Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International GCSE

Time 2 hours

Paper reference **4PM1/02**

Further Pure Mathematics
PAPER 2

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

P71666A

Q:1/1/1/1/1/

P 7 1 6 6 6 A 0 1 3 6

Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Find the set of values of k for which the equation

$$2kx^2 + 5kx + 5k - 3 = 0 \qquad \text{where } k \neq 0$$

has real roots.

(4)

(Total for Question 1 is 4 marks)

2 A particle P moves along the x-axis. At time t seconds, the displacement, x metres, of P from the origin O is given by

$$x = t^4 - 13.5t + 12$$

(a) Find the velocity, in m/s, of P when $t = 3$

(2)

(b) Find the value of t for which P is instantaneously at rest.

(2)

(c) Find the acceleration, in m/s^2, of P when $t = 2$

(2)

Question 2 continued

(Total for Question 2 is 6 marks)

3 O, A and B are fixed points such that

$$\overrightarrow{OA} = (p\mathbf{i} - 4\mathbf{j}) \qquad \overrightarrow{OB} = \mathbf{i} + (2p + 1)\mathbf{j}$$

Given that $\sqrt{2}\left|\overrightarrow{OA}\right| = \left|\overrightarrow{OB}\right|$ and $p > 0$

(a) find the value of p

(4)

Using this value of p

(b) find a unit vector that is parallel to $\overrightarrow{AB}$

(5)

Question 3 continued

(Total for Question 3 is 9 marks)

4

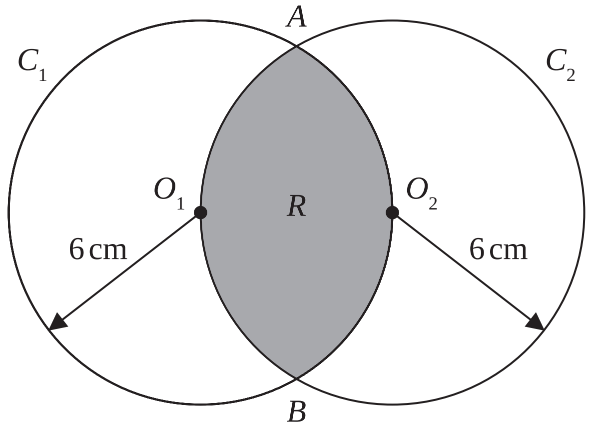


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows two circles, C_1 and C_2, each with a radius of 6 cm.

The centre of C_1 is O_1 such that O_1 lies on C_2

The centre of C_2 is O_2 such that O_2 lies on C_1

The circles intersect at the points A and B and enclose the region R, shown shaded in Figure 1

The area of region R is $P\,\text{cm}^2$

Find the exact value of P, giving your answer in the form $a\pi - b\sqrt{c}$ where a, b and c are integers.

(7)

Question 4 continued

Question 4 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

(Total for Question 4 is 7 marks)

5 The roots of the quadratic equation $2x^2 + (6 + 2p)x + 2p = 0$ are α and β

(a) Write down an expression in terms of p for

(i) $\alpha + \beta$ (ii) $\alpha\beta$

(2)

(b) Show that $(\alpha - \beta)^2 = 9 + 2p + p^2$

(4)

Given that $(\alpha - \beta) = 3$

(c) find the possible values of p

(3)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 9 marks)

6 (a) Using a formula from page 2, show that $\cos 2A = 1 - 2\sin^2 A$

(2)

The finite region R is bounded by the curve with equation $y = 3 + 2\sin x$, the x-axis, the y-axis and the line with equation $x = \dfrac{\pi}{4}$

The region R is rotated through 360° about the x-axis.

(b) Use calculus to find the volume of the solid generated.
Give your answer to the nearest integer.

(6)

Question 6 continued

(Total for Question 6 is 8 marks)

7 (i) (a) Using a formula from page 2, show that

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

(2)

Given that $\tan 2\alpha = 1$

(b) show that $\tan\alpha = a \pm \sqrt{b}$ where a and b are integers whose values need to be found.

(3)

(ii) (a) Using formulae from page 2, show that $\cos(x - 30)^\circ = \sin(x + 30)^\circ$ can be written as $\tan x^\circ = 1$

(4)

(b) Hence, or otherwise, solve

$$\cos(2y - 30)^\circ = \sin(2y + 30)^\circ \qquad \text{for } -90 < y \leqslant 90$$

(2)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 7 continued

PMT

Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

(Total for Question 7 is 11 marks)

8

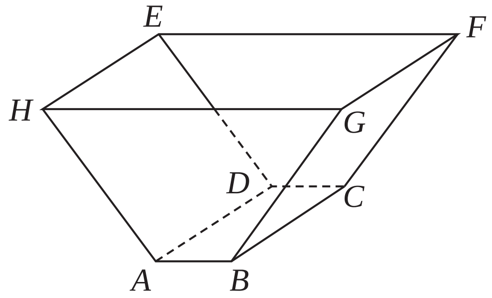


Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows a waste paper basket in the shape of a right prism with 5 faces and a cross section that is a trapezium. The top, *EFGH*, of the waste paper basket is open.

The base of the prism *ABCD* is a rectangle with

$$AB = DC = 2x\,\text{cm} \quad \text{and} \quad AD = BC = h\,\text{cm}$$

The cross sections *HGBA* and *EFCD* are such that

$$EF = HG = 8x\,\text{cm} \quad \text{and} \quad AH = BG = CF = DE = 5x\,\text{cm}$$

The top, *EFGH*, of the waste paper basket is such that

$$EH = FG = h\,\text{cm}$$

The volume of the waste paper basket is $2250\,\text{cm}^3$

The total surface area of the 5 faces of the waste paper basket is $S\,\text{cm}^2$

(a) Show that $S = 40x^2 + \dfrac{1350}{x}$

(5)

Given that x can vary,

(b) use calculus, to find, to 3 significant figures, the value of x for which S is a minimum.
Justify that this value of x gives a minimum value of S

(5)

(c) Find, to 3 significant figures, the minimum value of S

(2)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 12 marks)

9 The straight line L_1 passes through the point A with coordinates (4, 7) and has gradient m, where $m < 0$

Another straight line L_2 is perpendicular to L_1 and passes through the point B with coordinates $(4, k)$ where $k \neq 7$

The lines L_1 and L_2 intersect at the point C.

Given that the y coordinate of C is Y

(a) show that $Y = \dfrac{7 + m^2 k}{m^2 + 1}$

(7)

Given that the triangle ABC is isosceles,

(b) find the value of m

(5)

Question 9 continued

Question 9 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 9 continued

(Total for Question 9 is 12 marks)

10 Solve the equation

$$\log_4 x + \log_{16} x + \log_2 x = 10.5$$

Show your working clearly.

(5)

Question 10 continued

(Total for Question 10 is 5 marks)

11 A curve C has equation

$$y = \frac{(2a-1)x+1}{ax-6} \qquad \text{where } a \text{ is a constant and } x \neq \frac{6}{a}$$

(a) Find $\dfrac{dy}{dx}$

(3)

The curve crosses the y-axis at the point A.

The normal to C at the point A is the line l with equation $66y - 72x + 11 = 0$

Show that

(b) (i) $a = 3$

(4)

(ii) the equation of C is $y = \dfrac{5x+1}{3x-6}$ where $x \neq 2$

(1)

(c) Using the axes on the opposite page, sketch C, showing clearly the asymptotes with their equations and the coordinates of the points where C crosses the coordinate axes.

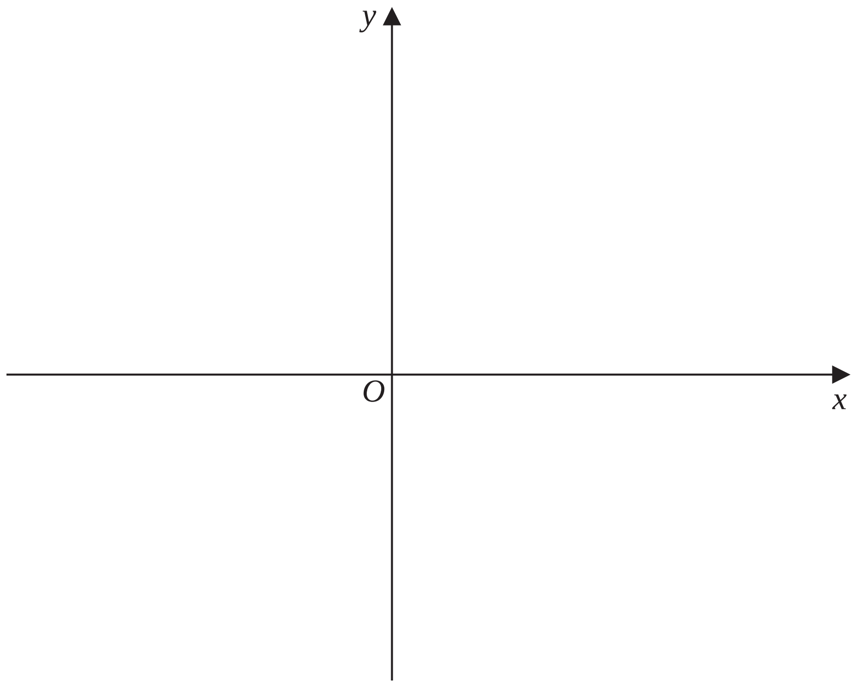
(5)

The line l meets C again at the point D.

(d) Find the x coordinate of D.
Give your answer as an improper fraction.

(4)

Question 11 continued



Question 11 continued

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 11 continued

Question 11 continued

(Total for Question 11 is 17 marks)

TOTAL FOR PAPER IS 100 MARKS