Write your name here Surname	Other name	es
Pearson Edexcel International GCSE	Centre Number	Candidate Number
Further Pu Level 2 Paper 2	ure Mathe	ematics
Sample assessment material for first Time: 2 hours	teaching September 2017	Paper Reference 4PM1/02
Calculators may be used.		Total Marks

Instructions

- Use black ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
 there may be more space than you need.
- You must NOT write anything on the formulae page.
 Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
 - use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times \text{slant height}$

Volume of sphere =
$$\frac{4}{3}\pi r^3$$

Series

Arithmetic series

Sum to *n* terms,
$$S_n = \frac{n}{2} [2a + (n-1)d]$$

Geometric series

Sum to *n* terms,
$$S_n = \frac{a(1-r^n)}{(1-r)}$$

Sum to infinity,
$$S_{\infty} = \frac{a}{1-r} |r| < 1$$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots$$
 for $|x| < 1, n \in \mathbb{Q}$

Calculus

Quotient rule (diferentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x} \left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)} \right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{\left[\mathrm{g}(x)\right]^2}$$

Trigonometry

Cosine rule

In triangle *ABC*: $a^2 = b^2 + c^2 - 2bc \cos A$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 The <i>n</i> th term of a geometric series is 3	$e^{(1-2n)}$
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Find the sum to infinity of this series.

Give your answer in the form $\frac{ae}{e^b-1}$ where a and b are integers to be found.	(5)

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(Total for Question 1 is 5 marks)

2	Find the set of values of x for which	
	(a) $3 + x < 2x - 1$	
		(1)
	(b) $x(x-1) > 6$	
		(3)
	(c) both $3 + x < 2x - 1$ and $x(x - 1) > 6$	
		(1)

3	$O, A \text{ and } B \text{ are fixed points such that}$ $\overrightarrow{OA} = 4\mathbf{i} + 3\mathbf{j} \qquad \overrightarrow{OB} = 8\mathbf{i} + p\mathbf{j} \text{ and } \left \overrightarrow{AB} \right = 2\sqrt{13}$	
	(a) Find the possible values of <i>p</i> .	(3)
	Given that $p > 0$ \longrightarrow	
	(b) find a unit vector parallel to \overrightarrow{AB}	(2)
_	(Total for Question 3	is 5 marks)

4	$f(x) = 2x^3 + px^2 + qx + 12$ $p, q \in \mathbb{Z}$	
	Given that $(x + 3)$ is a factor of $f(x)$ and that when $f'(x)$ is divided by $(x + 3)$ the remained	der is 37
	(a) show that $p = 1$ and find the value of q	
		(6)
	(b) hence factorise $f(x)$ completely	(2)
	(c) show that the equation $f(x) = 0$ has only one real root.	
		(2)
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(a) Show that cos(A - B) - cos(A + B) = 2 sin A sin B(2) (b) Hence express $2\sin 5x \sin 3x$ in the form $\cos mx - \cos nx$ where m and n are integers, giving the value of m and the value of n, (1)(c) (i) Find $\int 4\sin 5\theta \sin 3\theta \, d\theta$ (ii) Hence evaluate $\int_0^{\frac{\pi}{6}} 4\sin 5\theta \sin 3\theta \, d\theta$, giving your answer in the form $\frac{a\sqrt{b}}{c}$ where a, b and c are integers. (4)

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6	Solve the equation $\log_2 x + 6\log_x 2 = 7$

7 The curve *C* with equation

$$y = \frac{ax - 5}{x - b}$$

where a and b are integers, crosses the x-axis at the point (2.5, 0). The asymptote to C which is parallel to the y-axis has equation x = 1

- (a) (i) Show that a = 2
 - (ii) Find the value of b.

(3)

(b) Find the coordinates of the point where C crosses the y-axis.

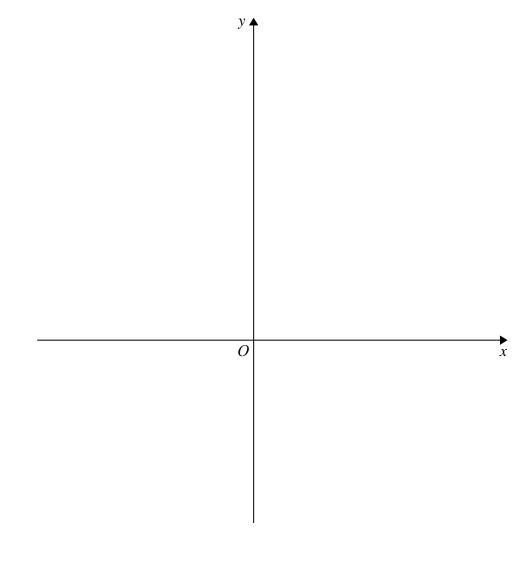
(1)

(c) Find the equation of the asymptote to C which is parallel to the x-axis.

(1)

(d) Using the axes below, sketch the curve C showing clearly the asymptotes and the coordinates of the points where C crosses the coordinate axes.

(3)



8 (a) Expand $\frac{3}{\sqrt{1-2x}}$ in ascending powers of x up to and including the term in x^3 and simplifying each term as far as possible.

(4)

(b) Write down the range of values of x for which this expansion is valid.

(1)

(c) Show that $\frac{3}{\sqrt{0.9}} = \sqrt{10}$

(1)

(d) Express $\frac{1}{\sqrt{10}-3}$ in the form $a\sqrt{10}+b$, where a and b are integers.

(2)

(e) Hence, using your expansion with a suitable value for x, obtain an approximation to 5 decimal places of $\frac{1}{\sqrt{10}-3}$

(3)

9 $f(x) = 7 + 4x - 2x^2$ Given that f(x) can be written in the form $P(x + Q)^2 + R$ where P, Q and R are constants, (a) find the value of P, the value of Q and the value of R. (3)(b) hence write down (i) the maximum value of f(x), (ii) the value of x for which this maximum occurs. (2)The curve C has equation $y = 7 + 4x - 2x^2$ The line *l* with equation y = 4 - x intersects *C* at two points. (c) Find the x coordinates of these two points. (3)The finite region bounded by the curve C and the line l is rotated 360° about the x-axis. (d) Use algebraic integration to find, to 3 significant figures, the volume of the solid generated. (5)

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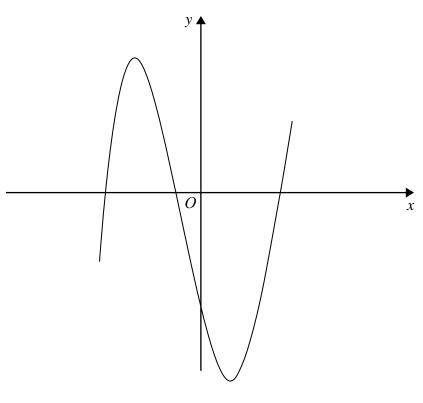


Figure 1

Figure 1 shows the curve M with equation $y = x^3 - 13x - 12$

The point P, with x coordinate -2, lies on M and line l_1 is the tangent to M at the point P.

(a) Find an equation for l_1

(5)

The point Q lies on M and the line l_2 is the tangent to M at the point Q.

Given that l_1 and l_2 are parallel,

(b) find an equation for l_2

(4)

The normal to M at P meets l_2 at the point R.

(c) Find the coordinates of R.

(4)

(d) Find the exact length of the line PR.

(2)

The tangent and normal at P and the tangent and normal at Q form a rectangle.

(e) Find the exact area of this rectangle.

(3)

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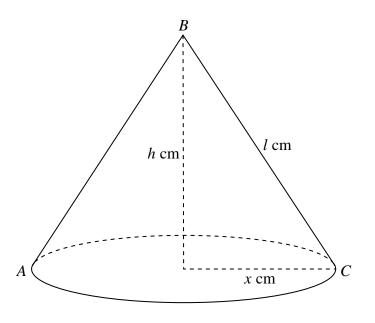


Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows a right circular cone with a base radius of x cm. The slant height of the cone is l cm and the height of the cone is h cm. The vertex of the cone is B and the points A and C, on the base of the cone, are such that AC is a diameter of the base.

The cone is increasing in size in such a way that the size of the angle ABC is constant at 60° and the **total** surface area of the cone is increasing at a constant rate of $10 \text{ cm}^2/\text{s}$.

Find the exact rate of increase of the volume of the cone when $x = 6$	(11)

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	(Total for Question 11 is 11 marks)
	TOTAL FOR PAPER IS 100 MARKS

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