

$\ell$	$::=$ $\mid$ $\mathcal{P}$ $\mid$ $\mathcal{S}$	Label
$\sigma$	$::=$ $\mid$ $\text{uint}_b$ $\mid$ $\text{bool}$	Base type
$\tau$	$::=$ $\mid$ $\sigma^\ell$ $\mid$ $\text{uint}_b^\ell[\ ]_n$	Type
$e$	$::=$ $\mid$ $n_b$ $\mid$ $x$ $\mid$ $e_1 \oplus e_2$ $\mid$ $e_1 ? e_2 : e_3$ $\mid$ $e_1 > e_2$ $\mid$ $x[\overline{e_i}^{i \in 1..n}]$ $\mid$ <b>mux</b> $e$ $e_1$ $e_2$ $\mid$ $e_1 \oplus_{\mathbf{s}} e_2$ $\mid$ $e_1 >_{\mathbf{s}} e_2$ $\mid$ $e \triangleright \mathbf{arith}$ $\mid$ $e \triangleright \mathbf{bool}$	Expression
$c$	$::=$ $\mid$ $\tau x$ $\mid$ $x := e$ $\mid$ <b>foreach</b> $x \in [n \dots m] \{c\}$ $\mid$ $x[\overline{e_i}^{i \in 1..n}] := e$ $\mid$ <b>if</b> $e$ $c_1$ $c_2$ $\mid$ <b>out</b> $e$ $\mid$ $c_1; c_2$	Command
$\Gamma$	$::=$ $\mid$ $\cdot$ $\mid$ $\Gamma, x : \tau$	Type environment

$$\boxed{\ell_1 \sqsubseteq \ell_2}$$

$$\overline{\ell \sqsubseteq \ell} \quad \text{L\_REFL}$$

$$\overline{\mathcal{P} \sqsubseteq \mathcal{S}} \quad \text{L\_PS}$$

$$\boxed{\Gamma \vdash e : \tau \rightsquigarrow e'}$$

$$\frac{}{\Gamma \vdash n_b : \text{uint}_b^{\mathcal{P}} \rightsquigarrow n_b} \text{ S\_CONST}$$

$$\frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau \rightsquigarrow x} \text{ S\_VAR}$$

$$\frac{\Gamma \vdash e_i : \text{uint}_b^{\mathcal{P}} \rightsquigarrow e'_i}{\Gamma \vdash e_1 \oplus e_2 : \text{uint}_b^{\mathcal{P}} \rightsquigarrow e'_1 \oplus e'_2} \text{ S\_PBINOP}$$

$$\frac{\Gamma \vdash e_i : \text{uint}_b^{\mathcal{S}} \rightsquigarrow e'_i}{\Gamma \vdash e_1 \oplus e_2 : \text{uint}_b^{\mathcal{S}} \rightsquigarrow e'_1 \oplus_{\mathbf{s}} e'_2} \text{ S\_SBINOP}$$

$$\frac{\Gamma \vdash e : \text{bool}^{\mathcal{P}} \rightsquigarrow e' \quad \Gamma \vdash e_i : \tau \rightsquigarrow e'_i}{\Gamma \vdash e ? e_1 : e_2 : \tau \rightsquigarrow e' ? e'_1 : e'_2} \text{ S\_PCOND}$$

$$\frac{\Gamma \vdash e : \text{bool}^{\mathcal{S}} \rightsquigarrow e' \quad \Gamma \vdash e_i : \tau \rightsquigarrow e'_i}{\Gamma \vdash e ? e_1 : e_2 : \tau \rightsquigarrow \mathbf{mux} \ e' \ e'_1 \ e'_2} \text{ S\_SCOND}$$

$$\frac{\Gamma \vdash e_i : \text{uint}_b^{\mathcal{P}} \rightsquigarrow e'_i}{\Gamma \vdash e_1 > e_2 : \text{bool}^{\mathcal{P}} \rightsquigarrow e'_1 > e'_2} \text{ S\_PGT}$$

$$\frac{\Gamma \vdash e_i : \text{uint}_b^{\mathcal{S}} \rightsquigarrow e'_i}{\Gamma \vdash e_1 > e_2 : \text{bool}^{\mathcal{S}} \rightsquigarrow e'_1 >_{\mathbf{s}} e'_2} \text{ S\_SGT}$$

$$\frac{\Gamma \vdash x : \text{uint}_b^{\ell}[\ ]_n \rightsquigarrow x \quad \Gamma \vdash e_i : \text{uint}_b^{\mathcal{P}} \rightsquigarrow e'_i}{\Gamma \vdash x[\overline{e_i}^{i \in 1..n}] : \text{uint}_b^{\ell} \rightsquigarrow x[\overline{e'_i}^{i \in 1..n}]} \text{ S\_AREAD}$$

$$\frac{\Gamma \vdash e : \sigma^{\ell_1} \rightsquigarrow e' \quad \ell_1 \sqsubseteq \ell_2}{\Gamma \vdash e : \sigma^{\ell_2} \rightsquigarrow e'} \text{ S\_SUB}$$