Question Number	Scheme	Marks
8(a)(i)	$\cos 2A = \cos^2 A - \sin^2 A$	M1
	$= \left(1-\sin^2 A\right) - \sin^2 A, = 1 - 2\sin^2 A$ *	M1,A1 (3)
(ii)	$\sin 2A = 2\sin A\cos A$	B1 (1)
(b)	$\sin 3A = \sin 2A\cos A + \cos 2A\sin A$	M1
	$= \sin A\left(1-2\sin^2 A\right) + 2\sin A\cos^2 A$	M1
	$\sin A - 2\sin^3 A + 2\left(1-\sin^2 A\right)\sin A$	M1
	$= 3\sin A - 4\sin^3 A$	A1 (4)
(c)	$\sin 3A = -\frac{1}{2}$	M1
	$3A = 210°, -30°, -150°$	M1 (any one)
	$A = 70°, -10°, -50°$	A1A1 (4)
(d)	(i) $\int \sin^3\theta \, d\theta = \frac{1}{4}\int \left(3\sin\theta - \sin 3\theta\right) d\theta$	M1
	$= \frac{1}{4}\left[-3\cos\theta + \frac{1}{3}\cos 3\theta\right]$	M1A1
	(ii) $\frac{1}{4}\left[-3\cos\frac{\pi}{4} + \frac{1}{3}\cos\frac{3\pi}{4} - \left(-3\cos 0 + \frac{1}{3}\cos 0\right)\right]$	
	$\frac{1}{4}\left[-\frac{3}{\sqrt{2}} - \frac{1}{3}\times\frac{1}{\sqrt{2}} - \left(-3 + \frac{1}{3}\right)\right]$	M1
	$\frac{8-5\sqrt{2}}{12}$ oe for 5,8,12	A1 (5)

(d)	ALT for (d) (i) $\int \sin^3\theta \, d\theta = \int \left(\sin\theta - \cos^2\theta \sin\theta\right) d\theta$	M1
	$= \left[-\cos\theta + \frac{1}{3}\cos^3\theta\right]$ (+ *c*)	M1A1
	(ii) $-\cos\frac{\pi}{4} + \frac{1}{3}\cos^3\frac{\pi}{4} - \left(\cos 0 + \frac{1}{3}\cos^3 0\right)$	
	$= -\frac{1}{\sqrt{2}} + \frac{1}{3} \times \frac{1}{2\sqrt{2}} - \left(1 + \frac{1}{3}\right) = \frac{8 - 5\sqrt{2}}{12}$	M1dd A1
		[17]

Notes

(a) (ii)

M1 for the correct expression for cos 2*A*

M1 for using $\cos^2 A + \sin^2 A = 1$, and substituting into the expression for cos 2*A*

A1 for the correct identity as shown. Note this is show question!

(ii)

B1 for the correct identity for sin 2*A*

(b)

M1 for substituting sin 2*A* into their expression in part (a) to give sin (2*A* +*A*)

M1 for using the given cos 2*A* and their sin 2*A* in their expression for sin3*A*

M1 for using $\cos^2 A + \sin^2 A = 1$

A1 for the final identity as shown. Note: this is a show question

(c)

M1 for $8\sin^3 A - 6\sin A = 1 \Rightarrow -2\left(3\sin A - 4\sin^3 A\right) = 1 \Rightarrow \sin 3A = k$, where $-1 \le k \le 1$

M1 for 3*A* equal to any one of $210^\circ, -30^\circ, -150^\circ$

A1 for any **two** correct angles

A1 for **all three** correct angles

If there are extra angles outside of range, ignore. If there are extra angles within the range deduct one A mark for each up to a maximum of 2 marks.

(d) (i)

M1 for re-arranging the GIVEN expression for sin 3*A* to make $\sin^3 A$ the subject

M1 for an attempt at integrating their re-arranged expression

As a minimum for this mark, $\int \sin 3A \, dA \Rightarrow \pm\frac{1}{3}\cos 3A \; (+c)$ (+c not required)

A1 for the correct integrated expression as shown (+c not required)

(ii)

M1dd for substituting $\frac{\pi}{4}$ **and** 0 into their integrated expression **and** attempting to evaluate (there must be a final answer given for this mark) .

A1 for the final answer as shown

ALT

(i)

M1 for finding $\sin^3 A = \sin A\left(1-\cos^2 A\right) = \sin A - \sin A\cos^2 A$

M1 for attempting to integrate. $\int \sin A\mathrm{d}A - \int \cos^2 A\sin A\mathrm{d}A = \left\{-\cos A + \frac{1}{3}\cos^3 A(+c)\right\}$

For a minimum attempt you need to see; $\int \cos^2 A\sin A\mathrm{d}A = \pm\frac{1}{3}\cos^3 A\ (+c)$

A1 for $-\cos A + \frac{1}{3}\cos^3 A(+c)$

(ii)

M1dd for substituting $\frac{\pi}{4}$ **and** 0 into their integrated expression **and** attempting to evaluate (there must be a final answer given for this mark) .

A1 for the final answer as shown

ALT (Using substitution)

(i)

M1 for writing the integral as $\sin\theta\sin^2\theta \Rightarrow \sin\theta\left(1-\cos^2\theta\right)$ and substituting $\cos\theta = u$ and differentiating to achieve $\frac{\mathrm{d}u}{\mathrm{d}\theta} = -\sin\theta$

M1 for substituting and integrating $-\int \sin\theta\left(1-u^2\right)\frac{\mathrm{d}u}{\mathrm{d}\theta}.\mathrm{d}\theta \Rightarrow -\int 1-u^2\mathrm{d}u \Rightarrow -\left[u-\frac{u^3}{3}\right]$

For definition of an attempt, see General Guidance

A1 for substituting $u = \cos\theta$ to give $-\cos A + \frac{1}{3}\cos^3 A(+c)$

(ii)

M1dd for substituting $\frac{\pi}{4}$ **and** 0 into their integrated expression **and** attempting to evaluate (there must be a final answer given for this mark).

A1 for the final answer as shown