

## **CBA: Practice Problem Set 2**

### **Topics: Sampling Distributions and Central Limit Theorem**

1. Examine the following normal Quantile plots carefully. Which of these plots indicates that the data

...

- I. Are nearly normal?

ANS: C

- II. Have a bimodal distribution? (One way to recognize a bimodal shape is a “gap” in the spacing of adjacent data values.)

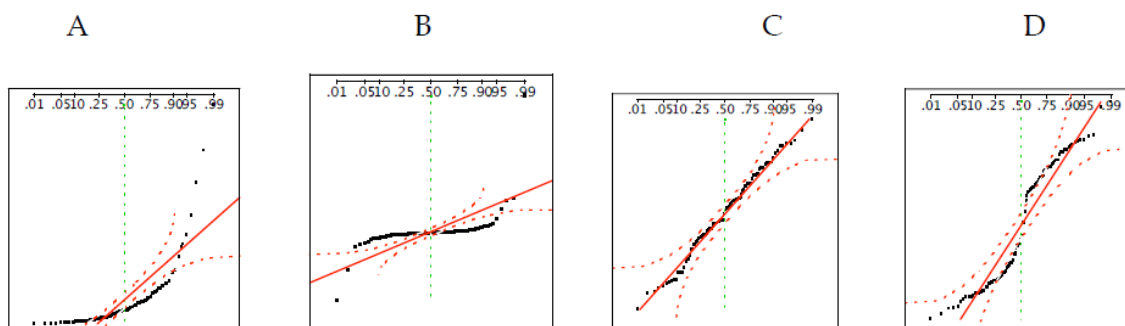
ANS: D

- III. Are skewed (i.e. not symmetric) ?

ANS : A

- IV. Have outliers on both sides of the center?

ANS: B



2. For each of the following statements, indicate whether it is True/False. If false, explain why.

The manager of a warehouse monitors the volume of shipments made by the delivery team. The automated tracking system tracks every package as it moves through the facility. A sample of 25 packages is selected and weighed every day. Based on current contracts with customers, the weights should have  $\mu = 22$  lbs. and  $\sigma = 5$  lbs.

- (i) Before using a normal model for the sampling distribution of the average package weights, the manager must confirm that weights of individual packages are normally distributed.

ANS: False

As long as the sample size is sufficiently large ( $n \geq 30$ ), the manager can rely on the normal model for the sampling distribution of the average package weights, even if the individual package weights are not normally distributed.

- (ii) The standard error of the daily average  $SE(\bar{x}) = 1$ .

ANS: True

The standard deviation of the population ( $\sigma$ ) is given as 5 lbs, and the sample size ( $n$ ) is 25 packages. Let's calculate the standard error:

$$SE(\bar{x}) = 5 / \sqrt{25} = 5 / 5 = 1$$

3. Auditors at a small community bank randomly sample 100 withdrawal transactions made during the week at an ATM machine located near the bank's main branch. Over the past 2 years, the average withdrawal amount has been \$50 with a standard deviation of \$40. Since audit investigations are typically expensive, the auditors decide to not initiate further investigations if the mean transaction amount of the sample is between \$45 and \$55. What is the probability that in any given week, there will be an investigation?

- A. 1.25%
- B. 2.5%
- C. 10.55%
- D. 21.1%**
- E. 50%

ANS: D

Given: Population mean ( $\mu$ ) = \$50 Population standard deviation ( $\sigma$ ) = \$40 Sample size ( $n$ ) = 100  
Sample mean ( $\bar{x}$ ) follows a normal distribution.

The standard error of the sample mean (SE) is given by:

$$SE = \sigma / \sqrt{n} = 40 / \sqrt{100} = 40 / 10 = 4$$

Now, we can calculate the Z-scores for the lower and upper bounds of the range:

$$Z_{\text{lower}} = (\text{Lower bound} - \mu) / SE = (45 - 50) / 4 = -1.25$$

$$Z_{\text{upper}} = (\text{Upper bound} - \mu) / SE = (55 - 50) / 4 = 1.25$$

Next, we find the probabilities associated with these Z-scores using a standard normal distribution table:

$$P(Z < -1.25) \approx 0.1056$$

$$P(Z > 1.25) \approx 0.1056$$

Finally, we find the probability that the sample mean falls outside the range \$45 to \$55:

$$P(\text{outside range}) = P(Z < -1.25) + P(Z > 1.25) = 0.1056 + 0.1056 = 0.2112$$

The probability that the sample mean falls outside the range \$45 to \$55 is 0.2112, which is equivalent to 21.12%.

4. The auditors from the above example would like to maintain the probability of investigation to 5%. Which of the following represents the minimum number transactions that they should sample if they do not want to change the thresholds of 45 and 55? Assume that the sample statistics remain unchanged.
- A. 144
  - B. 150
  - C. 196
  - D. 250
  - E. Not enough information

**Ans:** The correct answer is **option E**. Not enough information. The provided answer options do not include the calculated minimum sample size of 171.

5. An educational startup that helps MBA aspirants write their essays is targeting individuals who have taken GMAT in 2012 and have expressed interest in applying to FT top 20 b-schools. There are 40000 such individuals with an average GMAT score of 720 and a standard deviation of 120. The scores are distributed between 650 and 790 with a very long and thin tail towards the higher end resulting in substantial skewness. Which of the following is likely to be true for randomly chosen samples of aspirants?
- A. The standard deviation of the scores within any sample will be 120.
  - B. The standard deviation of the mean of across several samples will be 120.
  - C. The mean score in any sample will be 720.
  - D. The average of the mean across several samples will be 720.
  - E. The standard deviation of the mean across several samples will be 0.60

**Ans: D**

The average (or mean) of the means across several samples, also known as the grand mean, is expected to be close to the population mean (720). As the sample size increases and the number of samples becomes larger, the grand mean will tend to converge to the population mean.