

Question number	Scheme	Marks
10 (a)	$\frac{1}{2} + \sin 3x = 0 \Rightarrow \sin 3x = -\frac{1}{2} \Rightarrow 3x = -\frac{\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \Rightarrow x = \frac{7\pi}{18}, \frac{11\pi}{18}$ <p>Coordinates of <math>M</math> are <math>\left(\frac{7\pi}{18}, 0\right)^*</math></p> <p>Coordinates of <math>N</math> are <math>\left(\frac{11\pi}{18}, 0\right)</math></p> <p><b>ALT</b></p> $\frac{1}{2} + \sin\left(3 \times \frac{7\pi}{18}\right) = 0$ <p>Coordinates of <math>M</math> are <math>\left(\frac{7\pi}{18}, 0\right)^*</math></p> <p>Coordinates of <math>N</math> are <math>\left(\frac{11\pi}{18}, 0\right)</math></p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>[3]</p> <p>[M1</p> <p>A1</p> <p>A1]</p>
(b)	$\frac{dy}{dx} = 3 \cos 3x = 0 \Rightarrow 3x = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{6}$ $y = \frac{1}{2} + \sin 3\left(\frac{\pi}{6}\right) = 1.5$ <p>Coordinates of point <math>A</math> are <math>\left(\frac{\pi}{6}, 1.5\right)</math></p> <p><b>ALT</b></p> <p>Max of sine curve is 1 so that <math>\frac{1}{2} + 1 = \frac{3}{2} \Rightarrow y = \frac{3}{2}</math></p> $\sin 3\theta = 1 \Rightarrow 3\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{6}$ <p>Coordinates of point <math>A</math> are <math>\left(\frac{\pi}{6}, \frac{3}{2}\right)</math></p>	<p>M1A1</p> <p>dM1A1</p> <p>[4]</p> <p>[M1A1</p> <p>dM1A1]</p>
(c)	<p>Uses the given <math>x</math> coordinate for point <math>M</math></p> <p>At point <math>M</math> <math>\frac{dy}{dx} = 3 \cos\left(3 \times \frac{7\pi}{18}\right) = -\frac{3\sqrt{3}}{2}</math></p> $y - 0 = -\frac{3\sqrt{3}}{2}\left(x - \frac{7\pi}{18}\right) \Rightarrow 12y + 18\sqrt{3}x - 7\sqrt{3}\pi = 0 \text{ o.e.}$	<p>B1</p> <p>M1A1A1</p> <p>[4]</p>
(d)	$A = \int_0^{\frac{7\pi}{18}} \left(\frac{1}{2} + \sin 3x\right) dx + \left  \int_{\frac{7\pi}{18}}^{\frac{11\pi}{18}} \left(\frac{1}{2} + \sin 3x\right) dx \right $ $A = \left[ \frac{1}{2}x - \frac{\cos 3x}{3} \right]_0^{\frac{7\pi}{18}} + \left  \left[ \frac{1}{2}x - \frac{\cos 3x}{3} \right]_{\frac{7\pi}{18}}^{\frac{11\pi}{18}} \right $ $A = \left[ \left( \frac{1}{2} \times \frac{7\pi}{18} - \frac{\cos 3\left(\frac{7\pi}{18}\right)}{3} \right) - \left( 0 - \frac{\cos 3 \times 0}{3} \right) \right]$	<p>M1</p> <p>M1</p> <p>M1</p>

	$+ \left  \left[ \left( \frac{1}{2} \times \frac{11\pi}{18} - \frac{\cos 3 \left( \frac{11\pi}{18} \right)}{3} \right) - \left( \frac{1}{2} \times \frac{7\pi}{18} - \frac{\cos 3 \left( \frac{7\pi}{18} \right)}{3} \right) \right] \right $ $A = [(1.23287) + (0.22828)] = 1.46115... \approx 1.46$	A1 [4]
<b>Total 15 marks</b>		

Part	Mark	Notes
(a)	<b>M1</b>	Sets the equation equal to 0, solves using inverse sin and obtains a correct angle $\frac{1}{2} + \sin 3x = 0 \Rightarrow \sin 3x = -\frac{1}{2} \Rightarrow 3x = -\frac{\pi}{6}$ Condone working in degrees for M mark.
	<b>A1 cso</b>	For correctly obtaining $\left(\frac{7\pi}{18}, 0\right)$ * with no errors. Award for finding $\frac{7\pi}{18}$ and $\frac{11\pi}{18}$ but not shown as coordinates.
	<b>A1</b>	For correct coordinates of N: $\left(\frac{11\pi}{18}, 0\right)$
<b>Alternative method</b>		
	<b>M1</b>	For correct substitution of $\frac{7\pi}{18}$ into $\frac{1}{2} + \sin 3x$
	<b>A1 cso</b>	For correctly showing that $\frac{1}{2} + \sin\left(3 \times \frac{7\pi}{18}\right) = 0$ and stating $\left(\frac{7\pi}{18}, 0\right)$ * Award for showing $\frac{7\pi}{18}$ and finding $\frac{11\pi}{18}$ but not shown as coordinates
	<b>A1</b>	For correct coordinates of N: $\left(\frac{11\pi}{18}, 0\right)$
(b)	<b>M1</b>	For attempt to differentiate $y = \frac{1}{2} + \sin 3x$ , set equal to 0 and solve for $x$ . $\frac{dy}{dx} = 3 \cos 3x = 0 \Rightarrow 3x = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{6}$ Condone finding $x$ in degrees for this mark. Attempt at differentiation of two terms: $\frac{1}{2} \rightarrow 0$ $\sin(3x) \rightarrow k \cos(3x)$
	<b>A1</b>	For correctly obtaining $x = \frac{\pi}{6}$ <b>Note: Do not award this mark if <math>\frac{dy}{dx}</math> is incorrect.</b>
	<b>dM1</b>	Substitutes $x = \frac{\pi}{6}$ to find a value for $y$ . $y = \frac{1}{2} + \sin 3\left(\frac{\pi}{6}\right) = 1.5$ Condone working with $x$ in degrees for this mark.
	<b>A1</b>	For correct coordinates of A. $\left(\frac{\pi}{6}, \frac{3}{2}\right)$ Allow values equivalent to $\frac{3}{2}$
<b>Alternative method</b>		
	<b>M1</b>	For stating that the maximum of a sine curve is 1 and adding $\frac{1}{2}$
	<b>A1</b>	For correctly obtaining $y = \frac{3}{2}$
	<b>dM1</b>	For setting $\sin 3x$ equal to 1 and attempt to solve for $x$ $\sin 3x = 1 \Rightarrow 3x = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{6}$ Condone working with $x$ in degrees for this mark.

	<b>A1</b>	For correctly obtaining $x = \frac{\pi}{6}$
<b>(c)</b>	<b>B1</b>	For correct gradient at point $M$ $\left(\frac{dy}{dx}\right) = -\frac{3\sqrt{3}}{2}$
	<b>M1</b>	For a fully correct method of finding the equation of the tangent to $C$ at $M$ $y - 0 = -\frac{3\sqrt{3}}{2}\left(x - \frac{7\pi}{18}\right)$ Allow use of their $-\frac{3\sqrt{3}}{2}$ obtained from substitution of $x = \frac{7\pi}{18}$ into their $\frac{dy}{dx}$ .
	<b>A1</b>	For a correct equation of the tangent in any form: $y - 0 = -\frac{3\sqrt{3}}{2}\left(x - \frac{7\pi}{18}\right)$
	<b>A1</b>	For correct equation of the tangent in the required form: $12y + 18\sqrt{3}x - 7\sqrt{3}\pi = 0$ Accept integer multiples of this equation.
<b>(d)</b>	<b>M1</b>	For identifying the correct limits to find the area above the curve and the area below the curve: $\int_0^{\frac{7\pi}{18}} \left(\frac{1}{2} + \sin 3x\right) + \left  \int_{\frac{7\pi}{18}}^{\frac{11\pi}{18}} \left(\frac{1}{2} + \sin 3x\right) \right $ or $\int_0^{\frac{7\pi}{18}} \left(\frac{1}{2} + \sin 3x\right) - \int_{\frac{7\pi}{18}}^{\frac{11\pi}{18}} \left(\frac{1}{2} + \sin 3x\right)$ Allow use of their $\frac{11\pi}{18}$ . Needs to correctly indicate dealing with areas above and below axis.
	<b>M1</b>	For an attempt to integrate $\frac{1}{2} + \sin 3x$ obtaining: $\frac{1}{2}x - k \cos 3x$ where $k \neq -3$ For this mark ignore incorrect / absent limits.
	<b>M1</b>	For substituting limits correctly into their integrated expression (must be a changed expression). $A = \left[ \left( \frac{1}{2} \times \frac{7\pi}{18} - \frac{\cos 3\left(\frac{7\pi}{18}\right)}{3} \right) - \left( 0 - \frac{\cos 3 \times 0}{3} \right) \right]$ $+ \left  \left[ \left( \frac{1}{2} \times \frac{11\pi}{18} - \frac{\cos 3\left(\frac{11\pi}{18}\right)}{3} \right) - \left( \frac{1}{2} \times \frac{7\pi}{18} - \frac{\cos 3\left(\frac{7\pi}{18}\right)}{3} \right) \right] \right $ Allow use of their $\frac{11\pi}{18}$ . Must show substitution or M0.
	<b>A1</b>	For correctly obtaining awrt 1.46

Question number	Scheme	Marks
11 (a)	$f(x) = \int ax^2 - 14x - 10 \, dx = \frac{ax^3}{3} - \frac{14x^2}{2} - 10x + c$ $f(4) = \frac{a \times 4^3}{3} - \frac{14 \times 4^2}{2} - 10 \times 4 + c = 0 \Rightarrow \frac{64a}{3} - 152 + c = 0$ $f(-1) = \frac{a \times (-1)^3}{3} - \frac{14 \times (-1)^2}{2} - 10 \times (-1) + c = 25 \Rightarrow -\frac{a}{3} - 22 + c = 0$ $\frac{64a}{3} - 152 + c = 0$ $-\frac{a}{3} - 22 + c = 0$ $\Rightarrow 152 - \frac{64a}{3} = 22 + \frac{a}{3} \Rightarrow a = 6$	M1A1 M1 M1  M1A1 [6]
(b)	$c = 22 + \frac{6}{3} = 24$ $f(x) = 2x^3 - 7x^2 - 10x + 24$ $x - 4 \overline{) 2x^3 - 7x^2 - 10x + 24}$ $f(x) = (x - 4)(2x - 3)(x + 2) = 0$ $x = 4, \frac{3}{2}, -2$ <b>ALT</b> $2x^3 - 7x^2 - 10x + 24 = (x - 4)(ax^2 + bx + c)$ $a = 2, b = 1, c = -6$ $f(x) = (x - 4)(2x^2 + x - 6)$ $f(x) = (x - 4)(2x - 3)(x + 2) = 0$ $x = 4, \frac{3}{2}, -2$	B1  M1A1  dM1A1  A1 [6]  [M1  A1  dM1A1  A1]
<b>Total 12 marks</b>		

Part	Mark	Notes
(a)	<b>M1</b>	For an attempt to integrate $f'(x) = ax^2 - 14x - 10$ See General Guidance on what constitutes an attempt to integrate.
	<b>A1</b>	For correctly integrating $f'(x)$ to obtain $f(x) = \frac{ax^3}{3} - \frac{14x^2}{2} - 10x + c$ Must include $+c$ for this mark.
	<b>M1</b>	For substituting $x = \pm 4$ in their $f(x)$ and setting $=0$
	<b>M1</b>	For substituting $x = \pm 1$ in their $f(x)$ and setting $=25$

	<b>M1</b>	For correct method to solve the equations simultaneously to find $a$ A correct intermediate step is required e.g. $65a = 390$
	<b>A1</b> <b>cs0</b>	For fully correct working leading to $a = 6$
<b>(b)</b>	<b>B1</b>	For correctly identifying $c = 24$
	<b>M1</b>	For attempt to divide $f(x)$ by $x - 4$ Must get as far as $2x^2 + \dots$
	<b>A1</b>	For correct division of $f(x)$ by $x - 4$ to obtain $2x^2 + x - 6$
	<b>dM1</b>	For an attempt to factorise their 3TQ which must come from a cubic. See General Guidance on what constitutes an attempt to factorise.
	<b>A1</b>	For obtaining correct factorisation of the cubic: $(x - 4)(2x - 3)(x + 2)$
	<b>A1</b>	For all three correct solutions of the equation: $x = 4, \frac{3}{2}, -2$
<b>Alternative method</b>		
	<b>M1</b>	For an attempt to find the quadratic factor that multiplies $x - 4$ to give $f(x)$ Must get as far as $f(x) = (x - 4)(2x^2 + bx + c)$
	<b>A1</b>	For correctly comparing coefficients to obtain $2x^2 + x - 6$
	<b>dM1</b>	For an attempt to factorise their 3TQ which must come from a cubic. See General Guidance on what constitutes an attempt to factorise.
	<b>A1</b>	For obtaining correct factorisation of the cubic: $(x - 4)(2x - 3)(x + 2)$
	<b>A1</b>	For all three correct solutions of the equation: $x = 4, \frac{3}{2}, -2$
	<b>Note:</b> Correct solution seen with no working scores B0M0A0M0A0A0	

