

s	$::=$ \mid \mathcal{A} \mid \mathcal{B}	Secret label
ℓ	$::=$ \mid \mathcal{P} \mid s	Label
σ	$::=$ \mid \mathbf{uint}_b \mid \mathbf{bool}	Base type
τ	$::=$ \mid σ^ℓ \mid $\mathbf{uint}_b^\ell[\]_n$	Type
e	$::=$ \mid n_b \mid x \mid $e_1 \oplus e_2$ \mid $e_1 ? e_2 : e_3$ \mid $e_1 > e_2$ \mid $x[\overline{e_i}^{i \in 1..n}]$ \mid $e_1 \oplus_s e_2$ \mid $\mathbf{mux}_s e \ e_1 \ e_2$ \mid $e_1 >_s e_2$ \mid $e \triangleright \ell$	Expression
c	$::=$ \mid $\tau \ x$ \mid $x := e$ \mid $\mathbf{for} \ x \in [n \dots m] \ \mathbf{do} \ c$ \mid $x[\overline{e_i}^{i \in 1..n}] := e$ \mid $\mathbf{if} \ e \ c_1 \ c_2$ \mid $\mathbf{out} \ e$ \mid $c_1; c_2$	Command
Γ	$::=$ \mid $.$ \mid $\Gamma, x : \tau$	Type environment

$$\boxed{\ell_1 \sqsubseteq \ell_2}$$

$$\frac{}{\overline{\ell \sqsubseteq \ell}} \quad \text{L_REFL}$$

$$\frac{}{\overline{\mathcal{P} \sqsubseteq s}} \quad \text{L_PS}$$

$$\frac{}{s_1 \sqsubseteq s_2} \quad \text{L_SS}$$

$$\boxed{\Gamma \vdash e : \tau \rightsquigarrow e'}$$

$$\frac{}{\Gamma \vdash n_b : \text{uint}_b^{\mathcal{P}} \rightsquigarrow n_b} \quad \text{S_CONST}$$

$$\frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau \rightsquigarrow x} \quad \text{S_VAR}$$

$$\frac{\Gamma \vdash e_i : \text{uint}_b^{\mathcal{P}} \rightsquigarrow e'_i}{\Gamma \vdash e_1 \oplus e_2 : \text{uint}_b^{\mathcal{P}} \rightsquigarrow e'_1 \oplus e'_2} \quad \text{S_PBINOP}$$

$$\frac{\Gamma \vdash e_i : \text{uint}_b^{\mathcal{A}} \rightsquigarrow e'_i}{\Gamma \vdash e_1 \oplus e_2 : \text{uint}_b^{\mathcal{A}} \rightsquigarrow e'_1 \oplus_{\mathcal{A}} e'_2} \quad \text{S_SBINOP}$$

$$\frac{\Gamma \vdash e : \text{bool}^{\mathcal{P}} \rightsquigarrow e' \quad \Gamma \vdash e_i : \tau \rightsquigarrow e'_i}{\Gamma \vdash e ? e_1 : e_2 : \tau \rightsquigarrow e' ? e'_1 : e'_2} \quad \text{S_PCOND}$$

$$\frac{\Gamma \vdash e : \text{bool}^{\mathcal{B}} \rightsquigarrow e' \quad \Gamma \vdash e_i : \tau \rightsquigarrow e'_i}{\Gamma \vdash e ? e_1 : e_2 : \tau \rightsquigarrow \mathbf{mux}_{\mathcal{B}} e' e'_1 e'_2} \quad \text{S_SCOND}$$

$$\frac{\Gamma \vdash e_i : \text{uint}_b^{\mathcal{P}} \rightsquigarrow e'_i}{\Gamma \vdash e_1 > e_2 : \text{bool}^{\mathcal{P}} \rightsquigarrow e'_1 > e'_2} \quad \text{S_PGT}$$

$$\frac{\Gamma \vdash e_i : \text{uint}_b^{\mathcal{B}} \rightsquigarrow e'_i}{\Gamma \vdash e_1 > e_2 : \text{bool}^{\mathcal{B}} \rightsquigarrow e'_1 >_{\mathcal{B}} e'_2} \quad \text{S_SGT}$$

$$\frac{\Gamma \vdash x : \text{uint}_b^{\ell}[\]_n \rightsquigarrow x \quad \Gamma \vdash e_i : \text{uint}_b^{\mathcal{P}} \rightsquigarrow e'_i}{\Gamma \vdash x[\overline{e_i}^{i \in 1..n}] : \text{uint}_b^{\ell} \rightsquigarrow x[\overline{e'_i}^{i \in 1..n}]} \quad \text{S_AREAD}$$

$$\frac{\Gamma \vdash e : \sigma^{\ell_1} \rightsquigarrow e' \quad \ell_1 \sqsubseteq \ell_2}{\Gamma \vdash e : \sigma^{\ell_2} \rightsquigarrow e' \triangleright \ell_2} \quad \text{S_SUB}$$

$$\boxed{\Gamma \vdash c \rightsquigarrow c' \mid \Gamma'}$$

$$\frac{}{\Gamma \vdash \tau x \rightsquigarrow \tau x \mid \Gamma, x : \tau} \text{C_DECL}$$

$$\frac{\begin{array}{l} \Gamma(x) = \tau \\ \Gamma \vdash e : \tau \rightsquigarrow e' \end{array}}{\Gamma \vdash x := e \rightsquigarrow x := e' \mid \Gamma} \text{C_VASSGN}$$

$$\frac{\begin{array}{l} \Gamma, x : \text{uint}_b^{\mathcal{P}} \vdash c \rightsquigarrow c' \mid - \\ x \notin \text{modifies}(c) \end{array}}{\Gamma \vdash \text{for } x \in [n \dots m] \text{ do } c \rightsquigarrow \text{for } x \in [n \dots m] \text{ do } c' \mid \Gamma} \text{C_FOR}$$

$$\frac{\begin{array}{l} \Gamma \vdash x : \text{uint}_b^{\ell}[\]_n \rightsquigarrow x \\ \Gamma \vdash e_i : \text{uint}_b^{\mathcal{P}} \rightsquigarrow e'_i \\ \Gamma \vdash e : \text{uint}_b^{\ell} \rightsquigarrow e' \end{array}}{\Gamma \vdash x[\overline{e_i}^{i \in 1..n}] := e \rightsquigarrow x[\overline{e'_i}^{i \in 1..n}] := e' \mid \Gamma} \text{C_AWRITE}$$

$$\frac{\begin{array}{l} \Gamma \vdash e : \text{bool}^{\mathcal{P}} \rightsquigarrow e' \\ \Gamma \vdash c_1 \rightsquigarrow c'_1 \mid - \\ \Gamma \vdash c_2 \rightsquigarrow c'_2 \mid - \end{array}}{\Gamma \vdash \text{if } e \text{ } c_1 \text{ } c_2 \rightsquigarrow \text{if } e' \text{ } c'_1 \text{ } c'_2 \mid \Gamma} \text{C_IF}$$

$$\frac{\Gamma \vdash e : \tau \rightsquigarrow e'}{\Gamma \vdash \text{out } e \rightsquigarrow \text{out } e' \mid \Gamma} \text{C_OUT}$$

$$\frac{\begin{array}{l} \Gamma \vdash c_1 \rightsquigarrow c'_1 \mid \Gamma_1 \\ \Gamma_1 \vdash c_2 \rightsquigarrow c'_2 \mid \Gamma' \end{array}}{\Gamma \vdash c_1; c_2 \rightsquigarrow c'_1; c'_2 \mid \Gamma'} \text{C_SEQ}$$