Combinatorics Problem Set

(1) Can there be drawn on a circle of radius 1 a number of 1975 distinct points, so that the distance (measured on the chord) between any two points is a rational number?

(2) Prove that from a set of ten distinct two-digit numbers, it is always possible to find two disjoint subsets whose members have the same sum.

(3) Prove that for every positive integer m we can find a finite set S of points in the plane, such that given any point A of S, there are exactly m points in S at unit distance from A.

(4) Given a finite set of points in the plane, each with integer coordinates, is it always possible to color the points red or white so that for any straight line L parallel to one of the coordinate axes the difference (in absolute value) between the numbers of white and red points on L is not greater than 1?

(5) Consider 9 points in space, no four of which are coplanar. Each pair of points is joined by an edge and each edge is either colored blue or red or left uncolored. Find the smallest value of n such that whenever exactly n edges are colored, the set of colored edges necessarily contains a triangle all of whose edges have the same color.