Programming Assignment II : Metropolis Hastings Algorithm

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17CS30042

Dependencies and Variables : ¶

- n := number of iterations of algorithm
- K := number of bivariate gaussians for the mixture distribution
- sigma := standard deviation for stepping normal distribution
- accepted := points accepted by metropolis hastings algo
- rejected := pts rejected by metropolis algo
- samples := pts sampled by metroplois algo
- rv[i] := ith Gaussian distribution
- mu[i] := mean of rv[i]
- cov[i] := covariance of rv[i]
- x1 := 1st element of bivariate gaussian
- x2 := 2nd element of bivariate gaussian

In [1]:
```
import warnings
warnings.filterwarnings("ignore")

import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from sklearn.mixture import GaussianMixture
from scipy.stats import norm, multivariate_normal
import seaborn as sns
import math
```

Metropolis Algorithm

In [2]:
```
x=np.random.multivariate_normal([2,3], [[math.sqrt(1)/2,0],[0,math.sqrt(1)/2
]]).T
print(x)
```

```
[0.69830179 3.38973668]
```

In [3]:
```
def metropolis_hastings(p, n, sigma):
    x, y = np.random.multivariate_normal([2,3], [[math.sqrt(1)/2,0],[0,math.sq
rt(1)/2]]).T
    samples = np.zeros((n,2))  # has all the walk points both rej and acc
    accepted = []  # only new walk points
    rejected = []  # if no change has happended, predicteed pts xstar and ysta
r go to rej

    for i in range(n): # perform for n iterations
        x_star, y_star = np.array([x, y]) + np.random.normal(scale=(sigma), si
ze = 2)
        # sample a move such that d(x) ~ N(0,sigma)
        u=np.random.uniform(0,1)
        if u < p(x_star, y_star) / p(x, y): # accept and move to new pt
            x, y = x_star, y_star
            accepted.append(np.array([x, y]))
        else :  # reject
            rejected.append(np.array([x_star, y_star]))
        samples[i] = np.array([x, y])
    return np.array(accepted), np.array(rejected), np.array(samples)
```

Visualizing data

In [4]:
```
def visualize_data(rejected, samples, n, sigma, K):
    print("Visualized Data on (n, sigma, K):", n, sigma, K)
    plt.plot(rejected[:,0], rejected[:,1], 'rx', label = "Rejected", color =
"red")
    plt.plot(samples[:,0], samples[:,1], label = "Walk")
    plt.title("Walk of Metropolis Algorithm")
    plt.xlabel("x1")
    plt.ylabel("x2")
    plt.legend()
    plt.grid()
    plt.show()

    print("\n First Direction Plot against no. of iters:\n")
    iters=[i for i in range(1,n+1)]
    #print(yt)
    plt.plot(iters,samples[:,0])
    plt.xlabel('Iterations')
    plt.ylabel('%xdel1')
    plt.show()

    print("\n First Direction Frequency Plot against no. of iters:\n")
    plt.hist(samples[:,0],100)
    plt.xlabel('x1')
    plt.ylabel('Frequency of samples')
    plt.show()

    print("Sampled Points:\n")
    sns.jointplot(samples[:, 0], samples[:, 1])
    return
```

Building DataSet

We are required to form a mixture bivariate gaussian distribution (with K components)

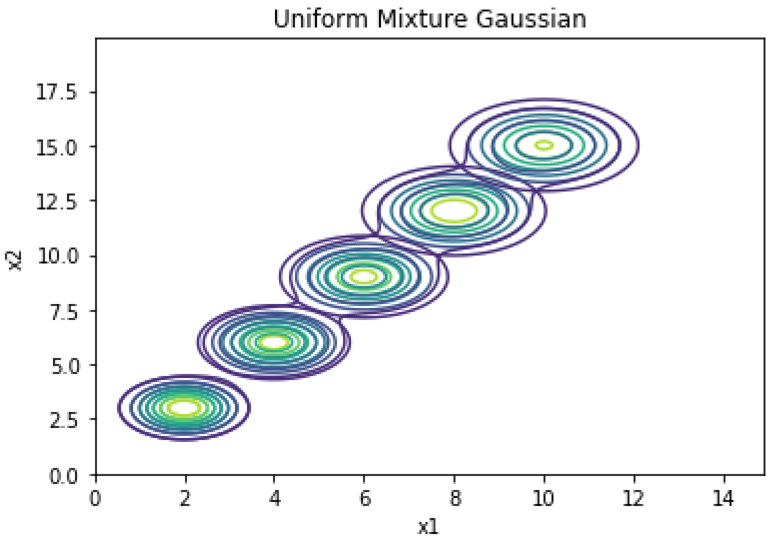
In [5]:
```
def usr_in():
    n = int(input("n - "))
    K = int(input("K - "))
    sigma = float(input("sigma - "))
    return n, K, sigma
```

In [6]:
```
def run_algo(n, K, sigma):
  # prepare position coordinates
  # these are just used to visualize data
  # no use in forming distibutions
  x, y = np.mgrid[0:3*K:.1, 0:4*K:.1]
  pos = np.dstack((x,y))

  # create k bivariate gaussians with required means and covariances
  rv = {}
  mx = np.zeros(np.shape(x))
  i = 1
  mu =[]
  cov =[]
  while i<=K:
      mui = [2*i, 3*i]
      sigmai = np.eye(2)*0.5*np.sqrt(i)
      rv[i] = multivariate_normal(mui, sigmai)
      mx = np.add( mx, rv[i].pdf(pos) )
      plt.contour(x,y,rv[i].pdf(pos))
      mu.append(mui)
      cov.append(sigmai)
      i+=1

  def mixture_GM_pdf(x1, x2):
    i = 1
    mx = 0.0
    while i<=K:
      mx += rv[i].pdf([x1,x2])
      i += 1
    mx = mx/K
    return mx

  plt.title("Uniform Mixture Gaussian")
  plt.contour(x,y,mx)
  plt.xlabel("x1")
  plt.ylabel("x2")
  plt.show()

  accepted, rejected, samples = metropolis_hastings(mixture_GM_pdf, n, sigma)

  visualize_data(rejected, samples, n, sigma, K)

  return
```

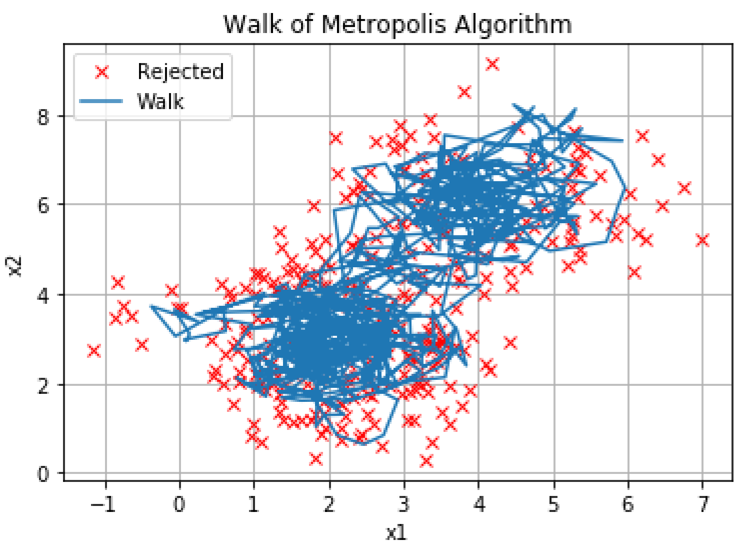
Run 1 of Algorithm

- n = 1000
- K = 5
- $sigma$ = 0.5

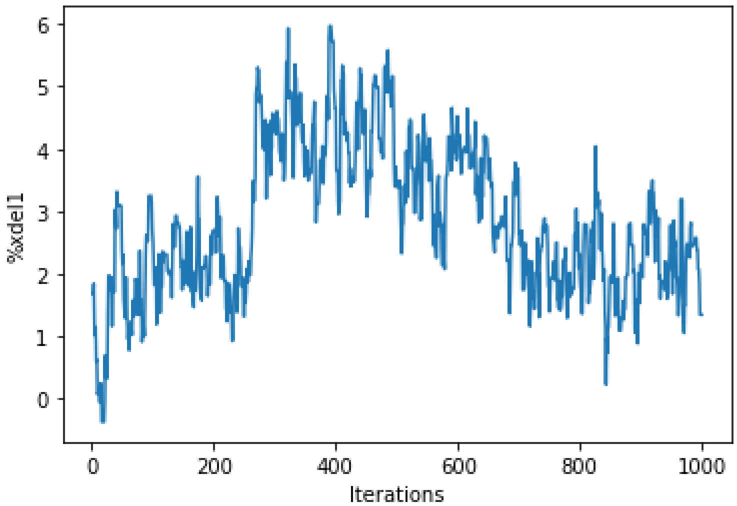
In [7]:
```
run_algo(1000, 5, 0.5)
```



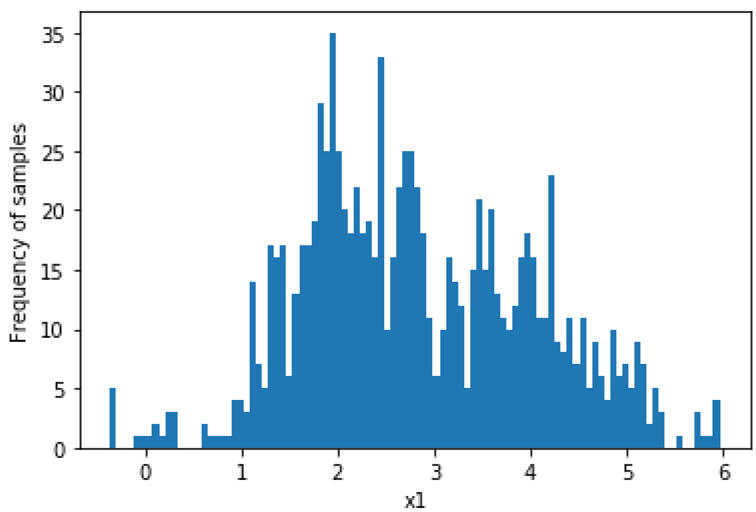
Visualized Data on (n, sigma, K): 1000 0.5 5



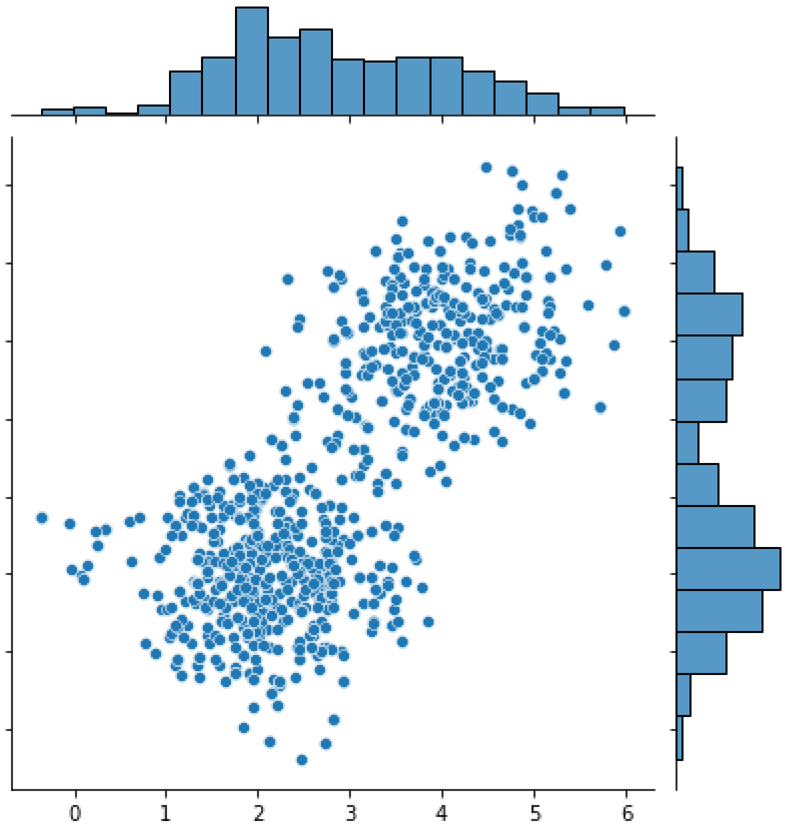
First Direction Plot against no. of iters:



First Direction Frequency Plot against no. of iters:



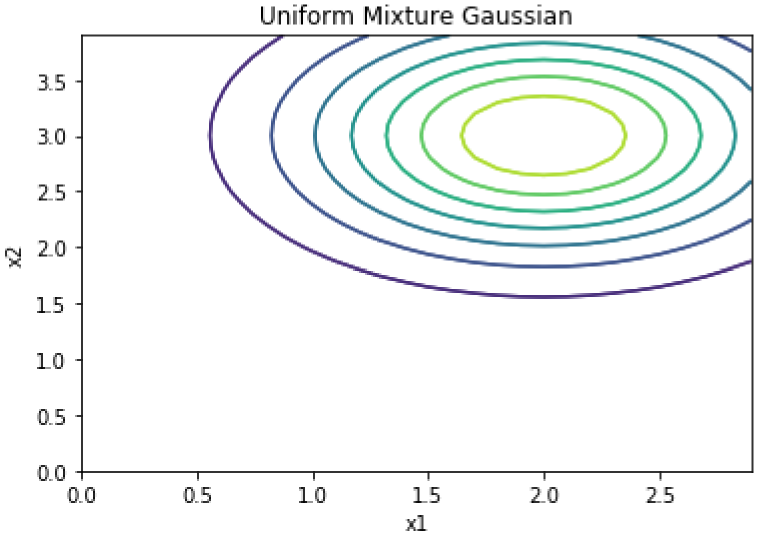
Sampled Points:



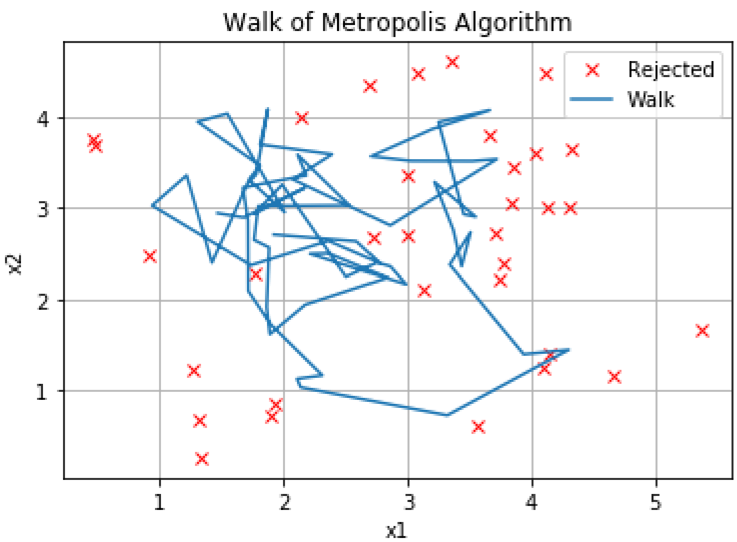
Run 2 of Algorithm

- n = 100
- K = 1
- $sigma$ = .5

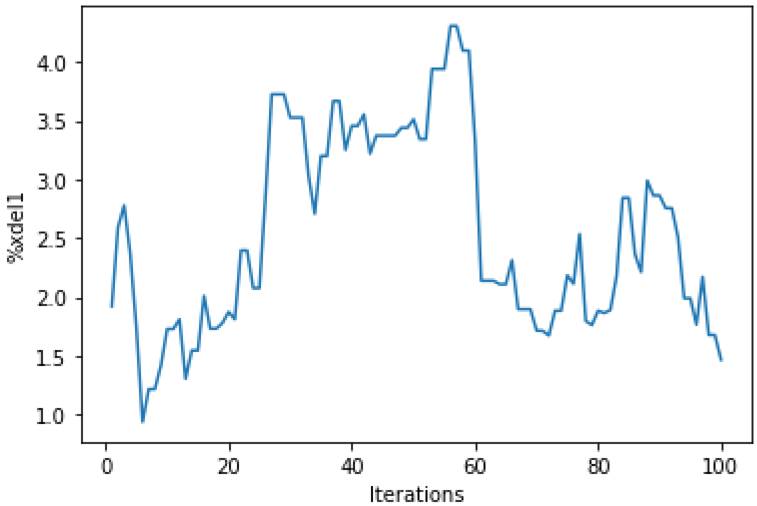
In [8]:
```
run_algo(100, 1, .50)
```



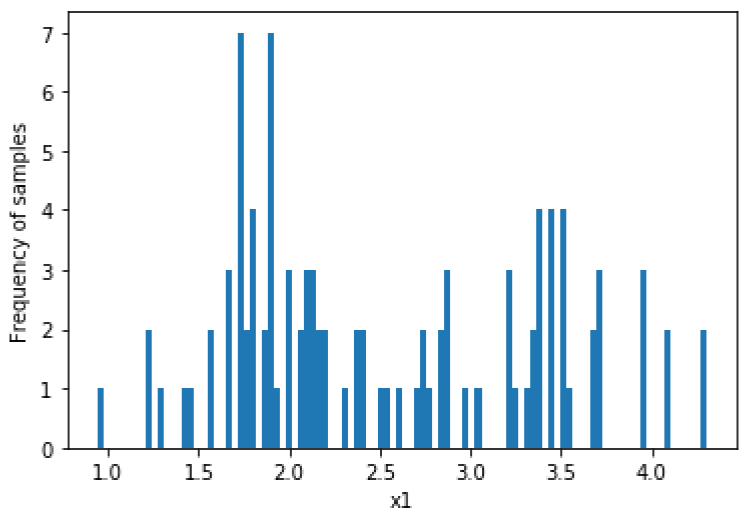

```
Visualized Data on (n, sigma, K): 100 0.5 1
```



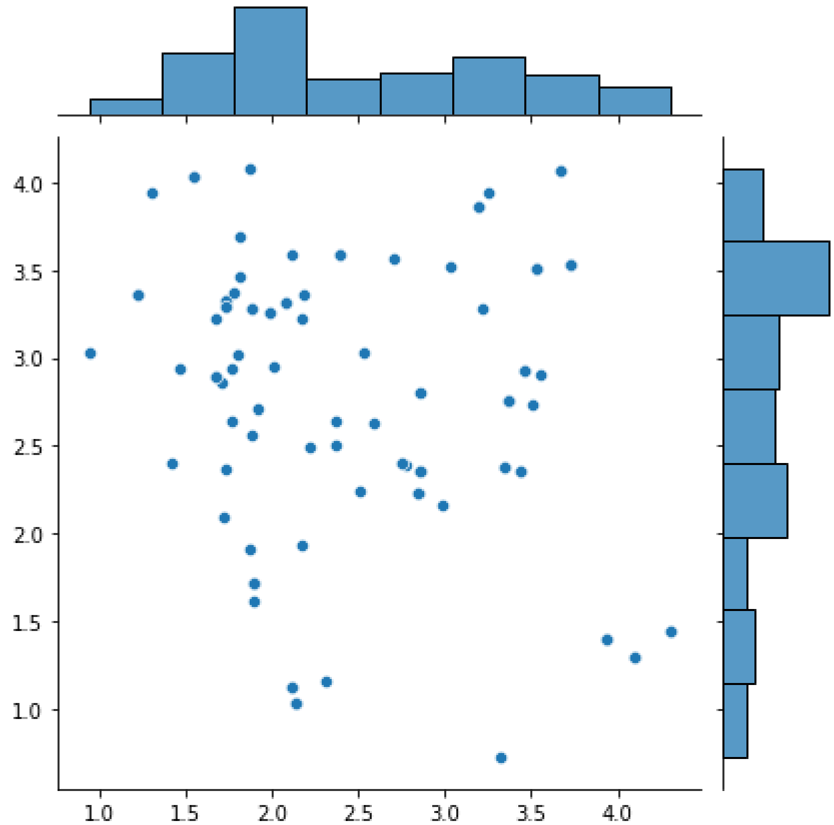

```
First Direction Plot against no. of iters:
```



First Direction Frequency Plot against no. of iters:



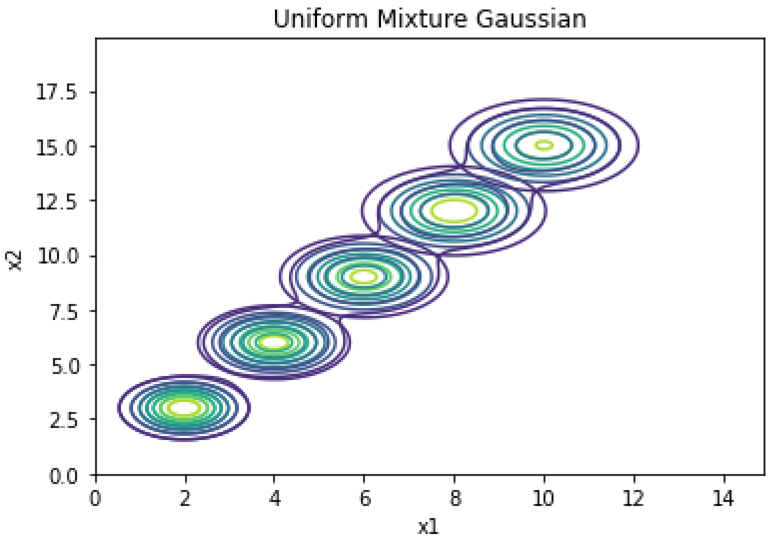
Sampled Points:



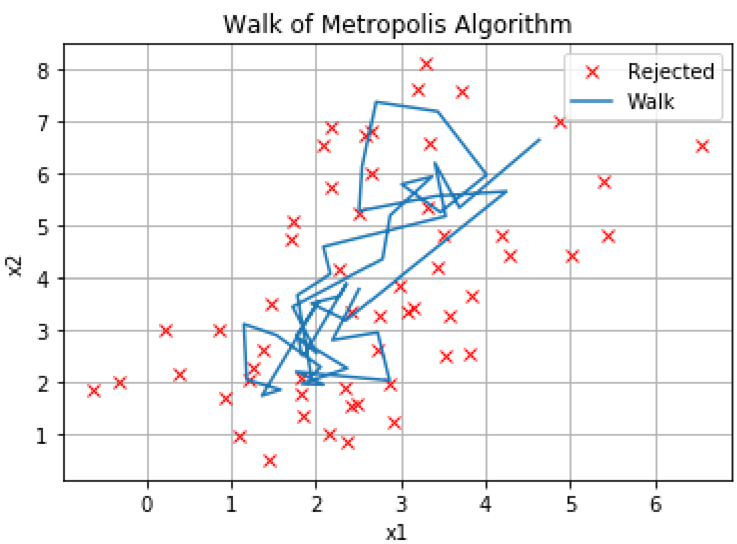
Run 3 of Algorithm

- n = 100
- K = 5
- $sigma$ = 1.0

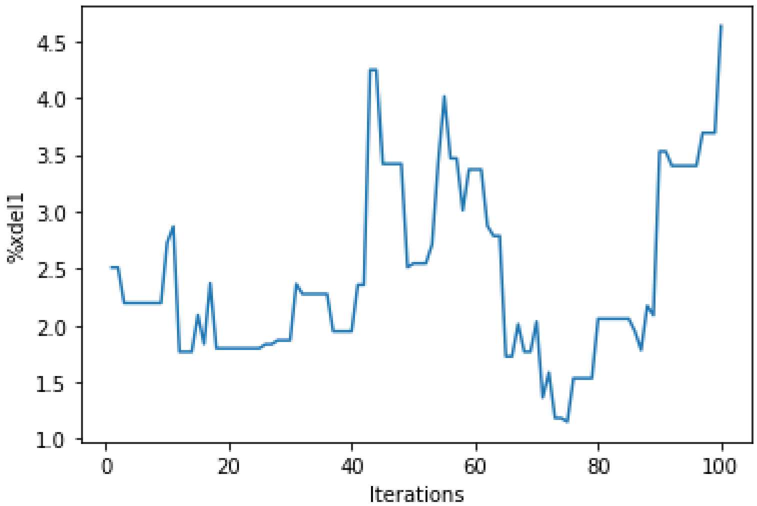
In [21]:
```
run_algo(100, 5, 1.0)
```



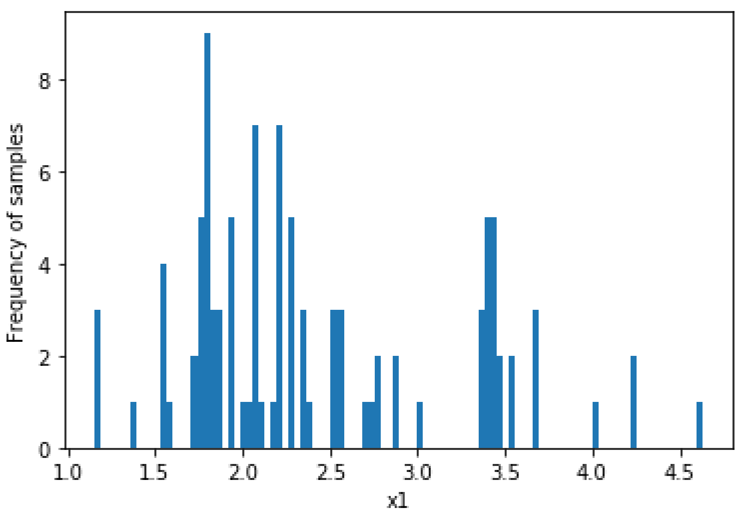

```
Visualized Data on (n, sigma, K): 100 1.0 5
```



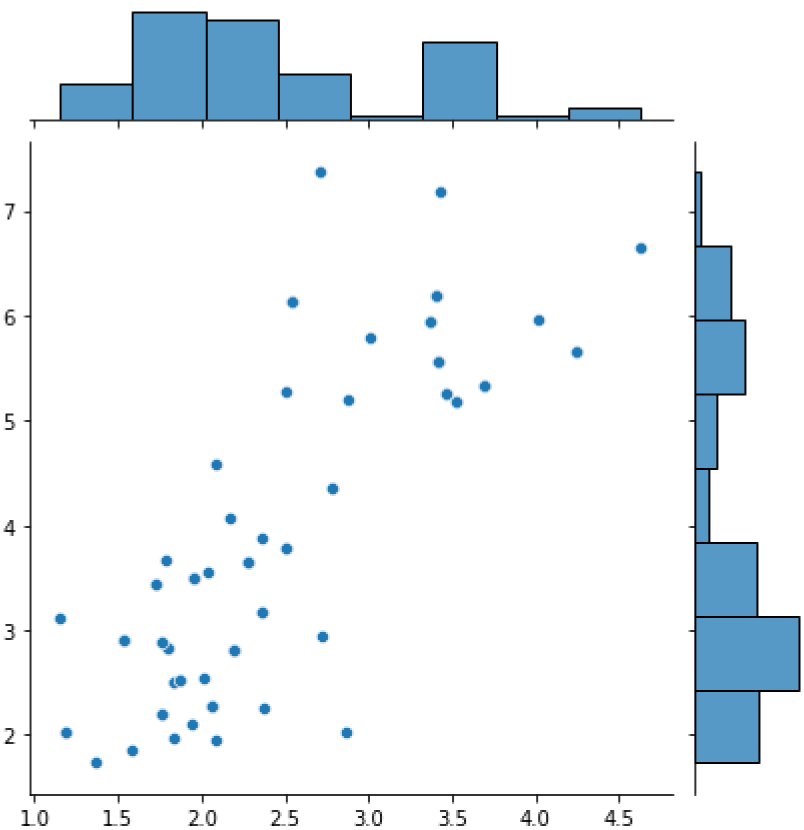

```
First Direction Plot against no. of iters:
```




```
First Direction Frequency Plot against no. of iters:
```



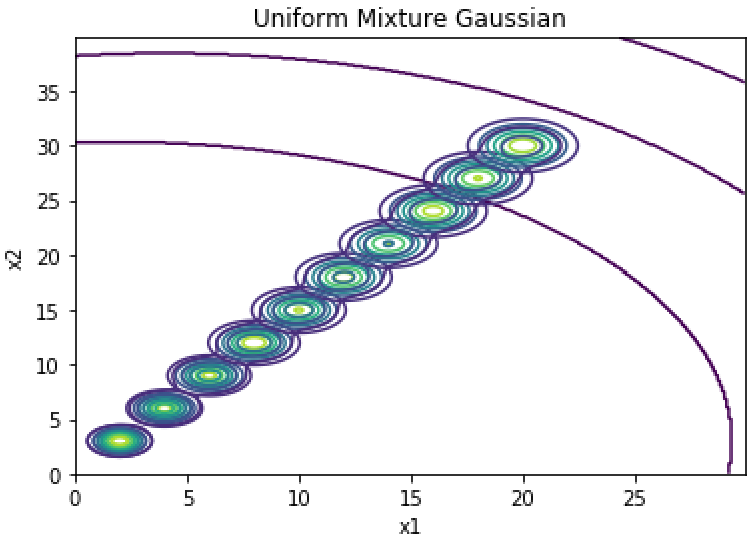

```
Sampled Points:
```



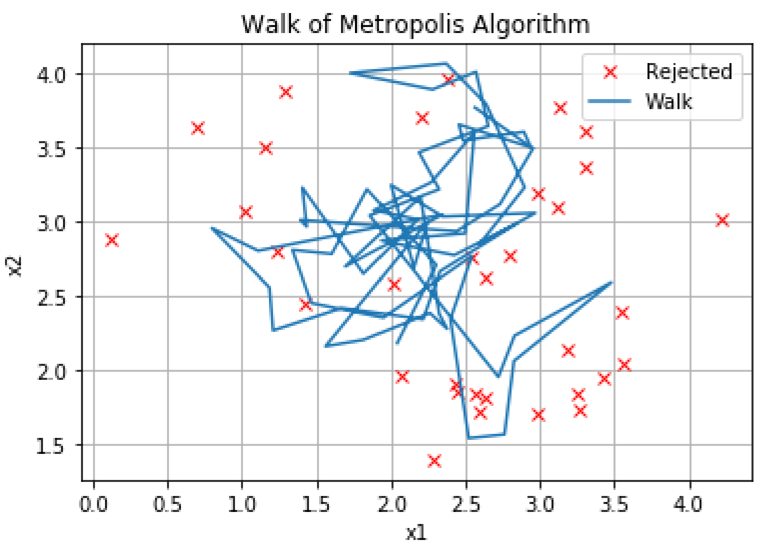
Run 4 of Algorithm

- n = 100
- K = 10
- $sigma$ = .5

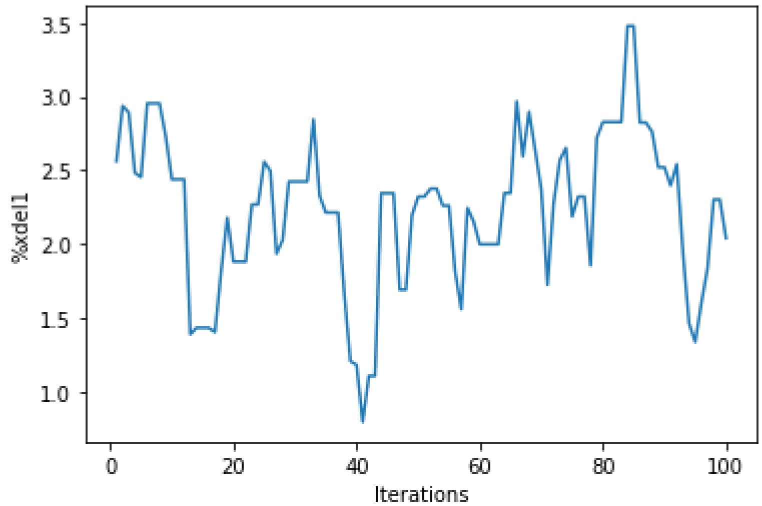
In [10]:
```
run_algo(100, 10, .5)
```



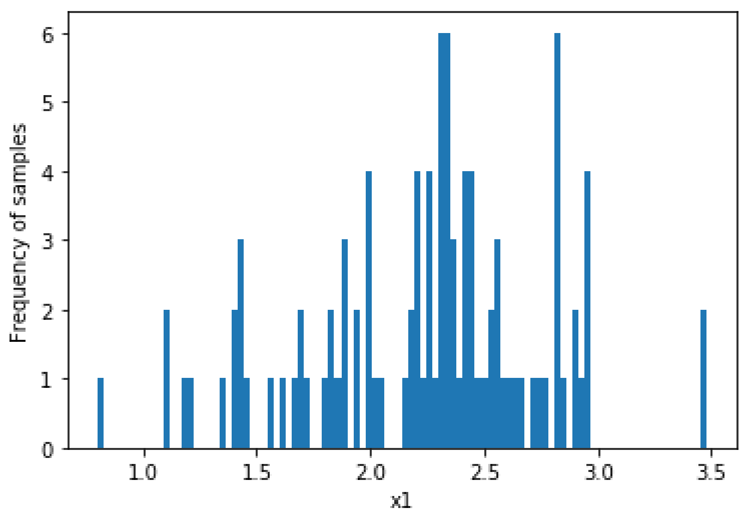

```
Visualized Data on (n, sigma, K): 100 0.5 10
```



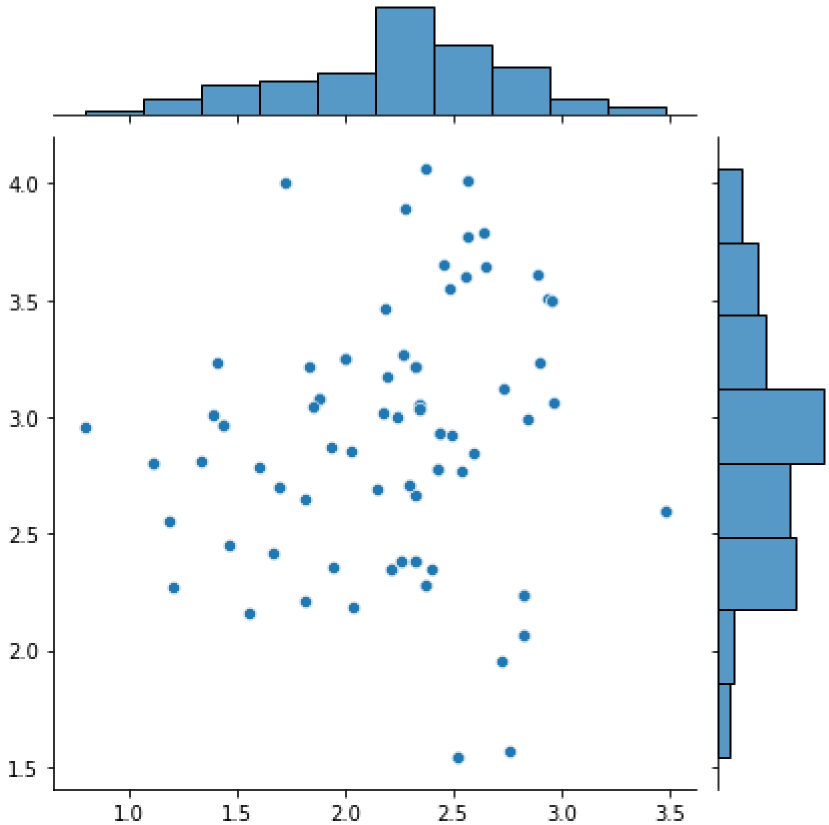

```
First Direction Plot against no. of iters:
```



First Direction Frequency Plot against no. of iters:



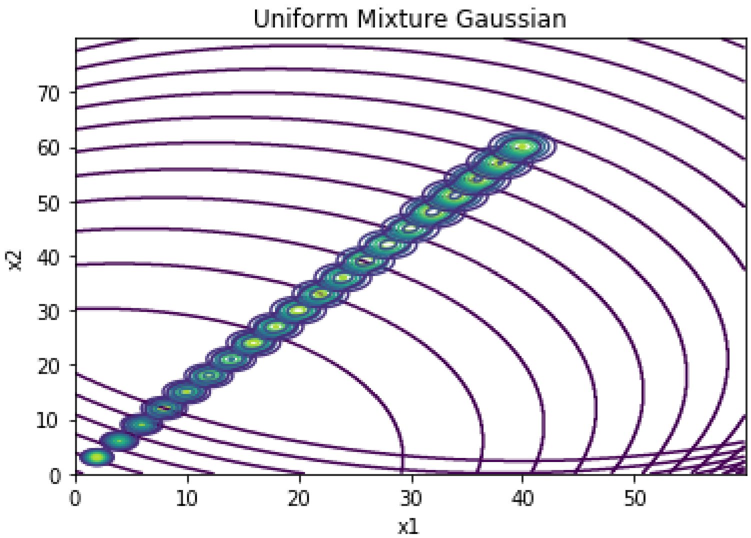
Sampled Points:



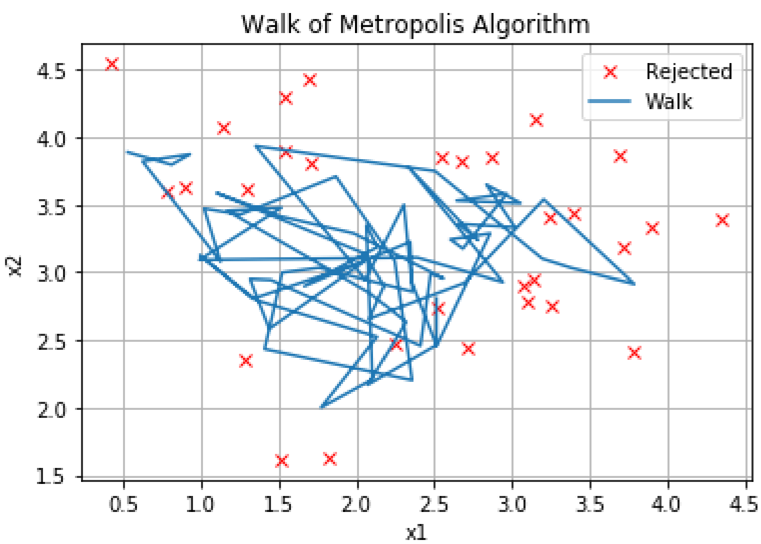
Run 5 of Algorithm

- n = 100
- K = 20
- $sigma$ = .5

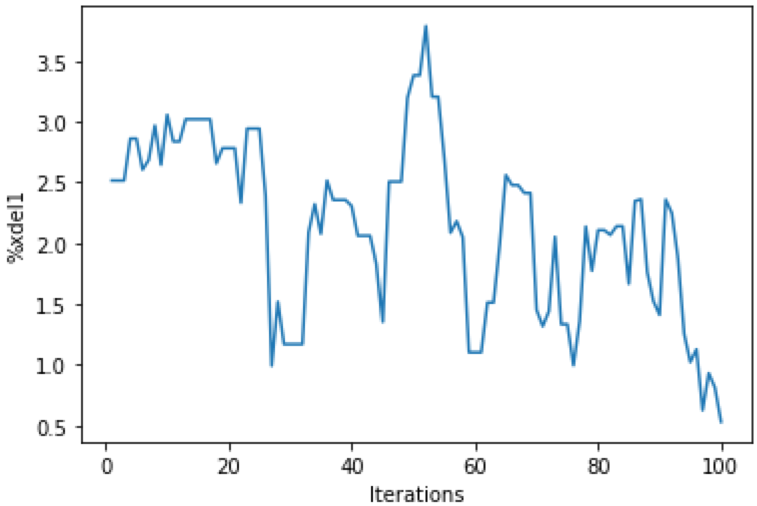
In [11]:
```
run_algo(100, 20, .5)
```



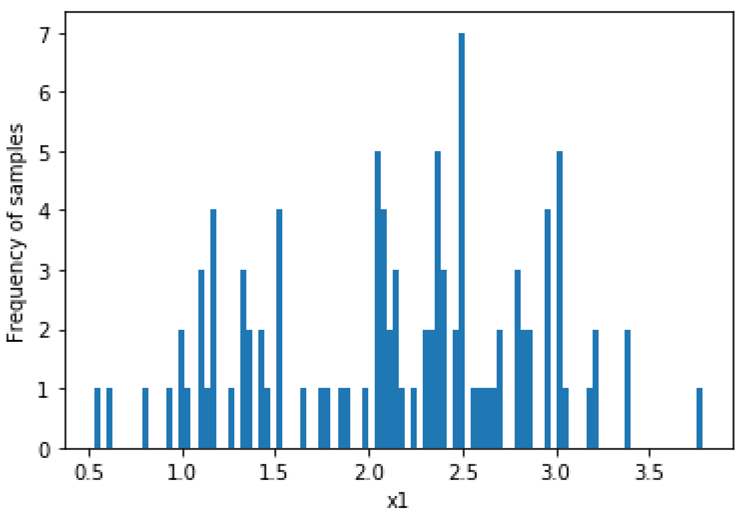

```
Visualized Data on (n, sigma, K): 100 0.5 20
```



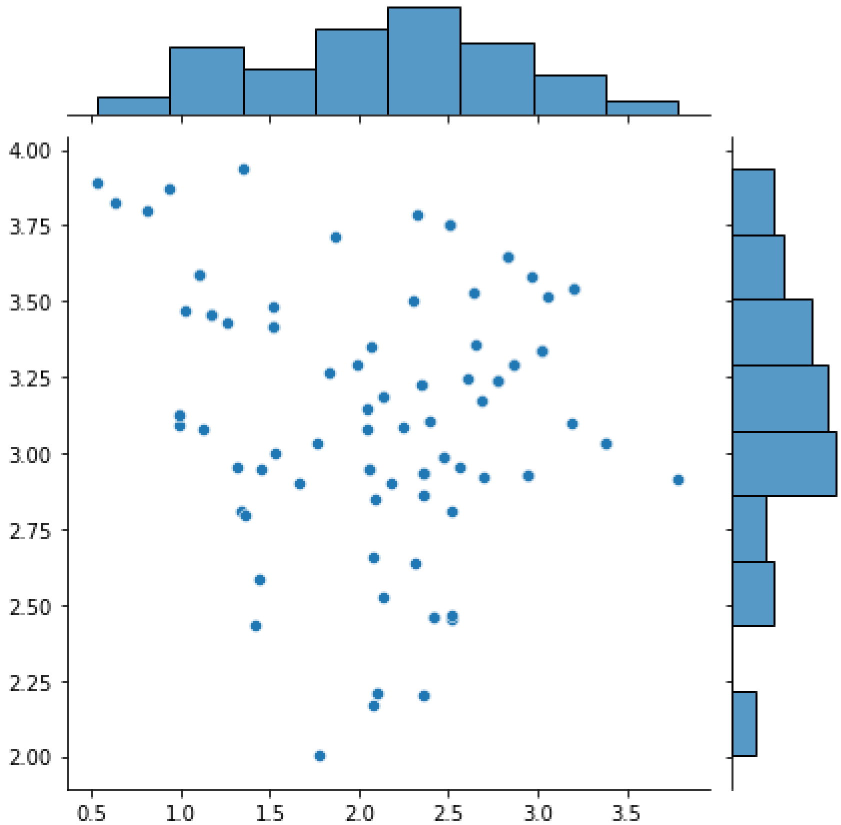

```
First Direction Plot against no. of iters:
```



First Direction Frequency Plot against no. of iters:



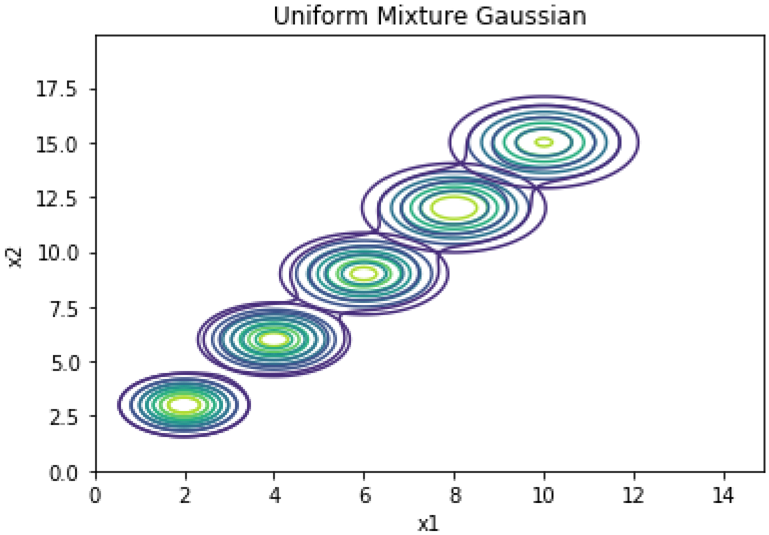
Sampled Points:



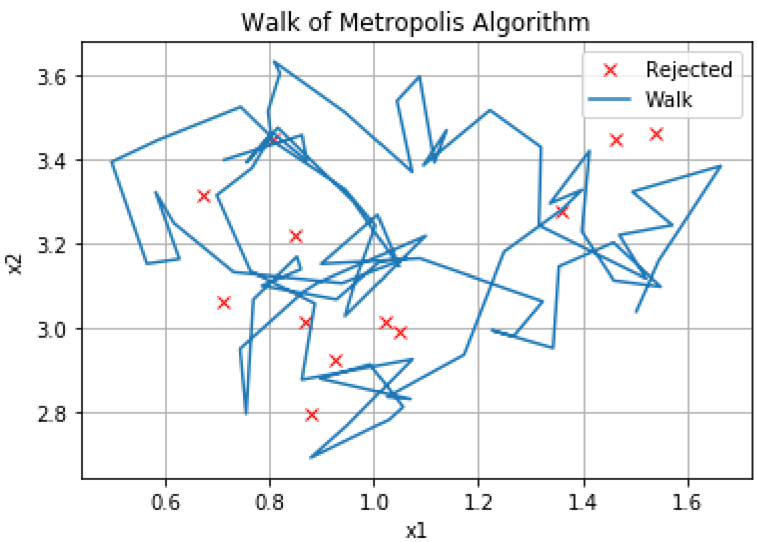
Run 6 of Algorithm

- n = 100
- K = 5
- $sigma$ = 0.01

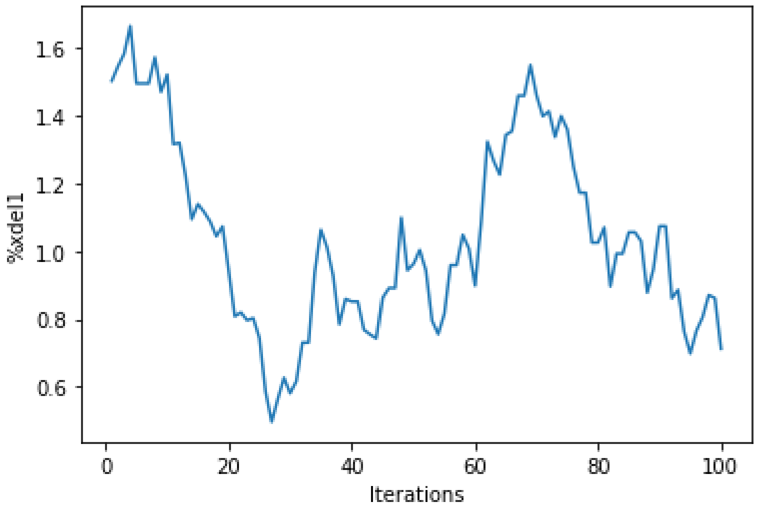
In [12]:
```
run_algo(100, 5, 0.1)
```



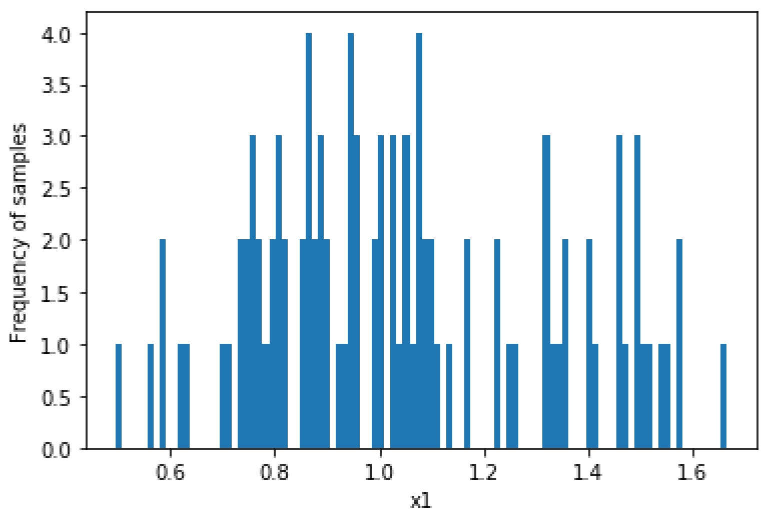

```
Visualized Data on (n, sigma, K): 100 0.1 5
```



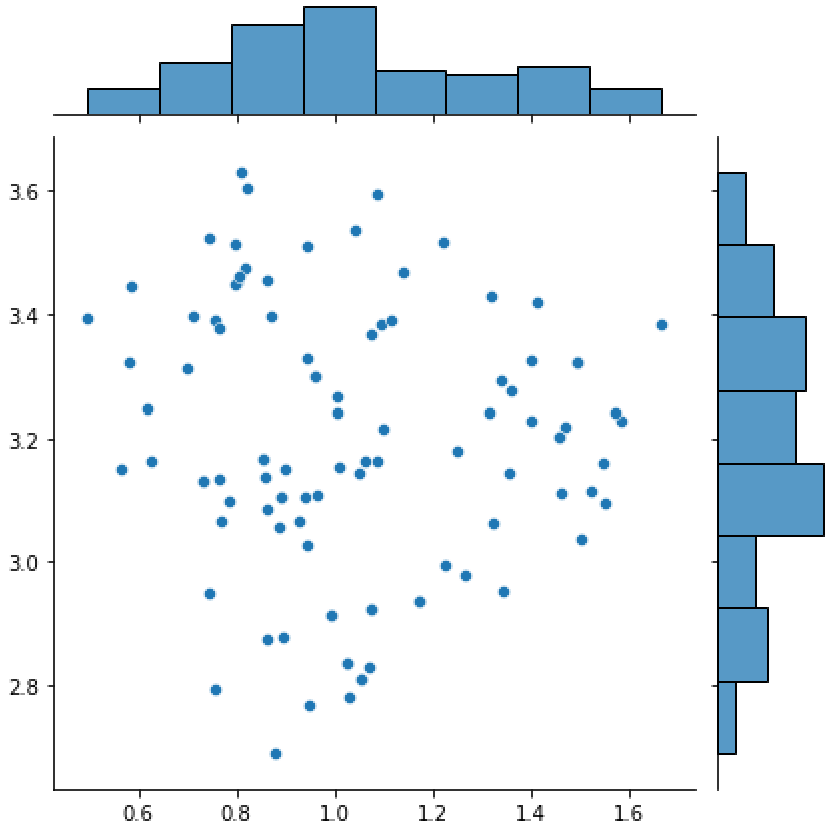

```
First Direction Plot against no. of iters:
```



First Direction Frequency Plot against no. of iters:



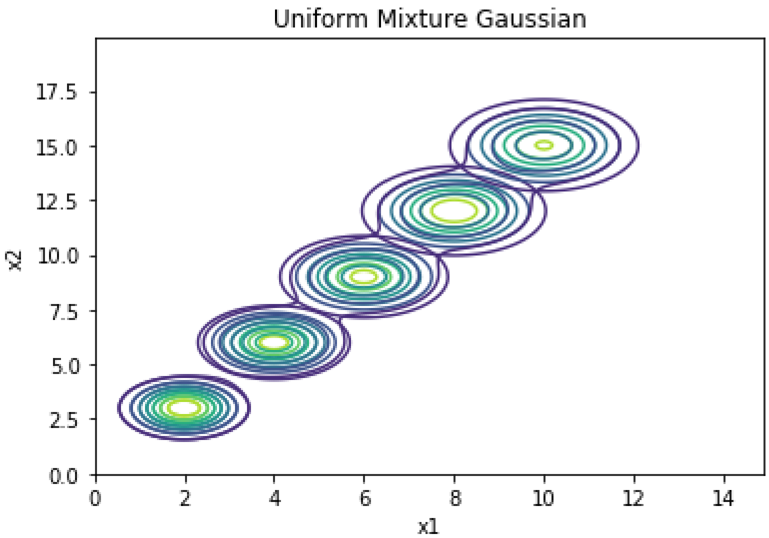
Sampled Points:



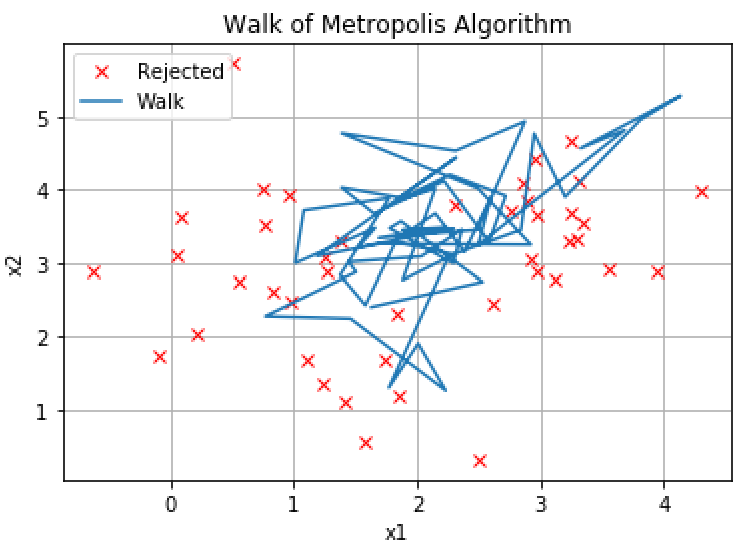
Run 7 of Algorithm

- n = 100
- K = 5
- $sigma$ = .75

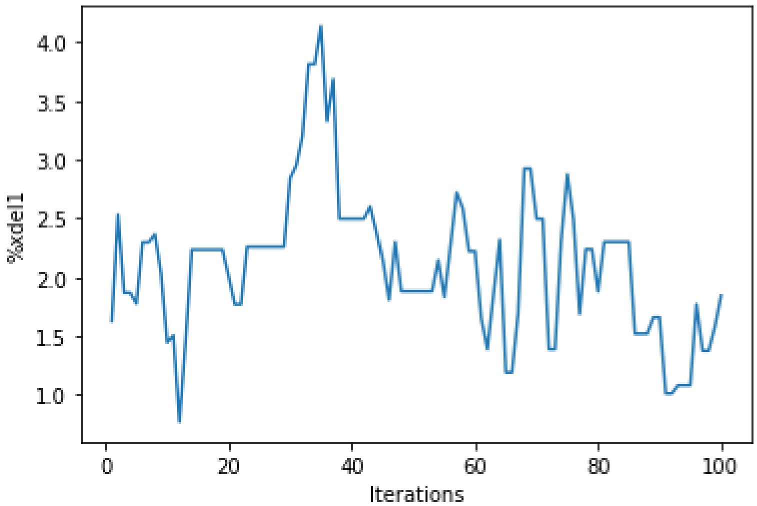
In [13]:
```
run_algo(100, 5, 0.75)
```



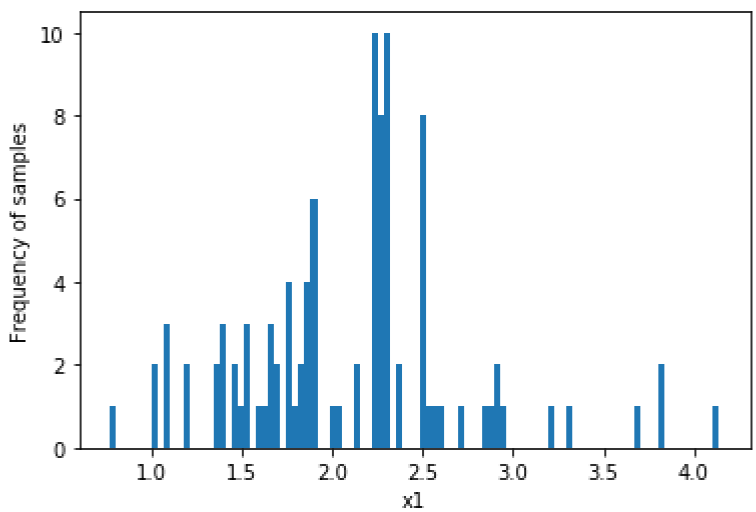

```
Visualized Data on (n, sigma, K): 100 0.75 5
```



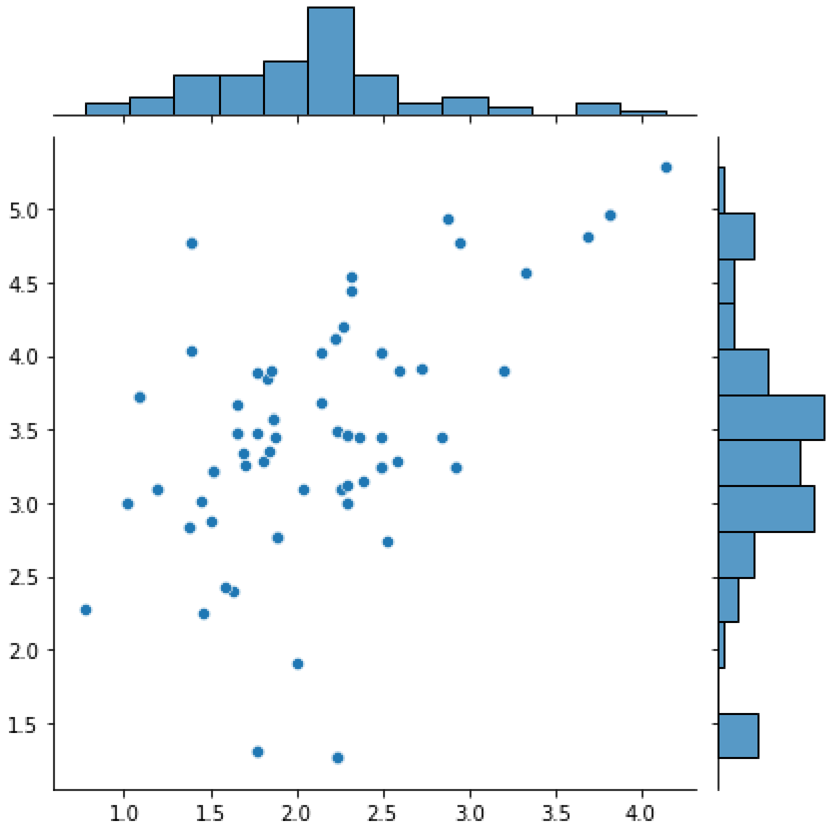

```
First Direction Plot against no. of iters:
```



First Direction Frequency Plot against no. of iters:



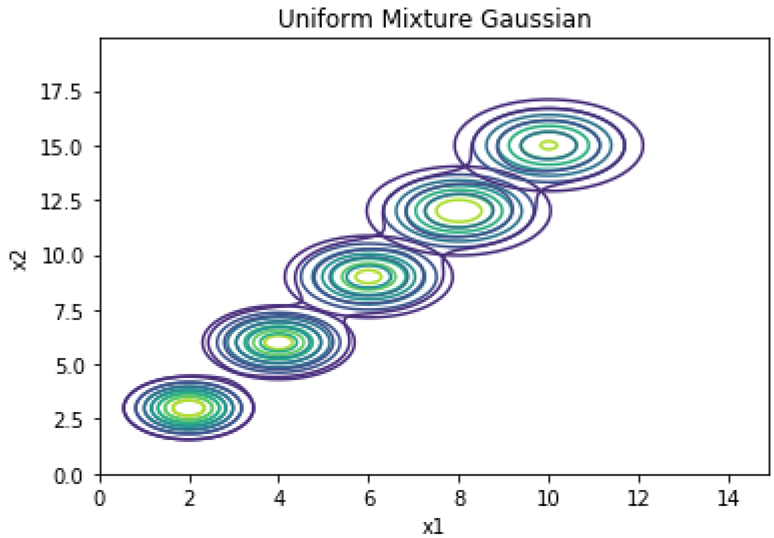
Sampled Points:



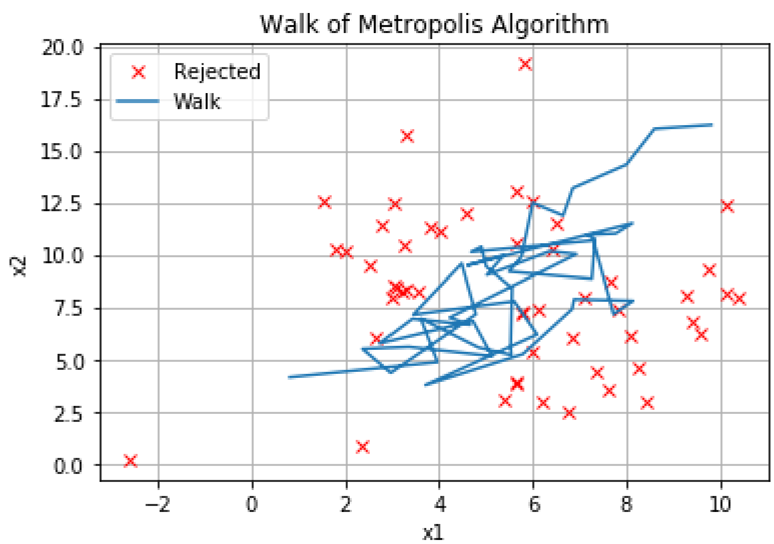
Run 8 of Algorithm

- n = 100
- K = 5
- $sigma$ = 2.0

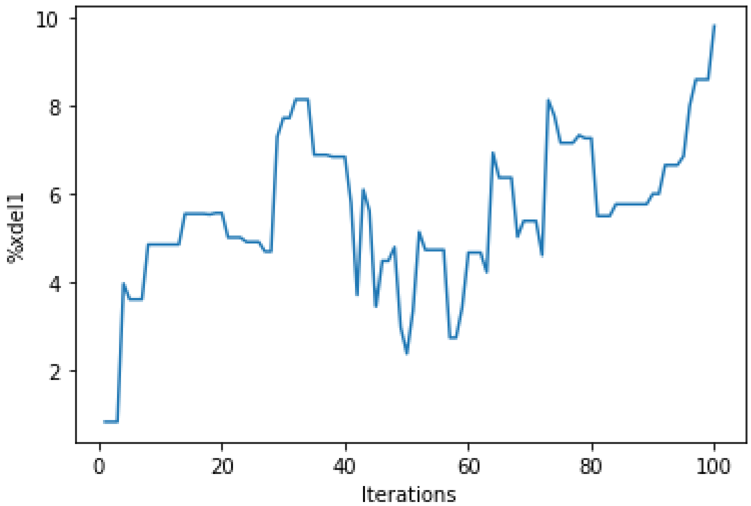
In [14]:
```
run_algo(100, 5, 2.0)
```



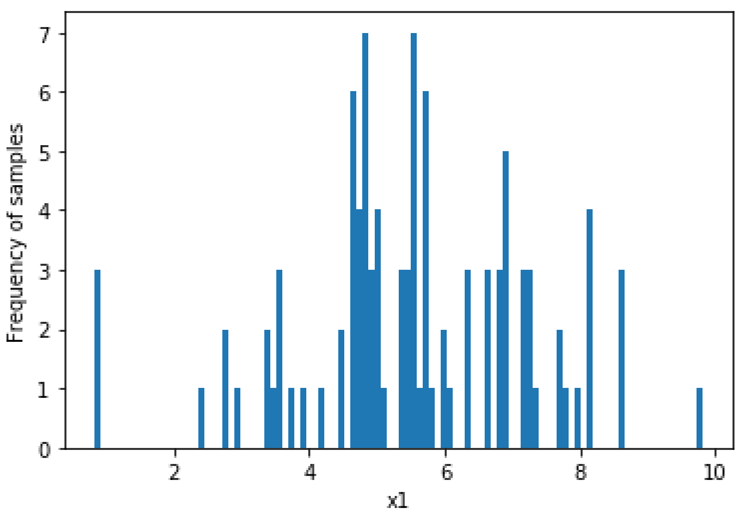

```
Visualized Data on (n, sigma, K): 100 2.0 5
```



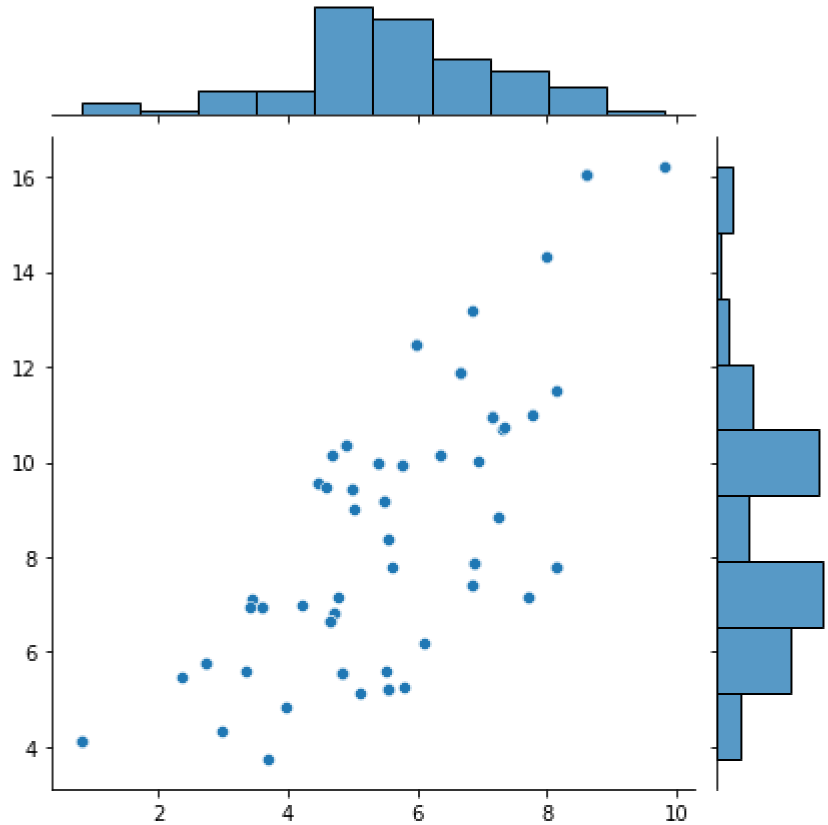

```
First Direction Plot against no. of iters:
```



First Direction Frequency Plot against no. of iters:



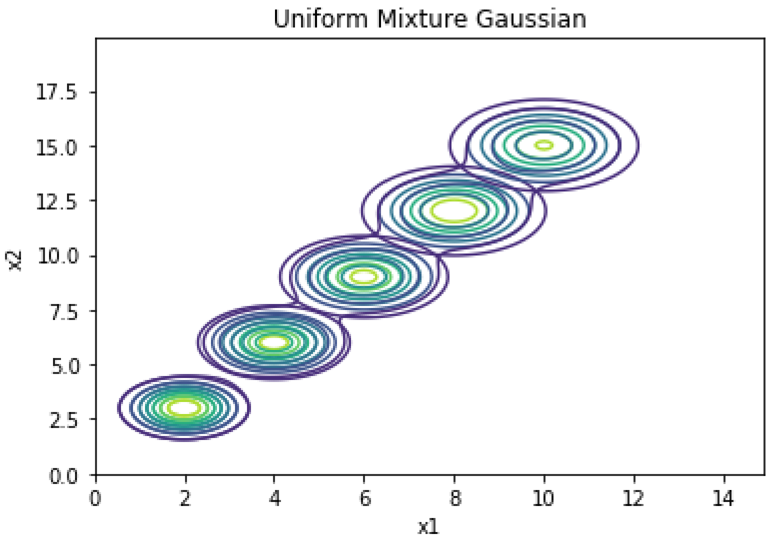
Sampled Points:



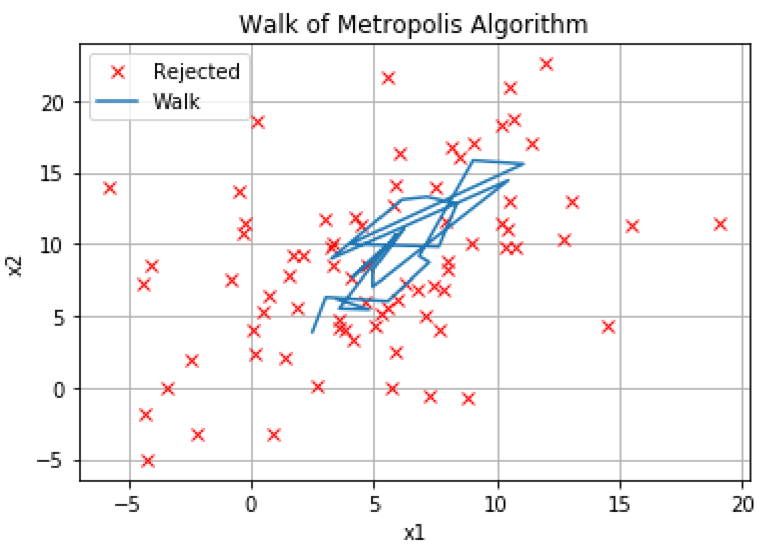
Run 9 of Algorithm

- n = 100
- K = 5
- $sigma$ = 4.0

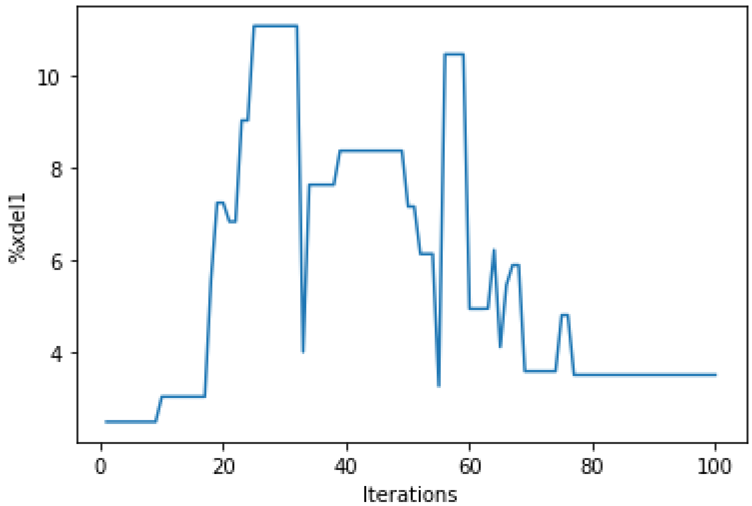
In [15]:
```
run_algo(100, 5, 4.0)
```



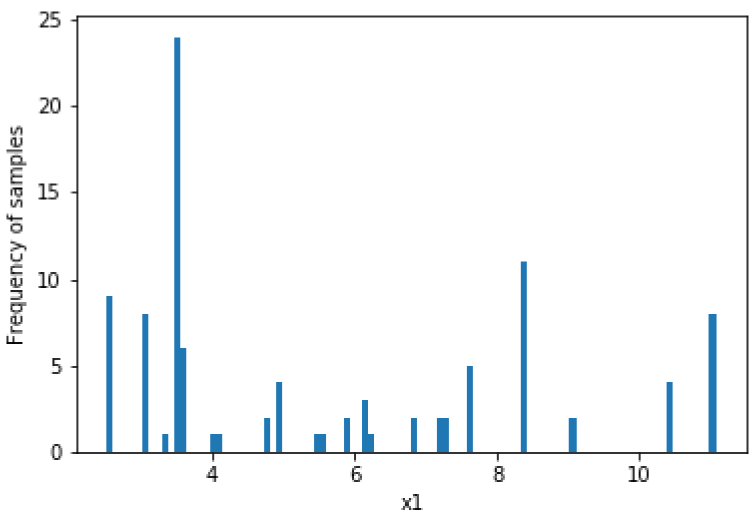

```
Visualized Data on (n, sigma, K): 100 4.0 5
```



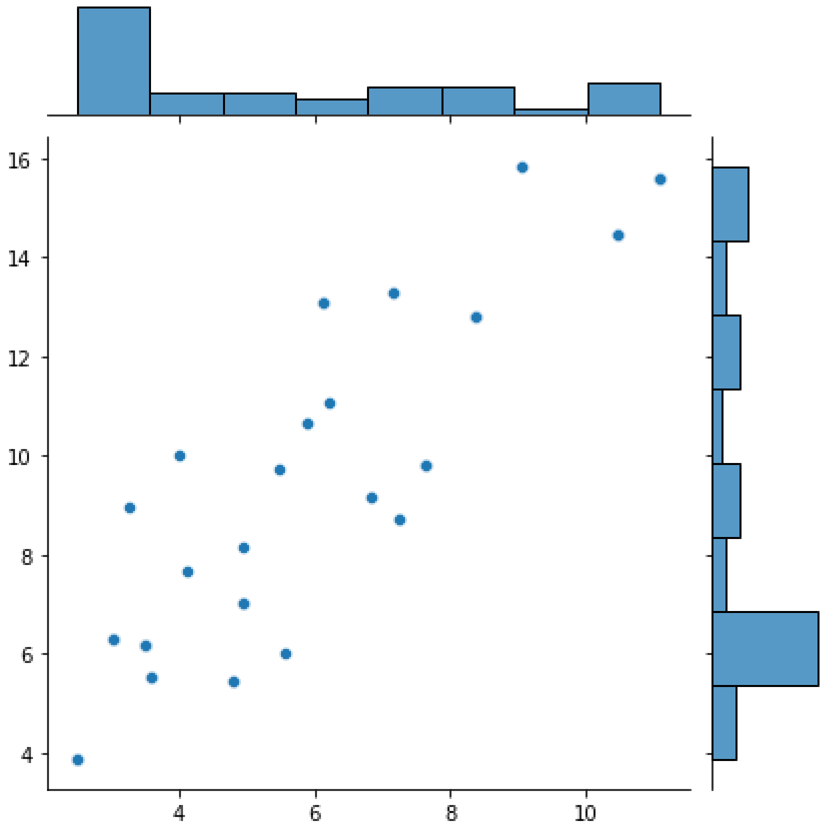

```
First Direction Plot against no. of iters:
```



First Direction Frequency Plot against no. of iters:



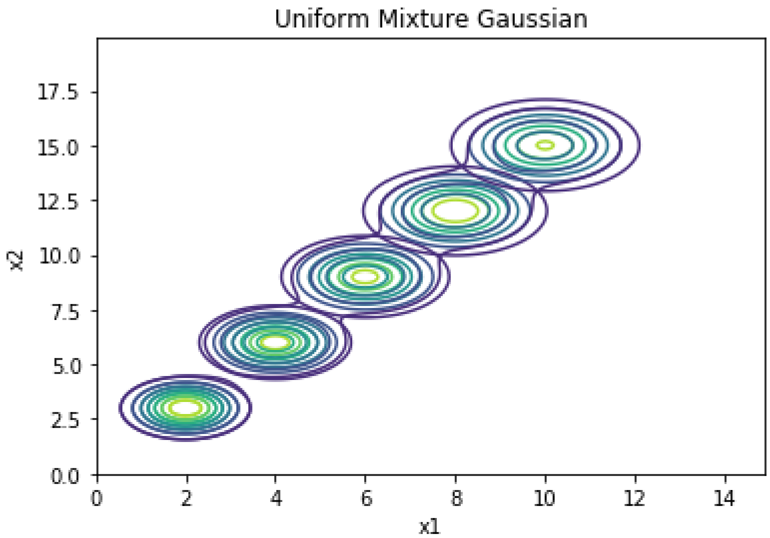
Sampled Points:



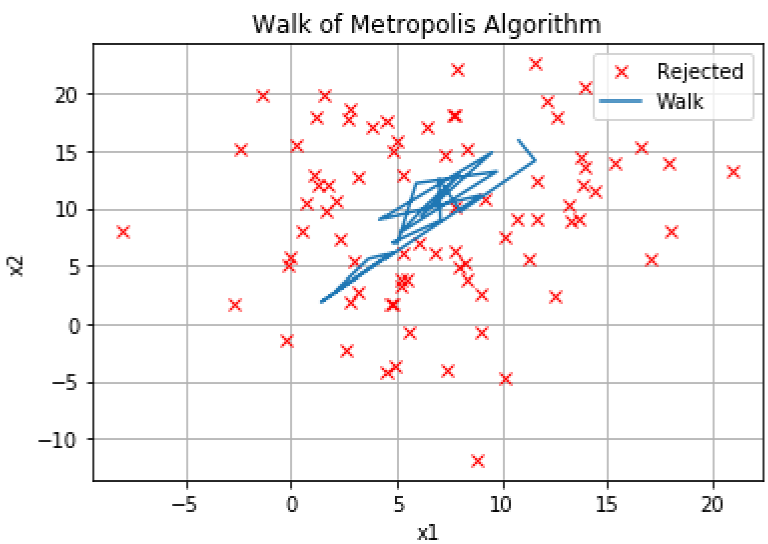
Run 10 of Algorithm

- n = 100
- K = 5
- $sigma$ = 4.5

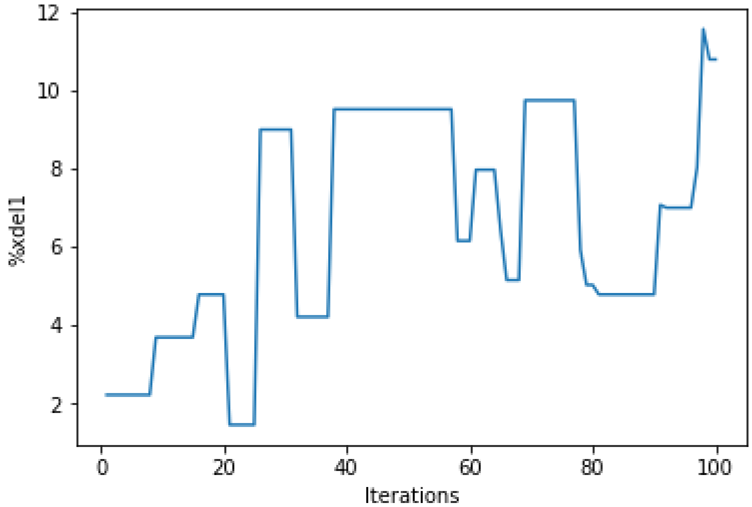
In [16]:
```
run_algo(100, 5, 4.5)
```



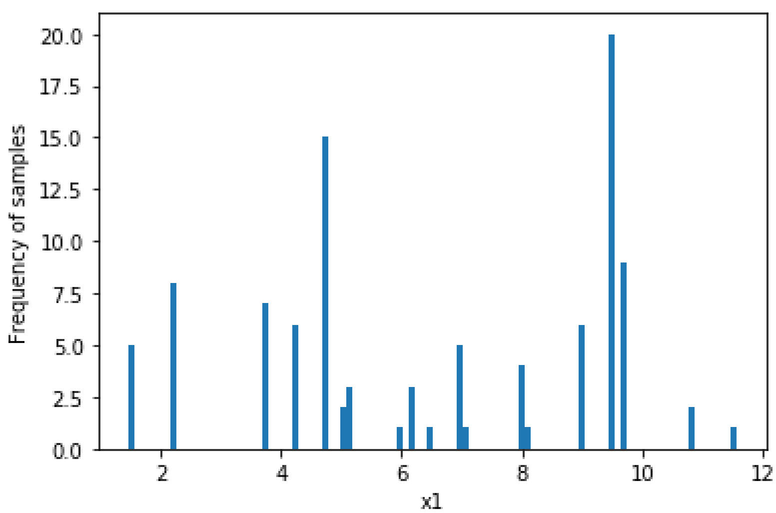

```
Visualized Data on (n, sigma, K): 100 4.5 5
```



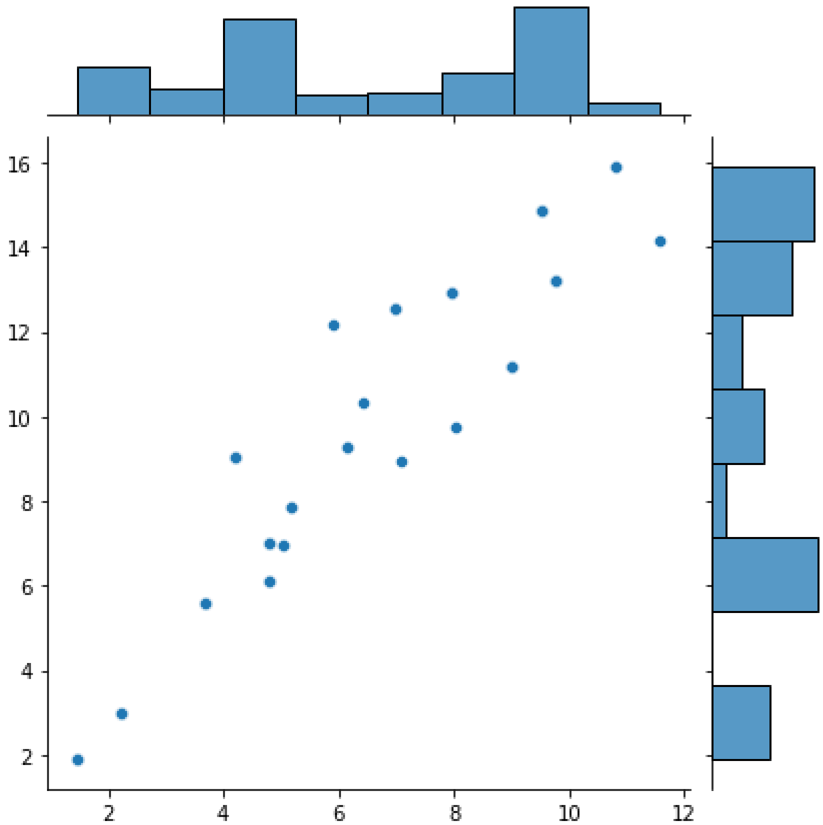

```
First Direction Plot against no. of iters:
```



First Direction Frequency Plot against no. of iters:



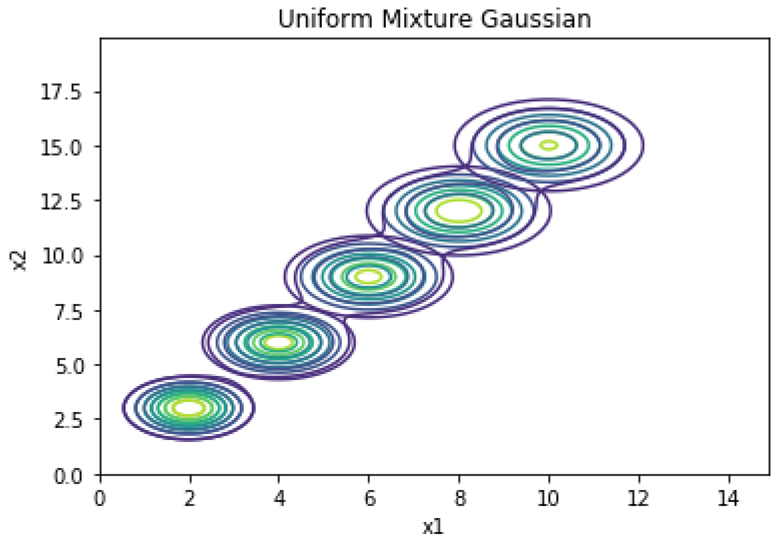
Sampled Points:



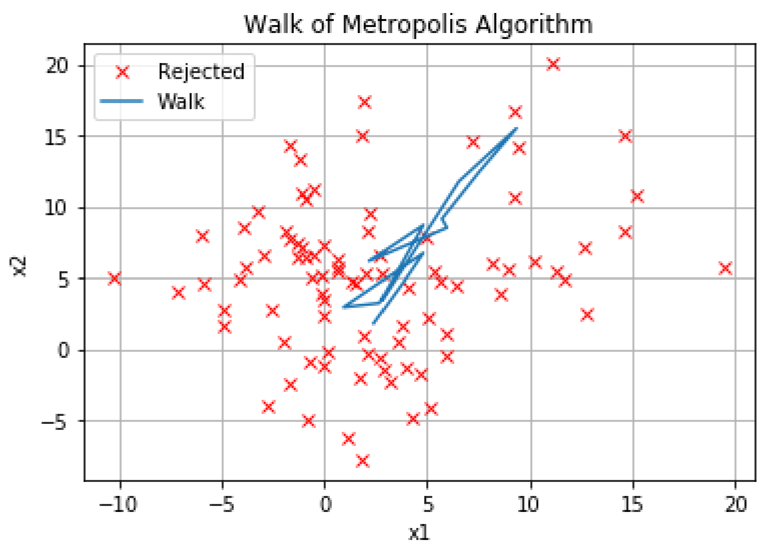
Run 11 of Algorithm

- n = 100
- K = 5
- $sigma$ = 5

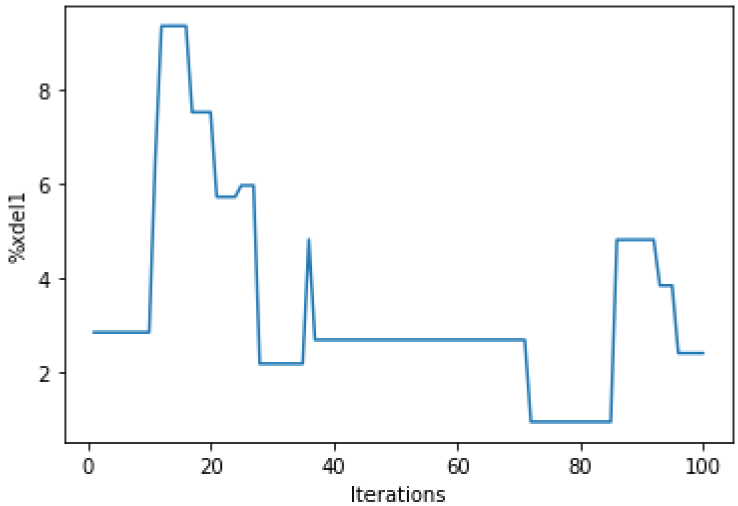
In [17]:
```
run_algo(100, 5, 5.0)
```



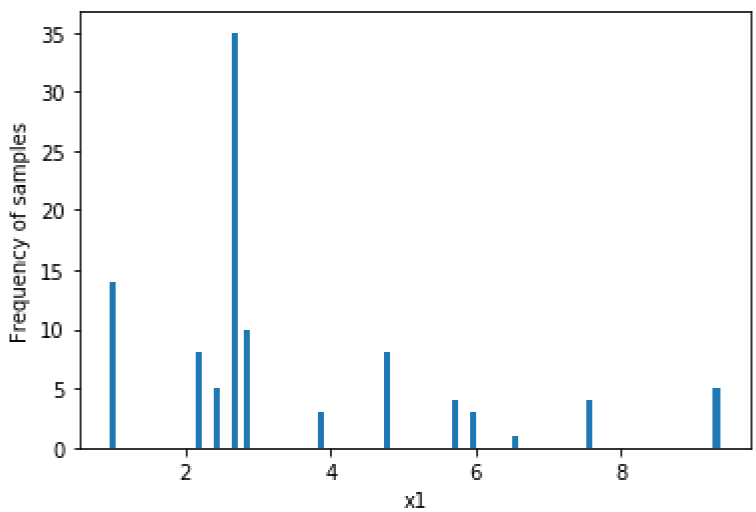

```
Visualized Data on (n, sigma, K): 100 5.0 5
```



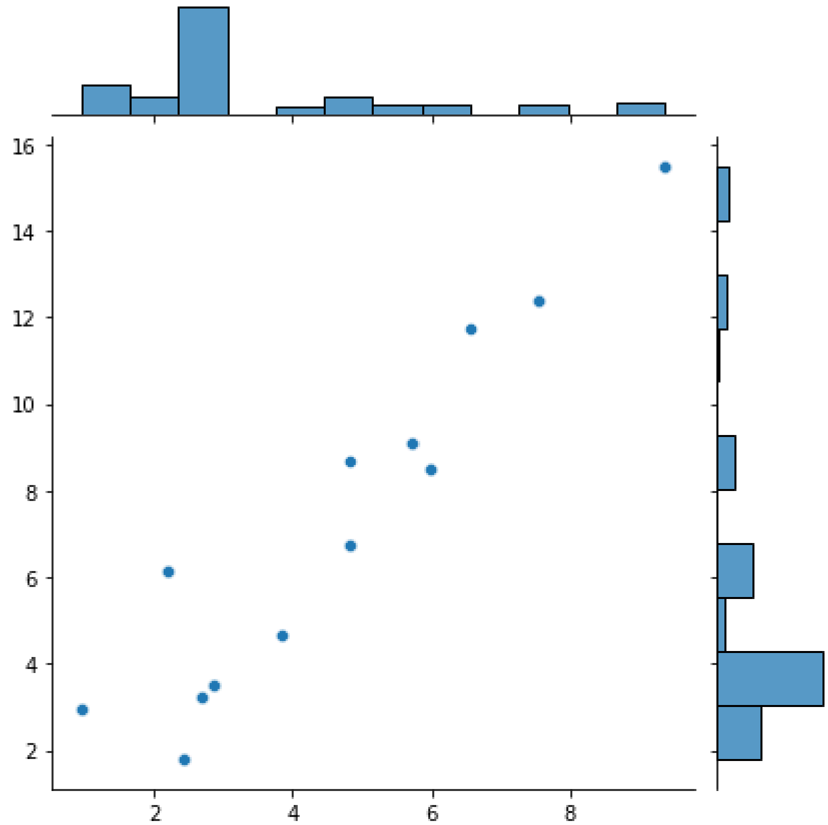

```
First Direction Plot against no. of iters:
```



First Direction Frequency Plot against no. of iters:



Sampled Points:

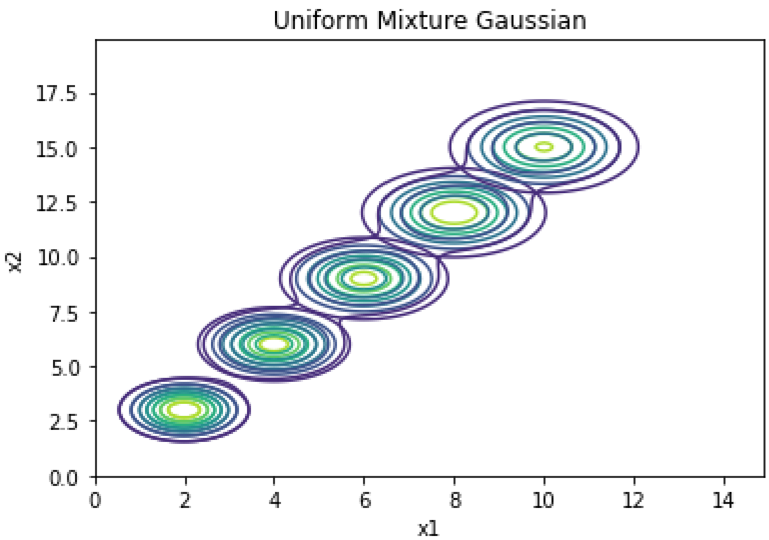


Run 12 of Algorithm

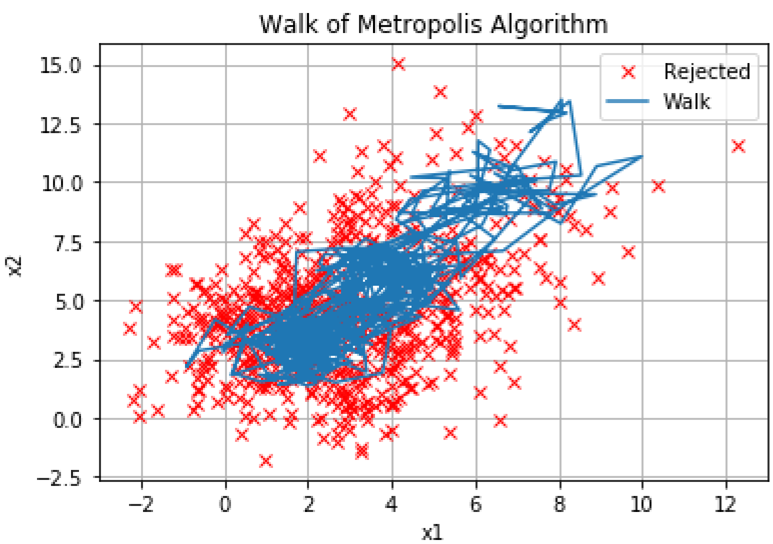
- n = user-defined
- K = user-defined
- $sigma$ = user-defined

In [18]:
```
n1, K1, sigma1 = usr_in()
run_algo(n1, K1, sigma1)
```

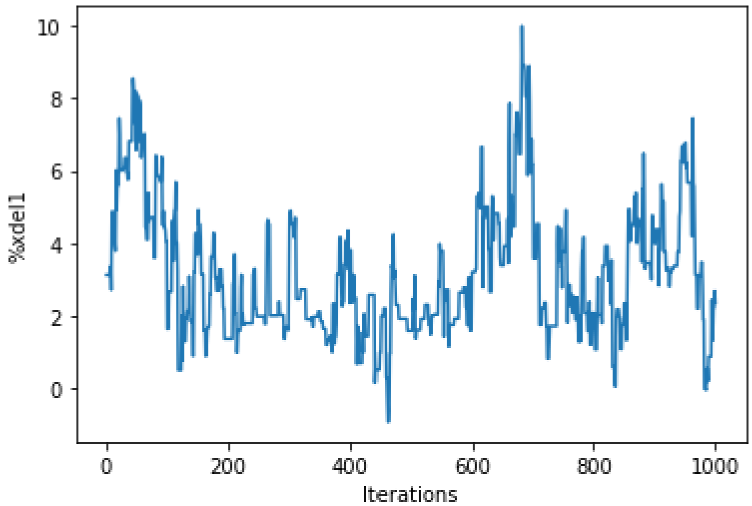
```
n - 1000
K - 5
sigma - 1.375
```



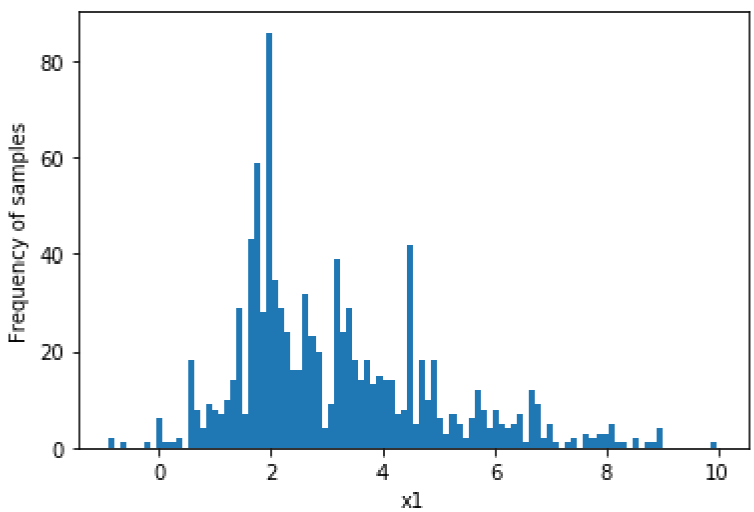

```
Visualized Data on (n, sigma, K): 1000 1.375 5
```



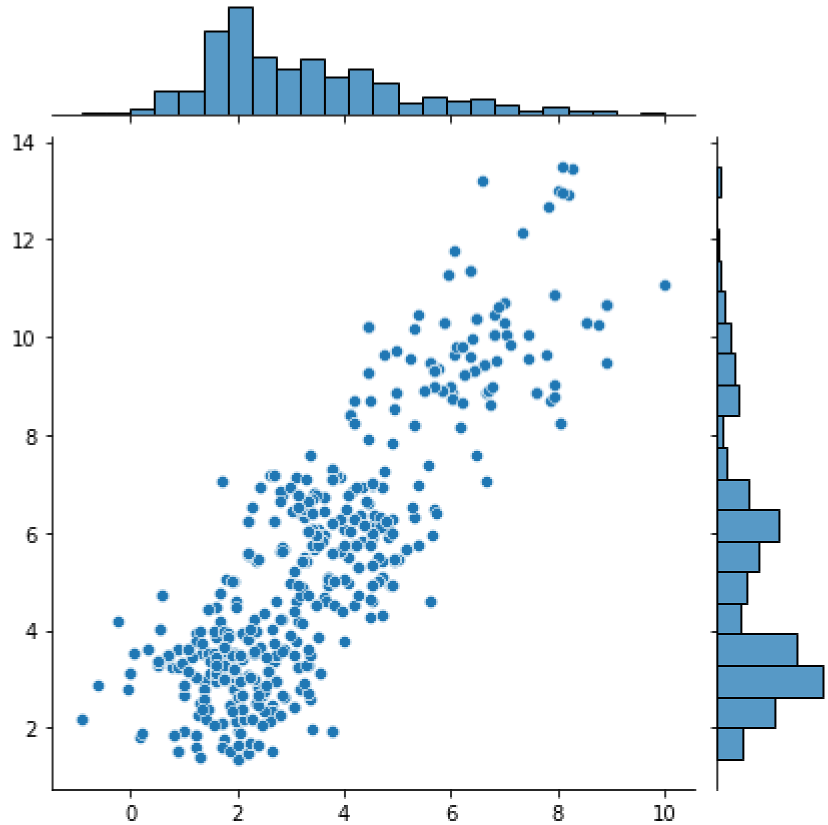

```
First Direction Plot against no. of iters:
```



First Direction Frequency Plot against no. of iters:



Sampled Points:


14
12
10
8
6
4
2
0
2
4
6
8
10