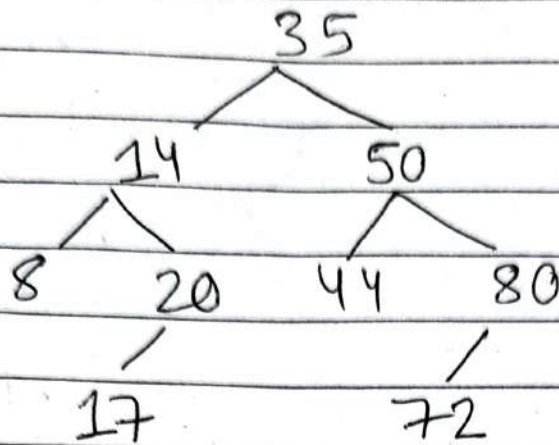


1

# Binary Search Tree (BST)

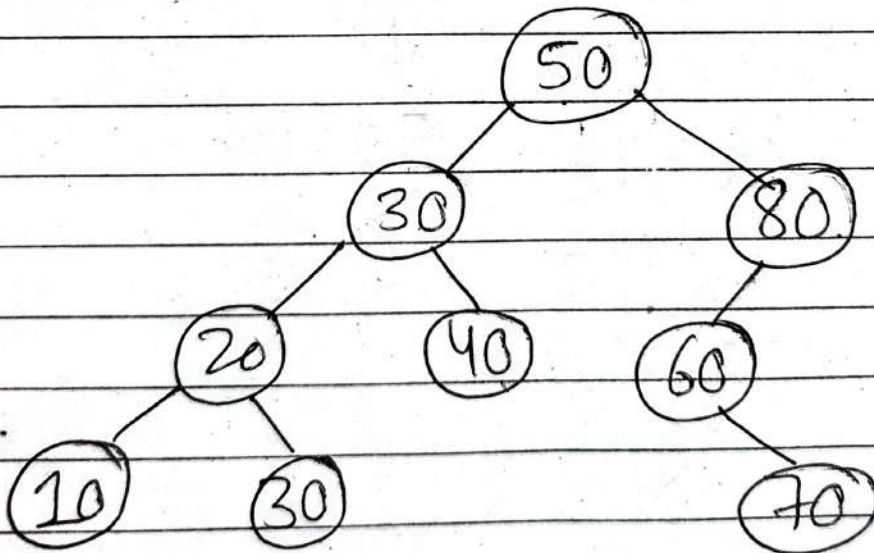


at most 2

Left  $\leq$  Data < Right  
(subtree) (subtree)

Example:

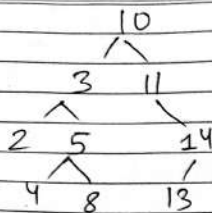
50, 30, 40, 20, 80, 30, 60, 70, 10



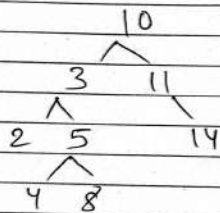


## Deletion

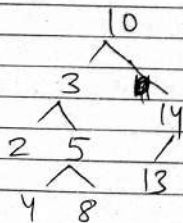
(4)



Case I: 4, 8, 13  $\Rightarrow$  any leaf node can be deleted.  
(13 is deleted.)



Case II: Node with 1 child are 11, 14.  
(11 is deleted.)



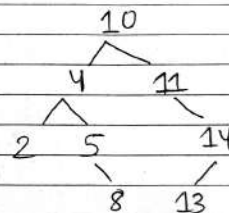
(5)

Case III: Node with 2 children is deleted.

(Nodes with 2 children are 10, 3, 5.)

2, 3, 4, 5, 8, 10, 11, 13, 14 (Inorder traversal)

$\Rightarrow$  If 3 is deleted then, adjacent element (4) will be used to replace (3).  
(inorder)



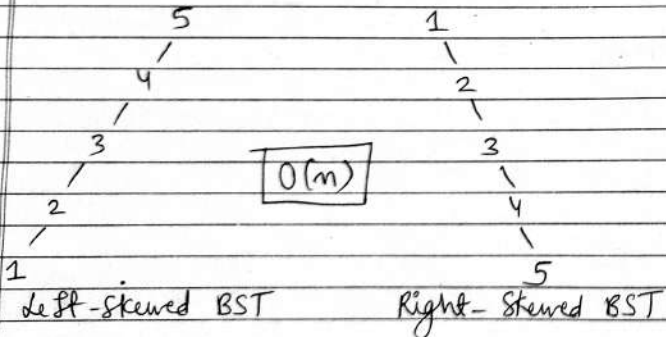
6

## \* Introduction to AVL Tree:

- also called as height balance tree.
- it is binary search tree.
- balance tree's concept was introduced in 1962 by ~~Adelson-Velsky-Landis~~ Adelson-Velsky & Landis
- self-balancing ~~tree~~ binary search tree.

$$\text{Balance Factor} = h(T^L) - h(T^R)$$

Problem with BST:

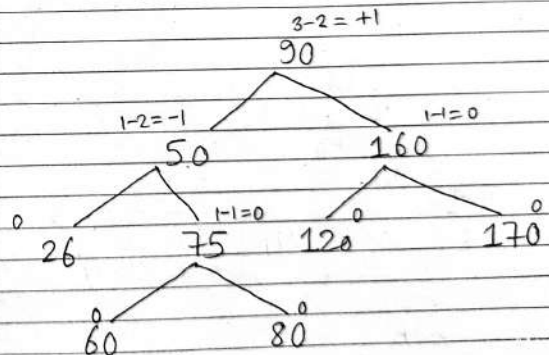
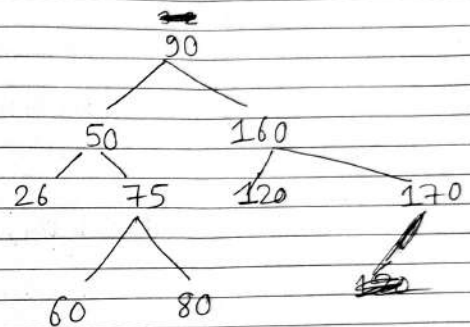


⇒ Time complexity of BST:  $\log(n)$ .

⇒ Worst case:  $O(n)$   
(Linear)

7

$$\text{Balance Factor} = h(T^L) - h(T^R) \quad [0, +1, -1]$$

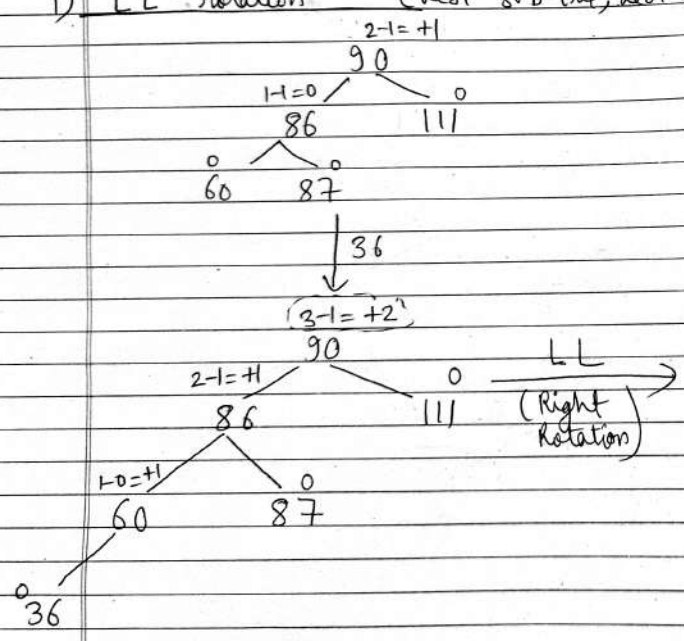


8

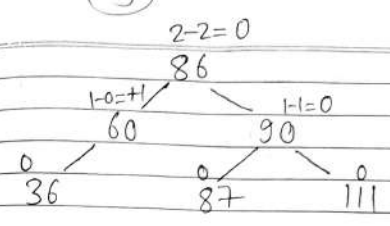
\* Insertion into an AVL tree  
(Rotations in AVL tree):

- i) LL rotation  $\rightarrow$  (right rotation) } single rotation
- ii) RR rotation  $\rightarrow$  (left rotation) }
- iii) LR rotation } double rotation
- iv) RL rotation }

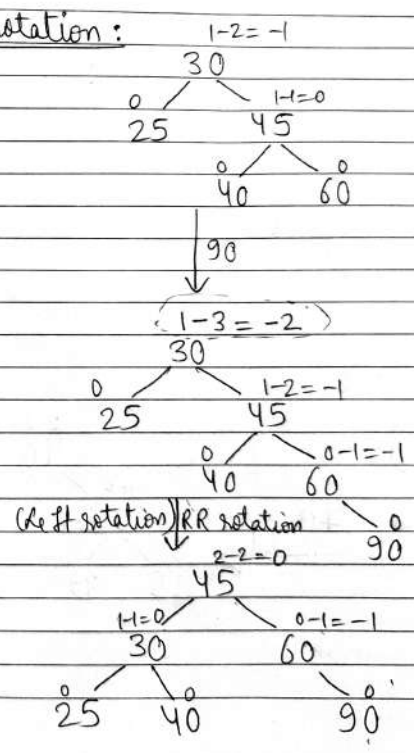
i) LL rotation (Left sub-tree, left node):



9



ii) RR rotation:





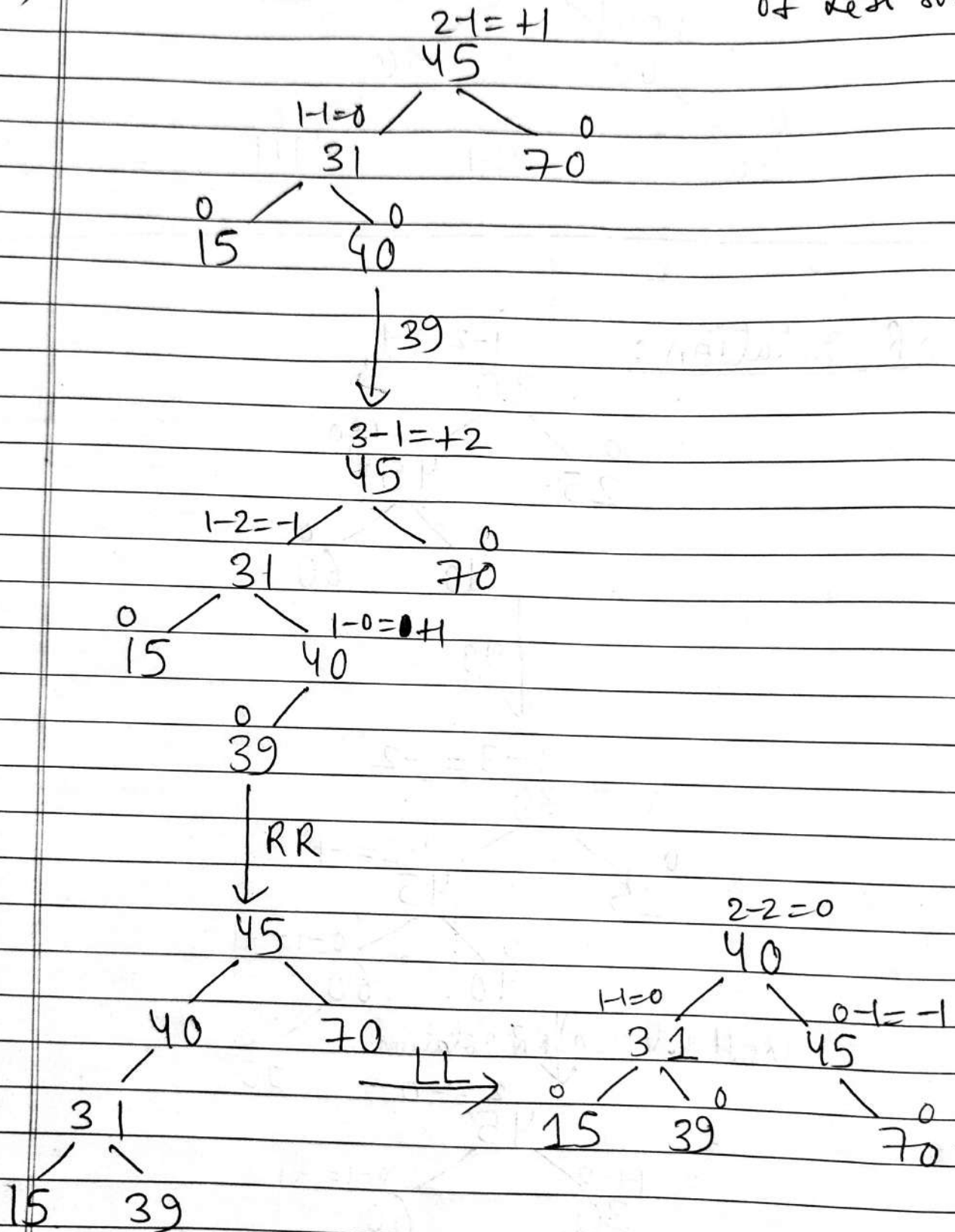
10

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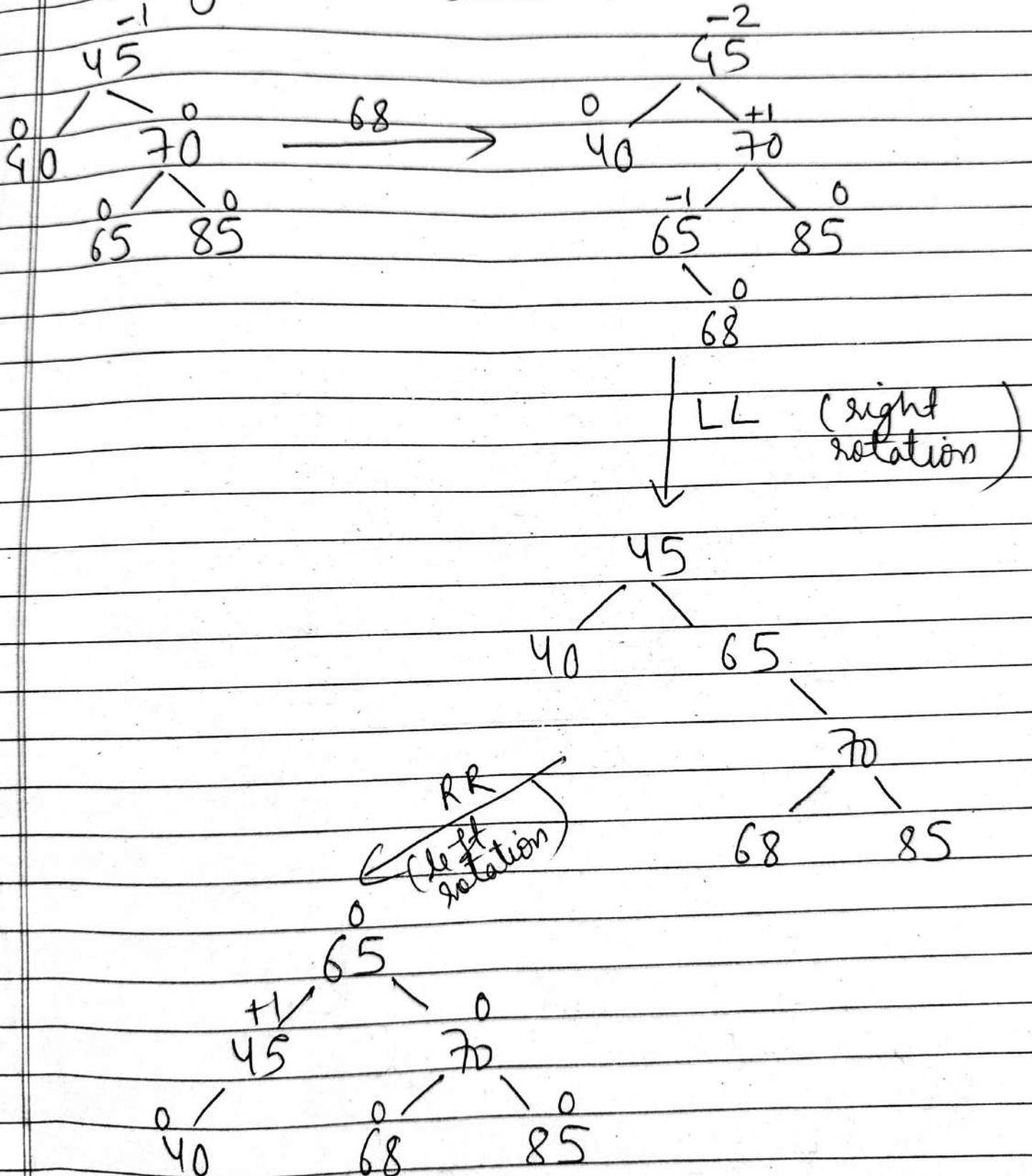
DATE :

(RR → LL)

iii) LR rotation : (Add new node to right node of left sub-tree)



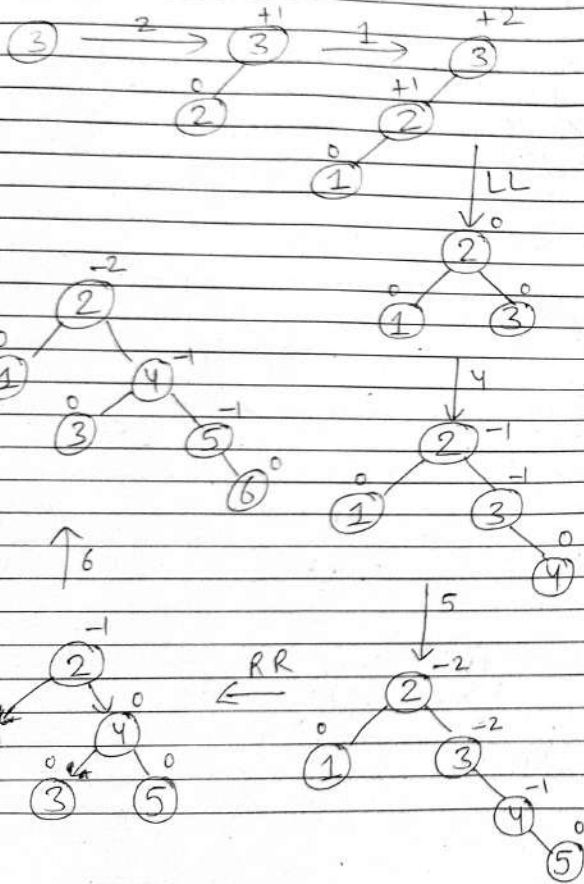
iv) RL rotation (LL → RR):  
(Add new node to left node of right subtree).



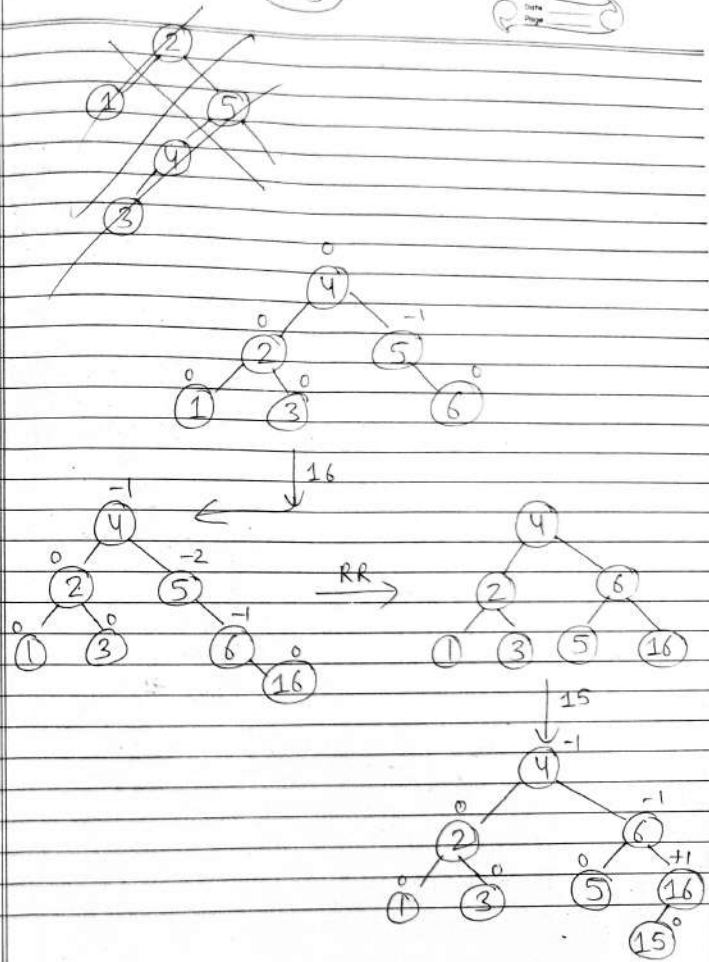
12

\* Construct AVL tree:

3, 2, 1, 4, 5, 6, 16, 15



13



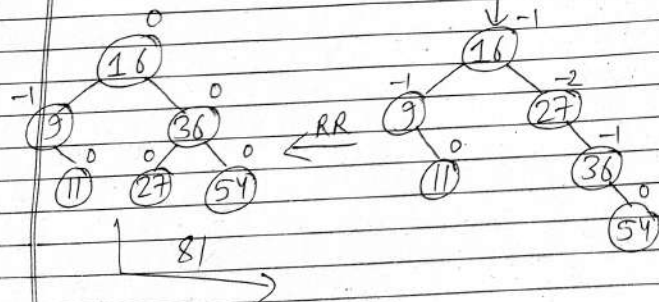
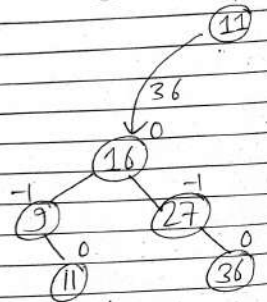
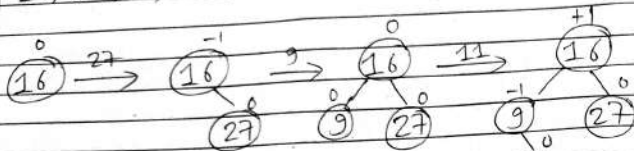


14

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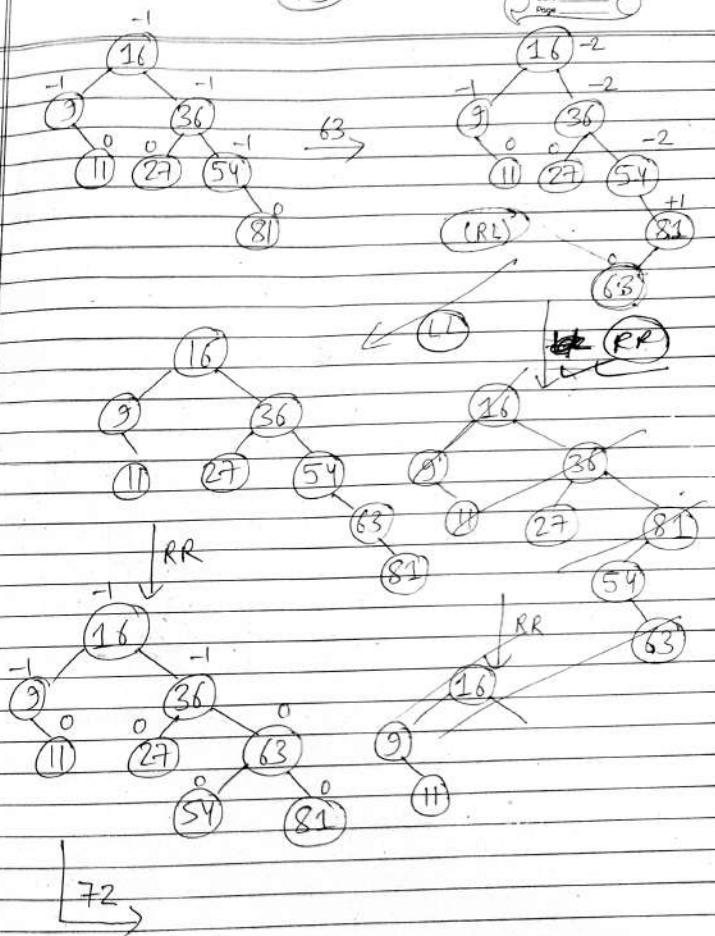
Questions: Construct AVL trees.

① 16, 27, 9, 11, 36, 54, 81, 63, 72



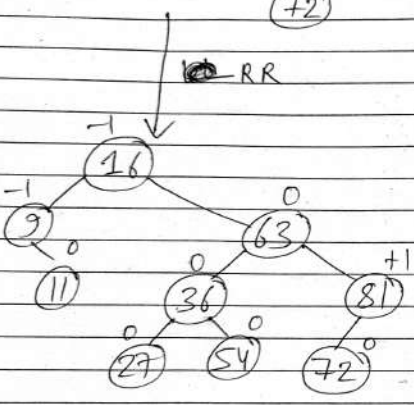
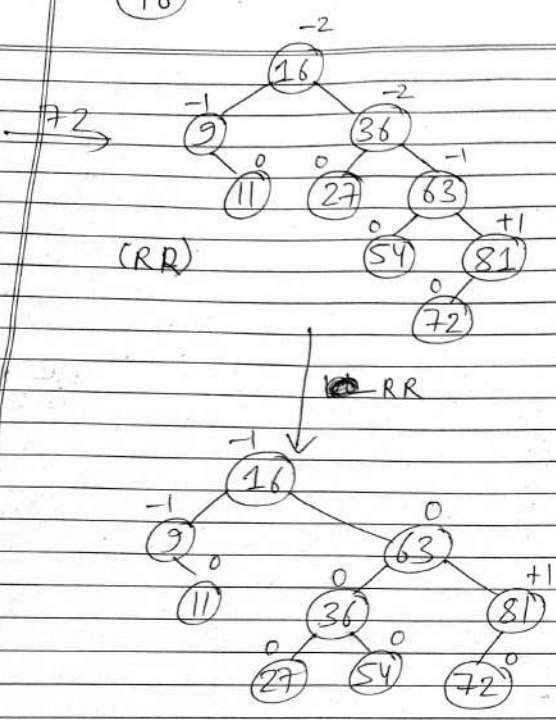
15

Date \_\_\_\_\_  
Page \_\_\_\_\_



16

Date \_\_\_\_\_  
Page \_\_\_\_\_

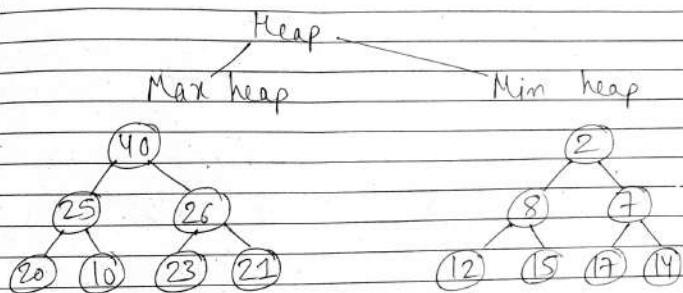


17

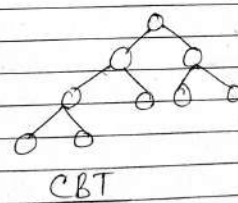
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# \* Heap

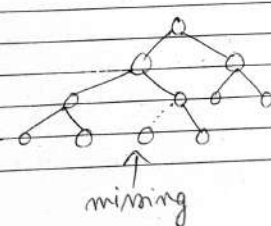
- It is a complete binary tree. (left possible)
- The value at node N is greater than or equal to the value at each of the children of N (max heap)



Complete Binary Tree: When new node is added, add to the left most first.



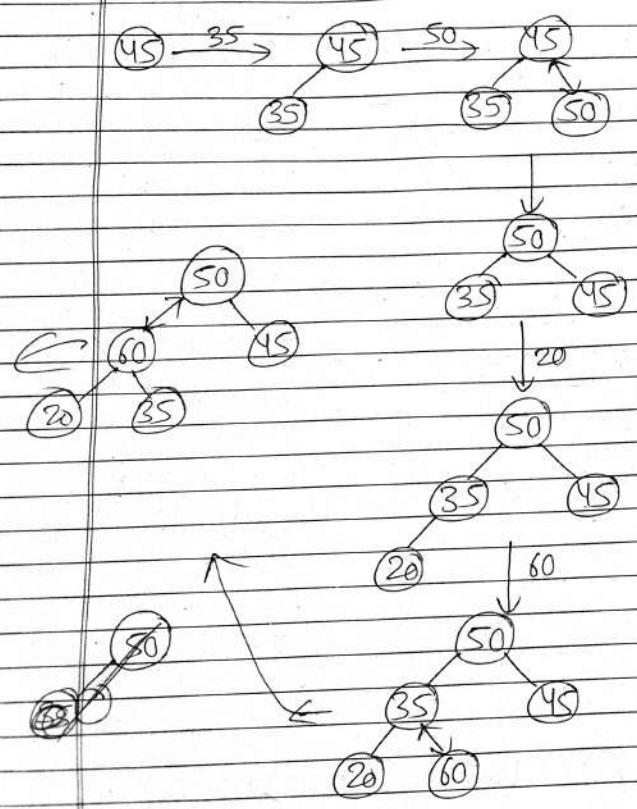
Not Complete Binary Tree:



18

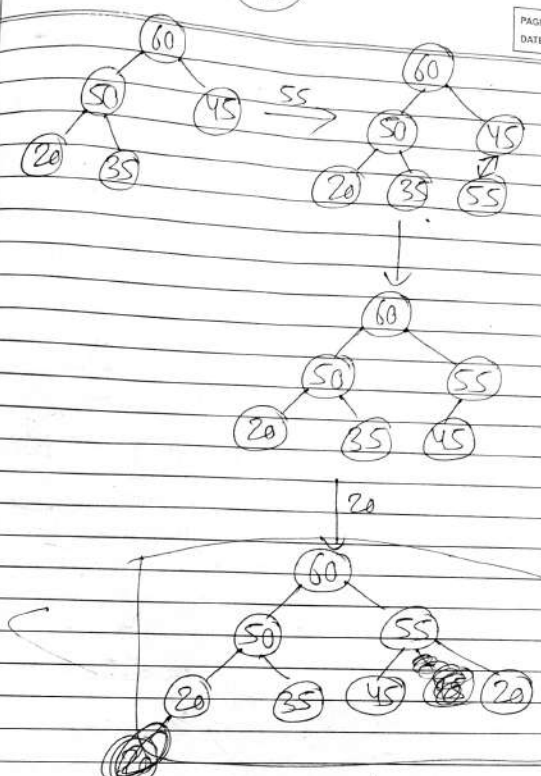
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\* Insertion in Heap: (Left most priority)  
(Max heap)  
45, 35, 50, 20, 60, 55, 20



19

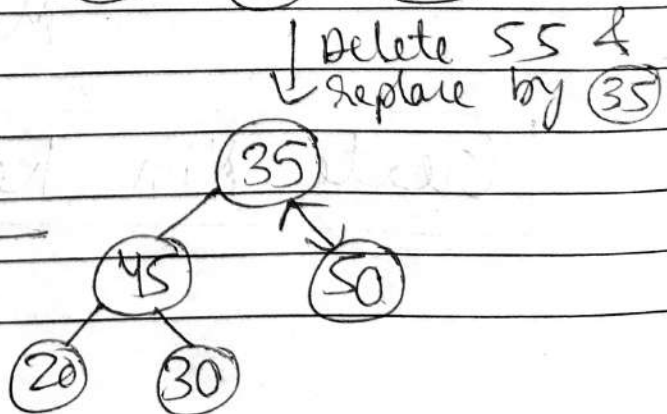
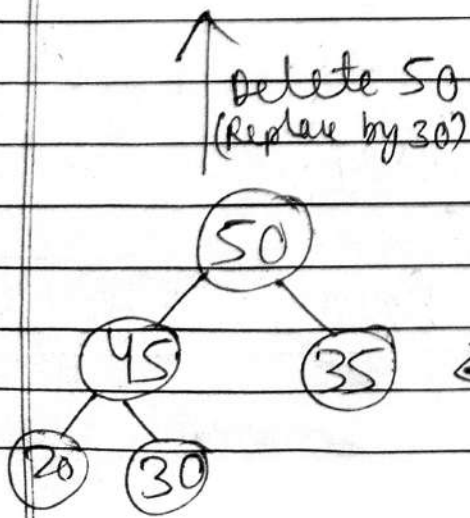
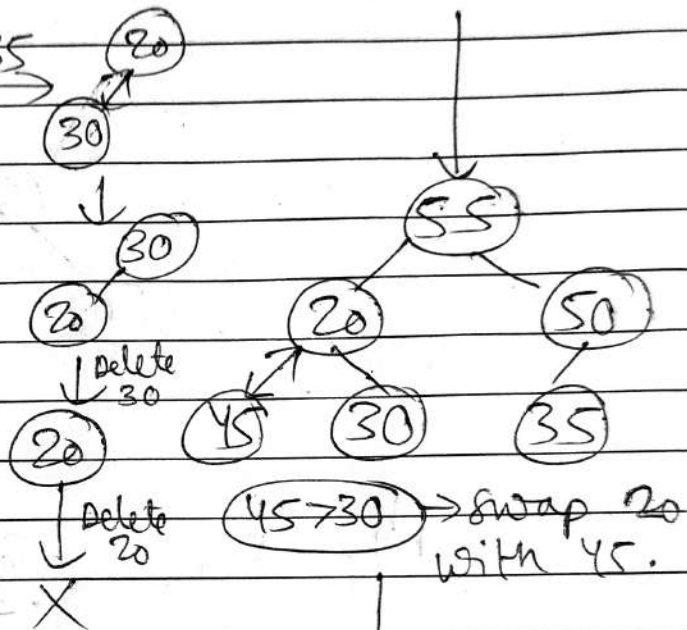
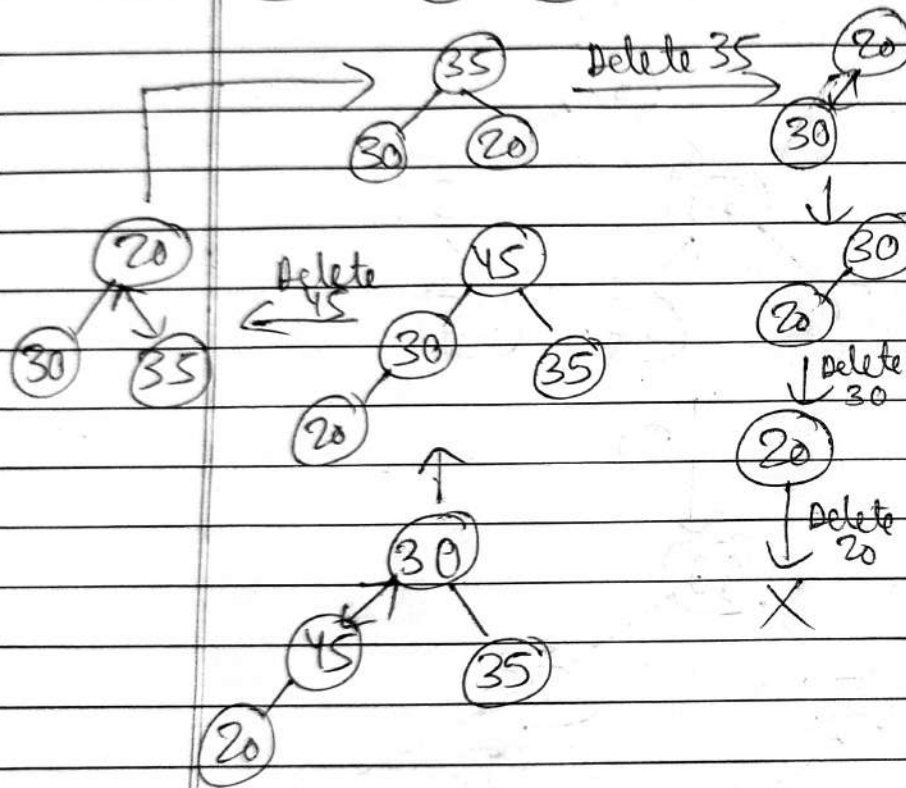
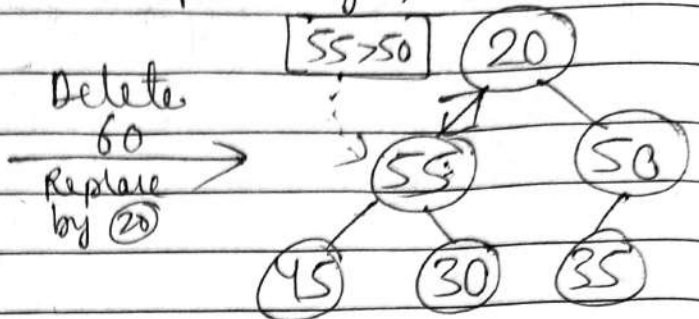
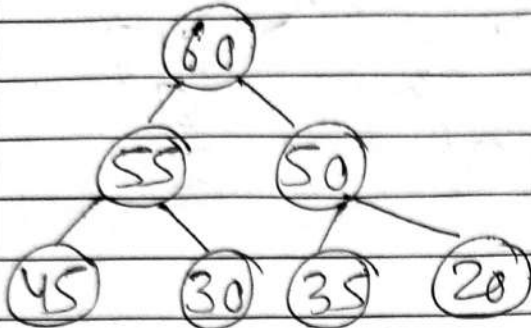
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Max heap.

Create Min heap from above data

\* Deletion in heap: (Delete the topmost & replace by last element)



x) B-Tree:

- It is a self-balanced search tree in which every node contains multiple keys (values) & has more than two children.
- all nodes (leaf) are at same level.  
[M-order]
- all nodes except root node have at least  $\lceil m/2 - 1 \rceil$  keys & max.  $(m-1)$  keys.  $\Rightarrow (m/2 \rightarrow \text{ceiling value})$
- no node contains more than  $(m-1)$  values.
- keys are arranged in defined order within the nodes. (ascending order)



22

Date \_\_\_\_\_  
Page \_\_\_\_\_

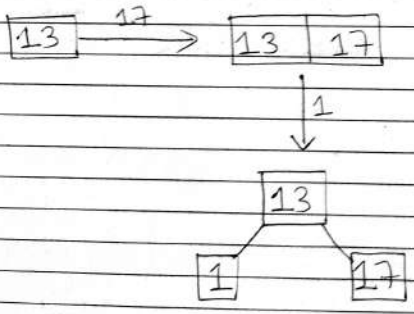
\* Construct B-Tree:

13, 17, 1, 14, 16, 23, 24

Order (m) = 3

81<sup>st</sup> min. keys (values) =  $\frac{m}{2} - 1$   
 $= \frac{3}{2} - 1$   
 $= 2 - 1$   
 $= 1 //$

max. keys = m-1  
 $= 3 - 1$   
 $= 2 //$



Rough  
 (since, leaf nodes should be at same level.)

23

14 | 16 | 17

Date \_\_\_\_\_  
Page \_\_\_\_\_

