Shivambu Dev Pandey

Advanced Algorithm Design (CS366)

Aug 4

Aug 5

→ Correctness of algorithms

Algorithm Correctness Proof

1. Counterexample

2. Induction

3. Loop Invariant

→ Time Complexity

$$n + \frac{n}{2} + \frac{n}{3} + \cdots \frac{n}{n}$$

$$n\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots \frac{1}{n}\right)$$

$$n \log n$$

$T(n) = O(n) + T(n-1)$

$T(n-1) = O(n-1) + T(n-2)$

$T(2) = O(2) + T(1)$

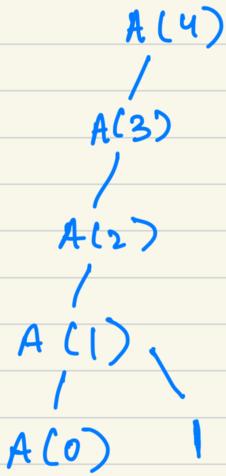
$T(1) = O(1)$

$T(n) = n + (n-1) + (n-2) + \dots + 2 \cdot 1$

$\dfrac{n(n+1)}{2} = O(n^2)$

T

- Space Complexity of Recursive Algo



0
A1
A2
A3
A4

n^2

A(n-1)
Print(n)
A(n-1)

→ Greedy Algo
→ Divide & Conquer

optimal substr
overlapping prob

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Knapsack Problem

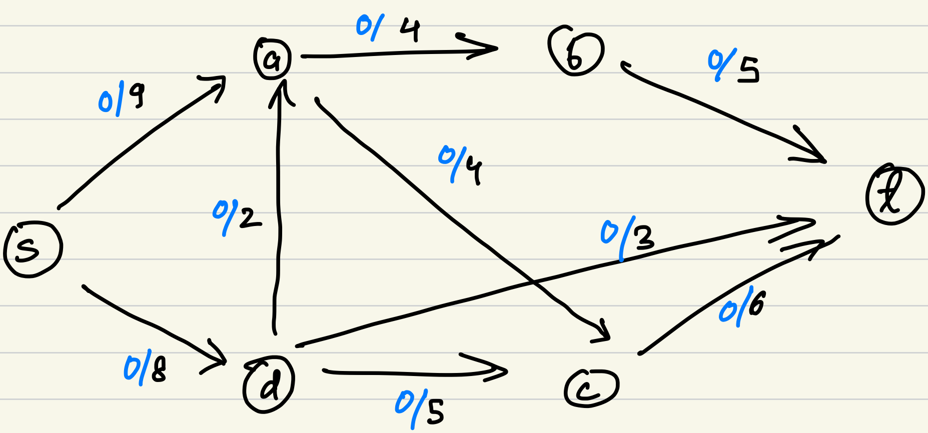
→ Flow Network

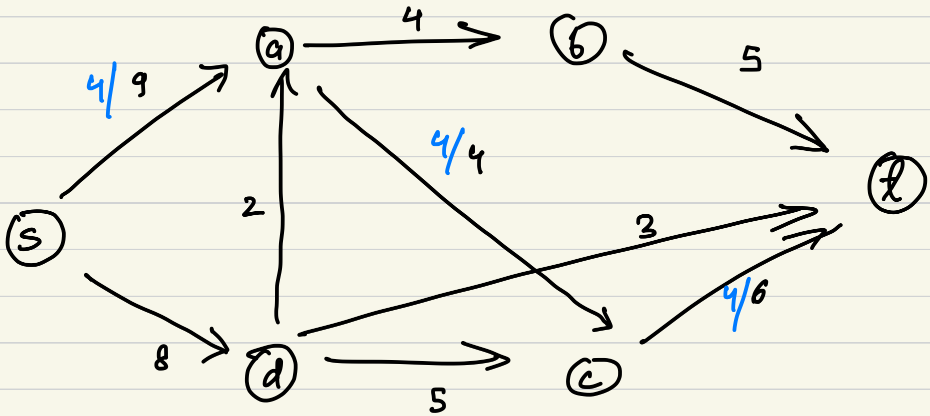
Source
sink

i) Capacity cond.

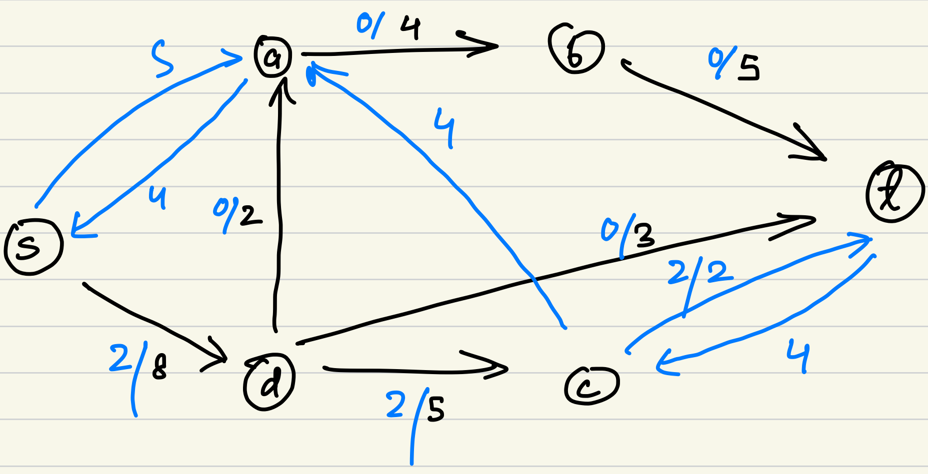
ii) Conservation cond.

Bottleneck (P, f)

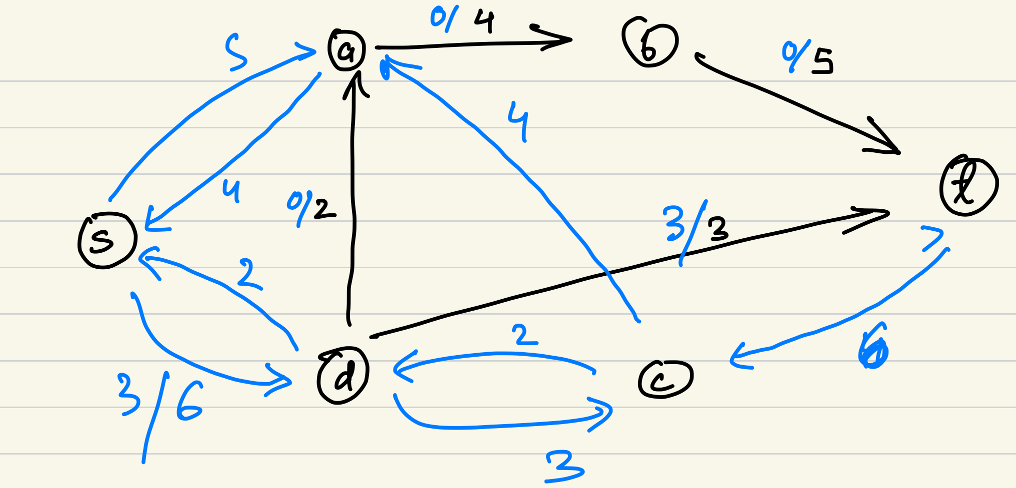




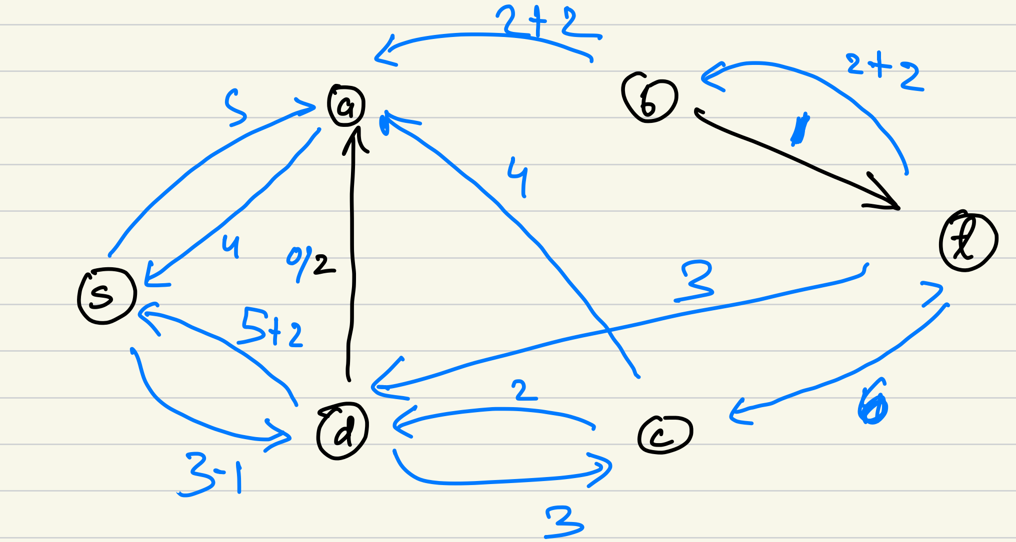
$s \xrightarrow{9} a \xrightarrow{4} c \xrightarrow{6} t$, $b = 4$, $\underline{f} = 0 + 4$



$s \xrightarrow{8} d \xrightarrow{5} c \xrightarrow{2} t$, $b = 2$, $f = 4 + 2$



$s \xrightarrow{6} d \xrightarrow{3} t$, $b = 3$, $f = 4+2+3$



$s \xrightarrow{3} d \xrightarrow{2} a \xrightarrow{4} b \xrightarrow{5} t$, $b = 2$, $f = 4+2+3+2$

$s \xrightarrow{5} a \xrightarrow{2} b \xrightarrow{2} t$, $b = 2$, $f = 4+2+3+2+2$

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m - edges n - vertices

$m \geq \frac{n}{2}$ $O(m+n)$

$O(m)$

Max flow $O(m \cdot C)$

7.1 — 7.8

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$$v(f) = f^{out}(A) - f^{in}(A)$$

$$\underset{\text{flow}}{v(f)} \leq \underset{\text{cut}}{c(A,B)}$$

Valid cut

$$\begin{cases} A^* = S \\ B^* = \emptyset \end{cases}$$

$$\underset{\text{maxflow}}{v(f)} = \underset{\text{mincut}}{c(A^*, B^*)}$$

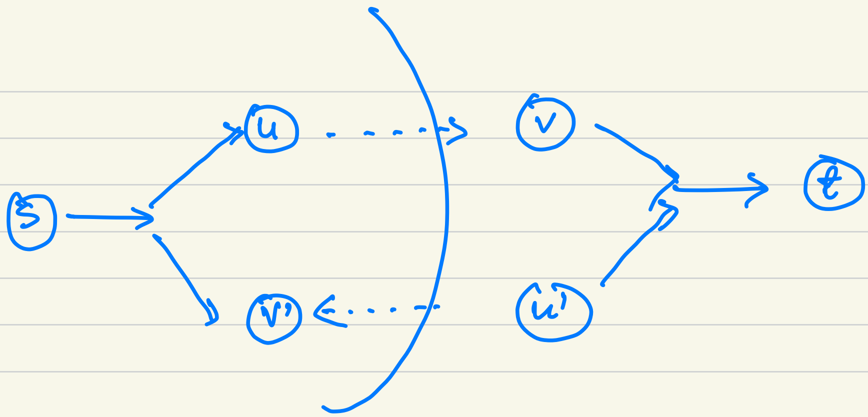
$$v(f) \leq c(A,B)$$

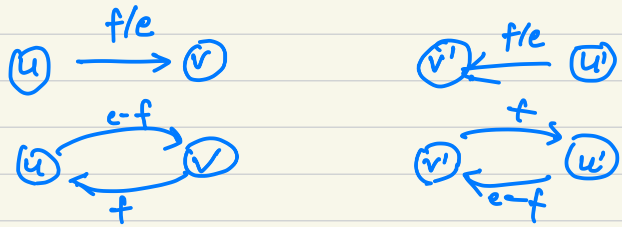
$$c(A^*, B^*) \leq c(A,B)$$

$$v(f') \leq c(A^*, B^*)$$

$$\leq v(f)$$

$$\boxed{\begin{aligned} v(f) &= f^{out}(A^*) - f^{in}(A^*) \\ &= \sum_{e \text{ out of } A^*} c_e - 0 \\ &= c(A^*, B^*) \end{aligned}}$$


s
u
v
v'
u'
t


f/e
u
v
e-f
f
f/e
v'
u'
f
e-f

Choosing good augmenting paths

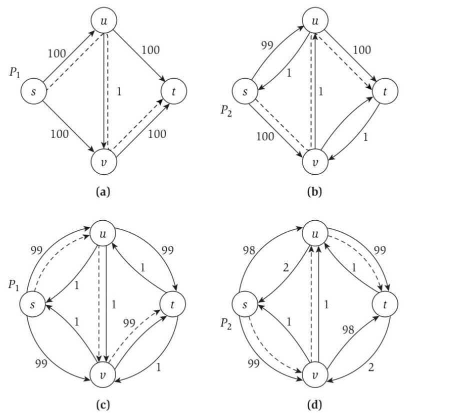
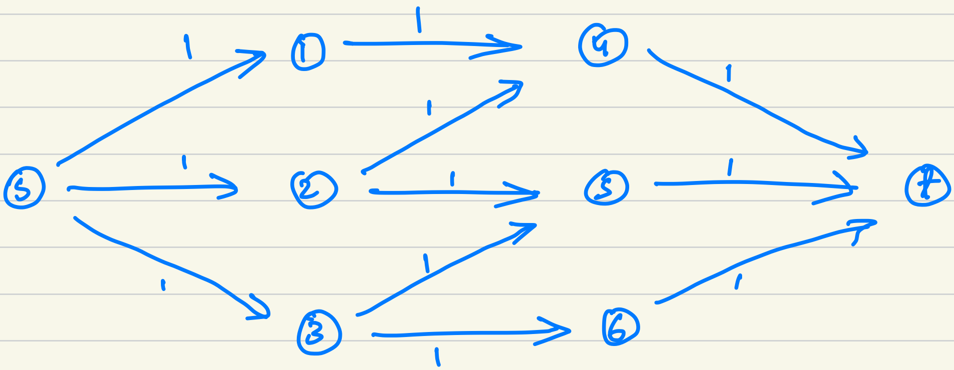


Figure 7.6 Parts (a) through (d) depict four iterations of the Ford-Fulkerson Algorithm using a bad choice of augmenting paths: The augmentations alternate between the path P_1 through the nodes s, u, v, t in order and the path P_2 through the nodes s, v, u, t in order.

$s \to u \to v \to t$, $b = 1$

$s \to v \to u \to t$, $b = 1$

Bipartite Matching



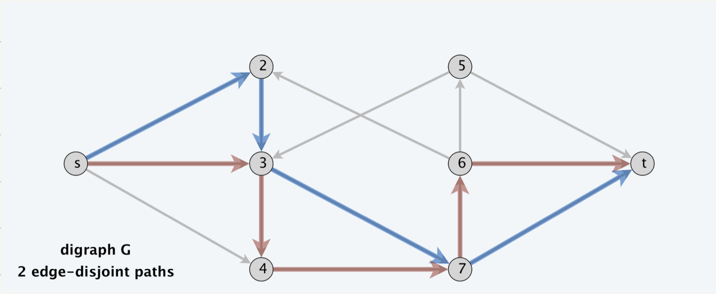
$s \to 2 \to 4 \to t$, $b = 1$

$s \to 3 \to 6 \to t$, $b = 1$

$s \to 1 \to 4 \to 2 \to 5 \to t$. $b = 1$

Edge-Disjoint paths

Nf11 p22



Backtracking

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243 SS skr. 2nd

[ ]

[1] [2] [3] [4]

[1,2] [1,3] [1,4] [2,1] [2,3] [2,4] [3,1] [3,2] [3,4] [4,1] [4,2] [4,3]

[1,2,3] [1,2,4] [1,3,2] [1,3,4]

[1,2,3,4]

Intractability

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Problems

Decidable
(Algo exist)

Undecidable
(No Algo exist)

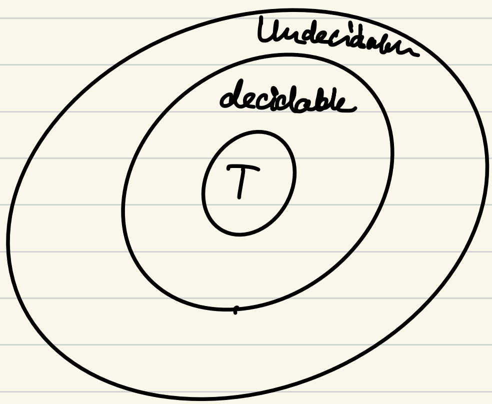
Decidable → Tractable, Intractable

Tractable
(Polynomial Time Algo Exist)

$O(n)$
$O(\log n)$

Intractable

$O(2^n)$
$O(2^{\log n})$



Shortest path	Longest path
Euler cycle	Hamiltonian cycle
2SAT	3SAT

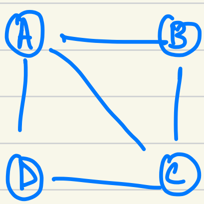
Optimization Problem

0/1 Knapsack $O(nc)$ pseudo polynomial
TSP

Opt. P $\rightarrow$ Dec. P

- Verification Algorithm

OP $\rightarrow$ DP, C



A $\rightarrow$ B $\rightarrow$ C $\rightarrow$ D $\rightarrow$ A

1) Covering all the vertices ?

2) All the edges are present ?

P class :

Set of all DP that are solvable in poly time

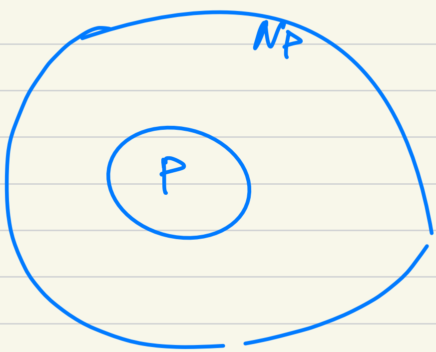
NP class :

Set of all DP for which poly time verification algo exist

Exponential:

Set of all DP that are not solvable in poly time

P = NP ?



Polynomial Time Reduction

$$A \le_p B$$

i) every instance 'α' of 'A' can be transferred to same instance 'β' of 'B' in poly time.

ii) Answer to α is 'yes' iff ans to β is 'yes'

$$A \xrightarrow{O(n^k)} B \qquad O(n^l)$$

$$O(n^k + n^l)$$

* If B is easy problem, then A is also easy

* If A is already proven as hard problem then B is also hard.

A) n boolean var with value $x_1, x_2, \cdots x_n$
does atleast one var have the 'True' value?

B) n integers $i_1, i_2 \dots i_n$, is there $\max_{j=1}^{n} i_j > 0$

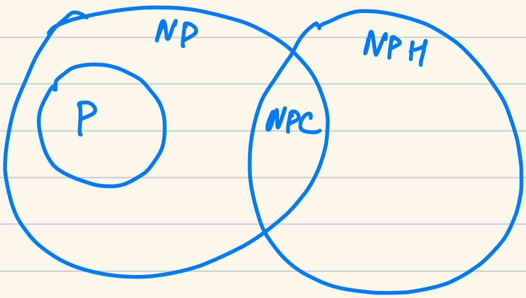
(T, f, f, f) $\rightarrow$ yes

$\downarrow$ $\qquad\qquad\qquad\qquad \uparrow$

$(1, 0, 0, 0) > 0 \longrightarrow$ yes

NPC = NPH + NP

NPH

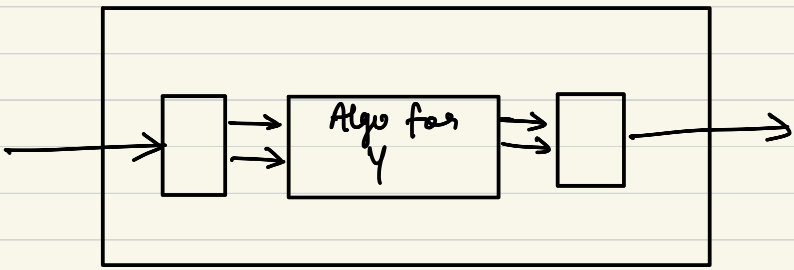


$X \leq_p Y$
$Y \leq_p Z$

$X \leq_p Z$

Sept 2

$X \leq_p Y$



Closest Pair $\leq_p$ Sorting

2, 25, 5, 18, 9

GCD $\leq_p$ LCM

Sort $\leq_p$ Convex Hull

$-2, 5, 1, 3, 2, -1$

$x \to x, x^2$

$-2 \to -2, 4$

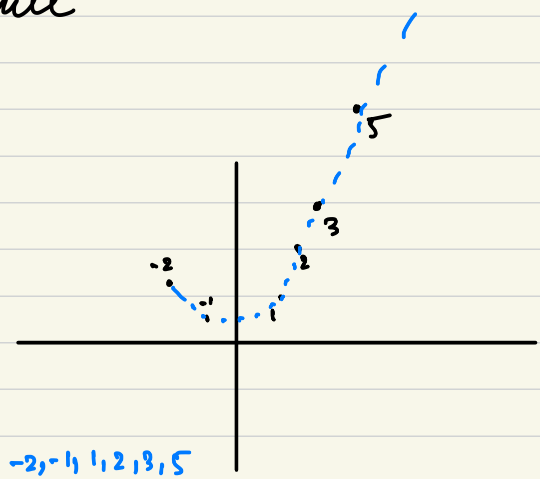
$5 \to 5, 25$

$1 \to 1, 1$

$3 \to 3, 9$

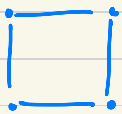
$2 \to 2, 4$

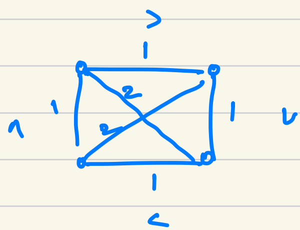
$-1 \to -1, 1$



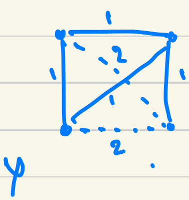
Hamiltonian Cycle $\leq_p$ TSP

Complete graph — always HC.





(TSP, 4)



(TSP, 4) — No

G

$\downarrow$

CG

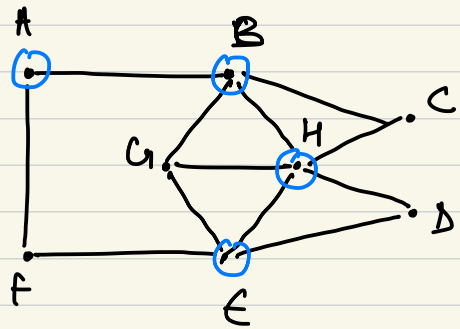
$\downarrow$

(TSP, n) ?

$\downarrow$

?

Independent Set $\leq_p$ Vertex Cover



$\{C, D, F, G\}$

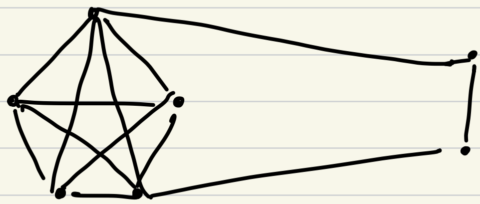
$IS = n - |VC|$

Independent Set $\le_p$ Clique

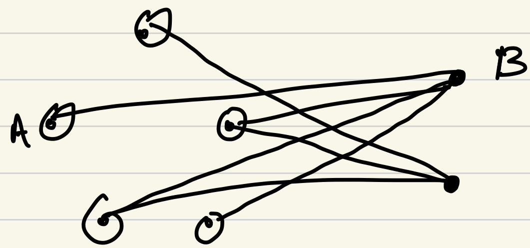
G, k

$\downarrow$

Complement Graph



$C \le_p IS$



$VC \le_p$ Clique

V G, k

$VC \le_p IS$

IS $G, n-k$

$IS \le_p$ Clique

Cliq $G', n-k$

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Set cover

$VC \le_p SC$

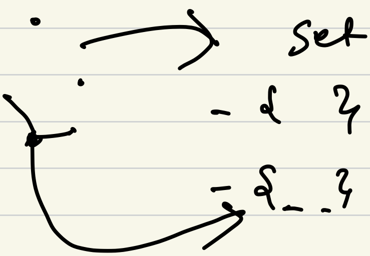
$U = \{1, 2, 3, 4, 5, 6, 7\}$

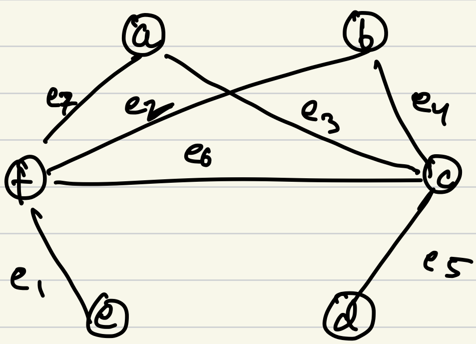
$S_A = \{3, 7\}$ $\quad$ $S_B = \{2, 4\}$

$S_C = \{3, 4, 5, 6\}$ $\quad$ $S_D = \{5\}$

$S_E = \{1\}$ $\quad$ $S_F = \{1, 2, 6, 7\}$

4





$U = \{e_1 \cdots e_7\}$

$S_a = \{e_7, e_3\}$

$S_b = \{e_2, e_4\}$

$S_c = \{e_4, e_3, e_6, e_5\}$

$S_d = \{e_5\}$

$S_e = \{e_1\}$

$S_f = \{e_7, e_2, e_6, e_1\}$

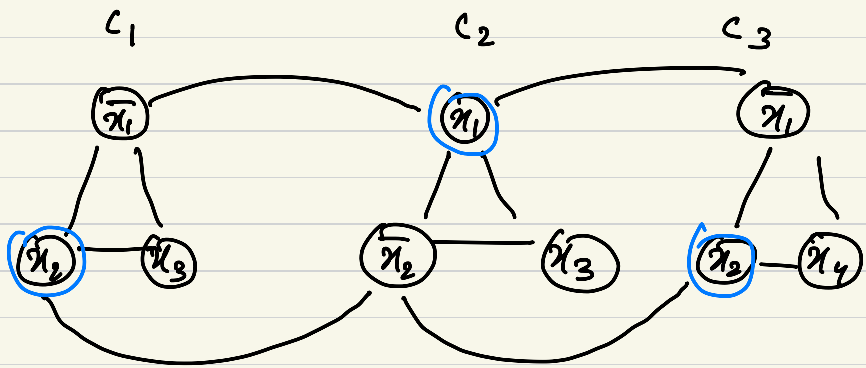
$\boxed{X} \leq_p IS \leq_p VC \leq_p SC$

Satisfiability Problem (SAT)

CNF-SAT

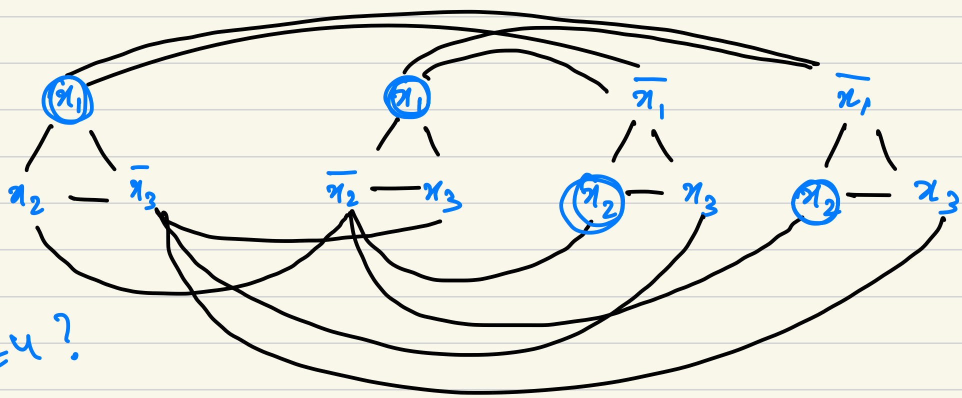
$$f = (\overline{x_1} \vee x_2 \vee x_3) \wedge (x_1 \vee \overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee x_2 \vee x_4)$$

$k=3$?



3CNF SAT

$$f = (x_1 \vee x_2 \vee \overline{x_3}) \wedge (x_1 \vee \overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee x_2 \vee x_3) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_3})$$



$k=4$?

→ Circuit Satisfiability ?

$$SAT \le_p 3SAT$$

→ K-map reduction

$1\ SAT \le_p 3\ SAT$
$2\ SAT \le_p 3\ SAT$
$3\ SAT \le_p 3\ SAT$
$4\ SAT \le_p 4\ SAT$

$F = (\bar{x}_1 \vee \bar{x}_2 \vee x_3 \vee x_4)$ $\qquad x_A = x_3 \vee x_4$

$F = (\bar{x}_1 \vee \bar{x}_2 \vee x_A) \wedge (x_A \Leftrightarrow (x_3 \vee x_4))$

$(x_A \Leftrightarrow (x_3 \vee x_4))$

$(x_A \Rightarrow x_3 \vee x_4) \wedge ((x_3 \vee x_4) \Rightarrow x_A)$

$(\bar{x}_A \vee x_3 \vee x_4) \wedge ((\bar{x}_3 \wedge \bar{x}_4) \vee x_A)$

$((\bar{x}_3 \vee x_A) \wedge (\bar{x}_4 \vee x_A))$

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$(A \vee B)$

$(\bar{A} \Rightarrow B) \wedge (\bar{B} \Rightarrow A)$

A	B	$A \Rightarrow B$
T	T	T
T	F	F
F	T	T
F	F	T

$x \rightsquigarrow \bar{x}$

$\cancel{x}$ $x = T$ $\quad$ $x = F$
$\bar{x} = T$

$\bar{x} \rightsquigarrow x$

Implication graph

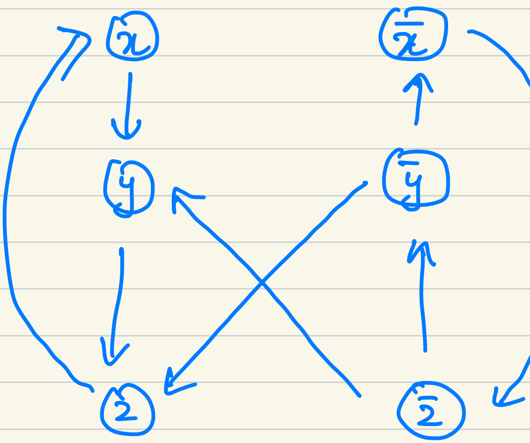
$\cancel{f}$ $\bar{x} = T$ $\quad$ $\bar{x} = F$
$x = T$ $\quad$ $x = T$

not satisfiable, when $x \to \bar{x}$
$\bar{x} \to x$
path exist.

$$f = (\bar{x} \vee y) \wedge (\bar{y} \vee z) \wedge (x \vee \bar{z}) \wedge (z \vee y)$$

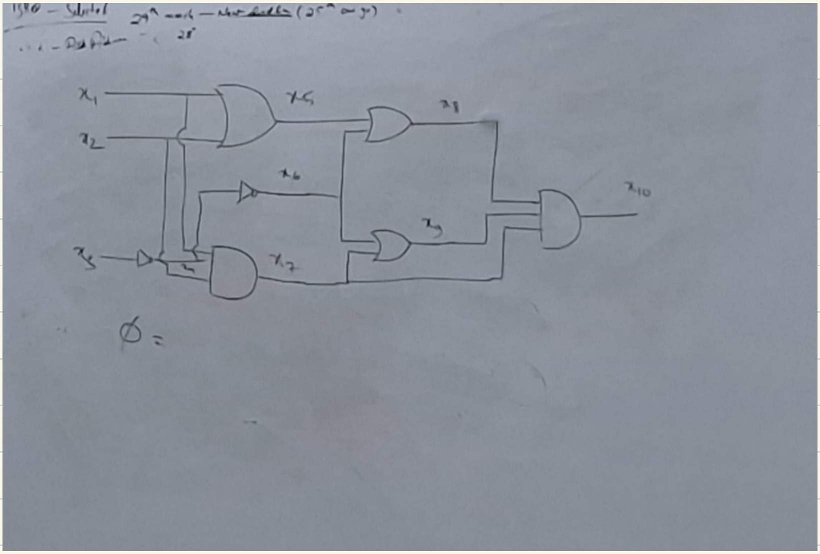
$(A \vee B)$

$$\bar{A} \rightarrow B$$
$$\bar{B} \rightarrow A$$



2SAT belongs to P class

3SAT : NP Complete



$$
\begin{aligned}
\phi = x_{10} \;&\wedge\; (x_{10} \Leftrightarrow (x_7 \wedge x_8 \wedge x_9)) \\
&\wedge\; (x_9 \Leftrightarrow (x_6 \vee x_7)) \\
&\wedge\; (x_8 \Leftrightarrow (x_5 \vee x_6)) \\
&\wedge\; (x_7 \Leftrightarrow (x_1 \wedge x_2 \wedge x_4)) \\
&\wedge\; (x_6 \Leftrightarrow \overline{x_4}) \\
&\wedge\; (x_5 \Leftrightarrow (x_1 \vee x_2)) \\
&\wedge\; (x_4 \Leftrightarrow \overline{x_3})
\end{aligned}
$$

P

1) $P \in NP$

2) $NH \leq_p P$

$NC \leq_p P$

$P \leq NP$