# Calculation of the crosshead velocity in mm/min required to achieve a specified stress rate in MPa s$^{-1}$ or an estimated strain rate in s$^{-1}$

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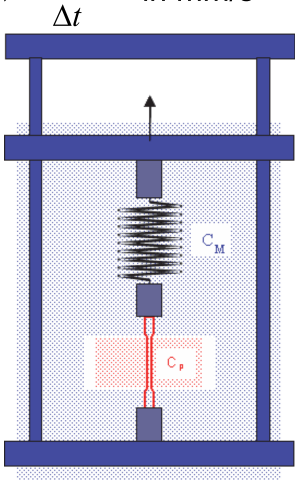
## Basics for setting the test speed

**Specification of crosshead speed**
Every tensile testing machine consists basically of a machine frame, a force-measuring device and fixturing devices. These machine parts undergo elastic deformations in tension. The sum of these elastic deformations describes the compliance $K_M$ of the machine. It represents the reciprocal value of the machine stiffness $C_M$. Figure 1 shows the basic design configuration. The spring represents machine compliance $K_M$ or stiffness $C_M$ respectively. Most electronically controlled testing machines allow the test speed to be specified as a crosshead speed or traverse speed for spindle machines, or as a piston speed for hydraulic machines (in the following, the term crosshead speed will be used for simplicity).

This traverse speed is defined as change of displacement per time interval:

$$v = \frac{\Delta s}{\Delta t} \quad \text{in mm/s} \qquad (1)$$




Configuration of a testing machine

Stiffness of testing equipment $C_M$:
$C_M = f$ (frame, load cell, clamping system, ...)

Stiffness of specimen $C_P$:
$C_P = f$ (slope of stress/strain curve, original cross-sectional area,
parallel length,...)

Stiffness of test configuration $C$:

$$\frac{1}{C} = \frac{1}{C_M} + \frac{1}{C_P}$$

**Speed for deformation of specimen during elastic range**

If the testing machine were equal ideally stiff the crosshead speed to be set on the machine could be calculated using Hooke's Law:

$\sigma = E * \varepsilon$ (1)

with $\varepsilon = \frac{\Delta L}{L_c}$ in m/m the result is:

$\sigma = E * \frac{\Delta L}{L_c}$ → $\Delta L = \frac{\sigma}{E} * L_c$ (2)

with (1) and (2) the result is:

$v = \frac{\delta}{\Delta t} * \frac{L_c}{E} = L_c * \frac{\dot{\sigma}}{E}$ in mm/s

or

$v_{\text{deform of specimen}} = 60 * L_c * \frac{\dot{\sigma}}{E}$ in mm/min (3)

This is the speed required for deformation of the specimen in the elastic range

**Speed for deformation of testing equipment**

In additional to specimen deformation the testing equipment (load frame, load cell, grips, etc.) must also be considered.

This means we must add to the speed for deforming the specimen the following formula for deformation of the equipment:

$\Delta l_{\text{equipment}} = \frac{F[N]}{C_M[N/mm]}$ (4)

Where $C_M$ = stiffness of testing equipment with (1) the result is

$$v_{deform\ equipment} = \frac{\Delta l_{equipment}}{\Delta t} = \frac{F}{C_M * \Delta t}$$

And with $F = \sigma * S_0$ the result is

$$v_{deform\ equipment} = \frac{\sigma * S_0}{\Delta t * C_M} = \dot{\sigma} * \frac{S_0}{C_M} \quad \text{in mm/s}$$

or

$$v_{deform\ equipment} = \dot{\sigma} * \frac{S_0}{C_M} * 60 \quad \text{in mm/min} \qquad (5)$$

**Finally the crosshead speed required to achieve a specified stress rate in the elastic range can be calculated using the formula**

$v_{\text{crosshead}} = v_{\text{deform specimen}} + v_{\text{deform equipment}}$

$$v_{crosshead} = 60 * \dot{\sigma}\left(\frac{L_C}{E} + \frac{S_0}{C_M}\right) \quad \text{in mm/min}$$

with
- $\dot{\sigma}$ = stress rate in MPa/sec
- $L_C$ = grip-to-grip separation (or parallel length of specimen) in mm
- $E$ = Young's modulus (slope of Hooke's Law graph) of specimen in N/mm²
- $S_0$ = cross-section of specimen in mm²
- $C_M$ = stiffness of equipment in N/mm

## Table of calculated speeds in elastic range for practical use

a) $v$ in mm/min for specimen deformation without equipment deformation:
(specimen with parallel length of 120 mm)

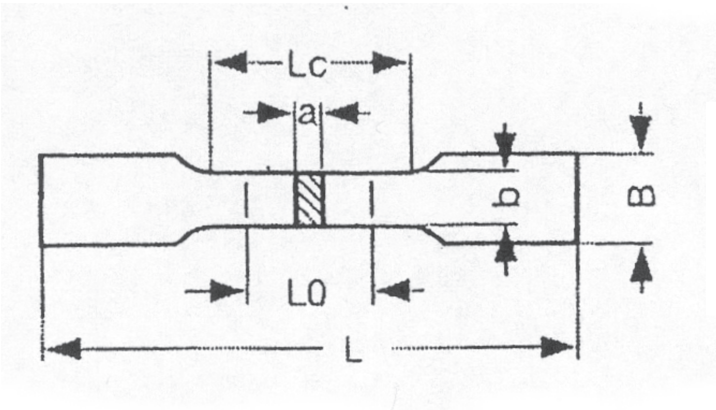
| | **Young's modulus in [N/mm²]** | | |
|---|---|---|---|
| **Stress rate in MPa/s** | **210000** | **175000** | **75000** |
| **30** | 1.02 | 1.23 | 2.88 |
| **20** | 0.68 | 0.82 | 2.92 |
| **10** | 0.34 | 0.41 | 0.96 |

b) $v$ in mm/min for equipment deformation calculated for a stress rate of 30 MPa/s

| | **Cross-section of specimen in mm²** | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Stiffness of equipment in N/mm** | **10** | **20** | **30** | **40** | **50** | **60** | **70** | **80** | **90** | **100** | **150** | **200** |
| **5800** | 3.10 | 6.20 | 9.31 | 12.41 | 15.51 | 18.62 | 21.72 | 24.82 | 27.93 | 31.03 | 46.55 | 62.06 |
| **10000** | 1.8 | 3.6 | 5.4 | 7.2 | 9 | 10.8 | 12.6 | 14.4 | 16.2 | 18 | 27 | 36 |
| **20000** | 0.9 | 1.8 | 2.7 | 3.6 | 4.5 | 5.4 | 6.3 | 7.2 | 8.1 | 9 | 13.5 | 18 |
| **30000** | 0.6 | 1.2 | 1.8 | 2.4 | 3.0 | 3.6 | 4.2 | 4.8 | 5.4 | 6 | 9 | 12 |
| **40000** | 0.45 | 0.9 | 1.35 | 1.8 | 2.25 | 2.7 | 3.15 | 3.6 | 4.05 | 4.5 | 6.75 | 9 |
| **50000** | 0.36 | 0.72 | 1.08 | 1.44 | 1.8 | 2.16 | 2.52 | 2.88 | 3.25 | 3.6 | 5.4 | 7.2 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **60000** | 0.3 | 0.6 | 0.9 | 1.2 | 1.5 | 1.8 | 2.1 | 2.4 | 2.7 | 3.0 | 4.5 | 6.0 |
| **70000** | 0.26 | 0.51 | 0.77 | 1.03 | 1.28 | 4.54 | 1.8 | 2.06 | 2.31 | 2.57 | 3.86 | 5.14 |
| **80000** | 0.23 | 0.45 | 0.67 | 0.9 | 1.13 | 1.35 | 1.58 | 1.8 | 2.03 | 2.25 | 3.37 | 4.5 |
| **90000** | 0.2 | 0.4 | 0.6 | 0.8 | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 | 3 | 4.0 |
| **100000** | 0.18 | 0.36 | 0.54 | 0.72 | 0.9 | 1.08 | 1.26 | 1.44 | 1.62 | 1.8 | 2.7 | 3.6 |
| **110000** | 0.16 | 0.32 | 0.49 | 0.65 | 0.81 | 0.98 | 1.14 | 1.30 | 1.47 | 1.64 | 2.45 | 3.27 |

## Summary of relationship:
## stress-rates ←→ stain-rates ←→ crosshead speeds

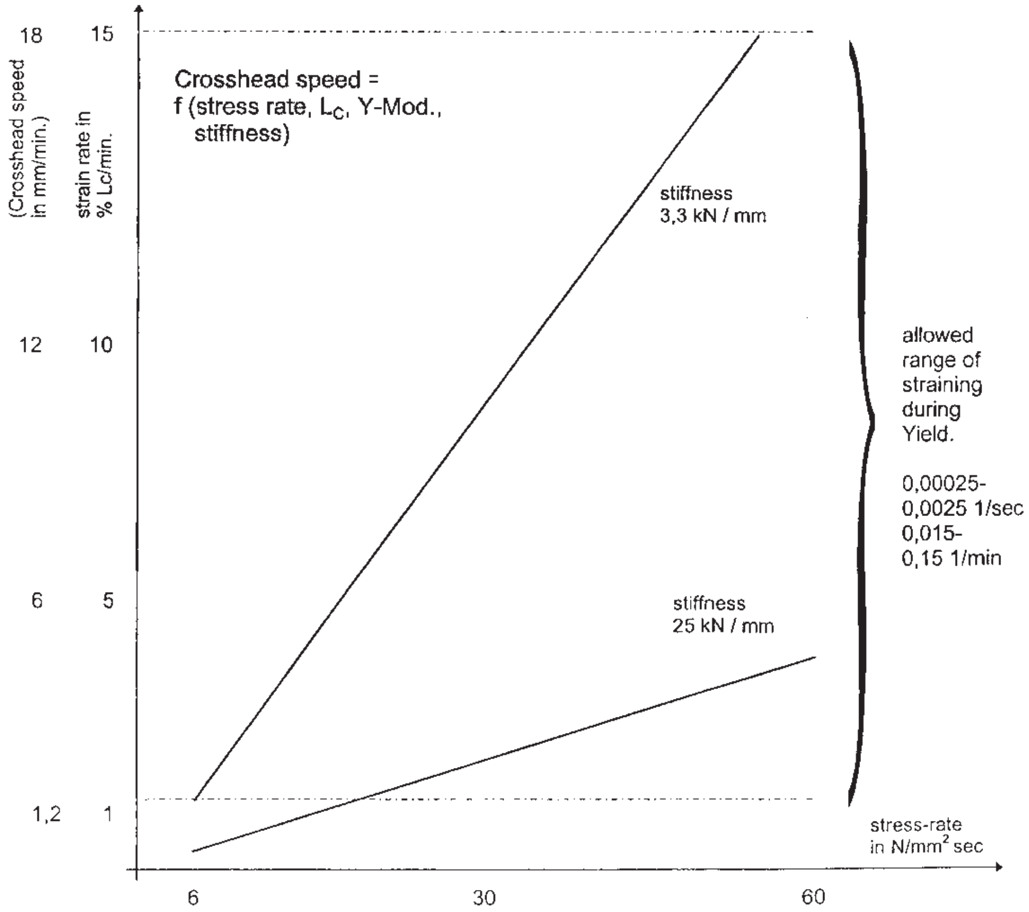



Specimen:
Material St. 15 (Y's mod.=210000 N/mm²)
b = 20mm ; a = 0.81mm → 16.02mm²
$Lc$ = 120mm
Stiffness of test equipment 3.3 or 25kN/mm

| **Stress-rate** | **6 N/mm² sec.** | | **30 N/mm² sec.** | | **60 N/mm² sec.** | |
|---|---|---|---|---|---|---|
| **Stiffness in kN/mm** | 3.3 | 25 | 3.3 | 25 | 3.3 | 25 |
| Crosshead speed for deformation **of specimen** in mm/min | 0.20 | 0.2 | 1.02 | 1.02 | 2.05 | 2.05 |
| Crosshead speed for deformation **of equipment** in mm/min | 1.74 | 0.23 | 8.73 | 1.15 | 17.4 | 2.3 |
| $\sum$ of crosshead speed for deformation in mm/min | 1.94 | 0.43 | 9.75 | 2.17 | 19.45 | 4.35 |
| $\sum$ of crosshead speed in **%** $Lc$ / min | 1.6 | 0.3 | 8.1 | 1.8 | 16.1 | 3.6 |

$\sum$ = speed for equipment + specimen deformation

## Relationship: crosshead speed ←→ strain-rate ←→ stress-rate




## Examples and utilities for calculation of crosshead speed for achieving a specified strain rate

As published in different reportsespecially the $R_p$ or $R_{eh}$ values in tensile tests, based on constant separation of the crossheads within defined stress rate limits, are influenced by the stiffness of the testing equipment and the specimen. To obtain more reproducible results the use of strain rate controlled tests is recommended.

Some test equipment, particularly older versions,  is not capable of controlling the strain rate, so a crosshead speed equivalent to the recommended strain rate can be used.

**On the basis of the above considerations, the crosshead speed required to achieve a specified strain rate can be calculated using the formula:**

$$v_C = 60 \cdot \dot{e}_m \left( \frac{m \cdot S_O}{C_M} + L_C \right) \text{ mm min}^{-1}$$

$v_C$ crosshead separation rate in mm $s^{-1}$

$\dot{e}_m$ resulting strain rate in the specimen in $s^{-1}$

$m$ slope of the stress/strain curve at a given moment of the test (e.g. around the area of interest such as $R_{p0,2}$) in MPa

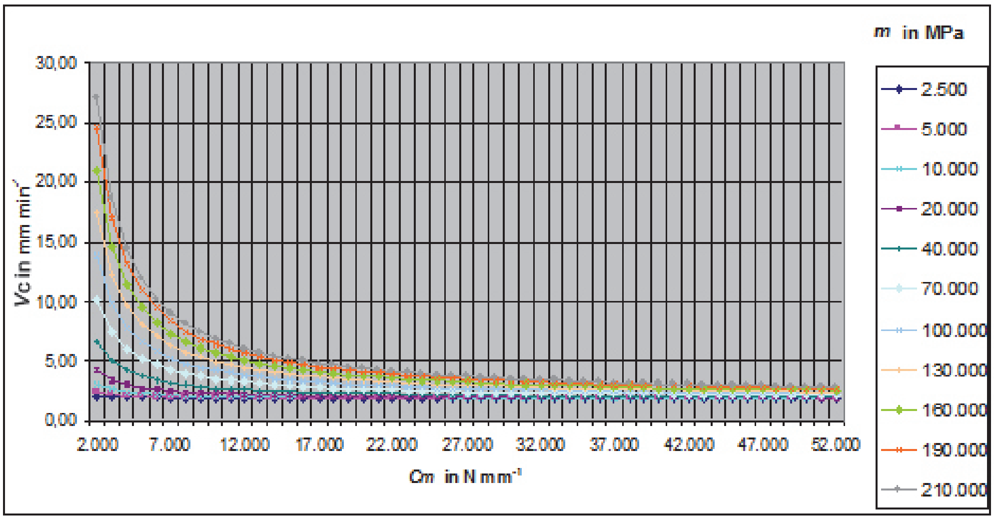
$S_O$ original cross-section area in $mm^2$

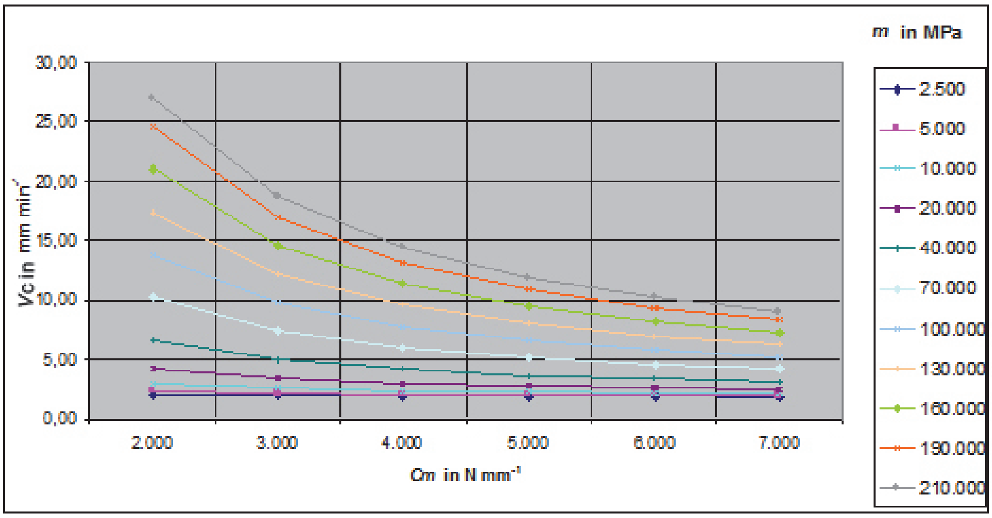
$L_C$ parallel length of the test piece in mm

$C_M$ stiffness of the testing equipment in N $mm^{-1}$ (around the point of interest such as $R_{p0,2}$, if stiffness is not linear, e.g. when using wedge grips)

Remark:
the use of $E$ (modulus of elasticity) as m (slope of stress strain-curve near the $R_{eh}$ or $R_p$ value) falsifies the result!

For diagrams of calculated crosshead speeds $V_C$ for practical use (based on the specimen dimensions on page 5 and a resulting strain rate of 0.00025 $s^{-1}$ ) see next page




With const $C_M$ and variable $S_0$: