Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International GCSE

Time 2 hours

Paper reference **4PM1/01**

Further Pure Mathematics
PAPER 1

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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Q:1/1/1/1/

P 7 1 6 6 1 A 0 1 3 2

Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\qquad$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\qquad$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$ $\qquad$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

Answer all TEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Given that $\dfrac{2\sqrt{3}-4}{3\sqrt{3}+5}$ can be written in the form $a + b\sqrt{3}$ where a and b are integers, find, without using a calculator, the value of a and the value of b

Show your working clearly.

(3)

(Total for Question 1 is 3 marks)

2

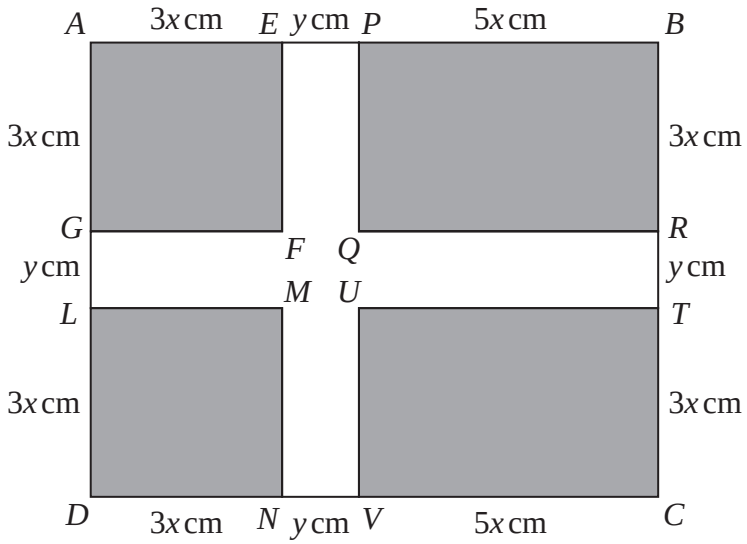

Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows the design for a flag consisting of a white cross on a grey background.

AEFG and *DLMN* are squares with sides of length $3x$ cm.
BPQR and *CTUV* are rectangles with sides of length $5x$ cm and $3x$ cm.
The width of the cross is y cm.

The total area of the flag is H cm^2

(a) Write down an expression, in terms of x and y, for H.

(1)

Given that the area of the cross is K cm^2

(b) show that $K = 14xy + y^2$

(3)

The total area of the flag is to be 3432 cm^2 and the area of the cross is to be 1080 cm^2

(c) Find the value of x and the value of y

(5)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 9 marks)

3

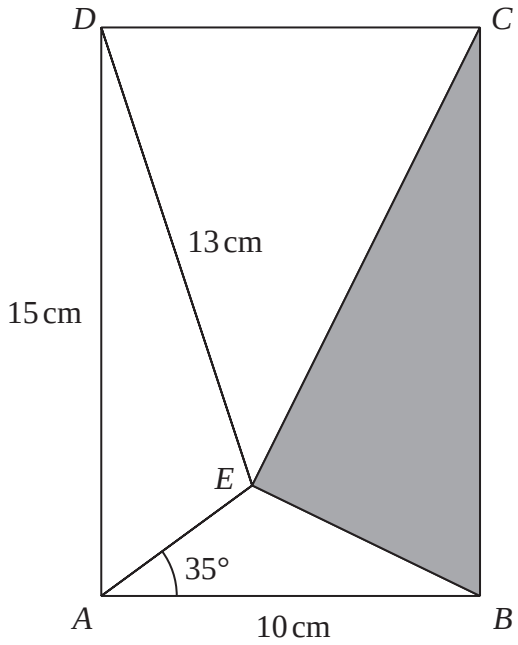

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Figure 2

Figure 2 shows a rectangle $ABCD$ with $AB = 10$ cm and $AD = 15$ cm.
E is the point inside the rectangle such that $DE = 13$ cm and angle $BAE = 35°$

Given that angle AED is obtuse,

find the area, in cm^2 to one decimal place, of triangle BCE.

(7)

Question 3 continued

(Total for Question 3 is 7 marks)

4 The common ratio of a geometric series G is positive.

The sum of the first 4 terms of G is 80
The sum to infinity of G is 81

Show that the sum of the first 7 terms of G differs from the sum to infinity of G by $\frac{1}{27}$

(7)

Question 4 continued

(Total for Question 4 is 7 marks)

5 Given that $(2 + 3x)^{-1}$ can be expressed in the form $p(1 + qx)^{-1}$ where p and q are constants,

(a) find the value of p and the value of q

(2)

(b) Hence expand $(2 + 3x)^{-1}$ in ascending powers of x up to and including the term in x^3, expressing each coefficient as an exact fraction in its lowest terms.

(3)

$$f(x) = \frac{1 + x}{2 + 3x}$$

(c) Obtain a series expansion for $f(x)$, in ascending powers of x up to and including the term in x^3, expressing each coefficient as an exact fraction in its lowest terms.

(2)

(d) Hence use algebraic integration to obtain an estimate, to 4 decimal places, of

$$\int_0^{0.5} f(x)\,dx$$

(4)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 11 marks)

6 (a) Complete the table of values for $y = 1 + 3e^{-x}$ giving your answers to 2 decimal places where appropriate.

(2)

x	0	0.25	0.5	1	1.5	2	3
y		3.34	2.82		1.67		1.15

(b) On the grid opposite, draw the graph of $y = 1 + 3e^{-x}$ for $0 \leqslant x \leqslant 3$

(2)

(c) By drawing an appropriate straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation

$$x = e^{-x} \quad \text{in the interval } 0 \leqslant x \leqslant 3$$

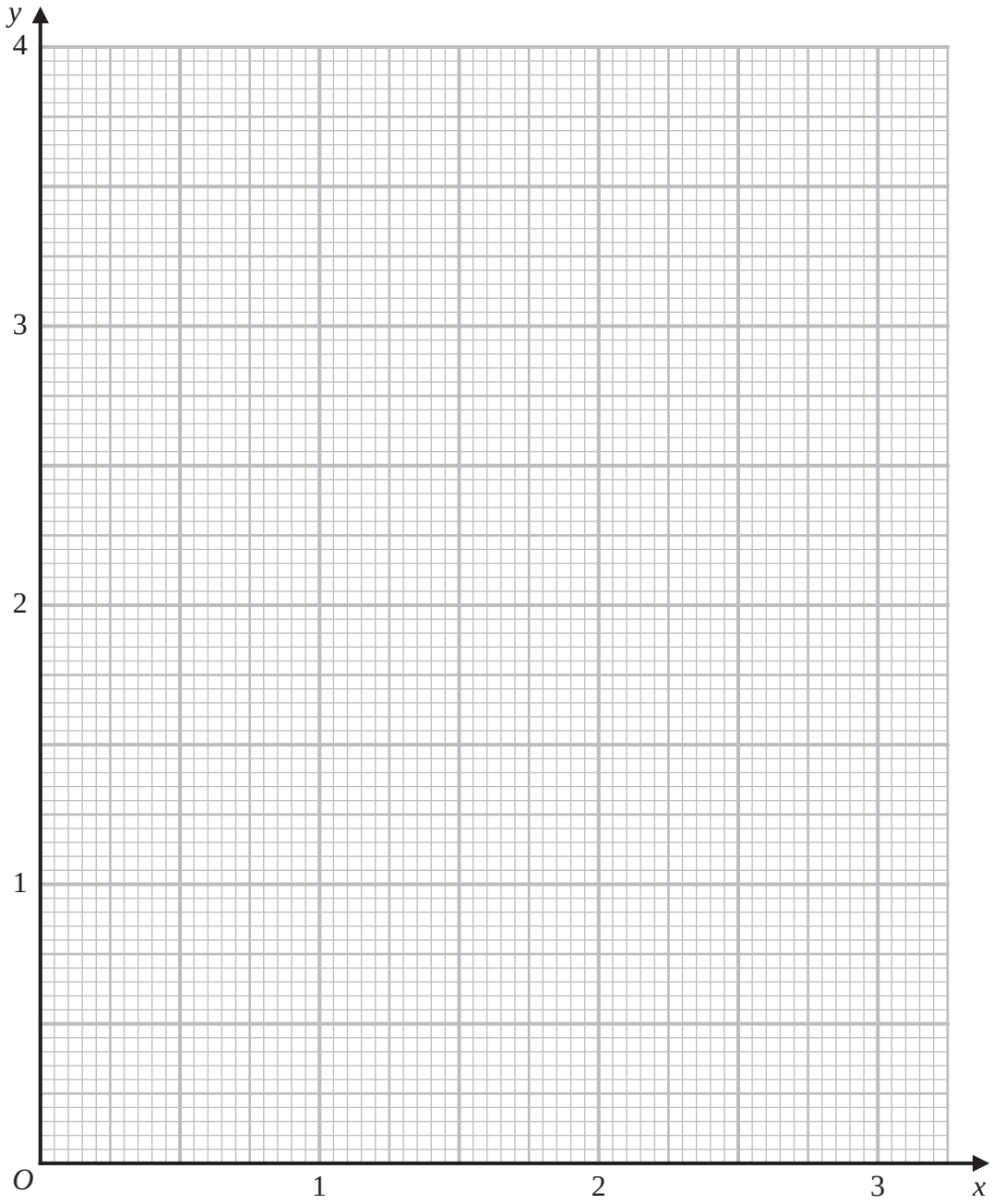
(3)

(d) By drawing an appropriate straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation

$$\ln(x - 1)^3 = -3x \quad \text{in the interval } 0 \leqslant x \leqslant 3$$

(4)

Question 6 continued

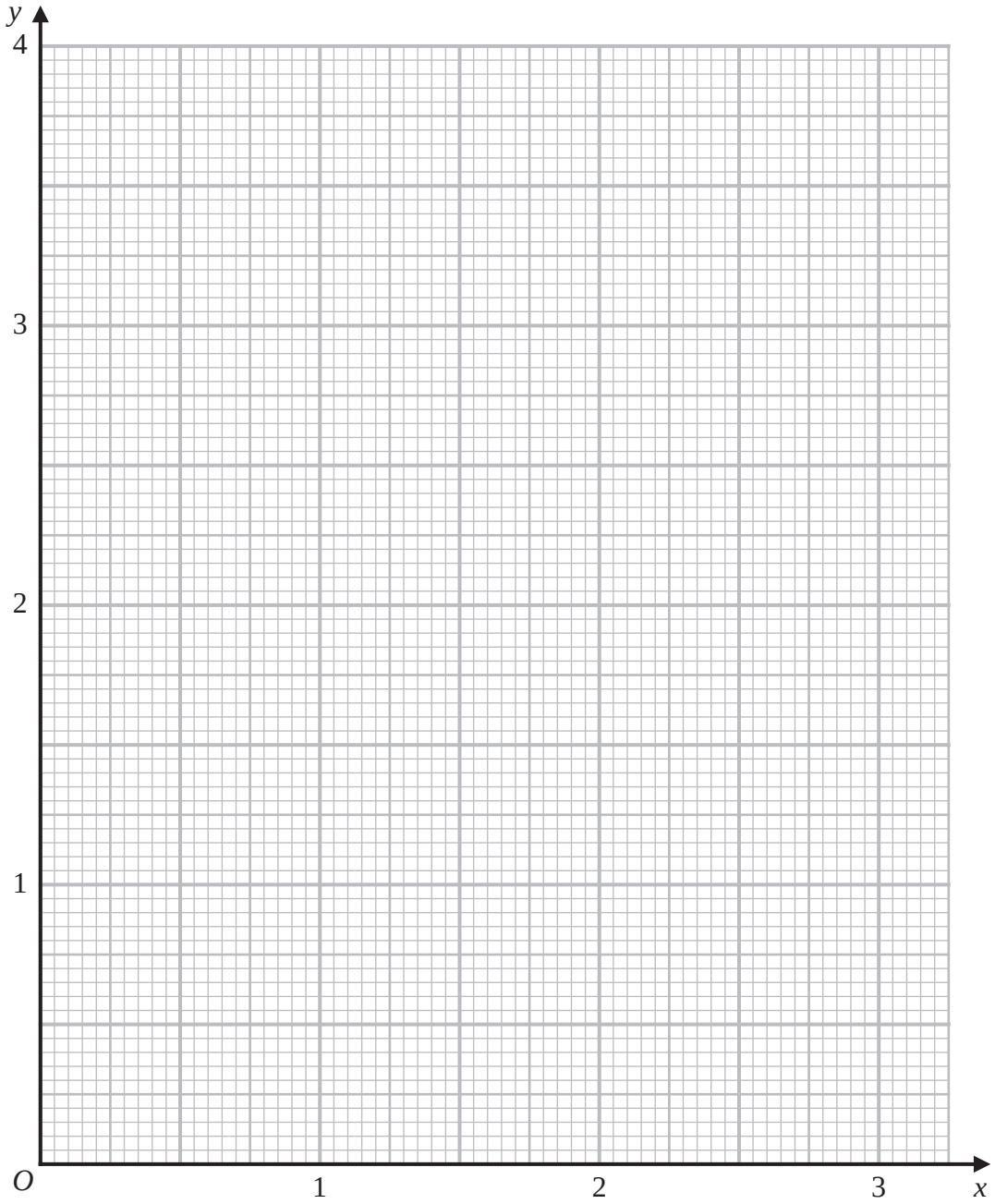


Turn over for a spare grid if you need to redraw your graph.

Question 6 continued

Question 6 continued

Only use this grid if you need to redraw your graph.



(Total for Question 6 is 11 marks)

7 The point with coordinates (4, –104) lies on the curve C with equation $y = f(x)$

Given that $f'(x) = 4x^3 - 12x^2 - 19x + 12$

(a) (i) show that C passes through the origin,

(4)

(ii) show that C has a maximum at the point on the curve where $x = 0.5$

(3)

The curve C has another turning point at A and another turning point at B.

Given that the x coordinate of A is negative,

(b) (i) find the coordinates of A and the coordinates of B,

(5)

(ii) determine the nature of these turning points.

(3)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 15 marks)

8 The volume of a sphere is $500\,\text{cm}^3$

(a) Calculate the radius, in cm to 3 significant figures, of the sphere.

(2)

The surface area of the sphere is increased by $20\,\text{cm}^2$

(b) Using calculus, find an estimate for the increase in the radius, in cm to 2 significant figures, of the sphere.

(5)

Question 8 continued

(Total for Question 8 is 7 marks)

9 $f(x) = 3x^4 + 4x^3 - 36x^2 + 64$

Given that $f(x)$ can be written in the form $(x - 2)^2(ax^2 + bx + c)$

(a) find the value of a, the value of b and the value of c

(4)

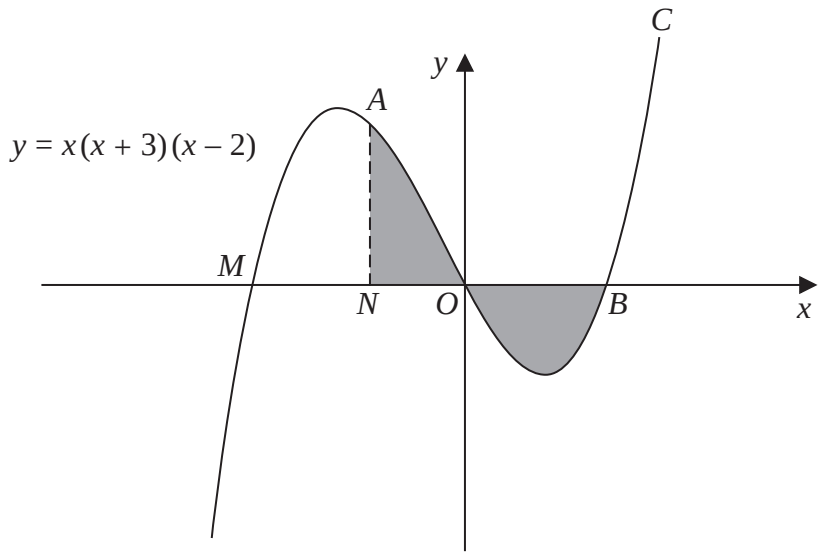


Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows a sketch of part of the curve C with equation $y = x(x + 3)(x - 2)$

The curve C crosses the x-axis at the point M, the origin and the point B.
The point N lies on the x-axis between M and O.
The point A lies on C such that AN is parallel to the y-axis.

The area of the shaded region bounded by the curve and OB is numerically equal to the area of the shaded region bounded by the curve, ON and NA.

Given that the coordinates of N are $(n, 0)$,

(b) use algebraic integration to show that n satisfies the equation

$$(x - 2)^2(3x^2 + 16x + 16) = 0$$

(7)

(c) Hence find the exact coordinates of A.

(5)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 16 marks)

10

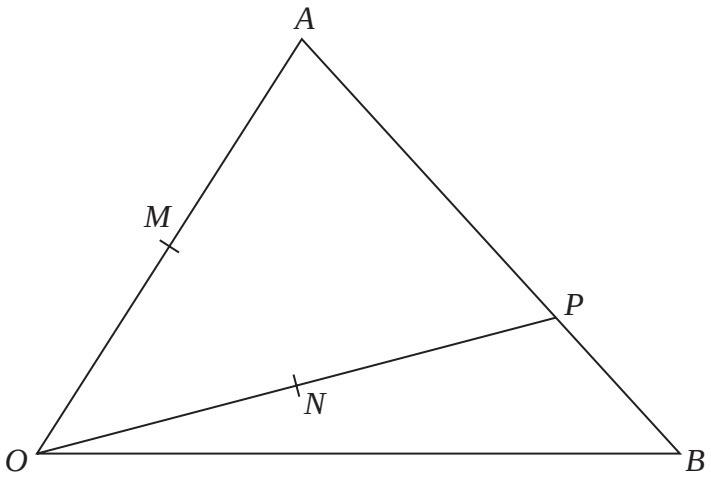


Diagram **NOT** accurately drawn

Figure 4

Figure 4 shows triangle OAB in which

$$\overrightarrow{OA} = \mathbf{a} \text{ and } \overrightarrow{OB} = \mathbf{b}$$

The point P lies on AB such that $AP : PB = 3 : 1$

The point M is the midpoint of OA and the point N is the midpoint of OP.

(a) Find, as simplified expressions in terms of **a** and **b**, the vector

(i) $\overrightarrow{OP}$ (ii) $\overrightarrow{MN}$

(4)

The point C lies on OB such that ANC is a straight line.

(b) Using a vector method, find the vector $\overrightarrow{OC}$ as a simplified expression in terms of **b**

(6)

Given that $\dfrac{\text{area of quadrilateral } AMNP}{\text{area of triangle } OAB} = K$

(c) find the exact value of K

(4)

Question 10 continued

Question 10 continued

(Total for Question 10 is 14 marks)

TOTAL FOR PAPER IS 100 MARKS