Cartesian Geometry Problem Set
(Arranged approximately by increasing difficulty)

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1. Prove that the line joining the midpoints of two adjacent sides of a triangle is parallel to the third side and half its length.

2. Prove that the three medians of a triangle are concurrent.

3. In $\triangle ABC$, $\angle A$ is right, $AB = 3$, $BC = 5$, and E is the midpoint of BC. Point D lies on AC such that $CD = 1$. Find the area of quadrilateral $CEFD$. (IMC 2020)

4. In square $ABCE$, $AF = 3FE$ and $CD = 3ED$. Find the ratio of the area of $\triangle BFD$ to the area of the square. (BDMO NAT)

5. Points K, L, M, N lie on sides AB, BC, CD, DA of square $ABCD$ such that area$(KLMN)$ is half of area$(ABCD)$. Prove some diagonal of $KLMN$ is parallel to a side of $ABCD$. (IGO 2021)

6. Points A, B, C lie on a line in this order. AB is diameter of semicircle ω_1, AC of semicircle ω_2, both on same side. Point D on ω_2 satisfies $BD \perp AC$. A circle centered at B with radius BD meets ω_1 at E. Point F on AC satisfies $EF \perp AC$. Prove $BC = BF$. (BDMO 2023)

7. Let ABC be a triangle with $AB = AC$ and orthocenter H. E is midpoint of AC, and D lies on BC with $3CD = BC$. Prove $BE \perp HD$. (IGO 2021)

8. In $\triangle ABC$ with orthocenter H and $AB = 13$, $BC = 14$, $CA = 15$, let G_A be centroid of $\triangle HBC$, and define G_B, G_C similarly. Find area$(G_A G_B G_C)$. (HMMT 2015)

9. In square $ABCD$, a circle with diameter AB and a circle centered at C with radius CB meet inside the square at P. Prove $DP = 2AP$. (BDMO NAT 2017)

10. Arcs AC and BC have centers at B and A. A circle tangent to both arcs and segment AB exists. Arc BC has length 12. Find the tangent circle's circumference. (BDMO NAT 2017)