9 $\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

Using the above identities

(a) show that $\cos 2\theta = 2\cos^2 \theta - 1$

(3)

(b) find a simplified expression for $\sin 2\theta$ in terms of $\sin \theta$ and $\cos \theta$

(1)

(c) show that $\cos 3\theta = 4\cos^3 \theta - 3\cos \theta$

(4)

Hence, or otherwise,

(d) solve, for $0 \leqslant \theta < \pi$ giving your answers in terms of π, the equation

$$6\cos \theta - 8\cos^3 \theta + 1 = 0$$

(4)

(e) find

(i) $\int \left(8\cos^3 \theta + 4\sin \theta\right) \mathrm{d}\theta$

(ii) the exact value of $\int_0^{\frac{\pi}{3}} \left(8\cos^3 \theta + 4\sin \theta\right) \mathrm{d}\theta$

(4)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 16 marks)