Binomial distribution

$$P(X=K) = {}^{M}C_{K}\, p^{K} (1-p)^{M-K}$$

Poisson

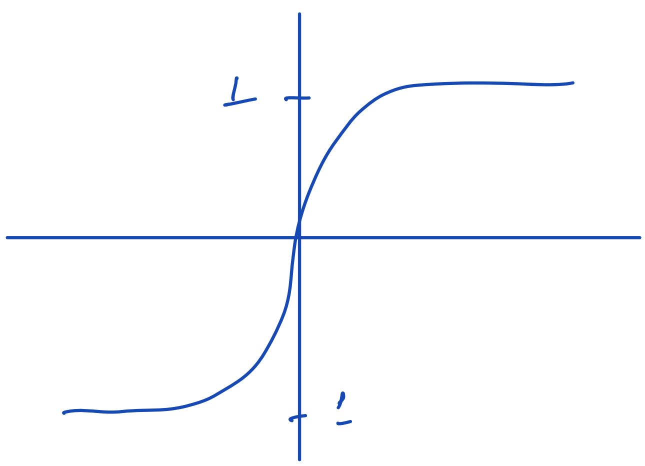
$$P(X=K) = \frac{e^{-\lambda} \lambda^{K}}{K!} \qquad \lambda = Mp$$

Bernoulli

$$P(X=K) = p$$

Geometric

$$P(X=K) = (1-p)^{K-1}\, p$$



① Uniform

RNG
Sampling
MC Methods

$$f_X(x) = \frac{1}{b-a} \qquad a \le x \le b$$

$$F_X(x; a, b) = \frac{x-a}{b-a} \qquad a \le x \le b$$

$$f_X(x) = \frac{d}{dx} F_X(x)$$

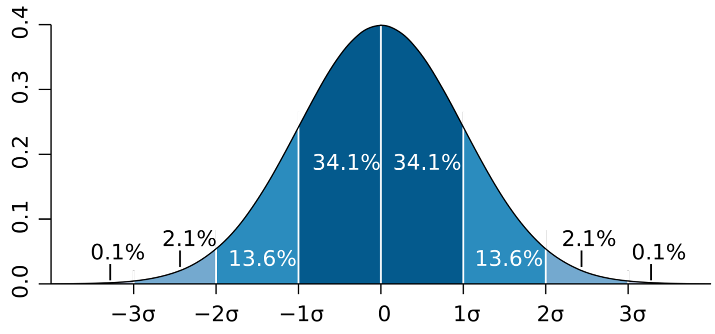
$$F_X(x) = \int_{-\infty}^{x} f_X(t)\, dt$$

$$= \int_{a}^{x} \frac{1}{b-a}\, dt = \frac{x-a}{b-a}$$

② Gaussian

$$f(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$F(x, \mu, \sigma^2) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x-\mu}{\sqrt{2}\sigma}\right)\right]$$



$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\,dt$$

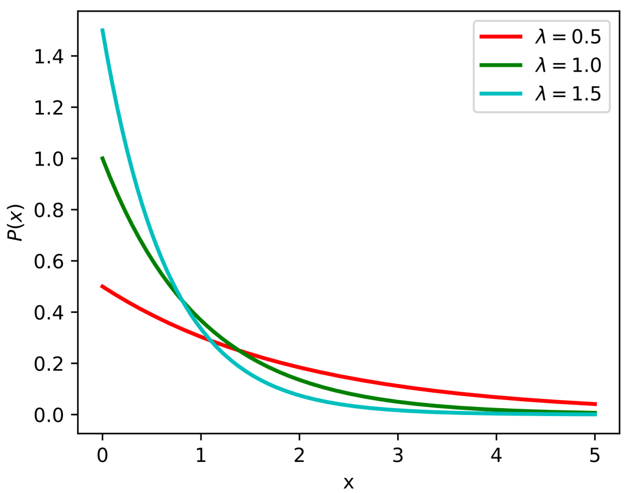
$$\operatorname{erf}(-x) = -\operatorname{erf}(x)$$

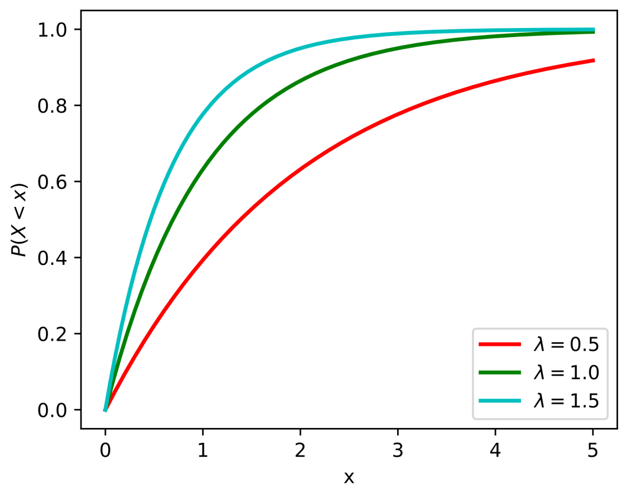
③ Exponential

average no. of events per unit time

$$f(x;\lambda) = \lambda e^{-\lambda x} \qquad x \geq 0 \qquad E[x] = \frac{1}{\lambda}$$

$$F(x;\lambda) = 1 - \lambda e^{-\lambda x} \qquad x \geq 0 \qquad V[x] = \frac{1}{\lambda^2}$$





→ Queuing systems → inter-arrival times or service times

→ Radioactive decays

→ Modelling natural disasters

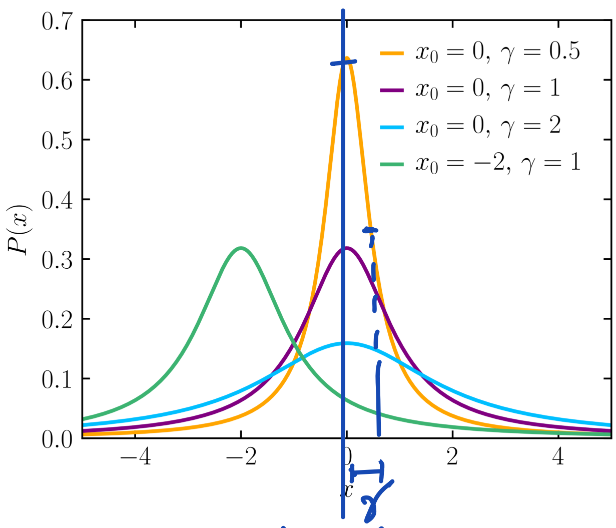
④ Cauchy

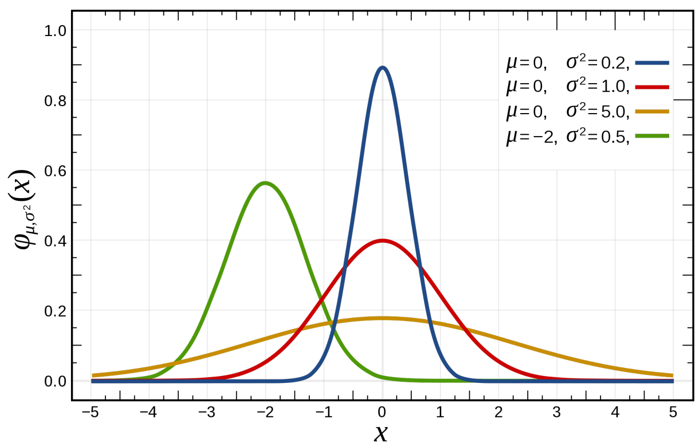
$$f(x;x_0,\gamma) = \frac{1}{\pi\gamma\left[1+\left(\frac{x-x_0}{\gamma}\right)^2\right]} \qquad -\infty < x < \infty$$

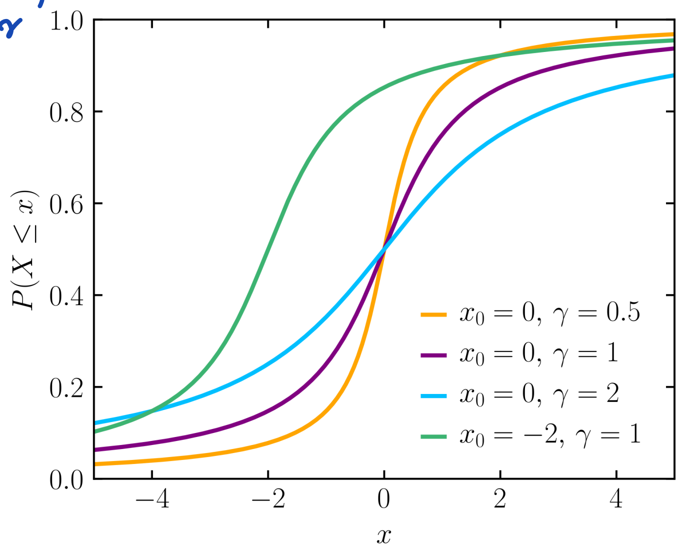
$$F(x;x_0,\gamma) = \frac{1}{\pi}\tan^{-1}\left(\frac{x-x_0}{\gamma}\right)+\frac{1}{2}$$

no mean $\qquad x_0 \to$ median

no variance $\qquad \gamma \to$ scale parameter $\qquad$ HWHM





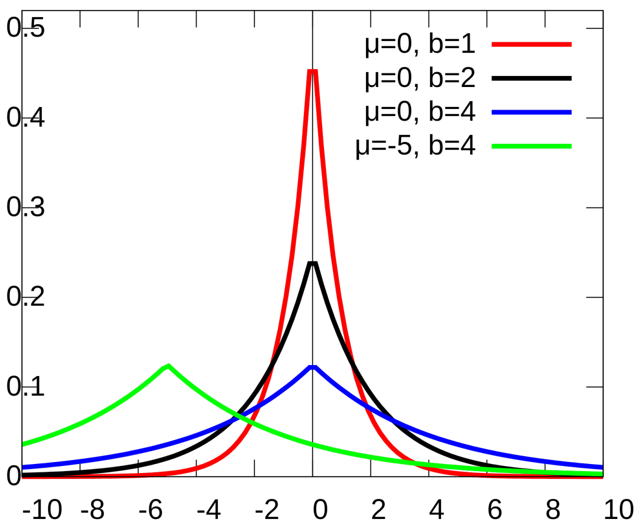


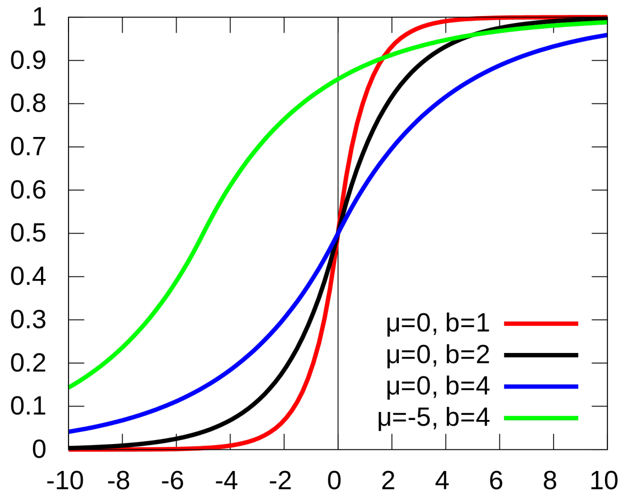
Stock market
Risk management
Extreme weather conditions

5 Laplacian distribution

$$f(x;\mu,b) = \frac{1}{2b}\exp\left(-\frac{|x-\mu|}{b}\right) \qquad -\infty < x < \infty$$

$$F(x;\mu,b) = \begin{cases} \frac{1}{2}\exp\left(\frac{x-\mu}{b}\right) & x < \mu \\ 1 - \frac{1}{2}\exp\left(-\frac{x-\mu}{b}\right) & x \geq \mu \end{cases}$$





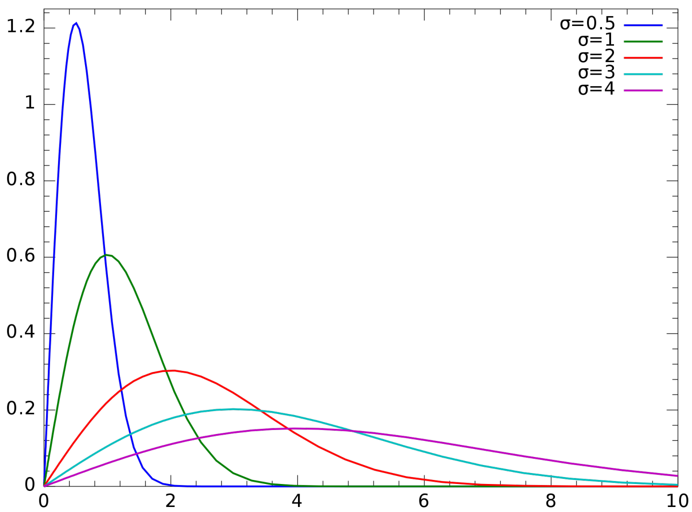
Data transmission → fading radio signals

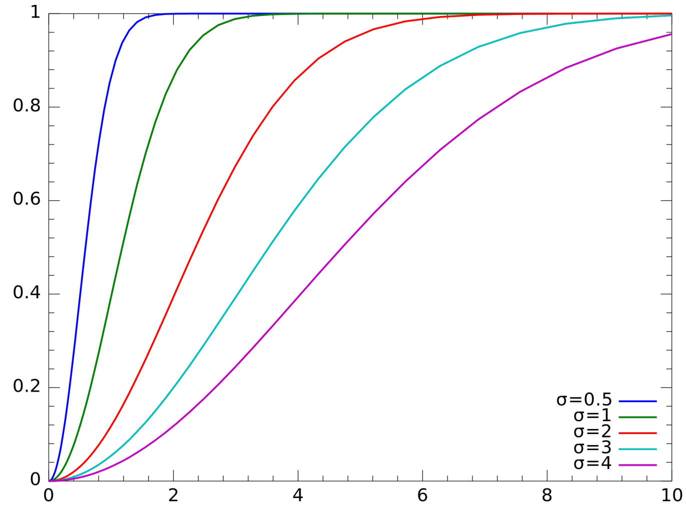
→ random fluctuations due to interference & multipath propagation

⑥ Rayleigh

$$f(x,\sigma) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} \qquad x \geq 0$$

$$F(x,\sigma) = 1 - e^{-\frac{x^2}{2\sigma^2}} \qquad x \geq 0$$





Underwater acoustics

Radar systems $\longrightarrow$ SNR

Wind speed analysis

$$F_X(x) = \sum_{k=-\infty}^{x} P[X=k] = \int_{-\infty}^{x} f_X(t)\,dt$$

$f_X(x) \longrightarrow F_X(x)$

DRV $\quad F_X(x) = \sum f_X(x) = \sum_{k=-\infty}^{i} P[X = x_i]$

$\hookrightarrow P[X \leq x_i]$

np.cumsum

CRV $\quad F_X(x) = \int_{-\infty}^{x} f_X(t)\, dt$

$f_X(x) \rightarrow$

$\downarrow$

CDF