Essentials of Computer Systems - Exercises #1 (with partial solutions)

1 Positional Number Systems

Exercise 1.1 *Convert into hexadecimal notation:*

(i.) 1581_{10} [$62D_{16}$]

(ii.) 1948_{10} [$79C_{16}$]

(iii.) 1453_{10} [$5AD_{16}$]

(iv.) 1811_{10} [713_{16}]

(v.) 1883_{10} [$75B_{16}$]

(vi.) 2265_{10} [$8D9_{16}$]

(vii.) 2373_{10} [945_{16}]

(viii.) 2122_{10} [$84A_{16}$]

(ix.) 2179_{10} [883_{16}]

(x.) 2381_{10} [$94D_{16}$]

Exercise 1.2 *Convert into binary notation:*

(i.) 170_{10} [010101010_2]

(ii.) 128_{10} [10000000_2]

(iii.) 384_{10} [0110000000_2]

(iv.) 81_{10} [01010001_2]

(v.) 63_{10} [0111111_2]

(vi.) 225_{10} [11111111_2]

(vii.) 74_{10} [1001010_2]

(viii.) 16_{10} [10000_2]

(ix.) 115_{10} [1110011_2]

(x.) 12_{10} [1100_2]

Exercise 1.3 *Convert into the given radix:*

(i.) 72_{10} *to* $r = 3$ [2200_3]

(ii.) 214_{10} *to* $r = 5$ [1324_5]

(iii.) 351_{10} *to* $r = 5$ [2401_5]

(iv.) 54_{10} *to* $r = 3$ [2000_3]

(v.) 14_{10} *to* $r = 2$ [1110_2]

(vi.) 92_{10} *to* $r = 8$ [134_8]

(vii.) 72_{10} *to* $r = 8$ [110_8]

(viii.) 15_{10} *to* $r = 7$ [21_7]

(ix.) 116_{10} *to* $r = 16$ [74_{16}]

(x.) 164_{10} *to* $r = 11$ [$13A_{11}$]

Exercise 1.4 *Convert into decimal notation:*

(i.) 01010101_2 [85_{10}]

(ii.) 101011_2 [43_{10}]

(iii.) 1101101_2 [109_{10}]

(iv.) $CAFE_{16}$ [51966_{10}]

(v.) $2D3_{16}$ [723_{10}]

(vi.) 201_3 [19_{10}]

(vii.) 231_6 [91_{10}]

(viii.) 414_5 [109_{10}]

(ix.) 315_8 [205_{10}]

(x.) 76_9 [69_{10}]

2 Boolean Algebra

Exercise 2.1 *Fill out the truth tables and draw the circuit diagrams below each table. Available gates: NOT, 2-input OR, 2-input AND gate. Derive the delay and the area of each circuit given the following assumptions:*

$A_{\texttt{NOT}}$ = 1 GE, $A_{\texttt{AND}}$ = 2 GE, $A_{\texttt{OR}}$ = 3 GE

$t_{\texttt{NOT}}$ = 0.5 ns, $t_{\texttt{AND}}$ = 0.7 ns, $t_{\texttt{OR}}$ = 0.7 ns

x	y	$\overline{x}(x+y)$
0	0	
0	1	
1	0	
1	1	

x	y	$xy + x\overline{x}$
0	0	
0	1	
1	0	
1	1	

x	y	$xy(\overline{x}+y)$
0	0	
0	1	
1	0	
1	1	

x	y	$x(\overline{xy} + \overline{x}\,\overline{y})$
0	0	
0	1	
1	0	
1	1	

x	y	z	$xyz + \overline{x}y + \overline{z}$
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

x	y	z	$(x+y+\overline{z})(\overline{x}+y)z$
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

x	y	z	$(\overline{x}+yz) + xy\overline{z}$
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Exercise 2.2 *Proof of Absorption:*

$$\underline{x + xy = x}$$

$$\begin{aligned} x + xy &= x \cdot 1 + xy && \ldots \text{identity} \\ &= x \cdot (1 + y) && \ldots \text{distributivity} \\ &= x \cdot (y + 1) && \ldots \text{commutativity} \\ &= x \cdot 1 && \ldots \text{null element} \\ &= x && \ldots \text{identity} \end{aligned}$$

$$\underline{x(x + y) = x}$$

$$\begin{aligned} x(x + y) &= (x + 0)(x + y) && \ldots \text{identity} \\ &= x + (0 \cdot y) && \ldots \text{distributivity} \\ &= x + (y \cdot 0) && \ldots \text{commutativity} \\ &= x + 0 && \ldots \text{null element} \\ &= x && \ldots \text{identity} \end{aligned}$$

Exercise 2.3 *Proof of consensus theorem:*

$$\underline{xy + \overline{x}z + yz = xy + \overline{x}z}$$

$$\underline{(x + y)(\overline{x} + z)(y + z) = (x + y)(\overline{x} + z)}$$

$$\begin{aligned} & xy + \overline{x}z + yz \\ &= xy + \overline{x}z + yz \cdot 1 && \ldots \text{identity} \\ &= xy + \overline{x}z + yz \cdot (x + \overline{x}) && \ldots \text{complement} \\ &= xy + \overline{x}z + yzx + yz\overline{x} && \ldots \text{distributivity} \\ &= xy + xyz + \overline{x}z + \overline{x}yz && \ldots \text{commutativity twice} \\ &= xy \cdot 1 + xyz + \overline{x}z \cdot 1 + \overline{x}zy && \ldots \text{identity twice, commut.} \\ &= xy(1 + z) + \overline{x}z(1 + y) && \ldots \text{distributivity twice} \\ &= xy + \overline{x}z && \ldots \text{null element and identity twice} \end{aligned}$$

Exercise 2.4 *Prove:* $\underline{\textit{if } a = b \textit{ if and only if } \overline{a}b + a\overline{b} = 0}$

Since 'if and only if' is the equivalence, i.e., implication in both directions, which is what we have to prove.

(⇒) we take l.h.s. $a = b$ *as hypothesis:*

$$\begin{aligned} \overline{a}b + a\overline{b} &= \overline{a}a + a\overline{b} && \ldots \text{substitue hypothesis} \\ &= \overline{a}a + b\overline{b} && \ldots \text{substitue hypothesis} \\ &= a\overline{a} + b\overline{b} && \ldots \text{commutativity} \\ &= 0 + 0 && \ldots \text{complement twice} \\ &= 0 && \ldots \text{idempotency} \end{aligned}$$

(⇐) we take r.h.s. $\overline{a}b + a\overline{b} = 0$ *as hypothesis, but we will perform two steps, both treating the hypothesis as an equation (not as mere expression as in previous examples):*

$$\begin{aligned} \overline{a}b + a\overline{b} &= 0 && \ldots \quad a + \text{ on both sides} \\ a + \overline{a}b + a\overline{b} &= a + 0 \\ a + a\overline{b} + \overline{a}b &= a && \ldots \text{commutativity on left, identity on right} \\ a + \overline{a}b &= a && \ldots \text{absorption on left} \\ a + b &= a && \ldots \text{simplification on left, (slide 9)} \end{aligned}$$

very important: we do NOT have cancellations!!!

$$\begin{aligned} \overline{a}b + a\overline{b} &= 0 && \ldots \quad b + \text{ on both sides} \\ b + \overline{a}b + a\overline{b} &= b + 0 \\ b + b\overline{a} + a\overline{b} &= b && \ldots \text{commutativity on left, identity on right} \\ b + \overline{b}a &= b && \ldots \text{absorption on left, absorption} \\ b + a &= b && \ldots \text{simplification on left, (slide 9)} \\ a + b &= b && \ldots \text{commutativity on left} \end{aligned}$$

very important: we do NOT have cancellations!!!

We have obtained $a + b = a$ *and* $a + b = b$ *and conclude that* $a = b$.

Exercise 2.5 *Simplify (using algebraic manipulation) the following Boolean functions:*

(i.) $xyz + xy\overline{z} + \overline{z}\overline{y}$ $[x + \overline{y} + \overline{z}]$

(ii.) $bc + b(ad + a\overline{d})$ $[b(a + c)]$

(iii.) $\overline{x_1}\,\overline{x_2}\,\overline{x_3} + \overline{x_1}\,\overline{x_2}x_3 + \overline{x_1}x_2\overline{x_3} + x_1\overline{x_2}x_3 + x_1x_2\overline{x_3}$ $[\overline{x_1}\,\overline{x_3} + \overline{x_2}x_3 + x_2\overline{x_3}]$

(iv.) $\overline{a}b + \overline{b}c + \overline{a}\,\overline{b}\,\overline{c}$ $[\overline{a} + \overline{b}c]$

(v.) $\overline{(\overline{a} + b)(\overline{b} + c)}$ $[a\overline{b} + b\overline{c}]$

(vi.) $\sum^3(1, 3, 6)$ $[\overline{x_1}x_3 + x_1x_2\overline{x_3}]$

(vii.) $\sum^3(0, 1, 2, 4, 6, 7)$ $[x_1x_2 + \overline{x_1}\,\overline{x_2} + \overline{x_3}]$

Exercise 2.6 *Write the* **normal** *SOP form for the following functions :*

(i.) $x_1x_2x_3 + \overline{x_1}x_2 + \overline{x_3}$ $\left[\sum^3(0, 2, 3, 4, 6, 7)\right]$

(ii.) $(x_1 + x_2 + \overline{x_3})(\overline{x_1} + x_2)x_3$ $\left[\sum^3(3, 7)\right]$

(iii.) $(\overline{x_1} + x_2x_3) + x_1x_2\overline{x_3}$ $\left[\sum^3(0, 1, 2, 3, 6, 7)\right]$

(iv.) $x_1x_2 + \overline{x_1}x_3 + x_2x_3$ $\left[\sum^3(1, 3, 6, 7)\right]$

Exercise 2.7 *Write the* **normal** *POS form for the following functions :*

(i.) $\overline{x_1\overline{x_2}x_3} + x_1x_2 + \overline{x_3}$ $\left[\prod^3(2)\right]$

(ii.) $x_1x_2x_3 + \overline{x_1}x_2 + \overline{x_3}$ $\left[\prod^3(2, 6)\right]$

(iii.) $(\overline{x_1} + x_2x_3) + x_1x_2\overline{x_3}$ $\left[\prod^3(2, 3)\right]$

(iv.) $x_1x_2 + \overline{x_1}x_3 + x_2x_3$ $\left[\prod^3(0, 1, 4, 6)\right]$

3 K-maps

Exercise 3.1 *Simplify the examples from exercise 2.5 using K-maps !*

Exercise 3.2 *Simplify using K-maps:*

(i.) $\overline{(x_1\overline{x_2})}(x_1 + \overline{x_2} + \overline{x_3}) + x_1(x_2 + \overline{x_3})$ $[x_1x_2 + \overline{x_1}\,\overline{x_2} + \overline{x_3}]$

(ii.) $\overline{(x_1\overline{x_2}x_3)}(x_1 + x_2)$ $[x_2 + x_1\overline{x_3}]$

(iii.) $x_1\overline{x_2}x_3 + x_1x_2 + \overline{x_3}$ $[x_1 + \overline{x_3}]$

(iv.) $x_1x_2\overline{x_3}x_4 + \overline{x_3}\,\overline{x_4} + x_2\overline{x_3}x_4 + x_1x_4$ $[x_1x_4 + \overline{x_3}\,\overline{x_4} + x_2\overline{x_3}]$