COMP 478/6771 Assignment 4 solutions – Fall 2024

Theory questions

Question 1. (8 points)

1. In Otsu's method for thresholding in Chapter 10, derive Eq. (10.3.15) by using Eqs. (10.3.10), (10.3.11) and (10.3.14). You should give all steps in your derivation.

We have the following equations:

$$P_1 m_1 + P_2 m_2 = m_G \tag{10.3.10}$$

$$P_1 + P_2 = 1 \tag{10.3.11}$$

$$\sigma_B^2 = P_1(m_1 - m_G)^2 + P_2(m_2 - m_G)^2 \tag{10.3.14}$$

We will derive Eq. (10.3.15) from Eqs. (10.3.10), (10.3.11) and (10.3.14)

$$\sigma_B^2 = P_1 P_2 (m_1 - m_2)^2 \tag{10.3.15}$$

Combining Eq. (10.3.14) and (10.3.10), we have:

$$\sigma_B^2 = P_1(m_1 - P_1 m_1 - P_2 m_2)^2 + P_2(m_2 - P_1 m_1 - P_2 m_2)^2$$

$$= P_1\big((1 - P_1)m_1 - P_2 m_2\big)^2 + P_2\big((1 - P_2)m_2 - P_1 m_1\big)^2 \quad (\text{since } P_1 + P_2 = 1)$$

$$= P_1(P_2 m_1 - P_2 m_2)^2 + P_2(P_1 m_2 - P_1 m_1)^2$$

$$= P_1 P_2^2 (m_1 - m_2)^2 + P_1^2 P_2 (m_1 - m_2)^2$$

$$= P_1 P_2 (m_1 - m_2)^2 \quad (\text{since } P_1 + P_2 = 1)$$

Note that each row of derivation that leads to the final result will take 2 points.

Question 2 (6 points). Each correct erosion result is worth 3 points

Notes for the solution:

1. We ONLY do pattern matching for the foreground pixels between the image and the structuring element.
2. The defined origin of the structuring element is where you leave the 1 or 0 for the post-processed image according to the rules of the morphological operation. As a result, for an erosion operation, it is possible that we could create new foreground pixels.
3. We move the structuring element pixel-by-pixel centered at the defined origin across the entire image. This means that different positionings of the origin in the structuring element can change the outcomes as well.

The results from structuring element b and c are shown in the first and second row below:
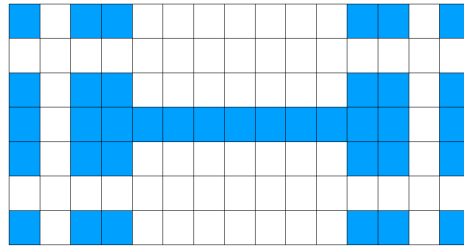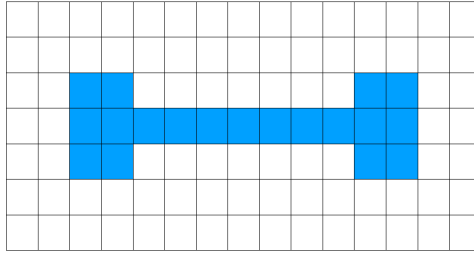
Part II: Programming questions

Question 1. (10 points: part a=7 points, part b = 5 points, part c = 3 points, part d = 5 points)

Part (a): Correct implementation of the algorithm = 5 points; demonstration of the result = 2 points; <u>For this question: the algorithm needs to be implemented from scratch</u>.

Part (b): Demonstration of the results =3 points; comments on the differences before and after averaging filtering = 2 points.

Part (c): Demonstration of results = 3 points (the result should be similar to Part b)

Part (d): Demonstration of 10 circles' center coordinates and radii = 4 points, visualization of the circles = 1 point