Identification and Information Transfer of Multidimensional Tactons Presented by a Single Vibrotactile Actuator

Yongjae Yoo, Juliette Regimbal, and Jeremy R. Cooperstock[1]

***Abstract*— Techniques designed to deliver information to users via the haptic channel often make use of multiple actuators spread over different body locations, sometimes employing multiple haptic modalities (e.g., vibration, skin stretch, temperature) in parallel. Instead, we sought to determine the information transfer capacity of a single vibrotactile actuator. To do so, we designed 90 multidimensional tactons using four parameters: the number of pulses, tempo, frequency, and amplitude, based on prior literature and results of pilot studies. We conducted a two-day experiment with 12 participants, which evaluated their accuracy in identifying each parameter and estimated the information transfer (IT) under continuously repeating and one-shot tacton presentation schemes. Participants achieved identification accuracy of individual parameters of approximately 70% for tempo, frequency, and amplitude, while their identification accuracy for number of pulses reached 90%. Although the overall tacton identification accuracy was only 34.12% and 38.66% for continuous and one-shot presentations, respectively, the high dimensionality of these tactons resulted in an estimated information transfer of 3.06 bits (for one-shot presentation), the highest IT achieved to date using only a single actuator. We provide a detailed analysis of these results, and drawing from the experiment, present guidelines for multidimensional tacton design.**

I. Introduction

Tactons, structural and abstract tactile stimuli used to convey information [1], [2], have been used in a wide range of human-computer interaction scenarios. Numerous studies have used them to deliver information to users via the haptic channel on various body sites. Use case scenarios include information delivery under cognitive load (e.g., vehicle or medical operations scenarios) [3], [4], [5], translation of audiovisual information [6] and even conveying semantic information via haptic language [7]. The literature is replete with designs for such applications employing multiple actuators that stimulate various locations on the user's body as a means of increasing the information capacity of haptic displays. This approach enables a wider design space that can make use of spatiotemporal tactile patterns, including illusionary moving sensations [8]. However, the associated larger, heavier, often cumbersome, and more power-demanding hardware of such multi-actuator systems poses several drawbacks, including potential discomfort to users, and the need for careful control and inter-actuator calibration.

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[1]The authors are with the Department of Electrical and Computer Engineering, Centre for Intelligent Machines (CIM), and Centre for Interdisciplinary Research in Music, Media and Technology (CIRMMT), McGill University, 3480 Rue University, Montréal, H3A 0E9, Quebec, Canada {yyoo, juliette, jer}@cim.mcgill.ca

A. Design of multidimensional tactons

Multiple design parameters have been considered to increase the information capacity of tactons without increasing the number of actuators. Determination of these parameters, e.g., vibrotactile frequency, amplitude, duration, and rhythm, along with their associated values, is an important problem in multidimensional tacton design [9]. In addition to traditional tacton parameters as listed above, past research has investigated variation of vibrotactile chords (superposition of different frequencies) [10], envelope variation [11], asymmetric vibration [12], and numerosity [13], [14] as approaches to enhancing the expressivity of tactons.

B. Presentation schemes of tactons

The choice between continuous and one-shot tacton presentation is another consideration for the designer. Most schemes present each tacton only a single time at the moment when the information to be delivered becomes available. However, some use cases, such as industrial processing or medical monitoring [5], in which the cost of missing a particular piece of information can be extremely high, warrant continuous feedback. Inspired by the latter use case, our past work demonstrated the feasibility of presenting multidimensional tactons representing patient vital signs, initially using multiple actuators distributed over the body [4], and later, using a single actuator [15].

C. Information transfer

Information transfer (IT), or mutual information, is a context-free measure of the amount of information provided by a communication signal. For multidimensional tactons, IT can be measured by the decrease of uncertainty, $log_2 \frac{P(S_i|R_j)}{P(S_i)}$, resulting from response R_j to stimulus S_i. IT is often estimated by the maximum likelihood estimation (MLE) method, using the empirical results of probabilities from an absolute identification experiment. However, it requires a large number of trials ($n \geq 5K^2$) to reduce estimation bias to a negligible level. Simulation-based [16] or theoretical estimation [17] alternatives have been derived to estimate IT from a reasonably small number of data points.

Using a single actuator, an IT was measured of 2.41 bits with eight different stimuli that varied in amplitude, frequency and pulse duration [18], 2.5 bits with 12 keyclicks that varied in amplitude, frequency, and number of pulses [19], and 2.23–2.67 bits with varied pulse rates and intensities [20]. IT generally increases when multiple actuator displays are used. An array of nine (3×3) vibration motors affixed to the palm demonstrated an IT of 2.46 bits [21], and

TABLE I: Tacton Design Parameters

Parameter	Levels
Tempo	120, 240, 480 BPM (Slow, Moderate, Fast)
Number of Pulses	1, 2, 3, 5, 7
Carrier Frequency	60+80 Hz, 160 Hz, 320+360 Hz (Low, Mid, High)
Amplitude	Weak, Strong (individually adjusted)

a waist belt with a 24-actuator array (4×6) achieved an IT of 2.98 bits [22].

Increased IT can be achieved through a combination of multiple actuators and multiple parameters for tacton design. Brown and Brewster designed 18 tactile stimuli with different rhythms, roughness levels, and spatial locations, presented via three voice coil actuators on the forearm, achieving an IT of 3.37 bits [23]. Lee and Starner developed 24 vibration patterns by changing their intensity, temporal pattern, starting point, and direction, using a three-actuator wrist band, reporting 4.28 bits of IT [24], and Tan et al. developed a three-finger friction-vibration multimodal display using 30 different waveforms, which achieved 6.50 bits [25]. To the best of our knowledge, the highest IT reported in the literature for vibrotactile patterns was the work of Park et al., who achieved an IT of 7.02 bits using 168 spatiotemporal vibrotactile patterns presented to the participant's hand [26]. Tan et al.'s review paper [27] suggested some general guidelines for increasing information capacity of tactons: use 1) multiple dimensions of the design space, 2) fewer levels for each dimension, and 3) illusory (phantom) sensations.

In pursuit of a higher information transfer rate, achievable via a single actuator, the main objective of our study is to characterize the effects on information capacity of various parameters of a multidimensional tacton. To this end, assuming a four dimensional information chunk, we selected four parameters—number of pulses, tempo, frequency and amplitude—to build 90 multidimensional tactons (see Sec. II-A), and conducted an identification experiment with 12 participants under two different presentation schemes. Similar to our previous monitoring scenario study [15], we assumed the use case of a wrist-based tactile display that communicates the levels of various signals, but here, we explore the potential to further increase the number of parameters and levels of each parameter without adversely affecting information transfer. Our contributions in this paper are a demonstration of the potential of high information transfer capacity from a single vibrotactile actuator, reaching a level that permits reliable identification of eight to nine different vibrotactile patterns, and our findings on the relative ease of discrimination of several vibrotactile parameters.

II. Experiment

The study was conducted remotely due to the current COVID-19 pandemic. All experimental materials were delivered to the participants' homes and instructions were provided via video calls. The experiment was approved by McGill's Research Ethics Board (REB #20-10-041), and conducted according to applicable public health directives.

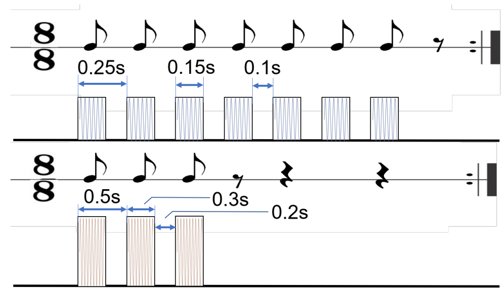


Fig. 1: Stimuli examples. Top: 240 BPM, seven pulses, weak. Bottom: 120 BPM, three pulses, strong vibrations.

A. Stimuli

The parameter levels used in each dimension are presented in Table I. We borrowed the representation of a musical eight-beat bar for our vibrotactile stimuli. Each tacton consists of $n \in \{1, 2, 3, 5, 7\}$ sequential vibrotactile eighth notes (pulses) followed by $8 - n$ rests to complete the bar. Rhythm and note duration were not varied. Tempo could take on one of three values (slow, moderate, and fast), and three frequency combinations were used to form vibrotactile chords [10], selected throughrigorous pilot tests of tacton identification with the requirement that no frequency component, including the beat frequency, be used twice. Additionally, two levels of amplitude, weak and strong, were selected by individual participants and equalized for perceived intensity on the frequencies used.

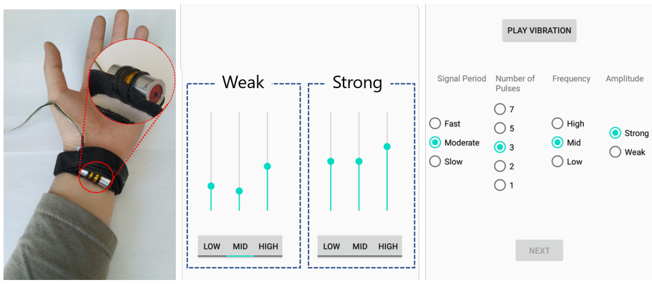
From this setup, an eighth note had a duration of 0.5, 0.25, and 0.125 s depending on tempo, with a duty cycle of 60 %. Thus the pulse widths for eighth notes were 0.3 s, 0.15 s, and 0.075 s for slow, moderate, and fast tempos respectively. Combining these four dimensions resulted in 90 ($5\times3\times3\times2$) multidimensional stimuli, as illustrated in Figure 1.

B. Participants

Eight adults (6M, 2F, aged 20–48 years, $\bar{X} = 26.63$) and four minors (3M, 1F, aged 11–16 years, $\bar{X} = 13.75$) with no known sensory disorders or disabilities participated in the study. Seven of the 12 participants reported no prior experience with haptic user studies, while the remainder had previously participated (1–5 times, $\bar{X} = 2.4$). All participants received 35 CAD as compensation for their time. The three participants with highest tacton identification accuracy received an additional incentive payment up to 30 CAD.

C. Apparatus

An application was developed to generate the stimuli and record responses from participants, and was run on an Android tablet (Xiaomi, Mi Pad 4/Teclast, M40). Vibrotactile signals were transmitted through an audio output channel of the tablet. Signals were amplified (SURE, PAM8803 amplifier) and displayed through a voice-coil actuator (Tactile Labs, Haptuator Original) worn on the glabrous part of the non-dominant wrist and secured with a Velcro strap. Participants wore noise-reduction earplugs (3M, 1100, Noise Reduction Ratio –29 dB) and over-ear headphones that



* (Play button appeared on Day 2 only.)

Fig. 2: Velcro strap with Haptuator (left) UIs for app used in this study (middle: intensity tuning, right: main experiment)

played pink noise to mask audible cues from the vibrations. Figure 2 show the Velcro strap and the user interface of the experimental app, respectively.

D. Procedure

The experiment was performed over two days. Prior to the start of the experiment, the apparatus was delivered to the participant, and a video call was held between the experimenter and the participant. The participant signed the consent form and then completed the pre-experiment questionnaire, which collected general demographic information and relevant prior haptic experience and musical training. The experimenter then introduced the experiment and explained the instructions.

Each experiment began with a stage for intensity matching across frequencies, and one for familiarization to the stimuli, carried out under experimenter guidance. For the former, participants were asked to adjust the amplitude to levels they perceived as equivalent across different frequencies, as illustrated in Figure 2 (middle). This was first done for the sliders corresponding to a designation of "weak" intensity, and then, for the "strong" intensity. The participant was then asked to experience and begin identifying the stimuli in the familiarization stage for at least 10 minutes. Once familiarized with the stimuli, the participants carried out the remainder of the experiment on their own, as follows.

1) Day 1: The first experimental session consisted of a training block and six testing blocks. The training block consisted of 40 trials, 20 randomly picked and 20 "difficult" stimuli, as identified in a pilot study. Correct answer feedback was provided after each answer. The six testing blocks (6×30 trials $=$ 180 trials in total) were divided equally into six blocks without feedback. As a result, each of the 90 tactons was tested twice per participant. Participants took inter-session breaks of a minimum of 2 min. During each trial, the tactons were repeated continuously and tacton features were identified using radio button inputs.

2) Day 2: This session was the same as Day 1, except that stimuli were presented for a fixed-length time of 4 s per trial. To do so, 480 BPM signals were repeated four times, while 120 BPM were only played once, to prevent the additional parameter of duration from affecting perception. In each trial, participants tapped the "Play" button to display the tacton and then entered their answers.

Experimental sessions on Days 1 and 2 lasted approximately 2 h and 1.5 h, respectively. After each session, participants completed a post-experimental questionnaire, ranking parameters in terms of identification difficulty and providing their written comments.

E. Data Analysis

We calculated tacton identification accuracy, both independently over the individual parameters, and for the combination of all four parameters, i.e., tactons as a whole, for Days 1 and 2. We then estimated the IT of the multidimensional tacton set using the results from the Day 2 session. Given the large number of stimuli (90), we found the lower and upper bounds of the true IT value using Houtsma's method [16] and Durlach's general additivity law [17]. The former simulates a large number of trials modeled from empirical results of the pooled data. The latter derives marginal confusion matrices to determine the IT_{est} of individual parameters and sums these to find the overall IT.

III. Results

A. Accuracy

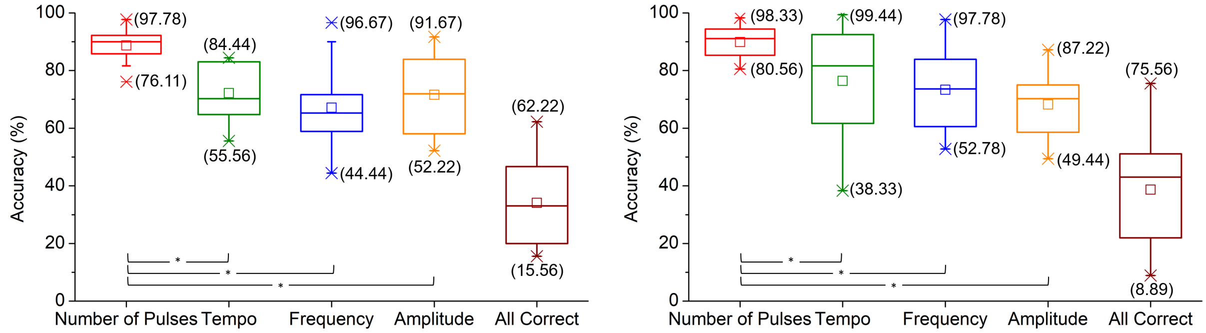
Figure 3 summarizes the accuracy results collected for both individual parameters and tactons as a whole over the two experimental sessions. ranged between a minimum of 67.13 % for frequency to a maximum of 89.91 % for number of pulses, with whole tacton identification accuracy of 34.12 % on Day 1 and 38.66 % on Day 2. Identification accuracy of number of pulses was significantly higher (pairwise t-tests, $p < 0.01$) than that for other parameters, both in Day 1 and 2. The differences between the two sessions, both for individual parameters and whole tacton identification, were not statistically significant.

B. Parameter-wise confusion matrices and IT_{est}

We derived the partial confusion matrices for each parameter using the pooled data of the 12 participants from Day 1 (continuous presentation) and Day 2 (one-shot presentation), respectively in Tables II and III. Each cell indicates the number of trials in which the participant responded with R_j to the presentation of stimulus S_i.

We performed a maximum likelihood estimation of the IT_{est} from the values of Table III for each of the parameters, obtaining 1.758 for number of pulses, 0.619 for tempo, 0.587 for frequency, and 0.101 bits for amplitude. Thus, the upper bound of IT_{est} was 3.065 bits, corresponding to 8.34 tokens of different information. The lower bound of IT was observed as 3.047 bits in Houtsma's method, with an error probability model of [0.11, 0.27, 0.26, 0.28] for the four parameters in a simulation of 200,000 trials (see Figure 4). The two methods resulted in almost identical IT of 3.047 bits and 3.065 bits, from which we can conclude that our multidimensional icons achieved an IT of approximately 3.06 bits.

We also derived participant IT_{est} using the same method. obtaining a range of 1.937–5.425 bits, with a geometric mean of 3.393 bits. These values suggest considerable individual differences in perception and/or identification ability for the multidimensional tactons used in our study.



Boxplot with 25% and 75% percentile range, and median as horizontal bar. Whiskers denote 95% confidence intervals, means are shown as small squares and minima and maxima by X. Asterisks indicate statistically significant ($p < 0.01$) differences.

Fig. 3: Boxplot accuracy results for Day 1 (top) and Day 2 (bottom) experiments

TABLE II: Confusion matrices of number of pulse, tempo, frequency, and amplitude (left to right) identification on Day 1.

S_i \ R_j	1	2	3	5	7	Sum
1	418	9	3	1	1	432
2	18	405	8	0	1	432
3	5	53	364	10	0	432
5	3	2	30	391	6	432
7	3	1	5	85	338	432
Sum	447	470	410	487	346	2160

(a) Number of pulses

S_i \ R_j	S	M	F	Sum
S	518	185	17	720
M	133	486	101	720
F	21	144	555	720
Sum	672	815	673	2160

(S: slow, M: moderate, F: fast)

(b) Tempo

S_i \ R_j	L	M	H	Sum
L	590	109	21	720
M	38	461	221	720
H	49	272	399	720
Sum	677	842	641	2160

(L: low, M: mid, H: high)

(c) Frequency

S_i \ R_j	W	S	Sum
W	770	310	1080
S	304	776	1080
Sum	1074	1086	2160

(W: weak, S: strong)

(d) Amplitude

TABLE III: Confusion matrices of number of pulse, tempo, frequency, and amplitude (left to right) identification on Day 2.

S_i \ R_j	1	2	3	5	7	Sum
1	407	16	5	4	0	432
2	13	399	20	0	0	432
3	2	36	382	12	0	432
5	2	5	17	397	11	432
7	1	3	2	69	357	432
Sum	425	459	426	482	368	2160

(a) Number of pulses

S_i \ R_j	S	M	F	Sum
S	469	210	41	720
M	80	562	78	720
F	37	64	619	720
Sum	586	836	738	2160

(S: slow, M: moderate, F: fast)

(b) Tempo

S_i \ R_j	L	M	H	Sum
L	590	102	29	720
M	33	520	167	720
H	31	214	475	720
Sum	653	836	671	2160

(L: low, M: mid, H: high)

(c) Frequency

S_i \ R_j	W	S	Sum
W	660	420	1080
S	266	814	1080
Sum	926	1234	2160

(W: weak, S: strong)

(d) Amplitude

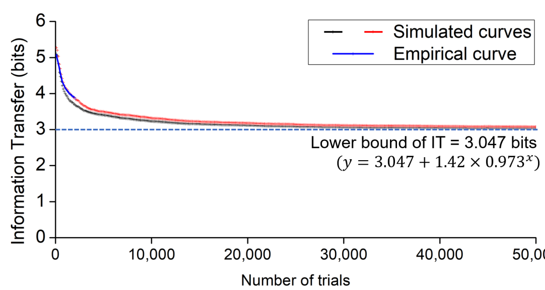


Fig. 4: Simulation results on IT's lower bound. Plot truncated at 50,000 trials for legibility.

TABLE IV: Comparison between perceived easiness (inverse of difficulty) rank and accuracy for parameters

Day	Data	Num.Pulses	Tempo	Frequency	Amplitude
Day 1	Rank	1.333	2.417	3.333	2.917
	Accuracy	88.70%	72.18%	67.13%	71.57%
Day 2	Rank	1.5	1.833	3	3.667
	Accuracy	89.91%	76.39%	73.33%	68.24%
	IT$_{est}$ (bit)	1.758	0.619	0.587	0.101

C. Post-experiment survey results

Participant responses to the post-experimental survey indicated that frequency was the hardest parameter to identify (6 out of 12 participants), followed by amplitude (3 participants) on Day 1, while amplitude was the hardest (9 participants), followed by frequency (3 participants) on Day 2. Free-form written comments also reflected this subjective difficulty; some mentioned that mid-weak and high-strong frequency-amplitude combinations felt similar and that it was difficult to discriminate frequency and amplitude in presentations with a small number of pulses or a fast tempo. Identification of the number of pulses was the easiest on both Days 1 and 2 (for 8 of the participants), followed by tempo. However, two of the participants noted that tempo was the hardest parameter to identify on Day 1, in particular when presented with a small number of pulses. The difficulty rank of the parameters are plotted in Figure 5. The perceived difficulty and accuracy results are highly correlated, as seen in Table IV, which compares average perceived ease of identification (inverse of difficulty rank) and accuracy for each parameter. Subjective comments from participants generally describe the difficulty of frequency and amplitude identification when presented in a few short pulses.

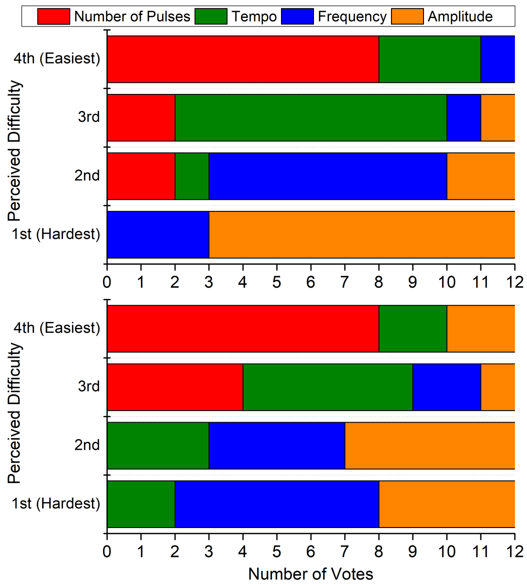


Fig. 5: Perceived difficulty ranks of parameters on Day 1 (top) and Day 2 (bottom).

IV. Discussion

A. Effects of presentation scheme on performance

Repeated tacton presentation did not increase identification accuracy: no significant difference was observed between continuous and single-instance presentation, provided that the tacton duration was sufficient, in this case 4 s.

While accuracy did not significantly differ between presentation schemes, amplitude was easier to identify than frequency in continuously presented tactons, while the opposite was true for single-instance presentation. However, continuous presentation could also cause adaptation and fatigue, which might adversely affect frequency identification.

B. Accuracy

Of the individual parameters, only the number of pulses demonstrated high identification accuracy (approximately 90 %). Accuracy of the other parameters ranged from 67.13 % to 76.39 %. This was likely due to a combination of two factors. First, the interdependence of parameters seems to adversely affect identification performance. For example, either a fast tempo or small number of pulses reduces tacton duration, meaning that the participants have less time to perceive the frequency and amplitude of vibrations. We tried to minimize the potential conflicts of vibrotactile parameters, such as how adopting a longer pulse length can improve frequency and amplitude perception, but limits the useful range of numerosity if an upper bound is imposed on tacton duration. Finding the optimum point in such a multidimensional parameter design space remains a challenge. Second, higher cognitive load, associated with the demands of discriminating the individual values within the multidimensional tactons, may itself impact identification performance [28], [3]. Therefore, although we obtained higher IT in this experiment with four-dimensional tactons than was the case for our previous study with three-dimensional tactons [15], the cost in cognitive load may be prohibitive.

TABLE V: Comparison table of IT achieved

Source	Parameter dimensions	Stimuli locations	Body site(s)	IS (bits)	IT (bits)
[19]	3	1	Hand (phone)	5.91	2.40
[18]	3	1	Forearm, finger	3.17	2.41
[20]	2	1	Finger	3.32	2.23–2.67
Current	4	1	Wrist	6.49	3.06
[23]	2	3	Forearm	4.75	2.98–3.37
[29]	3	2	Finger	6.97	4.00
[5]	2	8	Torso (belt)	6.17	4.18

*IS = information in source (log-scaled number of stimuli)

C. Information Transfer

The results, as described in Sec. III-B indicate that number of pulses and tempo—the two parameters related to temporal rhythm perception—contributed 2.377 bits of information, or approximately 77.6 %, of the total IT. The addition of frequency and amplitude parameters further contributed, albeit to a lesser extent. The subjective difficulty rankings and free-form feedback from participants echo these results; identifying number of pulses was the easiest, while discriminating levels of frequency or amplitude was the most difficult.

Frequency and amplitude, despite being commonly used parameters of tacton design, proved to be less effective for information delivery when applied in conjunction with other dimensions. Our choice of parameter levels provided for 2.584 bits of information (six possible combinations) for frequency and amplitude, but accounted for only 0.687 bits of information successfully delivered. This suggests that at most two levels of either frequency or amplitude can be used in multidimensional information delivery. As confirmed by Brown and Brewster, in such cases when accuracy is low, reducing the number of parameter levels can increase the effective information capacity by reducing uncertainty [23].

Table V compares our results with the IT_{est} achieved in previous studies with multidimensional tactons used for information delivery, with an emphasis on those employing only a single actuator. The IT we achieved of 3.06 bits means that approximately 8–9 items can be delivered using multidimensional tactons without loss. To the best of our knowledge, this represents the highest IT_{est} achieved to date using only a single actuator. Moreover, we used the wrist as the stimulus delivery location, while others used the whole hand or a finger, which are known to exhibit superior tactile sensitivity. It bears reminder, however, that IT values well in excess of 3.06 bits have been achieved by multi-actuator systems, which can exploit the benefits of spatial pattern mappings, which is not feasible with only a single actuator.

These results imply that in the absence of a rigorous training procedure to familiarize users with the vibrotactile patterns, utilizing less than ten easily discriminable tactons might be better than introducing a massive number of combinations in multiple dimensions.

D. General discussions and limitations

The main advantages of multidimensional tactons on a single actuator would be diversifying the tactons by providing multiple design dimensions and a larger information capacity from the large set of stimuli. However, due to high cognitive load, presenting many parameters simultaneously would undermine these benefits. Consistent with the findings of previous studies [17], [19], we note that designers may benefit from the more easily discriminated parameters of numerosity and tempo, rather than those of amplitude and frequency. To build the optimal stimuli set for information delivery, tradeoffs in parameter choice, as well as the influence of perceptual characteristics such as dissimilarity [1], must be considered.

Finally, we must consider the potential impact of remote experimentation. Although we provided detailed instructions in both printed and verbal form during a video call, and encouraged questions at any time, the constraints of testing under pandemic conditions may well have negatively affected the participants' concentration level, and in turn, the results obtained, compared to an in-person study. However, it also bears note that the results from a controlled lab experiment, whether conducted in-person or at-home, may not translate directly to real-world use conditions.

V. Conclusions and Future Work

This paper studied identification performance and information transfer capacity of four-dimensional tactons using a single actuator. Using 90 tactons, we demonstrated an overall accuracy of 38.44% and an estimated IT of 3.06 bits, exceeding the highest rate we could find in the literature for a single-actuator display. The accuracy of individual parameters was generally around 70%, except for numerosity perception (approximately 90%). Based on the results, some insights for designing tactons were discussed, such as 1) using temporal parameters—e.g., number of pulses and tempo,— 2) avoiding use of too many frequencies and amplitudes, and 3) reducing the stimuli set, similar to Brown and Brewster [23]. In the future, we plan to conduct a longitudinal study exploring the learnability of multidimensional tactons and the strategies of mapping information to tacton features. Finally, applying these theoretical results to real-world applications would serve to demonstrate the practical benefits of our approach.

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