9 The finite region R enclosed by the y-axis, the straight line with equation $y + 2x = 13$ and the curve with equation $y = x^2 - 2$, is defined for points with coordinates (x, y) with $x \geqslant 0$

The region R is rotated through 360° about the y-axis.

Use algebraic integration to find the volume of the solid generated.
Give your answer in terms of π.

(9)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 9 marks)

10 (a) Use the formula for $\cos(A + B)$ to show that $\cos 2A = 2\cos^2 A - 1$

(2)

(b) Show that $\cos 4A = 8\cos^4 A - 8\cos^2 A + 1$

(4)

(c) Solve the equation $\cos^2\left(\frac{\theta}{4} + \frac{\pi}{24}\right)\left[\cos^2\left(\frac{\theta}{4} + \frac{\pi}{24}\right) - 1\right] = -\frac{1}{16} \qquad 0 \leqslant \theta < 2\pi$

Give your answers in terms of π.

(5)

$$f(A) = 4\cos^4 A - 4\cos^2 A + 1$$

(d) Using calculus, find the exact value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} f(A)\,dA$

Give your answer in the form $a\pi - b\sqrt{c}$ where a and b are fractions in their lowest terms and c is a prime number.

(4)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 15 marks)

11 The quadratic equation $x^2 - px + q = 0$ where $p > 0$, has roots α and β.

Given that $2\alpha\beta = 3$ and that $4(\alpha^2 + \beta^2) = k^2 - 6k - 3$ where $k > 3$

(a) (i) write down the value of q,

(ii) find an expression, in terms of k, for p.

(5)

Given also that $7\alpha\beta = 3(\alpha + \beta)$

(b) find the value of k.

(2)

(c) Hence form an equation, with integer coefficients, which has roots

$$\frac{\alpha}{\alpha + \beta} \text{ and } \frac{\beta}{\alpha + \beta}$$

(5)

Question 11 continued

Question 11 continued

(Total for Question 11 is 12 marks)

TOTAL FOR PAPER IS 100 MARKS