Linear control system design SSY285

Assignment $M1$

Dynamic model of DC motor with flywheel

Pre-approval of solution is mandatory before submission from TA (tutorial session)

Problem

Consider the system shown in Figure 1 which consists of an electric DC-motor (having separate magnetization) that drives a flywheel and that is influenced by external torque.

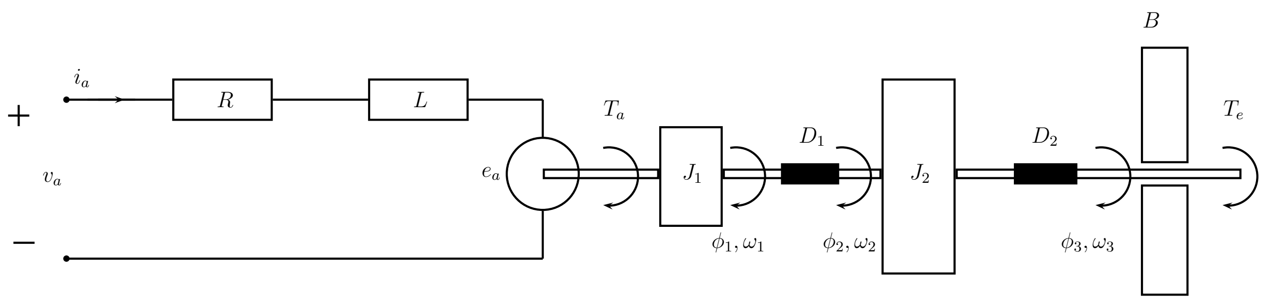

Figure 1: DC motor with flywheel

The system is charachterized by the following variables and parameters such as external voltage applied to the rotor v_1, rotor current , i_a, induced rotor voltage, e_a, rotor produced torque T_a, external torque applied to flywheel axis T_e, angles ϕ_1, ϕ_2, ϕ_3, angular speeds $\omega_1, \omega_2, \omega_3$.

Moreover, L is the rotor inductance, R is the rotor resistance, K_E is the coefficient related to induced voltage e_a to rotor speed, K_T is the rotor torque constant (driving torque to rotor current i), J_1 is the rotor inertia, J_2 is the flywheel inertia. Both D_1 and D_2 represent flexibility of the axis ("torsional springs") on each side of flywheel, B denotes dynamic (linear) friction proportional to the angular speed.

Questions

a) Given the relations $e_a = K_E\omega_1$ and $T_a = K_T i_a$, formulate a mathematical model for this system, using basic laws of electricity and mechanics. How many linear differential equations and at what order are needed to describe this system completely? Which are the system inputs in this case?

b) Assume that the inductance is very small, $L \approx 0$, then choose state variables to compose a state vector $x(t)$, and inputs for vector $u(t)$ of the underlying system. Pay attention to select the state and control inputs such that you formulate a continuous time state equation under the form of $\dot{x}(t) = Ax(t) + Bu(t)$. What are the matrices A and B?

c) The output $y(t)$ of the system in a state space model is related to state and input, as $y(t) = Cx(t) + Du(t)$. Give the matrices C and D for the following two cases; (1) $y_1(t) = \phi_2, y_2(t) = \omega_2$ and (2) $y_1(t) = i_a, y_2 = \omega_3$!

d) Assume the following parameter values:

$$R = 1\Omega,\ \ K_E = 10^{-1}Vs/rad,\ \ K_T = 10^{-1}Nm/A,\ \ J_1 = 10^{-5}kgm^2,\ \ J_2 = 4 \cdot 10^{-5}kgm^2$$
$$B = 2 \cdot 10^{-3}Nms,\ \ D_1 = 20Nm/rad,\ \ D_2 = 2Nm/rad$$

Calculate (with Matlab) the eigenvalues of the A-matrix. Is the system input-output stable for both of the subcases (1) and (2) of c)?

e) Suppose that the initial state, the applied rotor voltage and the external torque are all zero. Then the applied rotor voltage is stepwise changed from 0 to 10 Volt. Investigate by simulation, how the state vector components evolve in time. Suppose that all angular velocities have reached the steady state (angles are of course increasing unboundedly). Suddenly an external (speed reducing) torque of $0.1Nm$ is applied stepwise. Simulate the angular velocities as function of time.

f) Assume that the external torque is zero. Give the transfer function from the input, applied rotor voltage, to the output, corresponding to the case (2) in subproblem c). Calculate the poles and transmission zeros in this case. Is the system of minimum phase?

Pre-approval of solution is mandatory before submission from TA (tutorial session)