

1 Formal Development

Lemma 1 (Value inversion). *Inversion lemma for values:*

1. If $v : \text{uint}$, then $v = n$
2. If $v : \text{bool}$, then $v = \top$ or $v = \perp$
3. If $v : \sigma[n]$, then $v = [c_i]_n$ and $c_i : \sigma$

Lemma 2 (Consistency of source type and target type). *If $\psi \sim \tau$, then one of the following holds:*

1. $\psi = \sigma$ and $\tau = \sigma^\ell$.
2. $\psi = \sigma[n]$ and $\tau = \sigma^\ell[n]$.

Lemma 3 (Compilation of source environment). *If $\Gamma \vdash \rho \hookrightarrow \tilde{\rho}; \hat{\rho}_1, \hat{\rho}_2$ and $\Gamma(x) = \tau$, then one of the following holds:*

1. $\tau = \sigma^{\mathcal{P}}$, $\rho(x) = c$, $c : \sigma$, and $\tilde{\rho}(x) = c$.
2. $\tau = \sigma^m$, $\rho(x) = c$, $c : \sigma$, $\tilde{\rho}(x) = r$, and $(\hat{\rho}_1[r], \hat{\rho}_2[r]) = \mathcal{E}_m(c)$.
3. $\tau = \sigma^{\mathcal{P}}[n]$, $\rho(x) = [c_i]_n$, $\forall i \in \{0, n-1\}$. $c_i : \sigma$, and $\tilde{\rho}(x) = [c_i]_n$.
4. $\tau = \sigma^m[n]$, $\rho(x) = [c_i]_n$, $\forall i \in \{0, n-1\}$. $c_i : \sigma$, $\tilde{\rho}(x) = [r_i]_n$, $\forall i \in \{0, n-1\}$. $(\hat{\rho}_1[r_i], \hat{\rho}_2[r_i]) = \mathcal{E}_m(c_i)$

Lemma 4 (Soundness of public, scalar expressions). *If*

1. $\Gamma \vdash e : \sigma^{\mathcal{P}} \rightsquigarrow \tilde{e}$
2. $\Gamma \vdash \rho \hookrightarrow \tilde{\rho}; \hat{\rho}_1, \hat{\rho}_2$

Then

- (a) $\rho \vdash e \Downarrow c$
- (b) $c : \sigma$
- (c) $\tilde{\rho} \vdash \tilde{e} \Downarrow c$

Lemma 5 (Soundness of public, array expressions). *If*

1. $\Gamma \vdash e : \sigma^{\mathcal{P}}[n] \rightsquigarrow \tilde{e}$
2. $\Gamma \vdash \rho \hookrightarrow \tilde{\rho}; \hat{\rho}_1, \hat{\rho}_2$

Then

- (a) $\rho \vdash e \Downarrow [c_i]_n$
- (b) $\forall i \in \{0, n-1\}$. $c_i : \sigma$

$$(c) \tilde{\rho} \vdash \tilde{e} \Downarrow [c_i]_n$$

Lemma 6 (Soundness of secret, scalar expressions). *If*

1. $\Gamma \vdash e : \sigma^m \rightsquigarrow \tilde{e}$
2. $\Gamma \vdash \rho \hookrightarrow \tilde{\rho}; \hat{\rho}_1, \hat{\rho}_2$

Then

- (a) $\rho \vdash e \Downarrow c$
- (b) $c : \sigma$
- (c) $\tilde{\rho} \vdash \tilde{e} \Downarrow \kappa^e$
- (d) $\hat{\rho}_1, \hat{\rho}_2 \vdash \kappa^e \Downarrow b_1, b_2$
- (e) $c = \mathcal{D}_m(b_1, b_2)$

Lemma 7 (Soundness of secret, array expressions). *If*

1. $\Gamma \vdash e : \sigma^m[n] \rightsquigarrow \tilde{e}$
2. $\Gamma \vdash \rho \hookrightarrow \tilde{\rho}; \hat{\rho}_1, \hat{\rho}_2$

Then

- (a) $\rho \vdash e \Downarrow [c_i]_n$
- (b) $\forall i \in \{0, n-1\}. c_i : \sigma$
- (c) $\tilde{\rho} \vdash \tilde{e} \Downarrow [\kappa_i^e]_n$
- (d) $\forall i \in \{0, n-1\}. \hat{\rho}_1, \hat{\rho}_2 \vdash \kappa_i^e \Downarrow b_{1\ i}, b_{2\ i}, \text{ s.t. } c_i = \mathcal{D}_m(b_{1\ i}, b_{2\ i})$

Lemma 8 (Target semantics correspondence). *If:*

1. $\Gamma \vdash s \rightsquigarrow \tilde{s} \mid \Gamma'$
2. $\Gamma \vdash \rho \hookrightarrow \tilde{\rho}; \hat{\rho}_1, \hat{\rho}_2$
3. $\rho \vdash s \Downarrow \rho'; O$

then:

- (a) $\tilde{\rho} \vdash \tilde{s} \Downarrow \tilde{\rho}'; \kappa^s$
- (b) $\hat{\rho}_1, \hat{\rho}_2 \vdash \kappa^s \Downarrow \tilde{\rho}'_1, \tilde{\rho}'_2; O$
- (c) $\Gamma \vdash \rho' \hookrightarrow \tilde{\rho}'; \hat{\rho}'_1, \hat{\rho}'_2$

Lemma 9 (Soundness of source semantics). *If:*

1. $\Gamma \vdash s \rightsquigarrow \tilde{s} \mid \Gamma'$

$$2. \Gamma \vdash \rho \hookrightarrow \tilde{\rho}; \hat{\rho}_1, \hat{\rho}_2$$

then:

$$(a) \rho \vdash s \Downarrow \rho'; O$$

$$(b) \Gamma \vdash \rho' \hookrightarrow \tilde{\rho}'; \hat{\rho}'_1, \hat{\rho}'_2$$

Theorem 10 (Soundness). *If:*

$$1. \Gamma \vdash s \rightsquigarrow \tilde{s} \mid \Gamma'$$

$$2. \Gamma \vdash \rho \hookrightarrow \tilde{\rho}; \hat{\rho}_1, \hat{\rho}_2$$

then:

$$(a) \rho \vdash s \Downarrow \rho'; O$$

$$(b) \tilde{\rho} \vdash \tilde{s} \Downarrow \tilde{\rho}'; \kappa^s$$

$$(c) \hat{\rho}_1, \hat{\rho}_2 \vdash \kappa^s \Downarrow \hat{\rho}'_1, \hat{\rho}'_2; O$$

$$(d) \Gamma \vdash \rho' \hookrightarrow \tilde{\rho}'; \hat{\rho}'_1, \hat{\rho}'_2$$