International GCSE Futher Pure Mathematics - Paper 2 mark scheme

Paper 2						
Question	Working	Answer	Mark	AO	Notes	
1	First term = $3e^{-1}$ , Common Ratio = $e^{-2}$		B1B1	1		
	$\varsigma = 3e^{-1} \checkmark \left(\frac{e^2}{e^2}\right) = 3e^{-1}$	39	MIMI	C		
	$\begin{bmatrix} 5 & 1 - e^{-2} \ \end{bmatrix} \begin{pmatrix} e^2 \end{pmatrix} = e^2 - 1$	$S_{\infty} = \frac{3c}{e^2 - 1}$	A1	١		
	(So, $a = 3$ and $b = 2$ )					
			(S)			
2		<i>x</i> > 4	B1			
<b>(a)</b>	$3+x < 2x-1 \Rightarrow x > 4$		Ş	1		
	$x(x-1) > 6 \implies x^2 - x - 6 > 0 \implies (x-3)(x+2) > 0$		MI			
<b>(p</b> )		x > 3 OR $x$	M1A1	1	(Outside region)	
	critical values are $x = 3, -2$ so $x > 3$ OR $x < -2$	7_/				
3		7	B1			
<b>(</b> 2)	x > 3  OR  x < -2 , $x > 4  so$ $x > 4$	χ < χ	(5)			

Question	Working	Answer	Mark	AO	Notes
<b>e</b> (	$\overline{AB} = -(4\mathbf{i} + 3\mathbf{j}) + (8\mathbf{i} + p\mathbf{j}) = 4\mathbf{i} + (p-3)\mathbf{j}$		B1	-	
<b>(B</b> )	$ \overline{AB}  = \sqrt{52} = \sqrt{4^2 + (p-3)^2} \Rightarrow p^2 - 6p - 27 = (p-9)(p+3) = 0$		M1A1	- <del></del>	
	$\Rightarrow p = 9, p = -3$				
( <b>p</b> )		-	į		
	$\overline{AB} = 4\mathbf{i} + (p-3)\mathbf{j} = 4\mathbf{i} + 6\mathbf{j} \implies \frac{1}{2\sqrt{13}} (4\mathbf{i} + 6\mathbf{j}) = \frac{1}{\sqrt{13}} (2\mathbf{i} + 3\mathbf{j}) \text{ (oe)}$	$\frac{1}{2\sqrt{13}}(4\mathbf{i}+6\mathbf{j})$	B1B1 (5)	_ <del></del>	
4 (3	$f(-3) = 2 \times (-3)^2 + p \times (-3)^2 + q \times (-3) + 12 = 0$				Mark parts (i) and
(a)	$\Rightarrow 42 = 9p - 3q \Rightarrow 14 = 3p - q$		M1	3	(II) together
	$f'(x) = 6x^2 + 2px + q$		M1		
	f'(-3) = $6 \times (-3)^2 + 2p \times (-3) + q = 37$		MIAI		
	$\Rightarrow -6p+q=-17 \Rightarrow q=17-6p$				Solving
	$14 = 3p - 1(6p - 17) \Rightarrow p = 1, q = -11$	q = -111	M1A1		simultaneous equations (by any
<b>(p)</b>	$\frac{\left(2x^3 + x^2 - 11x + 12\right)}{\left(2x^3 + x^2 - 11x + 12\right)} = 2x^2 - 5x + 4$		M1A1	ŗ	method)
	$(x+3)$ $\Rightarrow (x+3)(2x^2 - 5x + 4)$			_ <del></del>	
(2)	$b^2 - 4ac = (-5)^2 - 4 \times 2 \times 4 = -7$ (7 hence no real roots for quadratic factor, so $x =$		M1A1	-	
	-3 only real root.)		(10)		

Question	Working	Answer	Mark	AO	Notes
<b>5</b> (a)	$\cos(A-B) - \cos(A+B) = \cos A \cos B + \sin A \sin B - \cos A \cos B + \sin A \sin B$ $\Rightarrow 2 \sin A \cos B$		M1A1	1	
( <b>q</b> )	$2\sin 5x\sin 3x = \cos 2x - \cos 8x$	$\cos 2x - \cos 8x$	B1	2	
(c)(i)	$\int 4\sin 5\theta \sin 3\theta d\theta = 2\int \cos 2\theta - \cos 8\theta d\theta = 2\left[\frac{\sin 2\theta}{2} - \frac{\sin 8\theta}{8}\right](+c)$	L	M1A1	$\kappa$	
<b>(ii</b> )	$\int_0^{\frac{\pi}{6}} 4\sin 5\theta \sin 3\theta  dx = 2 \int_0^{\frac{\pi}{6}} (\cos 2\theta - \cos 8\theta)  d\theta = 2 \left[ \frac{\sin 2\theta}{2} - \frac{\sin 8\theta}{8} \right]_0^{\frac{\pi}{6}} = \frac{5\sqrt{3}}{8}$	8 8	M1A1		
			<u>S</u>		
9	$\log_x 2 = \frac{\log_2 2}{\log_2 x} = \frac{1}{\log_2 x}$		B1	2, 3	
	$\log_2 x + \frac{6}{\log_2 x} = 7 \Rightarrow (\log_2 x)^2 - 7\log_2 x + 6 = 0$		M1M1 A1		
	$\Rightarrow (\log_2 x - 6)(\log_2 x - 1) = 0 \Rightarrow \log_2 x = 6, \log_2 x = 1$ \Rightarrow x = 64, x = 2	64, 2	M1M1 A1		
			(7)		

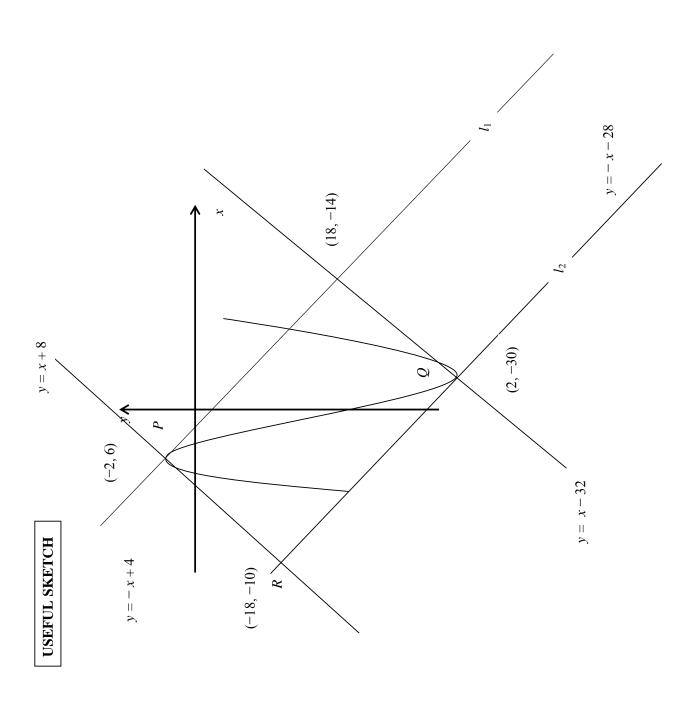
Question	Question   Working	Answer	Mark	AO	Notes
7 (a)	(i) $0 = \frac{ax - 5}{x^2} \Rightarrow ax - 5 = 0 \Rightarrow a = 2$		M1A1	2	
<u>.                                    </u>	(ii) $b = 1$	b = 1	B1	1 (	
<b>(9</b> )	When $x = 0$ , $y = \frac{-5}{-b} \Rightarrow y = \frac{5}{1} \Rightarrow y = 5$	(0, 5)	B1	2	Accept $y = 5$
(c)	y = 2	y = 2	B1	3	
(p)	x = 1			(	
				80	
			B1		Curve in correct
					quadrants
	u				
	2				
	7 = K				
	<b>^</b>				
	2.5				
			B1		Correct asymptotes
			B1		drawn
					Correct
			<b>8</b>		intersections with
					axes

Question	Working	Answer	Mark	OV	Notes
8 (a)	$\frac{3}{\sqrt{1-2x}} = 3(1-2x)^{-\frac{1}{2}} \Rightarrow$		B1	2	
	$=3\left\{1+\left(-\frac{1}{2}\right)(-2x)+\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\frac{(-2x)^2}{2!}+\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\frac{(-2x)^3}{3!}\dots\right\}$		M1A1		
	$\frac{3}{\sqrt{1-2x}} = 3+3x + \frac{9}{2}x^2 + \frac{15}{2}x^3$		A1		
(b)	$-\frac{1}{2} < x < \frac{1}{2} \text{ or }  x  < \frac{1}{2}$	$-\frac{1}{2} < x < \frac{1}{2}$	B1	7	
(c)	$\frac{3}{\sqrt{0.9}} = \frac{3}{\sqrt{9/10}} = \frac{3}{\sqrt{10}} = \sqrt{10}$		B1	3	
(p)	$\frac{1}{\sqrt{10} - 3} \times \frac{\left(\sqrt{10} + 3\right)}{\left(\sqrt{10} + 3\right)} = \frac{\left(\sqrt{10} + 3\right)}{1} = \left(\sqrt{10} + 3\right)$	$\left(\sqrt{10}+3\right)$	MIAI	$\kappa$	
(e)	$1 - 2x = 0.9 \Longrightarrow x = 0.05$		B1	3	
	$\sqrt{10} + 3 = 3 + 3 \times (0.05) + \frac{9}{2} \times (0.05)^2 + \frac{15}{2} \times (0.05)^3 + \dots + 3 = 3.1621875 + 3$ $\approx 6.16219$	6.16219	M1A1 (11)		

uestion	Question   Working	Answer	Mark	A0	Notes
9 (a)	$7 + 4x - 2x^{2} = -2\left[(x-1)^{2} - 1\right] + 7 = -2(x-1)^{2} + 9$		M1 A 1	1	
	V = -2, $Q = -1$ , $K = 9$		A1		
<b>(9</b> )	(i) 9 (ii) 1	(i) 9 (ii) 1	B1B1	2	
<i>,</i>					
<b>(</b> 2)	$7 + 4x - 2x^2 = 4 - x \Rightarrow 2x^2 - 5x - 3 = 0$			2	
	$\Rightarrow (2x+1)(x-3) = 0 \Rightarrow x = -\frac{1}{2},3$	$x = -\frac{1}{2}, x = 3$	M1M1 A1		
<b>(p</b> )	$V = \pi \int_{-0.5}^{3} (7 + 4x - 2x^2)^2 dx = \pi \int_{-0.5}^{3} (4 - x)^2 dx$		M1	8	
	$\Rightarrow \pi \int_{-0.5}^{3} 33 + 64x - 13x^{2} - 16x^{3} + 4x^{4} dx$		M1A1		
	$\Rightarrow \pi \left[ 33x + 32x^2 - \frac{13x^3}{3} - 4x^4 + \frac{4x^5}{5} \right]_{-0.5}^3 = \pi \frac{4459}{30} = 466.945 \approx 467$	467	M1A1		
			(13)		

Question	Working	Answer	Mark A	AO A	Notes
10 (a)	y = f(-2) = 6		B1		
	$\frac{dy}{dx} = 3x^2 - 13 \Rightarrow f'(-2) = 13 - 13 = -1$		MIAI		
	Equation of tangent $y-6=-1(x-(-2)) \Rightarrow y+x-4=0$	y + x - 4 = 0	M1A1		
<b>(9</b> )	$-1 = 3x^{2} - 13 \Rightarrow 3x^{2} - 12 = 0 \Rightarrow x^{2} - 4 = 0 \Rightarrow (x - 2)(x + 2) = 0$ at $Q, x = 2$		M1		
	f(2) = -30		B1 1	1, 2	
	$y30 = -(x - 2) \Rightarrow \{y = -x - 28\}$	y = -x - 28	M1A1		
(c)	Gradient of normal $= 1$		<u>8</u>		
	Equation of Normal $y-6=x-(-2) \Longrightarrow \{y=x+8\}$		2	6)	
	At R, $x+8=-x-28 \Rightarrow x=-18$		MI		
	When $x = -18$ , $y = -10$	(-18, -10)	A1A1		

Question	Working	Answer	Mark	A0	Notes
þ	{Coordinates of <i>R</i> (-18, -10)}				
( <b>n</b> )	$PR = \sqrt{(610)^2 + (-218)^2} = 16\sqrt{2}$	$16\sqrt{2}$	M1A1	3	
(e)	Area of rectangle				
	Length of $QR = \sqrt{(-18-2)^2 + (-10-30)^2} = 20\sqrt{2}$		M1	3	
	Area = $16\sqrt{2} \times 20\sqrt{2} = 640$	640	M1A1		
<b>(e)</b>	ALTERNATIVE  Equation of normal at $Q$ $y30 = x - 2 \Rightarrow y = x - 32$ Coordinates of $4^{th}$ vertex of rectangle $x - 32 = -x + 4 \Rightarrow x = 18 \text{ so } y = -14$		M1	33	
	Area = $\frac{1}{2}\begin{bmatrix} -2 & -18 & 2 & 18 & -2 \\ 6 & -10 & -30 & -14 & 6 \end{bmatrix} = 640$	640	M1A1		



Question   Working	Working	Answer	Mark	A0	Notes
11	Total Surface Area of the cone		Ž	,	
	$l = \frac{x}{1 + + \frac{x}{1 + x}{1 + \frac{x}{1 + \frac{x}{1 + \frac{x}{1 + x}{1 + \frac{x}{1 + \frac{x}{1 + x}{1 + \frac{x}{1 + x}}}}}}}}}}}}}}}}}}}}}}}} } } } } } }$		ВІ	2, 3	
	sin 30°		M1A1		
	$A = \pi r l + \pi r^2 \Rightarrow \pi x \times 2x + \pi x^2 = 3\pi x^2$		1471741		
	Volume of cone				
	$h = \frac{x}{t_{\text{and }} 30^{\circ}} = \sqrt{3}x$		B1		
	$\operatorname{Vol} = \frac{1}{2}\pi r^2 h \Rightarrow V = \frac{\sqrt{3}}{2}\pi x^3$		M1A1		
	$\frac{dA}{dx} = 10 \text{ (cm}^2 / s)$ $\frac{dV}{dx} = \sqrt{3\pi}x^2$ , $\frac{dA}{dx} = 6\pi x$		B1M1		M1 for
					differentiating
					either $\frac{\mathrm{d}V}{}$ or $\frac{\mathrm{d}A}{}$
	$\frac{dV}{dt} = \frac{dV}{dt} \times \frac{dA}{dt} \times \frac{dx}{dt}$		M1		dx dx
	$dt dx \hat{d}t \hat{d}A$				
	$dV = 6.2.10.1 = 5\sqrt{3} = 10.5 = 10.3 = 3.3$	$10\sqrt{3} \text{ (cm}^3/s) \mid \text{M1A1}$	M1A1		
	$\Rightarrow \frac{1}{\sqrt{1+x}} = \sqrt{3\pi}x \times 10 \times \frac{10}{\sqrt{1+x}} = \frac{10}{\sqrt{3}}x \Rightarrow 10\sqrt{3} \text{ (cm / s)}$				
			(11)		
		Total	100		