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2 a) class 1 = {0.5, 0.1, 0.2, 0.4, 0.3, 0.2, 0.2, 0.1, 0.35, 0.25}
class 2 = {0.9, 0.8, 0.75, 1.0}
σ_1^2 = 0.0149
σ_2^2 = 0.0092

To Fit a one-dimensional Gaussian using maximum Likelihood,
w.k.t Gaussian requires 2 parameters μ and σ^2 i.e.;
So we find μ using maximum likelihood from the formula

$$\mu = \frac{1}{m} \sum_{i=1}^{m} x_i$$

$$\mu_1 = \frac{1}{10}\left[0.5+0.1+0.2+0.4+0.3+0.2+0.2+0.1+0.35+0.25\right]$$

$$= \frac{1}{10}[2.6] \Rightarrow \boxed{\mu_1 = 0.26}$$

$$\mu_2 = \frac{1}{4}\left[0.9+0.8+0.75+1\right]$$

$$= \frac{1}{4}[3.45] \Rightarrow \boxed{\mu_2 = 0.8625}$$

If it is a gaussian distribution, wkt the prob. density function $\boxed{P(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)}$

The class probability p_1 and p_2 is given by
$\Rightarrow \dfrac{\text{Total points in that class}}{\text{Total points}}$

$\boxed{p_1 = \frac{10}{14}}$ $\boxed{p_2 = \frac{4}{14}}$

From conditional prob wkt

$$P(\text{class} / \text{datapoint}) = \frac{P(\text{datapoint} / \text{class}) \cdot P(\text{class})}{P(\text{datapoint})}$$

Here when we take the probability that 0.6 belongs to c_1
$\Rightarrow P(\text{class}) = P(p_1) = \frac{10}{14}$

$\Rightarrow P(\text{datapoint} / \text{class}) = \frac{1}{\sqrt{2\pi\sigma_1^2}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu_1}{\sigma_1}\right)^2\right)$.

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and

$P(\text{datapoint}) = P(x=0.6)$

$$= \sum_{i=1}^{2} P(p_i) \cdot P(\text{datapoint} / \text{class}_i)$$

$$P(x=0.6) = P(\text{class}1) * \frac{1}{\sqrt{2\pi\sigma_1^2}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu_1}{\sigma_1}\right)^2\right)$$

$$+ P(\text{class}2) * \frac{1}{\sqrt{2\pi\sigma_2^2}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu_2}{\sigma_2}\right)^2\right)$$

$$= \frac{10}{14} * \frac{1}{\sqrt{2\pi * 0.0149}} \exp\left(-\frac{1}{2}\left(\frac{(0.6-0.26)^2}{0.0149}\right)\right)$$

$$+ \frac{4}{14} * \frac{1}{\sqrt{2\pi * 0.0092}} \exp\left(-\frac{1}{2}\frac{(0.6-0.8625)^2}{0.0092}\right)$$

$$= \frac{5}{7} * \frac{1}{\sqrt{0.093}} \exp(-3.879) + \frac{2}{7} * \frac{1}{\sqrt{0.057}} \exp(-3.744)$$

$$= 0.0484 + 0.0283 = \underline{\underline{0.0767}}$$

$$P(x=0.6 / \text{class}=c_1) = P(\text{cla} \frac{1}{\sqrt{2\pi\sigma_1^2}} \exp\left(-\frac{1}{2}\frac{(x-0.26)^2}{0.0149}\right)$$

$$= \frac{1}{\sqrt{2\pi * 0.0149}} \exp(-3.879)$$

$$= \frac{\exp(-3.879)}{\sqrt{0.093}} = 0.0677$$

The probab of $x = 0.6$ lying in class 1 is

$P(\text{class}=1 / \text{datapoint}=0.6)$

$$= \frac{P(x=0.6 / \text{class}=1) \cdot P(\text{class}=1)}{P(x=0.6)}$$

$$= \frac{0.0677 * 0.714}{0.0767} = 0.6302$$

$$\therefore \boxed{P(\text{class}=1 \mid x=0.6) = 0.6302}$$

classmate

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2 b) $x = $ (goal, football, golf, defence, offence, wicked, office, strategy)

$$x_{politics} = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$x_{sport} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Prob. document $x = (1, 0, 0, 1, 1, 1, 1, 0)$ is about politics?

Total docs = 12

$\boxed{P(\text{politics}) = 1/2}$ $\boxed{P(\text{sports}) = 1/2}$

According to Naive Bayes Theorem where the attributes are independent of each other

$$P(\langle x_1\, x_2\, x_3 .. x_n \rangle / \text{class}) = \prod_{i=1}^{n} P(x_i / \text{class})$$

x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	Total docs
2	1	1	5	5	1	4	5	6
4	4	1	4	5	1	0	1	6

$P(\langle 10011110 \rangle / \text{politics}) = P(x_0 = 1/\text{politics}) \cdot P(x_1 = 0/\text{politics})$
$\cdot P(x_2 = 0/\text{politics}) \cdot P(x_3 = 1/\text{politics})$
$P(x_4 = 1/\text{politics}) \cdot P(x_5 = 1/\text{politics})$
$P(x_6 = 1/\text{politics}) \cdot P(x_7 = 0/\text{politics})$

P(doc/politics)

$$= \frac{2}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} \cdot \frac{4}{6} \cdot \frac{1}{6}$$

$||^{ly}$

$$P(doc / Sport) = \frac{4}{6} \cdot \frac{2}{6} \cdot \frac{5}{6} \cdot \frac{4}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{0}{6} \cdot \frac{1}{6} = 0 \quad -(1)$$

Now

$P(class = politics / document = x)$

$$= \frac{P(class = politics) \cdot P(document = x / class = politics)}{P(document = x)}$$

$$= \frac{P(class = 'p') \cdot P(doc = x / class = 'p')}{P(class = 'p') \cdot P(data = x / class = 'p') + P(class = 's') \cdot P(data = x / class = 's')}$$

$$= \frac{1/2 \cdot P(doc = x / class = 'p')}{1/2 \; P(doc = x / class = 'p') + 0}$$

[$\because$ From (1) we know that $P(data = x / class = sport) = 0$]

$$\Rightarrow \boxed{P(class = 'politics' / document) = 1}$$