

Question Number	Scheme	Marks
10(a)	$\cos(A+B) + \cos(A-B) = \cos A \cos B - \sin A \sin B + \cos A \cos B + \sin A \sin B$ $= 2 \cos A \cos B \quad *$	M1 A1 cso (2)
(b)	Let $A+B=P$, $A-B=Q \Rightarrow A=\frac{1}{2}(P+Q)$, $B=\frac{1}{2}(P-Q)$ As $\cos(A+B) + \cos(A-B) = 2 \cos A \cos B$ $\cos P + \cos Q = 2 \cos \frac{1}{2}(P+Q) \cos \frac{1}{2}(P-Q) \quad *$	M1 M1A1cso (3)
ALT	Let $A=\frac{1}{2}(P+Q)$, $B=\frac{1}{2}(P-Q) \Rightarrow A+B=P$, $A-B=Q$ As $2 \cos A \cos B = \cos(A+B) + \cos(A-B)$ $2 \cos \frac{1}{2}(P+Q) \cos \frac{1}{2}(P-Q) = \cos P + \cos Q \quad *$	M1 M1A1cso
(c)	$\cos 5\theta + \cos 7\theta = 2 \cos 6\theta \cos \theta = 0$ $\cos 6\theta = 0 \Rightarrow 6\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \Rightarrow \theta = \frac{\pi}{12}, \frac{\pi}{4} \left(\text{or } \frac{3\pi}{12} \right), \frac{5\pi}{12}$ $\cos \theta = 0 \Rightarrow \theta = \frac{\pi}{2}$	M1 A1A1 A1 (4)
(d)	$\cos 8x + 2 \cos 6x + \cos 4x = (\cos 8x + \cos 6x) + (\cos 6x + \cos 4x)$ $= 2 \cos 7x \cos x + 2 \cos 5x \cos x$ $= 2 \cos x (\cos 7x + \cos 5x) = 2 \cos x \times 2 (\cos 6x \cos x), = 4 \cos 6x \cos^2 x \quad *$	M1 dM1, A1cso (3)
ALT 1	$\cos 8x + 2 \cos 6x + \cos 4x = (\cos 8x + \cos 4x) + 2 \cos 6x$ $= 2 \cos 6x \cos 2x + 2 \cos 6x$ $= 2 \cos 6x (\cos 2x + 1) = 2 \cos 6x (2 \cos^2 x - 1 + 1) = 4 \cos 6x \cos^2 x \quad *$	M1 dM1A1cso (3)
ALT 2	Working from right to left $4 \cos 6x \cos^2 x = 4 \cos 6x \times \frac{1}{2} (\cos 2x + 1) = 2 \cos 6x \cos 2x + 2 \cos 6x$ $= \cos 8x + \cos 4x + 2 \cos 6x \quad *$	M1 dM1A1cso (3)

<p>(e)</p> <p>ALT</p>	$\int_0^{\frac{\pi}{3}} \cos 6x \cos^2 x \, dx = \frac{1}{4} \int_0^{\frac{\pi}{3}} (\cos 8x + 2 \cos 6x + \cos 4x) \, dx$ $= \frac{1}{4} \left[\frac{1}{8} \sin 8x + \frac{1}{3} \sin 6x + \frac{1}{4} \sin 4x \right]_0^{\frac{\pi}{3}}$ $= \frac{1}{4} \left[\frac{1}{8} \sin \frac{8\pi}{3} + \frac{1}{3} \sin 2\pi + \frac{1}{4} \sin \frac{4\pi}{3} - 0 \right] = -\frac{\sqrt{3}}{64} \quad (-0.02706... \text{ scores M1A0})$ <p>For the first M mark Award M1 only when the integrand has been changed (by a valid method) to a function which can be integrated</p> $\int_0^{\frac{\pi}{3}} \cos 6x \cos^2 x \, dx = \int_0^{\frac{\pi}{3}} \cos 6x \times \frac{1}{2} (\cos 2x + 1) \, dx$ $= \frac{1}{2} \int_0^{\frac{\pi}{3}} (\cos 6x \times \cos 2x + \cos 6x) \, dx = \frac{1}{2} \int_0^{\frac{\pi}{3}} \frac{1}{2} (\cos 8x + \cos 4x + 2 \cos 6x) \, dx$ <p>Rest as above.</p>	<p>M1</p> <p>A1</p> <p>dM1A1 (4)</p> <p>[16]</p> <p>M1</p>
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<p>(a)</p> <p>M1</p> <p>A1cso</p> <p>(b)</p> <p>M1</p> <p>M1</p> <p>A1cso</p> <p>ALT</p> <p>(c)</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>NB</p> <p>(d)</p> <p>M1</p> <p>dM1</p> <p>A1cso</p>	<p>Use the standard formulae. This is a “show that” question so these formulae must be written out in full. Both sides of the result must be seen although the working may appear between them as seen in the scheme.</p> <p>Final given result. Do not award if the expansions shown are not in the correct order (this suggests incorrect signs in the formulae).</p> <p>Use $A + B = P$, $A - B = Q$ to obtain A and B in terms of P and Q</p> <p>Substitute in the result from (a) to obtain an identity in P and Q only</p> <p>Correct result reached with no errors seen</p> <p>Working right to left: Notes similar to above</p> <p>Use the result from (b) to show that $(2) \cos 6\theta \cos(\pm\theta) = 0$</p> <p>Obtain one correct solution of $\cos 6\theta = 0$. Allow if in decimal form but must be radians.</p> <p>Two further correct solutions and no more within the range.</p> <p>If any of the 3 solutions of $\cos 6\theta = 0$ is not exact do not award this mark.</p> <p>State the solution of $\cos(\pm\theta) = 0$. Must be exact unless this is penalised above.</p> <p>If answers are given in degrees deduct 2A marks from any that would otherwise have been given.</p> <p>(Answers in degrees but then changed to radians are acceptable – mark the radian answers.)</p> <p>Ignore extra answers outside the stated range – any within are incorrect.</p> <p>Use the result from (a) or (b) once. Allow if one or both “2”s are missing</p> <p>Use the result from (a) or (b) again. Must include both “2”s this time. Depends on previous M mark.</p> <p>Reach the given result with no errors seen</p>
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ALT 1	
M1	Use the result from (a) or (b). Allow if the “2” is missing.
dM1	Factorise and use the <i>correct</i> double angle formula on $\cos 2x$ Depends on the previous M mark
A1cso	Reach the given result with no errors seen
ALT 2	
M1	Use the <i>correct</i> half angle formula on $\cos^2 x$
dM1A1	M1: Use the result from (a) or (b)
cso	A1: reach the given result with no errors seen
(e)M1	Obtain a function which can be integrated either by using the given result from (d) OR deriving the same result. Allow if $1/4$ is missing. (Integration by Parts – send to review)
A1	Correct integration (must have included the $1/4$) Limits not needed for these 2 marks.
dM1	Correct use of the given limits. All sines are 0 at the lower limit so these need not be shown
A1	Correct final answer which must be exact and stated as a single fraction .