Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International GCSE

Time 2 hours

Paper reference **4PM1/02R**

Further Pure Mathematics
PAPER 2R

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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P 7 1 8 2 0 A 0 1 3 6

Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\quad$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\quad$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$ $\quad$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 (a) Expand $\left(1+\frac{x}{4}\right)^8$ in ascending powers of x up to and including the term in x^3

Give each coefficient in its simplest terms.

(3)

(b) Use your expansion with a suitable value of x to obtain an approximation, to 4 decimal places, of $(1.035)^8$

(3)

(Total for Question 1 is 6 marks)

2 Find the set of values of x for which

(a) $3x - 8 < 5x + 3$

(1)

(b) $4x^2 - 7x + 1 > 6 - 2x^2$

(4)

(c) **both** $3x - 8 < 5x + 3$ **and** $4x^2 - 7x + 1 > 6 - 2x^2$

(1)

Question 2 continued

(Total for Question 2 is 6 marks)

3 Given that $y = e^{3x} \sin 2x$

show that $13y + \frac{d^2y}{dx^2} = 6\frac{dy}{dx}$

(8)

Question 3 continued

(Total for Question 3 is 8 marks)

4

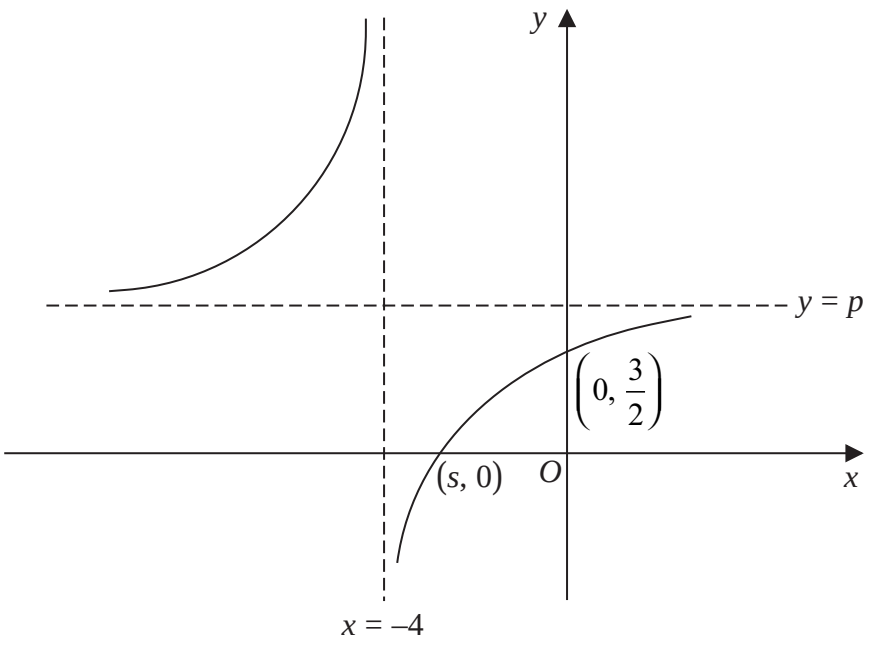

Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows part of the curve C with equation

$$y = \frac{2x + q}{x + r} \qquad x \neq -r$$

where q and r are integers.

The asymptote to C that is parallel to the y-axis has equation $x = -4$
The asymptote to C that is parallel to the x-axis has equation $y = p$

(a) Write down

(i) the value of p

(ii) the value of r

(2)

Given that C crosses the y-axis at the point with coordinates $\left(0, \frac{3}{2}\right)$

(b) find the value of q

(2)

Given that C crosses the x-axis at the point with coordinates $(s, 0)$

(c) find the value of s

(2)

.....

.....

.....

.....

Question 4 continued

(Total for Question 4 is 6 marks)

5 The line l with gradient $-\frac{1}{12}$ passes through the points A and B with coordinates $(p, 10)$ and $(123, 0)$ respectively.

(a) Show that $p = 3$

(2)

(b) Find an equation for l in the form $rx + sy + t = 0$ where r, s and t are integers.

(2)

The line k is perpendicular to l and passes through the point A.

(c) Find an equation for k in the form $y = mx + c$

(3)

Line k intersects the x-axis at the point C.

(d) Find the exact area of triangle ABC.

(4)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 11 marks)

6

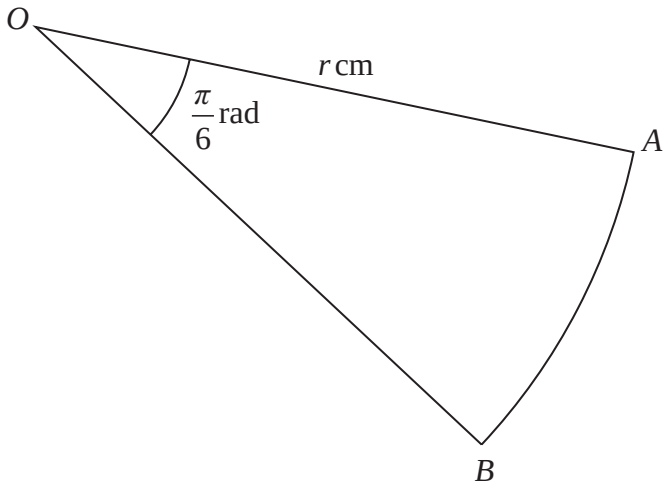

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Figure 2

Figure 2 shows the sector OAB of a circle with centre O and radius r cm.

$$\angle AOB = \frac{\pi}{6} \text{ radians} \qquad OA = OB = r \text{ cm}$$

The area of the sector is increasing in such a way that the size of $\angle AOB$ remains constant, and the lengths OA and OB are both increasing at a constant rate of $0.2\,\text{cm/s}$

Find the exact rate of change, in cm^2/s, of the area of the sector when the length of arc AB is $\frac{5\pi}{2}$

(6)

Question 6 continued

(Total for Question 6 is 6 marks)

7

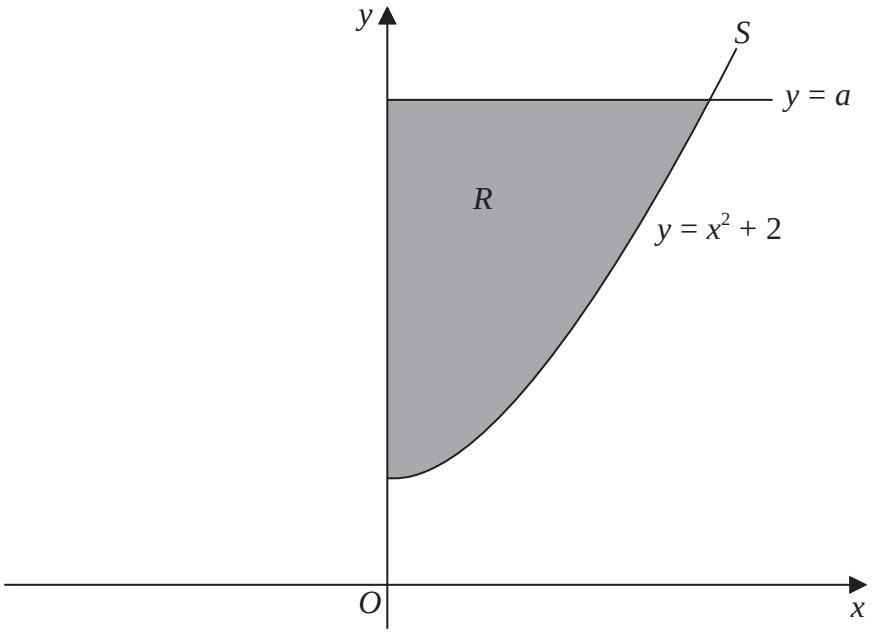

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Figure 3

Figure 3 shows part of the curve S with equation $y = x^2 + 2$

The finite region R, shown shaded in Figure 3, is bounded by S, the y-axis and the line with equation $y = a$ where $a > 2$

The region R is rotated through 360° about the y-axis to generate a solid with volume 18π

Use algebraic integration to find the value of a

(8)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 8 marks)

8 The quadratic equation $3x^2 - kx - 1 = 0$, where k is a positive integer, has roots α and β

(a) Show that $\alpha^2 + \beta^2 = \dfrac{k^2 + 6}{9}$

(3)

Given that $\alpha^4 + \beta^4 = \dfrac{466}{81}$

(b) find the value of k

(5)

(c) Hence form an equation, with integer coefficients, which has roots

$$\frac{\alpha^3 + \beta}{\beta} \text{ and } \frac{\beta^3 + \alpha}{\alpha}$$

(6)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 14 marks)

9 A geometric series G has common ratio r where $r > 0$

The third term of G is $\frac{27}{2}$ and the sum of the first three terms of G is $\frac{57}{2}$

Given that the sum to n terms of G is S_n

(a) show that $S_n = \sum_{j=1}^{n} 4\left(\frac{3}{2}\right)^j$

(8)

Given that $S_k > 50\,000$

(b) show that the least value of k is given by

$$k > \frac{\lg\left(\frac{12\,503}{3}\right)}{\lg\left(\frac{3}{2}\right)}$$

(3)

(c) Hence find the least value of k

(1)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 12 marks)

10 (a) Show that $\dfrac{9^{3y}}{243} = 3^{(6y-5)}$

(4)

(b) Solve the simultaneous equations

$$\frac{9^{3y}}{243} = 27^{(x-2)}$$

$$\log_{10}\sqrt{6xy} = \log_4 2$$

(9)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 13 marks)

11

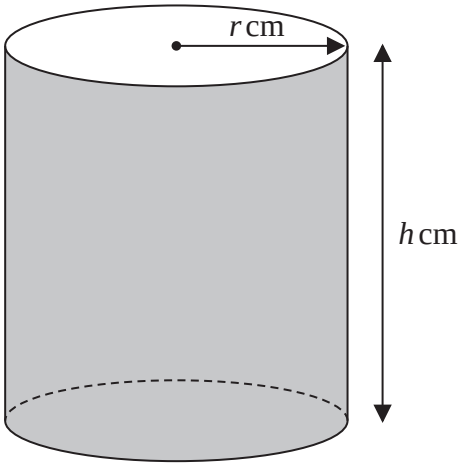

Diagram **NOT** accurately drawn

Figure 4

Figure 4 shows an open container in the shape of a cylinder with radius r cm and height h cm.

Given that the total surface area of the container is $625\pi\,\text{cm}^2$

(a) show that

$$h = \frac{625 - r^2}{2r}$$

(3)

The volume of the container is $V\,\text{cm}^3$

Given that r can vary,

(b) use calculus to find the value, to 3 significant figures, of r for which V is a maximum.
Justify that this value of r gives a maximum value of V

(6)

(c) For the value of r found in part (b), find the corresponding value, to 3 significant figures, of h

(1)

Question 11 continued

Question 11 continued

Question 11 continued

Question 11 continued

(Total for Question 11 is 10 marks)

TOTAL FOR PAPER IS 100 MARKS