$8 (a) \qquad 90\pi = \pi r^2 h \Longrightarrow h = \frac{90}{r^2}$	
$90\lambda = \lambda r \ h \rightarrow h = \frac{r}{r^2}$	
$S = 2\pi r^2 + 2\pi rh \Rightarrow S = 2\pi r^2 + 2\pi r \times \frac{90}{r^2}$	B1
$r^2$	M1
$S = 2\pi r^2 + \frac{2 \times 90\pi}{r} = 2\pi r^2 + \frac{180\pi}{r} *$	A1cso [3]
$\frac{\mathrm{d}S}{\mathrm{d}r} = 4\pi r - \frac{180\pi}{r^2}$	M1
$\frac{dS}{dr} = 0 \implies 4\pi r - \frac{180\pi}{r^2} = 0 \implies 4\pi r = \frac{180\pi}{r^2} \implies r^3 = 45 \implies r = \dots$ $r = 3.55689 \implies r \approx 3.56$	M1 A1
$\frac{d^2 S}{dr^2} = 4\pi + \frac{360\pi}{r^3}$ $\frac{d^2 S}{dr^2} = 4\pi + \frac{360\pi}{r^3} \Rightarrow \left(\frac{d^2 S}{dr^2} = 37.699\right)$	M1
$dr^{2} = R^{2} - r^{3} - dr^{2} = 37.055$ $37.699 > 0 \Rightarrow \text{ hence minimum}$	A1ft [5]
(c) $S = 2\pi \times 3.556^2 + \frac{2 \times 90\pi}{3.556} =$ $S = 238.4769 \Rightarrow S = 238 \text{ (cm}^2\text{)}$	M1 A1
$S = 238.4769 \Rightarrow S = 238 \text{ (cm}^2\text{)}$	[2]
	Total 10 marks

Part	Marks	Scheme
(a)	B1	For finding an expression for $h$ in terms of $r$
		$90\pi = \pi r^2 h \Rightarrow h = \frac{90}{r^2}$
		Award for finding an expression for <i>hr</i> in terms of <i>r</i>
		$90\pi = \pi r^2 h \Rightarrow hr = \frac{90}{r}$
	M1	For substituting their expression for <i>h</i> into a <b>correct</b> formula for the closed surface area of a cylinder
		$S = 2\pi r^2 + 2\pi rh \Rightarrow S = 2\pi r^2 + 2\pi r \times \frac{'90'}{r^2}$
		Or for substitution of their expression for $hr$ into a <b>correct</b> formula for the closed surface area of a cylinder
		$S = 2\pi r^2 + 2\pi rh \Rightarrow S = 2\pi r^2 + 2\pi \times \frac{'90'}{r}$
	A1 cso	For the correct expression for the area as shown

		$2 \times 90\pi$ $180\pi$
		$S = 2\pi r^2 + \frac{2 \times 90\pi}{r} = 2\pi r^2 + \frac{180\pi}{r}$
		Must have the $S = $ for this mark.
(b)	M1	For attempting to differentiate the given expression for $S$ at least one power to
(b)	1411	decrease and neither power to increase.
		$\frac{dS}{dt} = 4\pi r - \frac{180\pi}{2}$
		$\frac{dS}{dr} = 4\pi r - \frac{180\pi}{r^2}$ Sets their $\frac{dS}{dr} = 0$ and attempts to solve for $r$
	M1	Sets their $\frac{dS}{dr} = 0$ and attempts to solve for r
		dr
		$4\pi r - \frac{180\pi}{r^2} = 0 \Rightarrow 4\pi r = \frac{180\pi}{r^2} \Rightarrow r^3 = 45 \Rightarrow r = \dots$
	A1	For the correct value of $r = 3.55689 \Rightarrow r \approx 3.56$
		Accept awrt 3.56
	M1	For attempting to differentiate their expression for $\frac{dS}{dr}$ at least one power to
		decrease and neither power to increase.
		$d^2S = 360\pi$
		$\frac{d^2S}{dr^2} = 4\pi + \frac{360\pi}{r^3}$
	A1ft	For correct work throughout $\frac{d^2S}{dr^2} = 4\pi + \frac{180\pi}{r^3} \Rightarrow \left(\frac{d^2S}{dr^2} = 37.699\right)$
		$37.699 > 0 \Rightarrow$ hence minimum
		Evaluation not required as both terms positive so $\frac{d^2S}{dr^2} > 0$ hence minimum
		Indication of positive or >0 required.
		If $\frac{d^2S}{dr^2}$ evaluated incorrectly then do not award. If evaluated then accept awrt
		38
(c)	M1	For substituting their value of $r$ into the <b>given</b> expression for $S$
		$S = 2\pi \times 3.556'^2 + \frac{2 \times 90\pi}{3.556'} =$
	A 1	Their value of $r > 0$
	A1	$S = 238.4769 \Rightarrow S = 238 \text{ (cm}^2\text{)}$
		Accept awrt 238