

SEMESTER END EXAMINATIONS – JULY / AUGUST 2022

Program : B.E. : Computer Science and Engineering

Semester : IV

Course Name : Design and Analysis of Algorithms

Max. Marks : 100

Course Code : CS42

Duration : 3 Hrs

Instructions to the Candidates:

- Answer one full question from each unit.
- Draw diagram and algorithm wherever necessary.

UNIT- I

- Prove the following property of Asymptotic growth rate: CO1 (05)
Suppose that f and g are two functions such that for some other function h , we have $f = O(h)$ and $g = O(h)$. Then $f + g = O(h)$.
 - Write a recursive binary search algorithm and solve its recursive time CO1 (08)
complexity.
 - Illustrate with the help of an example Gale-Shapley stable matching CO1 (07)
algorithm and write an algorithm for the same.
- Prove the following property of Asymptotic growth rate: CO1 (05)
If $f_1(n)$ is $O(g_1(n))$ and $f_2(n)$ is $O(g_2(n))$ then $f_1(n) + f_2(n)$ is $O(\max(g_1(n), g_2(n)))$
 - Write a recursive insertion sort algorithm and solve its recursive time CO1 (07)
complexity.
 - Suppose the Girl's and Boy's preferences are given by the following table: CO1 (08)

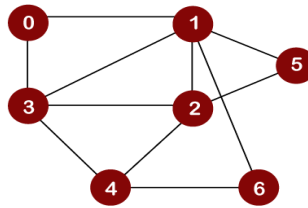
GIRL	1	2	3	4	5
Amy	Eric	Adam	Bill	Dan	Carl
Beth	Carl	Bill	Dan	Adam	Eric
Cara	Bill	Carl	Dan	Eric	Adam
Diane	Adam	Eric	Dan	Carl	Bill
Ellen	Dan	Bill	Eric	Carl	Adam

BOY	1	2	3	4	5
Adam	Beth	Amy	Diane	Ellen	Cara
Bill	Diane	Beth	Amy	Cara	Ellen
Carl	Beth	Ellen	Cara	Diane	Amy
Dan	Amy	Diane	Cara	Beth	Ellen
Eric	Beth	Diane	Amy	Ellen	Cara

Find a stable matching using the Gale-Shapley algorithm with girls making proposals.

UNIT – II

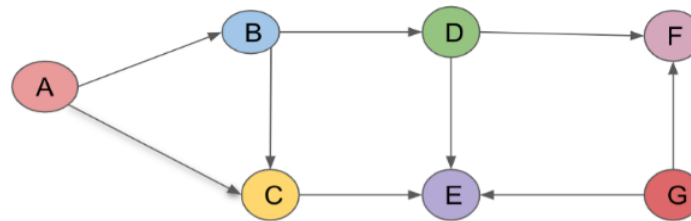
- Write an algorithm to sort the 'n' numbers using divide and conquer CO2 (06)
approach.
 - Compute Breadth First Search and Depth First Search for the following CO2 (08)
graph. Verify the bipartiteness of the given graph?



- c) Prove that if graph $G=(V,E)$ has a topological ordering, then G is a DAG. CO2 (06)
4. a) Illustrate the Depth First Search algorithm and find its worst case efficiency when graph is given by adjacency list representation. CO2 (06)
- b) Find the number of inversions for the following set of numbers. CO2 (08)

55	66	44	77	33	11	99	88	100
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- c) Explain the below approaches to solve recurrences: CO2 (06)
- Unrolling the Mergesort recurrence
 - Compute the topological ordering for the following graph

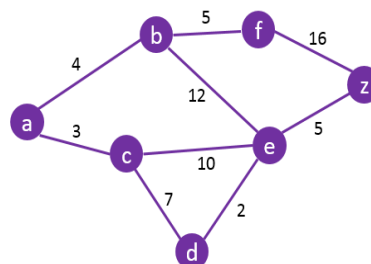


UNIT – III

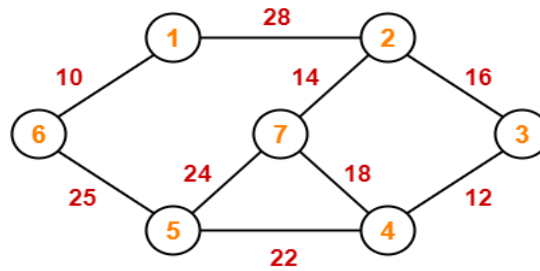
5. a) Find the average code word length for the table below using Huffman coding. CO2 (08)

character	Frequency
a	55
b	90
c	2
d	30
e	16
f	5

- b) Apply the single source shortest path algorithm for the given directed graph. Assume 'a' as source vertex. CO2 (06)



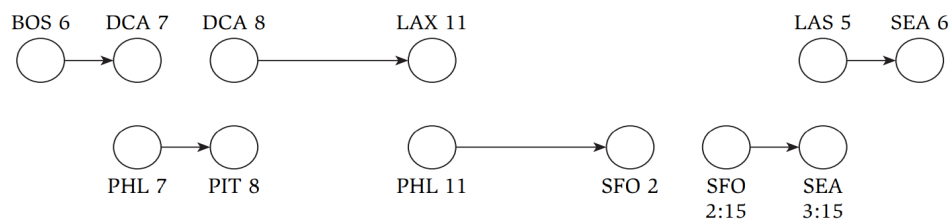
- c) Write the Dijkstra's algorithm to find the shortest path. CO4 (06)
6. a) Write the kruskals algorithm pseudocode and apply the same to the graph below to construct minimum spanning tree. CO2 (06)
- b) Explain with an example the working of prefix codes for data compression. CO2 (06)



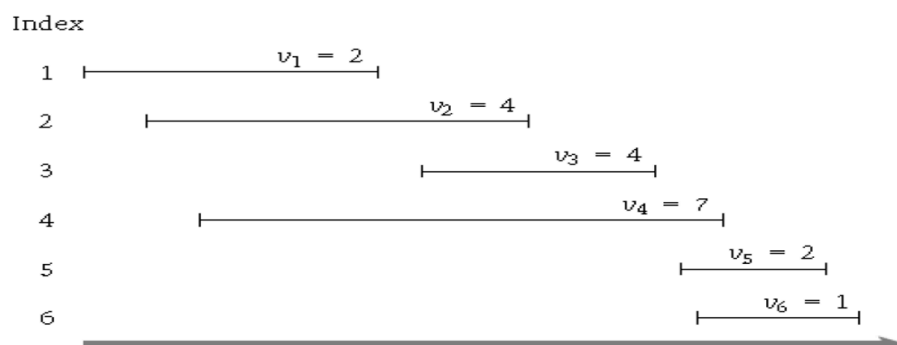
- c) Develop an algorithm for greedy based optimal caching problem. CO2 (08)

UNIT – IV

7. a) Using recurrence, prove that the Knapsack Problem can be solved in $O(nW)$ time. Knapsack size $W = 5$, items $w_1 = 2, w_2 = 2, w_3 = 3, v_1=20, v_2=10, v_3=30$. CO4 (10)
- b) Given a small instance of simple Airline Scheduling Problem below prove that there is a way to perform all flights using at most k planes if and only if there is a feasible circulation in the network G . CO4 (10)



8. a) Given a table M of the optimal values of the sub problems, prove that, the optimal set S can be found in $O(n)$ time. Illustrate the Subset Sum Problem algorithms iterations $i = 3$ on a sample given instance $W = 6$, items $w_1 = 2, w_2 = 2, w_3 = 3$. CO4 (10)
- b) Given the array M of the optimal values of the sub-problems, prove that $\text{Find_Solution}()$ returns an optimal solution in $O(n)$ time. Illustrate the proof with this example. CO4 (10)



UNIT – V

9. a) Prove the following claim: Suppose X is an NP-complete problem. Then X is solvable in polynomial time if and only if $P = NP$. CO5 (05)
- b) Explain the general strategy for proving new problems NP-Complete in detail. CO5 (08)
- c) Prove the following claim: Circuit Satisfiability is NP-complete. CO5 (07)
10. a) Illustrate with the help of an example, Traveling Salesman Problem in detail. CO5 (07)

- b) Bring out the difference between P problem and NP problem.
- c) Prove the following claims:
 - i) Independent Set \leq_P Vertex Cover.
 - ii) Vertex Cover \leq_P Independent Set.

CO5 (05)

CO5 (08)
