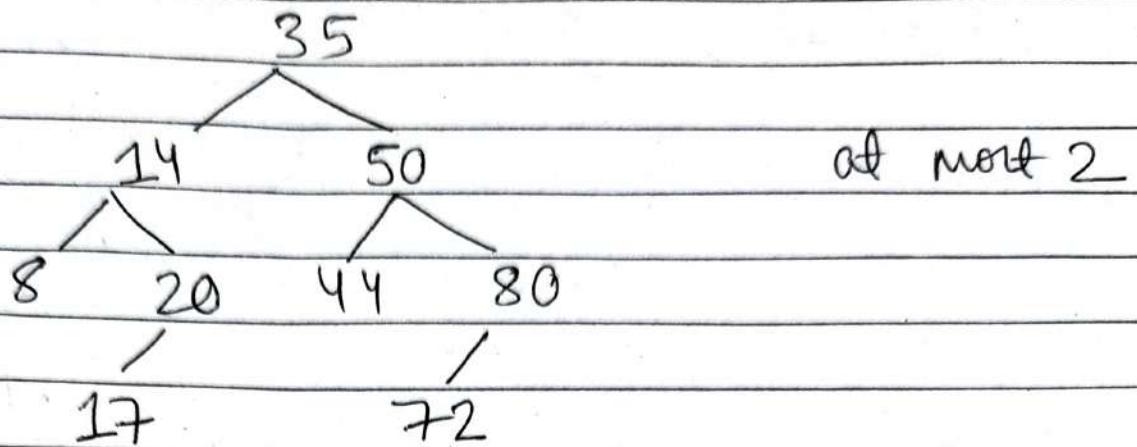


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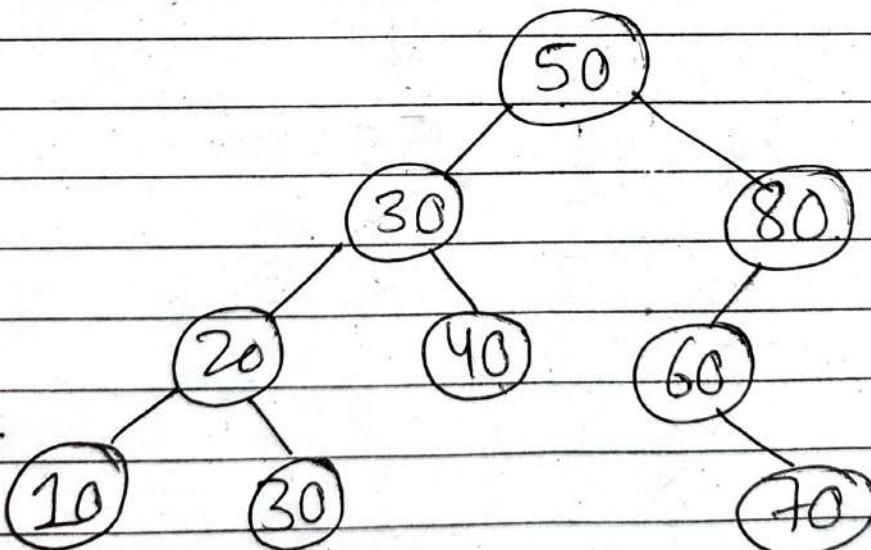
binary Search Tree (BST)

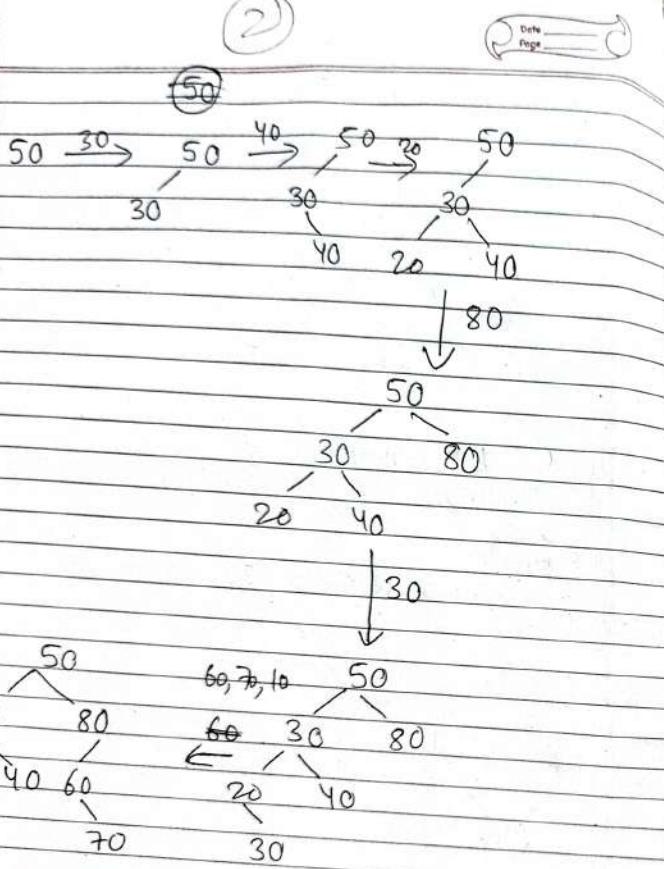


Left \leq Data < Right

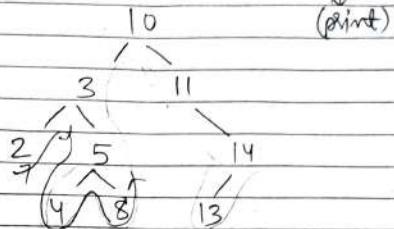
Example:

50, 30, 40, 20, 80, 30, 60, 70, 10





* In-Order Traversal (Left, Root, Right)



2, 3, 4, 5, 8, 10, 11, 13, 14 (sorted in ascending order)
 Left subtree Root Right subtree

* Deletion :

Three cases:

Case I: Deleting leaf node

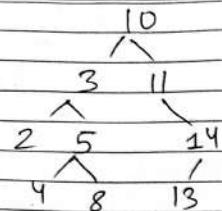
Case II: Deletion of node who has only one child

Case III: Deletion of node who has two children.

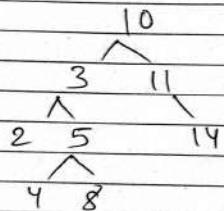
Deletion

(4)

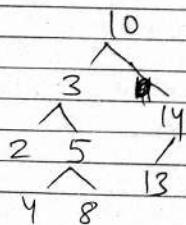
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Case I: $4, 8, 13 \rightarrow$ any leaf node can be deleted.
(13 is deleted.)



Case II: Node with 1 child are 11, 14.
(11 is deleted).



(5)

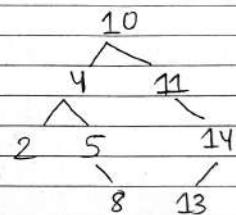
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Case III: Node with 2 children is deleted.

(Nodes with 2 children are 10, 3, 5.)

$2, 3, 4, 5, 8, 10, 11, 13, 14$ (Inorder traversal)

\Rightarrow If 3 is deleted then, adjacent element 4 will be used to replace 3.
(inorder)

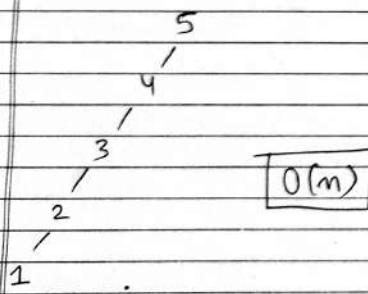


(6) Introduction to AVL Tree:

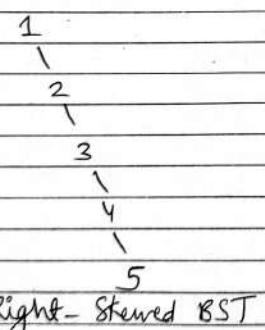
- also called as height balanced tree.
- it is binary search tree.
- balance tree's concept was introduced in 1962 by Adelson-Velsky & Landis
- self-balancing tree - binary search tree.

$$\text{Balance Factor} = h(T^L) - h(T^R)$$

Problem with BST:



Left-skewed BST



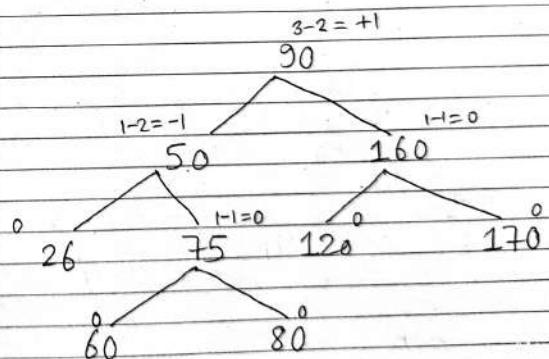
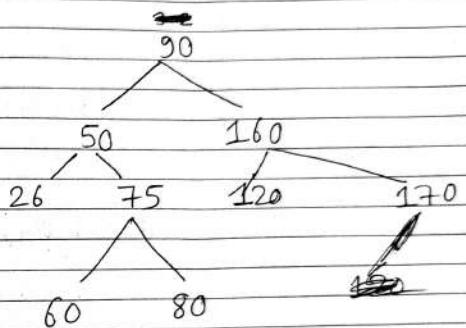
Right-skewed BST

⇒ Time Complexity of BST: $\log(n)$.

⇒ Worst case: $O(n)$
(Linear)

(7)

$$\text{Balance Factor} = h(T^L) - h(T^R) \quad [0, +1, -1]$$

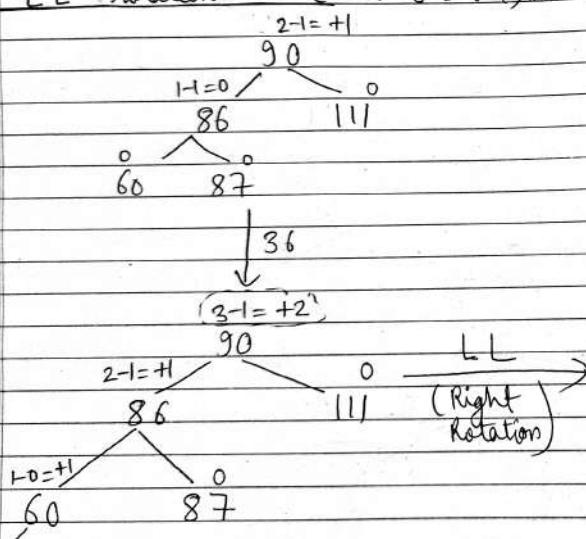


8

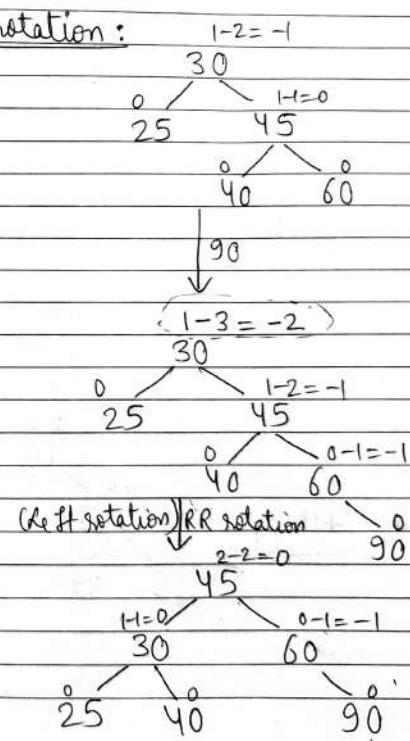
i) Insertion into an AVL tree
(Rotations in AVL tree):

- i) LL rotation \rightarrow single rotation
- ii) RR rotation \rightarrow single rotation
- iii) LR rotation \rightarrow double rotation
- iv) RL rotation \rightarrow double rotation

i) LL rotation (Left sub-tree, left node):

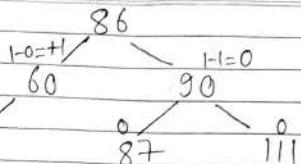


ii) RR rotation:



9

2-2=0



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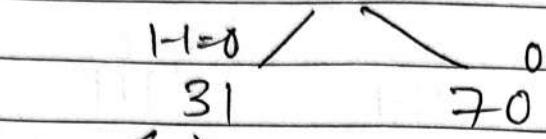
PAGE NO. : _____
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(RR → LL)

iii) LR rotation : (Add new node to right node of left sub-tree)

$$2-1=+1$$

45



$$1-1=0$$

$$31$$

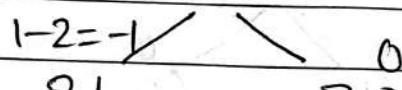
$$70$$



39

$$3-1=+2$$

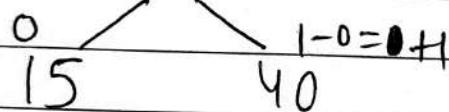
45



$$1-2=-1$$

$$31$$

$$70$$



$$1-0=+1$$

$$15$$

$$40$$

$$39$$

RR

$$45$$

$$40$$

$$70$$

$$31$$

$$39$$

$$LL \rightarrow$$

$$H=0$$

$$31$$

$$15$$

$$39$$

$$45$$

$$70$$

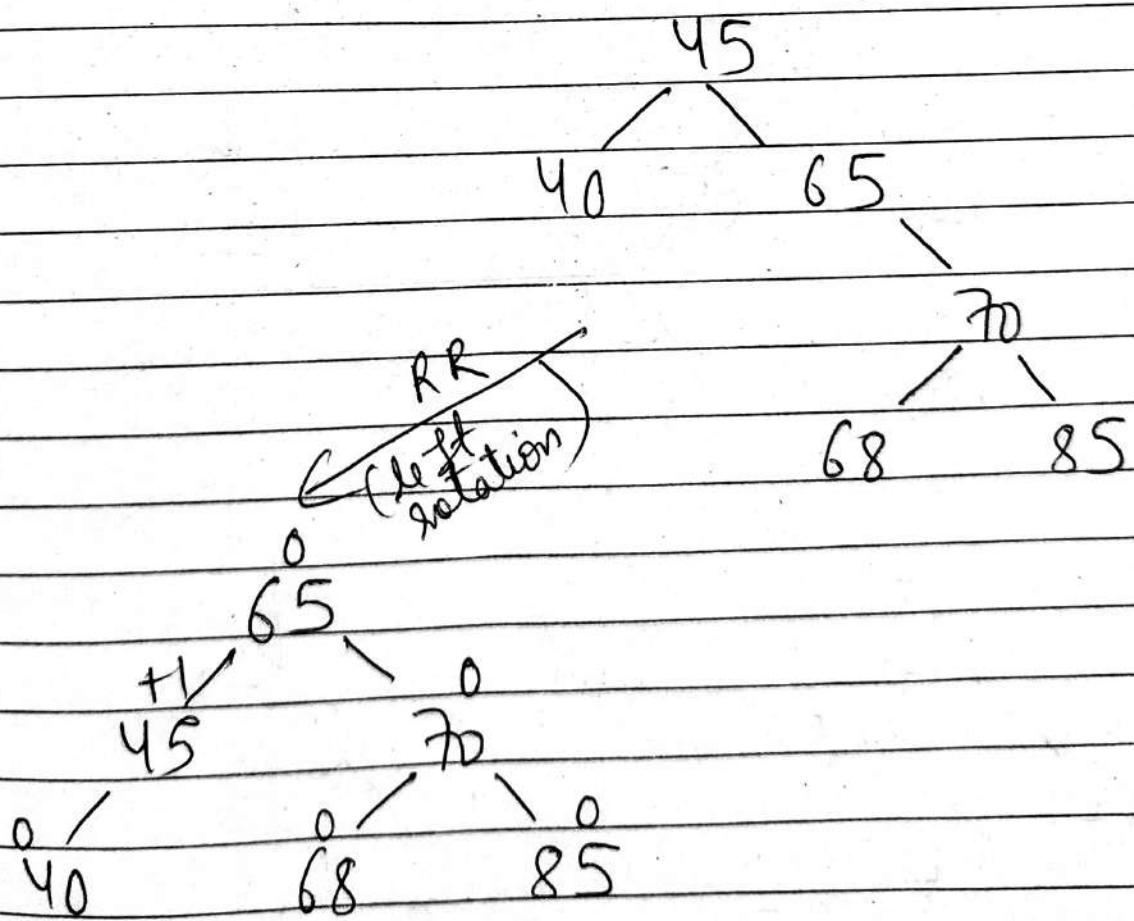
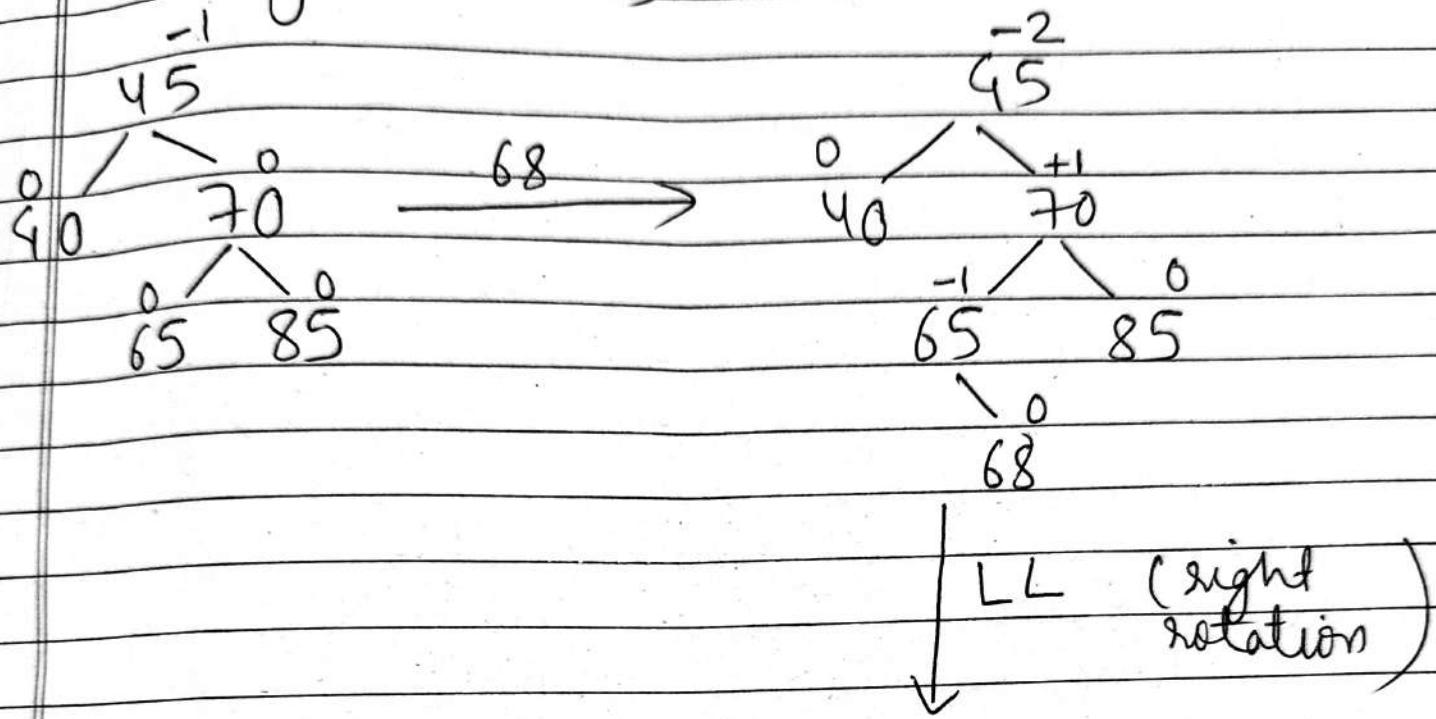
$$2-2=0$$

$$40$$

$$0-1=-1$$

iv) RL rotation ($LL \rightarrow RR$):

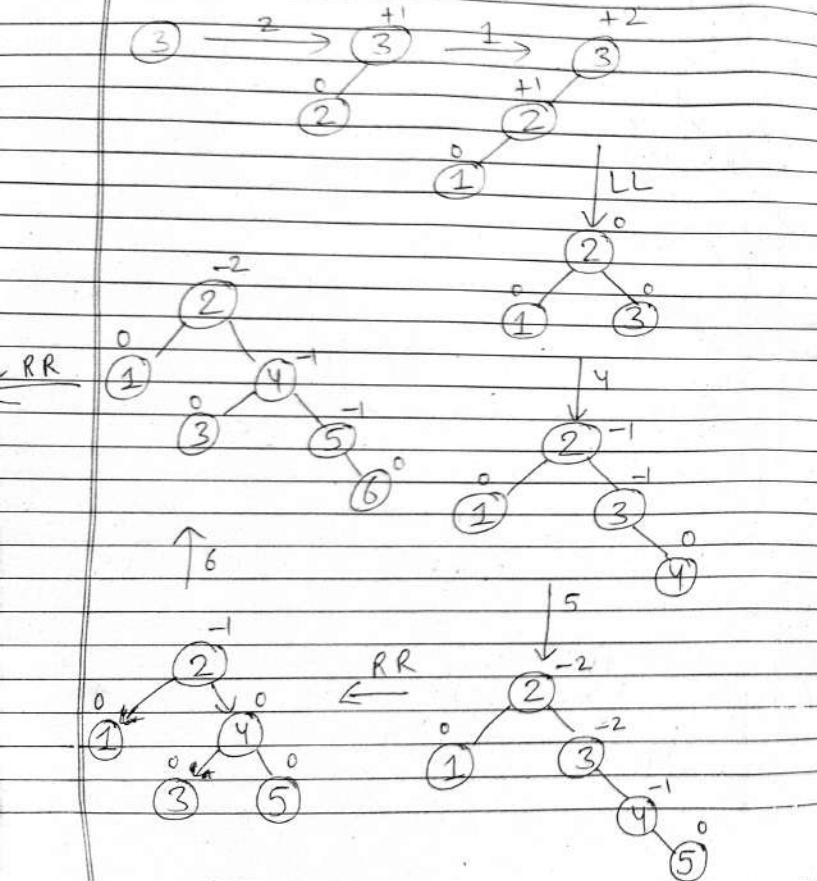
(Add new node to left node of right subtree).



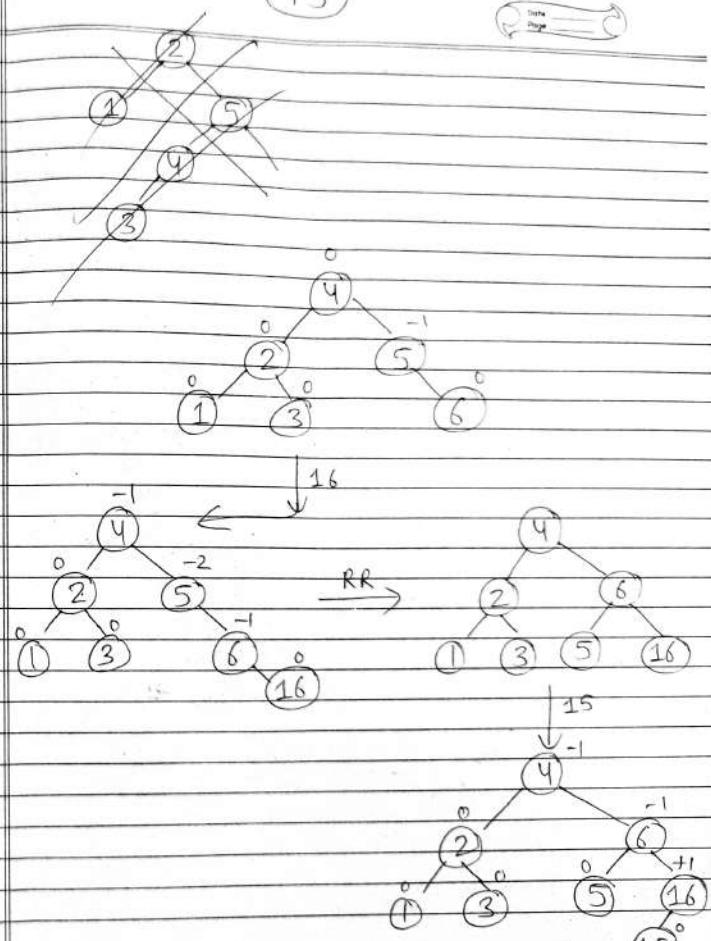
(12)

* Construct AVL tree:

3, 2, 1, 4, 5, 6, 16, 15



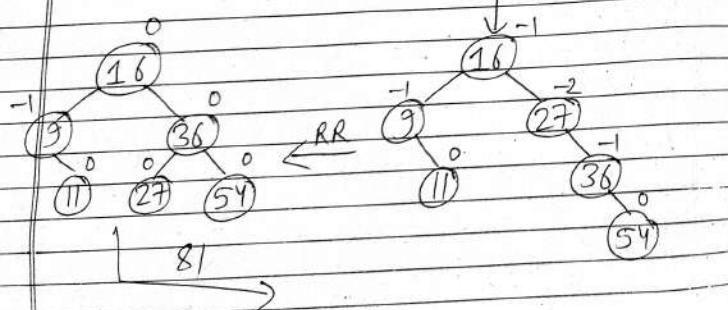
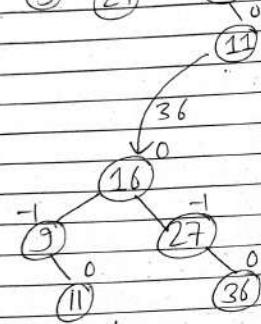
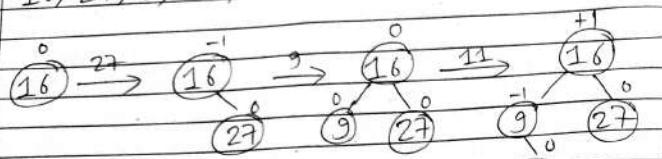
(13)



(14)

Questions: Construct AVL trees.

① 16, 27, 9, 11, 36, 54, 81, 63, 72



(15)

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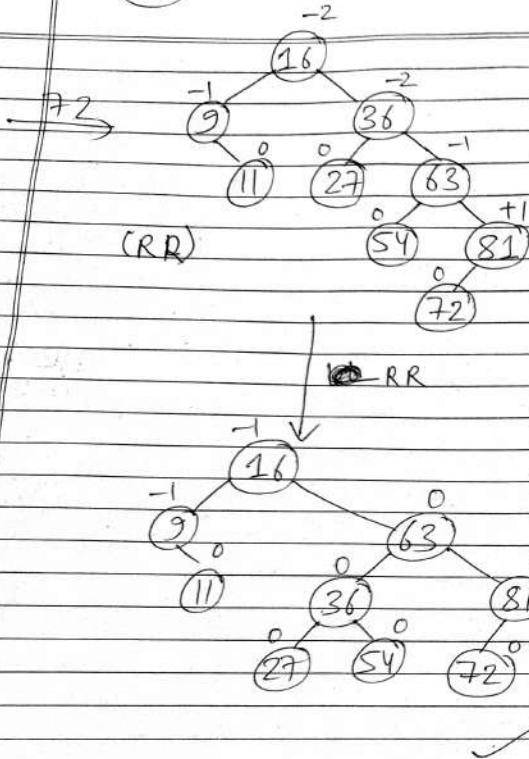
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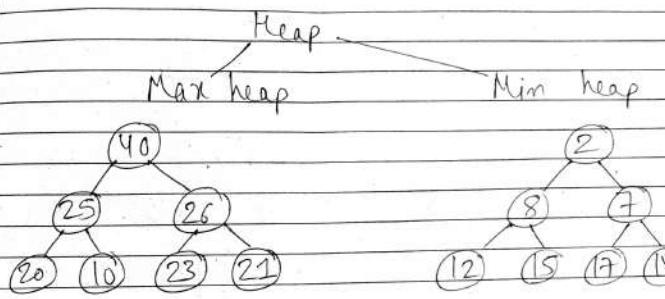
16



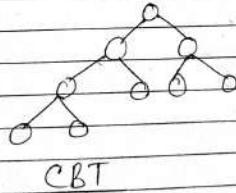
17

*) Heap

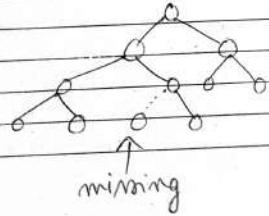
- It is a complete binary tree. (left possible)
- The value at node N is greater than or equal to the value at each of the children of N (max heap)



Complete Binary Tree: When new node is added, add to the left most first.



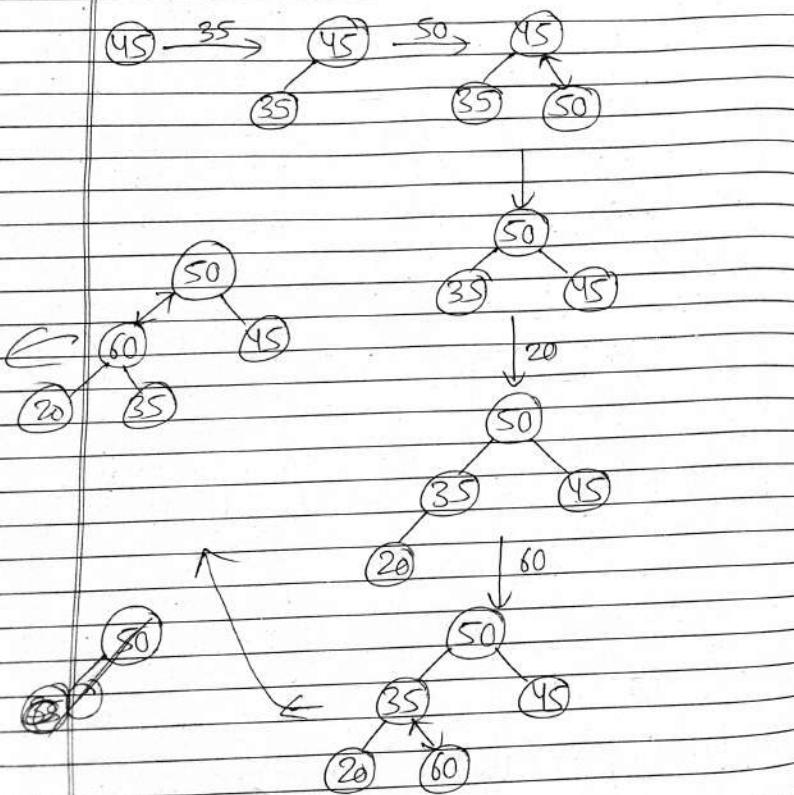
Not complete binary tree:



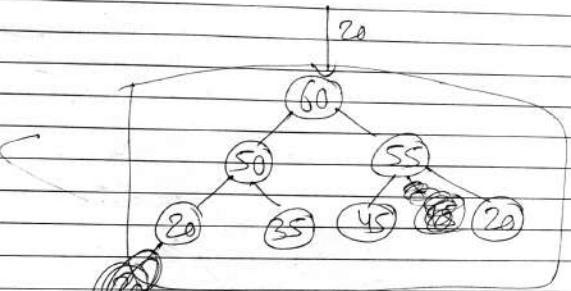
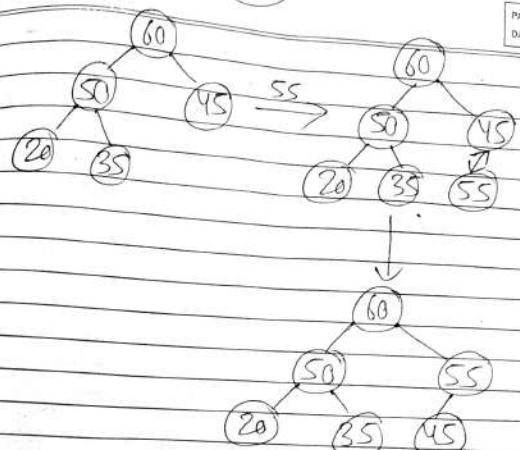
(18)

* Insertion in Heap: (left most priority)
(Max heap)

45, 35, 50, 20, 60, 55, 20



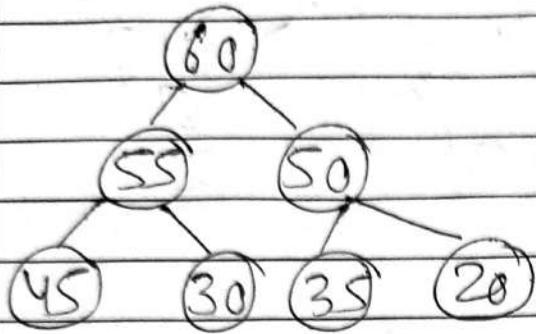
(19)



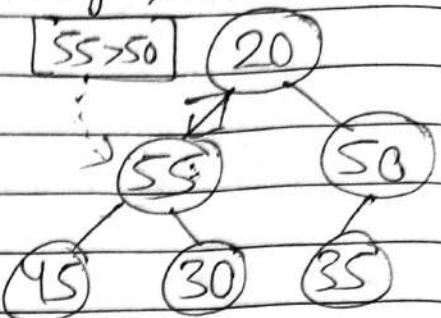
Max heap.

Create Min heap from above data.

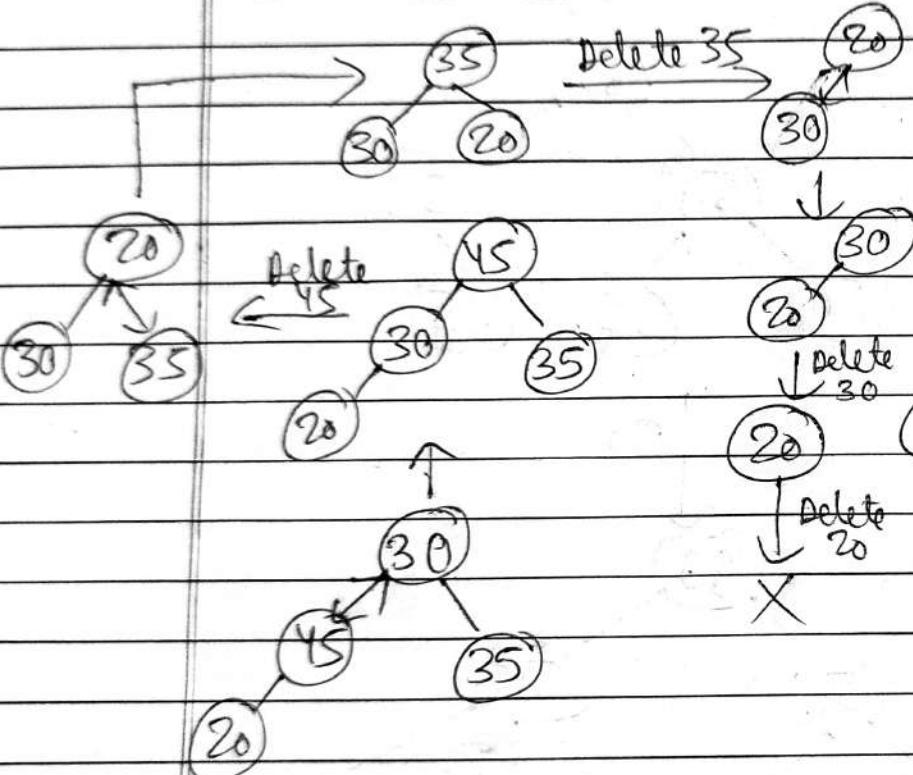
*) Deletion in heap: (Delete the top most)
& replace by last element



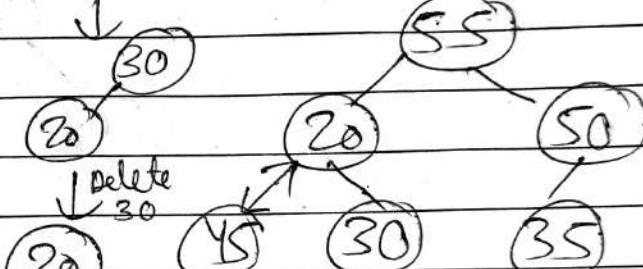
Delete 60
Replace by 20



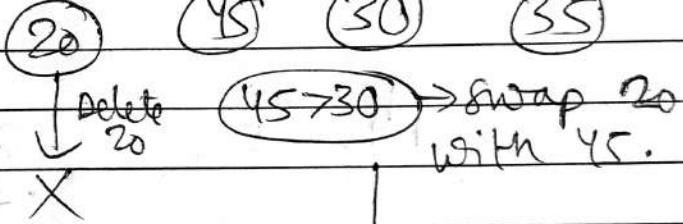
Delete 35



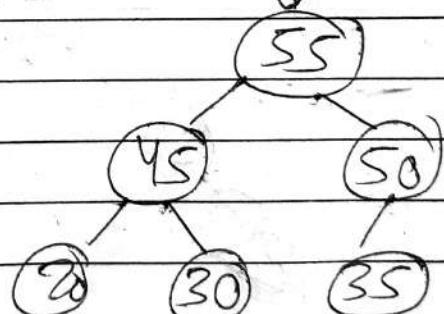
Delete 30



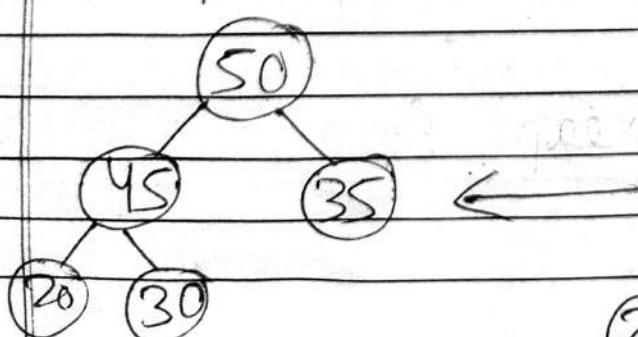
Delete 20



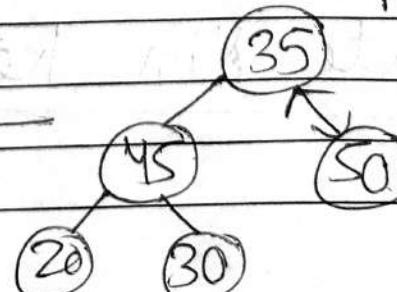
$45 > 30 \rightarrow$ swap 20 with 45.



Delete 50
(Replace by 30)



Delete 55 &
replace by 35



x) B-Tree:

- It is a self-balanced search tree in which every node contains multiple keys (values) & has more than two children.
- all nodes (leaf) are at same level.
[M-order]
- all nodes except root node have at least $\lceil \frac{m}{2} - 1 \rceil$ keys & max. $(m-1)$ keys. $\Rightarrow (\frac{m}{2} \rightarrow \text{ceiling value})$
- no node contains more than $(m-1)$ values.
- keys are arranged in defined order within the nodes. (ascending order)

(22)

Date _____
Page _____

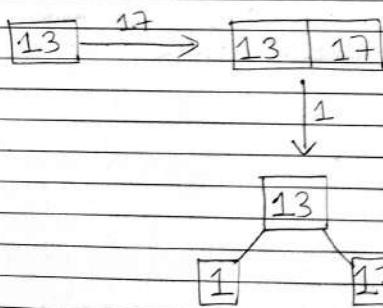
* Constraint B-Tree:

13, 17, 1, 14, 16, 23, 24

Order (m) = 3

$$\begin{aligned} \text{min. keys (values)} &= m/2 - 1 \\ &= 3/2 - 1 \\ &= 2 - 1 \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{max. keys} &= m - 1 \\ &= 3 - 1 \\ &= 2 \end{aligned}$$



Rough
↑

(since, leaf
nodes should
be at
same level.)

(23)

Date _____
Page _____

