

## Practical tasks

1. Write a computer program that solves:

$$a) \quad y' = x + \cos y, \quad y(1) = 30, \quad 1 \leq x \leq 2$$

$$b) \quad y' = x^2 + y^2, \quad y(2) = 1, \quad 1 \leq x \leq 2$$

Implement in the code numerical Runge-Kutta explicit methods of 2<sup>nd</sup> and 4<sup>th</sup> order with grid steps:  $h = 0.1, 0.05, 0.01, 0.005, 0.001$ .

For each numerical method compose the four arrays of the relative errors for the numbers  $N = 20, 100, 200, 1000$  in the same  $x$  coordinates:

$$N = 20: \quad |x_k(n_{20}) - x_k(n_{10})|, \quad k = \overline{1,10}$$

$$N = 100: \quad |x_k(n_{100}) - x_k(n_{20})|, \quad k = \overline{1,10}$$

$$N = 200: \quad |x_k(n_{200}) - x_k(n_{100})|, \quad k = \overline{1,10}$$

$$N = 1000: \quad |x_k(n_{1000}) - x_k(n_{200})|, \quad k = \overline{1,10}$$

Provide the functionality of printing the arrays of the relative errors onto the console in the format

-- in the first line print the title of the array (e.g. " $N_{20}$ "),

-- in the second line print the values stored in the array, separated by spaces, without the space after the last item.

For each numerical method compose the array of the absolute errors:

$$\left\{ \max_{k=\overline{1,10}} |x_k(n_{20}) - x_k(n_{10})|, \max_{k=\overline{1,10}} |x_k(n_{100}) - x_k(n_{20})|, \right. \\ \left. \max_{k=\overline{1,10}} |x_k(n_{200}) - x_k(n_{100})|, \max_{k=\overline{1,10}} |x_k(n_{1000}) - x_k(n_{200})| \right\}$$

Provide the functionality of plotting  $\text{Log}_2$  for the absolute errors of both methods together in one picture

2. Write a computer program that solves:

$$y'' = y \sin x, \quad y(0) = 0, \quad y'(0) = 1, \quad 0 \leq x \leq 1$$

Implement in the code numerical Runge-Kutta explicit methods of 2<sup>nd</sup> and 4<sup>th</sup> order with grid steps:  $h = 0.1, 0.05, 0.01, 0.005, 0.001$ .

For each numerical method compose the four arrays of the relative errors for the numbers  $N = 20, 100, 200, 1000$  in the same  $x$  coordinates:

$$N = 20: \quad |x_k(n_{20}) - x_k(n_{10})|, \quad k = \overline{1,10}$$

$$N = 100: \quad |x_k(n_{100}) - x_k(n_{20})|, \quad k = \overline{1,10}$$

$$N = 200: \quad |x_k(n_{200}) - x_k(n_{100})|, \quad k = \overline{1,10}$$

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Provide the functionality of plotting  $\text{Log}_2$  for the absolute errors of both methods together in one picture