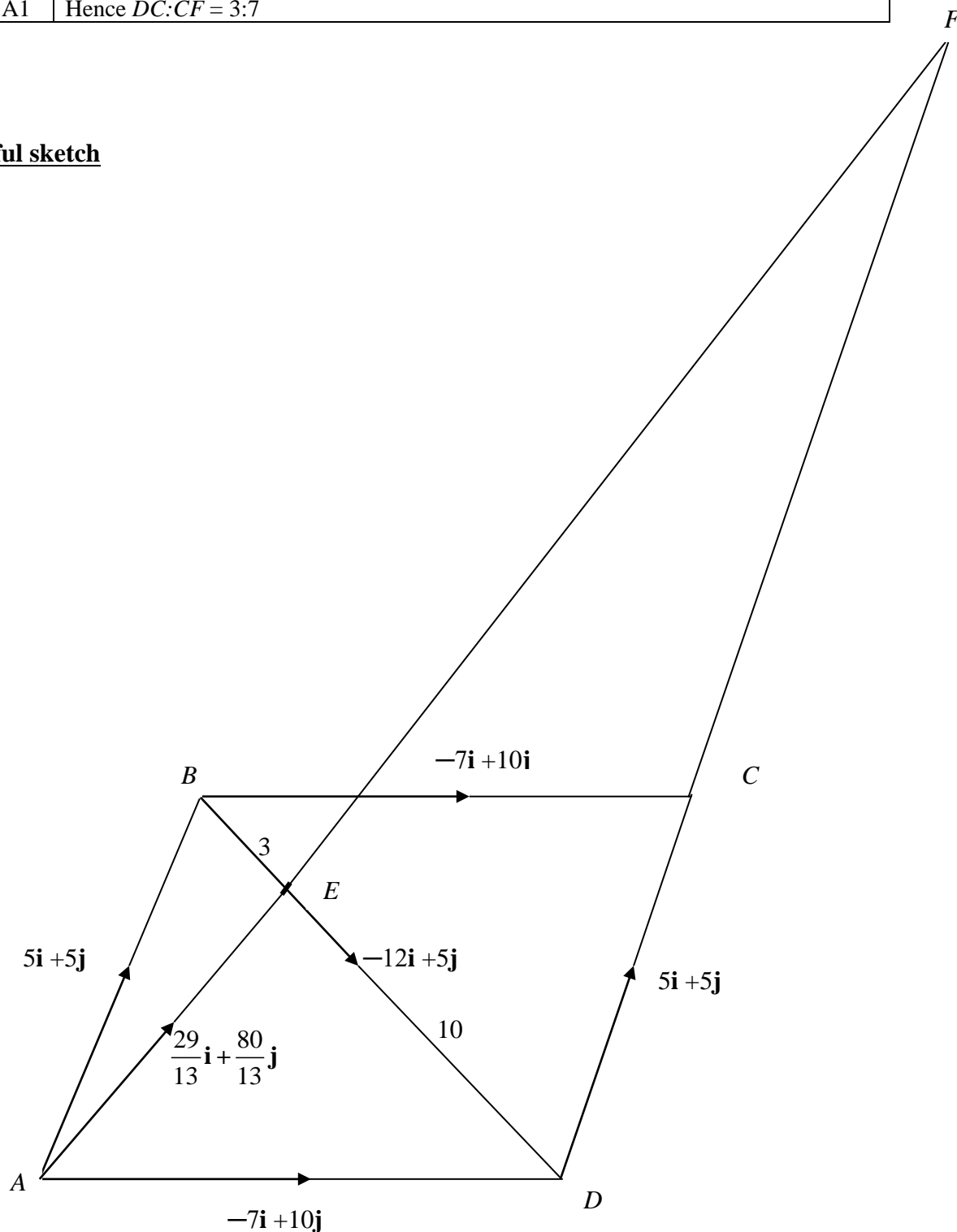


Question number	Scheme	Marks
10 (a) (i)	$\overrightarrow{DC} = \overrightarrow{DA} + \overrightarrow{AC}, = 7\mathbf{i} - 10\mathbf{j} - 2\mathbf{i} + 15\mathbf{j} = 5\mathbf{i} + 5\mathbf{j}$	M1,A1
(ii)	$\overrightarrow{DC} = \overrightarrow{AB}$ $\therefore ABCD$ is a parallelogram	M1 A1 [4]
(b)	$\overrightarrow{BD} = \overrightarrow{BA} + \overrightarrow{AD} = -5\mathbf{i} - 5\mathbf{j} - 7\mathbf{i} + 10\mathbf{j} = -12\mathbf{i} + 5\mathbf{j}$  unit vector $= (\pm)\frac{1}{13}, (-12\mathbf{i} + 5\mathbf{j})$	M1A1  B1ft,B1 [4]
(c)	$\overrightarrow{BE} = \frac{3}{13}(-12\mathbf{i} + 5\mathbf{j})$ $\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BE} = \frac{29}{13}\mathbf{i} + \frac{80}{13}\mathbf{j}$	M1A1 [2]
(d)	$\overrightarrow{AF} = \lambda \overrightarrow{AE} = \lambda \left( \frac{29}{13}\mathbf{i} + \frac{80}{13}\mathbf{j} \right)$ or $\lambda'(29\mathbf{i} + 80\mathbf{j})$ $\overrightarrow{AF} = \overrightarrow{AC} + \mu \overrightarrow{DC} = -2\mathbf{i} + 15\mathbf{j} + \mu(5\mathbf{i} + 5\mathbf{j})$  $\frac{29}{13}\lambda = -2 + 5\mu \quad \frac{80}{13}\lambda = 15 + 5\mu$  $\mu = \frac{7}{3} \quad \left( \lambda = \frac{13}{3}, \quad \lambda' = \frac{1}{3} \right)$  $DC : CF = 1 : \frac{7}{3} \quad (= 3 : 7)$	B1  M1A1      M1A1  A1 [6]
<b>Total 16 marks</b>		

Notes		
(a)	M1	For the correct vector statement for $\overrightarrow{DC}$ so $\overrightarrow{DC} = \overrightarrow{DA} + \overrightarrow{AC}$
	A1	For the correct simplified expression $\overrightarrow{DC} = 5\mathbf{i} + 5\mathbf{j}$
	M1	States $\overrightarrow{DC} = \overrightarrow{AB}$ Condone lack of arrows on vectors if they are clearly using vectors. i.e accept $DC = AB$
	A1	Conclusion required, therefore $ABCD$ is a parallelogram. Accept; a labelled diagram, shown, QED, or even a tick or # etc.
(b)	M1	For the correct vector statement for $\overrightarrow{BD} = \overrightarrow{BA} + \overrightarrow{AD}$ or $\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD}$
	A1	For the correct simplified expression $\overrightarrow{BD} = -12\mathbf{i} + 5\mathbf{j}$
	B1ft	For the correct magnitude of their vector. So that for $\overrightarrow{BD} = a\mathbf{i} + b\mathbf{j} \Rightarrow  \overrightarrow{BD}  = \sqrt{a^2 + b^2}$ , $ \overrightarrow{BD}  = 13$
	B1ft	Writes ' $\frac{1}{13}(a\mathbf{i} + b\mathbf{j})$ ' for their $\overrightarrow{BD}$
(c)	M1	For any correct path for $\overrightarrow{AE}$ with correct use of the ratio for $\overrightarrow{ED}$ or $\overrightarrow{BE}$
	A1	For $\overrightarrow{AE} = \frac{29}{13}\mathbf{i} + \frac{80}{13}\mathbf{j}$ oe
<b>Method 1 using triangle ACF</b> (they may use any letters for $\mu$ and $\lambda$ )		
(d)	B1	States $\overrightarrow{AF} = \lambda \overrightarrow{AE}$ or $\overrightarrow{AF} = \lambda \left( \frac{29}{13}\mathbf{i} + \frac{80}{13}\mathbf{j} \right)$ ,
	M1	$\overrightarrow{AF} = \overrightarrow{AC} + \mu \overrightarrow{DC}$
	A1	For the fully correct expression which need not be simplified $\overrightarrow{AF} = -2\mathbf{i} + 15\mathbf{j} + \mu(5\mathbf{i} + 5\mathbf{j})'$
	M1	Sets $\lambda \left( \frac{29}{13}\mathbf{i} + \frac{80}{13}\mathbf{j} \right)' = -2\mathbf{i} + 15\mathbf{j} + \mu(5\mathbf{i} + 5\mathbf{j})'$ and equates coefficients of $\mathbf{i}$ and $\mathbf{j}$ to form two equations in $\lambda$ and $\mu$ . Condone $\mathbf{i}$ and $\mathbf{j}$ in their equations.
	A1	For finding $\mu = \frac{7}{3}$
	A1	$DC : CF = 1 : \frac{7}{3}$ or $DC : CF = 3 : 7$
<b>Method 2 using triangle ADF</b> (they may use any letters for $\mu$ and $\lambda$ )		
	B1	States $\overrightarrow{AF} = \lambda \overrightarrow{AE}$ or $\overrightarrow{AF} = \lambda \left( \frac{29}{13}\mathbf{i} + \frac{80}{13}\mathbf{j} \right)$ ,
	M1	$\overrightarrow{AF} = \overrightarrow{AD} + \mu \overrightarrow{DC}$
	A1	For a fully correct expression which need not be simplified $\overrightarrow{AF} = -7\mathbf{i} + 10\mathbf{j} + \mu(5\mathbf{i} + 5\mathbf{j})'$
	M1	Sets $\lambda \left( \frac{29}{13}\mathbf{i} + \frac{80}{13}\mathbf{j} \right)' = -7\mathbf{i} + 10\mathbf{j} + \mu(5\mathbf{i} + 5\mathbf{j})'$ and equates coefficients of $\mathbf{i}$ and $\mathbf{j}$ to form two equations in $\mu$ and $\lambda$ . Condone $\mathbf{i}$ and $\mathbf{j}$ in their equations.
	A1	For finding $\mu = \frac{10}{3}$

	A1	$DC : DF = 1 : \frac{10}{3} \Rightarrow DC : CF = 1 : \frac{7}{3} \text{ or } DC : CF = 3 : 7$
<b>Method 3 using similar triangles</b>		
(d)	B1	States triangles $AEB$ and $DEF$ are similar. Can be implied from correct work.
	M1	$BE : ED = 3 : 10$ (given)
	A1	So therefore correspondingly $AB : DF = 3 : 10$
	M1	$AB = DC$ parallelogram
	A1	So $DC : DF = 3 : 10$
	A1	Hence $DC : CF = 3 : 7$

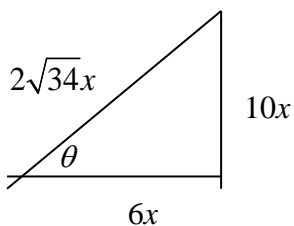
**Useful sketch**

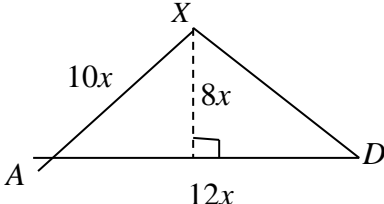
Question number	Scheme	Marks
11 (a)	$AC = 20x$ or $AX = 10x$ $EX = AX \tan 45^\circ = 10x$	B1 M1A1 [3]
(b)	$EA = \frac{EX}{\sin 45^\circ} = \frac{10x}{\sin 45^\circ} = \sqrt{200}x = (10\sqrt{2}x)$ Or use $\triangle EAC$ which is right-angled and isosceles	M1A1 [2]
(c)	$\tan \theta = \frac{EX}{\frac{1}{2}AD} = \frac{10}{6}$ $\theta = 59.0^\circ$	M1A1ft A1 [3]
(d)	Reqd angle is $AXD (= \phi)$ $\tan \frac{1}{2}\phi = \frac{6}{8}$ $\phi = 73.7^\circ$ must be acute <b>ALT:</b> Use cosine rule in triangle $AXD$	M1A1 A1 [3]
(e)	$Y$ is midpoint of $AD$ $EY = x\sqrt{(10\sqrt{2})^2 - 6^2} (= x\sqrt{164} \text{ or } 2x\sqrt{41})$ $\text{Area } \triangle AED = 6x^2\sqrt{164} = 250$ $x = 1.8037... = 1.804$	M1 M1 A1 [3]
<b>Total 14 marks</b>		

**In this question penalise ROUNDING of angles only once in parts (c) and (d).**

This applies only if both answers are correct but over-accurate, i.e. they would both round to the correct angle to 1 decimal place. For example; for an angle in (c) given as  $59.04^\circ$  award M1A1A0, but if the angle in (d) is then given as  $73.74^\circ$  do not penalise the angle in (d), and award M1A1A1.

If however the answer given in (c) is  $59^\circ$  without  $59.04^\circ$  seen, this is M1A1A0, and because it is under-accurate, if they then give the angle in (d) as  $73.74^\circ$ , then this is awarded M1A1A0 as well.

Notes		
(a)	B1	For using Pythagoras theorem to find either $AC = 20x$ or $AX = 10x$ <b>Do not accept</b> $AC = 20$ or $AX = 10$
	M1	For $EX = AX \tan 45^\circ = (10x)$ Ft their $AX$ but <b>not</b> if they use their $AC$ If there is no $x$ in their working it is M0 <b>UNLESS</b> they put $x$ into their final answer.
	A1	For $10x$
<b>ALT</b>		
	M1	Triangle $AEC$ is isosceles with $\angle EAX = \angle ECX = 45^\circ$ Hence triangle $AEX$ is also isosceles with $\angle EAX = \angle AEX = 45^\circ$  If there is no $x$ in their working it is M0 <b>UNLESS</b> they put $x$ into their final answer.
	A1	Hence $AX = EX$ so $EX = 10x$
<b>Accept</b> $EX = 10x$ just stated without working.		
(b)	M1	Uses Pythagoras theorem or any acceptable trigonometry to find length $AE$  $AE = \sqrt{(10x)^2 + (10x)^2} = (\sqrt{200}x)$ $AE = \frac{10x}{\sin 45^\circ} = (\sqrt{200}x) \text{ or } AE = \frac{10}{\cos 45^\circ} = (\sqrt{200}x)$ If there is no $x$ in their working it is M0 <b>UNLESS</b> they put $x$ into their final answer.
	A1	$AE = 10x\sqrt{2}$ or $\sqrt{200}x$ Also accept $14.1x$ or better  Accept unsimplified answers. Even accept answers given as $AE = \frac{10x}{\frac{\sqrt{2}}{2}}$
(c)	M1	Uses their $EX$ to find $\tan \theta = \frac{'EX'}{\frac{1}{2}AD} = \frac{'10x'}{6x} \Rightarrow \theta = \dots$  (Or any other complete method for the <b>required</b> angle)  <div style="text-align: center;">  </div> Accept working without $x$ 's as this is a ratio, but do not accept an $x$ in the numerator or denominator only.
	A1ft	A correct value of $\theta$ for their $EX$
	A1	$\theta = 59.0^\circ$ rounded correctly

(d)	M1		Angle required is $\angle AXD$ $\tan\left(\frac{1}{2}\angle AXD\right) = \frac{\frac{1}{2} \times AD}{\frac{1}{2} \times CD} = \frac{6x}{8x} \Rightarrow \angle AXD = \dots$ $\sin\left(\frac{1}{2}\angle AXD\right) = \frac{\frac{1}{2} \times AD}{10x} = \frac{6x}{10x} \Rightarrow \angle AXD = \dots$ $\cos\left(\frac{1}{2}\angle AXD\right) = \frac{\frac{1}{2} \times CD}{10x} = \frac{8x}{10x} \Rightarrow \angle AXD = \dots$
	Accept working without $x$ 's as this is a ratio, but do not accept $x$ in the numerator or denominator only.		
	A1	Uses correct values for their method	
	A1	$\angle AXD = 73.7^\circ$ (must be acute) <b>Note: Do not isw if both angles are given and the acute angle not identified.</b>	
<b>ALT (using cosine rule)</b>			
(d)	M1	$\angle AXD = \cos^{-1}\left(\frac{(10x)^2 + (10x)^2 - (12x)^2}{2 \times 10x \times 10x}\right) = \frac{56x^2}{200x^2} = \dots$ Accept working without $x$ 's as this is a ratio, but do not accept $x^2$ in the numerator or denominator only.	
	A1	Uses the correct value for $AX$	
	A1	$\angle AXD = 73.7^\circ$ ( must be acute) <b>Note: Do not isw if both angles are given and the acute angle not identified.</b>	
(e)	M1	Finds the length of $E$ to the midpoint of $AD$ ( $Y$ ) $EY = x\sqrt{('10\sqrt{2}')^2 - 6^2} (= x\sqrt{164} \text{ or } 2x\sqrt{41}) \text{ ft their } AE$ Any working without $x$ is M0.	
	M1	Equates area of $250 \text{ cm}^2$ to $\frac{1}{2} \times AD \times EY = \frac{1}{2} \times 12x \times x\sqrt{164} \Rightarrow x = \dots$	
	A1	$x = 1.804$ rounded correctly	
<b>ALT cosine rule and <math>\frac{1}{2}ab \sin C</math></b>			
(e)	M1	Finds angle $\angle AED$ $\angle AED = \cos^{-1}\left(\frac{('10\sqrt{2}x')^2 + ('10\sqrt{2}x')^2 - (12x)^2}{2 \times ('10\sqrt{2}x') \times ('10\sqrt{2}x')}Any working without x is M0.$	
	M1	Equates area of $250 \text{ cm}^2$ to expression for area of triangle: $250 = \frac{1}{2} \times AE \times ED \sin \angle AED = \frac{1}{2} \times 10\sqrt{2}x \times 10\sqrt{2}x \sin 50.208^\circ \Rightarrow x^2 = 3.2536\dots^\circ$ $\Rightarrow x = 1.8037\dots$	
	A1	$x = 1.804$ rounded correctly	