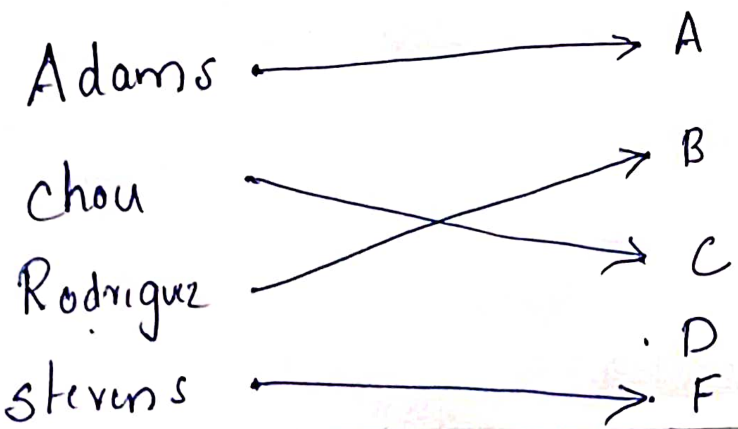
Functions

Let A and B be nonempty sets. A function F from A to B is an assignment of exactly one element of B to each element of A. $f(a) = b$, if b is the unique element of B assigned by the function f to the element a of A.

If F is a function from A to B

$$f : A \rightarrow B$$

Functions are sometime called as mappings or transformation.



Domain = {Adams, chou, Rodriguez, stevens}

Codomain = {A, B, C, F}

Range = {A, B, C, F}

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(1) Let f_1 and f_2 be functions from A to R. Then $f_1 + f_2$ and $f_1 f_2$ are also functions from A to R defined for all $x \in A$ by

$$(f_1 + f_2)(x) = f_1(x) + f_2(x)$$

$$(f_1 f_2)(x) = f_1(x) \cdot f_2(x)$$

(2) Let f_1 and f_2 be functions from R to R such that $f_1(x) = x^2$, $f_2(x) = x - x^2$. What are the functions $f_1 + f_2$ and $f_1 f_2$

$$(f_1 + f_2)(x) = f_1(x) + f_2(x) = x^2 + (x - x^2) = \underline{\underline{x}}$$

$$(f_1 f_2)(x) = f_1(x) \cdot f_2(x) = x^2 (x - x^2) = \underline{\underline{x^3 - x^4}}$$

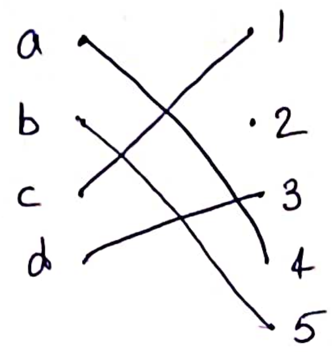
One to One and Onto Functions

One to One

Some functions never assign the same value to two different domains these functions are said to be one-to-one. Or injective.

Problem 1: Determine whether the function f from $\{a, b, c, d\}$ to $\{1, 2, 3, 4, 5\}$ with $f(a) = 4$, $f(b) = 5$, $f(c) = 1$, $f(d) = 3$ is one to one.

The function f is one-one because f takes on different values at the four elements of its domain.



Problem 2: $f(x) = x^2$ from set of integers to the set of integers.

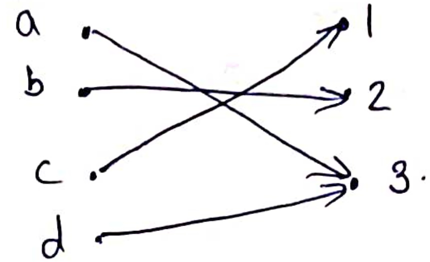
$f(1) = 1$, $f(-1) = 1$, 2 values in domain assigned to one value in co-domain

On to

A function f from A to B is called onto or a surjection if and only if for every element $b \in B$ there is an element $a \in A$ with $f(a) = b$. A function f is called surjective if it is onto.

Problem 1 :- Let f be the function from $\{a, b, c, d\}$ to $\{1, 2, 3\}$ defined by $f(a) = 3$, $f(b) = 2$, $f(c) = 1$ and $f(d) = 3$. Is f an onto function.

A the 3 codomain values has images of elements in domain.



Problem 2: If the function $f(x) = x^2$ from integers to the set of integers on to?

The function is not onto because there is no integer x with $x^2 = -1$, for instance.

Bijective

The function f is a one to one correspondence or a bijection, if it is both one-to-one and onto.

Problems 1 : Let f be the function from $\{a, b, c, d\}$ to $\{1, 2, 3, 4\}$ with $f(a) = 4$, $f(b) = 2$, $f(c) = 1$ and $f(d) = 3$. Is f is a bijection.

One to one → No two values in the domain are assigned. the same function values.

On to → Because all four elements of the codomain are images of elements in the domain.

Inverse Function

Let f be a one to one correspondence from set A to Set B. The inverse function of f is the function that assign to an element b belonging to B the unique element a ~~on~~ A. $f(a)=b$.

The Inverse function f is denoted by f^{-1} $f^{-1}(b) = a$ when $f(a)=b$

problem 1

Let f be the function from $\{a, b, c\}$ to $\{1, 2, 3\}$ such that $f(a) = 2$, $f(b) = 3$ and $f(c) = 1$. Is f invertible and if it is what is its inverse?

The function f is invertible because it is a one to one correspondence.

$f^{-1}(1) = c$, $f^{-1}(2) = a$, $f^{-1}(3) = b$.

Let f and g be the functions from set of integers to set of integers $f(x) = 2x + 3$ $g(x) = 3x + 2$. $f \circ g$? $g \circ f$? 34

$$f \circ g\,(x) = f(g(x)) = 2(3x+2) + 3$$
$$= 6x + 4 + 3 = 6x + 7$$

$$g \circ f\,(x) = g(f(x)) = 3(2x+3) + 2$$
$$= 6x + 9 + 2 = 6x + 11$$