10 The points A, B, C and D are the vertices of a quadrilateral such that

$$\overrightarrow{AB} = 3\mathbf{a} + 4\mathbf{b} \quad \overrightarrow{AC} = 7\mathbf{a} + 9\mathbf{b} \quad \overrightarrow{AD} = 4\mathbf{a} + 5\mathbf{b}$$

(a) Show that $ABCD$ is a parallelogram.

(3)

BC is extended to the point E such that BCE is a straight line.

Point F lies on CD such that $CF : FD = 1 : 2$

Given that A, F and E are collinear,

(b) find the vector $\overrightarrow{AE}$ in the form $X\mathbf{a} + Y\mathbf{b}$ where X and Y are rational numbers to be found.

(8)

P 7 6 5 0 8 A 0 2 6 3 2

Question 10 continued

Question 10 continued

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Question 10 continued

(Total for Question 10 is 11 marks)

P 7 6 5 0 8 A 0 2 9 3 2

11 Using formulae from page 2, show that

(a) (i) $\cos 2A = 2\cos^2 A - 1$

(3)

(ii) $\sin 2A = 2\sin A \cos A$

(1)

(b) Show that $\cos^3 A = \dfrac{\cos 3A + 3\cos A}{4}$
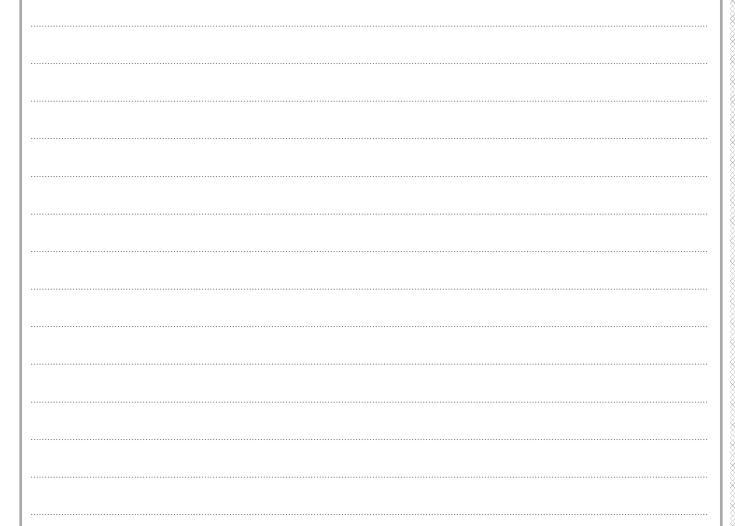
(4)

Hence, or otherwise,

(c) solve, giving exact values in terms of π

$$8\cos^3\left(\frac{\theta}{2}\right) - 6\cos\left(\frac{\theta}{2}\right) - 1 = 0 \qquad \text{for } 0 \leqslant \theta \leqslant 2\pi$$

(4)

(d) use algebraic integration to find the exact value of

$$\int_0^{\frac{\pi}{6}} \left(4\cos^3 \theta - \sin 2\theta\right)\, d\theta$$

(4)

Question 11 continued

P 7 6 5 0 8 A 0 3 1 3 2

Question 11 continued

(Total for Question 11 is 16 marks)

TOTAL FOR PAPER IS 100 MARKS

P 7 6 5 0 8 A 0 3 2 3 2