

Please check the examination details below before entering your candidate information

Candidate surname					Other names				
Centre Number					Candidate Number				


**Pearson Edexcel International GCSE**

Time 2 hours

Paper reference **4PM1/02**

**Further Pure Mathematics**

**PAPER 2**



**Calculators may be used.**

Total Marks

### Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.  
Anything you write on the formulae page will gain NO credit.

### Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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**Pearson**

## International GCSE in Further Pure Mathematics Formulae sheet

**Mensuration****Surface area of sphere**  $= 4\pi r^2$ **Curved surface area of cone**  $= \pi r \times \text{slant height}$ **Volume of sphere**  $= \frac{4}{3}\pi r^3$ **Series****Arithmetic series**Sum to  $n$  terms,  $S_n = \frac{n}{2}[2a + (n-1)d]$ **Geometric series**Sum to  $n$  terms,  $S_n = \frac{a(1-r^n)}{(1-r)}$ Sum to infinity,  $S_\infty = \frac{a}{1-r} \quad |r| < 1$ **Binomial series**

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad \text{for } |x| < 1, n \in \mathbb{Q}$$

**Calculus****Quotient rule (differentiation)**

$$\frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

**Trigonometry****Cosine rule**In triangle  $ABC$ :  $a^2 = b^2 + c^2 - 2bc \cos A$ 

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

**Logarithms**

$$\log_a x = \frac{\log_b x}{\log_b a}$$

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**Answer all ELEVEN questions.**

**Write your answers in the spaces provided.**

**You must write down all the stages in your working.**

**1** Find the set of values of  $x$  for which

(a)  $2(3x - 1) < 4 - 3x$

(2)

(b)  $3x^2 - 8x - 3 < 0$

(4)

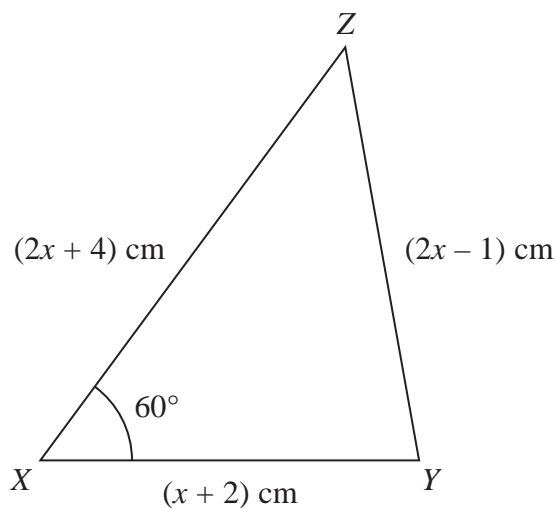
(c) **both**  $2(3x - 1) < 4 - 3x$  **and**  $3x^2 - 8x - 3 < 0$

(1)

**(Total for Question 1 is 7 marks)**



2

Diagram **NOT**  
accurately drawn**Figure 1**Figure 1 shows triangle  $XYZ$  in which

$$XY = (x + 2) \text{ cm} \quad XZ = (2x + 4) \text{ cm} \quad YZ = (2x - 1) \text{ cm} \quad \text{and} \quad \angle YXZ = 60^\circ$$

Find the value of  $x$ Give your answer in the form  $p + q\sqrt{3}$  where  $p$  and  $q$  are integers to be found.

(4)

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**Question 2 continued**

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**(Total for Question 2 is 4 marks)**



3

$$f(x) = 8x^2 + 10x - 3$$

Given that  $f(x)$  can be written in the form  $A(x + B)^2 + C$  where  $A$ ,  $B$  and  $C$  are constants,

- (a) find the value of  $A$ , the value of  $B$  and the value of  $C$ . (3)
- (b) Hence, or otherwise, find,
- (i) the value of  $x$  for which  $f(x)$  has a minimum,
  - (ii) the minimum value of  $f(x)$ . (2)

The curve  $C$  has equation  $y = f(x)$ .

- (c) Find the  $x$  coordinate of each of the points where  $C$  crosses the  $x$ -axis. (2)

The straight line  $l$  has equation  $y = 2x + 13$

- (d) Use algebra to find the coordinates of the two points of intersection of  $C$  and  $l$ . (4)

Using the same axes and the results of parts (b), (c) and (d),

- (e) sketch the curve  $C$  and the straight line  $l$ . (2)

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**Question 3 continued**

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**Question 3 continued**

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**Question 3 continued**

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**(Total for Question 3 is 13 marks)**



4 The equation of a curve is  $y = x^3 \sin x$

Find an equation of the tangent to the curve at the point on the curve where  $x = \frac{1}{2}$

Give your answer in the form  $y = mx + c$

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**Question 4 continued**

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**(Total for Question 4 is 7 marks)**



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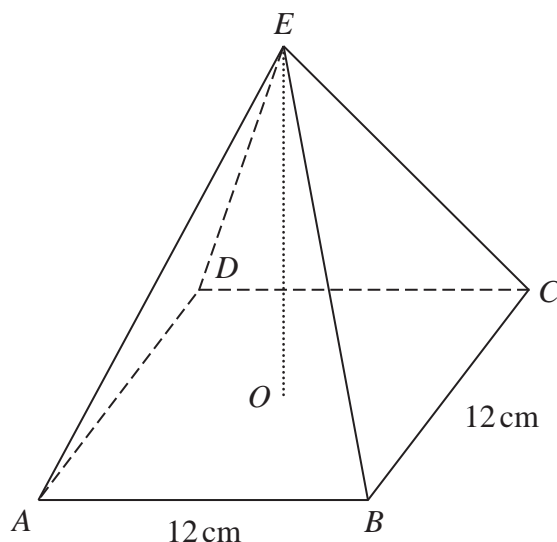


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### Figure 2

Figure 2 shows a right pyramid with a square base  $ABCD$  and vertex  $E$ .

The base of the pyramid is horizontal with  $AB = BC = 12$  cm.

The diagonals of the base intersect at the point  $O$ .

The vertex  $E$  of the pyramid is vertically above  $O$  and the angle between  $EA$  and the plane  $ABCD$  is  $30^\circ$

The height of the pyramid is  $h$  cm.

- (a) Find the exact value of  $h$

(3)

The point  $F$  lies on  $AD$  such that  $AF:FD = 1:4$

- (b) Calculate, to the nearest degree, the size of the angle  $EFO$ .

(4)



**Question 5 continued**

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**Question 5 continued**

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**Question 5 continued**

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**(Total for Question 5 is 7 marks)**



6 The  $n$ th term of a geometric series  $G$  is  $U_n$

The first three terms of  $G$  are given by

$$U_1 = q(4p + 1) \quad U_2 = q(2p + 3) \quad U_3 = q(2p - 3)$$

(a) Find the possible values of  $p$

(5)

Given that  $G$  is convergent with sum to infinity 250

(b) find the value of  $q$

(3)

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**Question 6 continued**

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**(Total for Question 6 is 8 marks)**



7

$$y = e^{2x} \cos 2x$$

(a) Show that

$$\frac{dy}{dx} = 2y - 2e^{2x} \sin 2x \quad (4)$$

(b) Hence show that

$$\frac{d^2y}{dx^2} = 4 \frac{dy}{dx} - 8y \quad (5)$$

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**Question 7 continued**

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**Question 7 continued**

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**Question 7 continued**

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**(Total for Question 7 is 9 marks)**



## 8 The quadratic equation

$$x^2 - 4k\sqrt{2}x + 2k^4 - 1 = 0$$

where  $k$  is a positive constant, has roots  $\alpha$  and  $\beta$

Given that  $\alpha^2 + \beta^2 = 66$  and that  $\alpha^3 + \beta^3 = p\sqrt{2}$  where  $p$  is an integer,

find the value of  $p$

(11)

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**Question 8 continued**

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**Question 8 continued**

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**Question 8 continued**

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**(Total for Question 8 is 11 marks)**



9 A cube has edges of length  $x$  cm.

The total surface area,  $A$  cm<sup>2</sup>, of the cube is increasing at a constant rate of 0.45 cm<sup>2</sup>/s

Find the rate of increase, in cm<sup>3</sup>/s, of the volume of the cube at the instant when the total surface area of the cube is 384 cm<sup>2</sup>

(7)

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**10** Using formulae given on page 2

(a) show that

(i)  $\sin 2\theta = 2\sin \theta \cos \theta$

(ii)  $\cos 2\theta = 2\cos^2 \theta - 1$

(5)

Given that  $\theta \neq (90^\circ + 180^\circ n)$  where  $n \in \mathbb{Z}$ (b) use the results from part (a) to show that  $\sin 2\theta - \tan \theta$  can be written as  $\tan \theta \cos 2\theta$ 

(4)

(c) Solve for  $0 < x < 360$ 

$$\sin 2x^\circ - \tan x^\circ = 0$$

(4)

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**Question 10 continued**

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**Question 10 continued**

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**Question 10 continued**

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**(Total for Question 10 is 13 marks)**



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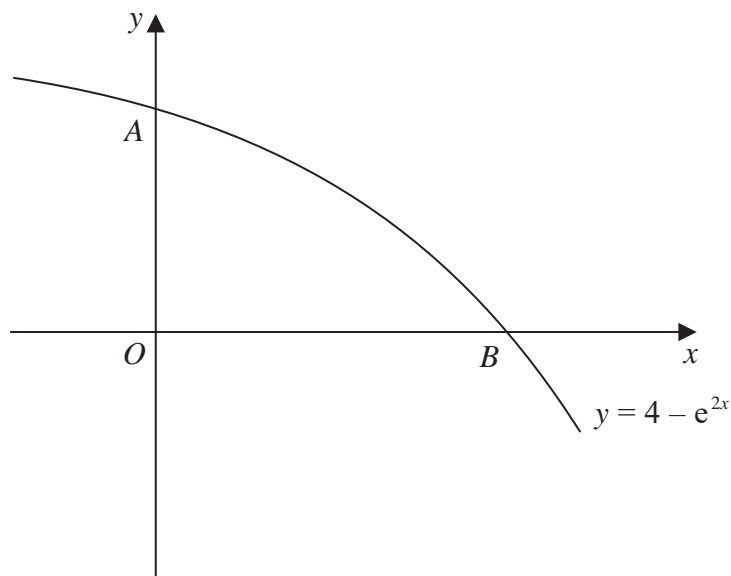
Diagram **NOT**  
accurately drawn**Figure 3**

Figure 3 shows part of the curve  $C$  with equation  $y = 4 - e^{2x}$ .  
The curve  $C$  crosses the  $y$ -axis at the point  $A$  and the  $x$ -axis at the point  $B$ .

(a) (i) Write down the  $y$  coordinate of point  $A$ .

(ii) Show that the  $x$  coordinate of  $B$  is  $x = \ln 2$

(3)

The line  $l$  is the normal to  $C$  at the point  $B$ .

(b) Find an equation for  $l$ , giving your answer in the form  $y = mx + c$

(4)

The finite region  $R$  is bounded by  $C$ ,  $l$  and the  $y$ -axis.

(c) Using calculus, find the area of  $R$ .

Give your answer to one decimal place.

(7)

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**Question 11 continued**

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**Question 11 continued**

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# Question 11 continued

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Handwritten solution for Question 11 continued:

$\frac{1}{2} \int_0^1 (2x^2 + 3x - 4) dx$   
 $= \frac{1}{2} \left[ \frac{2}{3}x^3 + \frac{3}{2}x^2 - 4x \right]_0^1$   
 $= \frac{1}{2} \left( \frac{2}{3} + \frac{3}{2} - 4 \right)$   
 $= \frac{1}{2} \left( \frac{4}{6} + \frac{9}{6} - \frac{24}{6} \right)$   
 $= \frac{1}{2} \left( \frac{4 + 9 - 24}{6} \right)$   
 $= \frac{1}{2} \left( \frac{-11}{6} \right)$   
 $= -\frac{11}{12}$



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**Question 11 continued**

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**(Total for Question 11 is 14 marks)****TOTAL FOR PAPER IS 100 MARKS**