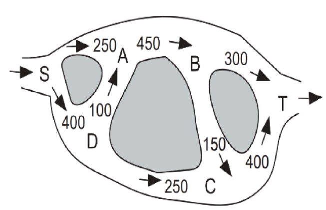
Dept. of CSE, IIIT Dharwad

CS 366: ADVANCED ALGORITHM DESIGN
Assignment 1:
Submission due date: 14/09/2025

- Assignment scores/markings depend on neatness and clarity.
- Write your answers with enough detail about your approach and concepts used, so that the grader will be able to understand it easily.
- The marking would be out of 100
- You are allowed to use only those concepts which are covered in the lecture class till date.
- Plagiarized assignments will be given a zero mark.

<table>
<tr><th>Sl. No.</th><th>Question</th></tr>
<tr><td>1.</td><td>Suppose you are given a directed graph G = (V, E), with a positive integer capacity C_e on each edge e, a designated source s ∈ V, and a designated sink t ∈ V. You are also given an integer maximum s-t flow in G, defined by a flow value f_e on each edge e.
Now suppose we pick a specific edge e ∈ E and increase its capacity by one unit. Show how to find a maximum flow in the resulting capacitated graph in time $(m + n)$, where m is the number of edges in G and n is the number of nodes.</td></tr>
<tr><td>2.</td><td>This diagram represents a road network. All vehicles enter at S and leave at T. The numbers represent the maximum flow rate in vehicles per hour in the direction from S to T.
a. What is the maximum number of vehicles which can enter and leave the network every hour?
b. Which single section of road could be improved to increase the traffic flow in the network?

</td></tr>
<tr><td>3.</td><td>The Jungle Junket food truck produces a large variety of different lunch menu items. Unfortunately, they can only produce their foods in limited quantities, so they often run out of popular items, making customers sad.
To minimize sadness, Jungle Junket is implementing a sophisticated lunch-ordering system. Customers text in their acceptable choices before lunch time. Then they can use an algorithm to preassign lunches to customers. Customers who do not get one of their choices should receive a Rs100 voucher. Jungle Junket would like to minimize the number of vouchers they give out.
Give an efficient algorithm for Jungle Junket to assign lunches to customers. In general, suppose that, on a given day, Jungle Junket has produced m types of food items $b_1, \ldots, b_m$, and the quantity of each type of food item b_j is exactly q_j . Suppose that n customers $a_1, \ldots, a_n$ text in their preferences, where each customer a_i submits a set A_i of one or more acceptable lunch choices. The algorithm should assign each customer either one of his/her choices or a Rs100 voucher. It should minimize the number of vouchers.
(Hint: Model this as a max flow problem.)</td></tr>
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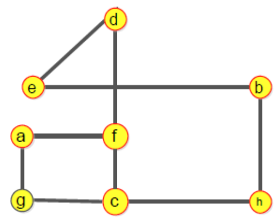
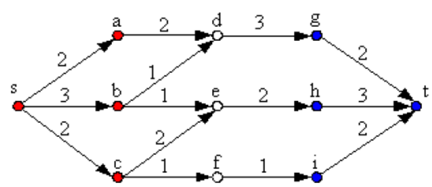
4.	Consider the following network flow $G(V,E)$, where $V=<s,a,b,c,d,t>$ and capacities of the edges in E are $c(s,a)$=13, $c(s,b)$=17,$c(a,c)$=18, $c(b,a)$=12, $c(c,t)$=19, $c(b,d)$=8, $c(d,t)$=14, $c(b,c)$=5, $c(c,d)$=12. Where s and t are the source and sink node respectively. a. Draw the above flow network. b. Show each residual graph. c. Compute the maximum flow of the above graph.
5.	A graph has a unique minimum cut if there is only one cut that whose weight is the minimum. Design an algorithm that finds if a graph has a unique minimum cut.
6.	a. You have given a graph G with 8 vertices and 9 edges. Check the graph G is bipartite or not? If yes, then find the maximum matching of the graph $(V,)$ and if no then explain why? b. Define the definition of perfect matching in a graph with a suitable example? If the graph G is a bipartite graph and has a maximum matching, then justify that the matching formed in G is perfect matching? c. Show that |f|=|M|, if possible of the above example, where f and M denotes the maximum flow in flow networks and maximum matching in bipartite graph. 
7.	a) Run the Ford Fulkerson to find the Maximum flow. Show each residual graph. b) Show the minimum-cut and find the capacity of minimum cuts. c) Justify it with Ford-Fulkerson algorithm (time complexity). 
8.	Consider the following network flow $G(V, E)$, where $V = <s, u, v, y, z, w, t>$ and capacities of the edges in E are $c(s, u) = 18$, $c(u, v) = 12$, $c(v, t) = 6$, $c(s, y) = 13$, $c(y, z) = 6$, $c(z, t) = 10$, $c(u, w) = 11$, $c(w, v) = 3$, $c(y, w) = 4$, $c(w, z) = 5$, Where s and t are the source and sink node respectively. a. Draw the above flow network. b. Show each residual graph. c. Compute the maximum flow of the above graph. d. If set $A = <s, u, y, w>$ and $B = <v, z, t>$, then what will be the capacity of the cut set (A, B).
9.	IIIT Dharwad requires an algorithm to schedule mid-semester exams for their courses each semester. There are *n* courses offered, *r* available rooms, and *m* time slots for exams. Given arrays *E[1...n]* and *S[1...r]*, *E[i]* represents the number of students enrolled in i^{th} course and *S[j]* represents the number of seats in each room respectively, an exam for course *i* can only be scheduled in room j if the number of enrolled students for that course is less than or equal to the number of seats available in the room ($E[i] \leq S[j]$). It's assumed that no two courses have overlapping enrollments. The goal is to develop an efficient algorithm for IIIT Dharwad to assign a room and a time slot to each course, or report if no such assignment is possible.
10.	Consider the maximum flow in a given network between two designated nodes s and t. For each of the following statements, either explain why it is true or provide a counterexample. A. If the capacity of every edge is even, then the value of the maximum flow must be even.

	B. If the capacity of every edge is even, then there is a maximum flow in which the flow on each edge is even. C. If the capacity of every edge is odd, then the value of the maximum flow must be odd. D. If the capacity of every edge is odd, then there is a maximum flow in which the flow on each edge is odd.
11.	Consider the maximum flow in a given network between two designated nodes s and t. For each of the following statements, either explain why it is true or provide a counterexample. A. Increasing the capacity of a single edge (u, v) by 1 can result in an increase of at most 1 in the max flow. B. Increasing the capacity of a single edge (u, v) by a positive integer k can result in an increase of at most k in the max flow. C. Decreasing the capacity of a single edge (u, v) by 1 can result in a decrease of at most 1 in the max flow. D. Decreasing the capacity of a single edge (u, v) by a positive integer k can result in a decrease of at most k in the max flow.
12.	Design the $construct\_candidates$ function to find the subset of a given set S whose sum is given K. Example: S={30,35,25,31,29}, K=60 Output: { {35,25},{31,29} }.
13.	$N-$Queen Problem: Place N queen in a $N \times N$ chess board such that no two queen place in same row, same column and diagonally. a. Draw the state space tree for N=4 using the above constraints. Assign each queen in different row. How many solutions are there? Also find the total number of backtrack are required to get the solution. b. How many nodes are in the state space tree of the $N-$queen problem with the constraint that no two queens place only same row and same column? c. How many leaf nodes are in the state space tree of the $N-$queen problem without any bound constraint (Assign each queen in different row)?
14.	Design the $construct\_candidates$ for printing all $K-$element subsets of n objects.