1. $\xi \sim R(0, \theta)$, $\theta > 0$ - ==вероятн. модель==

$\vec{x}_n$ - выборка

==Оценки параметра θ:==

· $\tilde{\theta}_1 = 2\overline{x}$

· $\tilde{\theta}_2 = x_{min}$

· $\tilde{\theta}_3 = x_{max}$

· $\tilde{\theta}_4 = x_1 + \dfrac{1}{n-1} \sum\limits_{i=2}^{n} x_i$

==Числовые хар-ки ξ==

$$M\xi = \int\limits_{-\infty}^{\infty} x \, dF(x, \theta) = \dfrac{\theta}{2}$$

$$M\xi^2 = \int\limits_{-\infty}^{\infty} x^2 \, dF(x, \theta) = \dfrac{\theta^2}{3}$$

$$D\xi = M\xi^2 - (M\xi)^2 = \dfrac{\theta^2}{12}$$

==Парам. модель==

несм.: $\forall \theta \in \Theta : M\tilde{\theta} = \theta$

сост.: $\forall \theta \in \Theta \quad \forall \varepsilon > 0 \; P(|\tilde{\theta} - \theta| \geq \varepsilon) \underset{n \to \infty}{\longrightarrow} 0$

эфф.: $\tilde{\theta}_1$ эфф $\tilde{\theta}_2$; 1) $\forall \theta \in \Theta \; D\tilde{\theta}_1 \leq D\tilde{\theta}_2$

$\qquad\qquad\qquad$ 2) $\overset{\exists}{\text{распр}} \theta \in \Theta : D\tilde{\theta}_1 \leq D\tilde{\theta}_2$

==Проверка оценок==

① · ==Несмещённость:==

$\forall \theta > 0 : M\tilde{\theta}_1 = M\left(\dfrac{2}{n} \sum x_i\right) \underset{\uparrow}{=} \dfrac{2}{n} \sum M x_i = \theta_{икл} \checkmark$

$\qquad\qquad x_i$ - незав сл вел., $x_i \sim R(0, \theta)$

· Дост. усл. сост.

$$\exists \forall \text{ распр. из в.м. } \tilde{\theta} \text{ несмещ и}$$

$$D\tilde{\theta} \xrightarrow[n\to\infty]{} 0$$

$$\Rightarrow \tilde{\theta} \text{ состоят.}$$

$\tilde{\theta}_1$ несмещ ✓

$$D\tilde{\theta}_1 = \frac{1}{n^2} D\left(\sum x_i\right) = \frac{1}{n^2} \sum D x_i = \frac{\theta^2}{3n} \xrightarrow[n\to\infty]{} 0 \checkmark$$

$$\Rightarrow \tilde{\theta}_1 \text{ состоят.}$$

② · **Несмещённость:**

$$\forall \theta > 0 \quad M\tilde{\theta}_2 = M x_{min} = ^*$$

$$\begin{cases} \xi \sim F(x) = R(x) \\ \xi_{min} \sim \underbrace{1 - (1 - F(x))^n}_{\Phi(x)} \end{cases} \qquad \varphi(x) = \Phi'(x) = n(1 - F(x))^{n-1} F'(x) = \begin{cases} \\ = n\left(1 - \frac{x}{\theta}\right)^{n-1}\frac{1}{\theta} \quad \{(0,\theta)\} \end{cases}$$

$$^* = \int_0^\theta x\, n\left(1 - \frac{x}{\theta}\right)^{n-1}\frac{1}{\theta}\,dx = \frac{\theta}{n+1} - \text{смещ. } \otimes$$

Исправл. $\tilde{\theta}_2' = (n+1) x_{min}$

$$\forall \theta > 0 \quad M\tilde{\theta}_2' = (n+1)M\tilde{\theta}_2 = \theta - \text{ несмещ. } \checkmark$$

· **Состоятельность**

· Дост. усл. сост.

$$\exists \forall \text{ распр. из в.м. } \tilde{\theta} - \text{несмещ и}$$

$$D\tilde{\theta} \xrightarrow[n\to\alpha]{} 0$$

$\Rightarrow \tilde{\theta}$ состоят

$\tilde{\theta}_2'$: несмещ $\otimes$

$$D\tilde{\theta}_2' = \underset{\theta}{D}((n+1)x_{min}) = (n+1)^2 D x_{min}$$

$$M x_{min}^2 = \int_0^\theta x^2 n \left(1 - \frac{x}{\theta}\right)^{n-1} \frac{1}{\theta} dx =$$
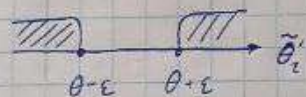
$$= \frac{2\theta^2}{(n+1)(n+2)}$$

$$D x_{min} = \frac{2\theta^2}{(n+1)(n+2)} - \frac{\theta^2}{(n+1)^2} = \frac{n\theta^2}{(n+1)^2(n+2)}$$

$$D\tilde{\theta}_2' = \frac{n\theta^2}{n+2} \underset{n\to\infty}{\not\to} 0 \qquad \bigotimes$$

· Опред. состоят.

$$\forall \theta > 0 \ \forall \varepsilon > 0 \ P(|\tilde{\theta}_2' - \theta| \geq \varepsilon) \underset{n\to\infty}{\longrightarrow} 0 \ \forall A$$

$$|\tilde{\theta}_2' - \theta| \geq \varepsilon \iff \left[\begin{array}{l} \tilde{\theta}_2' - \theta \leq -\varepsilon \\ \tilde{\theta}_2' - \theta \geq \varepsilon \end{array} \right.$$

$$P(|\tilde{\theta}_2' - \theta| \geq \varepsilon) \geq P(\tilde{\theta}_2' \geq \theta + \varepsilon) =$$

$$= P((n+1)x_{min} \geq \theta + \varepsilon) =$$

$$= P\left(x_{min} \geq \frac{\theta + \varepsilon}{n+1}\right) = 1 - \Phi\left(\frac{\theta + \varepsilon}{n+1}\right) =$$

в силу непрер. $\Phi(x)$

$$\underset{n \geq N}{=} \left(1 - \frac{\theta + \varepsilon}{\theta(n+1)}\right)^n \underset{n \to +\infty}{\longrightarrow} e^{-\frac{\theta + \varepsilon}{\theta}} > 0 \quad \bigotimes$$

$\Rightarrow$ <mark>$\tilde{\theta}_2'$ не явл. сост !</mark>

③ · <mark>Несмещённость</mark>

$$\forall \theta > 0 \quad M\tilde{\theta}_3' = M x_{max} = \frac{n}{(n+1)}\theta \quad \text{смещ} \quad \bigotimes$$

• Состоятельность

• Дост усл сост

$$\exists \forall \text{ распр. из в.м. } \tilde{\theta} \text{ несмещ и}$$

$$D\tilde{\theta} \xrightarrow[n\to\infty]{} 0$$

$$\Rightarrow \tilde{\theta} \text{ состоят.}$$

• $\tilde{\theta}_3'$: несмещ ✓

$$D\tilde{\theta}_3' = \left(\frac{n+1}{n}\right)^2 D x_{max}$$

$$M x_{max}^2 = \int_0^\theta x^2 n \left(\frac{x}{\theta}\right)^{n-1} \frac{1}{\theta} dx = \frac{n}{\theta^n} \frac{\theta^{n+2}}{n+2} = \frac{\theta^2 n}{n+2}$$

$$D x_{max} = \frac{n\theta^2}{(n+2)(n+1)^2}$$

$$D\tilde{\theta}_3' = \frac{\theta^2}{n(n+2)} \xrightarrow[n\to\infty]{} 0 \checkmark$$

$$\Rightarrow \tilde{\theta}_3' \text{ состоят}$$

• Опред состоят.

• $\tilde{\theta}_3'$: $\forall \theta > 0 \ \forall \varepsilon > 0 \ P(|\tilde{\theta}_3' - \theta| \geq \varepsilon) \xrightarrow[n\to\infty]{} 0$

$$0 \leq P(|\tilde{\theta}_3' - \theta| \geq \varepsilon) = P(|\tilde{\theta}_3' - M\tilde{\theta}_3'| \geq \varepsilon) \leq \frac{D\tilde{\theta}_3'}{\varepsilon^2} = \frac{\theta^2}{n(n+2)\varepsilon^2} \xrightarrow[n\to\infty]{} 0 \checkmark$$

$$\uparrow$$
$$\text{нерав-во Чебышева}$$

• $\tilde{\theta}_3$: $\forall \theta > 0 \ \forall \varepsilon > 0 \ P(|\tilde{\theta}_3' - \theta| \geq \varepsilon) \xrightarrow[n\to\infty]{} 0$

$$P(|\tilde{\theta}_3 - \theta| \geq \varepsilon) = P(|x_{max} - \theta| \geq \varepsilon) = P(x_{max} \leq \theta - \varepsilon) =$$

$$= \Phi(\theta - \varepsilon) = \begin{cases} 0, & \theta - \varepsilon \leq 0 \\ \left(\frac{\theta-\varepsilon}{\theta}\right)^n, & \theta - \varepsilon > 0 \end{cases}; \quad \left(\frac{\theta-\varepsilon}{\theta}\right)^n \xrightarrow[n\to\infty]{} 0 \checkmark$$

$$\Rightarrow \boxed{\tilde{\theta}_3 \text{ состоят.}}$$

④ • <mark>Несмещённость</mark>

$\forall \theta > 0 \quad M\tilde{\theta}_4 = \theta$ — <mark>несмещ</mark> ✓

• <mark>Состоятельность</mark>

• Дост. усл сост

∃ $\forall$ распр. из в.м $\tilde{\theta}$ несмещ и

$$D\tilde{\theta} \xrightarrow[n \to \infty]{} 0$$

⟹ $\tilde{\theta}$ состоят.

• $\tilde{\theta}_4$ несмещ. ✓

$$D\tilde{\theta}_4 = \frac{\theta^2}{12} + \frac{1}{n-1}\frac{\theta^2}{12} \not\to 0 \quad \otimes$$

• Опред состоят.

$\forall \theta \quad \tilde{\theta}_1 \xrightarrow{P} \theta$ 0A

$$\tilde{\theta}_4 = X_1 + \frac{1}{n-1}\sum_{i=2}^{n} X_i \xrightarrow{P} \zeta + \frac{\theta}{2} \quad \text{<mark>не явл. сост.</mark>} \ \otimes$$

Эффективность.

$$D\tilde{\theta}_2' = \frac{n\theta^2}{n+2}$$

$$D\tilde{\theta}_3' = \frac{\theta^2}{n(n+2)}$$

$\forall \theta \ \exists N. \ \forall n \geq N \to D\tilde{\theta}_2' < D\tilde{\theta}_3'$ — $\tilde{\theta}_3'$ эффектив

$N = 2$