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ICE ASSIGNMENT #1

Question 1

$$d_b = d_a \frac{\log(a)}{\log(b)}$$

(a) 0.89_{10}

(i) Base 2

$$d_b = 2\frac{\log 10}{\log 2}$$

$$= 6.64 \approx 7$$

$$0.89 \times \frac{2}{2}$$

$$= 1.78 \times 2^{-1}$$

$$= 1 \times 2^{-1} + 0.78 \times 2^{-1}$$

$$= 1 \times 2^{-1} + 1.56 \times 2^{-2}$$

$$= 1 \times 2^{-1} + 1 \times 2^{-2} + 0.56 \times 2^{-2}$$

$$= 1 \times 2^{-1} + 1 \times 2^{-2} + 1.12 \times 2^{-3}$$

$$= 1 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3} + 0.12 \times 2^{-3}$$

$$= 1 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3} + 0.24 \times 2^{-4}$$

$$= 1 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3} + 0 \times 2^{-4} + 0 \times 2^{-5} + 0.48 \times 2^{-5}$$

$$= 1 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3} + 0 \times 2^{-4} + 0 \times 2^{-5} + 0.96 \times 2^{-6}$$

$$= 1 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3} + 0 \times 2^{-4} + 0 \times 2^{-5} + 0 \times 2^{-6} + 1.92 \times 2^{-7}$$

$$= 1 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-7} + 0.92 \times 2^{-7}$$

$$= 1 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-7} + 1.84 \times 2^{-8}$$

$$= 0.1110001 + 1.84 \times 2^{-8}$$

$$\boxed{= 0.1110001_2}$$

(ii) Base 16

$$d_b = 2\frac{\log(10)}{\log(16)}$$

$$= 1.66 \approx 2$$

$$0.89 = 0.\underbrace{1110}_{E}|\underbrace{0011}_{3}{}_2$$

last 1 comes from (i)

$$\boxed{= 0.E3_{16}}$$

(iii) Base 5

$$d_b = 2\frac{\log(10)}{\log(5)} = 2.66 \approx 3$$

$0.89_{10} \times \frac{5}{5(d)}$

$= 4.45 \times 5^{-1}$

$= 4 \times 5^{-1} + 0.45 \times 5^{-1}$

$= 4 \times 5^{-1} + 2.25 \times 5^{-2}$

$= 4 \times 5^{-1} + 2 \times 5^{-2} + 0.25 \times 5^{-2}$

$= 4 \times 5^{-1} + 2 \times 5^{-2} + 1.25 \times 5^{-3}$

$= 4 \times 5^{-1} \times 2 \times 5^{-2} + 1 \times 5^{-3} + 0.25 \times 5^{-3}$

$= 4 \times 5^{-1} * 2 \times 5^{-2} + 1 \times 5^{-3} + 1.25 \times 5^{-4}$

$= 0.421_5 + 1.25_{10} \times 5^{-4}$

$\boxed{= 0.421_5}$

(iv) Base 8

$d_b = 2 \frac{\log(10)}{\log(8)}$

$= 2.21 \approx 3$

$0.89_{10} \times \frac{8}{8}$

$= 7.12 \times 8^{-1}$

$= 7 \times 8^{-1} + 0.12 \times 8^{-1}$

$= 7 \times 8^{-1} + 0.96 \times 8^{-2}$

$= 7 \times 8^{-1} + 0 \times 8^{-2} + 7.68 \times 8^{-3}$

$= 7 \times 8^{-1} + 0 \times 8^{-2} + 7 \times 8^{-3} + 0.68 \times 8^{-3}$

$= 7 \times 8^{-1} + 0 \times 8^{-2} + 7 \times 8^{-3} + 5.44 \times 8^{-4}$

$= 0.707_8 + 5.44_{10} \times 8^{-4}$

$\boxed{= 0.708_8}$

(b) 101111011101_2 $\quad d_b = 12 \frac{\log(2)}{\log(10)} = 3.61 \approx 4$

(i) Base 10

$= 2^{11} \times 1 + 2^{10} \times 0 + 2^9 \times 1 + 2^8 \times 1 + 2^7 \times 1 + 2^6 \times 1 + 2^5 \times 0 + 2^4 \times 1 +$
$2^3 \times 1 + 2^2 \times 1 + 2^1 \times 0 + 2^0 \times 1$

$= 2048 + 512 + 256 + 128 + 64 + 16 + 8 + 4 + 1$

$\boxed{= 3037_{10}}$

(ii) Base 16 $\qquad d_b = 12 \frac{\log(2)}{\log(16)} = 3$

$1011|1101|1101_2$

B D D

$\boxed{= BDD_{16}}$

(iii) Base 3 $\qquad d_b = 12 \frac{\log(2)}{\log(3)} = 7.57 \approx 8$

$101111011101_2 = 3037_{10}$

$Q^0 = 3037$

$Q^1 = 3037/3 = 1012 \qquad R^1 = 1$

$Q^2 = 1012/3 = 337 \qquad R^2 = 1$

$Q^3 = 337/3 = 112 \qquad R^3 = 1$

$Q^4 = 112/3 = 37 \qquad R^4 = 1$

$Q^5 = 37/3 = 12 \qquad R^5 = 1$

$Q^6 = 12/3 = 4 \qquad R^6 = 0$

$Q^7 = 4/3 = 1 \qquad R^7 = 1$

$Q^8 = 1/3 = 0 \qquad R^8 = 1$

$\boxed{3037_{10} \rightarrow 11011111_3}$

(iv) Base 4 $\qquad d_b = 12 \frac{\log(2)}{\log(4)} = 6$

$10|11|11|01|11|01_2$

2 3 3 1 3 1

$\boxed{= 233131_4}$

(c) $BA2F.A3_{16}$

(i) Base 2

$d_b = 6 \frac{\log(16)}{\log(2)}$

$= 24$

B	A	2	F	.	A	3
1011	1010	0010	1111	.	1010	0011

$= 1011101000101111.10100011_2$

(ii) Base 10

$$d_b = 6\,\frac{\log(16)}{\log(10)}$$

$$= 7.22 \approx 8$$

$$BA2F.A3_{16} = 16^3 \times 11 + 16^2 \times 10 + 16^1 \times 2 + 16^0 \times 15 + 16^{-1} \times 10 + 16^{-2} \times 3$$

$$\boxed{= 47663.637_{10}}$$

(iii) Base 5

$$d_b = 6\,\frac{\log(16)}{\log(5)}$$

$$= 10.34 \approx 11$$

$BA2F.A3_{16} = 47663.637_{10}$

Break into two parts → whole number = 47663

decimal number = 0.637

- Whole numbers ⇒

$Q^0 = 47663$	
$Q^1 = 47663/5 = 9532$	$R^1 = 3$
$Q^2 = 9532/5 = 1906$	$R^2 = 2$
$Q^3 = 1906/5 = 381$	$R^3 = 1$
$Q^4 = 381/5 = 76$	$R^4 = 1$
$Q^5 = 76/5 = 15$	$R^5 = 1$
$Q^6 = 15/5 = 3$	$R^6 = 0$
$Q^7 = 3/5 = 0$	$R^7 = 3$

$47663_{10} \rightarrow 3011123_5$

- Decimal numbers

$$0.637_{10} \times \frac{5}{5}$$

$$= 3.185 \times 5^{-1}$$

$$= 3 \times 5^{-1} + 0.185 \times 5^{-1}$$

$= 3 \times 5^{-1} + 0.925 \times 5^{-2}$

$= 3 \times 5^{-1} + 0 \times 5^{-2} + 4.625 \times 5^{-3}$

$= 3 \times 5^{-1} + 0 \times 5^{-2} + 4 \times 5^{-3} + 0.625 \times 5^{-3}$

$= 3 \times 5^{-1} + 0 \times 5^{-2} + 4 \times 5^{-3} + 3.125 \times 5^{-4}$

$= 3 \times 5^{-1} + 0 \times 5^{-2} + 4 \times 5^{-3} + 3 \times 5^{-4} + 0.125 \times 5^{-4}$

$= 3 \times 5^{-1} + 0 \times 5^{-2} + 4 \times 5^{-3} + 3 \times 5^{-4} + 0.625 \times 5^{-5}$

$= 0.3043_5 + 0.625 \times 5^{-5}$

$= 0.3043_5$

Putting both together $= 3011123_5 + 0.3043_5$

$\boxed{= 3011123.3043_5}$

(iv) Base 8

$$d_b = 6 \frac{\log(16)}{\log(8)}$$

$$= 8$$

$BA2F.A3_{16} = 47663.637_{10}$

Once again, we break it up into whole number and decimal components

- whole number

$47663_{10} \Rightarrow$

$Q^0 = 47663$	
$Q^1 = 47663/8 = 5957$	$R^1 = 7$
$Q^2 = 5957/8 = 744$	$R^2 = 5$
$Q^3 = 744/8 = 93$	$R^3 = 0$
$Q^4 = 93/8 = 11$	$R^4 = 5$
$Q^5 = 11/8 = 1$	$R^6 = 3$
$Q^6 = 1/8 = 0$	$R^7 = 1$

$47663_{10} \rightarrow 135057_8$

$0.637_{10} \times \frac{8}{8}$

$= 5.096 \times 8^{-1}$

$= 5 \times 8^{-1} + 0.096 \times 8^{-1}$

$= 5 \times 8^{-1} + 0.768 \times 8^{-2}$

$= 5 \times 8^{-1} + 0 \times 8^{-2} + 0.768 \times 8^{-2}$

$= 5 \times 8^{-1} + 0 \times 8^{-2} + 6.144 \times 8^{-3}$

$= 5 \times 8^{-1} + 0 \times 8^{-2} + 6 \times 8^{-3} + 0.144 \times 8^{-3}$

$= 0.506_8 + 0.144 \times 8^{-3}$

$= 0.51_8$

Putting them together $= 135057_8 + 0.51_8$

$\boxed{= 135057.51_8}$

(d) 17.341_8

(i) Base 10

$d_{10} = 5 \frac{\log(8)}{\log(10)}$

$= 4.51 \approx 5$

$17.341_8 = 8^1 \times 1 + 8^0 \times 7 + 8^{-1} \times 3 + 8^{-2} \times 4 + 8^{-3} \times 1$

$= 8 + 7 + 3/8 + 1/2 + 1/8$

$\boxed{= 15.439_{10}}$

(ii) Base 4

$d_b = 5 \frac{\log(8)}{\log(4)}$

$= 7.5 \approx 8$

1	7	.	3	4	1_2
001	111	.	011	100	001

$17.341_8 \rightarrow 001111.011100001_2$

$$11|11.01|11|00|00|10_2$$
$$3\ \ 3\ .\ 1\ \ 3\ \ 0\ \ 0\ \ 2$$

$$\boxed{1111.0011000001_2 \rightarrow 33.130020_4}$$

(iii) Base 3

$$d_b = 5\,\frac{\log(2)}{\log(3)}$$
$$= 9.46 \approx 10$$

$$17.341_{10} \rightarrow 15.439_{10}$$

Break into whole and decimal components

- Whole number

15_{10}

$$Q^0 = 15$$
$$Q^1 = 15/3 = 5 \qquad R^1 = 0$$
$$Q^2 = 5/3 = 1 \qquad R^2 = 2$$
$$Q^3 = 1/3 = 0 \qquad R^1 = 1$$

$$15_{10} \rightarrow 120_3$$

- Decimal number

0.439_{10}

$$0.439 \times \frac{3}{3}$$
$$= 1.317 \times 3^{-1}$$
$$= 1 \times 3^{-1} + 0.317 \times 3^{-1}$$
$$= 1 \times 3^{-1} + 0.951 \times 3^{-2}$$
$$= 1 \times 3^{-1} + 0 \times 3^{-2} + 2.853 \times 3^{-3}$$
$$= 1 \times 3^{-1} + 0 \times 3^{-2} + 2 \times 3^{-3} + 0.853 \times 3^{-3}$$
$$= 1 \times 3^{-1} + 0 \times 3^{-2} + 2 \times 3^{-3} + 2.559 \times 3^{-4}$$
$$= 1 \times 3^{-1} + 0 \times 3^{-2} + 2 \times 3^{-3} + 2 \times 3^{-4} + 0.559 \times 3^{-4}$$

$= 1\times 3^{-1} + 2\times 3^{-3} + 2\times 3^{-4} + 1.677\times 3^{-5}$

$= 1\times 3^{-1} + 2\times 3^{-3} + 2\times 3^{-4} + 1\times 3^{-5} + 0.677\times 3^{-5}$

$= 1\times 3^{-1} + 2\times 3^{-3} + 2\times 3^{-4} + 1\times 3^{-5} + 2.031\times 3^{-6}$

$= 1\times 3^{-1} + 2\times 3^{-3} + 2\times 3^{-4} + 1\times 3^{-5} + 2\times 3^{-6} + 0.031\times 3^{-6}$

$= 1\times 3^{-1} + 2\times 3^{-3} + 2\times 3^{-4} + 1\times 3^{-5} + 2\times 3^{-6} + 0.093\times 3^{-7}$

$= 1\times 3^{-1} + 2\times 3^{-3} + 2\times 3^{-4} + 1\times 3^{-5} + 2\times 3^{-6} + 0\times 3^{-7} + 0.279\times 3^{-8}$

$= 0.1022120_3 + 0.279\times 3^{-8}$

$= 0.1022121_3$

Put both together $\Rightarrow 120_3 + 0.1022121_3$

$\boxed{= 120.1022121_3}$

(iv) Base 16

$$d_b = 5\,\frac{\log(8)}{\log(16)}$$

$$= 3.75 \approx 4.$$

$17.841_8 \rightarrow 1111.011100001_2$

$1111.0111|0000|1000_2$

F . 7 0 8

$\boxed{1111.011100001_2 \rightarrow F.708_{16}}$

Question 2

(a) 0010 0010 0001 1110 1100 1110 0000 0000

$X = a \cdot 2^n$

bit 31 = 0 $\therefore$ sign = +

bit (30-23) = $0100\ 0100_2$

$= 2^6 \times 1 + 2^2 \times 1$

$= 68_{10}$

Subtract the offset of 127

$n = 68 - 127$

$\underline{n = -59}$.

bit (22-0) = 0011 1101 1001 1100 0000 000

3 D 9 C 0 0

# of Significant digits = 14

Add (1) for hidden value = 14 + 1

N = 15

# of significant digits required = N log(2)

= 15 log(2)

= 4.52

≈ 5 digits required

Convert to base 10

$a = 3 \times 16^{-1} + 13 \times 16^{-2} + 9 \times 16^{-3} + 12 \times 16^{-4} + 1$

hidden 1

$\underline{a = 1.24066_{10}}$

$x = a \cdot 2^n$

$= 1.24066 \times 2^{-59}$

$\boxed{x = 2.1522 \times 10^{-18}}$

(b) $0\times AC0396ED_{16}$

A	C	0	3	9	6	E	D
1010	1100	0000	0011	1001	0110	1110	1101

$X = a \cdot 2^n$

bit 31 = 1 ∴ sign = −

bit (30-23) = 01011000

$= 2^6 \times 1 + 2^4 \times 1 + 2^3 \times 1$

$= 88$

Subtract offset of 127

$n = 88 - 127$

$n = -39$

bit (22-0) = 0000 | 0111 | 0010 | 1101 | 1101 | 1010

0000	0111	0010	1101	1101	1010
0	7	2	D	D	A

# of significant digits = 23

Add (1) for hidden value = 23+1

N = 24

# of significant digits required = N log(2)

= 24 log(2)

= 7.22

≈ 8 digits required

Convert to base 10

$a = 1 + (16^{-1} \times 0\ 16^{-2} \times 7 + 16^{-3} \times 2 + 16^{-4} \times 13 + 16^{-5} \times 13 + 16^{-6} \times 10)$

$a = 1.0280434_{10}$

$x = a \cdot 2^n$

$= -1.0280434 \times 2^{-39}$

$\boxed{x = -1.8700000 \times 10^{-12}}$

Question 3

(a) $\underset{A}{E421_{16}} - \underset{B}{1ACE_{16}}$

E	4	2	1
1110	0100	0010	0001

→ 0001 1011 1101 1110
+0000 0000 0000 0001
0001 1011 1101 1111_2
= 7135_{10}

∴ $E421_{16} \rightarrow -7135_{10}$

1	A	C	E
0001	1010	1100	1110

$\rightarrow (2^{12} + 2^{11} + 2^{9} + 2^{7} + 2^{6} + 2^{3} + 2^{2} + 2) \times 1$
$= 6362_{10}$

∴ $1ACE_{16} \rightarrow 6362_{10}$

$E421_{16} - 1ACE_{16} = -7135_{10} - 6362_{10}$
$= -13997_{10}$ → Expected result

E421 → 1110 0100 0010 0001_2 — (A)
1ACE → 0001 1010 1100 1110_2 — (B)

Subtrahand (B)	0001 1010 1100 1110_2
Its complement	1110 0101 0011 0001_2
Add (1)	(+) 0000 0000 0000 0001_2
(−B)	1110 0101 0011 0010_2

Minuend (A)	1110 0100 0010 0001_2
(−B)	(+) 1110 0101 0011 0010_2
	1 1100 1001 0101 0011_2

overflow

$A + (-B) = 1100 \mid 1001 \mid 0101 \mid 0011_2$
C | 9 | 5 | 3

Hexadecimal: $\boxed{A - B = C953_{16}}$

Final Sum: 1100 1001 0101 0011_2
Complement: 0011 0110 1010 1100_2
Add (1) (+) 0000 0000 0000 0001_2
0011 0110 1010 1101_2 → 13997_{10}

To find its (−) value

Since the actual result is the same as the expected result, the computation was correct.

Decimal: $\boxed{A - B = -13997_{10}}$

(b) $4530_{16} + A123_{16}$

$\overset{\shortparallel}{A}$ $\quad\quad$ $\overset{\shortparallel}{B}$

4	5	3	0
0100	0101	0011	0000

$\rightarrow (2^{14} + 2^{10} + 2^{8} + 2^{5} + 2^{4}) \times 1$

$= 17712_{10}$

A	1	2	3
1010	0001	0010	0011

$\rightarrow 0101\ 1110\ 1101\ 1100_2$

$+\underline{0000\ 0000\ 0000\ 0001_2}$

$0101\ 1110\ 1101\ 1101_2$

$= 24285_{10}$

$\therefore 4530_{16} \rightarrow 17712_{10}$ $\quad\quad$ $\therefore A123_{16} \rightarrow -24285_{10}$

$4530_{16} + A123_{16} = 17712_{10} + (-24285_{10})$

$= -6573_{10} \rightarrow$ Expected Solution

$$\begin{array}{lrl}
4530_{16} \rightarrow & 0100\ 0101\ 0011\ 0000_2 & -(A) \\
A123_{16} \rightarrow & (+)\ 1010\ 0001\ 0010\ 0011_2 & -(B) \\
\hline
& 1110\ 0110\ 0101\ 0011_2 &
\end{array}$$

$A + B = 1110 \mid 0110 \mid 0101 \mid 0011$

$\quad\quad\quad\quad E \mid 6 \mid 5 \mid 3$

Hexadecimal : $\boxed{A + B = E653_{16}}$

$$\left.\begin{array}{lr}
\text{Final sum} : & 1110\ 0110\ 0101\ 0011_2 \\
\text{Complement} : & 0001\ 1001\ 1010\ 1100_2 \\
\text{Add (1)} & (+)\ \underline{0000\ 0000\ 0000\ 0001_2}
\end{array}\right\}\text{To find its (-) value}$$

$0001\ 1001\ 1010\ 1101_2$

$= 6573_{10}$

Decimal : $\boxed{A + B = -6573_{10}}$

Since the actual result and the expected solution was the same, the computation was correct.

(c) 3.4219087×10^{12}

• $X = a \cdot 2^n$

* $X = 3.4219087 \times 10^{12}$

$n = \log(X)/\log(2)$

$= \dfrac{\log(3.4219087 \times 10^{12})}{\log(2)}$

$= 41.638$

Rounded up $= 42$ Rounded down $= 41$

$a = X/2^{n_1}$ or $a = X/2^{n_2}$

$= \dfrac{3.4219087 \times 10^{12}}{2^{42}}$ or $= \dfrac{3.4219087 \times 10^{12}}{2^{41}}$

$= 0.778052$ $= 1.5561$ ⇐ Normalized

$\therefore n = 41$

$a = 1.5561$

$X = 1.5561 \times 2^{41}$

$n = 41$

Add offset of $127 = 41 + 127$

$= 168_{10}$

Convert to binary

Base 10	256	128	64	32	16	8	4	2	1
Base 2	0	1	0	1	0	1	0	0	0

* $168_{10} \rightarrow 10101000_2$ bits (30-23)

$a = 1.5561 - 1$ (subtract 1 for hidden digit)

$= 0.5561$

Original sig. digits $= 8$

# of sig. figures required $= \dfrac{8}{\log(16)}$

$= 6.64$

≈ 7 digits required

$0.5561 \times 16 = 8.8976 \longrightarrow 8$

$0.8976 \times 16 = 14.3616 \longrightarrow E$

$0.3616 \times 16 = 5.7856 \longrightarrow 5$

$0.7856 \times 16 = 12.5696 \longrightarrow C$

$0.5696 \times 16 = 9.1136 \longrightarrow 9$

$0.1136 \times 16 = 1.8176 \longrightarrow 1$

$0.8176 \times 16 = 13.0816 \longrightarrow D$

$0.0816 \times 16 = 1.3056 \longrightarrow 1$ ← for rounding

$\Rightarrow 8E5C91D_{16}$

Convert to binary ⇒

8	E	5	C	9	1	D
1000	1110	0101	1100	1001	0001	1101

* $100011100101110010010001\underbrace{1101}_{\text{extra values}}{}_2$

* Positive number ∴ bit 31 = 0

Combine ⇒ 0 10101000 10001110010111001001000

The above value is not the exact representation of the initial decimal value that we were asked to calculate in the question. Instead, it is a close approximation. This occurs due to the fact that we had more than 23 digits for the mantissa, and had to drop the last few, which explains the approximation.