Question Number	Scheme		Marks
4.	(a) $t_n = 1 \times r^{n-1} = r^{n-1}$		B1
	(b) $r^{n-1} + r^n = r^{n+1}$ $\Rightarrow 1 + r = r^2$		M1 A1
	$\Rightarrow r^2 - r - 1 = 0$		
	$\Rightarrow r = \frac{1 \pm \sqrt{1 + 4}}{2}$		M1
	$\Rightarrow r = \frac{1+\sqrt{5}}{2} \text{ since } r > 0$		A1
	(c) $t_1 = 1$ , $t_2 = r = \frac{1 + \sqrt{5}}{2}$		
	$t_3 = t_1 + t_2 = 1 + \frac{1 + \sqrt{5}}{2} \left( = \frac{3 + \sqrt{5}}{2} \right)$		M1 A1
	$t_4 = t_2 + t_3 = \frac{1 + \sqrt{5}}{2} + \frac{3 + \sqrt{5}}{2} = \frac{4 + 2\sqrt{5}}{2} = 2 + \sqrt{5}$ alternative		A1 (8)
	(c) $t_4 = \left(\frac{1+\sqrt{5}}{2}\right)^3 = \frac{1+3\sqrt{5}+3(\sqrt{5})^2+(\sqrt{5})^3}{8}$	M1	
	$=\frac{1+3\sqrt{5}+15+5\sqrt{5}}{8}$ oe	A1	
	$=\frac{16+8\sqrt{5}}{8}=2+\sqrt{5}$	A1	

## **Notes for Question 4**

(a) B1 for  $t_n = 1 \times r^{n-1}$  (or  $t_n = r^{n-1}$ )

(b)

- M1 for forming an equation  $r^{n-1} + r^n = r^{n+1}$  follow through their expression for  $t_n$  (ie their expression from (a) in  $t_n + t_{n+1} = t_{n+2}$ )
- A1 for dividing by  $r^{n-1}$  to obtain a correct 3 term quadratic terms in any order from a **correct** initial equation (ie eg  $t_n = r^n$  in (a) giving the equation  $r^n + r^{n+1} = r^{n+2}$  would score B0M1A0)

Alternative: Make r = 1 so  $t_1 + t_2 = t_3$  M1 and so  $1 + r = r^2$  A1

- M1 for solving **their** 3 term quadratic equation see start of doc for information about solving quadratic equations. Either the general formula must be quoted correctly or the full substitution seen (as answer given) **OR** subst.  $r = \frac{1+\sqrt{5}}{2}$  (M1) then if everything correct including a conclusion, give A1
- A1cso for  $r = \frac{1+\sqrt{5}}{2}$  \* as r > 0. Since this is a given answer we must see r > 0 somewhere. A correct

general formula with  $\pm$  needed or both solutions shown and the positive chosen with the reason. If  $r = \frac{1 + \sqrt{1 + 4}}{2} \implies r = \frac{1 + \sqrt{5}}{2} \quad r > 0 \text{ is given, award M1A0.}$ 

## **Notes for Question 4 Continued**

(c)

M1 for obtaining  $t_3$  by using  $t_3 = t_1 + t_2$  their numerical  $t_1$  and  $t_2$ 

A1 for 
$$t_3 = 1 + \frac{1 + \sqrt{5}}{2}$$
 oe

A1cao and cso for 
$$(t_4) = 2 + \sqrt{5}$$

Alternatives for (c)

M1 for using **their** formula found in (a) and attempting the expansion

A1 for 
$$\frac{1+3\sqrt{5}+15+5\sqrt{5}}{8}$$
 oe A1 for  $2+\sqrt{5}$ 

$$t_{4} = t_{3} + t_{2} = t_{2} + t_{1} + t_{2}$$

$$= 2 \times \frac{\left(1 + \sqrt{5}\right)}{2} + 1$$

$$= 2 + \sqrt{5}$$
M1A1
A1

By calculator:  $\left(\frac{1+\sqrt{5}}{2}\right)^3 = 2+\sqrt{5}$  scores M1A1A1, but an incorrect (or partially correct) answer scores

M0A0A0