Analysis Best Case

Basic operation: distance calculation of points within left and right in the partition phase which requires multiplication operation

In best case, there are equal number of points on top and bottom of the line between left and rightMost points

$C(n) = 2C(n/2) + n \quad , C(1) = 0 \qquad ; n = 2^k$

$= 2C(n/2) + 2 \cdot n/2$

$C(n) = 2[2C(n/4) + 2 \cdot n/4] + 2\, n/2$

$= 2^2 C(n/4) + 2^2\, n/4 + 2 \cdot n/2$

$= 2^2 C(n/4) + 2n$

$= 2^2 [2C(n/8) + 2 \cdot n/8] + 2n$

$= 2^3 C(n/8) + 3n$

$\therefore C(n) = 2^i C(n/2^i) + in$

$i = k \rightarrow C(n) = 2^k \underbrace{C(1)}_{0} + kn$

$= n \log_2 n$

$\in \Theta(n \log n)$

Using Master Theorem

$$\rightarrow \begin{cases} a < b^d & , T(n) \in \Theta(n^d) \\ a = b^d & , T(n) \in \Theta(n^d \log n) \\ a > b^d & , T(n) \in \Theta(n^{\log_b a}) \end{cases}$$

$C(n) = 2C(n/2) + n$

$a = 2,\ b = 2,\ d = 1$

here, $a = 2$ and $b^d = 2^1 = 2$

$a = b^d \rightarrow T(n) \in \Theta(n^1 \log n)$

$\in \Theta(n \log n)$

So, Master theorem verifies correctness