3. Data points $= \{x_i : i = 1, \dots n\}$ convex hull. $X = \sum_i \alpha_i x_i$
$\alpha_i \geq 0$
$\sum_i \alpha_i = 1$

$\{z_i : i = 1, \dots n\}$ convex hull $Z = \sum_i \beta_i z_i$ $\beta_i \geq 0$
$\sum_i \beta_i = 1$

Linear separable : $w^T x_i + w_0 > 0$ $\forall$ points $x_i \in X$
$w^T z_i + w_0 < 0$ $\forall$ points $z_i \in Z$

(i) if convex hull intersect, they cannot be linearly separable.

↳ If the convex hull intersect, then they must have common point in them. Let that point be 'a'. From the definition of linear separable for those 2 convex hulls, it comes as.

① – $w^T a + w_0 > 0$ [as the point a belongs to convex hull X]

② – $w^T a + w_0 < 0$ [as the point a belongs to convex hull Z]

But ① and ② cannot be true at the same time as they contradict each other.

∴ If the hulls intersect, then they are not linear separable

(ii) if the hull is linearly separable, then they cannot intersect

↳ As we know that the convex hulls are linearly separable, we know they follow the eqns

① – $w^T x_i + w_0 > 0$ $\forall$ $x_i \in X$

② – $w^T z_i + w_0 < 0$ $\forall$ $z_i \in Z$

For these hulls to intersect, we need to find one common point in both these hulls that satisfy the above 2 eqns.

Since ① & ② ~~are~~ contra with same point would contradict each other, they cannot ~~be~~ intersect each other