ECSE 543 – Numerical Methods
Assignment 3

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Question 1

You are given a list of measured BH points for M19 steel (Table 1), with which to construct a continuous graph of B versus H.

B (T)	H (A/m)
0.0	0.0
0.2	14.7
0.4	36.5
0.6	71.7
0.8	121.4
1.0	197.4
1.1	256.2
1.2	348.7
1.3	540.6
1.4	1062.8
1.5	2318.0
1.6	4781.9
1.7	8687.4
1.8	13924.3
1.9	22650.2

Table 1: BH Data for M19 Steel

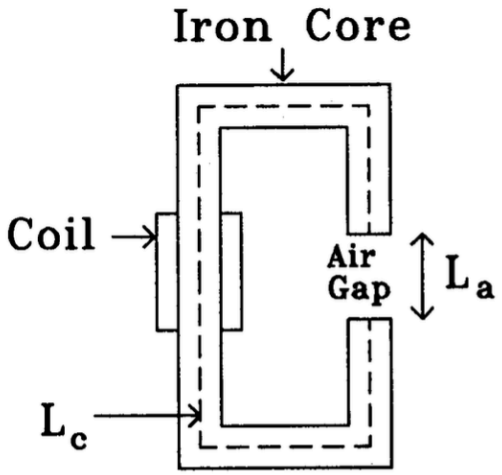
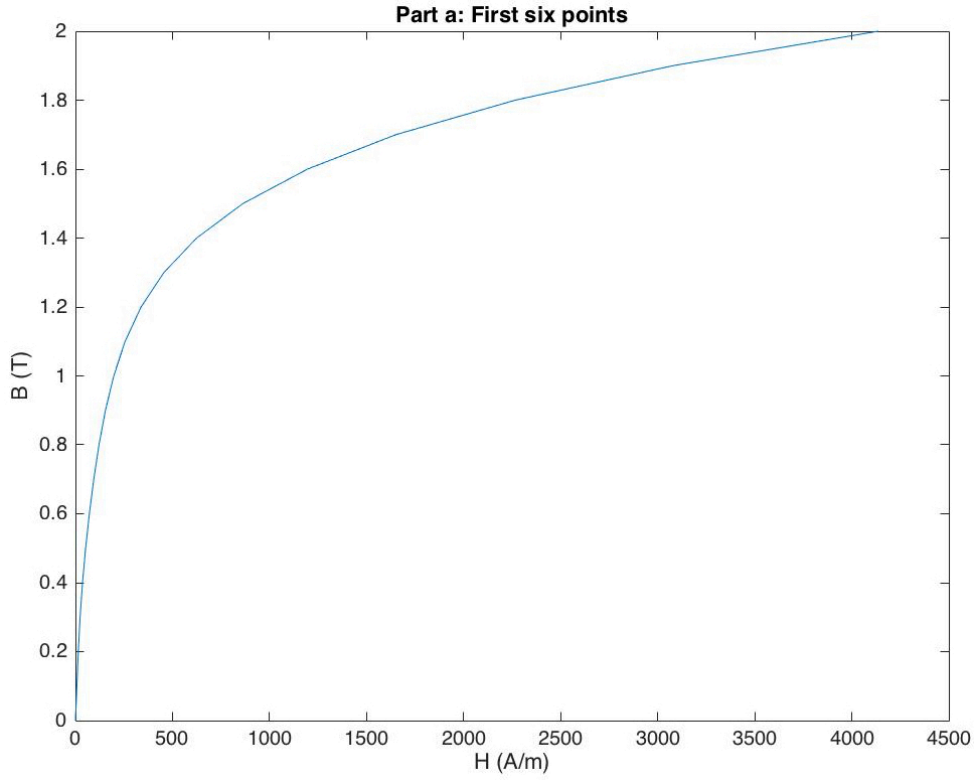


Figure 1

a) Interpolate the first 6 points using full-domain Lagrange polynomials. Is the result plausible, i.e. do you think it lies close to the true B versus H graph over this range?

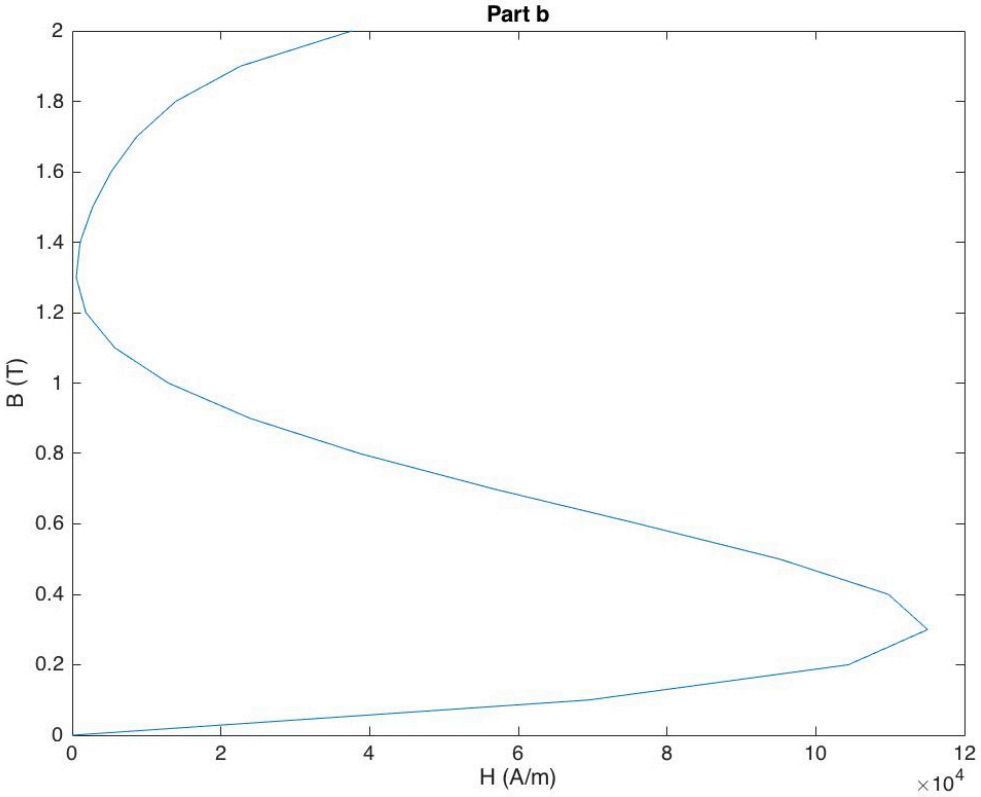
The code for this section is found in interpolate.py. A function was created that interpolates the given points and produces a graph of the points. For the first six points, the following chart was produced:



As we can see, the graph is really smooth, and there are no sharp turns or squiggles, and thus we can be pretty sure that the interpolation result is plausible and a good representation.

b) Now use the same type of interpolation for the 6 points at B = 0, 1.3, 1.4, 1.7, 1.8, 1.9. Is this result plausible?

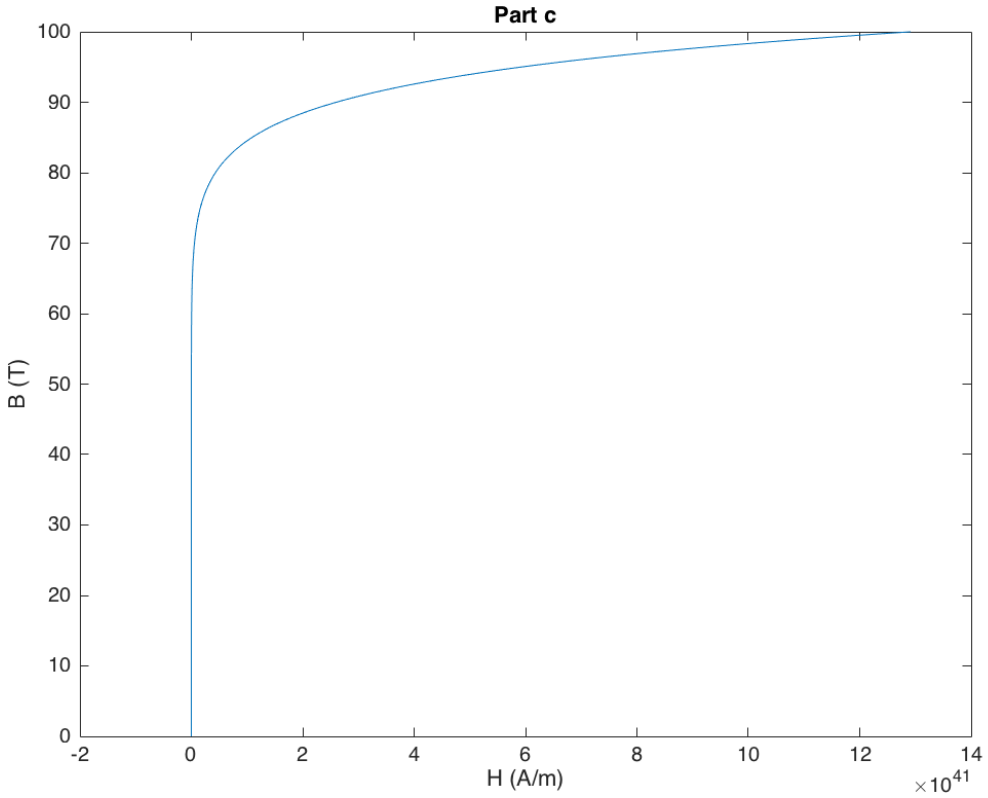
The code is found in interpolation.py. The graph that was generated is as follows:



The above graph was produced once the points were interpolated. The shape of the curve does not represent a B vs H graph, even though it passes through all the points provided. We can conclude that the result is NOT plausible.

c) An alternative to full-domain Lagrange polynomials is to interpolate using cubic Hermite polynomials in each of the 5 subdomains between the 6 points given in (b). With this approach, there remain 6 degrees of freedom - the slopes at the 6 points. Suggest ways of fixing the 6 slopes to get a good interpolation of the points. Test your suggestion and comment on the results.

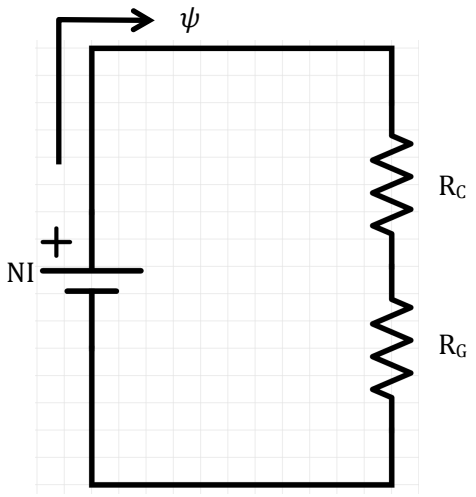
The code for cubic hermite is in interpolate.py. The function cubicHer takes in the H and B points, and produces a polynomial for the function, following the steps outlined in the lecture CV. The following graph was produced.



As we can see, the function is much smoother here, and we can conclude that cubic hermite produces a better curve, which matches the original function.

d) The magnetic circuit of Figure 1 has a core made of Ml9 steel, with a cross-sectional area of 1 cm². L_c =30 cm and L_a = 0.5 cm. The coil has N = 800 turns and carries a current I = 10 A. Derive a (nonlinear) equation for the flux y in the core, of the form f(y) = 0.

Using knowledge from ECSE 361 (Power engineering), the following equivalent circuit was produced. The steps for calculation of the equations and the values are outlined below:



$$R_G = \frac{l_G}{\mu_o * A_G} \qquad (1)$$

$$R_C = \frac{l_C}{\mu * A_C} \qquad (2)$$

$$NI = \psi(R_G + R_C) \qquad (3)$$

$$\psi = A * B \qquad (4)$$

$$\mu = \frac{B}{H} = \frac{\psi}{HA} \qquad (5)$$

Substituting equations (4) and (5) into (3), gives us the following:

$$NI = \psi\left(R_G + \frac{H * l_C}{\psi}\right) \qquad (6)$$

Expanding and simplifying the above equation gives us:

$$\psi R_G + H * l_C - NI = 0 \qquad (7)$$

Plugging in the values provided to us, generates the following nonlinear equation for the flux ψ in the core:

$$\boxed{f(\psi) = 0 = 3.978909 * 10^6\psi + 0.3H - 8000}$$

e) Solve the nonlinear equation using Newton-Raphson. Use a piecewise-linear interpolation of the data in Table 1. Start with zero flux and finish when $|f(y)/f(0)|<10^{-6}$. Record the final flux, and the number of steps taken.

The code for this can be found in M19circuit.py
We used the above equation and the steps outlined in lecture NL to solve for the flux.

$$f(\psi) = 0 = 3.978909 * 10^6\psi + 0.3H - 8000$$

$$f'(\psi) = 0 = 3.978909 * 10^6 + 0.3H'$$

Where H' is the change in slope calculated from the values in the table provided to us.
Performing Netwon Raphson, the following result was achieved:

Iterations: 3 Flux: $161.269369447*10^{-6}$ Wb

f) Try solving the same problem with successive substitution. If the method does not converge, suggest and test a modification of the method that does converge.

The code for this is also found in M19circuit.py

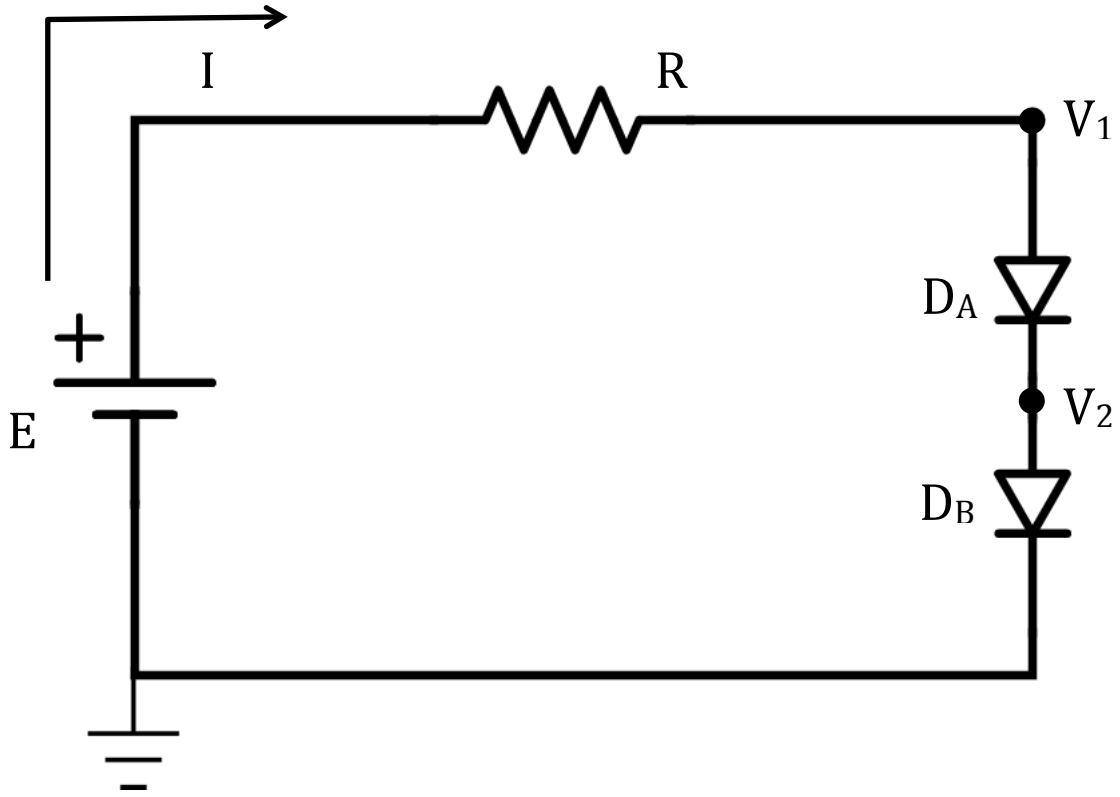
The steps for successive substitution is outlined in the lecture NL, and the function was created using that.

At first it was not converging, because of a division by zero, so I changed the initial conditions, so the initial flux is really small instead of zero.

And by doing so, I did get the same answer as part e.

Question 2

For the circuit shown below, the DC voltage E is 200 mV, the resistance R is 512 Ω, the reverse saturation current for diode A is I_{sA} = 0.8 μA, the reverse saturation current for diode B is I_{sB} = 1.1 μA, and assume kT/q = 25 mV.



a) Derive nonlinear equations for a vector of nodal voltages, v_n, in the form $f(v_n) = 0$. Give f explicitly in terms of the variables I_{sA} , I_{sB} , E, R and v_n.

First, we know the following relation for the current I:

$$I = \frac{E - V_1}{R} \quad (1)$$

And since all the elements are connected in series, the same current flows through them all. So the current I through the resistance and the diodes are the same.
Therefore, we get the following equations:

$$\text{Diode A} : I = I_{sA}\left(e^{\frac{q(V_1 - V_2)}{Kt}} - 1\right) \quad (2)$$

$$\text{Diode B} : I = I_{sB}\left(e^{\frac{qV_2}{Kt}} - 1\right) \quad (3)$$

Equating equations (1) and (2), we get:

$$\frac{E - V_1}{R} = I_{sA}\left(e^{\frac{q(V_1 - V_2)}{Kt}} - 1\right)$$

Solving for V_1, we get:

$$V_1 = E - RI_{sA}\left(e^{\frac{q(V_1 - V_2)}{Kt}} - 1\right)$$

$$\boldsymbol{f(V_1) = 0 = V_1 - E - RI_{sA}\left(e^{\frac{q(V_1 - V_2)}{Kt}} - 1\right)} \quad (4)$$

And we know that the current through the diodes are also equal, giving us the following equation:

$$I = I_A = I_B$$

$$I_{sA}\left(e^{\frac{q(V_1 - V_2)}{Kt}} - 1\right) = I_{sB}\left(e^{\frac{qV_2}{Kt}} - 1\right) \quad (5)$$

Rearranging equation (5), we get:

$$\boldsymbol{f(V_2) = 0 = I_{sA}\left(e^{\frac{q(V_1 - V_2)}{Kt}} - 1\right) - I_{sB}\left(e^{\frac{qV_2}{Kt}} - 1\right)} \quad (6)$$

b) Solve the equation f = 0 by the Newton-Raphson method. At each step, record f and the voltage across each diode. Is the convergence quadratic? [Hint: define a suitable error measure ε_k]

The code for solving the function is provided in newtonRaphson.py. To perform basic matrix operations such as inverse, matrix multiplication and addition are imported from basicDefinitions.py, which was created for assignment 1. f_1, f_2 and the jacobian was calculated by hand, and inputted into the code. The jacobian is shown below:

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial V_1} & \frac{\partial f_1}{\partial V_2} \\ \frac{\partial f_2}{\partial V_1} & \frac{\partial f_2}{\partial V_2} \end{bmatrix}$$

$$J = \begin{bmatrix} 1 + \frac{RI_{sA}}{0.025}e^{\frac{V_1 - V_2}{0.025}} & -\frac{RI_{sA}}{0.025}e^{\frac{V_1 - V_2}{0.025}} \\ \frac{I_{sA}}{0.025}e^{\frac{V_1 - V_2}{0.025}} & -\frac{I_{sA}}{0.025}e^{\frac{V_1 - V_2}{0.025}} - \frac{I_{sB}}{0.025}e^{\frac{V_2}{0.025}} \end{bmatrix}$$

The equation we have to solve to get the voltages at the nodes are:

$$J * \left[V_n^{(k+1)} - V_n^k\right] + f^k = 0 \quad (1)$$

Rearranging the above equation (1), we get:

$$V_n^{(k+1)} = -J^{-1} * f^k + V_n^k$$

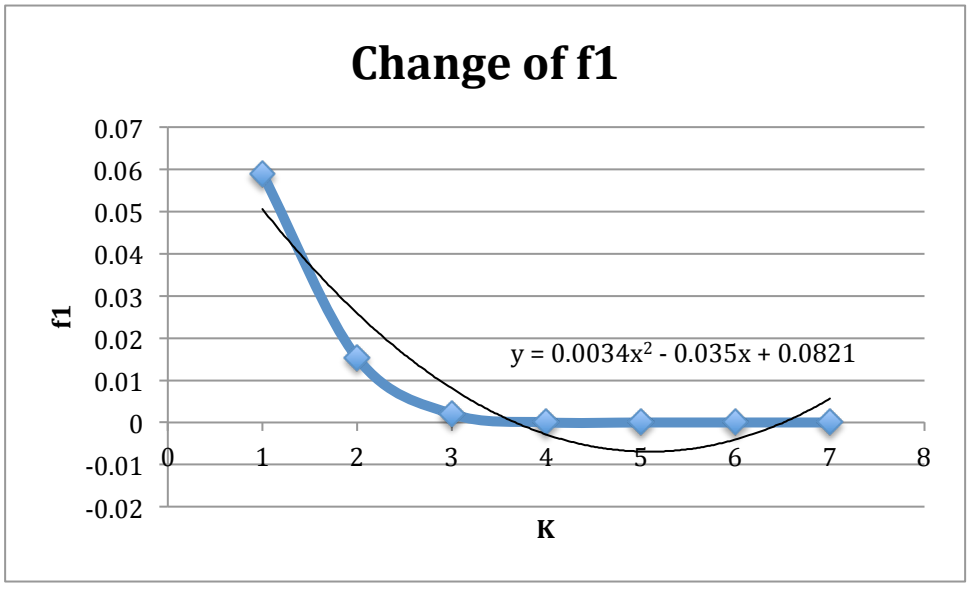
The following table shows the voltage and f values at each step:

k	f(v$_n$)		V$_n$	
	f1	f2	V_1	V_2
0	-0.2	0	0	0
1	0.058998671	0.000102706	0.198374913	0.066124971
2	0.015365126	2.84E-05	0.188435007	0.076313899
3	0.002043571	3.12E-06	0.184348331	0.082579242
4	3.55E-05	5.69E-08	0.183446499	0.083262944
5	1.16E-08	1.76E-11	0.183430837	0.083276563
6	1.17E-15	1.87E-18	0.183430832	0.083276567
7	-1.73E-17	-1.36E-20	0.183430832	0.083276567

Overall, the voltage across Diode A: $V_1 - V_2 = 100.15$ mV

Diode B: $V_2 - 0 = 83.28$ mV

The error function looks as follows:



And as we can see, the curve is indeed quadratic

Question 3

Write a program that accepts as input the values for the parameters x_0, x_N, and N and integrates a function f(x) on the interval $x = x_0$ to $x = x_N$ by dividing the interval into N equal segments and using one-point Gauss-Legendre integration for each segment.

The code for the Integrations is found in gaussLegendreOnePoint.py. There are two functions.
The first, "integral()" calculates the integral using N equal segments. Its inputs are the functions that we are going to calculate the integral for, the limits a and b, and finally, n, which is the number of segments.
The second, "integralUneven()", takes the functions as its inputs, as well as the limits a and b.

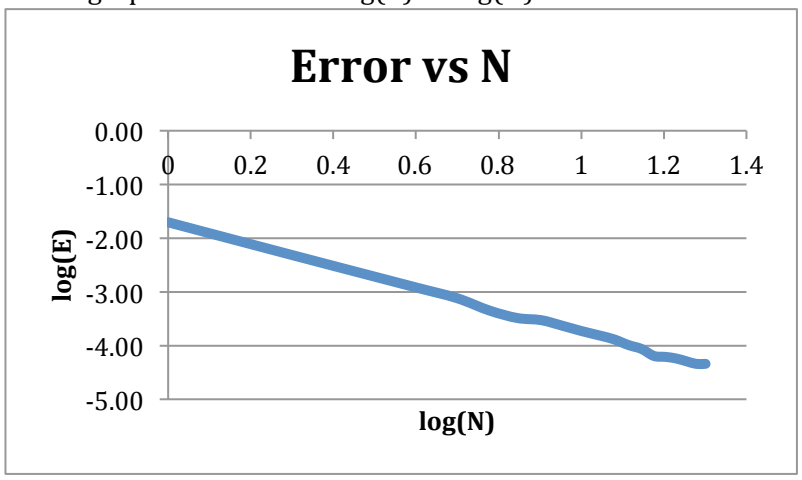
a) Use your program to integrate the function f(x) = sin(x) on the interval $x_0 = 0$ to $x_N = 1$ for N = 1, 2,..., 20. Plot $\log_{10}$(E) versus $\log_{10}$(N) for N=1,2,...,20, where E is the absolute error in the computed integral. Comment on the result.

The table below shows a few things. First it shows the value of i, which is the number of segments. Secondly, it shows the value it calculated after computing the integral for the given number of segments i, and finally, it shows error, which is how far off the computed value is to the actual value. The actual value is given in the first line, beside the function.

Integral of sin(x) = 0.45970

N	Integral	Error	log(N)	log(E)
1	0.4794255	0.0197255	0	-1.7049711
2	0.4645214	0.0048214	0.301029996	-2.3168305
3	0.4618328	0.0021328	0.477121255	-2.6710414
4	0.4608970	0.0011970	0.602059991	-2.9219024
5	0.4604648	0.0007648	0.698970004	-3.1164795
6	0.4607484	0.0004478	0.77815125	-3.3489313
7	0.4595501	0.0003258	0.84509804	-3.4870475
8	0.4599971	0.0002971	0.903089987	-3.5270784
9	0.4599342	0.0002342	0.954242509	-3.6303217
10	0.4549631	0.0001869	1	-3.7283172
11	0.4598560	0.0001560	1.041392685	-3.8067908
12	0.4598307	0.0001307	1.079181246	-3.8836066
13	0. 4597123	0.0001025	1.113943352	-3.9891777
14	0. 4597307	0.0000864	1.146128036	-4.0635373
15	0.4597002	0.0000640	1.176091259	-4.1939701
16	0.4597725	0.0000615	1.204119983	-4.2108411
17	0.4597640	0.0000574	1.230448921	-4.2414652
18	0.4597568	0.0000508	1.255272505	-4.2939914
19	0.5047716	0.0000456	1.278753601	-4.3411990
20	0.4597456	0.0000457	1.301029996	-4.3403535

The values of log(N) and log(E), which is the log value of the error are computed in the columns beside. The graph below shows log(E) vs log(N) :



The error obviously decreases with increasing N, but when the axes are scaled logarithmically, the error becomes more or less a linear function. The linearity means that increasing the number of segments N, does not reduce the absolute error of the integral.

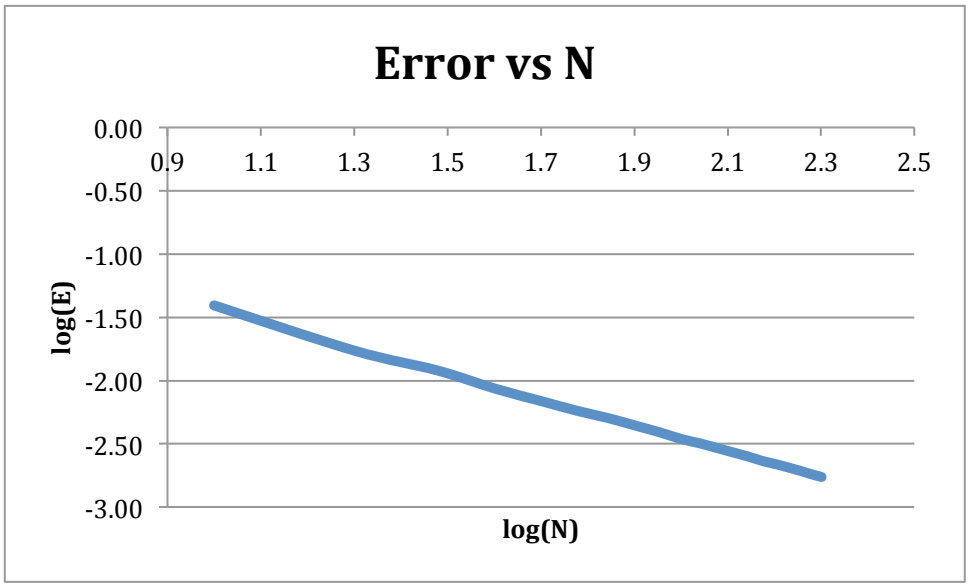
b) Repeat part (a) for the function f(x) = ln(x), only this time for N = 10, 20,..., 200. Comment on the result.

The table below shows the same items as for part a:

Integral of ln(x) = -1

N	Integral	Error	log(N)	log(E)
10	-0.9608800	0.0391200	1	-1.4076017
20	-0.9827755	0.0172245	1.301029996	-1.7638527
30	-0.9879429	0.0120571	1.477121255	-1.9187558
40	-0.9913617	0.0086383	1.602059991	-2.0635718
50	-0.9930852	0.0069148	1.698970004	-2.1602200
60	-0.9942353	0.0057647	1.77815125	-2.2392269
70	-0.9949558	0.0050442	1.84509804	-2.2972055
80	-0.9955965	0.0044035	1.903089987	-2.3561979
90	-0.9960928	0.0039072	1.954242509	-2.4081309
100	-0.9965384	0.0034616	2	-2.4607270
110	-0.9968115	0.0031885	2.041392685	-2.4964198
120	-0.9970801	0.0029199	2.079181246	-2.5346365
130	-0.9973070	0.0026930	2.113943352	-2.5697613
140	-0.9975011	0.0024989	2.146128036	-2.6022573
150	-0.9976914	0.0023086	2.176091259	-2.6366440
160	-0.9978160	0.0021840	2.204119983	-2.6607557
170	-0.9979455	0.0020545	2.230448921	-2.6872934
180	-0.9980605	0.0019395	2.255272505	-2.7123027
190	-0.9981771	0.0018229	2.278753601	-2.7392330
200	-0.9982682	0.0017318	2.301029996	-2.7614957

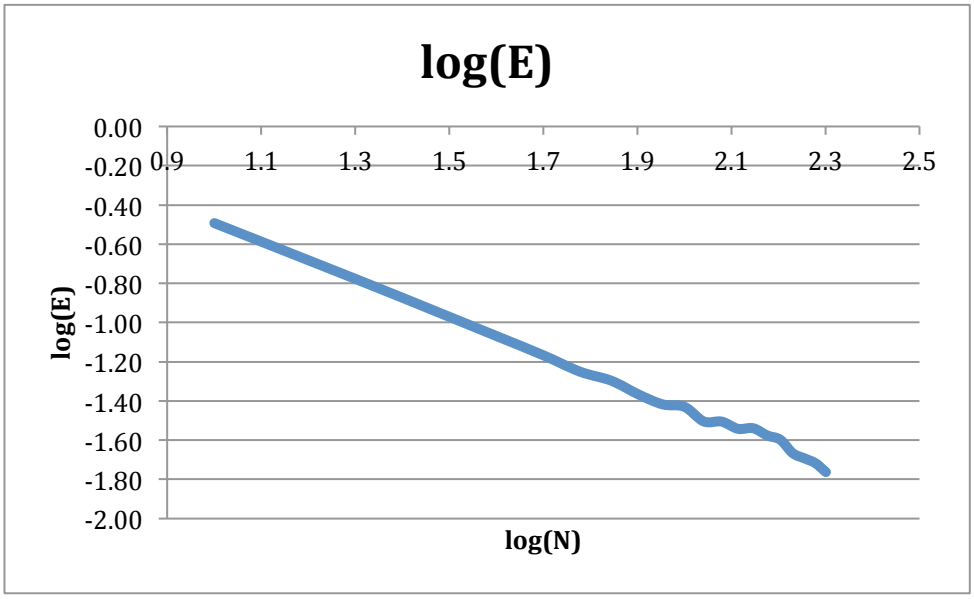
The log of N and errors were graphed, and as we can see bellow, produces a similar linear relation as part a.



c) Repeat part (b) for the function f(x) = ln(0.2|sin(x)1|). Comment on the result.

Integral of ln(0.2(|sin(x)|) : -2.66616

N	Integral	Error:	log(N)	log(E)
10	-2.3446046	0.3215554	1	-0.4927442
20	-2.4994320	0.1667280	1.301029996	-0.7779916
30	-2.5535800	0.1125800	1.477121255	-0.9485387
40	-2.5811784	0.0849816	1.602059991	-1.0706751
50	-2.5979101	0.0682499	1.698970004	-1.1658978
60	-2.6100165	0.0561435	1.77815125	-1.2507002
70	-2.6157180	0.0504420	1.84509804	-1.2972075
80	-2.6232539	0.0429061	1.903089987	-1.3674811
90	-2.6279347	0.0382253	1.954242509	-1.4176491
100	-2.6289799	0.0371801	2	-1.4296895
110	-2.6348724	0.0312876	2.041392685	-1.5046275
120	-2.6349164	0.0312436	2.079181246	-1.5052384
130	-2.6374627	0.0286973	2.113943352	-1.5421585
140	-2.6372534	0.0289066	2.146128036	-1.5390027
150	-2.6395996	0.0265604	2.176091259	-1.5757648
160	-2.6409193	0.0252407	2.204119983	-1.5978985
170	-2.6446017	0.0215583	2.230448921	-1.6663863
180	-2.6458640	0.0202960	2.255272505	-1.6925892
190	-2.6469866	0.0191734	2.278753601	-1.7173005
200	-2.6488963	0.0172637	2.301029996	-1.7628664



Similarly, the graph produces a linear relationship. This shows, that even if we integrate complicated functions, the resulting graph produced from log(N) vs log(E) remains the same

d) An alternative to dividing the interval into equal segments is to use smaller segments in more difficult parts of the interval. Experiment with a scheme of this kind, and see how accurately you can integrate f(x) in part (b) and (c) using only 10 segments. Comment on the results.

The segment width was decreased, as we got closer to the result. The values computed from 10 segments are as follows:

```
Unever integral
Integral of ln(x):
Integral = 0.98027383623 Error: 0.0197261637697

Integral of ln(0.2(|sin(x)|):
Integral = -2.64421714908 Error: 0.0219428509219
```

As we can see, for ln(x), using 10 segments, we get an error of 0.0197, where as using equal widths for segments, as done in part b) the error was 0.039. This shows that using this method gives more accurate results with less segments. The same thing is observed when we try to compute the integral of ln(0.2|sin(x)|). The error is only 0.0219 with uneven segment width, where as in part c) the error for 10 segments was 0.321

Appendix

```
#!/usr/bin/env python2
# -*- coding: utf-8 -*-
"""
Sharhad Bashar
ECSE 543
Assignment 3
interpolate.py
Contains the functions to perform Lagrange Interpolation and Cubic Hermite
"""
from sympy import *
def diff(f,x):
    n = len(x)
    y = []
    for i in range(n):
        if (i == n - 1):
            y.append((f[i]-f[i-1])/(x[2]-x[1]))
        else:
            y.append((f[i+1]-f[i]/(x[2]-x[1])))
    return y
def interLag(pointsX, pointsY):
    x = symbols('x')
    n = len(pointsX)
    Ljx = [None for j in range (n)]
    for j in range (n):
        Fjx = 1
        Fjxj = 1
        yx = 0
        for k in range (n):
```

```
            if (k != j):
                Fjx *= (x - pointsX[k])
                Fjxj *= (pointsX[j] - pointsX[k])
                Ljx[j] = Fjx.expand()/Fjxj
    # Now we have al the Lj(x)
    for j in range (n):
        yx += pointsY[j] * Ljx[j]
    return yx
def cubicHer(pointsX, pointsY):
    x = symbols('x')
    n = len(pointsX)
    Ljx = [None for j in range (n)]
    LjxPrime = [None for j in range (n)]
    Ujx = [None for j in range (n)]
    Vjx = [None for j in range (n)]
    b = [None for j in range (n)]
    yx = 0
    for j in range (n):
        Fjx = 1
        Fjxj = 1
        for k in range (n):
            if (k != j):
                Fjx *= (x - pointsX[k])
                Fjxj *= (pointsX[j] - pointsX[k])
                Ljx[j] = Fjx.expand()/Fjxj
                #now we have Lj(x)
    for j in range (n):
        LjxPrime[j] = Ljx[j].diff(x)
        #now we have Lj'(x)
    for j in range(n):
```

```
    Ujx[j] = ((1 - 2 * (LjxPrime[j] * (x - pointsX[j]))) * (Ljx[j] * Ljx[j])).expand()
    #now we have Uj(x)
    Vjx[j] = ((x - pointsX[j]) * (Ljx[j] * Ljx[j])).expand()
    #now we have Uj(x)
    if (j < 5):
      b[j] = (pointsY[j + 1] - pointsY[j]) / (pointsX[j + 1] - pointsX[j])
    elif (j == 5):
      b[j] = pointsY[j]/pointsX[j]
    #now we have the bj
  for j in range (n):
    yx += pointsY[j] * Ujx[j] + b[j] * Vjx[j]
  return yx
B = [0.0, 0.2,  0.4,  0.6,  0.8,  1.0,  1.1,  1.2,  1.3,  1.4,  1.5,  1.6,  1.7,  1.8,  1.9]
H = [0.0, 14.7, 36.5, 71.7, 121.4, 197.4, 256.2, 348.7, 540.6, 1062.8, 2318.0, 4781.9, 8687.4,
13924.3, 22650.2]
Bb = [0.0, 1.3,  1.4,  1.7,  1.8,  1.9]
Hb = [0.0, 540.6, 1062.8, 8687.4, 13924.3, 22650.2]
a = interLag (B[:6],H[:6])
b = interLag (Bb,Hb)
print ("First Six points: " + str(a))
print ('\n')
print ("Part b: " + str(b))
cubic = (cubicHer(Bb, Hb))
print cubic
%Plot the interpolated points
%First six points
a = @(x) 414.062500000009*x.^5 - 963.54166666669*x.^4 +
873.437500000015*x.^3 - 215.208333333338*x.^2 + 88.6500000000005*x;

%The six points given in part b
b = @(x) 156393.280524088*x.^5 - 966235.57224511*x.^4 +
2253820.22115058*x.^3 - 2337828.82945774*x.^2 + 906781.854422079*x;

%Cubic Hermite
c= @(x)1568971513379.42*x.^15 - 30309713506701.9*x.^14 +
268567396219957.0*x.^13 - 1.44432330293827e+15*x.^12 +
```

```
5.25560062780712e+15*x.^11 - 1.36509778845371e+16*x.^10 +
2.60061934819287e+16*x.^9 - 3.67319109930033e+16*x.^8 +
3.83801274237492e+16*x.^7 - 2.9210617380455e+16*x.^6 +
1.56706833559662e+16*x.^5 - 5.57371221336095e+15*x.^4 +
1.16503319968491e+15*x.^3 - 105922354752559.0*x.^2 +
415.846153846154*x;

%%%%%%%%%%%%%%%%%%%%%%%%%%
x_1 = 0:0.1:2;
x_2 = 0:0.1:100;
figure;
plot(a(x_1),x_1);
title('Part a: First six points');
xlabel('H (A/m)');
ylabel('B (T)');
figure;
plot(b(x_1),x_1);
title('Part b');
xlabel('H (A/m)');
ylabel('B (T)');
figure();
plot(c(x_2),x_2);
title('Part c');
xlabel('H (A/m)');
ylabel('B (T)');
%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
#!/usr/bin/env python2

# -*- coding: utf-8 -*-

"""

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ECSE 543

Assignment 3

M19circuit.py

Contains the functions to calculate the flux of the M19 circuit

"""

HvB = [[0.0, 0.0],[0.2,
14.7],[0.4,36.5],[0.6,71.7],[0.8,121.4],[1,197.4],[1.1,256.2],[1.2,348.7],[1.3,540.6],[1.4,1062.
8],[1.5,2318.0],[1.6,4781.9],[1.7,8687.4],[1.8,13924.3],[1.9,22650.2]]

def Hval(flux):

    B = flux/(1.0/pow(100,2))

    # interpolate for values outside domain

    if B > HvB[-1][0]:

        slope = (HvB[-1][1] - HvB[-2][1]) / (HvB[-1][0] - HvB[-2][0])
```

```
        return (B - HvB[-1][0]) * slope + HvB[-1][1]
    for i in range(len(HvB)):
        if HvB[i][0] > B:
            slope = (HvB[i][1] - HvB[i-1][1]) / (HvB[i][0] - HvB[i-1][0])
            return (B - HvB[i-1][0]) * slope + HvB[i-1][1]
    else: # must be smaller
        slope = (HvB[1][1] - HvB[0][1]) / (HvB[1][0] - HvB[0][0])
        return (B - HvB[0][0]) * slope + HvB[0][1]
def Hder(flux):
        B = flux/(1.0/pow(100,2))
        if B > HvB[-1][0]:
                return (HvB[-1][1] - HvB[-2][1]) / (HvB[-1][0] - HvB[-2][0])
        for i in range(len(HvB)):
                if HvB[i][0] > B:
                        slope = (HvB[i][1] - HvB[i-1][1]) / (HvB[i][0] - HvB[i-1][0])
                        return slope
        else: # must be smaller
                slope = (HvB[1][1] - HvB[0][1]) / (HvB[1][0] - HvB[0][0])
                return slope
def fFlux(flux):
        return 3.978873577e7 * flux + 0.3 * Hval(flux) - 8000
def fFluxDer(flux):
        return 3.978873577e7 + 0.3 * Hderivative(flux)/(1.0/pow(100,2))
def newRap(x, tolerance):
        i = 0
        while abs(fFlux(x)/fFlux(0)) > tolerance:
                i += 1
                x -= fFlux(x)/fFluxDer(x)

        print('Iterations: ' + str(i) + ' Flux: ' + str(x))
```

```
            return x
def fSubstitution(flux):
   return 8000/(39.78873577e6 + 0.3 * Hval(flux)/flux)
def succesSub (x,tolerance):
   i = 0
   while abs(fFlux(x)/fFlux(0)) > tolerance:
      i += 1
                        x = fSubstitution(x)
   print('Iterations: ' + str(i) + ' Flux: ' + str(x))
   return x
NR = newtonRaph(0.0, 1e-6)
#!/usr/bin/env python2
# -*- coding: utf-8 -*-
"""
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ECSE 543
Assignment 3
newtonRaphson.py
Computing the voltages using Newton Raphson
Dec 5th, 2016
"""
import math
from basicDefinitions import matrixAddorSub, matrixMult,
matInverse,is1Dor2D,matTranspose, scalarMult
#################################################################
def newRap (E,R,V1,V2,Isa,Isb,k):
   f1 = V1 - E + R * Isa * (math.exp((V1 - V2) / 0.025) - 1.0)
   f2 = Isa * ((math.exp((V1 - V2) / 0.025) - 1.0)) - Isb * (math.exp(V2 / 0.025) - 1.0)
   print ('f1 : ' + str(f1))
```

```
   print ('f2 : ' + str(f2))
   print ('Number of iterations : ' + str(k))
   print('')
   f1V1Prime = 1.0 + (R * Isa / 0.025) * (math.exp((V1 - V2) / 0.025))
   f1V2Prime = -1.0 * (R * Isa / 0.025) * (math.exp((V1 - V2) / 0.025))
   f2V1Prime = (Isa / 0.025) * (math.exp((V1 - V2) / 0.025))
   f2V2Prime = -1.0 * (Isa / 0.025) * (math.exp((V1 - V2) / 0.025)) - (Isb / 0.025) *
(math.exp(V2 / 0.025))
   V = [[V1],[V2]]
   f = [[f1],[f2]]
   J = [[None for x in range (2)] for y in range(2)]
   J[0][0] = f1V1Prime
   J[0][1] = f1V2Prime
   J[1][0] = f2V1Prime
   J[1][1] = f2V2Prime
   invJ = matInverse(J)
   V = matrixAddorSub(matTranspose(scalarMult(-1.0,matrixMult(invJ,f))),V,'a')
   print ('V1 : ' + str(V[0][0])+ ' V')
   print ('V2 : ' + str(V[1][0])+ ' V')
   return V
#################################################################
E = 0.2
R = 512.0
Isa = 0.0000006
Isb = 0.0000012
V1 = 0.0
V2 = 0.0
k = 0

print  ('V1 : ' + str(V1)+ ' V')
```

```
print ('V2 : ' + str(V2)+ ' V')


V = newRap(E,R,V1,V2,Isa,Isb,k)


f1 = V[0][0] - E + R * Isa *(math.exp((V[0][0] - V[1][0])/0.025) - 1.0)


while (f1 > 0):
   k += 1
   V = newRap(E,R,V[0][0],V[1][0],Isa,Isb,k)
   f1 = V[0][0] - E + R * Isa *(math.exp((V[0][0] - V[1][0])/0.025) - 1.0)
f1 = V[0][0] - E + R * Isa * (math.exp((V[0][0] - V[1][0]) / 0.025) - 1.0)
f2 = Isa * ((math.exp((V[0][0] - V[1][0]) / 0.025) - 1.0)) - Isb * (math.exp(V[1][0] / 0.025) -
1.0)
print ('f1 : ' + str(f1))
print ('f2 : ' + str(f2))
print ('Number of iterations : ' + str(k + 1))


#!/usr/bin/env python2
# -*- coding: utf-8 -*-
"""
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ECSE 543
Assignment 3
gaussLegendreOnePoint.py
Computing the Integral using Gauss Legendre One Point method
Dec 5th, 2016
"""
import math
```

```
def integral(func1,func2,n,a,b):
    segments = n
    n,a,b = float(n),float(a),float(b)
    wi = 2.0
    xi = 0.0
    summation = 0.0
    segSize = (b - a) / float(segments)
    if (func2 == 1):
        while (a < b):
            lowLim = a
            a += segSize
            highLim = a
            summation += (highLim - lowLim) * func1((highLim + lowLim)/2.0 * xi + (highLim + lowLim)/2.0)
    else:
        while (a < b):
            lowLim = a
            a += segSize
            highLim = a
            summation += (highLim - lowLim) * func2 (0.2 * func1((highLim + lowLim)/2.0 * xi + (highLim + lowLim)/2.0))
    return summation


def integralUneven(func1,func2, a, b):
    relativeWidths = [2**i for i in range (0,10)]
    a,b = float(a),float(b)
    scale = (b - a) / sum(relativeWidths)
    widths = [width * scale for width in relativeWidths]
    runningWidth = 0
    runningSum = 0
    if (func2 == 1):
```

```
        for w in widths:
            runningSum += func1(w/2 + runningWidth) * w
            runningWidth += w
    else:
        for w in widths:
            runningSum += func2(0.2*abs(func1(w/2 + runningWidth))) * w
            runningWidth += w
    return runningSum

print ("Integral of sin(x): 0.45970")
for i in range (1,21):
    error = 0.45970
    integ =  integral (math.sin,1,i,0,1)
    print ("i = " + str(i) + " Integral = " + str(integ) + " Error: " + str(integ - error))
print ('')
print ("Integral of ln(x): -1")
for i in range (1,21):
    error = -1
    integ =  integral (math.log,1,i * 10,0,1)
    print ("i = " + str(i*10) + " Integral = " + str(integ) + " Error: " + str(integ - error))
print ('')
print ("Integral of ln(0.2(|sin(x)|): -2.66616")
for i in range (1,21):
    error = -2.66616
    integ =  integral(math.sin,math.log,i,0,1)
    print ("i = " + str(i*10) + " Integral = " + str(integ) + " Error: " + str(integ - error))
print ('')
print ('Unever integral')
print ("Integral of ln(x): ")
integ =  integralUneven (math.log,1,0,1)
```

```
error = -1
print ("Integral = " + str(abs(integ)) + " Error: " + str(integ - error))
print ('')
print ("Integral of ln(0.2(|sin(x)|): ")
integ = integralUneven(math.sin,math.log,0,1)
error = -2.66616
print ("Integral = " + str(integ) + " Error: " + str(integ - error))
print ('')

# -*- coding: utf-8 -*-
"""
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ECSE 543
Assignment 3
basicDefinitions.py
A list of functions, used for this assignment
Dec 5th, 2016
"""

import math
from scipy import random
import numpy as np
import csv

#####################################################################
#Function that checks if a matrix is 1D or 2D
def is1Dor2D (A):
   while True:
      try:
```

```
            length = len(A[0]) #If true, A is 2D
            return A
            break
        except TypeError: #else A is 1D
            return [A]
            break
#####################################################################
#function that creates floats from lists
def list2float(A):
    length = len(A)
    floatA = [0 for x in range(length)]
    for i in range (length):
        numVal = ''
        stringVal = list(str(A[i]))
        for j in range (1,len(stringVal)-1,1):
            numVal = numVal + stringVal[j]
        floatVal = float(numVal)
        floatA [i] = floatVal
    return floatA
#####################################################################
#Function that transposes a Matrix
def matTranspose(A):
    A = is1Dor2D(A)
    rowsA = len(A)
    colsA = len(A[0])
    C = [[0 for rows in range (rowsA)] for cols in range(colsA)]
    for i in range (colsA):
        for j in range (rowsA):
            C[i][j] = A[j][i]
    return C
```

```
#################################################################
#Matrix that performs Inverse for a 2D matrix
def matInverse(A):
   detA = A[0][0] * A[1][1] - A[0][1] * A[1][0]
   invA = [[None for x in range (len(A))] for y in range(len(A))]
   invA[0][0] = A[1][1] / detA
   invA[1][1] = A[0][0] / detA
   invA[0][1] = -1 * A[0][1] / detA
   invA[1][0] = -1 * A[1][0] / detA
   return invA
#################################################################
#Function that adds or subtracts two matricies
def matrixAddorSub(A,B,operation):
   A = is1Dor2D(A)
   B = is1Dor2D(B)
   rowsA = len(A)
   colsA = len(A[0])
   rowsB = len(B)
   colsB = len(B[0])
   if (rowsA == rowsB and colsA == colsB):
      C = [[0 for row in range(colsA)] for col in range(rowsA)]
      if (operation == 'a'):
         for i in range (rowsA):
            for j in range (colsA):
               C[i][j] = A[i][j]+B[i][j]
      elif (operation == 's'):
         for i in range (rowsA):
            for j in range (colsA):
               C[i][j] = A[i][j]-B[i][j]
   return C
```

```
#################################################################
#function for scalar matrix multiplication
def scalarMult(scalar, A):
   newMat = [0 for x in range (len(A))]
   for i in range (len(A)):
      newMat[i] = scalar*A[i][0]
   return newMat
#################################################################
#Function that multiplies two matricies
def matrixMult (A, B):
   A = is1Dor2D(A)
   B = is1Dor2D(B)
   rowsA = len(A)
   colsA = len(A[0])
   rowsB = len(B)
   colsB = len(B[0])
   if (rowsA == colsB or colsA == rowsB):
      C = [[0 for row in range(colsB)] for col in range(rowsA)]


      for i in range(rowsA):
         for j in range(colsB):
            for k in range(colsA):
            # Create the result matrix
            # Dimensions would be rows_A x cols_B
               C[i][j] += A[i][k] * B[k][j]
   else:
    print ("Cannot multiply the two matrices. Incorrect dimensions.")
    return
   return C
#################################################################
```

```
#Function that creates a diagonal matrix
def diogMat (A):
   length = len(A)
   floatA = [0 for x in range(length)]
   for i in range (length):
      numVal = ''
      stringVal = list(str(A[i]))
      for j in range (1,len(stringVal)-1,1):
         numVal = numVal + stringVal[j]
      floatVal = float(numVal)
      floatA [i] = floatVal
   diognalMatrix = [[0 for x in range(length)] for y in range(length)]
   for i in range (length):
      diognalMatrix[i][i] = 1/floatA[i]
   return diognalMatrix
#################################################################
#Function that creates random A, given a length input
def randomSPD(length):
   A = [[0 for x in range(length)] for y in range(length)]
   L = random.rand(length,length)
   A = np.dot(L,L.T)
   return A
#################################################################
#Function that performs the Cholesky decomposition and returns L
def cholesky(A,length):
   global sum
   L = [[0 for x in range(length)] for y in range(length)]
   for i in range(length):
      for k in range(i + 1):
         sum = 0
```

```
            for j in range(k):
                sum += L[i][j] * L[k][j]
            if (i == k):
                L[i][k] = math.sqrt(abs(A[i][i] - sum))
            else:
                L[i][k] = (A[i][k] - sum) / L[k][k]
    return L
#####################################################################
#Function that solves y in Ly = b
def forwElim (L,b,length):
    b = list2float(b)
    global sum
    y = [0 for x in range (length)]
    for i in range (length):
        sum = 0.00
        for j in range (i):
            sum += L[i][j] * y[j]
        y[i] = (b[i]-sum)/L[i][i]
    return y
#####################################################################
#Function that solves for x in L^Tx = y
def backSub (L,y,length):
    global sum
    X = [0 for x in range(length)]
    for i in range (length - 1, -1, -1):
        sum = 0
        for j in range (i + 1, length, 1):
            sum += L[j][i] * X[j]
        X[i] = (y[i] - sum) / L[i][i]
    return X
```

##