 1$ above *x*-*y* plane.

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