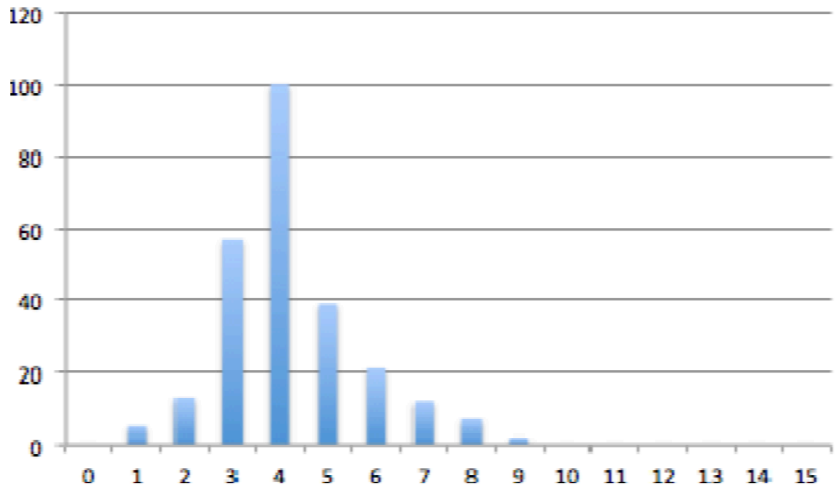
Theoretical questions

Q1 **(5 points)**

The median *m* of a set of numbers is such that half of the values in the set are below *m* and the other half are above it. A simple example will suffice to show that an operator that computes the median is nonlinear. Let $S_1 = \{1, -2, 3\}$ and $S_2 = \{4, 5, 6\}$ and a = b = 1, then $H(S_1)$ = median $\{1, -2, 3\} = 1$ and $H(S_2)$ = median $\{4, 5, 6\} = 5$. While $H(S_1 + S_2)$ = median$\{5, 3, 9\} = 5$ and $H(S_1 + S_2)$ is not equal to $H(S_1) + H(S_2)$ and the median operator is not linear.

Q2 (a) **(5 points)**

a) Plot of image histogram



(b) **(10 points): (i) and (ii) are 5 points each.**

b)

i) Calculating s_k and histogram equalization

Gray levels r_k	Number of pixels n_k	$p_r(r_k) = \frac{n_k}{n}$	$s_k = \frac{L-1}{n}\sum_{j=0}^{k} n_j$	Discrete values
0	0	0	0	0
1	5	0.02	0.293	0
2	13	0.05	1.055	1
3	57	0.22	4.395	4
4	100	0.39	10.25	10
5	39	0.15	12.54	13
6	21	0.08	13.77	14
7	12	0.05	14.47	14
8	7	0.03	14.88	15
9	2	0.01	15	15
10	0	0	15	15
11	0	0	15	15
12	0	0	15	15

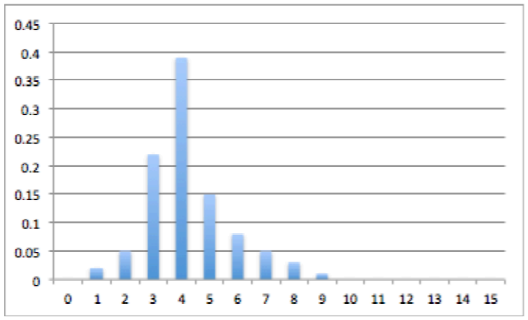
13	0	0	15	15
14	0	0	15	15
15	0	0	15	15

Which $L = 16$ and $n = 256$.

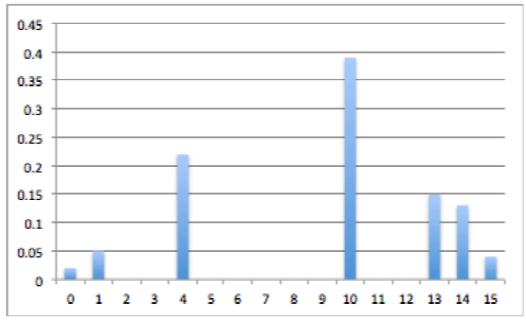
To do histogram equalization we map the gray level values by $S = T(r)$ transformation which yield the following table.

Gray levels s_k	Number of pixels n_{s_k}	$p_s(s_k) = \frac{n_{s_k}}{n}$
0	5	0.02
1	13	0.05
2	0	0
3	0	0
4	57	0.22
5	0	0
6	0	0
7	0	0
8	0	0
9	0	0
10	100	0.39
11	0	0
12	0	0
13	39	0.15
14	33	0.13
15	9	0.04

ii) Probability density function for $P_r(r_k)$ and $P_s(s_k)$

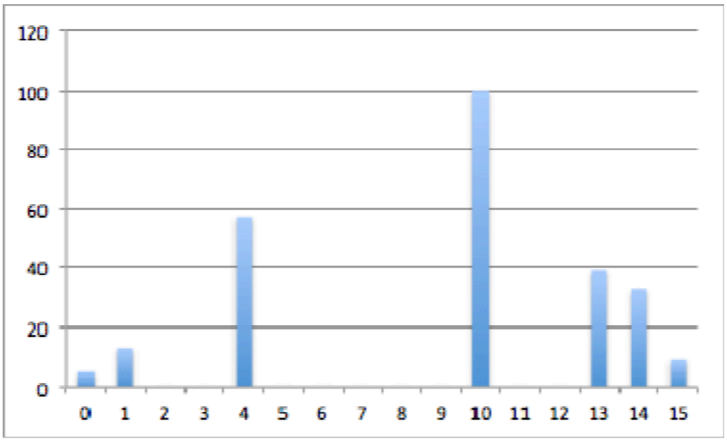


$P_r(r_k)$



$P_s(s_k)$

(c) **(5 points)** Histogram of image after histogram equalization



Q3 **(5 points)**

The question is meant to help the students understand that histogram carry no information about spatial information of the image content. For the case of $f(x,y) + g(x,y)$, the height of each bin for the histogram remains the same, but the entire histogram will be shifted to the right (**2 points**). For the case of $f(x,y)*g(x,y)$, the height of each bin of the histogram will still remain the same. However, their spacing between each other will increase, and thus the new histogram will be more spread out than the original (**3 points**).

Part II: programming questions

Q1
1- 1 point: correct script = 1 point
2- 3 points: correct script = 2 point, histogram display = 1 point
3- 2 points: execution of the function = 1 point, image comparison with comments = 1 point
4- 3 points: correct script = 2 point, result display = 1 point
5- 1 point: image comparison with comments = 1 point

Q2
1- 1 point: correct script = 1 point
2- 3 points: correct script for boxfilter = 1 point, correct script for the unsharp mask = 1 point, image demonstration = 1 point
3- 2 points: correct script = 1 point, showcase of result = 1 point
4- 2 points: each image demo is 1 point