Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International GCSE

Tuesday 31 October 2023

Morning (Time: 2 hours)

Paper reference **4PM1/01**

Further Pure Mathematics
PAPER 1

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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P 7 3 5 8 4 A 0 1 3 6

Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\quad$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\quad$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B} \qquad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

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Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1

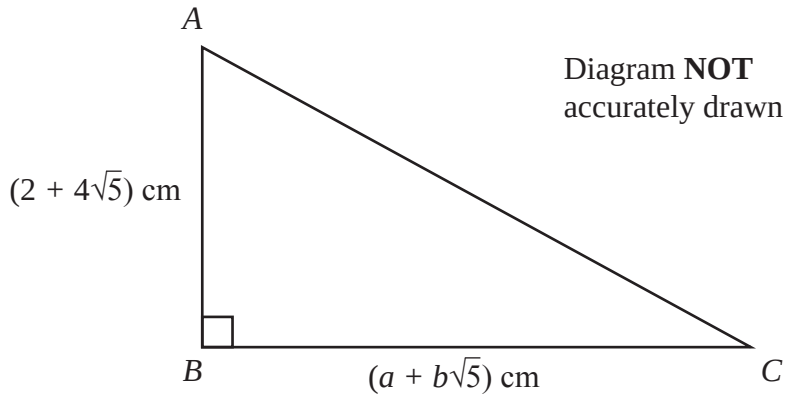

Figure 1

Figure 1 shows the triangle *ABC*

$\angle ABC = 90°$ $AB = \left(2 + 4\sqrt{5}\right)$ cm $BC = \left(a + b\sqrt{5}\right)$ cm where a and b are integers.

The area of triangle $ABC = \left(34 + 11\sqrt{5}\right)$ cm^2

Without using a calculator, find the value of a and the value of b

(4)

Question 1 continued

(Total for Question 1 is 4 marks)

2 $$g(x) = 2x^2 + \frac{1}{2}x - 3$$

(a) Express g(x) in the form $p(x+q)^2 + r$ where p, q and r are rational numbers to be found.

(3)

(b) Find

(i) the minimum value of g(x)

(ii) the value of x at which this minimum occurs.

(2)

$$h(x) = 2x^6 + \frac{1}{2}x^3 - 3$$

(c) Hence, or otherwise, write down

(i) the minimum value of h(x)

(ii) the value of x at which this minimum occurs.

(2)

Question 2 continued

(Total for Question 2 is 7 marks)

3 $g'(x) = mx^2 - 10x - 37$ where m is an integer

The curve $y = g(x)$ passes through the point with coordinates $(1, 20)$

Given that $(x - 5)$ is a factor of g(x)

(a) show that $g(x) = 2x^3 - 5x^2 - 37x + 60$

(5)

(b) Hence, or otherwise, use algebra to solve the equation $g(x) = 0$

(3)

Question 3 continued

(Total for Question 3 is 8 marks)

4 The point A with coordinates $(12, 14)$ and the point B with coordinates $(q, 2)$ where q is a constant, lie on the straight line with equation $3y - 2x - p = 0$ where p is a constant.

(a) Find the value of p and the value of q

(3)

The line L is perpendicular to AB and passes through the point X, which lies on AB such that $AX : XB = 1 : 2$

(b) Find an equation for L in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(6)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5

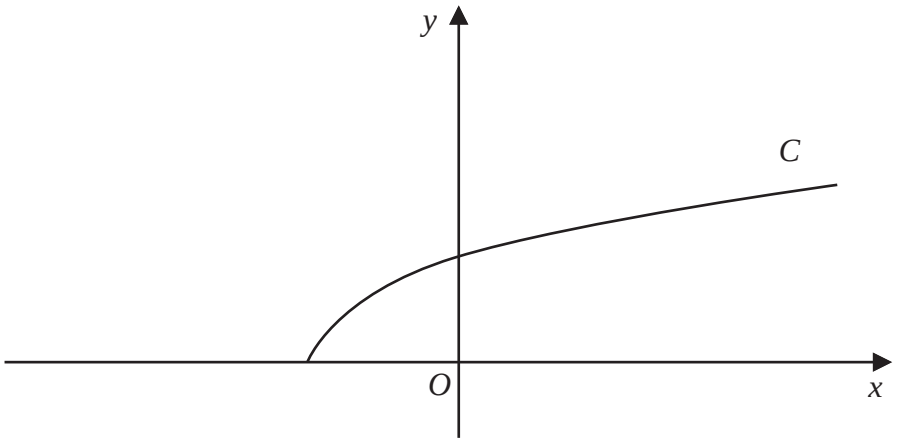

Figure 2

Figure 2 shows the graph of part of the curve C with equation $y = \sqrt{2x+6}$
The finite region enclosed by the curve C and the straight line with equation $3y - x = 3$ is rotated through 360° about the x-axis.

Use algebraic integration to find the exact volume of the solid generated.
Give your answer in terms of π

(8)

Question 5 continued

(Total for Question 5 is 8 marks)

6

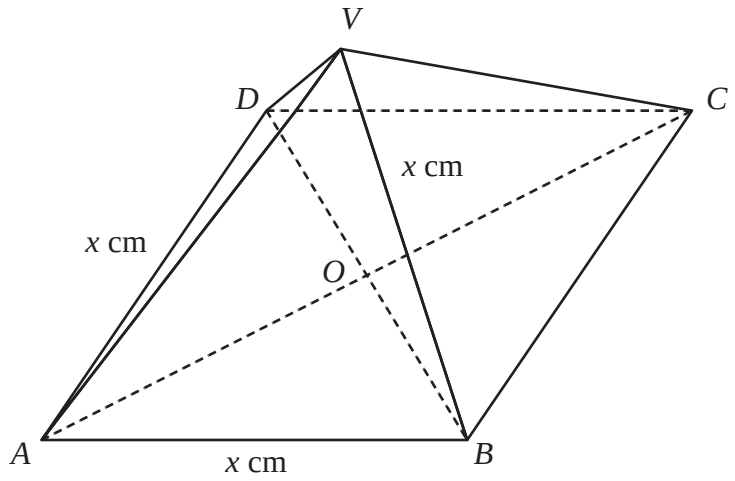

Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows a right pyramid with a horizontal square base.

$AB = BC = CD = DA = x$ cm

$AV = BV = CV = DV = x$ cm

O is the point of intersection of the diagonals of the base.
The vertex V of the pyramid is vertically above O

(a) Show that $VO = \frac{\sqrt{2}}{2}x$ cm

(3)

(b) Find, in degrees, the size of the angle AVC

(2)

(c) Find, in degrees to one decimal place, the size of the angle between the plane VAB and the plane VDC

(3)

The volume of the pyramid is 200 cm^3

Given that the volume of a pyramid $= \frac{1}{3} \times$ base area $\times$ height

(d) Find to 3 signficant figures, the value of x

(3)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 11 marks)

7 A geometric series G with common ratio r, has first term 16 and third term $\frac{2704}{625}$

(a) Find the two possible values of r

(2)

Given that $r > 0$

(b) find the sum to infinity of G

(2)

The sum to n terms of G is greater than 33

(c) Find, using logarithms, the least possible value of n
Show your working clearly.

(5)

Question 7 continued

(Total for Question 7 is 9 marks)

8 $$y = \frac{2e^{3x+1}}{5x^2}$$

(a) Find $\frac{dy}{dx}$

Give your answer in the form $\frac{Ae^{3x+1}(Bx - A)}{Cx^3}$ where A, B and C are prime numbers to be found.

(5)

The value of x increases by 2%

(b) Use your answer to part (a) to find an estimate, in terms of x, for the percentage change in y
Give your answer in the form $(Px - Q)$ where P and Q are integers.

(3)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 8 marks)

9 (a) Expand $\left(1-8x^2\right)^{-\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^6

giving each coefficient as an integer.

(3)

$$g(x)=\frac{a+bx}{\sqrt{1-8x^2}}$$ where a and b are prime numbers

Given that the fourth and fifth terms, in ascending powers of x, in the series expansion of g(x) are $20x^3$ and $48x^4$ respectively,

(b) find the value of a and the value of b

(4)

Using the first five terms, in ascending powers of x, in the series expansion of g(x)

(c) obtain an estimate, to 4 significant figures, of $\int_0^{0.2} g(x)\,dx$

(4)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 11 marks)

10 (a) Using formulae on page 2, show that

(i) $\sin 2A = 2\sin A\cos A$

(ii) $\cos 2A = 2\cos^2 A - 1$

(3)

$$\mathrm{f}(\theta) = \frac{2\tan\theta}{1+\tan^2\theta}$$

(b) Show that $\mathrm{f}(\theta) = \sin 2\theta$

(4)

(c) Solve, in radians to 3 significant figures, for $-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$, the equation

$$5\tan\left(x+\frac{\pi}{6}\right) = \left[1+\tan^2\left(x+\frac{\pi}{6}\right)\right]\left[1-2\cos^2\left(x+\frac{\pi}{6}\right)\right]$$

(6)

(d) Using calculus, find the exact value of

$$\int_0^{\frac{\pi}{2}} \left(\frac{4\tan\theta}{1+\tan^2\theta} - \cos 5\theta + 2\right) \mathrm{d}\theta$$

(4)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 17 marks)

11 Solve the simultaneous equations

$$2\log_4 x = \log_3 3y^2$$

$$\log_2 x^3 + 8\log_9 y = 13$$

Show your working clearly.

(8)

Question 11 continued

Question 11 continued

(Total for Question 11 is 8 marks)

TOTAL FOR PAPER IS 100 MARKS