Linear control system design SSY285

Assignment $M3$

Linear state estimation and control of DC-motor with flywheel

Pre-approval of solution is mandatory before submission from TA (tutorial session)

Problem

Consider the DC-motor with flywheel modeled and analyzed in the previous assignment $M2$. Starting point of this assignment is the sampled data linear state equation obtained in assignment $M2$ by (d, e). The angle and angular velocity of the flywheel are both assumed to be measured in this assignment.

Questions

a) Assume that (discrete time) white noises are added to both inputs, to the external torque T_e and to the applied motor voltage v_a. Suppose they are zero mean and uncorrelated sequences. The voltage disturbances, mostly due to variations in the power supply unit, are upper bounded by 0.3 V (normally distributed noise, account for 99.7% of the realisation as bound). The torque disturbance (normally distributed noise, account for 99.7% of the realisation as bound) is estimated to be less than 10% of the maximum applied external torque value (assumed to be 1 Nm). Propose a covariance matrix for the disturbance vector w (having the above two components). Which N-matrix should be used in $x(t+1) = Ax(t) + Bu(t) + Nw(t)$?

b) Measurement disturbances n_1, n_2 are added to the output. The disturbances are upper bounded by 0.02 radian and 0.01 radian per second, and are assumed to be discrete time, zero mean uncorrelated white noises (normally distributed, account for 99.7% of the realisation for bounds). Propose a covariance matrix for the measurement disturbance vector n.

c) Provided the cross spectrum between w and n is zero, compute a (discrete time) Kalman filter to estimate (the "current") $\hat{x}(k|k)$ of the system $x(k+1) = Ax(k) + Bu(k) + Nw(k), y(k) = Cx(k) + n(k)$. Find the the observer gain matrix, L! What is the covariance matrix of the state estimation error, P? What are the observer eigenvalues in this case?

d) Design a *discrete time* Linear Quadratic Gaussian controller and simulate the closed-loop answer to a step r_{ω_2} (jumping from an initial value 10 to 100) for the discrete time and noise corrupted system above. Use the previously computed Kalman filter gain, answered in *c*) to reconstruct system states. For the LQ controller, choose and use two appropriate dimensional weighting matrices Q_u and Q_x provided that you know the control input energy is "expensive".

   Hint: You may chose to use either integral states or a prefilter to ensure the reference is followed. However there may occur numerical problems when calculating the prefilter.

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