# Propositional Logic

* The rules of logic gives precise meaning to mathematical statements.

* These rules are used to distinguish between valid and invalid mathematical arguments.

## Proposition

* A proposition is a declarative statement.
* A declarative statement declares a fact.
* That can be either true or false. But it cannot be both.

ex: Declarative sentence are propositions

1. Washington D.C is capital of USA. (True)
2. $1 + 1 = 2$ (True)
3. $2 + 2 = 3$ (False)
4. Canada is the capital of Torronto (False)

Sentences not propositions

1. What time is it ? [Not a declarative sentence]

2. $x + y = 2$

$x = 1, y = 1$ then $1 + 1 = 2$ (True)

$x = 2, y = 1$ then $2 + 1 = 2$ (False)

A proposition can be either True or False, cannot be both.

## Terminologies

Propositional Variable → Variable that represent proposition.

Truth value of proposition → True → T

False → F

Propositional Logic - Area of logic that deals with proposition.

Propositional Logic was first developed by Greek philosopher Aristotle more than 2300 years ago.

## Definitions

### Negation of P

Let P be a proposition. The negation of p. is denoted by '$\neg P$' read as "not P"

(P is True $\neg P$ is false and vice versa)

Truth Table ($\neg P$)

| P | $\neg P$ |
|---|---|
| T | F |
| F | T |

### Conjunction of P and q

The conjunction $P \wedge q$ is true when both p and q are true and is false otherwise.

$P \wedge q \Rightarrow$ propostion "P and q"

Truth Table ($P \wedge q$)

| P | q | $P \wedge q$ |
|---|---|---|
| T | T | T |
| F | F | F |
| T | F | F |
| F | T | F |

## Disjunction of $p \vee q$

Let $p$ and $q$ be propositions. The disjunction of $p$ and $q$, denoted by $p \vee q$ is the proposition "$p$ or $q$". The disjunction $p \vee q$ is false when both $p$ and $q$ are false and is true otherwise.

### Truth Table for $p \vee q$.

| p | q | $p \vee q$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

## Exclusive - OR ($P \oplus q$)

The exclusive or of p and q denoted by $p \oplus q$ is the proposition that is true when exactly one of p and q is true and is false otherwise

| p | q | $P \oplus q$ |
|---|---|---|
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | F |

A student can have a salad with dinner and A student can have soup with dinner

⇒ The exclusive or of p and q is the statement that is true when exactly one of p and q is true.

$P \oplus q$ ⟶ A student can have soup or salad but not both with dinner.

Its a fair way of saying that taking both is not permitted.

## Conditional Statement or Implication

Let p and q be propositions.
The conditional statement $p \to q$ is the proposition "if p, then q".

$p \to q$ is false, p is true and q is false. and true otherwise.

$p \to$ hypothesis

$q \to$ conclusion.

## Truth Table

| p | q | $p \to q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

Let $p \to$ Maria learn discrete mathematics"
$q \to$ Maria will find a job

Express English meaning $p \to q$
$p \to q$ If maria learns discrete mathematics then she will find a Job

## Biconditional

Let p and q be propositions. The biconditional statement $p \leftrightarrow q$ is the proposition "p if and only if q".

$p \leftrightarrow q$ is true when p and q have the same truth value

$p \leftrightarrow q$

| p | q | $p \leftrightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

## Logic and Bit operation

A bit is symbol for two value

0 and 1

False True

Bit is represented Boolean variable.

## Bit String

A bit string is a sequence of zero or more bits. The length of this string is the number of bits in the string.

| | | | | |
|---|---|---|---|---|
| Bit string | P | 01 | 1011 | 0110 |
| | | 11 | 0001 | 1101 |
| Bitwise | OR | 11 | 1011 | 11.11 |
| | AND | 01 | 0001 | 0100 |
| | XOR | 10 | 1010 | 1011 |

## Compound Proposition

5 important logical connectives.

- → Conjunction.
- → disjunction.
- → exclusive or.
- → implication
- → biconditional operator.
- → Negation.

5

These connectives are used to build complicated compound propositions.

Construct the truth table for compound proposition.

$$(p \vee \neg q) \rightarrow (p \wedge q)$$

| $p$ | $q$ | $\neg q$ | $p \vee \neg q$ | $p \wedge q$ | $(p \vee \neg q) \rightarrow (p \wedge q)$ |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | [illegible] | 1 | 1 |

Precedence of logical operators

$\neg$, $\wedge$, $\vee$, $\rightarrow$ and $\leftrightarrow$

| Operator | precedence |
|---|---|
| $\neg$ | 1 |
| $\wedge$ | 2 |
| $\vee$ | 3 |
| $\rightarrow$ | 4 |
| $\leftrightarrow$ | 5 |

$\neg p \wedge q \rightarrow$ Negation applied first

$p \vee q \wedge r \rightarrow p \vee (q \wedge r) \rightarrow$ first

$p \wedge q \vee r \rightarrow (p \wedge q) \vee r$

$p \rightarrow q \vee r \longrightarrow p \rightarrow (q \vee r) \rightarrow$ first

Construct truth table for each of these compound proposition. (6)

(a) $p \wedge \neg q$ (e) $(q \to \neg p) \leftrightarrow (p \leftrightarrow q)$

| $p$ | $q$ | $\neg q$ | $p \wedge \neg q$ |
|---|---|---|---|
| F | F | T | F |
| T | F | T | T |
| F | T | F | F |
| T | T | F | F |

(b) $q \to \neg p \leftrightarrow (p \leftrightarrow q)$

| $p$ | $q$ | $q \to \neg p$ | $p \leftrightarrow q$ | $q \to \neg p \leftrightarrow (p \to q)$ |
|---|---|---|---|---|
| T | T | F | T | T |
| T | F | T | F | F |
| F | T | F | F | T |
| F | F | T | T | T |