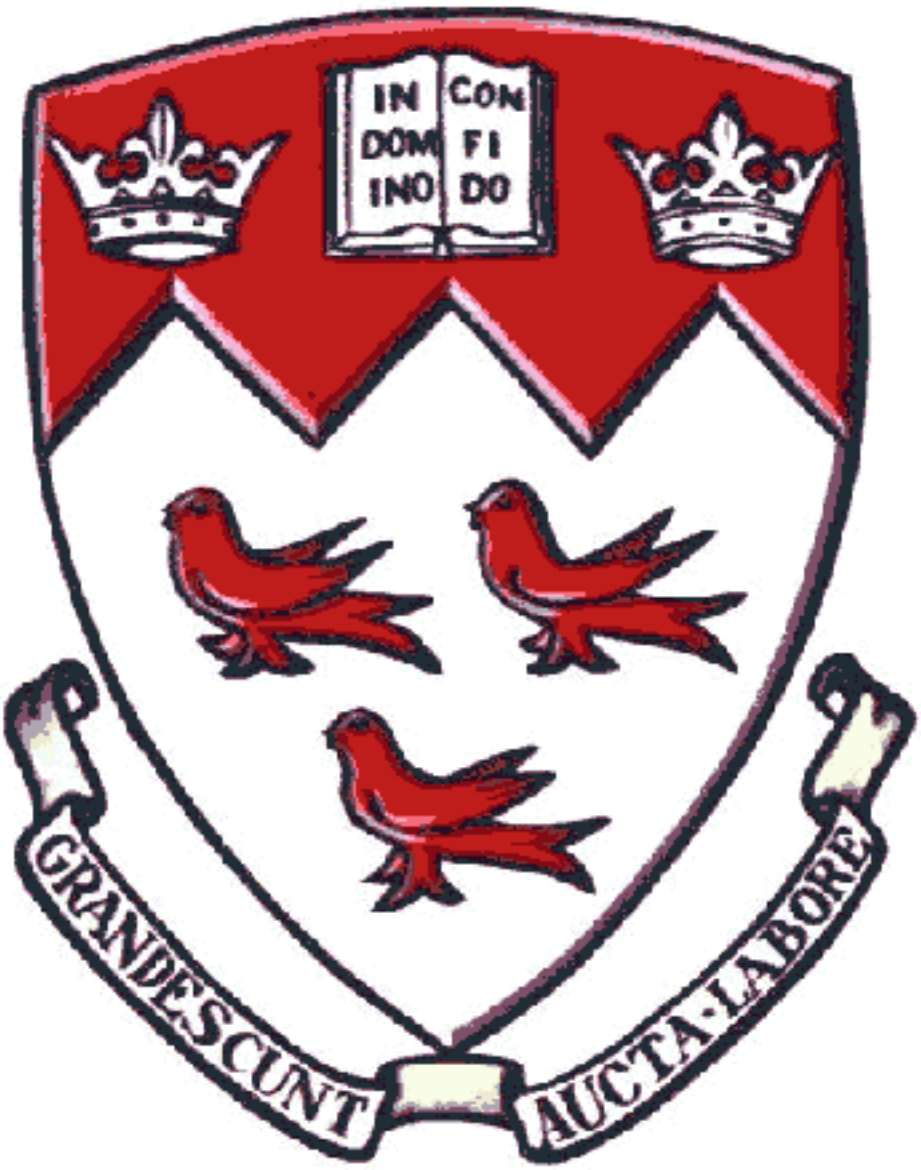

ECSE 543 – Numerical Methods
Assignment 2

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Question 1

Figure 1 shows two first-order triangular finite elements used to solve the Laplace equation for electrostatic potential. Find a local S-matrix for each triangle and a global S-matrix for the mesh, which consists of just these two triangles. The local (disjoint) and global (conjoint) node-numberings are shown in Figure 1(a) and (b), respectively. Also, Figure 1(a) shows the (x, y)-coordinates of the element vertices in meters.

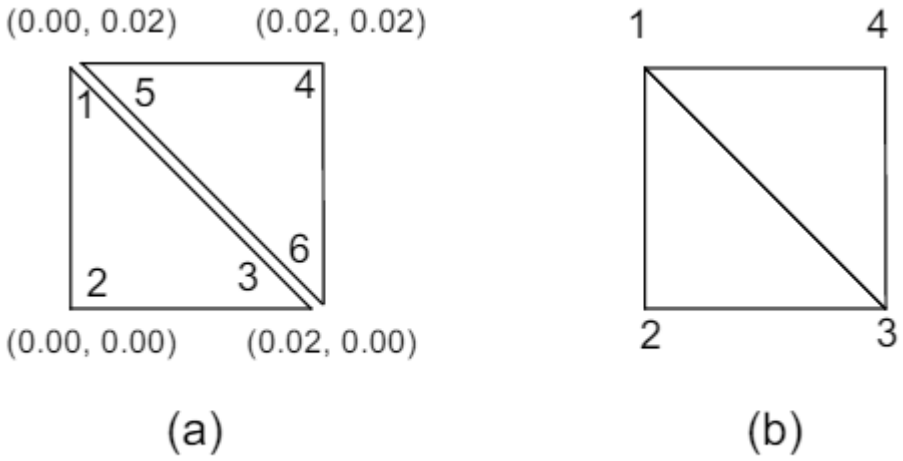


Figure 1

Local S-Matrices:

Energy is each triangle is given by the following equation (taken from lecture slides):

$$W^{(e)} = \frac{1}{2}\int_{\Delta_e} |\nabla U|^2 dS$$

$$= \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3} U_i U_j \int_{\Delta_e} \nabla\alpha_i \cdot \nabla\alpha_j dS$$

From the above equation, we can see that $S = \int \nabla\alpha_i \cdot \nabla\alpha_j \, dS$

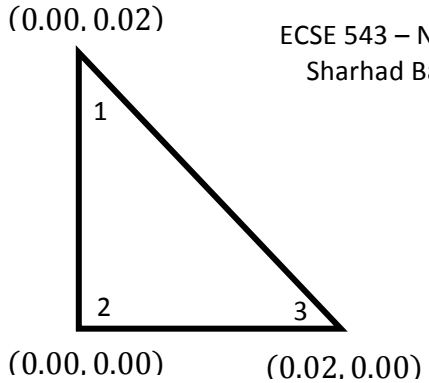
Using the example provided in the lecture FE1stOrder, a general form for S is formed:

$$S_{i,j} = \frac{1}{4A}[(y_j - y_{j+1})(y_{i+2} - y_i) + (x_{j+1} - x_j)(x_i - x_{i+2})]$$

We also need the Area of each of the triangles: $A = \frac{1}{2} b * h$

$$A = \frac{1}{2}(0.02)^2$$

$$A = 2 * 10^{-4}\, m^2 = 2\, cm^2$$



1. Triangle 1,2,3:
   Node 1: $(x_1, y_1) = (0.00, 0.02)$
   Node 2: $(x_2, y_2) = (0.00, 0.00)$
   Node 3: $(x_3, y_3) = (0.02, 0.00)$

   Calculation of alphas (values in cm):

$$\alpha_1(x_1, y_1) = \frac{1}{2A}[(x_2 y_3 - x_3 y_2) + x(y_2 - y_3) + y(x_3 - x_2)]$$

$$\boldsymbol{\alpha_1(x_1, y_1)} = 0.25\,(0x + 2y)$$

$$\alpha_2(x_2, y_2) = \frac{1}{2A}[(x_3 y_1 - x_1 y_3) + x(y_3 - y_1) + y(x_1 - x_3)]$$

$$\boldsymbol{\alpha_2(x_2, y_2)} = 0.25\,(4 - 2x - 2y)$$

$$\alpha_3(x_3, y_3) = \frac{1}{2A}[(x_1 y_2 - x_3 y_1) + x(y_1 - y_2) + y(x_2 - x_1)]$$

$$\boldsymbol{\alpha_3(x_3, y_3)} = 0.25\,(-4 + 2x - 0y)$$

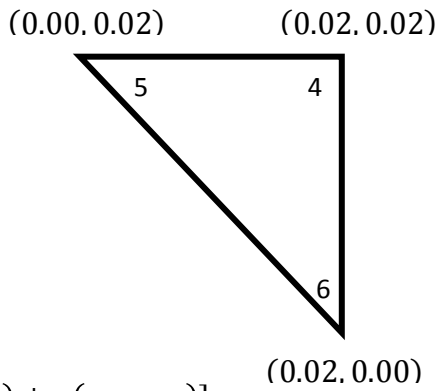
$$\boldsymbol{S_{i,j} = \int \nabla \alpha_i \cdot \nabla \alpha_j \; dS}$$

$$\boldsymbol{S_{i,j} = \frac{1}{4A} \nabla(\alpha_i \cdot \alpha_j)}$$

Using the above equation (which matches the general form above), the following local S – matrix was generated:

$$S^{(1,2,3)} = \begin{bmatrix} 0.5 & -0.5 & 0 \\ -0.5 & 1 & -0.5 \\ 0 & -0.5 & 0.5 \end{bmatrix}$$

2. Triangle (4,5,6):



Node 4: $(x_4, y_4) = (0.02, 0.02)$
Node 5: $(x_5, y_5) = (0.00, 0.02)$
Node 6: $(x_6, y_6) = (0.02, 0.00)$

Calculation of alphas (values in cm):

$$\alpha_4(x_4, y_4) = \frac{1}{2A}[(x_6y_5 - x_5y_6) + x(y_5 - y_5) + y(x_6 - x_5)]$$

$$\boldsymbol{\alpha_4(x_4, y_4)} = 0.25\,(4 + 2x + 2y)$$

$$\alpha_5(x_5, y_5) = \frac{1}{2A}[(x_4y_6 - x_6y_4) + x(y_6 - y_4) + y(x_4 - x_5)]$$

$$\boldsymbol{\alpha_5(x_5, y_5)} = 0.25\,(-4 - 2x + 0y)$$

$$\alpha_6(x_6, y_6) = \frac{1}{2A}[(x_5y_4 - x_4y_5) + x(y_4 - y_5) + y(x_5 - x_4)]$$

$$\boldsymbol{\alpha_6(x_6, y_6)} = 0.25\,(-4 + 0x - 2y)$$

$$\boldsymbol{S_{i,j} = \int \nabla\alpha_i \cdot \nabla\alpha_j \; dS}$$

$$\boldsymbol{S_{i,j} = \frac{1}{4A}\nabla(\alpha_i \cdot \alpha_j)}$$

Using the above equation (which matches the general form above), the following local S – matrix was generated:

$$S^{(4,5,6)} = \begin{bmatrix} 1 & -0.5 & -0.5 \\ -0.5 & 0.5 & 0 \\ -0.5 & 0 & 0.5 \end{bmatrix}$$

Global Matrix:

In order to find the Global Matrix, we use the following equation:

$$\boldsymbol{S = C^T S_{dis} C}$$

Where C is a conjoint numbering matrix and S_{dis} is a 6x6 matrix created by combining the two local S-matrices in the following manner:

$$S_{dis} = \begin{bmatrix} S^{1,2,3} & 0 \\ 0 & S^{4,5,6} \end{bmatrix}$$

Since each triangle has 3 nodes each separately, and when conjoint, there are 4 nodes (nodes 1 and 5 are common, and nodes 3 and 6 are common), C will be a 6x4 matrix:

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad C^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

And S_{dis}, a combination of the two local S-matrices, will be 6x6:

$$S_{dis} = \begin{bmatrix} 0.5 & -0.5 & 0 & & & \\ -0.5 & 1 & -0.5 & & 0 & \\ 0 & -0.5 & 0.5 & & & \\ & & & 1 & -0.5 & -0.5 \\ & 0 & & -0.5 & 0.5 & 0 \\ & & & -0.5 & 0 & 0.5 \end{bmatrix}$$

$$\boldsymbol{S = C^T S_{dis} C}$$

$$S = \begin{bmatrix} 1 & -0.5 & 0 & -0.5 \\ -0.5 & 1 & -0.5 & 0 \\ 0 & -0.5 & 1 & -0.5 \\ -0.5 & 0 & -0.5 & 1 \end{bmatrix}$$

Question 2

Figure 2 shows the cross-section of an electrostatic problem with translational symmetry: a rectangular coaxial cable. The inner conductor is held at 110 volts and the outer conductor is grounded.

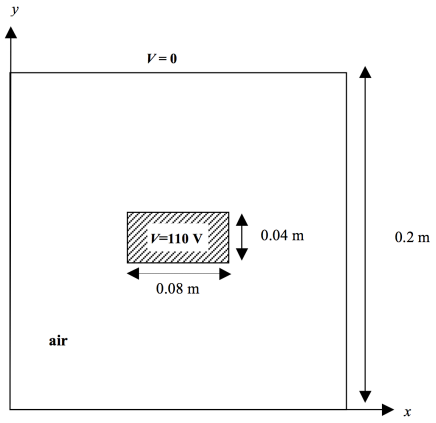
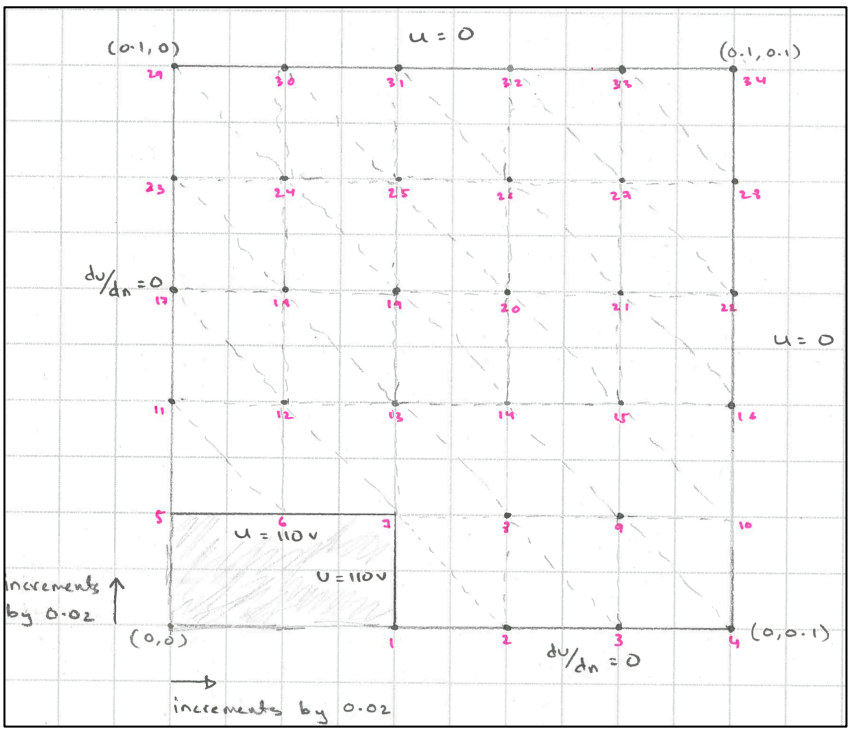


Figure 2.

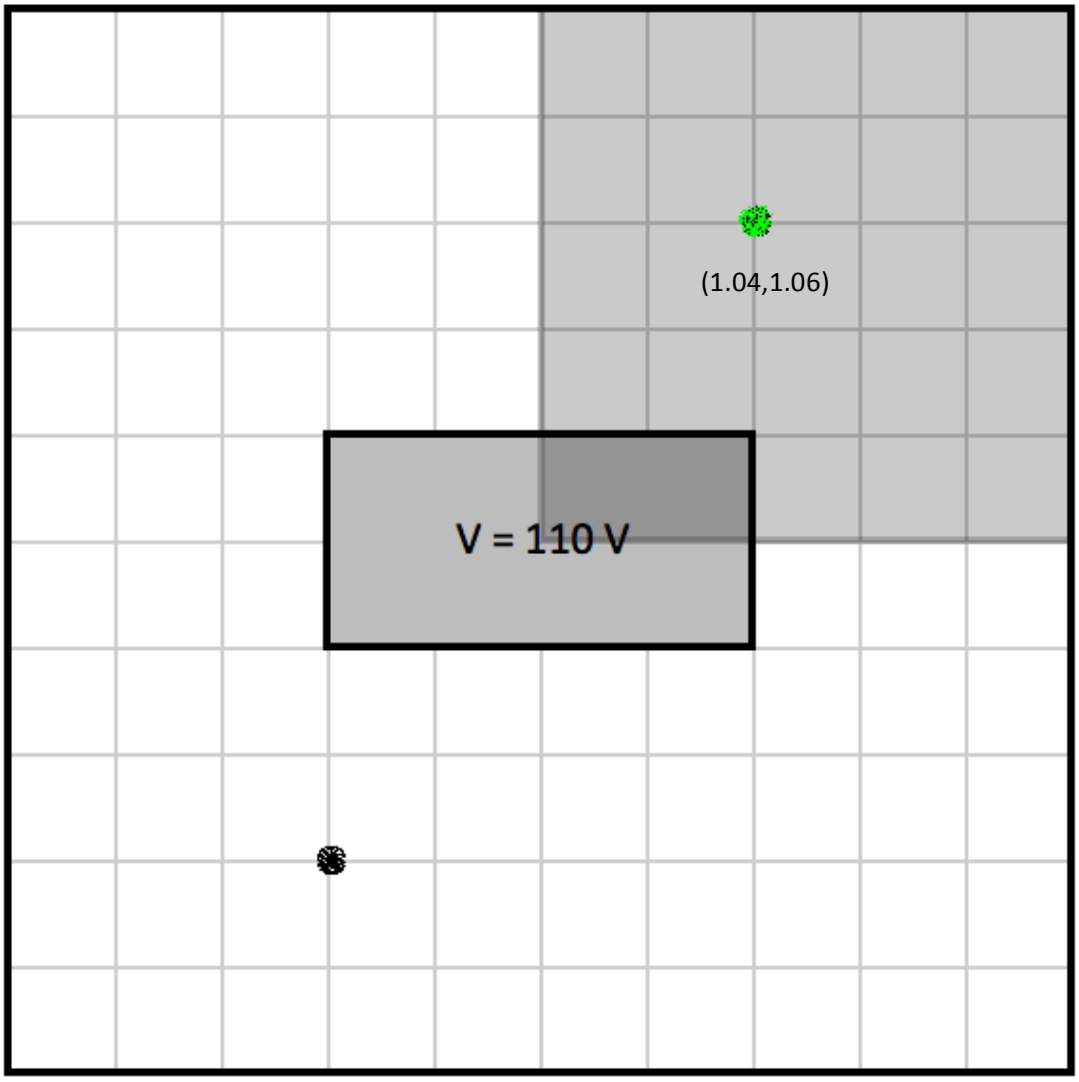
a) Use the two-element mesh shown in Figure 1(b) as a "building block" to construct a finite element meshes for one-quarter of the cross-section of the coaxial cable. Specify the mesh, including boundary conditions, in an input file following the format for the SIMPLE2D program as explained in the course notes.

Using the example provided in page 20 of the FE1stOrder lecture, the python code genFile4Simple2D.py was created to produce the input file for the simple2D program. This code generated the file fileForSimple2D.txt, and it indeed had 46 elements. Both the code and the input file are included in the appendix.
The top right quadrant was used cross-section of the coaxial cable was used. The following picture shows the node numbering of the triangles:



b) Use the SIMPLE2D program with the mesh from part (a) to compute the electrostatic potential solution. Determine the potential at (x, y) = (0.06, 0.04) from the data in the output file of the program.

The input file was fed into the Matlab version of the Simple2D program, and it generated an output file, similar to the one described in page 22 of the FE1stOrder lecture. The output file is included in the Appendix. The diagram below shows the cable. We were asked to compute the potential at (x, y) = (0.06,0.04). This point is marked in the black dot. Since the top right quadrant (grey are in the diagram) was used, the equivalent potential is located at (x, y) = (1.04,1.06) (if we only consider the quadrant, (x, y) = (0.04,0.06)) or node 19, which is marked in the green dot:



The potential at (x,y) = (0.06,0.04) was found to be **40.5265V**

c) Compute the capacitance per unit length of the system using the solution obtained from SIMPLE2D.

Energy is a capacitor is given by:

$$W = \frac{1}{2}\frac{Q^2}{C}$$

Which can also be written as follows:

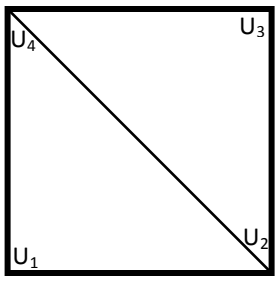
$$W = \frac{1}{2}\frac{CV^2}{\varepsilon_o}$$

Energy in a potential field is given by:

$$W = \frac{1}{2}U_{con}^T * S * U_{con}$$

Where S is from Question 1, U_{con} is a 1-D matrix with potential values from the four corners of the square from Figure 1(b), as shown below:

$U_{con} = [U_1, U_2, U_3, U_4]$



Equating the two energy equations from above, gives us the capacitance per length, (confirmed by [1]):

$$C = \frac{\varepsilon_o * (U_{con}^T * S * U_{con})}{V^2}$$

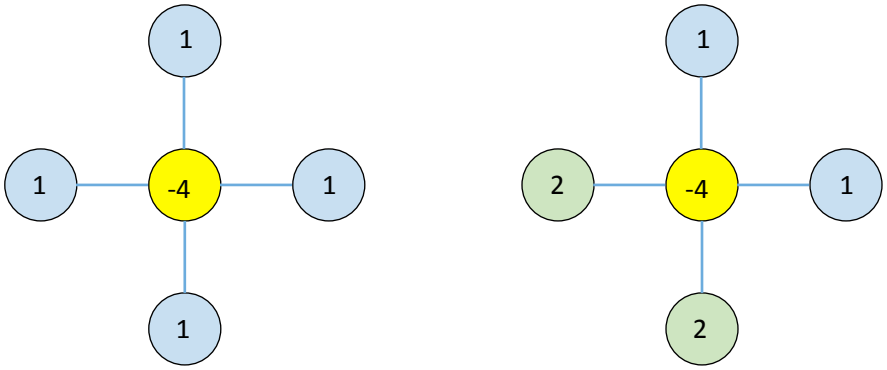
The file findCapacitance.py, calculates the capacitance per length, using the equation given above. Matrix multiplication and transpose operations were done using functions that were created in the basicDefinitions.py from Assignment 1 (both included in Appendix).

The Capacitance per length of the cable is: **5.2137e-11 F/m**

Question 3

Write a program implementing the conjugate gradient method (un-preconditioned). Solve the matrix equation corresponding to a finite difference node-spacing, h = 0.02m in x and y directions for the same one-quarter cross-section of the system shown in Figure 2 that considered in Question 2 above. Use a starting solution of zero.

The code for conjugate difference method is written in conjGradMethod.py. The Choleski decomposition is the same as the previous assignment, and can be found in basicDefinitions.py
The mesh is created similarly as the one from assignment 1, in q_3Functions.py.
The A and b matrices were also generated in conjGradMethod.py. There were a total of 19 free nodes, so A is a 19 x 19 matrix, and b is a column vector with 19 elements.
A was computed as outlined in the lecture: FD_LectureNotes.
Generating A:
4 nodes, as shown below, surround each free node:



Current node is the middle node (in yellow). A[currentNode][currentNode] = -4.0
Node Up, Node Down, Node Left, Node Right corresponds to the neighbouring nodes. The figure on the left shows free nodes are do not neighbour any bordering nodes. For these, A[currentNode][nodeUp] = A[currentNode][nodeDown] = A[currentNode][nodeLeft] = A[currentNode][nodeRight] = 1.0
For the nodes that do neighbour bordering nodes (below and to the right only), refer to the diagram on the right. For these, A[currentNode][nodeUp] and A[currentNode][nodeRight] are 2.0, where applicable.
The b coloumn vector has a voltage value of -110.0 for the nodes that border the inner cable, and zero otherwise.
All in all, the A and b matricies are shown below:

```
b = [-110,0,0,-110,0,0,-110,0,0,-110,-110,0,0,0,0,0,0,0,0]
```

```
A = [[-4, 1, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
     [1, -4, 1, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,],
     [0, 1, -4, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,],
     [1, 0, 0, -4, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,],
     [0, 1, 0, 1, -4, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,],
     [0, 0, 1, 0, 1, -4, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,],
     [0, 0, 0, 1, 0, 0, -4, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0,],
     [0, 0, 0, 0, 1, 0, 1, -4, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0,],
     [0, 0, 0, 0, 0, 1, 0, 1, -4, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0,],
     [0, 0, 0, 0, 0, 0, 0, 0, 0, -4, 2, 0, 0, 0, 1, 0, 0, 0, 0, 0,],
     [0, 0, 0, 0, 0, 0, 0, 0, 0, 1, -4, 1, 0, 0, 0, 1, 0, 0, 0, 0,],
     [0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, -4, 1, 0, 0, 0, 1, 0, 0, 0,],
     [0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, -4, 1, 0, 0, 0, 1, 0, 0,],
     [0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, -4, 0, 0, 0, 0, 1, 0,],
     [0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, -4, 2, 0, 0, 0, 0,],
     [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, -4, 1, 0, 0, 0,],
     [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, -4, 1, 0, 0,],
     [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, -4, 1, 0,],
     [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, -4, 1,],
     [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, -4,]]
```

a) Test your matrix using your Choleski decomposition program that you wrote for Question 1 of Assignment 1 to ensure that it is positive definite. If it is not, suggest how you could modify the matrix equation in order to use the conjugate gradient method for this problem.

The Matrix A is not Positive definite. This is because of the shared boundaries on both the x and y-axis. A visual check and an online decomposition was used to double check to make sure it indeed isn't positive definite, as shown below:

Input matrix:

```
-4.000  1.000  0.000  2.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000
 1.000 -4.000  1.000  0.000  2.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000
 0.000  1.000 -4.000  0.000  0.000  2.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000
 1.000  0.000  0.000 -4.000  1.000  0.000  1.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000
 0.000  1.000  0.000  1.000 -4.000  1.000  0.000  1.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000
 0.000  0.000  1.000  0.000  1.000 -4.000  0.000  0.000  1.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000
 0.000  0.000  0.000  1.000  0.000  0.000 -4.000  1.000  0.000  0.000  0.000  1.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000
 0.000  0.000  0.000  0.000  1.000  0.000  1.000 -4.000  1.000  0.000  0.000  0.000  1.000  0.000  0.000  0.000  0.000  0.000  0.000
 0.000  0.000  0.000  0.000  0.000  1.000  0.000  1.000 -4.000  0.000  0.000  0.000  0.000  1.000  0.000  0.000  0.000  0.000  0.000
 0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000 -4.000  2.000  0.000  0.000  0.000  1.000  0.000  0.000  0.000  0.000
 0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  1.000 -4.000  1.000  0.000  0.000  0.000  1.000  0.000  0.000  0.000
 0.000  0.000  0.000  0.000  0.000  0.000  1.000  0.000  0.000  0.000  1.000 -4.000  1.000  0.000  0.000  0.000  1.000  0.000  0.000
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 0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  1.000  0.000  0.000  0.000  1.000 -4.000  0.000  0.000  0.000  0.000  1.000
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 0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  1.000  0.000  0.000  0.000  1.000 -4.000  1.000
 0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000  1.000  0.000  0.000  0.000  1.000 -4.000
```

Cholesky Decomposition:

```
Not a symmetric positive definite matrix.
```

One way to rectify this issue is to multiply both A and b by A^T, transforming the equation to:
$A^T * A * x = A^T * b$
This creates a positive definite matrix, which can be decomposed

b) Once you have modified the problem so that the matrix is positive definite, solve the matrix equation first using the Choleski decomposition program from Assignment 1, and then the conjugate gradient program written for this assignment.

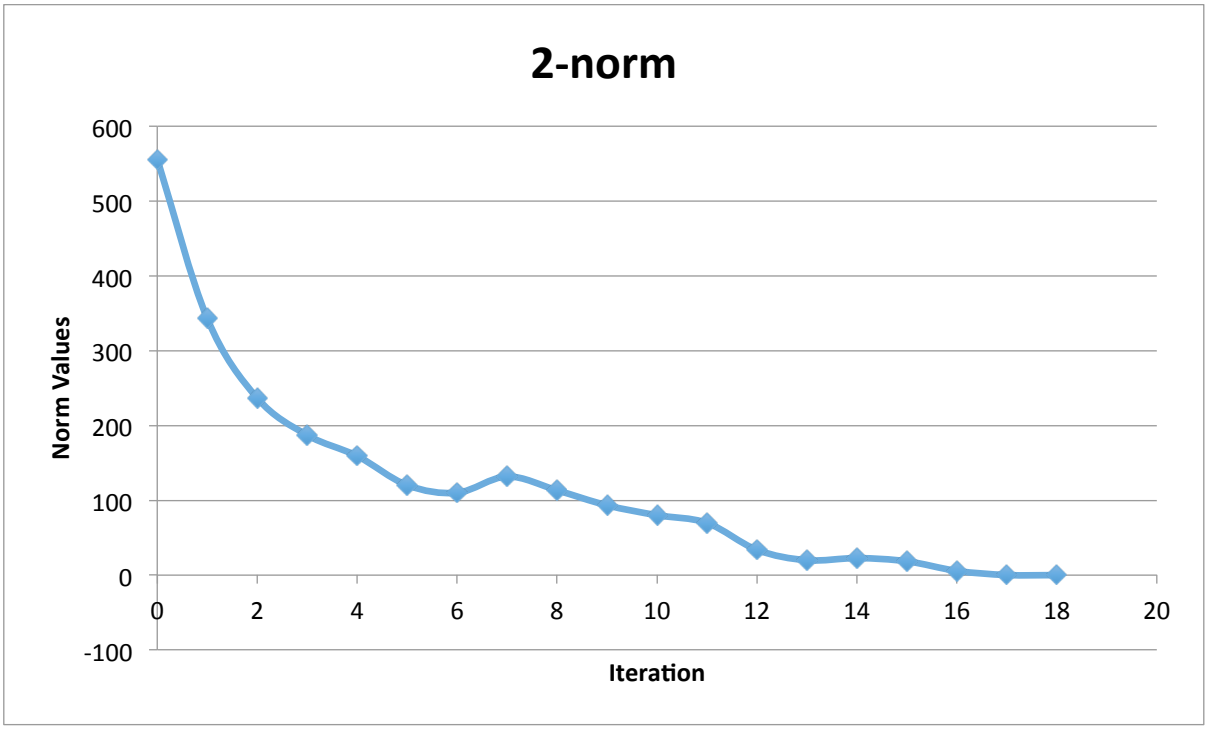
The code for both the decomposition and the Conjugate Gradient Method can be found in conjGradMethod.py. Multiplication and addition functions are in basicDefinitions.py.

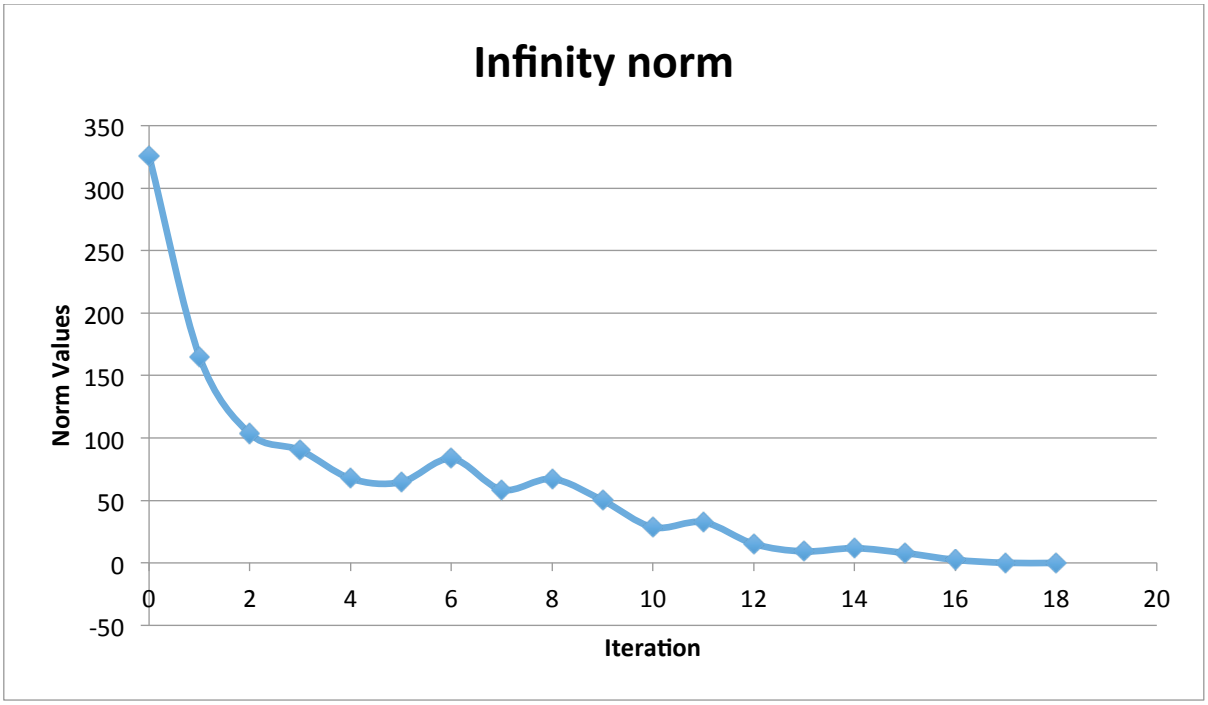
c) Plot a graph of the infinity norm and the 2-norm of the residual vector versus the number of iterations for the conjugate program.

The norm values are calculated in conjGradMethod.py:

Iteration	2-norm	Iteration	Infinity norm
0	555.1319626	0	325.8732212
1	343.0812586	1	165.2642705
2	236.703743	2	103.465448
3	187.1606422	3	90.15753512
4	159.2824468	4	67.53607946
5	120.256896	5	64.62750605
6	110.1450255	6	83.36937733
7	131.7943728	7	58.57329838
8	113.3462218	8	67.17612169
9	93.30339828	9	50.07314807
10	80.07526259	10	28.55090197
11	69.76736927	11	32.57103069
12	33.7521252	12	15.21885024
13	19.89542491	13	9.18304894
14	22.75210701	14	11.78157676
15	18.51914185	15	7.903589879
16	5.653268685	16	2.47321541
17	0.153755093	17	0.065569253
18	1.24E-05	18	5.14E-06

The graphs are shown below:


2-norm
600
500
400
300
200
100
0
-100
Norm Values
0
2
4
6
8
10
12
14
16
18
20
Iteration


Infinity norm
350
300
250
200
150
100
50
0
-50
Norm Values
0
2
4
6
8
10
12
14
16
18
20
Iteration

d) What is the potential at (x,y) = (0.06, 0.04), using the Choleski decomposition and the conjugate gradient programs, and how do they compare with the value you computed in Question 2(b) above. How do they compare with the value at the same (x,y) location and for the same node spacing that you computed in Assignment 1 using SOR.

After using the decomposition to solve the equation, the voltage value we received is: **40.5265V.**
Running the conjugate gradient program, we get the same value: **40.5265V.**
This value matches the values calculated in question 2 of this assignment above, and that from Assignment 1.
All the potential values for the nodes for both Choleski and Conjugate Gradient method have been included in the Appendix.

e) Suggest how you could compute the capacitance per unit length of the system from the finite difference solution.

After we compute A and b using finite difference method, we can compute the potential at each node using either Choleski or Conjugate Gradient. And once we have the voltages at each of the node, we can simply use the same equation used in question 2 in the findCapacitance.py to calculate the capacitance per length.

References

[1] Whiteman, J. R. (1973). The mathematics of finite elements and applications: Proceedings of the Brunel University conference of the Institute of Mathematics and Its Applications held in April 1972. London: Academic Press.

Appendix

```
#!/usr/bin/env python2

# -*- coding: utf-8 -*-

"""

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ECSE 543

Assignment 2

genFile4Simple2D.py

Creates the input file for the Simple2d_M program

"""

output = open("fileForSimple2D.txt", 'w') #opens the file

#All code below will go in here

#Also has print statements, so we can see in the IDE if proper numbers were printed

horNodes = 6

verNodes = 5

pot = 110.0

botBound = [(x+1) for x in range (horNodes - 2)]

mesh = [[((x + 5) + y * horNodes) for x in range(horNodes)] for y in range(verNodes)]

################################################################################
####

# First part of input

# Node -- xVal -- yVal

for i in range (horNodes - 2):
```

```
    print("{:<3} {:>6} {:>6}\n".format(botBound[i], (i + 2) * 0.02, i * 0.00))
    output.write("{:<3} {:>6} {:>6}\n".format(botBound[i], (i + 2) * 0.02, i * 0.00))
for y in range (verNodes):
    for x in range (horNodes):
        print("{:<3} {:>6} {:>6}\n".format(mesh[y][x], x * 0.02, (y + 1) * 0.02))
        output.write("{:<3} {:>6} {:>6}\n".format(mesh[y][x], x * 0.02, (y + 1) * 0.02))
##########################################################################################
####
print("\n")
output.write("\n")
##########################################################################################
####
#Second part of the input
# Node1 -- Node2 -- Node3 -- 0.000
for i in range (horNodes - 3):
    print("{:<3} {:<3} {:<3} {}\n".format(botBound[i],botBound[i + 1],mesh[0][i + 2],'0.000'))
    print("{:<3} {:<3} {:<3} {}\n".format(botBound[i + 1],mesh[0][i + 3],mesh[0][i + 2],'0.000'))
    output.write("{:<3} {:<3} {:<3} {}\n".format(botBound[i],botBound[i + 1],mesh[0][i + 2],'0.000'))
    output.write("{:<3} {:<3} {:<3} {}\n".format(botBound[i + 1],mesh[0][i + 3],mesh[0][i + 2],'0.000'))
for y in range (verNodes - 1):
    for x in range (horNodes - 1):
        print("{:<3} {:<3} {:<3} {:>6}\n".format(mesh[y][x],mesh[y][x + 1],mesh[y + 1][x],'0.000'))
        print("{:<3} {:<3} {:<3} {:>6}\n".format(mesh[y][x + 1],mesh[y + 1][x + 1],mesh[y + 1][x],'0.000'))
        output.write("{:<3} {:<3} {:<3} {:>6}\n".format(mesh[y][x],mesh[y][x + 1],mesh[y + 1][x],'0.000'))
        output.write("{:<3} {:<3} {:<3} {:>6}\n".format(mesh[y][x + 1],mesh[y + 1][x + 1],mesh[y +
1][x],'0.000'))
##########################################################################################
###
print("\n")
output.write("\n")
```

```
##########################################################################################
####

#Third part of the input

#Boundary Conditions

#Boundary Node -- Boundary Conditions

#Dirchlet Condition:

print ("{:<3} {}\n".format(1,pot))

output.write ("{:<3} {}\n".format(1,pot))

for i in range (horNodes):

   if (mesh[0][i] < 8):

      print ("{:<3} {}\n".format(mesh[0][i],pot))

      output.write ("{:<3} {}\n".format(mesh[0][i],pot))

for i in range (horNodes):

   print ("{:<3} {}\n".format(mesh[verNodes-1][i],'0.000'))

   output.write ("{:<3} {}\n".format(mesh[verNodes-1][i],'0.000'))

for i in range (verNodes - 1):

   print ("{:<3} {}\n".format(mesh[i][horNodes - 1],'0.000'))

   output.write ("{:<3} {}\n".format(mesh[i][horNodes - 1],'0.000'))

##########################################################################################

output.close() #closes the file
```

fileForSimple2D.txt

Input 1

1 0.04 0.0
2 0.06 0.0
3 0.08 0.0
4 0.1 0.0
5 0.0 0.02
6 0.02 0.02
7 0.04 0.02
8 0.06 0.02
9 0.08 0.02
10 0.1 0.02
11 0.0 0.04
12 0.02 0.04
13 0.04 0.04
14 0.06 0.04
15 0.08 0.04
16 0.1 0.04
17 0.0 0.06
18 0.02 0.06
19 0.04 0.06
20 0.06 0.06
21 0.08 0.06
22 0.1 0.06
23 0.0 0.08
24 0.02 0.08
25 0.04 0.08
26 0.06 0.08
27 0.08 0.08
28 0.1 0.08
29 0.0 0.1
30 0.02 0.1
31 0.04 0.1
32 0.06 0.1
33 0.08 0.1
34 0.1 0.1

Input 2

1 2 7 0.000
2 8 7 0.000
2 3 8 0.000
3 9 8 0.000
3 4 9 0.000
4 10 9 0.000
5 6 11 0.000
6 12 11 0.000
6 7 12 0.000
7 13 12 0.000
7 8 13 0.000
8 14 13 0.000
8 9 14 0.000
9 15 14 0.000
9 10 15 0.000
10 16 15 0.000
11 12 17 0.000
12 18 17 0.000
12 13 18 0.000
13 19 18 0.000
13 14 19 0.000
14 20 19 0.000
14 15 20 0.000
15 21 20 0.000
15 16 21 0.000
16 22 21 0.000
17 18 23 0.000
18 24 23 0.000
18 19 24 0.000
19 25 24 0.000
19 20 25 0.000
20 26 25 0.000
20 21 26 0.000
21 27 26 0.000
21 22 27 0.000
22 28 27 0.000
23 24 29 0.000
24 30 29 0.000
24 25 30 0.000
25 31 30 0.000
25 26 31 0.000
26 32 31 0.000
26 27 32 0.000
27 33 32 0.000
27 28 33 0.000
28 34 33 0.000

Input 3

1 110.0
5 110.0
6 110.0
7 110.0
29 0.000
30 0.000
31 0.000
32 0.000
33 0.000
34 0.000
10 0.000
16 0.000
22 0.000
28 0.000
4 0.000

Outfile.txt

Potential =

1.0000	0.0400	0	110.0000
2.0000	0.0600	0	66.6737
3.0000	0.0800	0	31.1849
4.0000	0.1000	0	0
5.0000	0	0.0200	110.0000
6.0000	0.0200	0.0200	110.0000
7.0000	0.0400	0.0200	110.0000
8.0000	0.0600	0.0200	62.7550
9.0000	0.0800	0.0200	29.0330
10.0000	0.1000	0.0200	0
11.0000	0	0.0400	77.3592
12.0000	0.0200	0.0400	75.4690
13.0000	0.0400	0.0400	67.8272
14.0000	0.0600	0.0400	45.3132
15.0000	0.0800	0.0400	22.1921
16.0000	0.1000	0.0400	0
17.0000	0	0.0600	48.4989
18.0000	0.0200	0.0600	46.6897
19.0000	**0.0400**	**0.0600**	**40.5265**
20.0000	0.0600	0.0600	28.4785
21.0000	0.0800	0.0600	14.4223
22.0000	0.1000	0.0600	0
23.0000	0	0.0800	23.2569
24.0000	0.0200	0.0800	22.2643
25.0000	0.0400	0.0800	19.1107
26.0000	0.0600	0.0800	13.6519
27.0000	0.0800	0.0800	7.0186
28.0000	0.1000	0.0800	0
29.0000	0	0.1000	0
30.0000	0.0200	0.1000	0
31.0000	0.0400	0.1000	0
32.0000	0.0600	0.1000	0
33.0000	0.0800	0.1000	0
34.0000	0.1000	0.1000	0

```
#!/usr/bin/env python2
# -*- coding: utf-8 -*-
"""
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ECSE 543
Assignment 2
```

findCapacitance.py

```
Computes the Capacitance per legnth of the cable
"""
#Function Imports
import numpy as np #numpy, for reading the output file
from basicDefinitions import matTranspose, matrixMult #multiplication and transpose functions
#variable declaration
totalEnergy = 0.0
nodeHeight = 6
nodeWidth = 6
pot = 110.0
UCon = [0 for x in range (4)]
#S, which was calculated in Q1
S = [[1.0,-0.5,0.0,-0.5],
    [-0.5,1.0,-0.5,0.0],
    [0.0,-0.5,1.0,-0.5],
    [-0.5,0.0,-0.5,1.0]]
################################################################################
#Create the Mesh of voltages from the output file of Simple2d_m
mesh = [[0 for x in range (nodeHeight)] for y in range(nodeWidth)]
values = np.loadtxt('outfile.txt', dtype = float, skiprows = 3, unpack= False)
for i in range (len(values)):
```

```
    xNode = int(values[i][1]/0.02)
    yNode = int(values[i][2]/0.02)
    mesh[yNode][xNode] = values[i][3]
#Voltage of the inner conductor
mesh[0][0] = pot
mesh[0][1] = pot
################################################################################
#Capacitance per lenght calculator
#c = eplison0*|divU|^2/V^2
#Total energy: |divU|^2 = Ucon^T * S * Ucon:
for y in range (nodeHeight - 1):
    for x in range(nodeWidth - 1):
        UCon[0] = mesh[y][x]
        UCon[1] = mesh[y][x + 1]
        UCon[2] = mesh[y + 1][x + 1]
        UCon[3] = mesh[y + 1][x]
        UConT = matTranspose(UCon)
        potential = matrixMult(matrixMult(UCon,S),UConT)
        totalEnergy += potential[0][0]
#Calculates the Capacitance per length
#multiplies by 4, to include all 4 quadrants
epsilon = 8.854187817620e-12
Vsquared = pot*pot
capPerLen = totalEnergy* epsilon * 4 / Vsquared
print (str(capPerLen) + ' F/m')
```

```
# -*- coding: utf-8 -*-
"""
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260519664
ECSE 543
Assignment 2
```

basicDefinition.py

```
A list of functions, used for this assignment
Nov 14th, 2016
"""
import math
import numpy as np
import csv


##############################################################################################
#Function that checks if a matrix is 1D or 2D
def is1Dor2D (A):
    while True:
        try:
            length = len(A[0]) #If true, A is 2D
            return A
            break
        except TypeError: #else A is 1D
            return [A]
            break
##############################################################################################
#function that creates floats from lists
def list2float(A):
    length = len(A)
```

```
    floatA = [0 for x in range(length)]
    for i in range (length):
        numVal = ''
        stringVal = list(str(A[i]))
        for j in range (1,len(stringVal)-1,1):
            numVal = numVal + stringVal[j]
        floatVal = float(numVal)
        floatA [i] = floatVal
    return floatA
##############################################################################################
#Function that transposes a Matrix
def matTranspose(A):
    A = is1Dor2D(A)
    rowsA = len(A)
    colsA = len(A[0])
    C = [[0 for rows in range (rowsA)] for cols in range(colsA)]
    for i in range (colsA):
        for j in range (rowsA):
            C[i][j] = A[j][i]
    return C
##############################################################################################
#Function that adds or subtracts two matricies
def matrixAddorSub(A,B,operation):
    A = is1Dor2D(A)
    B = is1Dor2D(B)
    rowsA = len(A)
    colsA = len(A[0])
    rowsB = len(B)
    colsB = len(B[0])
```

```
    if (rowsA == rowsB and colsA == colsB):
        C = [[0 for row in range(colsA)] for col in range(rowsA)]
        if (operation == 'a'):
            for i in range (rowsA):
                for j in range (colsA):
                    C[i][j] = A[i][j]+B[i][j]
        elif (operation == 's'):
            for i in range (rowsA):
                for j in range (colsA):
                    C[i][j] = A[i][j]-B[i][j]
    return C
##########################################################################################
#function for scalar matrix multiplication
def scalarMult(scalar, A):
    newMat = [0 for x in range (len(A))]
    for i in range (len(A)):
        newMat[i] = scalar*A[i][0]
    return newMat
##########################################################################################
#Function that multiplies two matricies
def matrixMult (A, B):
    A = is1Dor2D(A)
    B = is1Dor2D(B)
    rowsA = len(A)
    colsA = len(A[0])
    rowsB = len(B)
    colsB = len(B[0])
    if (rowsA == colsB or colsA == rowsB):
        C = [[0 for row in range(colsB)] for col in range(rowsA)]
```

```
        for i in range(rowsA):
            for j in range(colsB):
                for k in range(colsA):
                # Create the result matrix
                # Dimensions would be rows_A x cols_B
                    C[i][j] += A[i][k] * B[k][j]
    else:
      print ("Cannot multiply the two matrices. Incorrect dimensions.")
      return
    return C
##########################################################################################
#Function that creates a diagonal matrix
def diogMat (A):
    length = len(A)
    floatA = [0 for x in range(length)]
    for i in range (length):
        numVal = ''
        stringVal = list(str(A[i]))
        for j in range (1,len(stringVal)-1,1):
            numVal = numVal + stringVal[j]
        floatVal = float(numVal)
        floatA [i] = floatVal
    diognalMatrix = [[0 for x in range(length)] for y in range(length)]
    for i in range (length):
        diognalMatrix[i][i] = 1/floatA[i]
    return diognalMatrix
##########################################################################################
```

```
#Function that performs the Cholesky decomposition and returns L
def cholesky(A,length):
    global sum
    L = [[0 for x in range(length)] for y in range(length)]
    for i in range(length):
        for k in range(i + 1):
            sum = 0
            for j in range(k):
                sum += L[i][j] * L[k][j]
            if (i == k):
                L[i][k] = math.sqrt(abs(A[i][i] - sum))
            else:
                L[i][k] = (A[i][k] - sum) / L[k][k]
    return L
##########################################################################################
#Function that solves y in Ly = b
def forwElim (L,b,length):
    b = list2float(b)
    global sum
    y = [0 for x in range (length)]
    for i in range (length):
        sum = 0.00
        for j in range (i):
            sum += L[i][j] * y[j]
        y[i] = (b[i]-sum)/L[i][i]
    return y
##########################################################################################
#Function that solves for x in L^Tx = y
def backSub (L,y,length):
```

```
    global sum
    X = [0 for x in range(length)]
    for i in range (length - 1, -1, -1):
        sum = 0
        for j in range (i + 1, length, 1):
            sum += L[j][i] * X[j]
        X[i] = (y[i] - sum) / L[i][i]
    return X
```

```
################################################################################################
#!/usr/bin/env python2
# -*- coding: utf-8 -*-
"""
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ECSE 543
Assignment 2
```

conjGradMethod.py

```
Calculates the conjugate gradient for the cable
Nov 14th, 2016
"""
import numpy as np
from basicDefinitions import matrixMult, matTranspose, cholesky, forwElim, backSub, matrixAddorSub,
scalarMult
from q_3Functions import genMesh
import math


def genAb(h,freeNode):


cableHeight = 0.1
```

```
    cableWidth = 0.1
    coreHeight = 0.02
    coreWidth = 0.04
    corePot = 110.0
    mesh = genMesh(0.02)

    nodeHeight = (int)(cableHeight/h + 1)
    nodeWidth = (int)(cableWidth/h + 1)
    coreHeightNode = (int)(coreHeight/h + 1)
    codeWidthNode = (int)(coreWidth/h + 1)
    freeNode = (nodeHeight * nodeWidth) - (coreHeightNode * codeWidthNode) - (nodeHeight +
nodeWidth) + 1

    A = [[0 for x in range (freeNode)] for y in range (freeNode)]
    b = [0 for x in range(freeNode)]

    for y in range (nodeHeight):
        for x in range (nodeWidth):
            if (mesh[y][x][0] != None):
                cord = mesh[y][x][0]
                cordRight = mesh[y][x + 1][0]
                cordLeft = mesh[y][x - 1][0]
                cordUp = mesh[y + 1][x][0]
                cordDown = mesh[y - 1][x][0]
                A[cord][cord] = -4.0
                #print (cord, cordLeft, cordRight, cordUp, cordDown)
                #right
                if (cordRight != None):
                    A[cord][cordRight] = 1
```

```
            if (cord == 0):
                A[cord][cordRight] = 2
                b[cord] = -corePot
        #left
        if (cordLeft != None):
            A[cord][cordLeft] = 1
            if (cord == 1):
                A[cord][cordLeft] = 2
                b[cord + 1] = -corePot
            if (cordLeft == 4 or cordLeft == 9 or cordLeft == 14):
                A[cord][cordLeft] = 2
        #up
        if (cordUp != None):
            A[cord][cordUp] = 1
            if (cord == 4 or cord == 9):
                A[cord][cordUp] = 2
                if (cord == 4):
                    b[cord] = -corePot
                    b[cord + 1] = -corePot
                    b[cord + 2] = -corePot
        #down
        if (cordDown != None):
            A[cord][cordDown] = 1
            if (cord == 14 or cord == 9):
                A[cord][cordDown] = 2
            if (cordDown == 0 or cordDown == 1):
                A[cord][cordDown] = 2

return (A,b)
```

```
##################################################
#Choleski
def Cholesky (A, b):
   AT = matTranspose(A)
   b = matTranspose(b)
   newA = matrixMult(AT,A)
   newb = matrixMult(AT,b)
   length = 19#len(newA)
   L = cholesky(newA,length)
   y = forwElim (L, newb, length)
   X = backSub (L, y, length)
   print ('Choleski: ')
   print X
##################################################
#Conjugate Gradient
def conjGrad(A,b,freeNode):
   AT = matTranspose(A)
   b = matTranspose(b)
   newA = matrixMult(AT,A)
   newb = matrixMult(AT,b)
   X = [0 for x in range (freeNode)]
   X = matTranspose(X)
   r = matrixAddorSub(newb, matrixMult(newA,X),'s')
   p = r

   for i in range(freeNode):
      alpha =
float(matrixMult(matTranspose(p),r)[0][0])/float((matrixMult(matrixMult(matTranspose(p),newA),p))[0]
[0])
```

```
    X = matrixAddorSub(X, matTranspose(scalarMult(alpha,p)),'a')

    r = matrixAddorSub(newb, matrixMult(newA,X),'s')

    beta = (-
1)*float(matrixMult((matrixMult(matTranspose(p),newA)),r)[0][0])/float((matrixMult(matrixMult(matTr
anspose(p),newA),p))[0][0])

    p = matrixAddorSub(r, matTranspose(scalarMult(beta,p)),'a')


    #finding the norms

    infNorm = 0

    twoNorm = 0

    for j in range (freeNode):

      value = abs(r[j][0])

      if (value > infNorm):

        infNorm = value

      twoNorm += r[j][0]**2

    twoNorm = math.sqrt(twoNorm)

    print (i,twoNorm,infNorm)

  print X

#####################################################

cableHeight = 0.1

cableWidth = 0.1

coreHeight = 0.02

coreWidth = 0.04

corePot = 110.0


h = 0.02

nodeHeight = (int)(cableHeight/h + 1)

nodeWidth = (int)(cableWidth/h + 1)

coreHeightNode = (int)(coreHeight/h + 1)
```

```
codeWidthNode = (int)(coreWidth/h + 1)

freeNode = (nodeHeight * nodeWidth) - (coreHeightNode * codeWidthNode) - (nodeHeight + nodeWidth) + 1

 #Creates A and b

(A,b) = genAb(0.02,freeNode)

#Cholesky(A,b)

conjGrad (A,b,freeNode)
```

RESULT FROM CHOLESKI

[77.35922364524424,
48.498858387154726,
23.256867476258822,
75.46901809691109,
46.68967121355792,
22.264305758940274,
67.82717752884203,
40.52650261122562,
19.110684345944378,
66.67372442302747,
62.75498087537059,
45.31318940723139,
28.47847735655821,
13.651929013611632,
31.184935941368614,
29.033009671223503,
22.192121868154786,
14.422288394164239,
7.018554351943966]

RESULTS FROM CONJUGATE GRADIENT

[77.3592236913273],
[48.49885832259556],
[23.256867521606033],
[75.46901802963987],
[46.68967130700212],
[22.264305693582298],
[67.82717758581617],
[40.526502535612636],
[19.110684397876476],
[66.67372440401857],
[62.75498090994626],
[45.31318935318992],
[28.478477416035485],
[13.651928974644253],
[31.18493595888895],
[29.033009641909214],
[22.192121902250346],
[14.422288360411832],
[7.01855437327436]]

```
# -*- coding: utf-8 -*-
"""
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ECSE 543
Assignment 2
q_3Functions.py
Functons for generating the matrix from Assign 1
Nov 14th, 2016
"""
#using the top right quater of the cable, due to symmetry
import math
################################################################################################
#Generates the initial mesh, taking into considering the boundary conditions
def genMesh (h):
    cableHeight = 0.1
    cableWidth = 0.1
    coreHeight = 0.02
    coreWidth = 0.04
    corePot = 110.0
    nodeHeight = (int)(cableHeight/h + 1)
    nodeWidth = (int)(cableWidth/h + 1)
    #Create the mesh, with Dirchlet conditions
    mesh = [[(None,corePot) if x <= coreWidth/h and y <= coreHeight/h \
            else ((None,0.0) if x == nodeHeight - 1 or y == nodeWidth - 1 \
            else (0,0.0)) for x in range(nodeWidth)] for y in range(nodeHeight)]
    #update the mesh to take into account the Neuman conditions
    nodeForA = 0
    for y in range (nodeHeight):
```

```
    for x in range(nodeWidth):
        if (mesh[y][x][0] == 0):
            mesh[y][x] = (nodeForA,0.0)
            nodeForA += 1
  return mesh
##########################################################################################
#The Equation that calculates SOR
def SOR(mesh,h,w):
  cableHeight = 0.1
  cableWidth = 0.1
  coreHeight = 0.02
  coreWidth = 0.04
  nodeHeight = (int)(cableHeight/h + 1)
  nodeWidth = (int)(cableWidth/h + 1)
  for y in range (1,nodeHeight - 1):
    for x in range (1,nodeWidth - 1):
      if (x > (int)(coreWidth/h) or y > (int)(coreHeight/h)):
        mesh[y][x] = (1 - w) * mesh[y][x] + (w/4) * (mesh[y][x-1] + mesh[y][x+1] + mesh[y-1][x] +
mesh[y+1][x])
  return mesh
##########################################################################################
#The Equation that calculates Jacobian
def jacobian(mesh,h):
  cableHeight = 0.1
  cableWidth = 0.1
  coreHeight = 0.02
  coreWidth = 0.04
  nodeHeight = (int)(cableHeight/h + 1)
  nodeWidth = (int)(cableWidth/h + 1)
```

```
    for y in range (1,nodeHeight - 1):
        for x in range (1,nodeWidth - 1):
            if (x > (int)(coreWidth/h) or y > (int)(coreHeight/h)):
                mesh[y][x] = (1/4) * (mesh[y][x-1] + mesh[y][x+1] + mesh[y-1][x] + mesh[y+1][x])
    return mesh
##########################################################################################
#Equation that computes the residue
def computeMaxRes(mesh,h):
    cableHeight = 0.1
    cableWidth = 0.1
    coreHeight = 0.02
    coreWidth = 0.04
    nodeHeight = (int)(cableHeight/h + 1)
    nodeWidth = (int)(cableWidth/h + 1)
    maxRes = 0
    for y in range(1, nodeHeight - 1):
        for x in range(1, nodeWidth - 1):
            if (x > coreWidth/h or y > coreHeight/h):
                #calculate the residue of each free point
                res = mesh[y][x-1] + mesh[y][x+1] + mesh[y-1][x] + mesh[y+1][x] - 4 * mesh[y][x]
                res = math.fabs(res)
                if (res > maxRes):
                    #Updates variable with the biggest residue amongst the free point
                    maxRes = res
    return maxRes
##########################################################################################
#Function that computes the number of iterations
def numIteration (initialMesh,h,w,method):
    minRes = 0.0001
```

```
    iteration = 1
    if (method == 's'):
        mesh = SOR(initialMesh,h,w)
        while (computeMaxRes(mesh,h) >= minRes):
            mesh = SOR(mesh,h,w)
            iteration += 1
    elif (method == 'j'):
        mesh = jacobian(initialMesh,h)
        while (computeMaxRes(mesh,h) >= minRes):
            mesh = jacobian(mesh,h)
            iteration += 1
    print ('Number of iterations: '+ str(iteration))
    return(mesh)
##############################################################################################
#Function that returns the potential at a free node
def getPot(mesh, x, y, h):
    cableHeight = 0.1
    cableWidth = 0.1
    nodeHeight = (int)(cableHeight/h + 1)
    nodeWidth = (int)(cableWidth/h + 1)
    xNode = int(nodeWidth - x/h - 1)
    yNode = int(nodeHeight - y/h - 1)
    return mesh[yNode][xNode]
##############################################################################################
```