

s	$::=$ \mid \mathcal{A} \mid \mathcal{B}	Secret label
ℓ	$::=$ \mid \mathcal{P} \mid s	Label
σ	$::=$ \mid \mathbf{uint} \mid \mathbf{bool}	Base type
τ	$::=$ \mid σ^ℓ \mid $\mathbf{uint}^\ell[]$	Type
e	$::=$ \mid n \mid x \mid $e_1 \oplus e_2$ \mid $e_1 ? e_2 : e_3$ \mid $e_1 > e_2$ \mid $x[e]$ \mid $e_1 \oplus_s e_2$ \mid $\mathbf{mux}_s e \ e_1 \ e_2$ \mid $e_1 >_s e_2$ \mid $e \triangleright s$	Expression
c	$::=$ \mid $\tau \ x = e$ \mid $x := e$ \mid $\mathbf{for} \ x \in [n \dots m] \ \mathbf{do} \ c$ \mid $x[e_1] := e_2$ \mid $\mathbf{if} \ e \ c_1 \ c_2$ \mid $\mathbf{out} \ e$ \mid $c_1 ; c_2$	Command
Γ	$::=$ \mid $.$ \mid $\Gamma, x : \tau$	Type environment

$$\boxed{\ell_1 \sqsubseteq \ell_2}$$

$$\frac{}{\ell \sqsubseteq \ell} \quad \text{L_REFL}$$

$$\frac{}{\mathcal{P} \sqsubseteq s} \quad \text{L_PS}$$

$$\frac{}{s_1 \sqsubseteq s_2} \quad \text{L_SS}$$

$$\boxed{\Gamma \vdash e : \tau \rightsquigarrow e'}$$

$$\frac{}{\Gamma \vdash n : \mathbf{uint}^{\mathcal{P}} \rightsquigarrow n} \quad \text{S_CONST}$$

$$\frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau \rightsquigarrow x} \quad \text{S_VAR}$$

$$\frac{\Gamma \vdash e_i : \mathbf{uint}^{\mathcal{P}} \rightsquigarrow e'_i}{\Gamma \vdash e_1 \oplus e_2 : \mathbf{uint}^{\mathcal{P}} \rightsquigarrow e'_1 \oplus e'_2} \quad \text{S_PBINOP}$$

$$\frac{\Gamma \vdash e_i : \mathbf{uint}^{\mathcal{A}} \rightsquigarrow e'_i}{\Gamma \vdash e_1 \oplus e_2 : \mathbf{uint}^{\mathcal{A}} \rightsquigarrow e'_1 \oplus_{\mathcal{A}} e'_2} \quad \text{S_SBINOP}$$

$$\frac{\begin{array}{l} \Gamma \vdash e : \mathbf{bool}^{\mathcal{P}} \rightsquigarrow e' \\ \Gamma \vdash e_i : \tau \rightsquigarrow e'_i \end{array}}{\Gamma \vdash e ? e_1 : e_2 : \tau \rightsquigarrow e' ? e'_1 : e'_2} \quad \text{S_PCOND}$$

$$\frac{\begin{array}{l} \Gamma \vdash e : \mathbf{bool}^{\mathcal{B}} \rightsquigarrow e' \\ \Gamma \vdash e_i : \tau \rightsquigarrow e'_i \end{array}}{\Gamma \vdash e ? e_1 : e_2 : \tau \rightsquigarrow \mathbf{mux}_{\mathcal{B}} e' e'_1 e'_2} \quad \text{S_SCOND}$$

$$\frac{\Gamma \vdash e_i : \mathbf{uint}^{\mathcal{P}} \rightsquigarrow e'_i}{\Gamma \vdash e_1 > e_2 : \mathbf{bool}^{\mathcal{P}} \rightsquigarrow e'_1 > e'_2} \quad \text{S_PGT}$$

$$\frac{\Gamma \vdash e_i : \mathbf{uint}^{\mathcal{B}} \rightsquigarrow e'_i}{\Gamma \vdash e_1 > e_2 : \mathbf{bool}^{\mathcal{B}} \rightsquigarrow e'_1 >_{\mathcal{B}} e'_2} \quad \text{S_SGT}$$

$$\frac{\begin{array}{l} \Gamma \vdash x : \mathbf{uint}^{\ell}[\] \rightsquigarrow x \\ \Gamma \vdash e : \mathbf{uint}^{\mathcal{P}} \rightsquigarrow e' \end{array}}{\Gamma \vdash x[e] : \mathbf{uint}^{\ell} \rightsquigarrow x[e']} \quad \text{S_AREAD}$$

$$\frac{\begin{array}{l} \Gamma \vdash e : \sigma^{\ell} \rightsquigarrow e' \\ \ell \sqsubseteq s \end{array}}{\Gamma \vdash e : \sigma^s \rightsquigarrow e' \triangleright s} \quad \text{S_SUB}$$

$$\boxed{\Gamma \vdash c \rightsquigarrow c' \mid \Gamma'}$$

$$\frac{\Gamma \vdash e : \tau \rightsquigarrow e'}{\Gamma \vdash \tau x = e \rightsquigarrow \tau x = e' \mid \Gamma, x : \tau} \quad \text{C_DECL}$$

$$\frac{\begin{array}{l} \Gamma(x) = \tau \\ \Gamma \vdash e : \tau \rightsquigarrow e' \end{array}}{\Gamma \vdash x := e \rightsquigarrow x := e' \mid \Gamma} \quad \text{C_VASSGN}$$

$$\frac{\begin{array}{l} \Gamma, x : \text{uint}^{\mathcal{P}} \vdash c \rightsquigarrow c' \mid - \\ x \notin \text{modifies}(c) \end{array}}{\Gamma \vdash \text{for } x \in [n \dots m] \text{ do } c \rightsquigarrow \text{for } x \in [n \dots m] \text{ do } c' \mid \Gamma} \quad \text{C_FOR}$$

$$\frac{\begin{array}{l} \Gamma \vdash x : \text{uint}^{\ell}[] \rightsquigarrow x \\ \Gamma \vdash e_1 : \text{uint}^{\mathcal{P}} \rightsquigarrow e'_1 \\ \Gamma \vdash e_2 : \text{uint}^{\ell} \rightsquigarrow e'_2 \end{array}}{\Gamma \vdash x[e_1] := e_2 \rightsquigarrow x[e'_1] := e'_2 \mid \Gamma} \quad \text{C_AWRITE}$$

$$\frac{\begin{array}{l} \Gamma \vdash e : \text{bool}^{\mathcal{P}} \rightsquigarrow e' \\ \Gamma \vdash c_1 \rightsquigarrow c'_1 \mid - \\ \Gamma \vdash c_2 \rightsquigarrow c'_2 \mid - \end{array}}{\Gamma \vdash \text{if } e \text{ } c_1 \text{ } c_2 \rightsquigarrow \text{if } e' \text{ } c'_1 \text{ } c'_2 \mid \Gamma} \quad \text{C_IF}$$

$$\frac{\Gamma \vdash e : \tau \rightsquigarrow e'}{\Gamma \vdash \text{out } e \rightsquigarrow \text{out } e' \mid \Gamma} \quad \text{C_OUT}$$

$$\frac{\begin{array}{l} \Gamma \vdash c_1 \rightsquigarrow c'_1 \mid \Gamma_1 \\ \Gamma_1 \vdash c_2 \rightsquigarrow c'_2 \mid \Gamma' \end{array}}{\Gamma \vdash c_1; c_2 \rightsquigarrow c'_1; c'_2 \mid \Gamma'} \quad \text{C_SEQ}$$

$$\begin{array}{ll} w & ::= \\ & \mid n \\ & \mid \text{true} \\ & \mid \text{false} \\ & \mid w^{s,1} \\ & \mid w^{s,2} \end{array} \quad \text{Runtime base values}$$

$$\begin{array}{ll} v & ::= \\ & \mid w \\ & \mid [\overline{v_i}^i] \end{array} \quad \text{Runtime values}$$

$$\begin{array}{ll} \rho & ::= \\ & \mid \cdot \\ & \mid \rho[x \mapsto v] \end{array} \quad \text{Runtime environment}$$

$$\boxed{\rho_1, \rho_2 \vdash e \Downarrow v_1, v_2}$$

$$\frac{}{\rho_1, \rho_2 \vdash n \Downarrow n, n} \text{EE_CONST}$$

$$\frac{\rho_1[x] = \rho_2[x] = v}{\rho_1, \rho_2 \vdash x \Downarrow v, v} \text{EE_PVAR}$$

$$\frac{\begin{array}{l} \rho_1[x] = w^{s,1} \\ \rho_2[x] = w^{s,2} \end{array}}{\rho_1, \rho_2 \vdash x \Downarrow w^{s,1}, w^{s,2}} \text{EE_SVAR}$$

$$\frac{\rho_1, \rho_2 \vdash e_i \Downarrow n_i, n_i}{\rho_1, \rho_2 \vdash e_1 \oplus e_2 \Downarrow n_1 \oplus n_2, n_1 \oplus n_2} \text{EE_PBINOP}$$

$$\frac{\begin{array}{l} \rho_1, \rho_2 \vdash e \Downarrow \text{true}, \text{true} \\ \rho_1, \rho_2 \vdash e_1 \Downarrow v_1, v_2 \end{array}}{\rho_1, \rho_2 \vdash e ? e_1 : e_2 \Downarrow v_1, v_2} \text{EE_PCONDT}$$

$$\frac{\begin{array}{l} \rho_1, \rho_2 \vdash e \Downarrow \text{false}, \text{false} \\ \rho_1, \rho_2 \vdash e_2 \Downarrow v_1, v_2 \end{array}}{\rho_1, \rho_2 \vdash e ? e_1 : e_2 \Downarrow v_1, v_2} \text{EE_PCONDF}$$

$$\frac{\rho_1, \rho_2 \vdash e_i \Downarrow n_i, n_i}{\rho_1, \rho_2 \vdash e_1 > e_2 \Downarrow n_1 > n_2, n_1 > n_2} \text{EE_PGT}$$

$$\frac{\begin{array}{l} \rho_1, \rho_2 \vdash e \Downarrow n, n \\ \rho_1, \rho_2 \vdash x \Downarrow [\overline{v_1}^i], [\overline{v_2}^i] \end{array}}{\rho_1, \rho_2 \vdash x[e] \Downarrow v_{1n}, v_{2n}} \text{EE_AREAD}$$

$$\frac{\rho_1, \rho_2 \vdash e_i \Downarrow n_i^{s,1}, n_i^{s,2}}{\rho_1, \rho_2 \vdash e_1 \oplus_s e_2 \Downarrow (n_1 \oplus n_2)^{s,1}, (n_1 \oplus n_2)^{s,2}} \text{EE_SBINOP}$$

$$\frac{\begin{array}{l} \rho_1, \rho_2 \vdash e \Downarrow \text{true}^{s,1}, \text{true}^{s,2} \\ \rho_1, \rho_2 \vdash e_1 \Downarrow v_1, v_2 \end{array}}{\rho_1, \rho_2 \vdash \mathbf{mux}_s e e_1 e_2 \Downarrow v_1, v_2} \text{EE_SCONDT}$$

$$\frac{\begin{array}{l} \rho_1, \rho_2 \vdash e \Downarrow \text{false}^{s,1}, \text{false}^{s,2} \\ \rho_1, \rho_2 \vdash e_2 \Downarrow v_1, v_2 \end{array}}{\rho_1, \rho_2 \vdash \mathbf{mux}_s e e_1 e_2 \Downarrow v_1, v_2} \text{EE_SCONDF}$$

$$\frac{\rho_1, \rho_2 \vdash e_i \Downarrow n_i^{s,1}, n_i^{s,2}}{\rho_1, \rho_2 \vdash e_1 >_s e_2 \Downarrow (n_1 > n_2)^{s,1}, (n_1 > n_2)^{s,2}} \text{EE_SGT}$$

$$\frac{\rho_1, \rho_2 \vdash e \Downarrow v_1, v_2}{\rho_1, \rho_2 \vdash e \triangleright s \Downarrow v_1 \triangleright s, v_2 \triangleright s} \text{EE_COERCE}$$

$$\boxed{\rho_1, \rho_2 \vdash c \longrightarrow \rho'_1, \rho'_2}$$

$$\frac{\rho_1, \rho_2 \vdash e \Downarrow v_1, v_2}{\rho_1, \rho_2 \vdash \tau x = e \longrightarrow \rho_1[x \mapsto v_1], \rho_2[x \mapsto v_2]} \quad \text{EC_DECL}$$

$$\frac{\rho_1, \rho_2 \vdash e \Downarrow v_1, v_2}{\rho_1, \rho_2 \vdash x := e \longrightarrow \rho_1[x \mapsto v_1], \rho_2[x \mapsto v_2]} \quad \text{EC_ASSGN}$$