1. Two particles A and B, of mass $5m$ kg and $2m$ kg respectively, are moving in opposite directions along the same straight horizontal line. The particles collide directly. Immediately before the collision, the speeds of A and B are 3 m s^{-1} and 4 m s^{-1} respectively. The direction of motion of A is unchanged by the collision. Immediately after the collision, the speed of A is 0.8 m s^{-1}.

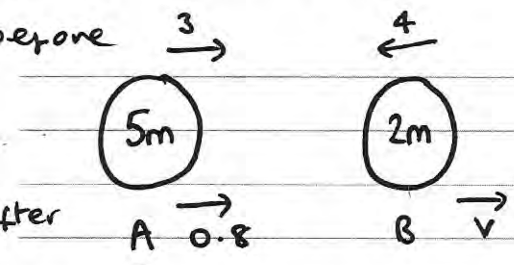
(a) Find the speed of B immediately after the collision.

(3)

In the collision, the magnitude of the impulse exerted on A by B is 3.3 N s.

(b) Find the value of m.

(3)



CLM $\Rightarrow 15m - 8m = 4m + 2mV$

$3\cancel{m} = 2\cancel{m}V \Rightarrow V = 1.5$

b) Mom A before $= 15m$ $\quad \therefore$ Impulse $= 11m$

Mom A after $= 4m$

$11m = 3.3 \Rightarrow m = 0.3$

2.

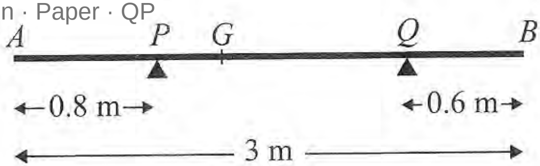


Figure 1

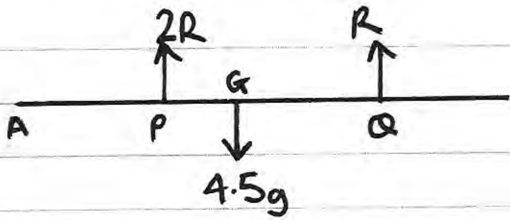
A non-uniform rod AB has length 3 m and mass 4.5 kg. The rod rests in equilibrium, in a horizontal position, on two smooth supports at P and at Q, where $AP = 0.8$ m and $QB = 0.6$ m, as shown in Figure 1. The centre of mass of the rod is at G. Given that the magnitude of the reaction of the support at P on the rod is twice the magnitude of the reaction of the support at Q on the rod, find

(a) the magnitude of the reaction of the support at Q on the rod,

(3)

(b) the distance AG.

(4)



$RF \uparrow = \downarrow \quad 3R = 4.5g$

$\therefore R = 1.5g\,N$

b) $A \circlearrowright \quad 4.5g \times AG = 3g \times 0.8 + 1.5g \times 2.4$

$4.5\cancel{g} \times AG = 6\cancel{g} \quad \Rightarrow AG = \frac{4}{3}$

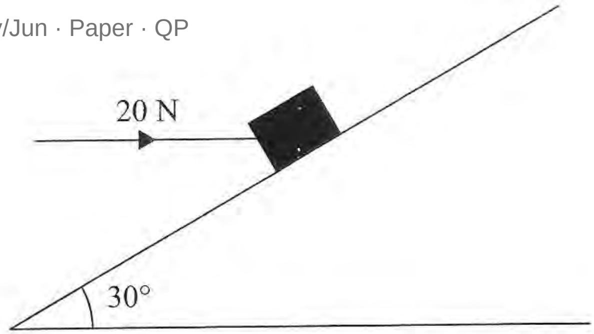


Figure 2

A box of mass 5 kg lies on a rough plane inclined at 30° to the horizontal. The box is held in equilibrium by a horizontal force of magnitude 20 N, as shown in Figure 2. The force acts in a vertical plane containing a line of greatest slope of the inclined plane.
The box is in equilibrium and on the point of moving down the plane. The box is modelled as a particle.

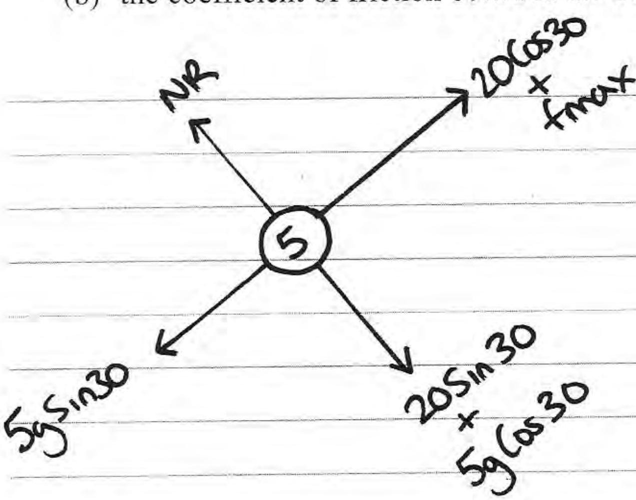
Find

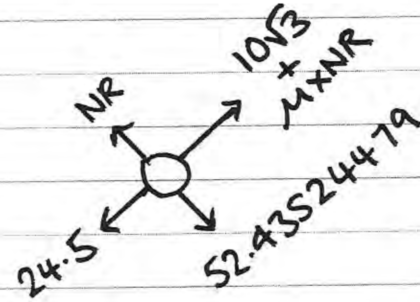
(a) the magnitude of the normal reaction of the plane on the box,

(4)

(b) the coefficient of friction between the box and the plane.

(5)





$Rf\uparrow = 0 \Rightarrow NR = 52.4$ (3sf)

$Rf \nearrow = 0 \quad 10\sqrt{3} + \mu(52.43\ldots) = 24.5$

$\mu = 0.137$ (3sf)

4. A car is moving on a straight horizontal road. At time $t = 0$, the car is moving with speed 20 m s^{-1} and is at the point A. The car maintains the speed of 20 m s^{-1} for 25 s. The car then moves with constant deceleration 0.4 m s^{-2}, reducing its speed from 20 m s^{-1} to 8 m s^{-1}. The car then moves with constant speed 8 m s^{-1} for 60 s. The car then moves with constant acceleration until it is moving with speed 20 m s^{-1} at the point B.

(a) Sketch a speed-time graph to represent the motion of the car from A to B.

(3)

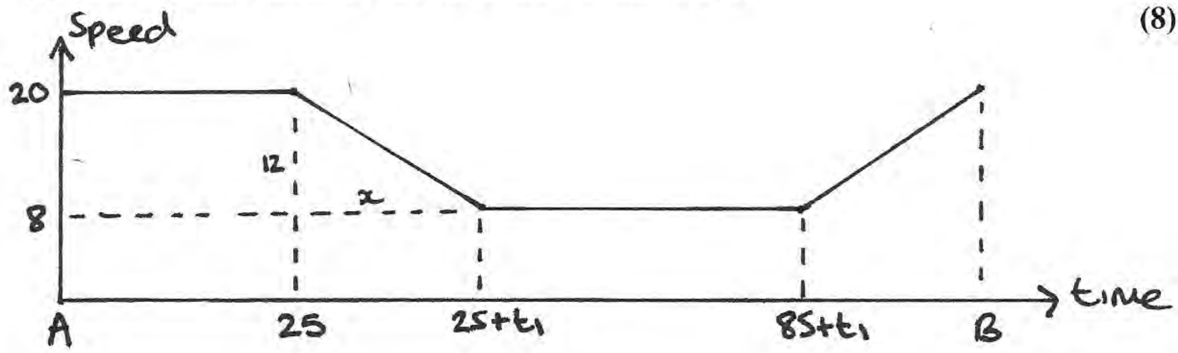
(b) Find the time for which the car is decelerating.

(2)

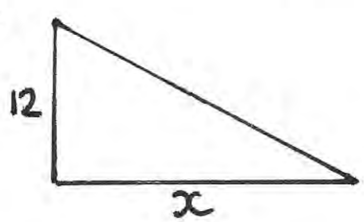
Given that the distance from A to B is 1960 m,

(c) find the time taken for the car to move from A to B.

(8)



b)

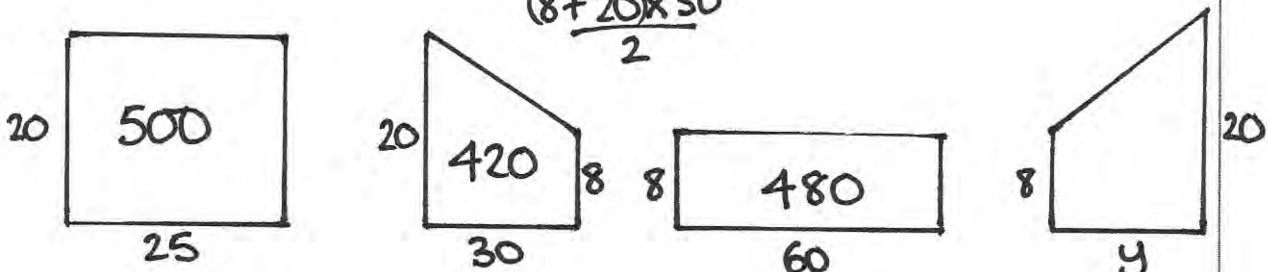


gradient = $-0.4 = \dfrac{-12}{x} \Rightarrow x = \dfrac{12}{0.4}$
acc

$x = 30$ sec

c)

$\dfrac{(8+20)\times 30}{2}$



$500 + 420 + 480 = 1400$

$1960 - 1400 = 560$

$\therefore \dfrac{(8+20)\times y}{2} = 560$

$28y = 1120$

$y = 40$

$\therefore$ total time $= 25 + 30 + 60 + 40$

$= \underline{155 \text{ sec}}$

5. A particle P is projected vertically upwards from a point A with speed u m s^{-1}. The point A is 17.5 m above horizontal ground. The particle P moves freely under gravity until it reaches the ground with speed 28 m s^{-1}.

(a) Show that $u = 21$

(3)

At time t seconds after projection, P is 19 m above A.

(b) Find the possible values of t.

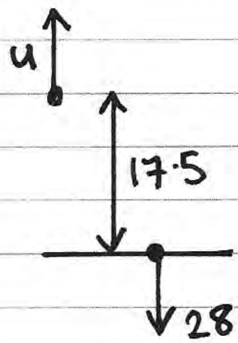
(5)

The ground is soft and, after P reaches the ground, P sinks vertically downwards into the ground before coming to rest. The mass of P is 4 kg and the ground is assumed to exert a constant resistive force of magnitude 5000 N on P.

(c) Find the vertical distance that P sinks into the ground before coming to rest.

(4)

a)



$s = -17.5$ $\quad v^2 = u^2 + 2as$

u

$v = -28$ $\quad 784 = u^2 + 343$

$a = -9.8$

t $\quad u^2 = 441 \Rightarrow u = 21$ #

b) $s = 19$ $\quad s = ut + \frac{1}{2}at^2$ $\quad t = \dfrac{21 \pm \sqrt{21^2 - 4(4.9)(19)}}{9.8}$

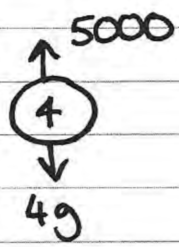
$u = 21$

v $\quad 19 = 21t - 4.9t^2$

$a = -9.8$ $\quad t_1 = 2.99$ (3sf)

t $\quad 4.9t^2 - 21t + 19 = 0$ $\quad t_2 = 1.30$ (3sf)

c)



$Rf\downarrow = ma \Rightarrow 39.2 - 5000 = 4a$

$\Rightarrow a = -1240.2$

$u = 28, v = 0$

$v^2 = u^2 + 2as \Rightarrow 0 = 784 - 2480.4s$

$\Rightarrow s = 0.316$ m (3sf)

6. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.*]

A ship S is moving with constant velocity $(-12\mathbf{i} + 7.5\mathbf{j})$ km h^{-1}.

(a) Find the direction in which S is moving, giving your answer as a bearing. **(3)**

At time t hours after noon, the position vector of S is $\mathbf{s}$ km. When $t = 0$, $\mathbf{s} = 40\mathbf{i} - 6\mathbf{j}$.

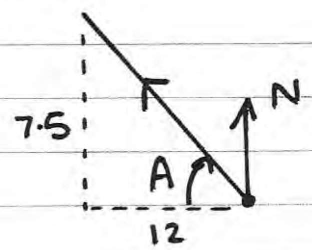
(b) Write down $\mathbf{s}$ in terms of t. **(2)**

A fixed beacon B is at the point with position vector $(7\mathbf{i} + 12.5\mathbf{j})$ km.

(c) Find the distance of S from B when $t = 3$ **(4)**

(d) Find the distance of S from B when S is due north of B. **(4)**

a)



bearing $= 270 + A$

$= 270 + \tan^{-1}\left(\frac{7.5}{12}\right)$

$= \underline{302°}$

b) pos after t = orig pos + t(vel)

$S = \begin{pmatrix} 40 \\ -6 \end{pmatrix} + t\begin{pmatrix} -12 \\ 7.5 \end{pmatrix}$

c) $t = 3$ $\quad S = \begin{pmatrix} 40 - 36 \\ -6 + 22.5 \end{pmatrix}$ $\quad S = \begin{pmatrix} 4 \\ 16.5 \end{pmatrix}$

$\overrightarrow{BS} = \underline{S} - \underline{B} = \begin{pmatrix} 4 \\ 16.5 \end{pmatrix} - \begin{pmatrix} 7 \\ 12.5 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

dist $= |\overrightarrow{BS}| = \sqrt{3^2 + 4^2} = 5$ km.

d) due North $\Rightarrow$ i component of $\overrightarrow{BS} = 0$ (i parts of S = 7!)

$(40 - 12t) = 7 \quad 12t = 33 \quad t = \frac{11}{4}$

$t = \frac{11}{4}$ $\quad S = \begin{pmatrix} 40 - 12\left(\frac{11}{4}\right) \\ -6 + 7.5\left(\frac{11}{4}\right) \end{pmatrix} = \begin{pmatrix} 7 \\ \frac{117}{8} \end{pmatrix}$

$\vec{SB} = \frac{117}{8} - 12.5 = \frac{17}{8}$ km

7.

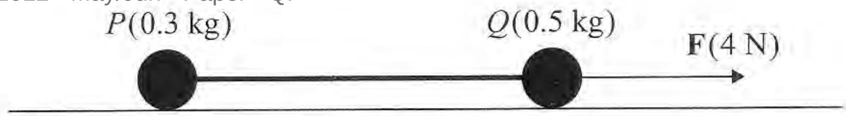


Figure 3

Two particles P and Q, of mass 0.3 kg and 0.5 kg respectively, are joined by a light horizontal rod. The system of the particles and the rod is at rest on a horizontal plane. At time $t = 0$, a constant force **F** of magnitude 4 N is applied to Q in the direction PQ, as shown in Figure 3. The system moves under the action of this force until $t = 6$ s. During the motion, the resistance to the motion of P has constant magnitude 1 N and the resistance to the motion of Q has constant magnitude 2 N.

Find

(a) the acceleration of the particles as the system moves under the action of **F**,
(3)

(b) the speed of the particles at $t = 6$ s,
(2)

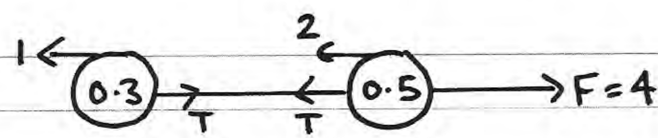
(c) the tension in the rod as the system moves under the action of **F**.
(3)

At $t = 6$ s, **F** is removed and the system decelerates to rest. The resistances to motion are unchanged. Find

(d) the distance moved by P as the system decelerates,
(4)

(e) the thrust in the rod as the system decelerates.
(3)

a)



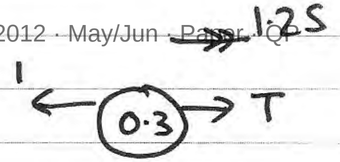
whole system $\vec{Rf} = ma$ $4-2-1 = 0.8a$

$\therefore a = \underline{1.25\ \text{ms}^{-2}}$

b) $u=0$ $t=6$ $a=1.25$ $v=u+at$

$v = 0 + 1.25 \times 6$ $\Rightarrow v = \underline{7.5}\ \text{ms}^{-1}$

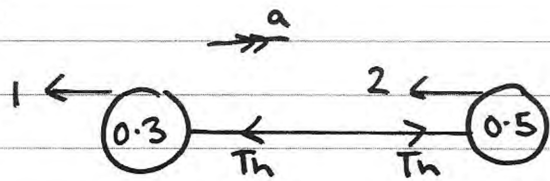
c)



$\overrightarrow{Rf} = ma$

$T - 1 = 0.3 \times 1.25 \quad \Rightarrow T = \frac{11}{8}\text{N}$

d)



whole system $\overrightarrow{Rf} = ma \qquad -1 - 2 = 0.8a$

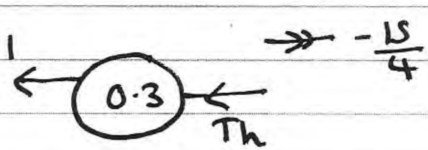
$\Rightarrow a = -\frac{15}{4}$

$u = 7.5 \quad v = 0 \quad a = -\frac{15}{4}$

$v^2 = u^2 + 2as \quad \Rightarrow \quad 0 = 56.25 - \frac{15}{2}s$

$\therefore s = 7.5\text{m}$

e)



$\overrightarrow{Rf} = ma$

$-1 - Th = 0.3\left(-\frac{15}{4}\right)$

$-1 - Th = -\frac{9}{8}$

$-Th = -\frac{1}{8}$

$\therefore \text{Thrust} = \underline{\frac{1}{8}\text{N}}$