5 Given that $\alpha + \beta = 5$ and $\alpha^2 + \beta^2 = 19$

(a) show that $\alpha\beta = 3$

(2)

(b) Hence form a quadratic equation, with integer coefficients, which has roots α and β

(2)

(c) Form a quadratic equation, with integer coefficients, which has roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$

(5)

Question 5 continued

(Total for Question 5 is 9 marks)

6

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\frac{\sin A}{\cos A} = \tan A$$

Using the above formulae, show that

(a) $\sin 2x = 2 \sin x \cos x$

(1)

(b) $\cos 2x = \cos^2 x - \sin^2 x$

(1)

(c) $\dfrac{\sin 2x}{1 + \cos 2x} = \tan x$

(4)

Question 6 continued

(Total for Question 6 is 6 marks)