Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International GCSE

Time 2 hours

Paper reference **4PM1/01**

Further Pure Mathematics
PAPER 1

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\quad$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\quad$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$ $\quad$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

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Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 The nth term of an arithmetic series is a_n where

$$a_{10} + a_{11} + a_{12} = 129 \quad \text{and} \quad a_{19} + a_{20} + a_{21} = 237$$

Find a_1

(4)

(Total for Question 1 is 4 marks)

2 The point A has coordinates $(-5, 3)$, the point B has coordinates $(4, 0)$ and the point C has coordinates $(-1, 5)$.

The line l passes through C and is perpendicular to AB.

(a) Find an equation of l.
Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(4)

The line l intersects AB at the point D.

(b) Show that the coordinates of D are $(-2, 2)$.

(3)

(c) Show that l is not the perpendicular bisector of AB.

(2)

(d) Find the value of $\tan\angle ABC$.
Give your answer in its simplest form.

(4)

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Question 2 continued

Question 2 continued

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Question 2 continued

(Total for Question 2 is 13 marks)

3 Curve C has equation $y = \dfrac{ax + 3}{1 - 2x}$ where $x \neq \dfrac{1}{2}$ and a is a constant.

The asymptote to C that is parallel to the x-axis has equation $y = 4$

(a) Find the value of a

(2)

(b) Write down the equation of the asymptote to C that is parallel to the y-axis.

(1)

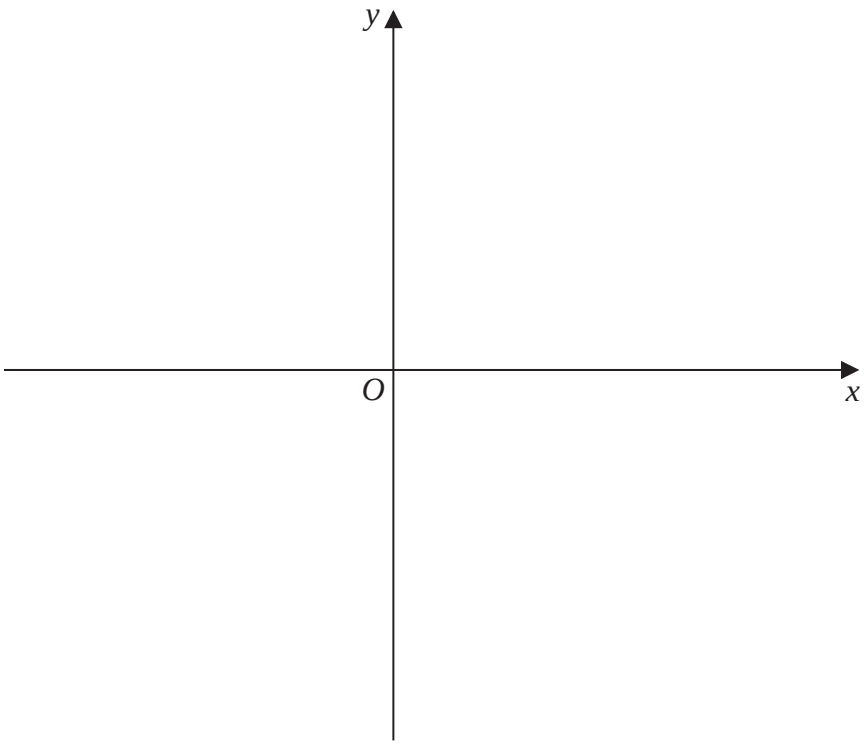
(c) Find the coordinates of the point where C crosses

(i) the x-axis, (ii) the y-axis.

(2)

(d) Using the axes below, sketch C, showing clearly the asymptotes and the coordinates of the points where C crosses the coordinate axes.

(4)



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Question 3 continued

(Total for Question 3 is 9 marks)

4 $f(x) = x^3 + px^2 + qx + 6$ where p and q are constants.

Given that $(x - 1)$ is a factor of $f(x)$ and that when $f(x)$ is divided by $(x + 1)$ the remainder is 8

(a) (i) show that $p = -2$

(ii) find the value of q

(6)

(b) Hence, solve the equation $f(x) = 0$

(3)

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Question 4 continued

(Total for Question 4 is 9 marks)

5 Given that k is a non-zero constant

curve C has equation $kx^2 - xy + (k + 1)x = 1$

straight line l has equation $y = \frac{k}{2}x + 1$

The point A is the only point that lies on both C and l.

(a) Find the value of k

(6)

(b) Hence, find the coordinates of A.

(2)

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Question 5 continued

(Total for Question 5 is 8 marks)

6 Given that $(8 + 3x)^{\frac{1}{3}}$ can be expressed in the form $p(1 + qx)^{\frac{1}{3}}$ where p and q are constants,

(a) find the value of p and the value of q

(2)

(b) Hence, expand $(8 + 3x)^{\frac{1}{3}}$ in ascending powers of x up to and including the term in x^2, expressing each coefficient as an exact fraction in its lowest terms.

(3)

Using the expansion found in part (b) with a suitable value of x

(c) show that $\sqrt[3]{9} \approx \frac{599}{288}$

(2)

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Question 6 continued

(Total for Question 6 is 7 marks)

7 (a) Complete the table of values for

$$y = 0.5^{\left(\frac{x}{3}+1\right)} + 2$$

giving each value to 2 decimal places where appropriate.

x	–6	–5	–4	–3	–2	–1	0
y	4	3.59	3.26				2.5

(2)

(b) On the grid opposite, draw the graph of $y = 0.5^{\left(\frac{x}{3}+1\right)} + 2$ for $-6 \leqslant x \leqslant 0$

(2)

(c) By drawing a suitable straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation

$$\log_2(2x + 2)^3 + x + 3 = 0 \text{ in the interval } -6 \leqslant x \leqslant 0$$

(6)

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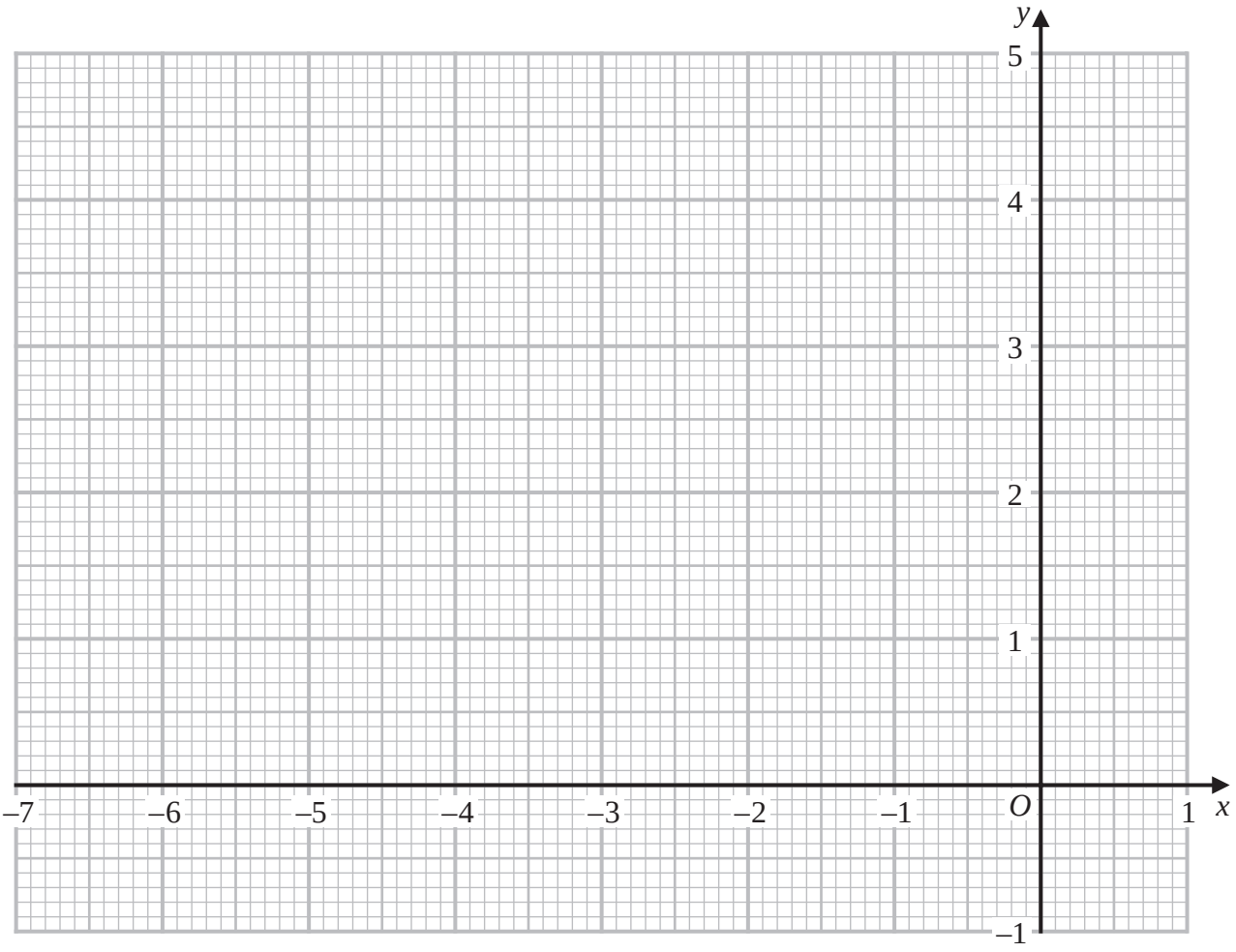
Question 7 continued



Turn over for a spare grid if you need to redraw your graph.

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Question 7 continued

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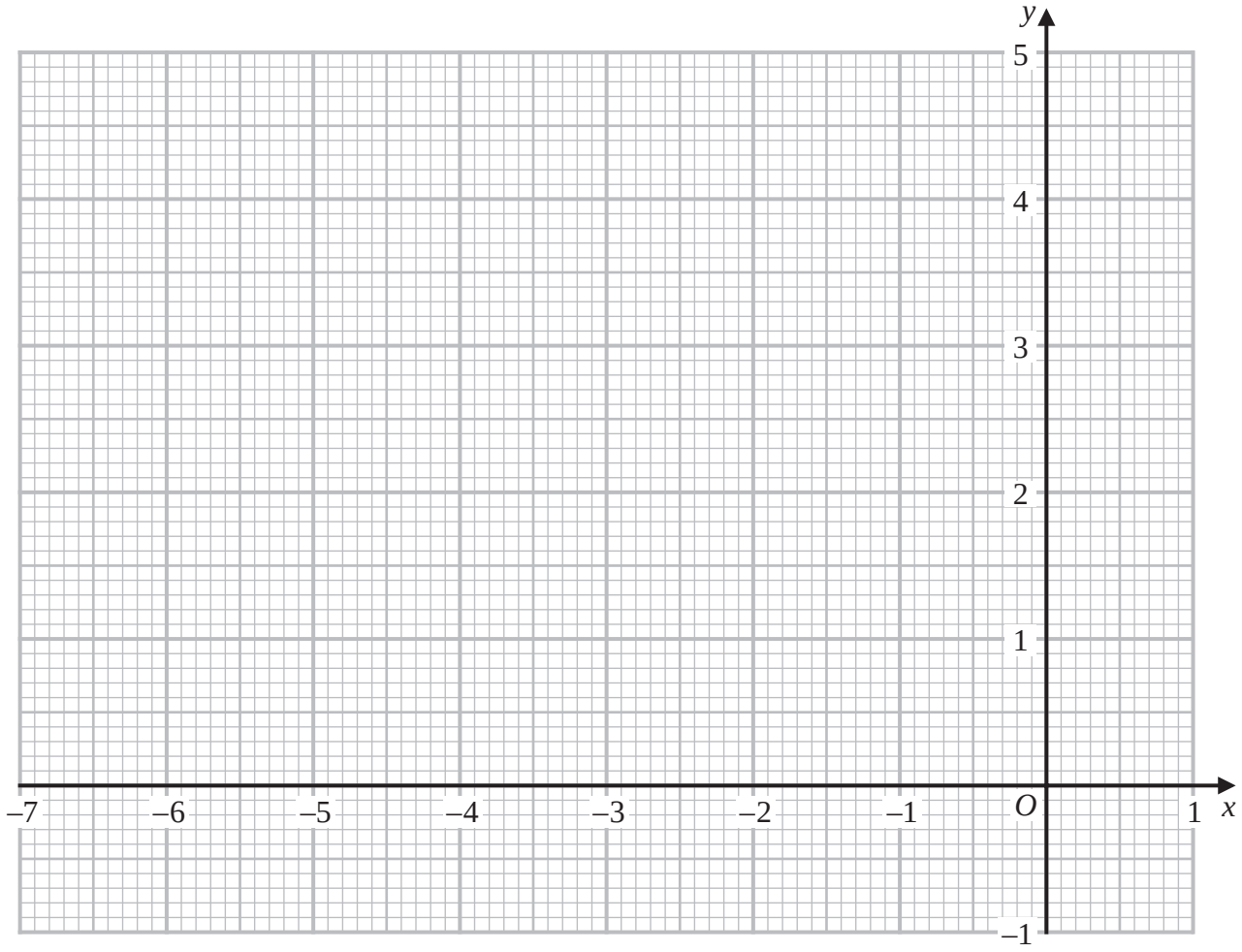
Only use this grid if you need to redraw your graph.



(Total for Question 7 is 10 marks)

8

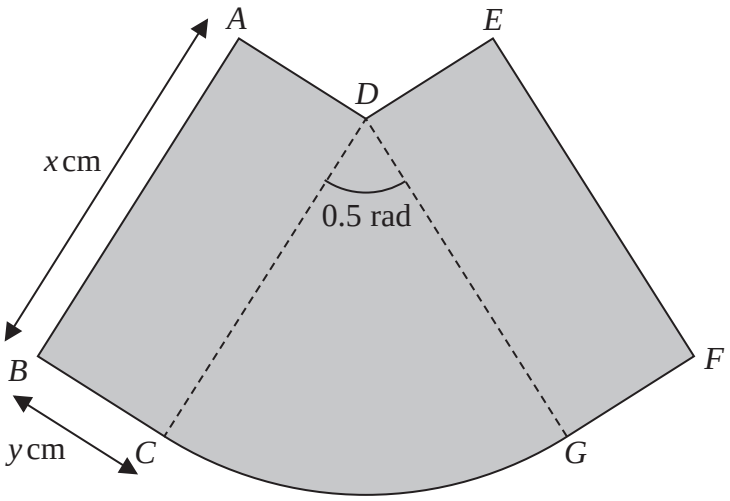


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows a badge, shown shaded, made from two identical rectangles, *ABCD* and *DEFG*, and a sector *DCG* of a circle with centre *D*.

Each rectangle measures x cm by y cm.
The radius of the sector is x cm and the angle *CDG* is 0.5 radians.

The area of the badge is $50\,\text{cm}^2$
The perimeter of the badge is P cm.

(a) Show that

$$P = 2x + \frac{100}{x}$$

(5)

Given that x can vary,

(b) use calculus, to find the exact value of x for which P is a minimum.
Justify that this value of x gives a minimum value for P

(6)

(c) Find the minimum value of P
Give your answer in the form $k\sqrt{2}$, where k is an integer to be found.

(2)

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Question 8 continued

Question 8 continued

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Question 8 continued

(Total for Question 8 is 13 marks)

9 Giving each value in your solution to 2 decimal places, solve the simultaneous equations

$$e^{2y} - x + 2 = 0$$

$$\ln(x + 3) - 2y - 1 = 0$$

(8)

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Question 9 continued

(Total for Question 9 is 8 marks)

10

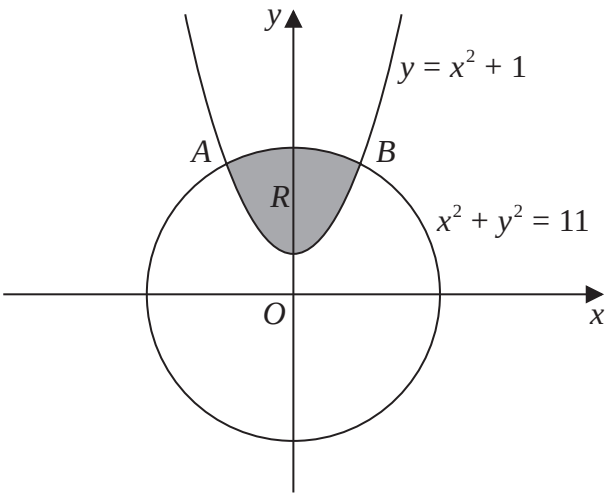

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Figure 2

The region R, shown shaded in Figure 2, is bounded by the curve with equation $y = x^2 + 1$ and the curve with equation $x^2 + y^2 = 11$

The two curves intersect at the point A and at the point B.

(a) Find the x coordinate of the point A and the x coordinate of the point B.

(4)

The region R is rotated through 360° about the x-axis.

(b) Use algebraic integration to find the volume, to 2 decimal places, of the solid generated.

(5)

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Question 10 continued

Question 10 continued

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Question 10 continued

(Total for Question 10 is 9 marks)

11

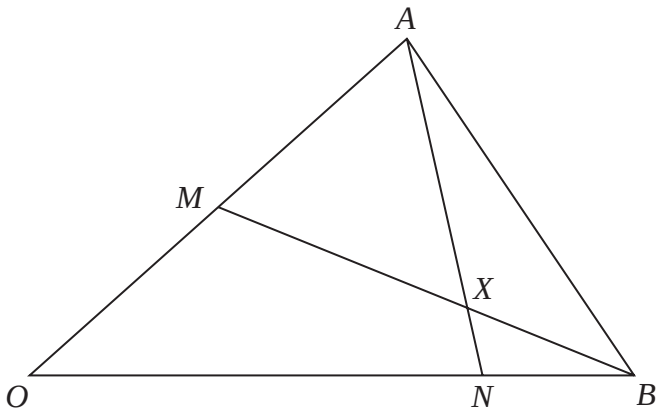

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Figure 3

Figure 3 shows triangle OAB with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

M is the midpoint of OA.

N is the point on OB such that $ON : NB = 3 : 1$

The lines AN and BM intersect at the point X.

(a) Find expressions, in terms of **a** and **b**, for

(i) $\overrightarrow{AN}$ (ii) $\overrightarrow{BM}$

(3)

(b) Using a vector method, find $AX : XN$

(7)

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Question 11 continued

Question 11 continued

(Total for Question 11 is 10 marks)

TOTAL FOR PAPER IS 100 MARKS

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