

| Question              | Scheme  | Marks  |
|-----------------------|---|--|
| <b>6(a)</b>           | $\tan 30^\circ = \frac{r}{h} \Rightarrow r = \frac{h}{\sqrt{3}}$ $V = \frac{1}{3}\pi r^2 h \Rightarrow V = \frac{1}{3}\pi \left(\frac{h}{\sqrt{3}}\right)^2 h \Rightarrow V = \frac{1}{9}\pi h^3 *$   | B1<br><br>M1A1<br>cso<br>[3]                             |
| <b>(b)</b>            | $\frac{dV}{dt} = -4$ $A = \pi r^2 \Rightarrow A = \pi \left(\frac{h}{\sqrt{3}}\right)^2 = \frac{\pi h^2}{3}$ $\frac{dA}{dh} = \frac{2\pi h}{3} \quad \frac{dV}{dh} = \frac{3\pi h^2}{9} = \frac{\pi h^2}{3}$ $\frac{dA}{dt} = \frac{dA}{dh} \times \frac{dh}{dt} \times \frac{1}{\frac{dV}{dh}}$ $\frac{dA}{dt} = \frac{2\pi h}{3} \times (-4) \times \frac{3}{\pi h^2} = -\frac{8}{h} \Rightarrow \frac{dA}{dt} = -\frac{8}{24} = -\frac{1}{3} \quad (\text{cm}^2/\text{s})$   | B1<br><br>B1<br><br>M1M1A1<br><br>M1<br><br>dM1A1<br>[8] |
|                       | <b>ALT – working in terms of <math>r</math> rather than <math>h</math></b>  |  |
|                       | $\frac{dV}{dt} = -4$ $V = \frac{1}{3}\pi r^2 h \Rightarrow V = \frac{1}{3}\pi r^2(\sqrt{3}r) = \frac{\sqrt{3}}{3}\pi r^3$ $A = \pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r \quad V = \frac{\sqrt{3}}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = \sqrt{3}\pi r^2$ $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{1}{\frac{dV}{dr}} \times \frac{dV}{dt}$ $\frac{dA}{dt} = 2\pi r \times \frac{1}{\sqrt{3}\pi r^2} \times -4 = -\frac{8}{\sqrt{3}r} \Rightarrow \frac{dA}{dt} = -\frac{8}{\sqrt{3} \times \frac{24}{\sqrt{3}}} = -\frac{1}{3}$ | [B1<br><br>B1<br><br>M1M1A1<br><br>M1<br><br>dM1A1]      |
| <b>Total 11 marks</b> |   |  |

| Part | Mark       | Notes  |
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| (a)  | B1         | For $r = \frac{h}{\sqrt{3}}$ or $r^2 = \frac{1}{3}h^2$<br>$r = \frac{h}{\sqrt{3}}$ may be implied by substitution into the correct formula for volume of a cone.   |
|      | M1         | For using the correct volume of a cone and substituting in their expression for $r$ or $r^2$ to find an expression for the volume in terms of $h$  |
|      | A1         | For the correct expression with no errors seen.<br>Must have $V =$   |
| (b)  | B1         | For stating $\frac{dV}{dt} = -4$ (allow +)<br>May be embedded in working.  |
|      | B1         | For finding the correct expression for the area of a circle in terms of $h$<br>Note: if differentiation is attempted before writing in terms of $h$ then award<br>1 <sup>st</sup> M1 for $\frac{dA}{dh} = \frac{dA}{dr} \times \frac{dr}{dh}$ with attempts at $\frac{dA}{dr}, \frac{dr}{dh}$ found and substituted in and attempt to substitute for $r$ in terms of $h$ .<br>B1 once $\frac{dA}{dh}$ found completely in terms of $h$ |
|      | M1         | For attempting to differentiate their expression for $A$ in terms of $h$<br>See general guidance for what constitutes an attempt to differentiate.<br>$A$ must have the form $A = kh^2$ (dimensionally correct for area).  |
|      | M1         | For attempting to differentiate the given $V$<br>See general guidance for what constitutes an attempt to differentiate.  |
|      | A1         | Both derivatives correct. $\frac{dA}{dh}$ and $\frac{dV}{dh}$  |
|      | M1         | For a correct expression of chain rule to find $\frac{dA}{dt}$   |
|      | dM1        | For substituting their derivatives into a correct chain rule.<br>Dep on previous three M marks.  |
|      | A1         | For the correct $\frac{dA}{dt} = -\frac{1}{3} \text{ (cm}^2/\text{s)}$<br>Allow $-0.\dot{3}$ but not $-0.3$ .<br>ISW once exact answer found.  |
|      | <b>ALT</b> |  |
|      | B1         | For stating $\frac{dV}{dt} = -4$ (allow +)<br>May be embedded in working.  |
|      | B1         | For finding the correct expression for the volume in terms of $r$<br>Must be working with correct volume expression as given in (a).   |
|      | M1         | For attempting to differentiate expression for $A$ in terms of $r$<br>See general guidance for what constitutes an attempt to differentiate.<br>$A$ must have the form $A = kr^2$ (dimensionally correct for area).  |
|      | M1         | For attempting to differentiate expression for $V$ in terms of $r$<br>See general guidance for what constitutes an attempt to differentiate.   |

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|  | A1  | Both derivatives correct. $\frac{dA}{dr}$ and $\frac{dV}{dr}$  |
|  | M1  | For a correct expression of chain rule to find $\frac{dA}{dt}$   |
|  | dM1 | For substituting their derivatives into a correct chain rule.<br>Dep on previous three M marks.  |
|  | A1  | For the correct $\frac{dA}{dt} = -\frac{1}{3} \text{ (cm}^2 \text{ / s)}$<br>Allow $-0.\dot{3}$ but not $-0.3$ .<br>ISW once exact answer found. |