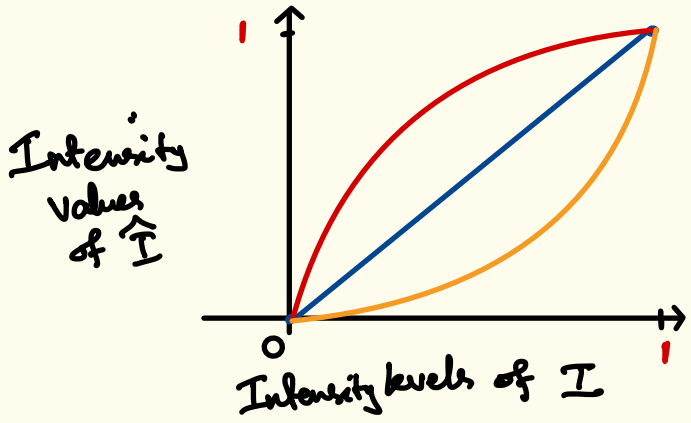
Point Operations

Input $I(i,j)$ → Point Operation → Output $\hat{I}(i,j)$



$[0, 255] \rightarrow [0, 1]$

8-bit Gray level

$\gamma > 0$

$\gamma = 1$

$\gamma < 1$ Bright

$\gamma > 1$ Dark

$\hat{I} = I^{\gamma}$

1. Histogram Equalization

2. Histogram Specification (Matching)

$$\frac{\text{Histogram bin value}}{\text{Total number of values}} \rightarrow \text{Probability Mass Function}$$

$$\frac{\#\text{ pixels with a given intensity value}}{\text{Total number of pixels}}$$

Discrete Random Variable

Transformation of Random Variable

Continuous

$$Y = g(X)$$

$X, Y \rightarrow$ Continuous Random Variables

$g(\cdot) \rightarrow$ One-one and onto function (Invertible)

$\rightarrow g^{-1}(\cdot)$ exists.

$p_X(x) \rightarrow$ Probability Density Function of X

CDF $$F_X(x) = \int_{-\infty}^{x} p_x(x')\, dx'$$

$$p_x(x) = \frac{d F_x(x)}{dx} \qquad x = g^{-1}(y)$$

$$p_Y(y) = p_x(g^{-1}(y)) \left| \frac{d g^{-1}(y)}{dy} \right|$$

1. Histogram Equalization

I	$\hat{I}$
Any	Uniform

$$X \xrightarrow{g} Y \qquad p_y(y) = \begin{cases} \frac{1}{b-a}, & a \le y \le b \\ 0, & \text{o/w} \end{cases} \qquad 0 \le y \le 1$$

$p_x(x)$ Given $\qquad p_y(y)$ Uniform,

$$F_x(x) = \int_{-\infty}^{x} p_x(x')\,dx' = P[X \le x] \qquad g^{-1}$$

$$y = g(x) = F_x(x) \Rightarrow x = F_x^{-1}(y)$$

$$p_y(y) = p_x\left(g^{-1}(y)\right) \left| \frac{d\, g^{-1}(y)}{dy} \right|$$

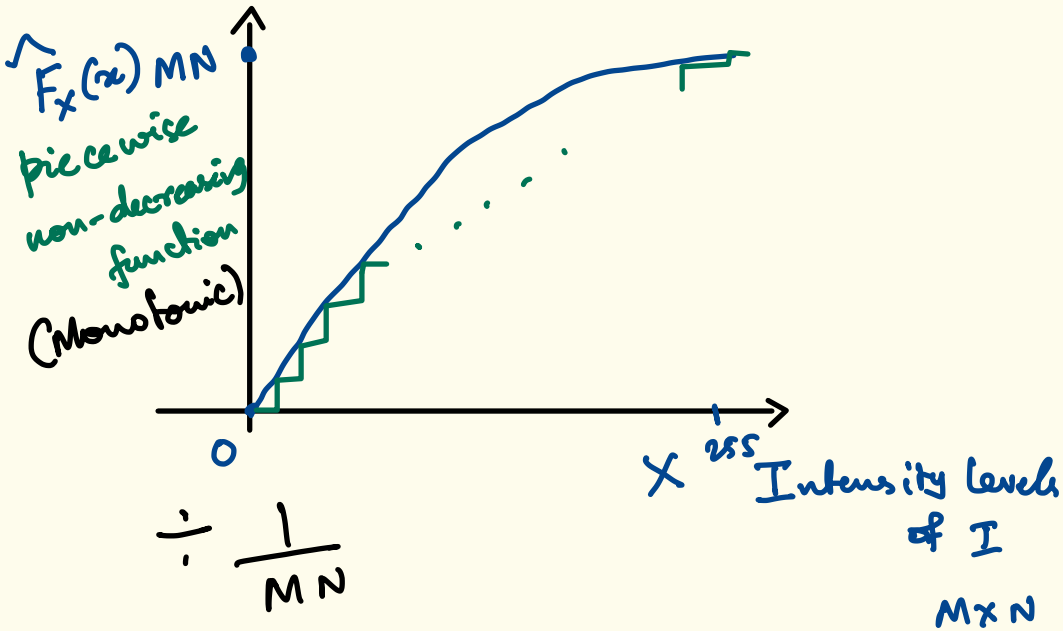
$\frac{d}{dy}$

$$F_y(y) = F_x\left(g^{-1}(y)\right) \rightarrow$$ Monotonically non-decreasing function

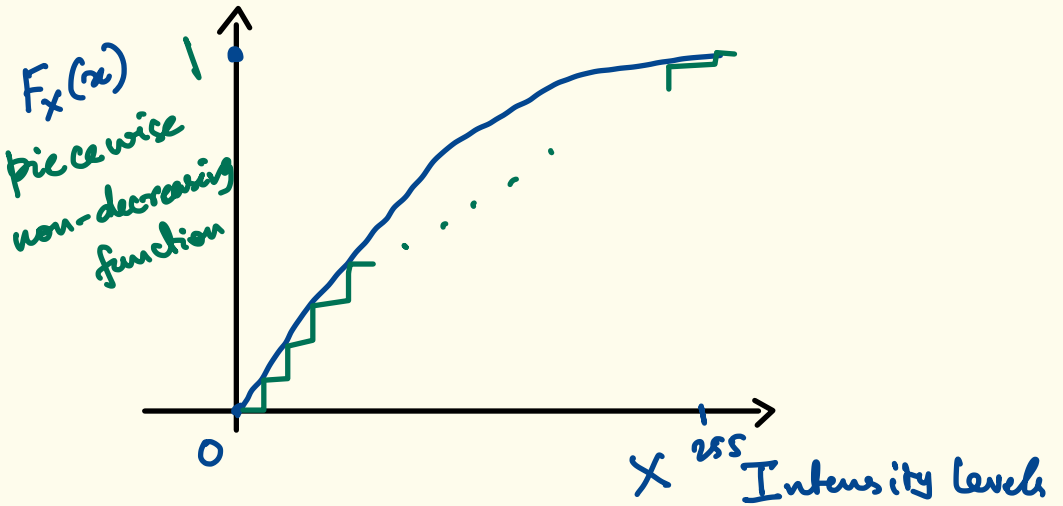
$$F_y(y) = F_x\left(F_x^{-1}(y)\right)$$

$$F_y(y) = y$$

$$p_y(y) = 1 \Longrightarrow \text{Uniform}$$



$\div \frac{1}{MN}$

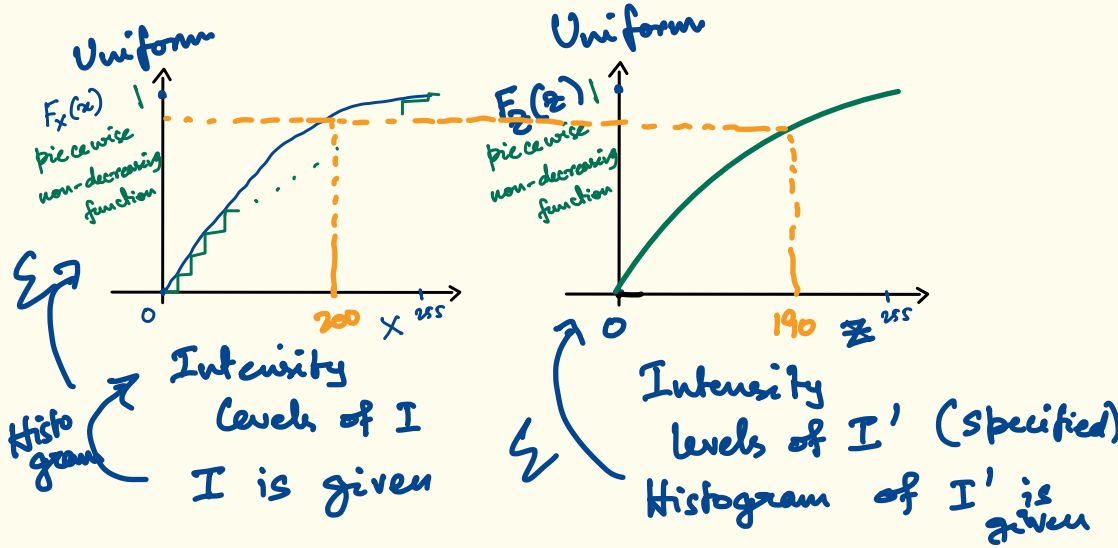


for each pixel in I,

$$\hat{I}(i,j) = \text{floor}\left(255 \times F_x^{(i,j)}(I(i,j))\right)$$

2. Histogram Specification

history



For each pixel (i,j) in I,

$$I'(i,j) = F_z^{-1}\left(F_x(I(i,j))\right)$$

$$I' = g(I) \quad \text{where}$$

$$g(\cdot) = F_z^{-1}(F_x(\cdot))$$