Question 1

1. $(n^2+1)^{10}$
   $(n^2+1)^{10} \approx (n^2)^{10} = n^{20} \in \Theta(n^{20})$

   Proof:

   $$\lim_{n\to\infty}(n^2+1)^{10}/n^{20} = \lim_{n\to\infty}(n^2+1)^{10}/(n^2)^{10} = \lim_{n\to\infty}((n^2+1)/(n^2))^{10} = \lim_{n\to\infty}(1+1/n^2)^{10} = 1$$

2. $\sqrt{10n^2+7n+3}$
   $\sqrt{10n^2+7n+3} \approx \sqrt{10n^2} = \sqrt{10}n \in \Theta(n)$

   Proof:

   $$\lim_{n\to\infty}\sqrt{10n^2+7n+3}/n = \lim_{n\to\infty}\sqrt{(10n^2+7n+3)/n^2} = \lim_{n\to\infty}\sqrt{10+7/n+3/n^2} = \sqrt{10}$$

3. $2n\lg(n+2)^2 + (n+2)^2\lg(n/2)$
   $2n\lg(n+2)^2+(n+2)^2\lg(n/2) = (2n)2\lg(n+2)+(n+2)^2(\lg n-\lg 2) \in \Theta(n\lg n)+\Theta(n^2\lg n)$
   $= \Theta(n^2\lg n)$

   Proof:

   First prove $(2n)2\lg(n+2) \in \Theta(n\lg n)$ and $(n+2)^2(\lg n - \lg 2) \in \Theta(n^2\lg n)$ by using limits.

   Second use the theorem: If $t_1(n) \in \Theta(g_1(n))$ and $t_2(n) \in \Theta(g_2(n))$ then $t_1(n)+t_2(n) \in \Theta(\max\{g_1(n), g_2(n)\})$ to prove.

4. $2^{n+1}+3^{n-1}$
   $2^{n+1}+3^{n-1} = 2(2^n)+(1/3)(3^n) \in \Theta(2^n)+\Theta(3^n) = \Theta(3^n)$.

   Proof:

   First prove $2(2^n) \in \Theta(2^n)$ and $(1/3)(3^n) \in \Theta(3^n)$ by using limits.

   Second use the theorem: If $t_1(n) \in \Theta(g_1(n))$ and $t_2(n) \in \Theta(g_2(n))$ then $t_1(n)+t_2(n) \in \Theta(\max\{g_1(n), g_2(n)\})$ to prove.

5. $\lfloor \log_2 n \rfloor$
   $\lfloor \log_2 n \rfloor \approx \log_2 n \in \Theta(\log n)$.

   Proof:
   $\lim_{n\to\infty}(\lfloor \log_2 n\rfloor/\log_2 n) = \lim_{n\to\infty}((\log_2 n - \epsilon)/\log_2 n) =$
   $\lim_{n\to\infty}(\log_2 n/\log_2 n - \epsilon/\log_2 n) = 1$
   where $0 \le \epsilon < 1$.

Question 2

$(n-2)!$, $5\lg(n+100)^{10}$, 2^{2n}, $0.001n^4+3n^3+1$, $\ln^2 n$, $\sqrt[3]{n}$, 3^n.

$(n-2)! \in \Theta(n!)$, $5\lg(n+100)^{10} \in \Theta(\log n)$, $2^{2n} \in \Theta(4^n)$, $0.001n^4+3n^3+1 \in \Theta(n^4)$, $\ln^2 n \in \Theta(\log^2 n)$, $\sqrt[3]{n} \in \Theta(n^{1/3})$, $3^n \in \Theta(3^n)$.

The order should be
$5\lg(n+100)^{10}$, $\ln^2 n$, $\sqrt[3]{n}$, $0.001n^4 + 3n^3 + 1$, 3^n. 2^{2n}, $(n-2)!$,

Question 3

1. $1+3+5+7+...+999 = \sum_{i=1}^{500}(2i-1) = \sum_{i=1}^{500} 2i - \sum_{i=1}^{500} 1 = 2(500 \times 501)/2 - 500 = 250,000$
2. $2+4+8+16+...+1,024 = \sum_{i=1}^{10} 2^i = \sum_{i=0}^{10} 2^i - 1 = (2^{11}-1) - 1 = 2046$
3. $\sum_{i=3}^{n+1} 1 = (n+1) - 3 + 1 = n-1$
4. $\sum_{i=3}^{n+1} i = \sum_{i=1}^{n+1} i - \sum_{i=1}^{2} i = (n+1)(n+2)/2 - (1+2) = (n^2+3n-4)/2$
5. $\sum_{i=0}^{n-1} i(i+1) = \sum_{i=0}^{n-1} i^2 + \sum_{i=0}^{n-1} i = (n-1)n(2n-1)/6 + (n-1)n/2 = n(n^2-1)/3$
6. $\sum_{j=1}^{n} 3^{j+1} = 3\sum_{j=1}^{n} 3^j = 3[\sum_{j=0}^{n} 3^j - 1] = 3[(3^{n+1}-1)/(3-1) - 1] = (3^{n+2}-9)/2$
7. $\sum_{i=1}^{n}\sum_{j=1}^{n} ij = \sum_{i=1}^{n} i \sum_{j=1}^{n} j = \sum_{i=1}^{n} i[n(n+1)/2] = [n(n+1)/2]\sum_{i=1}^{n} i$
   $= [n(n+1)/2][n(n+1)/2] = n^2(n+1)^2/4$
8. $\sum_{i=1}^{n} 1/[i(i+1)] = \sum_{i=1}^{n}[(1/i) - (1/(i+1))] = (1-1/2) + (1/2-1/3) + ... + (1/n - 1/(n+1)) = 1 - 1/(n+1) = n/(n+1)$

Question 4

1. $\sum_{i=0}^{n-1}(i^2+1)^2 = \sum_{i=0}^{n-1}(i^4+2i^2+1) = \sum_{i=0}^{n-1} i^4 + \sum_{i=0}^{n-1} 2i^2 + \sum_{i=0}^{n-1} 1 \in \Theta(n^5) + \Theta(n^3) + \Theta(n) = \Theta(n^5)$
2. $\sum_{i=2}^{n-1} \lg i^2 = \sum_{i=2}^{n-1} 2\lg i = 2\sum_{i=2}^{n-1} \lg i = 2\sum_{i=1}^{n} \lg i - 2\lg n \in \Theta(n\log n) - \Theta(\log n) = \Theta(n \log n)$
3. $\sum_{i=1}^{n}(i+1)2^{i-1} = \sum_{i=1}^{n} i2^{i-1} + \sum_{i=1}^{n} 2^{i-1} = (1/2)\sum_{i=1}^{n} i2^i + \sum_{j=0}^{n-1} 2^j \in \Theta(n2^n) - \Theta(2^n) = \Theta(n2^n)$
4. $\sum_{i=0}^{n-1}\sum_{j=0}^{i-1}(i+j) = \sum_{i=0}^{n-1}[\sum_{j=0}^{i-1} i + \sum_{j=0}^{i-1} j] = \sum_{i=0}^{n-1}[i^2 + ((i-1)i/2)] = \sum_{i=0}^{n-1}[(3/2)i^2 + (1/2)i] = (3/2)\sum_{i=0}^{n-1} i^2 - (1/2)\sum_{i=0}^{n-1} i \in \Theta(n^3) - \Theta(n^2) = \Theta(n^3)$