COMP 478/6771 Assignment 2 solutions

Theoretical questions

Q1. Image sharpening questions

a) [**3 points**] Comparing filters A and B, although the center elements are both -4, the edges to be sharpened are of different directions. While filter A enhances edges (intensity transitions) in the diagonal directions filter (**1 point**), B enhances those in the horizontal and vertical directions in the image (**1 point**). Filter C detects the edge information in all orientations and thus have a better sharpening result (**1 point**).

b) [**7 points**] In general, the sum of all elements in the filter should be zero (**1 point**). With the center element being a negative number, the positive entries should be symmetric around the center element (**1 point**). In this way, no additional information is introduced through the filtering operation.

If the student presents a solution of the 5x5 Laplacian-like filter that satisfies the conditions mentioned above, give **2 points**.

An example is here:

0	0	1	0	0
0	1	1	1	0
1	1	-12	1	1
0	1	1	1	0
0	0	1	0	0

In general, increasing the size of the "Laplacian-like" mask produces blurring. This is because a larger "Laplacian-like" mask does not follow the definition of a second derivative. Consider an image consisting of two vertical bands, a black band (intensity =0) on the left and a white band (intensity = 255) on the right, with the transition between the bands occurring through the center of the image. That is, the image has a sharp vertical edge through its center. We know that a second derivative should produce a double edge in the region of the vertical edge when a 3×3 Laplacian mask is centered on the edge. As the center of the mask moves more than two pixels on either side of the edge the entire mask will encompass a constant area and its response would be zero. However, suppose that the mask is much larger. As its center moves through, say, the black area, the mask can still be contained in the white area. The sum of the products will therefore be different from 0, making the region of filtered edges more blurry (**3 points**).

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Q2. Fourier Transform questions

Q2a (**7 points**)

1 point = correct formula for Fourier transform

2 points = correct integration following the Fourier transform

2 points = correct re-organization of equation to obtain the sin() function

2 points = correct final result

a)

$$
\begin{aligned}
F(\mu) &= \int_{-\infty}^{\infty} f(t) e^{-j2\pi\mu t} dt \\
&= \int_{0}^{W} A e^{-j2\pi\mu t} dt \\
&= \frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu t}\right]_0^W \\
&= \frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu W} - 1\right] \\
&= \frac{A}{j2\pi\mu}\left[e^{j\pi\mu W} - e^{-j\pi\mu W}\right] e^{-j\pi\mu W} \\
&= \frac{A}{\pi\mu}\sin(\pi\mu W)\, e^{-j\pi\mu W} \\
&= AW\left[\frac{\sin(\pi\mu W)}{\pi\mu W}\right] e^{-j\pi\mu W}
\end{aligned}
$$

The only difference between this result and the result in Example 4.1 is the exponential term. It is a phase term that accounts for the shift in the function. The magnitude of the Fourier transform is the same in both cases, as expected.

For the correct final answer, just need to plug in A=W=2.

Q2b (**4 points**)

2 points = correction statement of the convolution theorem/direct use of the multiplication

2 points = correct final result

b) Since the tent function is the convolution of two equal box functions, by the convolution theorem, the Fourier transform of the spatial convolution of two functions is the product their transforms. Recall that the trans- form of a box is a sinc function. Therefore, the Fourier transform of a tent function is a sinc function squared. From Example 4.1, we have

$$F(\mu) = AW\frac{\sin(\pi\mu W)}{\pi\mu W}$$

The Fourier transform of the tent function is

$$F(\mu)F(\mu) = (AW)^2\frac{\sin^2(\pi\mu W)}{(\pi\mu W)^2}$$

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Part II: Programming

Q1 (**22 points**)
8 points: The program functions correctly
6 points: Demonstrate of results
4 points: Discussion about the choice of parameters and filters
4 points: Compare the results with ***adaptthresh()*** function in MATLAB

Q2 (**2 points**)
1 point for correct script and **1 point** for image demonstration