Essentials of Computer Systems - Exercises #1

1 Positional Number Systems

Exercise 1.1 *Convert into hexadecimal notation:*

(i.) 1581_{10}

(ii.) 1948_{10}

(iii.) 1453_{10}

(iv.) 1811_{10}

(v.) 1883_{10}

(vi.) 2265_{10}

(vii.) 2373_{10}

(viii.) 2122_{10}

(ix.) 2179_{10}

(x.) 2381_{10}

Exercise 1.2 *Convert into binary notation:*

(i.) 170_{10}

(ii.) 128_{10}

(iii.) 384_{10}

(iv.) 81_{10}

(v.) 63_{10}

(vi.) 225_{10}

(vii.) 74_{10}

(viii.) 16_{10}

(ix.) 115_{10}

(x.) 12_{10}

Exercise 1.3 *Convert into the given radix:*

(i.) 72_{10} *to* $r = 3$

(ii.) 214_{10} *to* $r = 5$

(iii.) 351_{10} *to* $r = 5$

(iv.) 54_{10} *to* $r = 3$

(v.) 14_{10} *to* $r = 2$

(vi.) 92_{10} *to* $r = 8$

(vii.) 72_{10} *to* $r = 8$

(viii.) 15_{10} *to* $r = 7$

(ix.) 116_{10} *to* $r = 16$

(x.) 164_{10} *to* $r = 11$

Exercise 1.4 *Convert into decimal notation:*

(i.) 01010101_2

(ii.) 101011_2

(iii.) 1101101_2

(iv.) $CAFE_{16}$

(v.) $2D3_{16}$

(vi.) 201_3

(vii.) 231_6

(viii.) 414_5

(ix.) 315_8

(x.) 76_9

2 Boolean Algebra

Exercise 2.1 *Fill out the truth tables and draw the circuit diagrams below each table. Available gates: NOT, 2-input OR, 2-input AND gate. Derive the delay and the area of each circuit given the following assumptions:*

$A_{\texttt{NOT}}$*=1 GE,* $A_{\texttt{AND}}$*=2 GE,* $A_{\texttt{OR}}$*=3 GE*

$t_{\texttt{NOT}}$*=0.5 ns,* $t_{\texttt{AND}}$*=0.7 ns,* $t_{\texttt{OR}}$*=0.7 ns*

x	y	$\overline{x}(x+y)$
0	0	
0	1	
1	0	
1	1	

x	y	$xy + x\overline{x}$
0	0	
0	1	
1	0	
1	1	

x	y	$xy(\overline{x}+y)$
0	0	
0	1	
1	0	
1	1	

x	y	$x(\overline{xy} + \overline{x}\,\overline{y})$
0	0	
0	1	
1	0	
1	1	

x	y	z	$xyz + \overline{x}y + \overline{z}$
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

x	y	z	$(x+y+\overline{z})(\overline{x}+y)z$
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

x	y	z	$(\overline{x}+yz)+xy\overline{z}$
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Exercise 2.2 *Proof of Absorption:*

$$x + xy = x \qquad\qquad x(x + y) = x$$

Exercise 2.3 *Proof of consensus theorem:*

$$xy + \overline{x}z + yz = xy + \overline{x}z \qquad\qquad (x + y)(\overline{x} + z)(y + z) = (x + y)(\overline{x} + z)$$

Exercise 2.4 *Prove:* $a = b$ *if and only if* $\overline{a}b + a\overline{b} = 0$

Since 'if and only if' is the equivalence, i.e., implication in both directions, which is what we have to prove.

Exercise 2.5 *Simplify (using algebraic manipulation) the following Boolean functions:*

(i.) $xyz + xy\overline{z} + \overline{zy}$

(ii.) $bc + b(ad + a\overline{d})$

(iii.) $\overline{x_1}\,\overline{x_2}\,\overline{x_3} + \overline{x_1}\,\overline{x_2}x_3 + \overline{x_1}x_2\overline{x_3} + x_1\overline{x_2}x_3 + x_1x_2\overline{x_3}$

(iv.) $\overline{a}\,b + \overline{b}\,c + \overline{a}\,\overline{b}\,\overline{c}$

(v.) $\overline{(\overline{a} + b)(\overline{b} + c)}$

(vi.) $\sum^3(1, 3, 6)$

(vii.) $\sum^3(0, 1, 2, 4, 6, 7)$

Exercise 2.6 *Write the* **normal** *SOP form for the following functions :*

(i.) $x_1x_2x_3 + \overline{x_1}x_2 + \overline{x_3}$

(ii.) $(x_1 + x_2 + \overline{x_3})(\overline{x_1} + x_2)x_3$

(iii.) $(\overline{x_1} + x_2x_3) + x_1x_2\overline{x_3}$

(iv.) $x_1x_2 + \overline{x_1}x_3 + x_2x_3$

Exercise 2.7 *Write the* **normal** *POS form for the following functions :*

(i.) $\overline{x_1\overline{x_2}x_3} + x_1x_2 + \overline{x_3}$

(ii.) $x_1x_2x_3 + \overline{x_1}x_2 + \overline{x_3}$

(iii.) $(\overline{x_1} + x_2x_3) + x_1x_2\overline{x_3}$

(iv.) $x_1x_2 + \overline{x_1}x_3 + x_2x_3$

3 K-maps

Exercise 3.1 *Simplify the examples from exercise 2.5 using K-maps !*

Exercise 3.2 *Simplify using K-maps:*

(i.) $\overline{(x_1\overline{x_2})}(x_1 + \overline{x_2} + \overline{x_3}) + x_1(x_2 + \overline{x_3})$

(ii.) $\overline{(x_1\overline{x_2}x_3)}(x_1 + x_2)$

(iii.) $x_1\overline{x_2}x_3 + x_1x_2 + \overline{x_3}$

(iv.) $x_1x_2\overline{x_3}x_4 + \overline{x_3}\,\overline{x_4} + x_2\overline{x_3}x_4 + x_1x_4$