Assigment 5

Upload your code (.ipynb) on Learn dropbox and submit pdfs of the code and to Crowdmark.

Classification

In [880]:
```
# Download the LIBSVM package from here: https://www.csie.ntu.edu.tw/
~cjlin/libsvm/#download
# If your download is successfull you should have the folder with nam
e: libsvm-3.24.
# We will use this package to load datasets.

# Enter the downloaded folder libsvm-3.24 through your terminal.
# Run make command to compile the package.

# Load this auxiliary package.
import sys

# add here your path to the folder libsvm-3.24/python
path = "/home/tempo/Desktop/Fall 2019/CS 794/a5/libsvm-3.24/python"
# Add the path to the Python paths so Python can find the module.
sys.path.append(path)

# Load the LIBSVM module.
from svmutil import *

# Add here your path to the folder libsvm-3.24
path = "/home/tempo/Desktop/Fall 2019/CS 794/a5/libsvm-3.24"

# Test that it works. This will load the data "heart_scale"
# and it will store the labels in "b" and the data matrix in "A".
b, A = svm_read_problem(path + '/heart_scale')
print('Loaded data: Heart Scale')
# Use "svm_read_problem" function to load data for your assignment.

# Note that matrix "A" stores the data in a sparse format.
# In particular matrix "A" is a list of dictionaries.
# The length of the list gives you the number of samples.
# Each entry in the list is a dictionary. The keys of the dictionary
 are the non-zero features.
# The values of the dictionary for each key is a list which gives you
the feature value.
```

```
Loaded data: Heart Scale
```

In [881]:
```
import matplotlib.pyplot as plt

# Numpy is useful for handling arrays and matrices.
import numpy as np
from scipy.sparse import coo_matrix
import time
from random import randrange as rnd
import sklearn
import sklearn.metrics
import random
import pandas as pd
```

In [882]:
```
path_news20 = r'/home/tempo/Desktop/Fall 2019/CS 794/a5/news20.binar
y'
b_news20, A_news20 = svm_read_problem(path_news20) #(19996, 1355191)

def splitData(A, b):
    n = A.shape[0]
    test_split = 15996
    valid_split = 2000
    train_split = 2000

    ATrain = A[0 : test_split]
    bTrain = b[0 : test_split]

    AValid = A[test_split : test_split + valid_split]
    bValid = b[test_split : test_split + valid_split]

    ATest = A[n - train_split :]
    bTest = b[n - train_split :]

    return ATrain, AValid, ATest, bTrain, bValid, bTest

def toSparse(A, cols):
    row = []
    col = []
    data = []
    for i in range(len(A)):
        for key, val in A[i].items():
            row.append(i)
            col.append(key - 1)
            data.append(val)
    return coo_matrix((data, (row, col)), shape=(len(A), cols)).tocsr
()

def bConverter(b):
    for i in range(len(b)):
        if (b[i] == 2):
            b[i] = -1
    return b

A_news20 = toSparse(A_news20, 1355191)
b_news20 = np.array(b_news20)
b_news20 = b_news20.reshape((b_news20.shape[0], 1))
print(A_news20.shape)

ATrain_news20, AValid_news20, ATest_news20, bTrain_news20, bValid_new
s20, bTest_news20 = splitData(A_news20, b_news20)

print('News data set loaded')
```

```
(19996, 1355191)
News data set loaded
```

In [883]:
```
initParams = {
    'lambda_': 0.001,
    'epsilon': 1e-2,
    'max_iter': 2000,
    'alpha0': 1,
    't': 1,
    'beta0': np.random.uniform(-0.5, 0.5),
    'theta': 0.5,
    'x0_news20': np.random.uniform(-1, 1, size = (ATrain_news20.shape
[1], 1)),
    'sigma': 1e-12,
    'delta': 1e-12
}
```

In [884]:
```
proxGradData = []
accelProxGradData = []
```

Download the dataset news20.binary dataset: https://www.csie.ntu.edu.tw/~cjlin/libsvmtools/datasets/binary.html#news20.binary (https://www.csie.ntu.edu.tw/~cjlin/libsvmtools/datasets/binary.html#news20.binary) Check Assignment 4 for instructions about loading this dataset.

Question 1

Solve the l1-regularized logistic regression problem

$$\text{minimize}_{x\in\mathbb{R}^d,\beta\in\mathbb{R}}\ \lambda\|x\|_1 + \frac{1}{n}\sum_{i=1}^{n}\log(1+\exp(-b_i(a_i^T x+\beta)))$$

using

1) Proximal gradient descent

2) Accelerated proximal gradient descent

3) Proximal coordinate descent

4) Accelerated proximal coordinate descent

Tune the model parameter λ and any parameters that the algorithms have. You only have to tune λ once and use the same λ for all algorithms. Use similar techniques to Assignment 4 to tune λ. Plot the objective function (y-axis) vs running time in sec (x-axis) for all algorithms in the same plot. Make sure that the plots are clean and use appropriate legends. Report your generalization error. This should be measured in the same way as Assignment 4.

For training split the data into training (the first 15996 datapoints), validation (the next 2000 datapoints) and testing (the next 2000 datapoints). Do not use the testing data to influence training in any way. This means that you should compute the generalization error only once when you finish your work with this part of the assignment.

Marks: 15

In [885]:
```
def lineSearch(beta, xCurrent, gradF, A, b):
    lambda_ = initParams['lambda_']
    alpha = initParams['alpha0']
    theta = initParams['theta']
    logisticRegression = LogisticRegression()
    proxGradDesc = ProxGradDesc()

    #Compute F(x(alpha)):
    x_alpha = proxGradDesc.proxAG(alpha, xCurrent, gradF)
    F_x_alpha = logisticRegression.objFunc(x_alpha, A, b, beta)

    #Compute F(x_k):
    F_x_k = logisticRegression.objFunc(xCurrent, A, b, beta)

    #Compute theta * (l(x_k) - l(x(alpha))):
    l_x_k = F_x_k
    l_x_alpha = logisticRegression.gx(x_alpha) + logisticRegression.f
x(xCurrent, A, b, beta) + np.dot(gradF.T, x_alpha - xCurrent)

    while (F_x_alpha > F_x_k - theta * (l_x_k - l_x_alpha)):
        alpha /= 2

        #Compute F(x(alpha)):
        x_alpha = proxGradDesc.proxAG(alpha, xCurrent, gradF)
        F_x_alpha = logisticRegression.objFunc(x_alpha, A, b, beta)
        l_x_alpha = logisticRegression.gx(x_alpha) + logisticRegressi
on.fx(xCurrent, A, b, beta) + np.dot(gradF.T, x_alpha - xCurrent)

    return alpha

def validationError(bTrue, bPredict):
    t = len(bTrue)
    totalError = 0
    for i in range (t):
        totalError += np.abs(bPredict[i] - bTrue[i])
    return totalError / (2 * t)

def calcAccuracy(bTrue, bPredict):
    return 1 - .5 * validationError(bTrue, bPredict)
```

In [886]:
```
class LogisticRegression(object):

    def __init__(self):
        self.lambda_ = initParams["lambda_"]

    def __str__(self):
        return 'L1-regularized Logistic Regression'

    def gx(self, x):
        lambda_ = self.lambda_
        return lambda_ * np.linalg.norm(x, 1)

    def fx(self, x, A, b, beta):
        n = A.shape[0]
        aTx = A @ x
        sumLogistic = sum(np.log(1 + np.exp(-b * (aTx + beta))))
        return sumLogistic / n

    def objFunc(self, x, A, b, beta):
        g_x = self.gx(x)
        f_x = self.fx(x, A, b, beta)
        return g_x + f_x

    def gradient(self, x, A, b, beta):
        n = A.shape[0]
        aTx = A @ x
        denom = 1 / (1 + np.exp(b * (aTx + beta)))

        gradBeta = sum(-b * denom) / n
        gradFx = A.T @ (-b * denom)/n
        return gradFx, gradBeta

    def predict(self, x, A, beta):
        aTx = A @ x
        sigma = 1 + np.exp(-(aTx + beta))
        return 2 * (sigma < 2) - 1
```

In [887]:
```
class ProxGradDesc():
    def __init__(self):
        self.lambda_ = initParams['lambda_']
        self.epsilon = initParams['epsilon']
        self.beta = initParams['beta0']
        self.logisticRegression = LogisticRegression()

    def __str__(self):
        return 'Proximal Gradient Descent'

    def proxAG(self, alpha, x, gradF):
        lambda_ = self.lambda_
        xNext = np.zeros((x.shape[0], 1))
        la = lambda_ * alpha
        u = x - lambda_ * alpha * gradF

        xNext[u >= la] = u[u >= la] - la
        xNext[np.abs(u) <= la] = 0
        xNext[u <= -la] = u[u <= -la] + la

        return xNext

    def proxGrad(self, xCurrent, A, b):
        beta = self.beta
        max_iters = initParams['max_iter']
        start = time.time()
        for i in range(max_iters):
            gradF, gradB = self.logisticRegression.gradient(xCurrent,
A, b, beta)
            alpha = lineSearch(beta, xCurrent, gradF, A, b)

            proxGradData.append([self.logisticRegression.objFunc(xCur
rent, A, b, beta), time.time() - start])
            if (i % 200 == 0):
                print('Iteration:', i, 'out of', max_iters, 'iteratio
ns')
            xNext = self.proxAG(alpha, xCurrent, gradF)

            Gx = (xCurrent - xNext) / alpha

            if (np.linalg.norm(Gx) < self.epsilon):
                return xNext, beta
            else:
                xCurrent = xNext
                beta -= alpha * gradB

        return xNext, beta
```

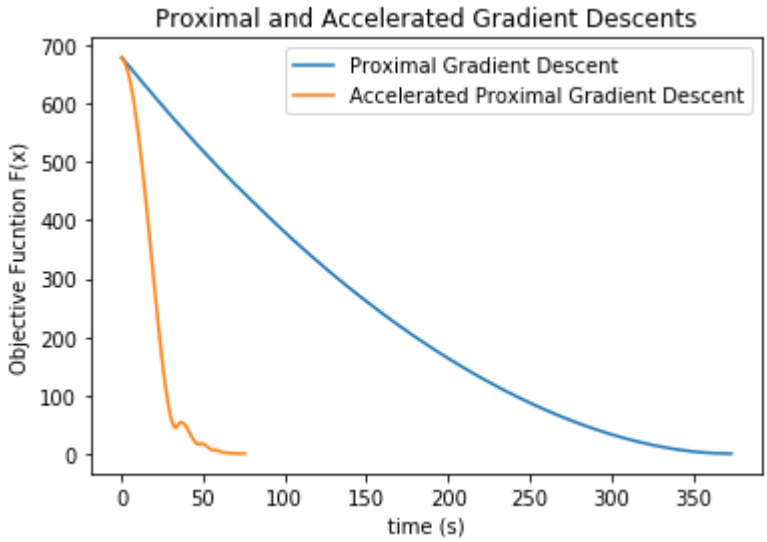
In [888]:
```
class AccelProxGradDesc():
    def __init__(self, lambda_ = initParams['lambda_']):
        self.lambda_ = lambda_
        self.epsilon = initParams['epsilon']
        self.beta = initParams['beta0']
        self.t = initParams['t']
        self.logisticRegression = LogisticRegression()

    def __str__(self):
        return 'Accelerated Proximal Gradient Descent'

    def accelProxAG(self, alpha, y, gradF):
        lambda_ = self.lambda_
        xNext = np.zeros((y.shape[0], 1))
        la = lambda_ * alpha
        u = y - lambda_ * alpha * gradF

        xNext[u >= la] = u[u >= la] - la
        xNext[np.abs(u) <= la] = 0
        xNext[u <= -la] = u[u <= -la] + la

        return xNext

    def accelProxGrad(self, xCurrent, A, b):
        beta = self.beta
        max_iters = initParams['max_iter']
        tCurrent = self.t
        yCurrent = xCurrent
        start = time.time()
        for i in range(max_iters):
            gradF, gradB = self.logisticRegression.gradient(xCurrent,
A, b, beta)
            alpha = lineSearch(beta, xCurrent, gradF, A, b)

            accelProxGradData.append([self.logisticRegression.objFunc
(xCurrent, A, b, beta), time.time() - start])
            if (i % 200 == 0):
                print('Iteration:', i, 'out of', max_iters, 'iteratio
ns')
            xNext = self.accelProxAG(alpha, yCurrent, gradF)
            tNext = (1 + np.sqrt(1 + 4 * tCurrent ** 2)) / 2
            yCurrent = xNext + ((tCurrent - 1)/tNext) * (xNext - xCur
rent)

            Gx = (xCurrent - xNext) / alpha

            if (np.linalg.norm(Gx) < self.epsilon):
                return xNext, beta
            else:
                xCurrent = xNext
                tCurrent = tNext
                beta -= alpha * gradB

        return xNext, beta
```

In [889]:
```
def calcError(x, beta):
    logisticRegression = LogisticRegression()
    bPredict = logisticRegression.predict(x, AValid_news20, beta)
    error = validationError(bValid_news20, bPredict)
    accuracy = calcAccuracy(bValid_news20, bPredict)
    return error, accuracy
```

In [890]:
```
x0 = initParams['x0_news20']
print('Starting', ProxGradDesc().__str__())
start = time.time()
xProxGrad, betaProxGrad = ProxGradDesc().proxGrad(x0, ATrain_news20,
bTrain_news20)
end = time.time() - start
print('Done', ProxGradDesc().__str__(), 'in', end, 's')
error, accuracy = calcError(xProxGrad, betaProxGrad)
print('Error:', error, 'Accuracy:', accuracy)

print()

print('Starting', AccelProxGradDesc().__str__())
start = time.time()
xAccelProxGrad, betaAccelProxGrad = AccelProxGradDesc().accelProxGrad
(x0, ATrain_news20, bTrain_news20)
end = time.time() - start
print('Done', AccelProxGradDesc().__str__(), 'in', end, 's')
error, accuracy = calcError(xAccelProxGrad, betaAccelProxGrad)
print('Error:', error, 'Accuracy:', accuracy)
```

```
Starting Proximal Gradient Descent
Iteration: 0 out of 2000 iterations
Iteration: 200 out of 2000 iterations
Iteration: 400 out of 2000 iterations
Iteration: 600 out of 2000 iterations
Iteration: 800 out of 2000 iterations
Iteration: 1000 out of 2000 iterations
Done Proximal Gradient Descent in 373.2762362957001 s
Error: [1.] Accuracy: [0.5]

Starting Accelerated Proximal Gradient Descent
Iteration: 0 out of 2000 iterations
Done Accelerated Proximal Gradient Descent in 75.85138416290283 s
Error: [1.] Accuracy: [0.5]
```

In [891]:
```
proxGradData = np.asarray(proxGradData)
accelProxGradData = np.asarray(accelProxGradData)
plt.plot(proxGradData[:, 1], proxGradData[:, 0], label = ProxGradDesc
().__str__())
plt.plot(accelProxGradData[:, 1], accelProxGradData[:, 0], label = Ac
celProxGradDesc().__str__())
plt.xlabel('time (s)')
plt.ylabel('Objective Fucntion F(x)')
plt.title('Proximal and Accelerated Gradient Descents')
plt.legend()
plt.show()
```



Did not have time to do Coordinate Descent

Question 2

Pick one algorithm that you want and plot the number of nonzeros in the output of the algorithm against λ. Explain in a couple of sentences (or less) what do you observe.

Marks: 10

In [842]:
```
class NumNonZero():
    def __init__(self, lambda_):
        self.lambda_ = lambda_
        self.epsilon = initParams['epsilon']
        self.beta = initParams['beta0']
        self.t = initParams['t']
        self.logisticRegression = LogisticRegression()

    def __str__(self):
        return 'Accelerated Proximal Gradient Descent'

    def accelProxAG(self, alpha, y, gradF):
        lambda_ = self.lambda_
        xNext = np.zeros((y.shape[0], 1))
        la = lambda_ * alpha
        u = y - lambda_ * alpha * gradF

        xNext[u >= la] = u[u >= la] - la
        xNext[np.abs(u) <= la] = 0
        xNext[u <= -la] = u[u <= -la] + la

        return xNext

    def accelProxGrad(self, xCurrent, A, b):
        beta = self.beta
        max_iters = 200
        tCurrent = self.t
        yCurrent = xCurrent
        start = time.time()
        for i in range(max_iters):
            gradF, gradB = self.logisticRegression.gradient(xCurrent,
A, b, beta)
            alpha = lineSearch(beta, xCurrent, gradF, A, b)

            xNext = self.accelProxAG(alpha, yCurrent, gradF)
            tNext = (1 + np.sqrt(1 + 4 * tCurrent ** 2)) / 2
            yCurrent = xNext + ((tCurrent - 1)/tNext) * (xNext - xCur
rent)

            Gx = (xCurrent - xNext) / alpha

            if (np.linalg.norm(Gx) < self.epsilon):
                return xNext, beta
            else:
                xCurrent = xNext
                tCurrent = tNext
                beta -= alpha * gradB

        return xNext, beta
```

In [859]:
```
lambda_list = np.power(10, np.linspace(-6, 0, 30))

numNon0X = np.zeros_like(lambda_list)

for i in range(len(lambda_list)):
    numNonZero = NumNonZero(lambda_list[i])

    print('Lambda =', lambda_list[i])
    print('Starting', numNonZero.__str__(), 'number', i, 'of', len(la
mbda_list))

    start = time.time()
    xAccelProxGrad, betaAccelProxGrad = numNonZero.accelProxGrad(x0,
ATrain_news20, bTrain_news20)
    end = time.time() - start

    numNon0X[i] = np.count_nonzero(xAccelProxGrad)

    print('Done', numNonZero.__str__(), 'number', i, 'of', len(lambda
_list), 'in', end, 's')
    print('Number of non zeros:', np.count_nonzero(xAccelProxGrad))

    error, accuracy = calcError(xAccelProxGrad, betaAccelProxGrad)

    print('Error:', error, 'Accuracy:', accuracy)

    print()
```

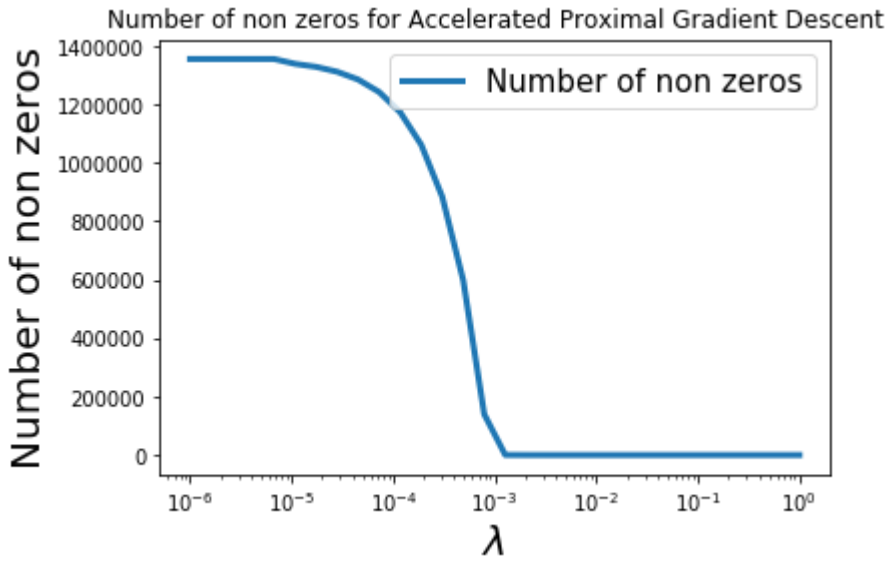
```
Lambda = 1e-06
Starting Accelerated Proximal Gradient Descent number 0 of 30
Done Accelerated Proximal Gradient Descent number 0 of 30 in 0.384655
237197876 s
Number of non zeros: 1355187
Error: [0.528] Accuracy: [0.736]

Lambda = 1.6102620275609392e-06
Starting Accelerated Proximal Gradient Descent number 1 of 30
Done Accelerated Proximal Gradient Descent number 1 of 30 in 0.354690
0749206543 s
Number of non zeros: 1355187
Error: [0.528] Accuracy: [0.736]

Lambda = 2.592943797404667e-06
Starting Accelerated Proximal Gradient Descent number 2 of 30
Done Accelerated Proximal Gradient Descent number 2 of 30 in 0.356843
9483642578 s
Number of non zeros: 1355184
Error: [0.528] Accuracy: [0.736]

Lambda = 4.1753189365604e-06
Starting Accelerated Proximal Gradient Descent number 3 of 30
Done Accelerated Proximal Gradient Descent number 3 of 30 in 0.367183
4468841553 s
Number of non zeros: 1355180
Error: [0.528] Accuracy: [0.736]

Lambda = 6.723357536499335e-06
Starting Accelerated Proximal Gradient Descent number 4 of 30
Done Accelerated Proximal Gradient Descent number 4 of 30 in 0.354260
4446411133 s
Number of non zeros: 1355177
Error: [0.528] Accuracy: [0.736]

Lambda = 1.082636733874054e-05
Starting Accelerated Proximal Gradient Descent number 5 of 30
Done Accelerated Proximal Gradient Descent number 5 of 30 in 70.87307
31010437 s
Number of non zeros: 1338571
Error: [0.835] Accuracy: [0.5825]

Lambda = 1.7433288221999873e-05
Starting Accelerated Proximal Gradient Descent number 6 of 30
Done Accelerated Proximal Gradient Descent number 6 of 30 in 71.22521
686553955 s
Number of non zeros: 1328280
Error: [0.8445] Accuracy: [0.57775]

Lambda = 2.8072162039411757e-05
Starting Accelerated Proximal Gradient Descent number 7 of 30
Done Accelerated Proximal Gradient Descent number 7 of 30 in 73.61372
73311615 s
Number of non zeros: 1311522
Error: [0.8725] Accuracy: [0.56375]

Lambda = 4.520353656360241e-05
```

```
Starting Accelerated Proximal Gradient Descent number 8 of 30
Done Accelerated Proximal Gradient Descent number 8 of 30 in 74.36868
786811829 s
Number of non zeros: 1284874
Error: [0.9135] Accuracy: [0.54325]

Lambda = 7.278953843983146e-05
Starting Accelerated Proximal Gradient Descent number 9 of 30
Done Accelerated Proximal Gradient Descent number 9 of 30 in 73.06985
092163086 s
Number of non zeros: 1242130
Error: [0.9655] Accuracy: [0.51725]

Lambda = 0.00011721022975334806
Starting Accelerated Proximal Gradient Descent number 10 of 30
Done Accelerated Proximal Gradient Descent number 10 of 30 in 70.7144
0815925598 s
Number of non zeros: 1173004
Error: [1.] Accuracy: [0.5]

Lambda = 0.00018873918221350977
Starting Accelerated Proximal Gradient Descent number 11 of 30
Done Accelerated Proximal Gradient Descent number 11 of 30 in 70.7552
11353302 s
Number of non zeros: 1062055
Error: [1.] Accuracy: [0.5]

Lambda = 0.0003039195382313198
Starting Accelerated Proximal Gradient Descent number 12 of 30
Done Accelerated Proximal Gradient Descent number 12 of 30 in 70.5761
3897323608 s
Number of non zeros: 883938
Error: [1.] Accuracy: [0.5]

Lambda = 0.0004893900918477494
Starting Accelerated Proximal Gradient Descent number 13 of 30
Done Accelerated Proximal Gradient Descent number 13 of 30 in 70.4008
5554122925 s
Number of non zeros: 597989
Error: [1.] Accuracy: [0.5]

Lambda = 0.0007880462815669912
Starting Accelerated Proximal Gradient Descent number 14 of 30
Done Accelerated Proximal Gradient Descent number 14 of 30 in 68.6278
0714035034 s
Number of non zeros: 139617
Error: [1.] Accuracy: [0.5]

Lambda = 0.0012689610031679222
Starting Accelerated Proximal Gradient Descent number 15 of 30
Done Accelerated Proximal Gradient Descent number 15 of 30 in 57.0101
9215583801 s
Number of non zeros: 263
Error: [1.] Accuracy: [0.5]

Lambda = 0.0020433597178569417
Starting Accelerated Proximal Gradient Descent number 16 of 30
```

```
Done Accelerated Proximal Gradient Descent number 16 of 30 in 43.5631
00814819336 s
Number of non zeros: 12
Error: [1.] Accuracy: [0.5]

Lambda = 0.0032903445623126675
Starting Accelerated Proximal Gradient Descent number 17 of 30
Done Accelerated Proximal Gradient Descent number 17 of 30 in 30.7975
84533691406 s
Number of non zeros: 0
Error: [1.] Accuracy: [0.5]

Lambda = 0.005298316906283708
Starting Accelerated Proximal Gradient Descent number 18 of 30
Done Accelerated Proximal Gradient Descent number 18 of 30 in 22.7023
16761016846 s
Number of non zeros: 0
Error: [1.] Accuracy: [0.5]

Lambda = 0.008531678524172805
Starting Accelerated Proximal Gradient Descent number 19 of 30
Done Accelerated Proximal Gradient Descent number 19 of 30 in 15.9521
72756195068 s
Number of non zeros: 0
Error: [1.] Accuracy: [0.5]

Lambda = 0.013738237958832637
Starting Accelerated Proximal Gradient Descent number 20 of 30
Done Accelerated Proximal Gradient Descent number 20 of 30 in 12.1214
38980102539 s
Number of non zeros: 0
Error: [1.] Accuracy: [0.5]

Lambda = 0.022122162910704502
Starting Accelerated Proximal Gradient Descent number 21 of 30
Done Accelerated Proximal Gradient Descent number 21 of 30 in 8.55515
9091949463 s
Number of non zeros: 0
Error: [1.] Accuracy: [0.5]

Lambda = 0.03562247890262444
Starting Accelerated Proximal Gradient Descent number 22 of 30
Done Accelerated Proximal Gradient Descent number 22 of 30 in 6.21954
607963562 s
Number of non zeros: 0
Error: [1.] Accuracy: [0.5]

Lambda = 0.05736152510448681
Starting Accelerated Proximal Gradient Descent number 23 of 30
Done Accelerated Proximal Gradient Descent number 23 of 30 in 4.27811
9087219238 s
Number of non zeros: 0
Error: [1.] Accuracy: [0.5]

Lambda = 0.09236708571873865
Starting Accelerated Proximal Gradient Descent number 24 of 30
Done Accelerated Proximal Gradient Descent number 24 of 30 in 3.21043
```

```
4675216675 s
Number of non zeros: 0
Error: [1.] Accuracy: [0.5]

Lambda = 0.14873521072935117
Starting Accelerated Proximal Gradient Descent number 25 of 30
Done Accelerated Proximal Gradient Descent number 25 of 30 in 2.51329
56504821777 s
Number of non zeros: 0
Error: [1.] Accuracy: [0.5]

Lambda = 0.2395026619987486
Starting Accelerated Proximal Gradient Descent number 26 of 30
Done Accelerated Proximal Gradient Descent number 26 of 30 in 1.59848
0224609375 s
Number of non zeros: 0
Error: [1.] Accuracy: [0.5]

Lambda = 0.38566204211634725
Starting Accelerated Proximal Gradient Descent number 27 of 30
Done Accelerated Proximal Gradient Descent number 27 of 30 in 1.25519
82402801514 s
Number of non zeros: 0
Error: [1.] Accuracy: [0.5]

Lambda = 0.6210169418915616
Starting Accelerated Proximal Gradient Descent number 28 of 30
Done Accelerated Proximal Gradient Descent number 28 of 30 in 0.92773
4375 s
Number of non zeros: 0
Error: [1.] Accuracy: [0.5]

Lambda = 1.0
Starting Accelerated Proximal Gradient Descent number 29 of 30
Done Accelerated Proximal Gradient Descent number 29 of 30 in 0.59047
38903045654 s
Number of non zeros: 0
Error: [1.] Accuracy: [0.5]
```

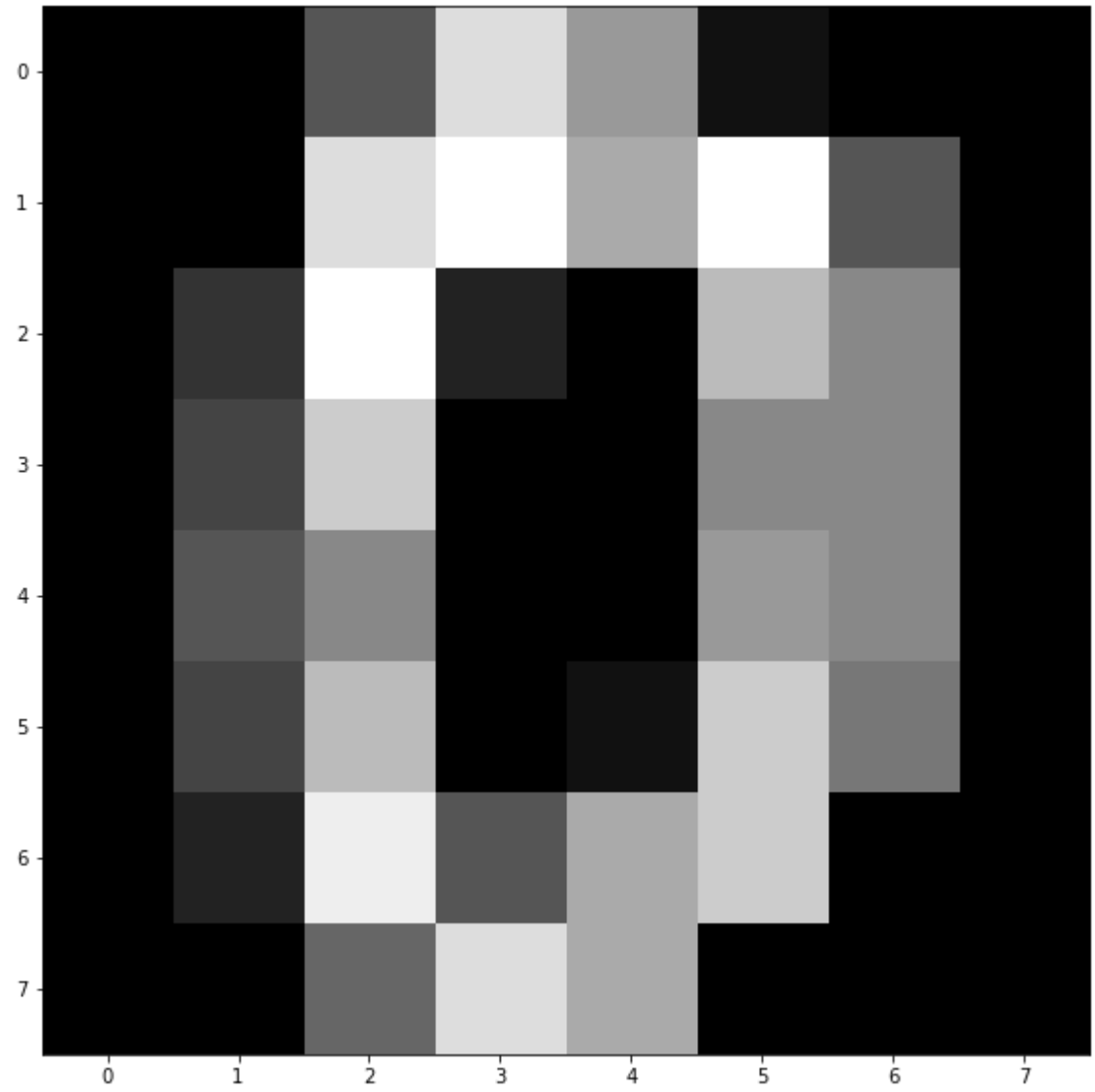
In [860]:
```
plt.plot(lambda_list, numNon0X, label = "Number of non zeros", linewi
dth = 3)
plt.legend(fontsize = 15)
plt.xlabel("$\lambda$", fontsize = 20)
plt.ylabel("Number of non zeros", fontsize = 20)
plt.xscale('log')
plt.title('Number of non zeros for Accelerated Proximal Gradient Desc
ent')
plt.show()
```



From the graph and the printed data, we can see that the number of non zero outputs goes to zero as lambda goes to increases. This is expected since lambda is the weight for the regulization term. As we are minimizing the objective function, the regulization term will be at its lowest when x is all zeros. So if we increase lambda, the minimum of the function will most likely be found when x is all zeros.

Label Propagation

In [892]:
```
# Load datasets from sklearn
from sklearn import datasets

# Load the digits dataset.
# See here for a distription of this dataset:
# https://scikit-learn.org/stable/modules/generated/sklearn.datasets.
load_digits.html#sklearn.datasets.load_digits
digits = datasets.load_digits()

# Briefly, this dataset has 1797 images of digits. The digits are fro
m 0 to 9.
# There are about 180 images per digit.

# digits.data includes the vectorized images of the digits
# digits.target includes the labels of the images
# digits.images includes the images in non-vectorized format

# Example: number 0
import matplotlib.pyplot as plt
plt.figure(1, figsize=(10, 10))
plt.imshow(digits.images[0], cmap='gray')
plt.show()
```



Let x_i and x*j be two vectorized images. Compute their pairwise similarity by $$ w*{ij} = \exp(-\gamma |x_i-x_j|_2^2), $$ where γ is parameter tha controls the importance of the distance $|x_i - x_j|_2^2$. If $\gamma = 1/(2\sigma^2)$ then this is called the Gaussian kernel and σ^2 represents the variance. You can compute the similarities by using: https://scikit-learn.org/stable/modules/generated/sklearn.metrics.pairwise.rbf_kernel.html (https://scikit-learn.org/stable/modules/generated/sklearn.metrics.pairwise.rbf_kernel.html). Alterantively, you can compute the pairwise similarities manually, by using the above formula.

Sparsify the graph by setting small values to zero. The thresholding tolerance is a parameter that you will have to tune. Generally, there is a consensus that sparse similarity matrices result in better performance. So, you should expect that, but have in mind that this is only an empirical observation.

In [893]:
```
# Example
# Pairwise weights matrix
W = sklearn.metrics.pairwise.rbf_kernel(digits.data)

# Remove self-similarity
W = W - np.eye(len(W))

# Sparsify (tune the tolerance parameter for your problem)
tol = 1e-9
W[abs(W) < tol] = 0.0
```

This dataset provides labels for every image of a digit. However, in this assignment we want to check how the label propagation model works. Use all images, i.e., 1797 images of digits, but use only 10 labels per type of digit, e.g., use the first 10 labels for digit 0, the first 10 labels of digit 1 etc. This means that you should use 100 labels in total out of 1797. Do not use the remaining labels for training the model. The remaining labels will be used only to measure accuracy of the model.

In [894]:
```
# digits.data includes the vectorized images of the digits
# digits.target includes the labels of the images
# digits.images includes the images in non-vectorized format

def createDataSet(digits, size, numbers = 10):
    knownSetLabel = np.array([])
    knownSetInd = np.array([])
    unknownSetInd = np.array([])
    for i in range(numbers):
        knownSetInd = np.append(knownSetInd, np.where(digits.target =
= i)[0][: size])

        unknownSetInd = np.append(unknownSetInd, np.where(digits.targ
et == i)[0][size :])
    knownSetLabel = np.append(knownSetLabel, digits.target[knownSetIn
d.astype(int)])
    return knownSetInd, knownSetLabel, unknownSetInd
```

Question 3

Train the label propagation model

$$\text{minimize}_{b\in\mathbb{R}^n} \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}(b_i - b_j)^2 + \sum_{i=1}^{m-n}\sum_{j=1}^{n} \bar{w}_{ij}(\bar{b}_i - b_j)^2$$

using the coordinate method that was discussed in Lecture 14 for the label propagation problem. Make sure that you understand the derivation of the algorithm before you apply it to train the label propagation model.

The output of the algorithm should be the soft labels b for each image. However, these soft labels are not going to be integers (actual labels). Therefore, we have to round the output labels b to get integer labels for each image. One possible way to round the output is to simply find the integer label (possible options 0 to 9) that a soft label is closest to. Then pick this label as the label of the image.

After you label all images, use the function in https://scikit-learn.org/stable/modules/generated/sklearn.metrics.precision_recall_fscore_support.html (https://scikit-learn.org/stable/modules/generated/sklearn.metrics.precision_recall_fscore_support.html) to measure precision, recall and f1_score. You only have to provide the true and the predicted labels when using this function. Do not change the default parameters.

If you want to learn more about precision, recall and f1_score check Wikipedia: precision and recall: https://en.wikipedia.org/wiki/Precision_and_recall (https://en.wikipedia.org/wiki/Precision_and_recall) f1_score: https://en.wikipedia.org/wiki/F1_score (https://en.wikipedia.org/wiki/F1_score).

Report precision, recall and f1score of your method.

Marks: 20

In [895]:
```
def labelProp(bBar, bBarI, bI, W, max_iter = 200):
    b = np.random.randint(1, 10, W.shape[0]).astype(float)
    b[bBarI.astype(int)] = bBar
    for i in range(max_iter):
        for j in bI:
            j = int(j)
            b[j] = np.dot(W[:, j], b) / np.sum(W[:, j])
    b = np.round(b).astype(int)

    return b
```

In [896]:
```
import pprint
bBarI, bBar, bI = createDataSet(digits, 10)
b = labelProp(bBar, bBarI, bI, W)

precision, recall, f1_score, support = sklearn.metrics.precision_recall_fscore_support(digits.target, b)
pd.DataFrame(data = {'Precision': precision, 'Recall': recall, 'f1 Score': f1_score, 'Num of Labels': support})
```

Out[896]:

	Precision	Recall	f1 Score	Num of Labels
0	1.000000	0.994382	0.997183	178
1	0.960000	0.263736	0.413793	182
2	0.593750	0.107345	0.181818	177
3	0.430341	0.759563	0.549407	183
4	0.729167	0.966851	0.831354	181
5	0.541254	0.901099	0.676289	182
6	0.648746	1.000000	0.786957	181
7	0.491124	0.927374	0.642166	179
8	0.466667	0.120690	0.191781	174
9	1.000000	0.055556	0.105263	180

Question 4

Increase the number of labels that are used in the model from 10 per digit to 20 per digit. Re-train and report precision, recall and f1score of your method. Explain in a couple of sentences what do you observe.

Marks: 5

In [897]:
```
bBarI, bBar, bI = createDataSet(digits, 20)
b = labelProp(bBar, bBarI, bI, W)

precision, recall, f1_score, support = sklearn.metrics.precision_recall_fscore_support(digits.target, b)
pd.DataFrame(data = {'Precision': precision, 'Recall': recall, 'f1 Score': f1_score, 'Num of Labels': support})
```

Out[897]:

	Precision	Recall	f1 Score	Num of Labels
0	1.000000	0.994382	0.997183	178
1	0.953125	0.335165	0.495935	182
2	0.745614	0.960452	0.839506	177
3	0.659898	0.710383	0.684211	183
4	0.660377	0.966851	0.784753	181
5	0.826389	0.653846	0.730061	182
6	0.628472	1.000000	0.771855	181
7	0.666667	0.927374	0.775701	179
8	0.328000	0.235632	0.274247	174
9	0.950000	0.316667	0.475000	180

Local Graph Clustering

In [898]:
```
import numpy as np

# Import networkx to load the data
import networkx as nx

# Provide the path of the dataset JohnsHopkins55
path = 'JohnsHopkins55.graphml'

# This dataset is a Facebook anonymized dataset on a particular day i
n September 2005
# for a student social network at John Hopkins university. The graph
 is unweighted
# and it represents ``friendship'' ties. This graph has 5157 nodes an
d 186572 edges.
# This dataset comes along with 6 features, i.e., second major, high
 school, gender, dorm, major index and year.
# We construct ``ground truth'' clusters by using the features for ea
ch node.
# In particular, we consider nodes with the same value of a feature t
o be a cluster, e.g., students of year 2009.
# For this assignment we will attempt to find the students of year 20
09 using local graph clustering.

# Load the data as a graph
G = nx.read_graphml(path).to_undirected()

# Construct the adjacency matrix
A = nx.adjacency_matrix(G).astype(np.float64)
ALocalGraphCluster = A
# Indices of nodes (students) of class year 2009
# There should be 910 students of class year 2009.
# We will use these students to measure performance of the model
students_of_year_2009 = []
ct = 0
for node in G.nodes(data=True):
    if node[1]['year'] == 2009:
        students_of_year_2009.append(ct)
    ct += 1
print(len(students_of_year_2009))
```

```
910
```

The target cluster is the students of year 2009 in the list: students_of_year_2009.

For this problem you will have to use the coordinate algorithm that was introduced in Lecture 14.

The coordinate algorithm returns a vector p. To get an actual cluster from vector p you will have to round the output of the coordinate algorithm using the rounding technique that was discussed in Lecture 14.

To measure accuracy of the rounded output you can use number of true positives and the number of false positives of the predicted cluster. True positives are the predicted nodes that are also part of class year 2009. False positive are the number of predicted nodes that are not part of the class year 2009.

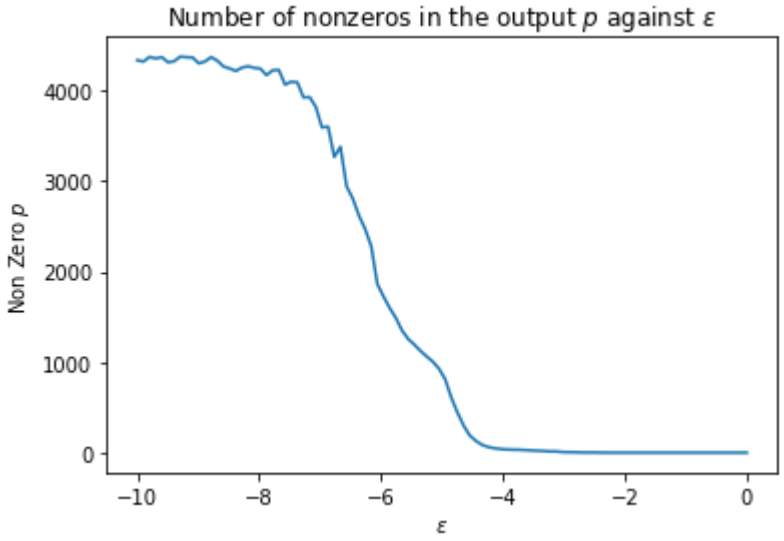
In [899]:
```
#A: Adjacency Martix
#degree: array of degrees of each nodes

def localGraphCluster(A, degree, seed, alpha, e, max_iters = 10000):
    n = A.shape[0]
    p = np.zeros(n)
    r = np.zeros(n)
    nodes = {seed}
    r[seed] = 1
    i = 0
    while i < max_iters and len(nodes) > 0:
        u = random.sample(nodes, 1)[0]
        p[u] = p[u] + alpha * r[u]
        v = A[u].indices
        r[v] += (0.5 * (1 - alpha) * r[u]) / degree[u]
        r[u] *= 0.5 * (1 - alpha)

        if r[u] < e * degree[u]:
            nodes.remove(u)

        nodes.update({i for i in v if r[i] >= e * degree[i]})
        i += 1
    return p

def getCluster(p, A, degree):
    non0Nodes = np.argwhere(p > 0).flatten()
    non0Nodes = non0Nodes[np.argsort(-p[non0Nodes]/degree[non0Nodes
])]
    edges = np.zeros_like(p)
    degreeSum = 0
    conductances = []

    for i, node in enumerate(non0Nodes):
        neighbours = A[node].indices
        degreeSum += len(neighbours)
        edges[neighbours] = 1
        edges[non0Nodes[: i + 1]] = 0
        conductances.append(sum(edges) / degreeSum)
    cluster = non0Nodes[: np.argmin(conductances) + 1]
    return cluster
```

Question 5

1) Report the number of nonzero nodes in the output of the coordinate algorithm (vector p) as the tolerance parameter ϵ goes to zero. In particular, plot the number of nonzeros in the output p against ϵ. The number of nonzeros should increase as ϵ goes to zero.

Marks: 10

In [900]:
```
seed = 5000
alpha = 0.1
non0P = []
A = ALocalGraphCluster
#epsilon from 1e-10 to 1
epsilonList = np.power(10, np.linspace(0, -10, 100))
degree = np.array([A[i].sum() for i in range(A.shape[0])], dtype = in
t)

for epsilon in epsilonList:
    p = localGraphCluster(A, degree, seed, alpha, epsilon)
    non0P.append(p[p > 0].size)

plt.figure()
plt.plot(np.log10(epsilonList), non0P)
plt.xlabel('$\epsilon$')
plt.ylabel('Non Zero $p$')
plt.title('Number of nonzeros in the output $p$ against $\epsilon$')
plt.show()
```



Question 6

2) Run the coordinate algorithm using as seed node the node with index 5000. Round the output to get an actual cluster back. Measure true positives and false positives of the predicted cluster. Report these two numbers.

Marks: 10

In [901]:
```
epsilon = 1e-5
seed5000 = 5000
alpha = 0.1
p5000 = localGraphCluster(A, degree, seed5000, alpha, epsilon)
cluster = getCluster(p5000, A, degree)

truePos = 0
falsePos = 0
for i in cluster:
    if i in students_of_year_2009:
        truePos += 1
    else:
        falsePos += 1

print('True and False positives for seed:', seed5000)
print('True Positive:', truePos)
print('False Positive:', falsePos)
```

```
True and False positives for seed: 5000
True Positive: 744
False Positive: 71
```

Question 7

3) Run the coordinate algorithm using as seed node the node with index 146. Round the output to get an actual cluster back. Measure true positives and false positive of the predicted cluster. Report these two numbers. How do they compare to the results in 2) ? Are they different? If yes, why do you think this might have happened? Explain intuitively in a couple of sentences why you think this might happen.

Marks: 5

In [902]:
```
seed146 = 146
p146 = localGraphCluster(A, degree, seed146, alpha, epsilon)
cluster = getCluster(p146, A, degree)

truePos = 0
falsePos = 0
for i in cluster:
    if i in students_of_year_2009:
        truePos += 1
    else:
        falsePos += 1
print('True and False positives for seed:', seed146)
print('True Positive:', truePos)
print('False Positive:', falsePos)
```

```
True and False positives for seed: 146
True Positive: 572
False Positive: 111
```

Compare nodes 5000 and 146

Yes the results for Seed node 5000 and Seed node 146 are indeed different. The number of true positives decrease, and we get more False Positives when we start with node 146. This is because node 146 only has 3 neightbours, thus we miss out on more classes

Reccomender System

Download the data matrix A from the webiste https://grouplens.org (https://grouplens.org). Navigate to the dataset tab https://grouplens.org/datasets/movielens/ (https://grouplens.org/datasets/movielens/) and download the dataset MovieLens 100K. Read the description of the dataset carefully. This dataset includes 5 data matrices A. We only need to use one of these in this assignment. In particular, use the dataset u1.base. Use the corresponding testing data u1.test to measure the accuracy of your model. For this question only it's ok if you use the testing data to tune parameters. Ignore the timestap feature in the dataset. We do not need this for this problem.

In [903]:
```
train = np.loadtxt('ml-100k/u1.base')[:, :3]
test = np.loadtxt('ml-100k/u1.test')[:, :3]

def getA(data, n, m):
    data[:, :2] = data[:, :2] - 1
    A = np.zeros((n, m))
    for i in range(len(data)):
        user = int(data[i][0])
        movie = int(data[i][1])
        rating = data[i][2]
        A[user][movie] = rating
    return A
ATrainRS = getA(train, 943, 1682)
ATestRS = getA(test, 943, 1682)
x0 = np.random.randint(1, 6, ATrainRS.shape)

knownValues = ATrainRS != 0
knownTestValues = ATestRS != 0
```

Question 8

Solve the penalized problem

$$\text{minimize } \lambda\|X\|_* + \sum_{ij \mid A_{ij}\, is\ known} (X_{ij} - A_{ij})^2,$$

where $\|X\|_*$ is the nuclear norm, using proximal gradient descent. This algorithm requires thresholding the singular values of X at each iteration. You can use https://docs.scipy.org/doc/scipy/reference/generated/scipy.linalg.svd.html (https://docs.scipy.org/doc/scipy/reference/generated/scipy.linalg.svd.html) (sparse matrices) or https://docs.scipy.org/doc/numpy/reference/generated/numpy.linalg.svd.html (https://docs.scipy.org/doc/numpy/reference/generated/numpy.linalg.svd.html) (dense matrices).

To check your solution against u1.test, report your score as follows. Let X be the output of the proximal gradient method, let A' and Ω' be the data matrix and the non-zero entry list for the test data. Then the score is

$$\frac{1}{|\Omega'|} \sum_{(i,j)\in\Omega'} |X_{ij} - A'_{ij}|.$$

In other words, the average absolute deviation of the computed ratings from the predicted ratings. Report your score.

Marks: 15

In [904]:
```
def prox(lambda_, alpha, xCurrent, gradF):
    z = xCurrent - alpha * gradF
    u, s, vT = np.linalg.svd(z)

    s = np.maximum(s - alpha * lambda_, 0)

    sDense = np.zeros((u.shape[1], vT.shape[0]))
    sDense[:len(s), :len(s)] = np.diag(s)

    xNext = (u.dot(sDense)).dot(vT)
    return xNext, s

def lineSearchRS(xCurrent, A, gradF):
    alpha = initParams['alpha0']
    theta = initParams['theta']
    lambda_ = 50

    rs = ReccomenderSystem()

    #F(x_k)
    s = np.linalg.svd(xCurrent, compute_uv = False)
    F_x_k = rs.objFunc(xCurrent, A, s)

    #l(x_k)
    l_x_k = F_x_k

    #x(alpha)
    x_alpha, s = prox(lambda_, alpha, xCurrent, gradF)

    #F(x(alpha))
    F_x_alpha = rs.objFunc(x_alpha, A, s)

    #l(x(alpha))
    l_x_alpha = rs.gx(x_alpha, s) + rs.fx(xCurrent, A) + (gradF * (x_
alpha - xCurrent)).sum()

    while (F_x_alpha > F_x_k - theta * (l_x_k - l_x_alpha)):
        alpha /= 2

        #x(alpha)
        x_alpha, s = prox(lambda_, alpha, xCurrent, gradF)

        #F(x(alpha))
        F_x_alpha = rs.objFunc(x_alpha, A, s)

        #l(x(alpha))
        l_x_alpha = rs.gx(x_alpha, s) + rs.fx(xCurrent, A) + (gradF *
(x_alpha - xCurrent)).sum()

    return alpha

def error(x, ATest):
    x = np.round(x)
    error = np.linalg.norm(x[knownTestValues] - ATest[knownTestValues
], 1)/knownTestValues.sum()
    return error
```

In [905]:
```
class ReccomenderSystem():
    def __init__(self):
        self.lambda_ = 50 #initParams["lambda_"]

    def __str__(self):
        return 'Reccomender System'

    def gx(self, X, s):
        lambda_ = self.lambda_
        return lambda_ * np.sum(s)

    def fx(self, X, A):
        return np.sum((X[knownValues] - A[knownValues]) ** 2)

    def objFunc(self, X, A, s):
        g_x = self.gx(A, s)
        f_x = self.fx(X, A)
        return g_x + f_x

    def gradient(self, X, A):
        gradF = np.zeros_like(A)
        gradF[knownValues] = 2 * (X[knownValues] - A[knownValues])
        return gradF
```

In [906]:
```
def penalizer(xCurrent, A):
    rs = ReccomenderSystem()
    max_iters = 200
    lambda_ = 10
    for i in range(max_iters):

        gradF = rs.gradient(xCurrent, A)
        alpha = lineSearchRS(xCurrent, A, gradF)

        xNext, s = prox(lambda_, alpha, xCurrent, gradF)
        if (i % 20 == 0):
            print('Iteration:', i, 'out of', max_iters, 'iterations')
            print('Objective Function:', rs.objFunc(xNext, A, s))

        Gx = (xCurrent - xNext) / alpha

        if (np.linalg.norm(Gx) < 1e-2):
            return xNext
        else:
            xCurrent = xNext

    return xNext
```

In [667]:
```
rs = ReccomenderSystem()
x0[knownValues] = ATrainRS[knownValues]
x = penalizer(x0, ATrainRS)
score = error(x, ATest)
print('Score:', score)
```

```
Iteration: 0 out of 200 iterations
Objective Function: 2224034.5471557504
Iteration: 20 out of 200 iterations
Objective Function: 342708.27298988577
Iteration: 40 out of 200 iterations
Objective Function: 332658.91948083654
Iteration: 60 out of 200 iterations
Objective Function: 327585.0407599081
Iteration: 80 out of 200 iterations
Objective Function: 324214.87051402545
Iteration: 100 out of 200 iterations
Objective Function: 321716.4282558738
Iteration: 120 out of 200 iterations
Objective Function: 319755.3231732809
Iteration: 140 out of 200 iterations
Objective Function: 318164.5651491285
Iteration: 160 out of 200 iterations
Objective Function: 316847.3353249789
Iteration: 180 out of 200 iterations
Objective Function: 315742.0705507721
Score: 0.7265
```

Question 9

Now consider the factorized problem

$$\text{minimize}_{L,R} \ \frac{\lambda}{2}\left(\|L\|_F^2 + \|R\|_F^2\right) + \|W \circ (LR^T - A)\|_F^2$$

See Lecture 15 about details for this problem. Solve this problem and obtain the matrix $X = LR^T$. Use X as the predicted rating and report your score. For calculating derivatives of matrix functions you can have a look here: https://www.math.uwaterloo.ca/~hwolkowi/matrixcookbook.pdf (https://www.math.uwaterloo.ca/~hwolkowi/matrixcookbook.pdf).

Marks: 10

In [907]:
```
A = ATrainRS
m, n = A.shape
W = np.zeros_like(A)
W[knownValues] = 1
k = 100
L0 = np.random.rand(m, k)
R0 = np.random.rand(n, k)
class FactorizedReccomenderSystem():
    def __init__(self, lambda_):
        self.lambda_ = lambda_

    def __str__(self):
        return 'Factorized Reccomender System'

    def objFunc(self, L, R, W, A):
        lambda_ = self.lambda_
        Lnorm = np.linalg.norm(L, ord = 'fro') ** 2
        Rnorm = np.linalg.norm(R, ord = 'fro') ** 2
        right = np.linalg.norm(W * (L @ R.T - A), ord = 'fro') ** 2
        return (lambda_ / 2) * (Lnorm + Rnorm) + right

    def gradL(self, L, R, W, A):
        lambda_ = self.lambda_
        return lambda_ * L + 2 * (W * (L @ R.T - A)) @ R

    def gradR(self, L, R, W, A):
        lambda_ = self.lambda_
        return lambda_ * R + 2 * ((W * (L @ R.T - A)).T @ L)

    def LLineSearch(self, L, R, W, A):
        lambda_ = self.lambda_
        alpha = 1
        f_x_k = objFunc(lambda_, L, R, W, A)
        gL = gradL(lambda_, L, R, W, A)
        LNext = L - alpha * gL
        while(objFunc(lambda_, LNext, R, W, A) > f_x_k):
            alpha /= 2
            LNext = L - alpha * gL
        return alpha

    def RLineSearch(self, L, R, W, A):
        lambda_ = self.lambda_
        alpha = 1
        f_x_k = objFunc(lambda_, L, R, W, A)
        gR = gradR(lambda_, L, R, W, A)
        RNext = R - alpha * gR
        while(objFunc(lambda_, L, RNext, W, A) > f_x_k):
            alpha /= 2
            RNext = R - alpha * gR
        return alpha

    def factorizedPenalizer(self, L, R, W, A):
        lambda_ = self.lambda_
        max_iters = initParams['max_iter']
        for i in range(max_iters):
            #Fix L and solve wrt R
```

```
            alphaR =  RLineSearch(lambda_, L, R, W, A)
            gR = gradR(lambda_, L, R, W, A)

            R = R - alphaR * gR

            #Fix R and solve wrt L
            alphaL = LLineSearch(lambda_, L, R, W, A)
            gL = gradL(lambda_, L, R, W, A)
            L = L - alphaL * gL

            if (i % 200 == 0):
                print('Iteration:', i, 'out of', max_iters, 'iteratio
ns')
                print('Objective Function:', objFunc(lambda_, L, R, W
, A))

        return L, R
```

In [908]:
```
lambda_ = 10
factorizedReccomenderSystem = FactorizedReccomenderSystem(lambda_)
L, R = factorizedReccomenderSystem.factorizedPenalizer(L0, R0, W, A)
x = L @ R.T
score = error(x, ATest)
print('Score:', score)
```

```
Iteration: 0 out of 2000 iterations
Objective Function: 10669077.629436143
Iteration: 200 out of 2000 iterations
Objective Function: 81709.61436792536
Iteration: 400 out of 2000 iterations
Objective Function: 72281.12131890278
Iteration: 600 out of 2000 iterations
Objective Function: 70697.90595178634
Iteration: 800 out of 2000 iterations
Objective Function: 70166.04706762842
Iteration: 1000 out of 2000 iterations
Objective Function: 69894.50762842332
Iteration: 1200 out of 2000 iterations
Objective Function: 69730.00617448597
Iteration: 1400 out of 2000 iterations
Objective Function: 69620.10686353974
Iteration: 1600 out of 2000 iterations
Objective Function: 69541.97711507631
Iteration: 1800 out of 2000 iterations
Objective Function: 69484.5006241859
Score: 0.75265
```

Nonnegative Matrix Factorization: facial feature extraction

In [909]:
```
# This piece of code is for loading data and visualizing
# the first 6 images in the dataset.

# Useful packages for loading the data and plotting
from numpy.random import RandomState
import matplotlib.pyplot as plt
from sklearn.datasets import fetch_olivetti_faces

n_row, n_col = 2, 3
image_shape = (64, 64)
rng = RandomState(0)

# Useful function for plotting
def plot_gallery(title, images, n_col=n_col, n_row=n_row, cmap=plt.cm
.gray):
    plt.figure(figsize=(2. * n_col, 2.26 * n_row))
    plt.suptitle(title, size=16)
    for i, comp in enumerate(images):
        plt.subplot(n_row, n_col, i + 1)
        vmax = max(comp.max(), -comp.min())
        plt.imshow(comp.reshape(image_shape), cmap=cmap,
                   interpolation='nearest',
                   vmin=-vmax, vmax=vmax)
        plt.xticks(())
        plt.yticks(())
    plt.subplots_adjust(0.12, 0.05, 0.99, 0.75, 0.04, 0.)

# Load faces data
dataset = fetch_olivetti_faces(shuffle=True, random_state=rng)

# Store the vectorized images. Each image has dimensions 64 x 64.
faces = dataset.data

print("Dataset consists of %d faces" % len(faces))

plot_gallery("First 6 faces from the dataset", faces[:6], 6, 1)
```

```
Dataset consists of 400 faces
```

In [910]:
```
class NonNegMatFac():
    def __init__(self, data):
        self.sigma = initParams['sigma']
        self.delta = initParams['delta']
        self.X = data.T

    def __str__(self):
        return 'Non-negative Matrix Factorization'

    def objFunc(self, W, H):
        X = self.X
        return 0.5 * np.linalg.norm(W @ H - X, ord='fro') ** 2

    def gradH(self, W, H):
        X = self.X
        return W.T @ (W @ H - X)

    def gradW(self, W, H):
        X = self.X
        return (W @ H - X) @ H.T

    def HBar(self, W, H):
        gradH = self.gradH(W, H)
        HBar = np.where(gradH >= 0, H, np.maximum(H, self.sigma * np.
ones((H.shape))))
        return HBar

    def WBar(self, W, H):
        gradW = self.gradW(W, H)
        WBar = np.where(gradW >= 0, W, np.maximum(W, self.sigma * np.
ones((W.shape))))
        return WBar

    def bars(self, W, H):
        HBar = self.HBar(W, H)
        WBar = self.WBar(W, H)
        return HBar, WBar
```

Question 10

Use the data in "faces" from the above code for the nonnegative matrix factorization problem

$$\begin{aligned} &\text{minimize}_{W,H} \ \|WH - X\|_F^2 \\ &\text{subj. to } W_{ij} \geq 0 \ \forall i,j \\ &\quad H_{ij} \geq 0 \ \forall i,j \end{aligned}$$

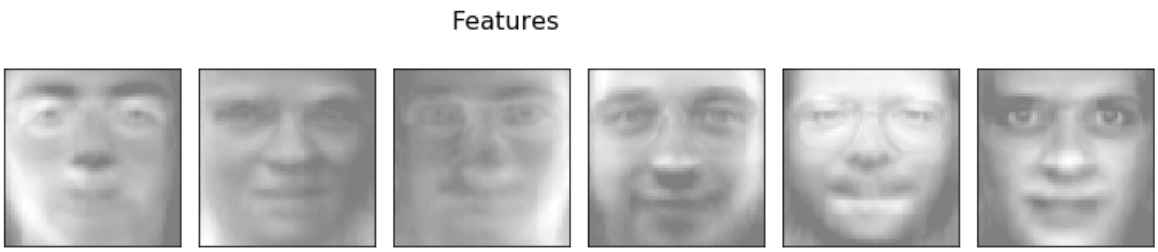
where $W \in \mathbb{R}^{m \times r}$ and $H \in \mathbb{R}^{r \times n}$. Set parameter $r = 6$ in the nonnegative factorization problem. Extract the features matrix W by solving the non-negative matrix factorization problem. Report the 6 features of the faces dataset, i.e., the 6 columns of matrix W. You can report the features by visualizing them in a similar way to the above example.

Marks: 15

In [911]:
```
def nonNegMatFac(data, r = 6):
    m, n = data.T.shape
    max_iters = initParams['max_iter']
    nnmf = NonNegMatFac(data)
    W = np.random.uniform(0, 1, size = (m, r))
    H = np.random.uniform(0, 1, size = (r, n))
    delta = initParams['delta']

    for i in range(max_iters):
        HBar, WBar = nnmf.bars(W, H)

        H = H - np.divide(HBar, ((W.T @ W @ HBar) + delta)) * nnmf.gr
adH(W, H)
        W = W - np.divide(WBar, ((WBar @ H @ HBar.T) + delta)) * nnmf
.gradW(W, H)

        #Normalize
        S = np.eye(r) / np.sum(W, 0)
        W = W @ S
        H = np.linalg.inv(S) @ H
        if (i % 200 == 0):
            print('Iteration:', i, 'out of', max_iters, 'iterations')
            print('Objective Function:', nnmf.objFunc(W, H))
    return W, H
```

In [912]:
```
W, H = nonNegMatFac(faces)
plot_gallery("Features", W.T[:6], 6, 1)
```

```
Iteration: 0 out of 2000 iterations
Objective Function: 25793.681538200235
Iteration: 200 out of 2000 iterations
Objective Function: 7010.241055904761
Iteration: 400 out of 2000 iterations
Objective Function: 6910.017278901699
Iteration: 600 out of 2000 iterations
Objective Function: 6891.487934288256
Iteration: 800 out of 2000 iterations
Objective Function: 6882.712619091637
Iteration: 1000 out of 2000 iterations
Objective Function: 6877.521901832734
Iteration: 1200 out of 2000 iterations
Objective Function: 6873.825724841319
Iteration: 1400 out of 2000 iterations
Objective Function: 6871.02090566685
Iteration: 1600 out of 2000 iterations
Objective Function: 6868.694681355642
Iteration: 1800 out of 2000 iterations
Objective Function: 6866.78572314461
```

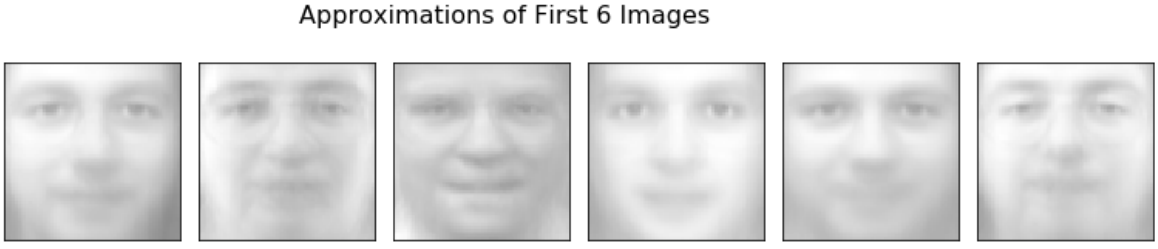


Question 11

Visualize the approximation of the first 6 images using the feature matrix W and the importance matrix H. Note that each face can be approaximated by a linear combination of features (columns) from matrix W. The positive coefficients of this linear combination are given by matrix H.

Marks: 5

In [913]:
```
WH = W @ H
plot_gallery('Approximations of First 6 Images', WH.T[:6], 6, 1)
```



Question 12

Use matrix H to find the 3 most important features of the first 6 faces. To find the 3 most important features for an image pick the columns of W with the largest coefficients in matrix H.

Marks: 5

In [914]:
```
first6 = faces[: 6]
top3Features = []
for i in range(6):
    top3Features.append(np.argsort(-H[:,i])[:3])
top3Features = np.asarray(top3Features)
for i in range(3):
    first6 = np.append(first6, W.T[top3Features.T[i]], 0)
for i in range(4):
    if (i == 0):
        plot_gallery('Original', first6[0: (i + 1) * 6], 6, 1)
    else:
        plot_gallery('Feature ' + str(i), first6[i * 6: (i + 1) * 6],
6, 1)
```