Application of Propositional Logic

1. Translating English Sentences

English language is ambiguous.
Converting English sentences in to compound statements of proposition removes ambiguity.

example : You are not allowed to drive vehicles if your age is less than 18 years or you have no age proof

step 1 : Find connectives which are connecting two proposition together.

step 2 : Rename the proposition.

q : You are allowed to drive vehicles

r : yours age is less than 18

s : You have age proof

$$\boxed{(r \vee \neg s) \rightarrow \neg q}$$

System Specification

Translating sentences of natural language in to logical expression is required for hardware and software system.

Eg, The automated reply, Cannot be sent when file system is full.

p → The automated reply can be sent

q → The file system is full

$$\boxed{q \rightarrow \neg p}$$

Boolean Searches

Logical Connections are used extensively in searches of large collection of Information. from Indexes of Web pages ⇒ Boolean Searches

eg: In Boolean Searches the connective AND is used to match record, that contain both of two Search term.

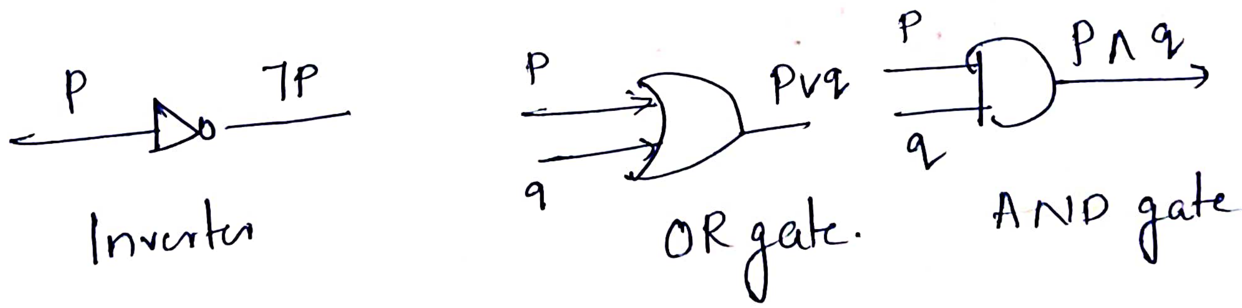
Logic Puzzles

Puzzles that can be solved using logical reasoning are known as logic puzzles. Its done with rule of logic.

Design of Logic Circuit

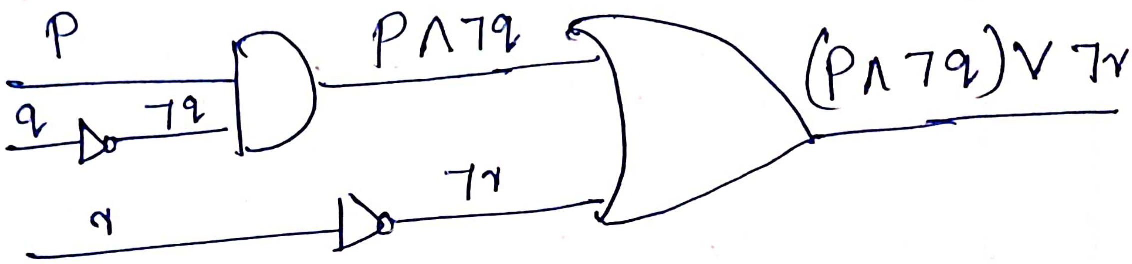
A logic circuit receives input signal $p_1, p_2 \cdots p_n$ each a bit and produce output signals $s_1, s_2 \cdots s_n$. each a bit.

Digital circuits can be constructed from 3 basic circuits called gates.

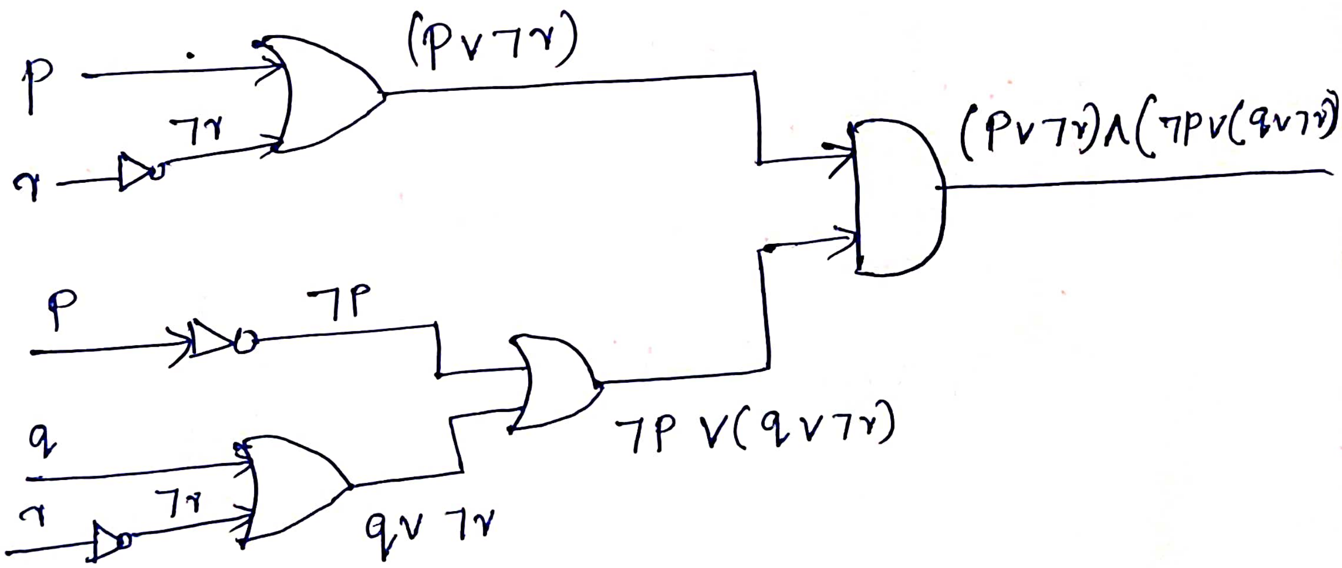


Design of Circuit

① $(p \land \neg q) \lor \neg r$.



② $(p \lor \neg r) \land (\neg p \lor (q \lor \neg r))$



Propositional Equivalences.

A compound proposition that is always true No matter what the truth values of the propositional variable in it is called tautology.

A compound proposition that is always false is contradiction.

A compound proposition that is neither tautology or contradiction is called Contigency:

tautology ↓ contradiction ↓

P	$\neg P$	$P \vee \neg P$	$P \wedge \neg P$
T	F	T	F
F	T	T	F

Logical Equivalence

- Compound proposition that have same truth value in all possible cases are called logically equivalent.
- The compound proposition p and q are logically equivalent if $p \leftrightarrow q$ is a tautology.
- The notation $p \equiv q$ denotes that p and q are logically equivalent.

$$\neg(p \land q) \equiv \neg p \lor \neg q$$

$$\neg(p \lor q) \equiv \neg p \land \neg q.$$

$\equiv$

p	q	$p \lor q$	$\neg(p \lor q)$	$\neg p$	$\neg q$	$\neg p \land \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

① $p \to q \equiv \neg p \lor q$

② $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$

Equivalence	Name
$p \wedge T \equiv p$ $p \vee F \equiv p$	Identity Law
$p \vee T \equiv T$ $p \wedge F \equiv F$	Domination Law
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent Law
$\neg(\neg p) \equiv p$	Double Negation Law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative Law
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative Law
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	Demorgans Law
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption Law
$p \vee \neg p \equiv T$ $p \wedge \neg p \equiv F$	Negation. Laws

More explanation on Predicate Logic

Proposition can be represented as Pred (arg₁, .. argn)

p: John is tall.

tall (John) → Either true or false.

q: Jane loves Paul.

Love (Jane, Paul)   Love' (paul, Jane)

r: The teacher saw

s: that children is reading a book.

See (Teacher, read (children, book))

All the below three statement have same predicate

p: $\boxed{\text{All}}$ girls love Paul

Love (girls, paul).

Mini Rini Pimi Sini

p: $\boxed{\text{Some}}$ girls love Paul

Love (girls, paul).

Mini Rini

Pimi Sini

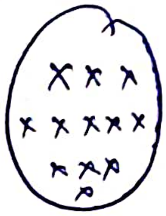
p: Girls don't love Paul.

Love (girls, paul)

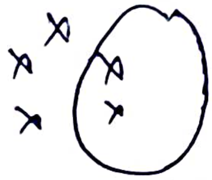
Min Rin Pimi Sin

So we use Quantifiers for specific meaning of statements

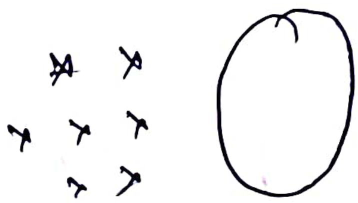
Quantifiers

1. $\forall x$  → All x it holds that.

Universal quantifier → All, Every

2. $\exists x$  → There exist at least one element x such that

Existential Quantifier

3. $\neg x$  For no x holds that . . . .

Negative Quantifier

p: All linguists are bald.

$\forall x\, (\text{Linguist}(x) \rightarrow \text{Bald}(x))$

q: All girls love Paul.

$\forall x\, (\text{Girls}(x) \rightarrow \text{Love}(x, paul))$.

Some linguist are bald.

$\exists x\, (\text{linguist}(x) \;\&\; \text{Bald}(x))$

Some girls love paul

$\exists x\, (\text{girl}(x) \;\&\; \text{Love}(x, paul))$.

$\neg$ or $\sim$ are symbols of Negative

p: No linguistic are bald.

$\neg \forall x \, (\text{Linqust}(x) \rightarrow \text{Bald}(x))$

No girls love paul.

$\neg x \, (\text{Girl}(x) \rightarrow \text{Love}(x, \text{paul}))$

Set of entities that are effected.

Everybody is happy.

$\forall x \, (\text{Person}(x) \rightarrow \text{Happy}(x))$

Nobody is Happy

$\neg x \, (\text{Person}(x) \rightarrow \text{Happy}(x))$

Negating Quantified Expression

$$\boxed{\neg \forall x\, P(x) \equiv \exists x\, \neg P(x)}$$

$$\boxed{\neg \exists x\, Q(x) \equiv \forall x\, \neg Q(x)}$$

Sequences and Summation.

Sequences

A sequence is a discrete structure used to represent an ordered list.

1, 2, 3, 5, 8 is a sequence of five terms

1, 3, 9, 27, 81.... 3^n is an infinite Sequence.

$\{a_n\}$ → used to describe a sequence.

example: 1

$$a_n = \frac{1}{n}.$$

The list of terms of this sequence begining with a_1

$$a_1, a_2, a_3, a_4 \ldots\ldots$$

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$$

Geometric Progression.

A geometric progression is a sequence of the form $a, ar, ar^2, \ldots ar^{n-1} \ldots$

where initial term a and common ratio r are real numbers

$$f(x) = a r^{n-1}$$

$4 \overset{\times 3}{,} 12 \overset{\times 3}{,} 36 \overset{\times 3}{,}$ $\quad a = 4 \quad r = 3.$

Arithmetic Progression

Arithmetic Progression is a sequence of the form $a, a+d, a+2d, a+(n-1)d \ldots$ where initial term a and common difference d are real numbers.

Geometric progression

$2000 \rightarrow 10\% \xrightarrow{12 \text{ years}} ?$

Principal Amount $\quad$ Rate of Interest

End of I^{st} year $= 2000 + \frac{2000 \times 10}{100} = 2000\left(1 + \frac{10}{100}\right)$

$= 2000\left(\frac{11}{10}\right)$

End of 2^{nd} year $=$

$= 2000\left(\frac{11}{10}\right) + \frac{10}{100} \times 2000\left(\frac{11}{10}\right)$

$= 2000\left(\frac{11}{10}\right)\left[1 + \frac{10}{100}\right] = 2000\left(\frac{11}{10}\right)\left(\frac{11}{10}\right) = 2000\left(\frac{11}{10}\right)^2$

Amount at end of 3 years = $2000 \times \left(\frac{11}{10}\right)^3$

Geometric Sequence.

$$2000 \times \left(\frac{11}{10}\right) + 2000 \times \left(\frac{11}{10}\right)^2 + 2000 \times \left(\frac{11}{10}\right)^3 \cdots$$

$$a = 2000 \times \left(\frac{11}{10}\right) \quad r = \frac{11}{10} \quad n = 12$$

Amount after n years $= ar^{n-1}$

$$= 2000\left(\frac{11}{10}\right) \times \left(\frac{11}{10}\right)^{12-1}$$

$$A = 2000\left(\frac{11}{10}\right)^{12}.$$

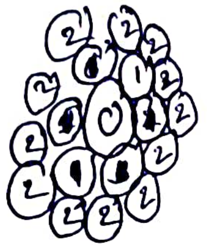
$A = 6277$

Arithmetic Progression

Best example is honey comb.

~~0th cell → 1~~

~~2nd~~



0^{th} ring → 1 cell ×

1^{st} ring → 6 cell

2^{nd} ring → 12 cell

3^{rd} ring → 18 cell

6, 12, 18, 24, 30

7^{th} ring how many cells.

$a = 1$

~~a+nd~~. $a + (n-1)d.$

$6 + 6 \times 6 = 42$ cells

Which term of arithmatic progression

3, 8, 13, 18 . . . . is 78

$a = 3 \quad d = 8 - 3 = 5$

$a_n = a + (n-1)d$

$78 = 3 + (n-1) \times 5$

$78 = 3 + 5n - 5$

$5n = 78 - 3 + 5 = 80$

$n = \frac{80}{5} = \underline{\underline{16}}$

The eight term of an AP is half its second term and eleventh term exceeds one third of its fourth term by 1. Find the fifteenth term.

$a_n = a + (n-1)d.$ $\qquad a_{15} = ?$

$a_8 = \frac{1}{2}(a_2) \longrightarrow (1)$ $\qquad a_{15} = a + 14d$

$a_{11} = \frac{1}{3}(a_4) + 1 \rightarrow (2)$

$a_8 = a + 7d.$

$a_2 = a + d.$

$a_8 = \frac{1}{2} \times a_2.$

$a + 7d = \frac{1}{2}(a+d)$

$2(a + 7d) = a + d.$

$2a + 14d = a + d.$

$a + 13d = 0. \longrightarrow (4)$

$a + 10d = \frac{1}{3}(a+3d)+1$

$3a + 30d = a + 3d + 3$

$2a + 27d = 3 \longrightarrow (5)$

$a + 13d = 0 \longrightarrow (4)$

(4) ×2 $\quad 2a + 26d = 0 \longrightarrow (6)$

(6) - (5) $\quad 2a + 27d = 3$

$-1d = -3$ $\quad \boxed{d = 3.}$

$a + 13 \times 3 = 0$ $\quad \boxed{a = -39}$

$a_{15} = -39 + 14 \times 3$

$= \underline{3}$