Set Identities

Identity Laws

1. $A \cap U = A$
2. $A \cup \phi = A$

Domination Laws

1. $A \cup U = U$
2. $A \cap \phi = \phi$

Idempotent Laws

1. $A \cup A = A$
2. $A \cap A = A$

Complementation Law

1. $\overline{(\overline{A})} = A$

Commutative Laws

1. $A \cup B = B \cup A$
2. $A \cap B = B \cap A$

Associative Law

1. $A \cup (B \cup C) = (A \cup B) \cup C$
2. $A \cap (B \cap C) = (A \cap B) \cap C$

Distributive Law

1. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
2. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Demorgan's Law

1. $\overline{A \cap B} = \overline{A} \cup \overline{B}$
2. $\overline{A \cup B} = \overline{A} \cap \overline{B}$

Complement Laws

$A \cup \overline{A} = \cancel{\phi}\ U$

$A \cap \overline{A} = \phi$

Absorption Laws

1. $A \cup (A \cap B) = A$
2. $A \cap (A \cup B) = A$

Methods of Proving Identities

Subset Method → show that each side of identity is a subset of the other side.

Membership Table → For each possible combination of the atomic sets, s.t an element in exactly these atomic sets must either belong to both sides or belong to neither side.

Apply existing Identities → Start with one side, transform to other side, using a sequence of steps by applying an established Identity.

1 a. $\overline{A \cap B} = \bar{A} \cup \bar{B}$ (17)

$\overline{A \cap B} = \{x \mid x \notin A \cap B\}$ by def of compliment

$= \{x \mid \neg(x \in (A \cap B))\}$ by def $\notin$ symbol.

$= \{x \mid \neg(x \in A \wedge x \in B)\}$ by def of intersection.

$= \{x \mid \neg(x \in A) \vee \neg(x \in B)\}$ by demorgan law for logical equivalence.

$= \{x \mid x \notin A \vee x \notin B\}$ by definition of does not belong symbol.

$= \{x \mid x \in \bar{A} \vee x \in \bar{B}\}$ by definition of compliment.

$= \{x \mid x \in \bar{A} \cup \bar{B}\}$ by def of union

$= \bar{A} \cup \bar{B}$ by meaning of set builder

1b) Membership table.

$\overline{A \cap B} = \bar{A} \cup \bar{B}$ using membership table

A	B	$\bar{A}$	$\bar{B}$	$\boxed{\bar{A} \cup \bar{B}}$	$A \cap B$	$\boxed{\overline{A \cap B}}$
0	0	1	1	1	0	1
0	1	1	0	1	0	1
1	0	0	1	1	0	1
1	1	0	0	0	1	0

2) By Membership table P.T $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

A	B	C	$B \cup C$	$A \cap (B \cup C)$	$A \cap B$	$A \cap C$	$(A \cap B) \cup (A \cap C)$
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

1 b) Membership table.

$\overline{A \cap B} = \bar{A} \cup \bar{B}$ using membership table

A	B	$\bar{A}$	$\bar{B}$	$\boxed{\bar{A} \cup \bar{B}}$	$A \cap B$	$\boxed{\overline{A \cap B}}$
0	0	1	1	1	0	1
0	1	1	0	1	0	1
1	0	0	1	1	0	1
1	1	0	0	0	1	0.

2) By Membership table P.T $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

A	B	C	$B \cup C$	$A \cap (B \cup C)$	$A \cap B$	$A \cap C$	$(A \cap B) \cup (A \cap C)$
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

3) Prove $\overline{A \cup (B \cap C)} = (\overline{C} \cup \overline{B}) \cap \overline{A}$

$\overline{A \cap B} = \overline{A} \cup \overline{B}$
$\overline{A \cup B} = \overline{A} \cap \overline{B}$ } By demorgans Law

$\overline{A \cup (B \cap C)} = \overline{A} \cap \overline{(B \cap C)}$

$= \overline{A} \cap (\overline{B} \cup \overline{C})$ (By demorgans Law)

$= (\overline{B} \cup \overline{C}) \cap \overline{A}$ (By Commutative Law)

$A \cup B = B \cup A.$
$A \cap B = B \cap A.$

$= (\overline{C} \cup \overline{B}) \cap \overline{A}$

L.H.S = R.H.S (Hence Proved.)

Membership Table

A	B	C	$B \cap C$	$A \cup B \cap C$	$\overline{A \cup B \cap C}$	$\bar{A}$	$\bar{B}$	$\bar{C}$	$\bar{C} \cup \bar{B}$	$\bar{A} \cap (\bar{C} \cup \bar{B})$
0	0	0	0	0	1	1	1	1	1	1
0	0	1	0	0	1	1	1	0	1	1
0	1	0	0	0	1	1	0	1	1	1
0	1	1	1	1	0	1	0	0	0	0
1	0	0	0	1	0	0	1	1	1	0
1	0	1	0	1	0	0	1	0	1	0
1	1	0	0	1	0	0	0	1	1	0
1	1	1	1	1	0	0	0	0	0	0

L.H.S = R.H.S Hence Proved.

Computer Representation of Sets : Bit String

Will Assume U is finite.

$U = \{a_1, a_2 \cdots a_n\}$

$A \subset U$ with bit string of length n

i^{th} bit in string is 1 $a_i \in A$

0 $a_i \notin A$

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Subsets of length Max 10

Bit string represent set of all odd integers : 1 0 1 0 1 0 1 0 1 0

10 1010 1010

Bit string represent set of all even integers : 01 0101 0101

Set of all integers in U that donot exceeds : 11 1110 0000

1. IF $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 2, 3, 4, 5\}$

$B = \{1, 3, 5, 7, 9\}$

(a) Find bit string for A and B.

(b) Find complement of A and B

(c) Find Union and intersection of A and B

(a) $A = 11111\,0\,0000$ $\quad \bar{A} = 00000\,11111$

$B = 1010\,10\,1010$ $\quad \bar{B} = 0101010101$

$A \cup B$ $\quad 11111\,0\,1010 = \{1, 2, 3, 4, 5, 7, 9\}$

$A \cap B$ $\quad 1010100000 = \{1, 3, 5\}$

(2) Suppose that Universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

find the set specified by each of these bit string.

(a) 11 1100 1111 $\Longrightarrow$ $\{1, 2, 3, 4, 7, 8, 9, 10\}$

(b) 01 0111 1000 $\Longrightarrow$ $\{2, 4, 5, 6, 7\}$

(c) 10 0000 0001 $\Longrightarrow$ $\{1, 10\}$.

(3) What subsets of a finite Universe set do the bit string represent

(a) the string with all zero $\Longrightarrow$ null set

(b) the string with all ones. $\Longrightarrow$ Universal set

Multiset

A multiset is an unordered collection of elements, where an element can occur as a member more than once.

Multiset is denoted by $\{a, a, a, b, b\}$.

$\Rightarrow$ element a thrice.

$\Rightarrow$ element b twice

Notation for multiset

$\{m_1 \cdot a_1, m_2 \cdot a_2 \ldots\ldots m_r \cdot a_r\}$ $\Rightarrow$ a_1 occurs m_1 times, a_2 occurs m_2 times and a_r occurs m_r times.

↳ multiplicities

Example $P = \{4 \cdot a, 1 \cdot b, 3 \cdot c\}$

$Q = \{3 \cdot a, 4 \cdot b, 2 \cdot d\}$

$P \cup Q = \{\max(4,3) \cdot a, \max(1,4) \cdot b, \max(3,0) \cdot c, \max(0,2) \cdot d\}$

$= \{4 \cdot a, 4 \cdot b, 3 \cdot c, 2 \cdot d\}$

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(2) Suppose that Universal Set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ find the set specified by each of these bit string.

(a) 11 1100 1111 $\Longrightarrow$ $\{1, 2, 3, 4, 7, 8, 9, 10\}$

(b) 01 0111 1000 $\Longrightarrow$ $\{2, 4, 5, 6, 7\}$

(c) 10 0000 0001 $\Longrightarrow$ $\{1, 10\}$.

(3) What subsets of a finite Universe set do the bit string repre

(a) the string with all zero $\Longrightarrow$ null set

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24 a)

$P \cap Q = \{ \min(4,3) \cdot a, \min(1,4) \cdot b, \min(3,0) \cdot c, \min(0,2) \cdot d \}$

$= \{3 \cdot a, 1 \cdot b, 0 \cdot c, 0 \cdot d\} = \underline{\underline{\{3 \cdot a, 1 \cdot b\}}}$

$P - Q = \{ \max(4-3, 0) \cdot a, \max(1-4, 0) \cdot b, \max(3-0, 0) \cdot c,$

$\max(0-2, 0) \cdot d \} = \{1 \cdot a, 0 \cdot b, 3 \cdot c, 0 \cdot d\}$

$= \underline{\underline{\{1 \cdot a, 3 \cdot c\}}}$

Sum of P and Q

$P + Q = \{ (4+3) \cdot a, (1+4) \cdot b, (3+0) \cdot c, (0+2) \cdot d \}$

$= \{7 \cdot a, 5 \cdot b, 3 \cdot c, 2 \cdot d\}$

A and B are multiset $\{3\cdot a, 2\cdot b, 1\cdot c\}$ and $\{2\cdot a,$

Find:

(a) $A \cup B = \{3\cdot a,\ 3\cdot b,\ 1\cdot c,\ 4\cdot d\}$

(b) $A \cap B = \{2\cdot a,\ 2\cdot b,\ 0\cdot c,\ 0\cdot d\}$

(c) $A - B = \{1\cdot a,\ 0\cdot b,\ 1\cdot c,\ 0\cdot d\}$

(d) $B - A = \{0\cdot a,\ 1\cdot b,\ 0\cdot c,\ 4\cdot d\}$

(e) $A + B = \{5\cdot a,\ 5\cdot b,\ 1\cdot c,\ 4\cdot d\}$.

A and B are multiset $\{3 \cdot a, 2 \cdot b, 1 \cdot c\}$ and $\{2 \cdot a, 3 \cdot b, 4 \cdot d\}$ (Q4 b)

Find.

(a) $A \cup B = \{3 \cdot a, 3 \cdot b, 1 \cdot c, 4 \cdot d\}$

(b) $A \cap B = \{2 \cdot a, 2 \cdot b, 0 \cdot c, 0 \cdot d\}$

(c) $A - B = \{1 \cdot a, 0 \cdot b, 1 \cdot c, 0 \cdot d\}$

(d) $B - A = \{0 \cdot a, 1 \cdot b, 0 \cdot c, 4 \cdot d\}$

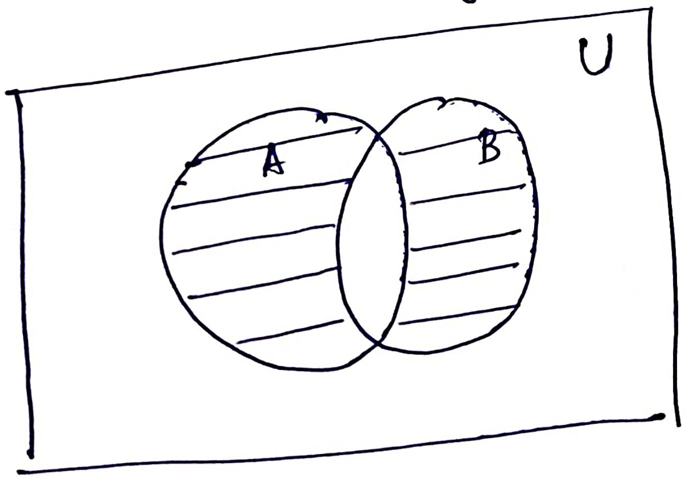
e) $A + B = \{5 \cdot a, 5 \cdot b, 1 \cdot c, 4 \cdot d\}$

Symmetric Difference

Symmetric difference of A and B denoted by $A \oplus B$ is the set containing those elements in either A or B not in both A and B.

Problem -: 1

Draw a Venn Diagram for the symmetric Difference of A & B



$$A \oplus B = (A \cup B) - (A \cap B)$$

$$A \oplus B = (A - B) \cup (B - A)$$

<u>problem</u> : Find Symmetric Difference of $\{1, 3, 5\}$ and $\{1, 2, 3\}$

Let $A = \{1, 3, 5\}$

$B = \{1, 2, 3\}$

$A \cup B = \{1, 2, 3, 5\}$

$A \cap B = \{1, 3\}$

$$A \oplus B = (A \cup B) - (A \cap B)$$

$$= \{1, 2, 3, 5\} - \{1, 3\}$$

$$= \underline{\underline{\{2, 5\}}}$$