

Inversion Count Brute Force (A)

// A[0..n-1] array of distinct numbers of size n

// inversion means, $A[i] > A[j]$ for $i < j$

count $\leftarrow 0$

for each $i \leftarrow 0$ to $n-2$ do

for each $j \leftarrow i+1$ to $n-1$ do

if $A[i] > A[j]$

count \leftarrow count + 1

return count

Analysis

Basic Operation: Comparison

$$T(n) = \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} 1$$

$$= \sum_{i=0}^{n-2} (n-1-i-1+1)$$

$$= \sum_{i=0}^{n-2} (n-1) - \sum_{i=0}^{n-2} i$$

$$= (n-1)(n-2-0+1) - \frac{(n-2)(n-2+1)}{2}$$

$$= (n-1)(n-1) - \frac{(n-2)(n-1)}{2}$$

$$= (n-1) \frac{(2n-2-n+2)}{2}$$

$$= \frac{(n-1)n}{2}$$

$$\in \Theta(n^2)$$

convex Hull Brute force (A)

// A[0..n-1] array of points of size n

hull $\leftarrow \{ \}$, collinear_points $\leftarrow \{ \}$

Swap the first point with leftmost extreme point

for each $i \leftarrow 0$ to $n-2$ do

for each $j \leftarrow i+1$ to $n-1$ do

previousSign $\leftarrow 0$, farthest point $A[j]$, rem_points = A - {A[i], A[j]}

$a = A[j].y - A[i].y$, $b = A[i].x - A[j].x$, $c = A[i].x \cdot A[j].y - A[i].y \cdot A[j].x$

for each $k \leftarrow 0$ to $n-2$ do

curSign = $a \cdot \text{rem_points}[k].x + b \cdot \text{rem_points}[k].y - c$

if curSign = 0 do // select farthest collinear point

if distance of A[i] to rem_points[k] > distance of A[i] to farthest pt

collinear_points U {farthest point}

farthest_point = rem_point[k]
continue

if prev-sign is 0 do:

prev-sign = cur-sign
continue

if prev-sign \neq cur-sign

break

if $k = n-2$: hull \leftarrow hull U {A[i], farthest point}

(collinear_points)

return hull

Analysis

Basic operation: Sign detection

which has 2 multiply and 3 add/sub

So, multiply most expensive operation

happening every iteration

$$M(n) = \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=0}^{n-2} 1 = \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} (n-2+1)$$

$$= \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} (n-1) = \sum_{i=0}^{n-2} (n-1)(n-1-i-1+1)$$

$$= (n-1) \sum_{i=0}^{n-2} (n-1-i) = (n-1)^2 \sum_{i=0}^{n-2} 1$$

$$= (n-1)^2 (n-2+1) - \frac{(n-2)(n-2+1)}{2}$$

$$= (n-1)^3 - \frac{(n-1)^3}{2} - \frac{(n-2)^2}{2} \in \Theta(n^3)$$