Write your name here	1	
Surname	Other name	es
Pearson Edexcel International GCSE	Centre Number	Candidate Number
Further Pu	ire Mathe	ematics
Monday 22 January 2018 Time: 2 hours	– Morning	Paper Reference 4PM0/02

Instructions

- Use **black** ink or ball-point pen.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
 - there may be more space than you need.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
 - use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶







Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1

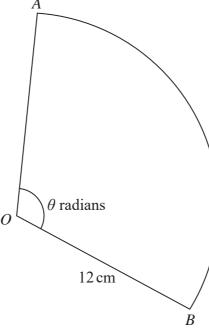


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Figure 1

Figure 1 shows the sector AOB of a circle with centre O and radius 12 cm. The angle AOB is θ radians and the area of the sector is $192 \, \mathrm{cm}^2$

Calculate

(a) the value of θ ,

(2)

(b) the length, in cm, of the arc AB.

(2)



2	(a) Show that $\sum_{r=1}^{n} (3r+2) = \frac{n}{2} (7+3n)$	(2)
	(b) Hence, or otherwise, evaluate $\sum_{r=10}^{20} (3r+2)$	(-)
	r=10	(3)



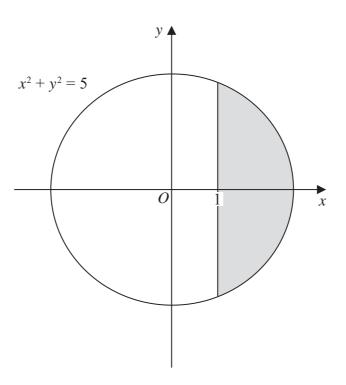


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Figure 2

The region enclosed by the circle with equation $x^2 + y^2 = 5$ and the straight line with equation x = 1, shown shaded in Figure 2, is rotated through 360° about the y-axis.

Use algebraic integration to find the exact volume of the solid generated.

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	Question 3 continued
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	(Total for Question 3 is 5 marks)



4	Here is a quadratic equation $3x^2 + px + 4 = 0$ where p is a constant.	
	(a) Find the set of values of p for which the equation has two real distinct roots.	(5)
	(b) List all the possible integer values of p for which the equation has no real roots.	(1)

	Question 4 continued
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	(Total for Question 4 is 6 marks)



5	Given that $y = 2e^x(3x^2 - 6)$					
	show that $\frac{d^2}{dx}$	$\frac{dy}{dx^2} - 2 \frac{dy}{dx} + y = 12e^x$	(7)			

Question 5 continued		
	 (Total for Ques	tion 5 is 7 marks)



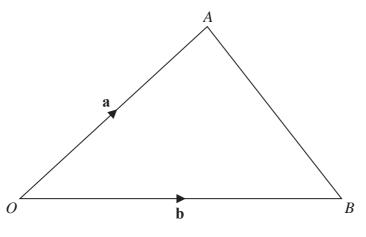


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Figure 3

Figure 3 shows the triangle \overrightarrow{OAB} with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Find \overrightarrow{AB} in terms of **a** and **b**.

(1)

The point P is such that $\overrightarrow{OP} = \frac{3}{4} \overrightarrow{OA}$, and the point Q is the midpoint of AB.

(b) Find \overrightarrow{PQ} as a simplified expression in terms of **a** and **b**.

(2)

The point R is such that PQR and OBR are straight lines where

$$\overrightarrow{QR} = \overrightarrow{\mu PQ}$$
 and $\overrightarrow{BR} = \overrightarrow{\lambda OB}$

- (c) Express \overrightarrow{QR} in terms of
 - (i) **a**, **b** and μ
 - (ii) **a**, **b** and λ

(3)

- (d) Hence find the value of
 - (i) μ
 - (ii) λ

(4)

Question 6 continued		



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Question 6 continued	



7 (i) Solve the equation $\frac{\left(8^x\right)^x}{32^x} = 4$ (ii) Solve the equation $\log_x 64 + 3\log_4 x - \log_x 4 = 5$

Question 7 co	itiiiueu			



Question 7 continued	



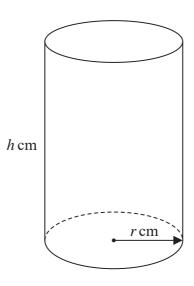


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Figure 4

A solid right circular cylinder has radius r cm and height h cm, as shown in Figure 4. The cylinder has a volume of 355 cm^3 and a total surface area of $S \text{ cm}^2$

(a) Show that
$$S = 2\pi r^2 + \frac{710}{r}$$

(4)

Given that r can vary,

(b) using calculus find, to 3 significant figures, the minimum value of S.

(5)

(c) Verify that your answer to part (b) does give the minimum value of S.

(2)

	Question 8 continued
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Question 8 continued	



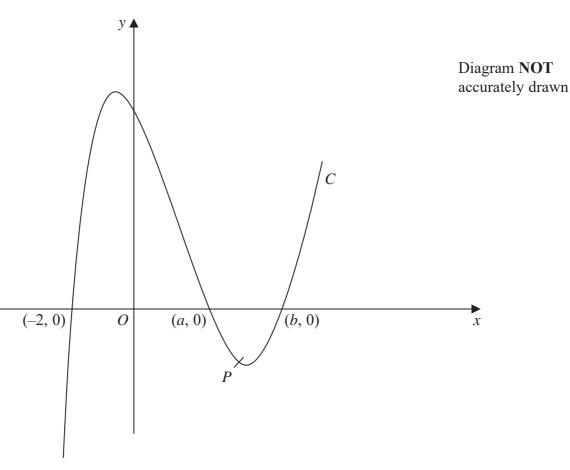


Figure 5

Figure 5 shows the curve C with equation $y = x^3 - 2x^2 - 5x + 6$

The curve C crosses the x-axis at the points with coordinates (-2, 0), (a, 0) and (b, 0)

- (a) (i) Show that a = 1
 - (ii) Find the value of b.

(4)

The point P on C has x coordinate 2 and the line l is the tangent to C at P.

(b) Show that l crosses the x-axis at the point with coordinates (-2, 0)

(6)

(c) Use algebraic integration to find the exact area of the finite region bounded by C and l.

(4)

Q	uestion 9 continued



Question 9 continued		



10	The point A has coordinates $(-6, -4)$ and the point B has coordinates $(4, 1)$ The line I passes through the point A and the point B .	
	(a) Find an equation of l .	(2)
	The point P lies on l such that $AP:PB=3:2$	
	(b) Find the coordinates of <i>P</i> .	(2)
		(2)
	The point Q with coordinates (m, n) lies on the line through P that is perpendicular to l .	
	Given that $m < 0$ and that the length of PQ is $3\sqrt{5}$	
	(c) find the coordinates of Q .	(5)
	The point R has coordinates $(-13, 0)$	
	(d) Show that	
	(i) AB and RQ are equal in length,	
	(ii) AB and RQ are parallel.	
		(4)
	(e) Find the area of the quadrilateral <i>ABQR</i> .	(2)
		(2)

Question 10 continued		



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Question 10 continued



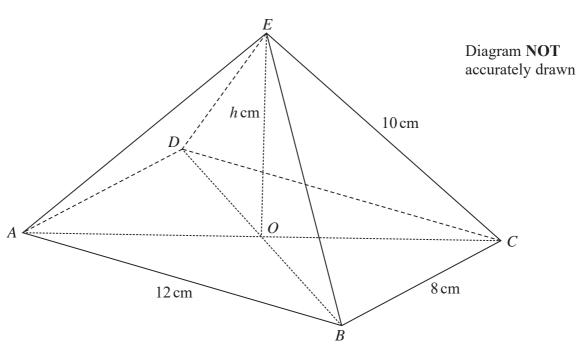


Figure 6

A pyramid with a rectangular base ABCD and vertex E is shown in Figure 6.

The rectangular base is horizontal with $AB = 12 \,\mathrm{cm}$ and $BC = 8 \,\mathrm{cm}$.

The diagonals of the base intersect at the point O.

The vertex E of the pyramid is vertically above O.

The height of the pyramid is h cm and AE = BE = CE = DE = 10 cm.

(a) Show that $h = 4\sqrt{3}$

(3)

(b) Find, in degrees to 1 decimal place, the size of angle OCE.

(2)

The angle between OE and the plane CBE is θ°

(c) Show that
$$\cos \theta^{\circ} = \frac{2\sqrt{7}}{7}$$

(3)

The point P is the midpoint of BE and the point Q is the midpoint of CE.

(d) Find, in degrees to 1 decimal place, the size of the angle between the plane OPQ and the plane EPQ.

(4)



	Question 11 continued
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Question 11 continued		

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Question 11 continued	
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	TOTAL FOR PAPER IS 100 MARKS