fluid

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Citing

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1 Flow past a cylinder

Let ρ_0 be the radius of the cylinder, ρ, ξ the polar coordinate system in use. One possible solution of this problem have these potential and streamline functions:

$$\phi(\rho, \xi) = \left(\rho + \frac{\rho_0^2}{\rho}\right) \cos \xi \tag{1}$$

$$\psi(\rho, \xi) = \left(\rho - \frac{\rho_0^2}{\rho}\right) \sin \xi \tag{2}$$

Although these equations are deeply coupled, the radius ρ and the phase ξ can be obtained as a function of the other quantities. For our purposes, we use ψ.

$$\rho(\psi, \xi) = \frac{\psi + \sqrt{\psi^2 + 4\rho_0^2 \sin^2 \xi}}{2 \sin \xi} \tag{3}$$

$$\xi(\psi, \rho) = \arcsin\left(\frac{\rho\, \psi}{\rho^2 - \rho_0^2}\right) \tag{4}$$

2 Conformal mapping

From the reference plane, which is equivalent to a two-pole machine, we use a complex map to obtain the quantities in the actual plane. Let p be the number of pole pairs. Then:

$$\begin{aligned} \zeta &\xrightarrow{\mathcal{M}} z = \sqrt[p]{\zeta} \\ \rho\,\mathrm{e}^{\mathrm{j}\xi} &\xrightarrow{\mathcal{M}} r\,\mathrm{e}^{\mathrm{j}\vartheta} = \sqrt[p]{\rho}\,\mathrm{e}^{\mathrm{j}\xi/p} \\ \chi + \mathrm{j}\eta &\xrightarrow{\mathcal{M}} x + \mathrm{j}y \end{aligned} \tag{5}$$

It is easy to find the inverse map:

$$\mathcal{M}\colon \sqrt[p]{\cdot} \qquad \mathcal{M}^{-1}\colon (.)^p \tag{6}$$

3 Computation of flux-barrier base points

Let the flux-barrier and flux-carrier thicknesses be given. Then the base points for the flux-barriers can be computed easily. Let D_r be the rotor outer diameter, $w_\mathrm{rib,t}$ the tangential iron rib width, $w_{\mathrm{c},k}$ the k-th flux-carrier width, and $t_{\mathrm{c},k}$ the k-th flux-barrier thickness.[1] Then

$$\begin{aligned} R_\mathrm{r} &= \frac{D_\mathrm{r}}{2} - w_\mathrm{rib,t} \\ R_{\mathsf{A}_1} &= R_\mathrm{r} - w_{\mathrm{c},1} \\ R_{\mathsf{B}_1} &= R_{\mathsf{A}_1} - t_{\mathrm{c},1} \\ &\vdots \end{aligned} \tag{7}$$

So, in general:

$$\begin{aligned} R_{\mathsf{A}_\mathsf{k}} &= R_{\mathsf{B}_{\mathsf{k}-1}} - w_{\mathrm{c},k} \\ R_{\mathsf{B}_\mathsf{k}} &= R_{\mathsf{A}_{\mathsf{k}-1}} - t_{\mathrm{c},k} \end{aligned} \tag{8}$$

with the exception $R_{\mathsf{B}_0} = R_\mathrm{r}$.

[1] You may wonder why the main dimensions of the flux-carrier and flux-barrier differ in the name (width versus thickness). This is due to a choice of mine, because I prefer to refer to width when the flux flows perpendicularly to the dimension, and to thickness when it flows in parallel.

Now we know both the radii and the angle – always $\pi/(2p)$ – of the flux-barrier internal points. So we can compute their respective streamline value.

3.1 Magnet insertion

$$w_{\mathrm{rib},k} = w_{\mathrm{rib},k} + w_{\mathrm{m},k}$$

where $w_{\mathrm{m},k}$ is the k-th magnet width.

3.2 Central base points

We refer to points A and B.

The line describing the q-axis is

$$\begin{aligned} y &= mx + q \\ m &= \tan\frac{\pi}{2p} \\ q &= \frac{w_{\mathrm{rib}}}{\cos\frac{\pi}{2p}} \end{aligned} \tag{9}$$

$$\begin{cases} y_{\mathsf{A}'} - m x_{\mathsf{A}'} - q = 0 \\ x_{\mathsf{A}'} - r_{\mathsf{A}'}(\psi_{\mathsf{A}}, \vartheta_{\mathsf{A}'}) \cos\vartheta_{\mathsf{A}'} = 0 \\ y_{\mathsf{A}'} - r_{\mathsf{A}'}(\psi_{\mathsf{A}}, \vartheta_{\mathsf{A}'}) \sin\vartheta_{\mathsf{A}'} = 0 \end{cases} \tag{10}$$

where ϑ_{A} is used as the third degree of freedom and r_{A} is then a function of it. The solution of such system can be determined solving the single equation

$$r_{\mathsf{A}'}(\psi_{\mathsf{A}}, \vartheta_{\mathsf{A}'}) \big(\sin\vartheta_{\mathsf{A}'} - m\cos\vartheta_{\mathsf{A}'}\big) - q = 0 \tag{11}$$

in the unknown $\vartheta_{\mathsf{A}'}$.

The same equation can be written for point B with the proper substitution and repeated for all the flux-barriers.

4 Outer base points

We refer to points C, D, and E. If the flux-barrier angle, α_{b}, is given, then

$$\begin{aligned} x_{\mathsf{E}} &= R_r \cos(\tfrac{\pi}{2p} - \alpha_{\mathrm{b}}) \\ y_{\mathsf{E}} &= R_r \sin(\tfrac{\pi}{2p} - \alpha_{\mathrm{b}}) \end{aligned} \tag{12}$$

Points C and D results from the connection of the flux-barrier sideline and point E. This connection should be as smooth as possible in order to avoid dangerous mechanical stress concentrations. We are going to use circular arcs to make this connection. So we impose the tangency between the flux-barrier sideline and the arc, between the arc and the radius through point E.