Question number	Scheme	Marks
7 a (i)	$\text{f}(x) = \int (4x^3 - 12x^2 - 19x + 12)\,\text{d}x = \frac{4}{4}x^4 - \frac{12}{3}x^3 - \frac{19}{2}x^2 + 12x + D$ oe For the point $(4, -104)$ it follows that $-104 = (4)^4 - 4(4)^3 - \frac{19}{2}(4)^2 + 12(4) + D$ * $-104 = 256 - 256 - 152 + 48 + D \Leftrightarrow D = 0$*	M1 A1 M1 A1* cso (4)
a (ii)	$x = 0.5 \quad \text{f}'(x) = 4(0.5)^3 - 12(0.5)^2 - 19(0.5) + 12 = 0$ $\text{f}''(x) = 12x^2 - 24x - 19$ $x = 0.5 \quad \text{f}''(x) = 12(0.5)^2 - 24(0.5) - 19(= -28) < 0$ Therefore maximum *	M1 M1 A1 cso (3)
b (i)	$\text{f}'(x) = (2x-1)(2x^2 - 5x - 12)$ $\text{f}'(x) = (2x-1)(2x+3)(x-4) \rightarrow x =$ $x = -\frac{3}{2}$ or $x = 4$, $\left[x = \frac{1}{2}\right]$ $A\left(-\frac{3}{2}, -\frac{333}{16}\right) \qquad B(4, -104)$	M1 M1 A1 A1 A1 (5)
b (ii)	$\text{f}''(x) = 12(-1.5)^2 - 24(-1.5) - 19(= 44) > 0$ Therefore minimum $x = 4 \quad \text{f}''(x) = 12(4)^2 - 24(4) - 19(= 77) > 0$ Therefore minimum (Read notes carefully for allocation of these marks) **Alternative** As we have a maximum at $x = 0.5$ and C is a continuous curve $x = -\frac{3}{2}$ is a minimum and $x = 4$ is a minimum	ddM1 A1 A1 (3) {ddM1} {A1}{A1} (3)
		Total 15 marks

Part	Marks	Notes
(a) (i)	**M1**	Attempt to integrate – see general guidance for minimally acceptable attempt – at least one term must be fully correct in this integration (unsimplified).
	A1	Fully correct integration – does not need to include a constant of integration, need not be simplified.
	M1	Substitution of $x = 4$ and $y = -104$ into their expression, which must include a constant of integration.
	A1 cso	Obtains the given result, making it clear the constant of integration is 0 or listing fully the function at the end after fully correct working. There must be no incorrect working for this final mark to be awarded.
Note: candidates may assume $D = 0$ & show, by substituting, (4, -104) lies on the curve **final M1 A1**		
(a) (ii)	**M1**	Substitution of $x = 0.5$ into $\mathrm{f}'(x)$ and shows $\mathrm{f}'(x) = 0$. A candidate may also solve f'(x) = 0 and solve to get $x = 0.5$.
	M1	For finding $\mathrm{f}''(x)$ - see general guidance for minimally acceptable attempt at differentiation. At least one term must be fully correct for this differentiation.
	A1 cso*	Substitutes $x = 0.5$ into a fully correct second derivative, shows that the result is negative and draws a conclusion. This can be as simple as "shown" or #. It is not necessary to work out the actual value of $\mathrm{f}''(0.5)$ if clear substitution is shown, but if calculated, it must be correct. No incorrect work for this mark to be awarded. Note: as always, this A mark must follow 2 M1 marks.

(b)	Part (i) and (ii) may be marked together	
(i)	**M1**	A clear attempt to divide by $(2x-1)$ or compare coefficients. The student must arrive at a quadratic factor of the form $(2x^2 + Ax - 12)$ if dividing It is also possible to divide by $\left(x - \frac{1}{2}\right)$ and arrive at a quadratic factor of the form $4x^2 + Bx - 24$
	M1	Uses a complete method to solve their 3TQ, see general guidance for a minimally acceptable attempt. Must progress to $x =$.
	A1	$x = -\frac{3}{2}$ or $x = 4$ Award M1 M1 A1 if **both** correct values are given without working.
	A1	$A\left(-\frac{3}{2}, -\frac{333}{16}\right)$ or $B(4, -104)$ Allow $x =$ and $y =$ (clearly paired) for **both** final A marks.
	A1	$A\left(-\frac{3}{2}, -\frac{333}{16}\right)$ and $B(4, -104)$ accept 20.8 or better – answer is 20.8125
	Factorising in (a)(ii) and then using in this part can be awarded the marks above.	
(b) (ii)	**ddM1**	Correctly substitutes $x = -\frac{3}{2}$ or $x = 4$ into their $f''(x)$. Their values Dependent on both previous M marks in part (b). A fully correct evaluation of the second derivative can imply this mark ie sight of 77 and 44. Part (i) and (ii) may be marked together
	A1	Substitutes $x = -1.5$ **or** $x = 4$ into a fully correct second derivative, shows that the result is positive and draws the conclusion this is a minimum. It is not necessary to work out the actual value of $f''(x)$ if clear substitution is shown, but if calculated, it must be correct.
	A1	Substitutes $x = -1.5$ **and** $x = 4$ into a fully correct second derivative, shows that the result is positive and draws the conclusion that both values of x give a minimum. It is not necessary to work out the actual value of $f''(x)$ if clear substitution is shown, but if calculated, it must be correct.
ALT	**ddM1**	For stating the curve is continuous and there is a maximum at $x = 0.5$ Dep both previous M
	A1	For stating "as we have a maximum at $x = 0.5$" and progressing to state either A or B must be a minimum.
	A1	Fully correct conclusion, stating A and B must both be minimum points.
There **MUST** be an appropriate justification for these marks to be awarded, such as that given in M1		

Question number	Scheme	Marks
8 a	$\frac{4}{3}\pi r^3 = 500$ so $r^3 = \frac{3\times 500}{4\pi}$ therefore $r = 4.92$ accept awrt 4.92	M1 A1 (2)
b	$\delta A = 20$	B1
	$(A = 4\pi r^2) \to \left(\frac{\mathrm{d}A}{\mathrm{d}r}\right) = 8\pi r$	M1
	$\delta r \approx \frac{\mathrm{d}r}{\mathrm{d}A}\delta A = 20 \times \frac{1}{8\pi r}$	M1
	When $r = 4.92$ $\delta r \approx \frac{20}{8\pi(4.92)} = 0.16(16204507)$	dM1
	or 0.1617428283 if 4.92 used 0.16 cm accept awrt to 0.16	A1 (5)
		Total 7 marks

Part	Marks	Notes
(a)	**M1**	For correct substitution into the formula for volume of a sphere and correct rearrangement to give r or r^3.
	A1	For $r = 4.92$
(b)	**B1**	For $\delta A = 20$ - may be stated explicitly or implicit in working. Accept $\frac{\mathrm{d}A}{\mathrm{d}t} = 20$
	M1	For $8\pi r$ A may be replaced with another variable, eg S
	M1	For $\delta r \approx \frac{\mathrm{d}r}{\mathrm{d}A}\delta A$ and substitution of 20 and their expression for $\frac{\mathrm{d}A}{\mathrm{d}r}$ Condone poor notation if substitution and their expression for $\frac{\mathrm{d}A}{\mathrm{d}r}$ is correct. Eg accept $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\mathrm{d}r}{\mathrm{d}A}\times\frac{\mathrm{d}A}{\mathrm{d}t}$ if using their expression for $\frac{\mathrm{d}A}{\mathrm{d}r}$
	dM1	For substitution of their value for r into their expression (as given above, poor notation condoned) for δr Dependent on previous method mark.
	A1	For 0.16 cm (units not required)
	Without appropriate calculus – no marks may be awarded for this question.	