COMP 478/6771 Assignment 3 solutions – Fall 2024

Question 1. (16 points = 8 points for Part a and 8 points for Part b)

(a) From Section 2.6, we know that an operator, O, is linear if $O(af_1 + bf_2) = aO(f_1) + bO(f_2)$. From the definition of the Radon transform in Eq. (5.11-3),

$$
\begin{aligned}
O(af_1 + bf_2) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (af_1 + bf_2)\delta(x\cos\theta + y\sin\theta - \rho)dxdy \\
&= a\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_1\delta(x\cos\theta + y\sin\theta - \rho)dxdy \\
&\qquad + b\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_2\delta(x\cos\theta + y\sin\theta - \rho)dxdy \\
&= aO(f_1) + bO(f_2)
\end{aligned}
$$

thus showing that the Radon transform is a linear operation.

(b) Let $p = x - x_0$ and $q = y - y_0$. Then $dp = dx$ and $dq = dy$. From Eq. (5.11-3), the Radon transform of $f(x - x_0, y - y_0)$ is

$$
\begin{aligned}
g(\rho,\theta) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x - x_0, y - y_0)\delta(x\cos\theta + y\sin\theta - \rho)dxdy \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(p,q)\delta\left[(p + x_0)\cos\theta + (q + y_0)\sin\theta - \rho)\right]dpdq \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(p,q)\delta\left[p\cos\theta + q\sin\theta - (\rho - x_0\cos\theta - y_0\sin\theta)\right]dpdq \\
&= g(\rho - x_0\cos\theta - y_0\sin\theta, \theta).
\end{aligned}
$$

Part II: Programming

Q1 (10 points)
4 points = completion of the script
2 points = successful demonstration of the outcome
2 points = successful box filter results
2 points = correct comments: adaptive filtering better reserves high frequency images (e.g., edges)

Q2 (18 points)

1) 4 points
2) 4 points
3) 4 points
4) 6 points: result comparison 4 points, appropriate comments 2 points