Aim- To write a code and run it for minimum spanning tree suing Prim's and Kruskal's Algorithm

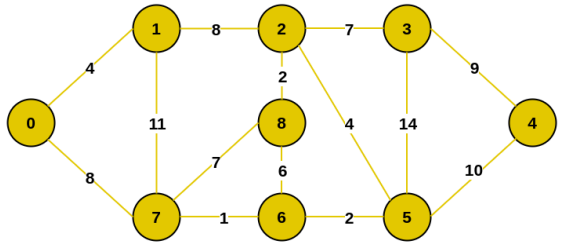
Theory- In Kruskal's algorithm, sort all edges of the given graph in increasing order. Then it keeps on adding new edges and nodes in the MST if the newly added edge does not form a cycle. It picks the minimum weighted edge at first at the maximum weighted edge at last. Thus we can say that it makes a locally optimal choice in each step in order to find the optimal solution. Hence this is a Greedy Algorithm.

Like Kruskal's algorithm, Prim's algorithm is also a Greedy algorithm. This algorithm always starts with a single node and moves through several adjacent nodes, in order to explore all of the connected edges along the way. A group of edges that connects two sets of vertices in a graph is called cut in graph theory. So, at every step of Prim's algorithm, find a cut, pick the minimum weight edge from the cut, and include this vertex in MST Set (the set that contains already included vertices).

Example-

Prim's-

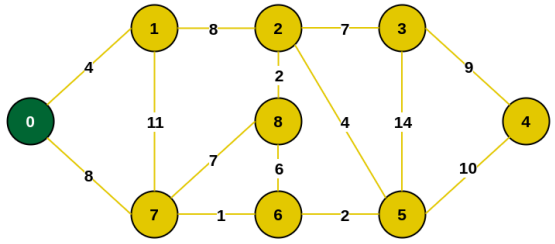
Consider the following graph as an example for which we need to find the Minimum Spanning Tree (MST).



Example of a Graph

Example of a graph

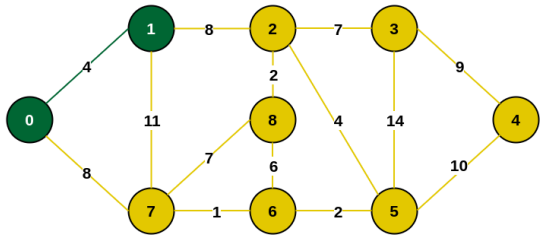
Step 1: Firstly, we select an arbitrary vertex that acts as the starting vertex of the Minimum Spanning Tree. Here we have selected vertex 0 as the starting vertex.



Select an arbitrary starting vertex. Here we have selected 0

0 is selected as starting vertex

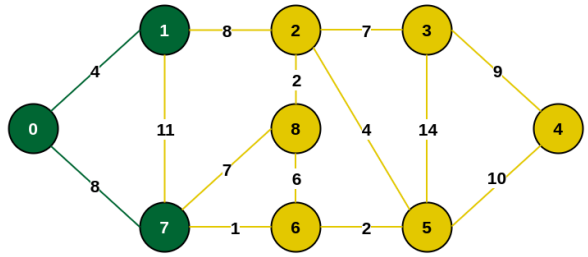
Step 2: All the edges connecting the incomplete MST and other vertices are the edges {0, 1} and {0, 7}. Between these two the edge with minimum weight is {0, 1}. So include the edge and vertex 1 in the MST.



Minimum weighted edge from MST to other vertices is 0-1 with weight 4

1 is added to the MST

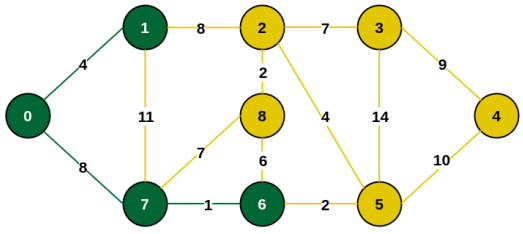
Step 3: The edges connecting the incomplete MST to other vertices are {0, 7}, {1, 7} and {1, 2}. Among these edges the minimum weight is 8 which is of the edges {0, 7} and {1, 2}. Let us here include the edge {0, 7} and the vertex 7 in the MST. [We could have also included edge {1, 2} and vertex 2 in the MST].



Minimum weighted edge from MST to other vertices is 0-7 with weight 8

7 is added in the MST

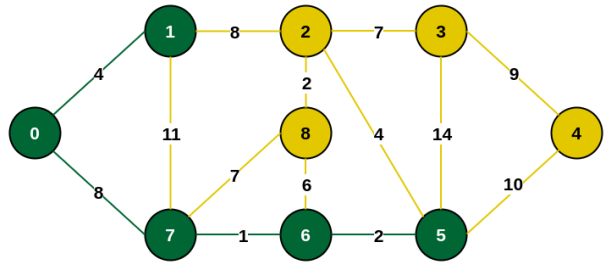
Step 4: The edges that connect the incomplete MST with the fringe vertices are {1, 2}, {7, 6} and {7, 8}. Add the edge {7, 6} and the vertex 6 in the MST as it has the least weight (i.e., 1).



Minimum weighted edge from MST to other vertices is 7-6 with weight 1

6 is added in the MST

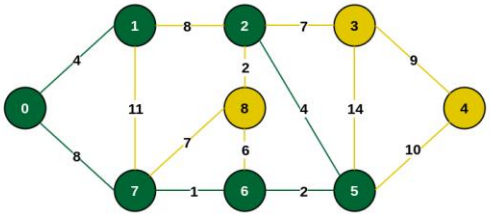
Step 5: The connecting edges now are {7, 8}, {1, 2}, {6, 8} and {6, 5}. Include edge {6, 5} and vertex 5 in the MST as the edge has the minimum weight (i.e., 2) among them.



Minimum weighted edge from MST to other vertices is 6-5 with weight 2

Include vertex 5 in the MST

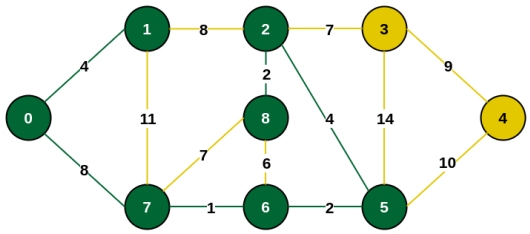
Step 6: Among the current connecting edges, the edge {5, 2} has the minimum weight. So include that edge and the vertex 2 in the MST.



Minimum weighted edge from MST to other vertices is 5-2 with weight 4

Include vertex 2 in the MST

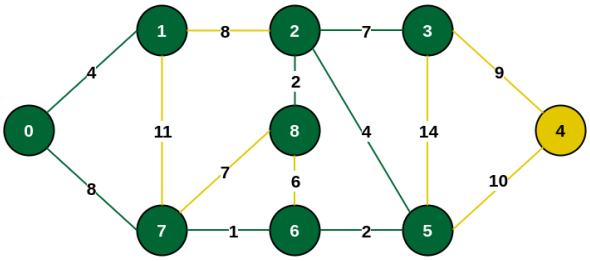
Step 7: The connecting edges between the incomplete MST and the other edges are {2, 8}, {2, 3}, {5, 3} and {5, 4}. The edge with minimum weight is edge {2, 8} which has weight 2. So include this edge and the vertex 8 in the MST.



Minimum weighted edge from MST to other vertices is 2-8 with weight 2

Add vertex 8 in the MST

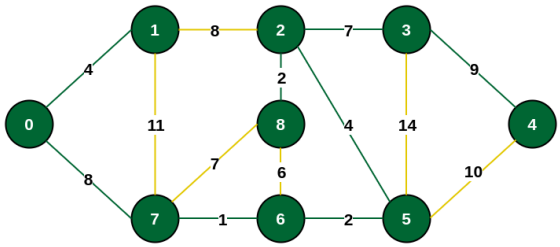
Step 8: See here that the edges {7, 8} and {2, 3} both have same weight which are minimum. But 7 is already part of MST. So we will consider the edge {2, 3} and include that edge and vertex 3 in the MST.



Minimum weighted edge from MST to other vertices is 2-3 with weight 7

Include vertex 3 in MST

Step 9: Only the vertex 4 remains to be included. The minimum weighted edge from the incomplete MST to 4

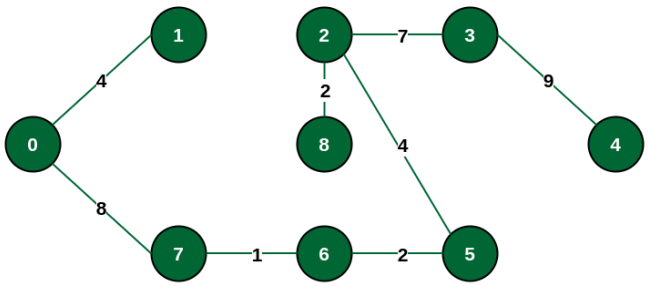


Minimum weighted edge from MST to other vertices is 3-4 with weight 9

is {3, 4}.

Include vertex 4 in the MST

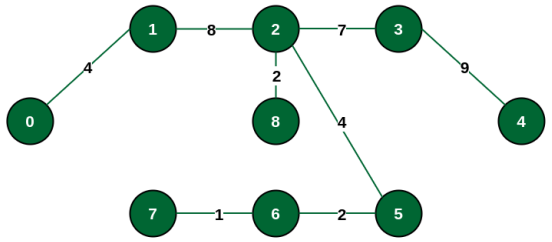
The final structure of the MST is as follows and the weight of the edges of the MST is (4 + 8 + 1 + 2 + 4 + 2 + 7 + 9) = 37.



The final structure of MST

The structure of the MST formed using the above method

Note: If we had selected the edge {1, 2} in the third step then the MST would look like the following.

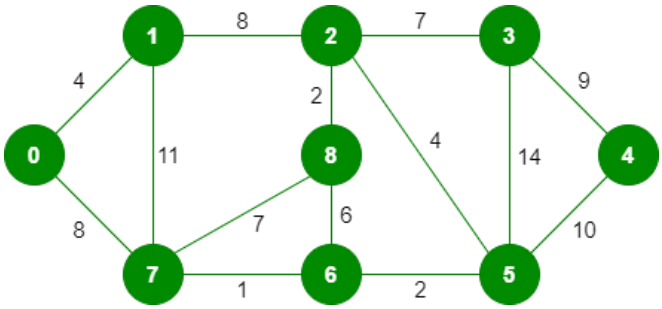


Alternative MST structure

Structure of the alternate MST if we had selected edge {1, 2} in the MST

Kruskal's-

Input Graph:



The graph contains 9 vertices and 14 edges. So, the minimum spanning tree formed will be having (9 – 1) = 8 edges.

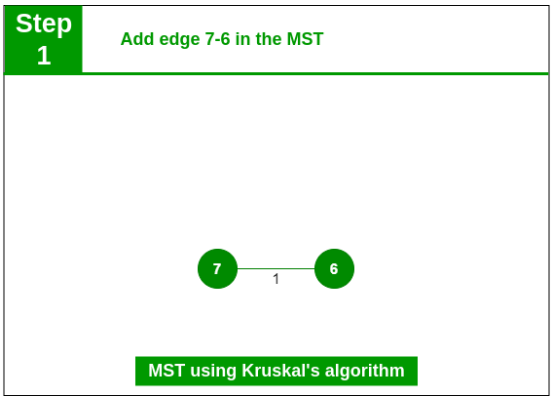
After sorting:

Weight	Source	Destination
1	7	6
2	8	2
2	6	5
4	0	1
4	2	5

6	8	6
7	2	3
7	7	8
8	0	7
8	1	2
9	3	4
10	5	4
11	1	7
14	3	5

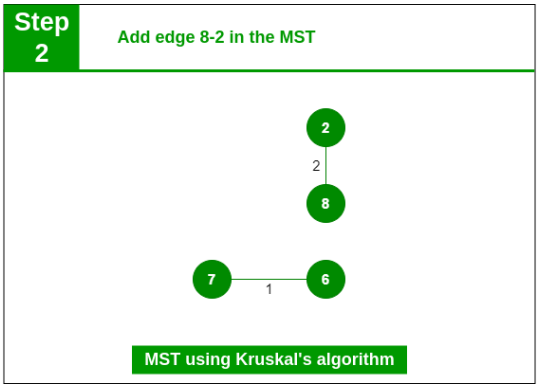
Now pick all edges one by one from the sorted list of edges

Step 1: Pick edge 7-6. No cycle is formed, include it.



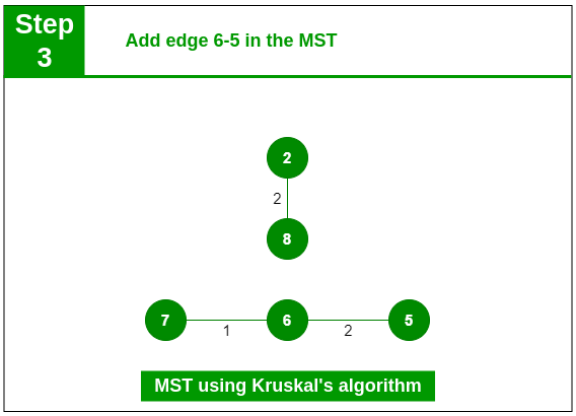
Add edge 7-6 in the MST

Step 2: Pick edge 8-2. No cycle is formed, include it.



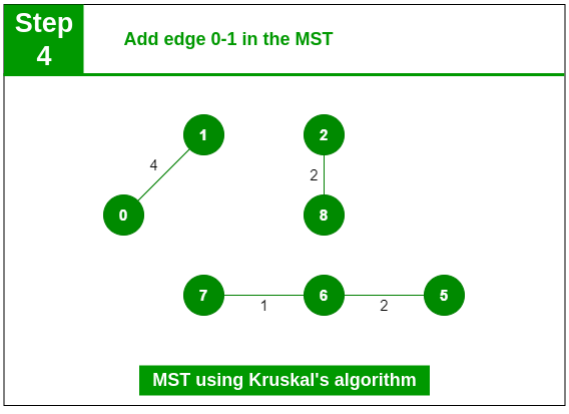
Add edge 8-2 in the MST

Step 3: Pick edge 6-5. No cycle is formed, include it.



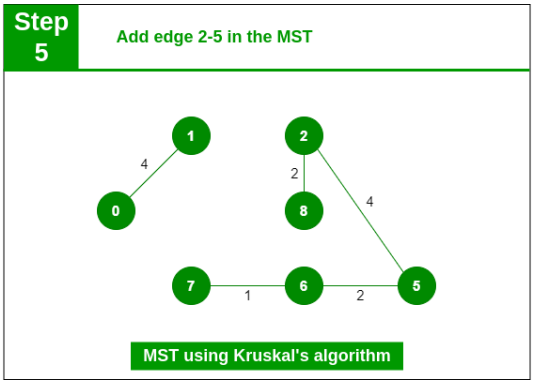
Add edge 6-5 in the MST

Step 4: Pick edge 0-1. No cycle is formed, include it



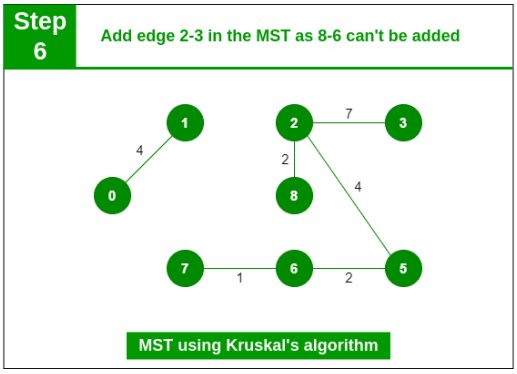
Add edge 0-1 in the MST

Step 5: Pick edge 2-5. No cycle is formed, include it.



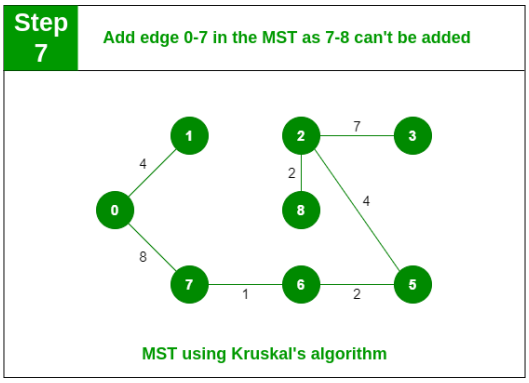
Add edge 2-5 in the MST

Step 6: Pick edge 8-6. Since including this edge results in the cycle, discard it. Pick edge 2-3: No cycle is formed, include it.



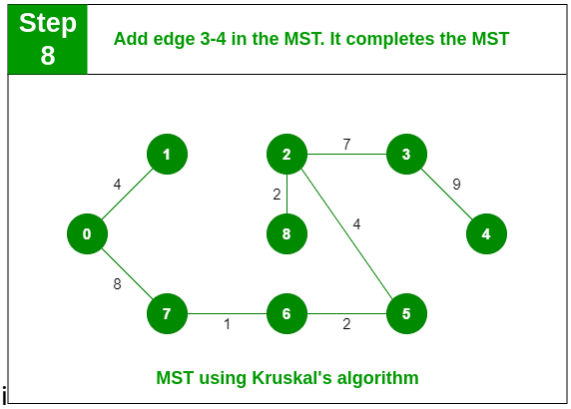
Add edge 2-3 in the MST

Step 7: Pick edge 7-8. Since including this edge results in the cycle, discard it. Pick edge 0-7. No cycle is formed, include it.



Add edge 0-7 in MST

Step 8: Pick edge 1-2. Since including this edge results in the cycle, discard it. Pick edge 3-4. No cycle



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Add edge 3-4 in the MST

Note: Since the number of edges included in the MST equals to (V – 1), so the algorithm stops here

Algorithm-

Prim's-

MST-PRIM (G, w, r)

1. for each u ∈ V [G]
2. do key [u] ← ∞
3. π [u] ← NIL
4. key [r] ← 0
5. Q ← V [G]
6. While Q? Ø
7. do u ← EXTRACT - MIN (Q)
8. for each v ∈ Adj [u]
9. do if v ∈ Q and w (u, v) < key [v]
10. then π [v] ← u
11. key [v] ← w (u, v)

Kruskal's-

MST- KRUSKAL (G, w)

1. A ← Ø
2. for each vertex v ∈ V [G]
3. do MAKE - SET (v)
4. sort the edges of E into non decreasing order by weight w
5. for each edge (u, v) ∈ E, taken in non-decreasing order by weight
6. do if FIND-SET (μ) ≠ if FIND-SET (v)
7. then A ← A ∪ {(u, v)}
8. UNION (u, v)
9. return A

Code-

Prim's

```
#include <limits.h>
#include <stdbool.h>
#include <stdio.h>

// Number of vertices in the graph
#define V 5

// A utility function to find the vertex with
// minimum key value, from the set of vertices
// not yet included in MST
int minKey(int key[], bool mstSet[])
{
   // Initialize min value
   int min = INT_MAX, min_index;

   for (int v = 0; v < V; v++)
      if (mstSet[v] == false && key[v] < min)
         min = key[v], min_index = v;

   return min_index;
}

// A utility function to print the
// constructed MST stored in parent[]
int printMST(int parent[], int graph[V][V])
{
   printf("Edge \tWeight\n");
   for (int i = 1; i < V; i++)
      printf("%d - %d \t%d \n", parent[i], i,
            graph[i][parent[i]]);
}

// Function to construct and print MST for
// a graph represented using adjacency
// matrix representation
void primMST(int graph[V][V])
{
   // Array to store constructed MST
   int parent[V];
   // Key values used to pick minimum weight edge in cut
   int key[V];
   // To represent set of vertices included in MST
   bool mstSet[V];

   // Initialize all keys as INFINITE
   for (int i = 0; i < V; i++)
      key[i] = INT_MAX, mstSet[i] = false;
```

```
    // Always include first 1st vertex in MST.
    // Make key 0 so that this vertex is picked as first
    // vertex.
    key[0] = 0;

    // First node is always root of MST
    parent[0] = -1;

    // The MST will have V vertices
    for (int count = 0; count < V - 1; count++) {

        // Pick the minimum key vertex from the
        // set of vertices not yet included in MST
        int u = minKey(key, mstSet);

        // Add the picked vertex to the MST Set
        mstSet[u] = true;

        // Update key value and parent index of
        // the adjacent vertices of the picked vertex.
        // Consider only those vertices which are not
        // yet included in MST
        for (int v = 0; v < V; v++)

            // graph[u][v] is non zero only for adjacent
            // vertices of m mstSet[v] is false for vertices
            // not yet included in MST Update the key only
            // if graph[u][v] is smaller than key[v]
            if (graph[u][v] && mstSet[v] == false
                && graph[u][v] < key[v])
                parent[v] = u, key[v] = graph[u][v];
    }

    // print the constructed MST
    printMST(parent, graph);
}

// Driver's code
int main()
{
    int graph[V][V] = { { 0, 2, 0, 6, 0 },
                        { 2, 0, 3, 8, 5 },
                        { 0, 3, 0, 0, 7 },
                        { 6, 8, 0, 0, 9 },
                        { 0, 5, 7, 9, 0 } };

    // Print the solution
    primMST(graph);

    return 0;
}
```

Kruskal's

```
#include <stdio.h>
#include <stdlib.h>

// Comparator function to use in sorting
int comparator(const void* p1, const void* p2)
{
   const int(*x)[3] = p1;
   const int(*y)[3] = p2;

   return (*x)[2] - (*y)[2];
}

// Initialization of parent[] and rank[] arrays
void makeSet(int parent[], int rank[], int n)
{
   for (int i = 0; i < n; i++) {
      parent[i] = i;
      rank[i] = 0;
   }
}

// Function to find the parent of a node
int findParent(int parent[], int component)
{
   if (parent[component] == component)
      return component;

   return parent[component]
         = findParent(parent, parent[component]);
}

// Function to unite two sets
void unionSet(int u, int v, int parent[], int rank[], int n)
{
   // Finding the parents
   u = findParent(parent, u);
   v = findParent(parent, v);

   if (rank[u] < rank[v]) {
      parent[u] = v;
   }
   else if (rank[u] > rank[v]) {
      parent[v] = u;
   }
   else {
      parent[v] = u;

      // Since the rank increases if the ranks of two sets are same
```

```
      rank[u]++;
   }
}

// Function to find the MST
void kruskalAlgo(int n, int edge[n][3])
{
   // First we sort the edge array in ascending order
   // so that we can access minimum distances/cost
   qsort(edge, n, sizeof(edge[0]), comparator);

   int parent[n];
   int rank[n];

   // Function to initialize parent[] and rank[]
   makeSet(parent, rank, n);

   // To store the minimun cost
   int minCost = 0;

   printf("Following are the edges in the constructed MST\n");
   for (int i = 0; i < n; i++) {
      int v1 = findParent(parent, edge[i][0]);
      int v2 = findParent(parent, edge[i][1]);
      int wt = edge[i][2];

      // If the parents are different that
      // means they are in different sets so
      // union them
      if (v1 != v2) {
         unionSet(v1, v2, parent, rank, n);
         minCost += wt;
         printf("%d -- %d == %d\n", edge[i][0],
               edge[i][1], wt);
      }
   }

   printf("Minimum Cost Spanning Tree: %d\n", minCost);
}
int main()
{
   int edge[5][3] = { { 0, 1, 10 },
                  { 0, 2, 6 },
                  { 0, 3, 5 },
                  { 1, 3, 15 },
                  { 2, 3, 4 } };

   kruskalAlgo(5, edge);

   return 0;
}
```

Output-

Prim's

```
Output

/tmp/d8kpQJGQyl.o
Edge    Weight
0 - 1   2
1 - 2   3
0 - 3   6
1 - 4   5
```

Kruskal's

```
Output

/tmp/d8kpQJGQyl.o
Following are the edges in the constructed MST
2 -- 3 == 4
0 -- 3 == 5
0 -- 1 == 10
Minimum Cost Spanning Tree: 19
```

Conclusion-Thus we have used an example to illustrate Minimum spanning tree using Prim's and Kruskal's algorithm