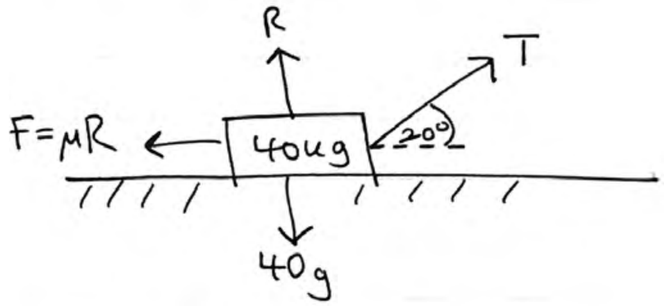
M1 October 2017 (IAL) (MA)

Q1)



N2L(suitcase) $\uparrow^{+}$: $T\sin 20 + R - 40g = 40(0)$

$R = 40g - T\sin 20$ //

N2L(suitcase) $\xrightarrow{+}$: $T\cos 20 - F = 40(0)$

$\therefore \; T\cos 20 = \mu R = \frac{3}{4}R$

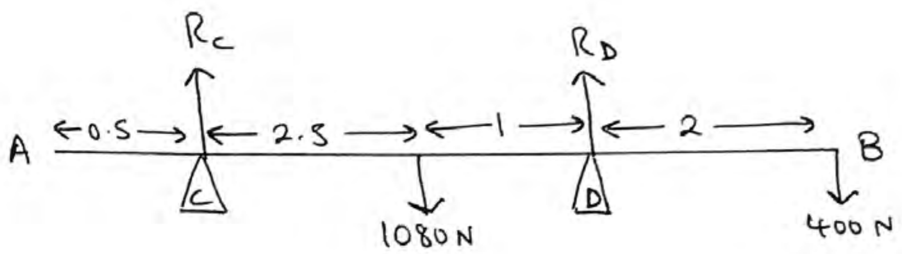
so $T\cos 20 = \frac{3}{4}(40g - T\sin 20)$

$T\cos 20 = 30g - \frac{3}{4}T\sin 20$

$T(\cos 20 + \frac{3}{4}\sin 20) = 30g$

$\therefore T = \dfrac{30g}{\cos 20 + \frac{3}{4}\sin 20} = \boxed{246\text{N}}$ (3 s.f)

Q2ai)



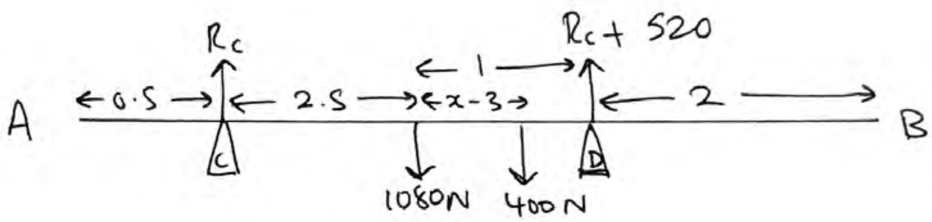
$\underline{M(D)}$: $R_C(3.5) + 400(2) = 1080(1)$

$\Rightarrow R_C = \dfrac{1080 - 400(2)}{3.5} = \boxed{80N}$

ii) $\underline{M(C)}$: $R_D(3.5) = 1080(2.5) + 400(5.5)$

$\Rightarrow R_D = \dfrac{1080(2.5) + 400(5.5)}{3.5} = \boxed{1400N}$

b)



$\underline{R(\updownarrow)}$: $R_C + (R_C + 520) = 1080 + 400$

$\Rightarrow R_C = \dfrac{1080 + 400 - 520}{2}$

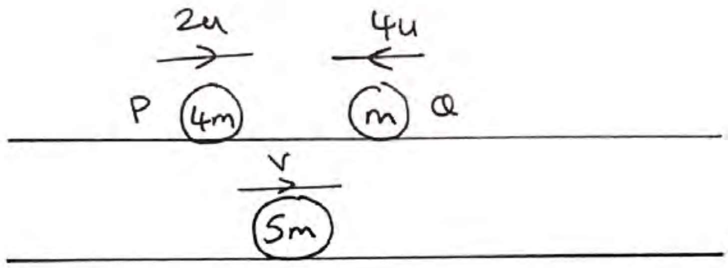
$= 480N$ //

$\underline{M(A)}$: $R_C(0.5) + (R_C + 520)(4) = 1080(3) + 400(x)$

$\therefore x = \dfrac{480(0.5) + (480 + 520)(4) - 1080(3)}{480}$

$= \boxed{2.5m}$

Q3)



$\overrightarrow{C.L.M}$: $4m(2u) - m(4u) = 5m(v)$

$8u - 4u = 5v$

$\Rightarrow \quad v = \frac{4}{5}u$ //

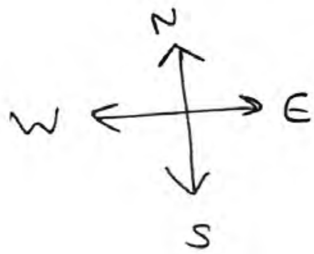
v is +ve so the final particle travels in a direction opposite to which Q was initially travelling in.

Impulse on $Q = m(v - u)$

$= m\left(\frac{4}{5}u - -4u\right) = \boxed{4.8mu}$

Q4a)

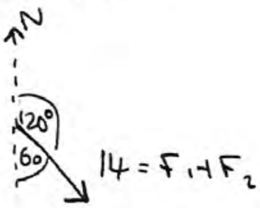
$\xrightarrow{F_1 = 8N}$



(ie) $F_1 = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$ in vector form.

and $F_2 = \begin{pmatrix} x \\ y \end{pmatrix}$

we are told $F_1 + F_2$ can be represented as :



or in vector form :

$$F_1 + F_2 = \begin{pmatrix} 14\sin 60 \\ -14\cos 60 \end{pmatrix}$$

$$\therefore \begin{pmatrix} 8 \\ 0 \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 14\sin 60 \\ -14\cos 60 \end{pmatrix}$$

$\frac{11}{5}mg - \frac{1}{4}(R) = 7ma$

R($\nearrow^+$) for A : $R - 3mg\cos\alpha = 0$

$R = 3mg\cos\alpha = 3mg\left(\frac{4}{5}\right)$
$= \left(\frac{12mg}{5}\right)$

so $\frac{11}{5}mg - \frac{12mg}{5\times 4} = 7ma$

$\therefore a = \dfrac{\frac{11}{5}g - \frac{12g}{20}}{7} = \boxed{\frac{8g}{35}}$

c) Particles have the same acceleration ...

d) find speed when B hits the ground:

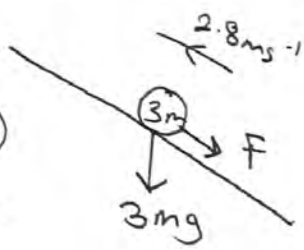
$s = 1.75$
$u = 0$
$v = v$
$a = 8g/35$
t

$v^2 = u^2 + 2as$

$\Rightarrow v^2 = 0^2 + 2\left(\frac{8g}{35}\right)(1.75)$

$\Rightarrow v = \sqrt{2\left(\frac{8g}{35}\right)(1.75)} = \boxed{2.8\text{ms}^{-1}}$

new model of A's motion once B hits the ground (string is now slack)



N2L(A)$\nwarrow^+$: $-F - 3mg\sin\alpha = 3m(a)$

$-\frac{1}{4}\left(\frac{12mg}{5}\right) - 3mg\left(\frac{3}{5}\right) = 3ma$

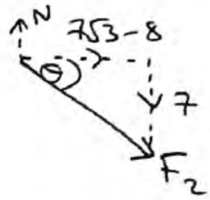
$\therefore a = -\frac{4}{5}g$

So $x = 14\sin 60 - 8 = 7\sqrt{3} - 8$
$y = -14\cos 60 = -7$

$\therefore |F_2| = \sqrt{x^2+y^2} = \sqrt{(-7)^2 + (7\sqrt{3}-8)^2}$

$= \boxed{8.12N}$

ii) $F_2 = \begin{pmatrix} 7\sqrt{3} - 8 \\ -7 \end{pmatrix}$

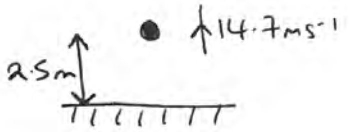


so $\tan\theta = \dfrac{7}{7\sqrt{3}-8}$

$\therefore$ Bearing required $= 90 + \tan^{-1}\left(\dfrac{7}{7\sqrt{3}-8}\right)$

$= \boxed{149°}$

(Q5a)



$+\uparrow$
$s = h$
$u = 14.7ms^{-1}$
$v = 0$
$a = -g$
$t =$

$v^2 = u^2 + 2as$
$0^2 = 14.7^2 - 2gh$
$h = \dfrac{14.7^2}{2g}$ //

So greatest height above ground will be $2.5 + \dfrac{14.7^2}{2g} = \boxed{13.5m}$

displacement from starting position will be -1.5m!

b) $\uparrow^{+}$
$s = -1.5$
$u = 14.7$
$v =$
$a = -9$
$t = t$

$s = ut + \frac{1}{2}at^2$

$-1.5 = 14.7t - \frac{9}{2}t^2$

$4.9t^2 - 14.7t - 1.5 = 0$

By Quadratic Formula: $t_1 = 3.0988\ldots$ s
$t_2 = -0.0988\ldots$ s

$t > 0$ so $\boxed{t = 3.10\text{s}}$

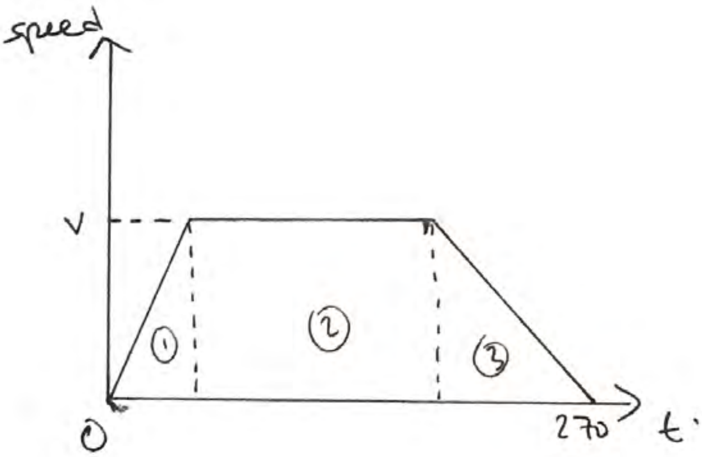
c) $\uparrow^{+}$
$s = -2.5$
$u = 14.7$
$v = v$
$a = -9$
$t =$

$v^2 = u^2 + 2as$

$v^2 = 14.7^2 + 2(-9)(-2.5)$
$v^2 = 265.09$

so $v = \sqrt{265.09} = \boxed{16.3 \text{ ms}^{-1}}$

Q6a)



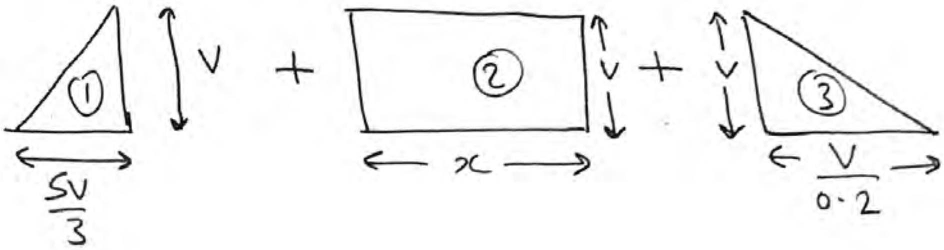
b) $a = \frac{v - u}{t} \Rightarrow 0.6 = \frac{v}{t}$

$\therefore t = \frac{v}{0.6} = \boxed{\frac{5v}{3}}$ //

c) area under graph = distance.

So... total area = 1500

$\text{Area}_{\text{TOTAL}} =$



$$x = 270 - \left(\frac{5v}{3} + \frac{v}{0.2}\right) = 270 - \frac{20v}{3}$$

$$\therefore \text{Area}_{\text{TOTAL}} = \frac{1}{2}\left(\frac{5v}{3}\right)(v) + \left(270 - \frac{20v}{3}\right)(v) + \frac{1}{2}(v)\left(\frac{v}{0.2}\right) = 1500$$

$$\Rightarrow \frac{5v^2}{6} + 270v - \frac{20v^2}{3} + \frac{v^2}{0.4} = 1500$$

$$\Rightarrow v^2\left(\frac{5}{6} - \frac{20}{3} + \frac{1}{0.4}\right) + v(270) - 1500 = 0$$

$$\Rightarrow -\frac{10v^2}{3} + 270v - 1500 = 0$$

$$\times\left(-\frac{3}{10}\right): \quad v^2 - 81v + 450 = 0$$

d) $V^2 - 81V + 450 = 0$

By Quadratic Formula: $V = 75$ or $V = 6$

$\left(\begin{array}{l} a=1 \\ b=-81 \\ c=450 \end{array}\right)$ or by factorisation:

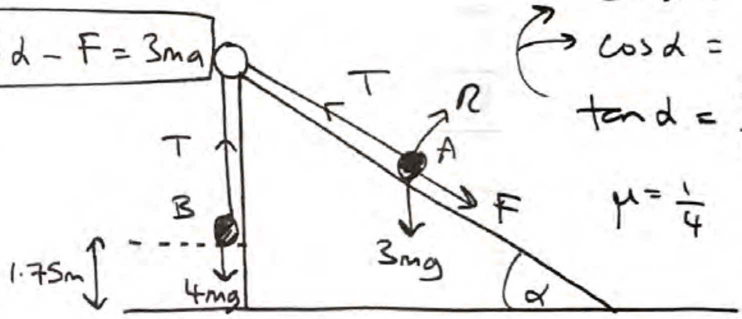
$(V-6)(V-75) = 0$

$\Rightarrow V = 75$ or $V = 6$

the correct answer is $\boxed{V=6}$.

This can be checked by simply inputting $V=75$ back into any equation for the area of ①, ② or ③ and you will see the answer will be greater than 1500m, so $V=75$ is clearly false.

Q7a) N2L(A) ↖+ : $\boxed{T - 3mg\sin\alpha - F = 3ma}$ ①



$\sin\alpha = \frac{3}{5}$
$\cos\alpha = \frac{4}{5}$
$\tan\alpha = \frac{3}{4}$

$\mu = \frac{1}{4}$

②

N2L(B) ↓+ : $\boxed{4mg - T = 4ma}$

b) ① + ② : $4mg - 3mg\sin\alpha + \cancel{T} - \cancel{T} - F = 7ma$

$4mg - 3mg\left(\frac{3}{5}\right) - \mu R = 7ma$

use suvat for a now from the moment B hits the ground til A comes to rest:

$(\nwarrow^{+})$
$s = d$
$u = 2.8$
$v = 0$
$a = -\frac{4g}{5}$
$t =$

$$v^2 = u^2 + 2as$$

$$0^2 = 2.8^2 - \frac{8gd}{5}$$

$$\frac{8gd}{5} = 2.8^2$$

$$\Rightarrow \quad d = \frac{(2.8^2)(5)}{8 \times 9.8} = 0.5\text{m} //$$

so total distance $= 1.75 + 0.5 = \boxed{2.25\text{m}}$

(if B travels 1.5m then so does A!)