Pearson
Edexcel

Mark Scheme (Results)

Summer 2024

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1) Paper 02

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
    - M marks: method marks
    - A marks: accuracy marks – can only be awarded when relevant M marks have been gained
    - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
    - cao – correct answer only
    - cso – correct solution only
    - ft – follow through
    - isw – ignore subsequent working
    - SC - special case
    - oe – or equivalent (and appropriate)
    - dep – dependent
    - indep – independent
    - awrt – answer which rounds to
    - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers may score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**

If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.

If a candidate misreads a number from the question: eg. uses 252 instead of 255; follow through their working and deduct 2A marks from any gained provided the work has not been simplified. (Do not deduct any M marks gained.)

If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes clear the method that has been used

Examiners should send any instance of a suspected misread to review (but see above for simple misreads).

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. incorrect cancelling of a fraction that would otherwise be correct.

  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$(x^2 + bx + c) = (x + p)(x + q)$,where $|pq| = |c|$ leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$ where$|pq| = |c|$ and $|mn| = |a|$leading to $x = \ldots$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x = \ldots$

3. Completing the square:

$x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$ leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \rightarrow x^{n-1})$

2. Integration:

Power of at least one term increased by 1. $(x^n \rightarrow x^{n+1})$

Use of a formula:

Generally, the method mark is gained by **either** quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values **or**, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks". General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the is rule may allow the mark to be awarded before the final answer is given.

Question	Scheme	Marks
1(a)	$\mathrm{f}\left(\frac{2}{3}\right)=6\times\left(\frac{2}{3}\right)^3-13\times\left(\frac{2}{3}\right)^2+a\times\left(\frac{2}{3}\right)-10=0\Rightarrow\frac{2a}{3}=14$	M1
	$\Rightarrow a=21*$	A1 cso [2]
	Alt – Polynomial division	
	$\begin{array}{r}2x^2-3x+5\\3x-2\overline{\big)6x^3-13x^2+ax-10}\end{array}\Rightarrow a-6=15\Rightarrow a=21$	[M1A1]
(b)	**Polynomial division**	
	$\begin{array}{r}2x^2-3x+5\\3x-2\overline{\big)6x^3-13x^2+21x-10}\end{array}$	M1A1
	Equates coefficients	
	$(3x-2)(ax^2+bx+c)=6x^3-13x^2+21x-10$	
	$\Rightarrow 3a=6\Rightarrow a=2$	
	$\Rightarrow 3b-2a=-13\Rightarrow b=-3$	[M1
	$\Rightarrow -2c=-10\Rightarrow c=5$	
	Quadratic factor is $2x^2-3x+5$	A1]
	Conclusion	M1
	$b^2-4ac=(-3)^2-4\times2\times5=-31$	
	The discriminant is negative hence the quadratic has no real solutions and the only intersection with the *x*-axis is at $x=\frac{2}{3}$	A1 [4]
	ALT	
	Completes square:	
	$2x^2-3x+5=0\Rightarrow\left(x-\frac{3}{4}\right)^2=-\frac{31}{16}$	[M1
	$\left[\Rightarrow x=\frac{3}{4}\pm\sqrt{-\frac{31}{16}}\right]$	
	$-\frac{31}{16}<0$ so the quadratic has no real roots	
	The only intersection [root] with the *x*-axis is at $x=\frac{2}{3}$	A1]
		Total 6 marks

Part	Mark	Notes
(a)	M1	Substitutes $\pm\frac{2}{3}$ into $\mathrm{f}(x)$ and sets = 0
	A1	For using $x = \frac{2}{3}$ with no errors seen.
	ALT 1 – accepting a substitution	
	M1	Substitutes $\pm\frac{2}{3}$ into $\mathrm{f}(x)$, $a = 21$
	A1	Shows that $\mathrm{f}(x) = 0$
	ALT 2 – Polynomial division	
	M1	For obtaining as a minimum $a - k = 15$
	A1	For obtaining $a = 21$ with no errors seen
(b)	**Polynomial division**	
	M1	Uses polynomial division to achieve as a minimum $2x^2 \pm 3x \pm k$ where k is a constant **NB** Award this seen in part (a) if they use polynomial division
	A1	For the correct quadratic factor $2x^2 - 3x + 5$
	OR Equates coefficients	
	M1	Achieves the correct values for a and b
	A1	For the correct quadratic factor $2x^2 - 3x + 5$
	Uses the discriminant	
	M1	Finds the value of the discriminant using their QE Allow the discriminant to be embedded as part of the Quadratic formula. If they do not achieve the correct 3TQ – please check their discriminant carefully. **NB: It must be evaluated,**
	A1	For a value of –31 for the discriminant with a conclusion. Minimally acceptable conclusion: • Discriminant < 0 hence no real roots. • The only intersection [root/solution] is at $x = \frac{2}{3}$
	Completes square	
	M1	Completes the square on their 3TQ as far as: $\left(x - \frac{3}{4}\right)^2 = -\frac{31}{16}$
	A1	For a value of $-\frac{31}{16}$ with a conclusion • $-\frac{31}{16} < 0$ so the quadratic has no real roots • The only intersection [root/solution] with the x-axis is at $x = \frac{2}{3}$
	Complex roots	
	The question states clearly 'Show Algebraically' so just quoting the complex roots is M0. If they go through the process of finding them fully, please send to REVIEW	

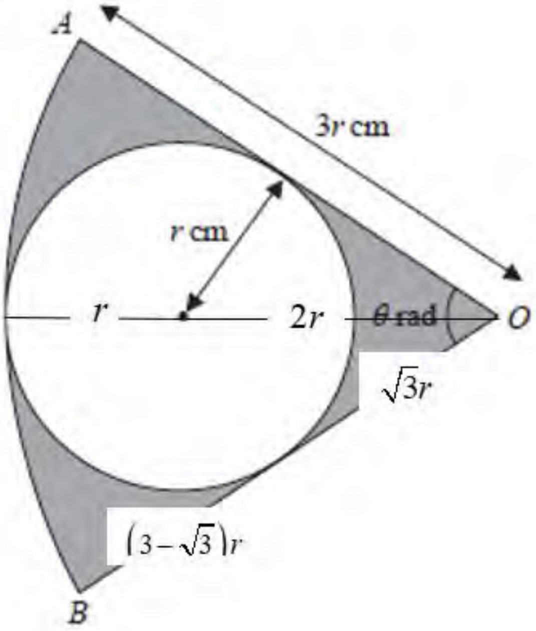
Question	Scheme	Marks
2	$\alpha+\beta=-\left(-\frac{5}{3}\right)=\frac{5}{3},\quad \alpha\beta=\frac{1}{3}$	B1,B1
	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta\Rightarrow\left(\frac{5}{3}\right)^2-2\times\frac{1}{3}=\frac{19}{9}$	B1
	PRODUCT: $\frac{\alpha}{2\beta}\times\frac{\beta}{2\alpha}=\frac{1}{4}$	B1
	SUM: $\frac{\alpha}{2\beta}+\frac{\beta}{2\alpha}=\frac{2\alpha^2+2\beta^2}{4\alpha\beta}=\left[\frac{\alpha^2+\beta^2}{2\alpha\beta}\right]\Rightarrow\frac{\alpha}{2\beta}+\frac{\beta}{2\alpha}=\frac{\frac{19}{9}}{\frac{2}{3}}=\frac{19}{6}$	M1A1
	EQUATION: $x^2-\frac{19}{6}x+\frac{1}{4}=0\Rightarrow 12x^2-38x+3=0$ o.e.	M1A1 [8]
		Total 8 marks

Mark	Notes
B1	For the correct sum of $\frac{5}{3}$ This may not be explicitly seen – look for it in their working
B1	For the correct product of $\frac{1}{3}$ This may not be explicitly seen – look for it in their working
B1	For the correct value of $\alpha^2+\beta^2$ $\left[\frac{19}{9}\right]$ but does not need to be evaluated so accept $\left[\left(\frac{5}{3}\right)^2-2\times\frac{1}{3}\right]$ seen anywhere in their working. This can be embedded anywhere in their work This can also be implied from a correct value of the SUM $\frac{\alpha}{2\beta}+\frac{\beta}{2\alpha}=\frac{19}{6}$
B1	For the correct PRODUCT of $\frac{1}{4}$
M1	For the correct algebra $\left[\frac{2\alpha^2+2\beta^2}{4\alpha\beta}\right]$ **and** substitution of their values of $\alpha^2+\beta^2$ [even if the algebra on $\alpha^2+\beta^2$ is incorrect] to find the SUM. Ft their $\frac{19}{9}$ even if it comes from incorrect algebra on $\alpha^2+\beta^2$ earlier.
A1	For the value of $\frac{19}{6}$
M1	For a correct equation in any form. Need not = 0 for this mark
A1	For a correct equation with integer coefficients = 0

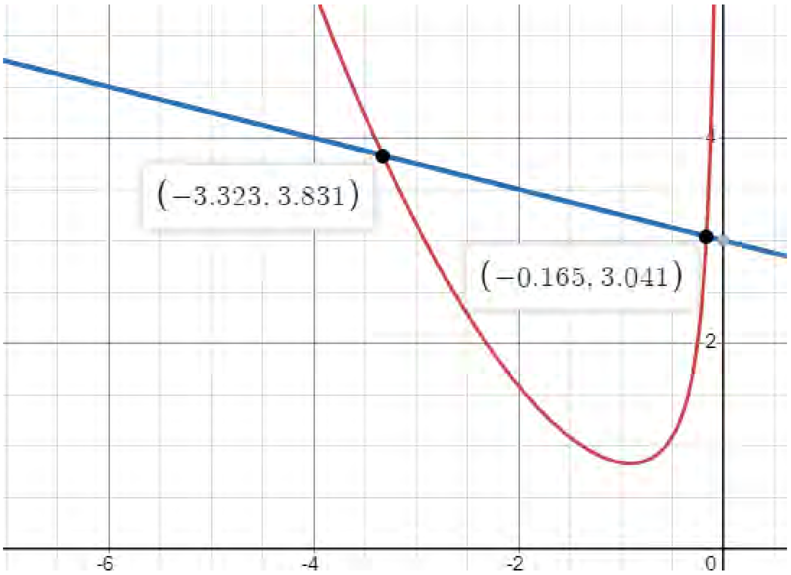
Question	Scheme	Marks
3(a)	$\sin\left(\frac{\theta}{2}\right)=\frac{r}{2r}\Rightarrow\frac{\theta}{2}=\frac{\pi}{6}$	M1
	$\Rightarrow\theta=\frac{\pi}{3}$	A1 [2]
	ALT works in degrees	
	$\sin\left(\frac{\theta}{2}\right)=\frac{r}{2r}\Rightarrow 30^{\circ}$	[M1
	$\Rightarrow\theta=\frac{\pi}{3}$	A1]
(b)	Area of circle = πr^2	B1
	Area of sector = $\frac{\left(\frac{\pi}{3}\right)}{2}\times(3r)^2=\frac{3}{2}\pi r^2$	M1
	Shaded area: $8\pi=\frac{3}{2}\pi r^2-\pi r^2=\frac{\pi r^2}{2}\Rightarrow r^2=16\Rightarrow r=4$ (cm)	dM1A1 [4]
	ALT works in degrees	
	Area of circle = πr^2	[B1
	Area of sector = $\frac{60^{\circ}}{360^{\circ}}\times\pi\times(3r)^2=\frac{3}{2}\pi r^2$	M1
	Shaded area: $8\pi=\frac{3}{2}\pi r^2-\pi r^2=\frac{\pi r^2}{2}\Rightarrow r^2=16\Rightarrow r=4$ (cm)	dM1A1]
		Total 6 marks

Part	Mark	Notes
(a)	M1	For the correct use of sine with r and $2r$ to obtain $\frac{\theta}{2}=\frac{\pi}{6}$ Allow working in degrees for the M mark only. i.e $\frac{\theta}{2}=30^{\circ}$ **Any responses not based on trigonometry are M0A0** If you are not sure, please send to Review
	A1	For $\theta=\frac{\pi}{3}$ **ONLY**
(b)	B1	For the correct area of the circle seen anywhere in part (b)
	M1	For using the correct formula for the area of a sector with their θ **provided** θ is within the specified range. $\left[0<\theta<\frac{\pi}{2},\quad 0<\theta<1.57..\right]$ Allow working in degrees for the sector provided it is correct. Do not allow use of the radian formula for a sector with degrees.
	dM1	For subtracting the area of the circle from the area of the sector and finding a value for r $r=\sqrt{\frac{8\pi}{\left(\frac{1}{2}\times(3)^{2}\times'\frac{\pi}{3}'-\pi\right)}}$ **This mark is dependent on the previous M mark**
	A1	For $r=4$

USEFUL SKETCH



Question	Scheme	Marks
4	$4x^3+3x^2-36x-6=0 \quad \to \div 12x$	
	$\Rightarrow \frac{x^2}{3}+\frac{x}{4}-3-\frac{1}{2x}=0 \Rightarrow \frac{x^2}{3}-\frac{1}{2x}=3-\frac{x}{4}$	M1A1
	ALT	
	$\frac{x^2}{3}-\frac{1}{2x}=ax+b \Rightarrow 2x^3-6ax-6bx-3=0$	
	$\Rightarrow 4x^3-12ax-12bx-6=4x^3+3x-36x-6$	[M1
	$\Rightarrow a=-\frac{1}{4} \quad b=3$	
	$\Rightarrow$ line required is $y=3-\frac{x}{4}$	A1]
	Draws the line with equation $y=3-\frac{x}{4}$	M1
	$x=-0.2 \quad x=-3.3$ or -3.4	A1 [4]
		Total 4 marks

Mark	Notes
Note - Correct values **without** any evidence of valid working or a line is M0A0M0A0 - Correct values **with** a correct line drawn without valid working is M0A0M0A0 - Correct values **with** a line drawn **and** the correct equation of a line without evidence of any valid working is SC M0A0M1A0	
M1	For dividing through by $12x$ [you may well see this in stages – e.g., first by 4, then by 3, then by x etc] and attempting to rearrange the equation to give as a minimum $y=k\pm\frac{x}{4}$ or $y=3\pm\frac{x}{m}$ $2\leqslant m\leqslant 6$ **OR** Sets the given equation of the curve $= ax + b$ and solves for a and b to find as a minimum to give as a minimum $y=k\pm\frac{x}{4}$ or $y=3\pm\frac{x}{m}$ $2\leqslant m\leqslant 6$
A1	For the correct straight line.
M1	Draws their line correctly on the grid. Look for (0, 3) and (–4, 4) provided it is of the form $y=k\pm\frac{x}{4}$ You MUST see the equation of the line WITH the drawn line. A correct line without an equation is M0 
A1	For both $x=-0.2$ and $x=-3.3$ **Answers must be given to 1 dp only** [2 or more dp is A0] **NB:** Calculator values are -0.165, -3.323

Question	Scheme	Marks
5	$\frac{\mathrm{d}y}{\mathrm{d}x}=2\mathrm{e}^{2x}\left(x^2-5x\right)+\mathrm{e}^{2x}\left(2x-5\right)\Rightarrow\left[\mathrm{e}^{2x}\left(2x-5\right)=\frac{\mathrm{d}y}{\mathrm{d}x}-2y\right]$	M1A1A1
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=4\mathrm{e}^{2x}\left(x^2-5x\right)+2\mathrm{e}^{2x}\left(2x-5\right)+2\mathrm{e}^{2x}\left(2x-5\right)+2\mathrm{e}^{2x}$	M1A1
	$\left[\text{OR }\ \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2\frac{\mathrm{d}y}{\mathrm{d}x}+2\mathrm{e}^{2x}\left(2x-5\right)+2\mathrm{e}^{2x}\right]$	
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2\frac{\mathrm{d}y}{\mathrm{d}x}+2\left[\frac{\mathrm{d}y}{\mathrm{d}x}-2y\right]+2\mathrm{e}^{2x}$	M1
	$\Rightarrow 2\mathrm{e}^{2x}=\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-4\frac{\mathrm{d}y}{\mathrm{d}x}+4y\quad *$	A1 cso [7]
	ALT 1 **LHS = RHS**	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=2\mathrm{e}^{2x}\left(x^2-5x\right)+\mathrm{e}^{2x}\left(2x-5\right)=2\mathrm{e}^{2x}x^2-10x\mathrm{e}^{2x}+2\mathrm{e}^{2x}x-5\mathrm{e}^{2x}$	
	$\left[=2\mathrm{e}^{2x}x^2-8\mathrm{e}^{2x}x-5\mathrm{e}^{2x}\right]$	[M1A1A1
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=4\mathrm{e}^{2x}x^2+4\mathrm{e}^{2x}x-8\mathrm{e}^{2x}-16\mathrm{e}^{2x}x-10\mathrm{e}^{2x}$	
	$\left[=4\mathrm{e}^{2x}x^2-12\mathrm{e}^{2x}x-18\mathrm{e}^{2x}\right]$	M1A1
	RHS $=4\mathrm{e}^{2x}x^2-12\mathrm{e}^{2x}x-18\mathrm{e}^{2x}-4\left(2\mathrm{e}^{2x}x^2-8\mathrm{e}^{2x}x-5\mathrm{e}^{2x}\right)+4\left(\mathrm{e}^{2x}x^2-5\mathrm{e}^{2x}\right)$	
	RHS $=4\mathrm{e}^{2x}x^2-12\mathrm{e}^{2x}x-18\mathrm{e}^{2x}-8\mathrm{e}^{2x}x^2+32\mathrm{e}^{2x}x+20\mathrm{e}^{2x}+4\mathrm{e}^{2x}x^2-20\mathrm{e}^{2x}x$	M1
	RHS $=2\mathrm{e}^{2x}=$ **LHS** $*$	A1]
	ALT 2 **LHS = RHS**	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=2\mathrm{e}^{2x}\left(x^2-5x\right)+\mathrm{e}^{2x}\left(2x-5\right)=\left[\mathrm{e}^{2x}\left(2x^2-8x-5\right)\right]$	[M1A1A1
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\mathrm{e}^{2x}\left(4x-8\right)+2\mathrm{e}^{2x}\left(2x^2-8x-5\right)=\left[\mathrm{e}^{2x}\left(4x^2-12x-18\right)\right]$	M1A1
	RHS $=\mathrm{e}^{2x}\left(4x^2-12x-18\right)-4\left(\mathrm{e}^{2x}\left(2x^2-8x-5\right)\right)+4\left(\mathrm{e}^{2x}\left(x^2-5x\right)\right)$	
	RHS $=4\mathrm{e}^{2x}x^2-12\mathrm{e}^{2x}x-18\mathrm{e}^{2x}-8\mathrm{e}^{2x}x^2+32\mathrm{e}^{2x}x+20\mathrm{e}^{2x}+4\mathrm{e}^{2x}x^2-20\mathrm{e}^{2x}x$	M1
	RHS $=2\mathrm{e}^{2x}=$ **LHS** $*$	A1]
		Total 7 marks

Mark	Notes
General principles of marking this question. - You must read every line of working carefully. - The first 3 marks are for the correct application of Product Rule - The next 2 marks are for the correct reapplication of Product rule. - The final M mark is effectively for rearranging/substituting to the required form. - The final mark is only to be awarded if everything is correct**. This is a show question.** **Some candidates may use implicit differentiation, this is absolutely fine.**	
M1	For an attempt at product rule. - There must be an acceptable attempt to differentiate both terms. - The correct formula must be applied. - Minimally acceptable differentiation is as follows: $e^{2x} \Rightarrow \pm ke^{2x}$ and $x^2 - 5x \Rightarrow \pm(2x-5)$
A1	For one term correct within product rule
A1	For a fully correct derivative.
M1	For differentiating their first derivative a second time to find the second derivative. There are several different ways this can be completed. Please apply the same rules for the second derivative as for the first. There must be only 4 terms here out of which, award this mark for at **least two** terms correct.
A1	For a fully correct $\frac{d^2y}{dx^2}$ in any form
M1	For substituting $e^{2x}(2x-5) = \frac{dy}{dx} - 2y$
A1cso	For the correct final expression with no errors.
ALT 1 and 2	
M1	For an attempt at product rule. - There must be an acceptable attempt to differentiate both terms. - The correct formula must be applied. - Minimally acceptable differentiation is as follows: $e^{2x} \Rightarrow \pm ke^{2x}$ and $x^2 - 5x \Rightarrow \pm(2x-5)$
A1	For one term correct within product rule
A1	For both terms fully correct.
M1	For differentiating their first derivative a second time to find the second derivative. There are several different ways this can be completed. Please apply the same rules for the second derivative as for the first. Look for two out of four terms correct If their starting point is $\frac{dy}{dx} = e^{2x}(2x^2 - 8x - 5)$ or $2e^{2x}x^2 - 8e^{2x}x - 5e^{2x}$ check their differentiation for the M mark and apply 1 out of 2, or 2 out of 4 correct..
A1	For a fully correct $\frac{d^2y}{dx^2}$ in any form
M1	Collects up all terms on the RHS to obtain $Ke^{2x} \quad K \neq 0$ **Please check their working carefully.**
A1cso	For the correct final expression with no errors.

Question	Scheme	Marks
6(a)	$2x^2 = \frac{1}{4x} \Rightarrow x^3 = \frac{1}{8} \Rightarrow x = \frac{1}{2}$	M1A1
	$y = 2\times\left(\frac{1}{2}\right)^2 = \frac{1}{2} \Rightarrow \left(\frac{1}{2}, \frac{1}{2}\right)$	B1 [3]
(b)	$y = 2x^2 \Rightarrow x^2 = \frac{y}{2}, \quad y = \frac{1}{4x} \Rightarrow x^2 = \frac{y^{-2}}{16}$ or $\left(\frac{1}{4y}\right)^2$ o.e.	B1,B1
	$V = \pi\int_{0.5}^{4}\left(\frac{y}{2}\right)\mathrm{d}y - \pi\int_{0.5}^{4}\left(\frac{y^{-2}}{16}\right)\mathrm{d}y$	M1
	$V = \pi\left[\frac{y^2}{4}\right]_{0.5}^{4} - \pi\left[\frac{y^{-1}}{-16}\right]_{0.5}^{4}$	M1A1
	$V = \frac{\pi}{4}\left(4^2 - \left(\frac{1}{2}\right)^2\right) + \frac{\pi}{16}\left(\frac{1}{4} - \frac{1}{0.5}\right) = \frac{245\pi}{64}$	M1A1 [7]
	[The decimal equivalent of the area is 12.0264….]	
		Total 10 marks

Part	Mark	Notes
(a)	M1	For setting the two equations together and attempting to find a value for *x* A minimally acceptable attempt is reaching at least. $x^3 = \frac{1}{8}$
	A1	For $x = \frac{1}{2}$
	B1	For the correct *y* coordinate $y = \frac{1}{2}$
(b)	B1	For rearranging the equation for *S* to $x^2 = \frac{y}{2}$ seen explicitly or embedded
	B1	For rearranging the equation for *C* to $x^2 = \frac{y^{-2}}{16}$ or $\frac{1}{16y^2}$ or $\left(\frac{1}{4y}\right)^2$ Seen explicitly or embedded
	M1	For a correct expression for the volume with the correct limits [ft their *y* coord of $\frac{1}{2}$] and π Accept the **[correct only]** expressions either way around. You may see π added in at the end. That is fine and please award this mark if that is the case. Ignore poor notation as long as the intention is clear. For example, ignore missing d*y*
	M1	For an attempt to integrate at least one of their two only expressions. Ignore limits and π for this mark. See General Guidance.
	A1	For a fully correct integrated expression for the volume.

		Ignore limits and π
	M1	For substituting their limits the correct way around into their integrated expression. You must see this if their integrated expression is incorrect, or their limits are incorrect. If the integration and limits are all correct, the correct volume seen (either in exact or in decimal form i.e. 12.0….) scores this mark. Accept partly processed, for example $\frac{\pi}{4}\left(16-\frac{1}{4}\right)+\frac{\pi}{16}\left(\frac{1}{4}-\frac{1}{0.5}\right)$ as long as you can see **four** calculations/terms as above. Ignore π for this mark.
	A1	For $\frac{245\pi}{64}$
	SC rotates around the *x* axis. Maximum score is B0B0M1M1A0M1A0	
	B0B0	Not available
	M1	For a correct expression for the volume with the correct calculated limits for x $\left(\sqrt{2} \text{ and } \frac{1}{16}\right)$ and π Accept the [**correct only]** expressions either way around. You may see π added in at the end. That is fine and please award this mark if that is the case. $V=\pi\int_{\frac{1}{16}}^{\sqrt{2}}\left(\frac{x^{-1}}{4}\right)^2 \mathrm{d}x-\pi\int_{\frac{1}{16}}^{\sqrt{2}}\left(2x^2\right)^2 \mathrm{d}x$
	M1	For an attempt to integrate one of their two only expressions. $V=\pi\left[-\frac{x^{-1}}{16}-\frac{4x^5}{5}\right]_{\frac{1}{16}}^{\sqrt{2}}$ Ignore π and limits for this mark
	A0	Not available
	M1	For substituting their values into the integrated expression correctly the correct way around. $V=\pi\left[\left(-\frac{\left(\sqrt{2}\right)^{-1}}{16}-\frac{4\left(\sqrt{2}\right)^5}{5}\right)-\left(-\frac{\left(\frac{1}{16}\right)^{-1}}{16}-\frac{4\left(\frac{1}{16}\right)^5}{5}\right)\right]$
	A0	Not available

Question	Scheme	Marks
7(a)	$(1+2x^2)^{-\frac{3}{4}}=1+\left(-\frac{3}{4}\right)(2x^2)+\frac{\left(-\frac{3}{4}\right)\left(-\frac{7}{4}\right)(2x^2)^2}{2!}+\frac{\left(-\frac{3}{4}\right)\left(-\frac{7}{4}\right)\left(-\frac{11}{4}\right)(2x^2)^3}{3!}$	M1
	$=1-\frac{3}{2}x^2+\frac{21}{8}x^4-\frac{77}{16}x^6$	A1A1 [3]
(b)	$\mathrm{f}(x)=(2+kx)\left(1-\frac{3}{2}x^2+\frac{21}{8}x^4-\frac{77}{16}x^6\right)$	
	$=2+kx-3x^2-\frac{3k}{2}x^3+\frac{21}{4}x^4+\frac{21k}{8}x^5$	M1A1 [2]
(c)	$14\times(-3)=\frac{21k}{8}\Rightarrow k=\frac{14\times(-3)\times 8}{21}=-16$	M1A1 [2]
		Total 7 marks

Part	Mark	Notes
(a)	M1	For an attempt at the binomial expansion. • The expansion must begin with 1 and there must be an attempt at terms up to $(2x^2)^3$ • The denominators must be correct. • The powers of $(2x^2)$ must be correct.
	A1	For at least one term in *x* correct and simplified, subject to the conditions for the M mark!
	A1	For a fully correct and simplified expansion in the correct order of ascending powers.
(b)	M1	For multiplying their expansion by $(2+kx)$ provided their expansion has at least 2 terms in *x* with differing powers.
	A1	For a fully correct expansion.
(c)	M1	For setting $14\times(\text{their coeff of }x^2)=(\text{their coeff of }x^5)$ and attempting to solve to find a value for *k*
	A1	For the correct value of *k*

Question	Scheme	Marks

8(a)	Finds the length *DE* $x^2=\left(DE\right)^2+\left(DE\right)^2\Rightarrow DE=\frac{x}{\sqrt{2}}$ or $DE^2=\frac{x^2}{2}$	B1
	Volume of prism $3.6=\left(\frac{x}{\sqrt{2}}\right)^2\times\frac{1}{2}\times y\Rightarrow\left[y=\frac{72}{5x^2}\right]$	M1
	Surface area of prism $S=2\times\frac{x^2}{4}+2\times\frac{x}{\sqrt{2}}\times\frac{72}{5x^2}+x\times\frac{72}{5x^2}=\left[\frac{x^2}{2}+\sqrt{2}\times\frac{72}{5x}+\frac{72}{5x}\right]$	M1
	$\Rightarrow S=\frac{x^2}{2}+\frac{72\left(\sqrt{2}+1\right)}{5x}$ *	A1 cso [4]
(b)	$\frac{\mathrm{d}S}{\mathrm{d}x}=x-\frac{72\left(\sqrt{2}+1\right)}{5x^2}=0$	M1
	$\Rightarrow\frac{72\left(\sqrt{2}+1\right)}{5x^2}=x$	M1
	$\Rightarrow x=\sqrt[3]{\frac{72\left(\sqrt{2}+1\right)}{5}}=3.26371...\approx 3.26\text{ (cm)}$	A1
	$\frac{\mathrm{d}^2S}{\mathrm{d}r^2}=1+\frac{144\left(\sqrt{2}+1\right)}{5x^3}$ $\Rightarrow$ positive + positive = positive $\Rightarrow$ is always greater than 0, hence minimum	B1ft [4]
(c)	$S_{(\text{min})}=\frac{3.26371^2}{2}+\frac{72\left(\sqrt{2}+1\right)}{5\times 3.26371}=15.97779...\approx 16\ \left(\text{cm}^2\right)$	M1A1 [2]
		Total 10 marks

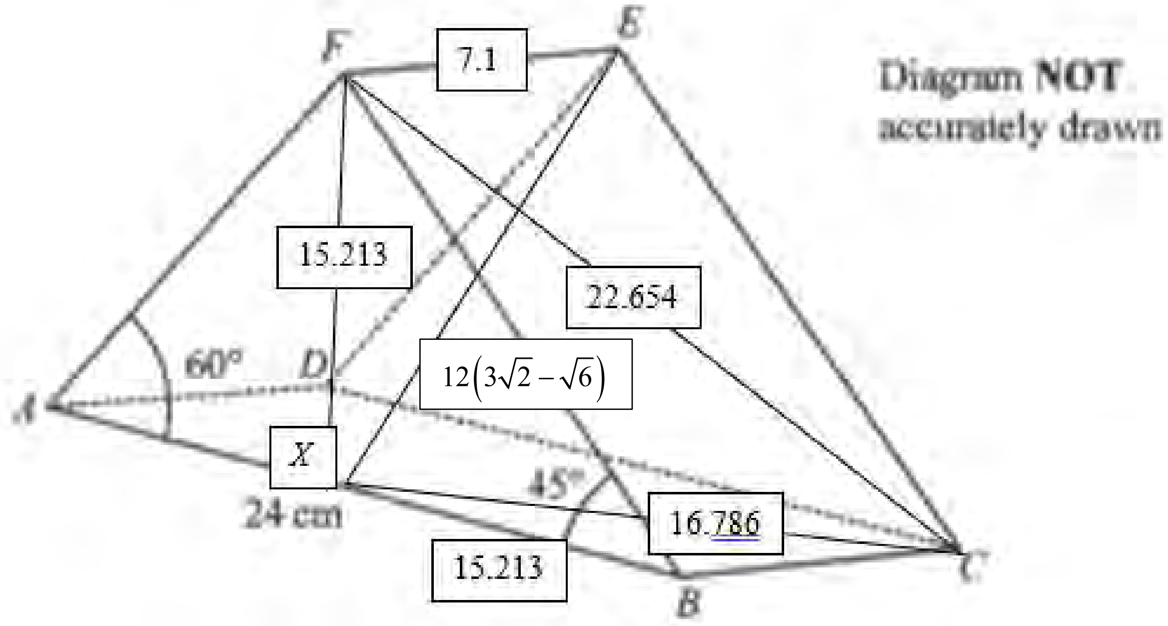
Part	**Mark**	**Notes**

(a)	B1	For using Pythagoras theorem or any appropriate trigonometry to find the length *DE* (or any equivalent side)
	M1	For using the correct formula to find the volume of a prism and rearranging to find an expression for *y* minimally of the form $y=\dfrac{A}{Bx^2}$ This need not be simplified.
	M1	For forming an expression for *S* using their *y* The basic expression for the surface area must be correct, with correct substitution of their values. $S=2\times{'}DE{'}+2\times{'}DE{'}\times y+xy\Rightarrow S=2\times{'}\dfrac{x^2}{4}{'}+2\times{'}\dfrac{x}{\sqrt{2}}{'}\times\left({'}\dfrac{72}{5x^2}{'}\right)+x\times\left({'}\dfrac{72}{5x^2}{'}\right)$
	A1 cso	For $S=\dfrac{x^2}{2}+\dfrac{72\left(\sqrt{2}+1\right)}{5x}$ as written
(b)	M1	For differentiating the **given** expression for *S* Minimally acceptable differentiated expression is as follows: $\dfrac{dS}{dx}=x\pm Kx^{-2}$ oe. where *K* is a constant
	M1	For setting their $\dfrac{dS}{dx}=0$ and attempting to solve to obtain a value for *x*
	A1	For the correct value of *x* Accept awrt 3.26 (cm)
	B1ft	For differentiating their $\dfrac{dS}{dr}$ to minimally obtain $\dfrac{d^2S}{dr^2}=1\pm Lx^{-3}$ and coming to an appropriate conclusion. The justification has to be correct and complete. If they refer/use a substitution it must be seen and it must be correct.. That is, $\dfrac{d^2S}{dr^2}=1+\dfrac{144\left(\sqrt{2}+1\right)}{5\times3.26^3}=3.00...$ $3>0$ so positive, hence minimum Alternatively: *x* is positive so; $\text{positive}+\dfrac{\text{positive}}{\text{positive}}\Rightarrow\text{positive}$ hence minimum
(c)	M1	For substituting their value of *x* (provided it is positive) and attempting to find a value of *S*
	A1	For awrt *S* = 16

Question	Scheme	Marks
9(a)	$\sin 75^\circ = \sin\left(45^\circ + 30^\circ\right) = \sin 45^\circ \cos 30^\circ + \sin 30^\circ \cos 45^\circ$	M1
	$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{\sqrt{2}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}\ *$	M1A1 cso [3]
	ALT 1	
	$\sin 75^\circ = \sin\left(120^\circ - 45^\circ\right) = \sin 120^\circ \cos 45^\circ - \sin 45^\circ \cos 120^\circ$	[M1
	$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \times \left(-\frac{1}{2}\right) = \frac{\sqrt{6}+\sqrt{2}}{4}\ *$	M1A1]
	ALT 2	
	$\sin 75^\circ = \sin\left(60^\circ + 15^\circ\right) = \sin 60^\circ \cos 15^\circ + \sin 15^\circ \cos 60^\circ$	[M1
	$= \frac{\sqrt{3}}{2} \times \left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) + \left(\frac{\sqrt{6}-\sqrt{2}}{4}\right) \times \frac{1}{2}$	M1
	$= \frac{3\sqrt{2}+\sqrt{6}}{8} + \frac{\sqrt{6}-\sqrt{2}}{8} = \frac{\sqrt{6}+\sqrt{2}}{4}$	A1]
(b)	$\left[\angle AFC = 180^\circ - 60^\circ - 45^\circ = 75^\circ\right]$	
	$\frac{24}{\sin 75^\circ} = \frac{BF}{\sin 60^\circ}$	
	$\Rightarrow \frac{24}{\sin 75^\circ} = \frac{BF}{\sin 60^\circ} \Rightarrow \frac{24}{\frac{\sqrt{6}+\sqrt{2}}{4}} = \frac{BF}{\frac{\sqrt{3}}{2}}, \Rightarrow BF = \frac{48\sqrt{3}}{\sqrt{6}+\sqrt{2}}$	M1,M1A1
	$BF = \frac{48\sqrt{3}}{\left(\sqrt{6}+\sqrt{2}\right)} \times \frac{\left(\sqrt{6}-\sqrt{2}\right)}{\left(\sqrt{6}-\sqrt{2}\right)} = \frac{48\sqrt{3}\sqrt{6} - 48\sqrt{6}}{6-2} = 12\left(3\sqrt{2}-\sqrt{6}\right)*$	M1A1 cso [5]
(c)	Let the vertical point from F to the line AB be X and the corresponding point on CD be Y. The required angle is EXY	
	$FX = 12\left(3\sqrt{2}-\sqrt{6}\right)\sin 45^\circ = 36 - 12\sqrt{3} = (15.2153...)$	M1
	OR Area $ABC = \frac{1}{2} \times 24 \times FX = \frac{1}{2} \times 24 \times 12\left(3\sqrt{2}-\sqrt{6}\right)\sin 45$ $\Rightarrow FX = 12\left(3\sqrt{2}-\sqrt{6}\right)\sin 45^\circ = 36 - 12\sqrt{3} = (15.2153...)$	M1A1 [3]
	$EF = \frac{36-12\sqrt{3}}{\tan 65^\circ} = 7.09505.... \approx 7.1$ (cm)	
(d)	Identifies the required angle FCX	B1
	$FC = \sqrt{\left[12\left(3\sqrt{2}-\sqrt{6}\right)\right]^2 + '7.09505'^2} = 22.65735.....$	M1
	$\angle FCX = \sin^{-1}\left(\frac{15.2153}{'22.6573'}\right) = 42.186...^\circ \approx 42.2^\circ$	M1A1 [4]
		Total 15 marks

Part	Mark	Notes
(a)	M1	For the correct use of the addition formula for sine with $\sin\left(45^\circ+30^\circ\right)$ Accept also $\sin\left(120^\circ-45^\circ\right)$ and $\sin\left(60^\circ+15^\circ\right)$ **but not** $\sin\left(90^\circ-15^\circ\right)$ or $\sin\left(180^\circ-105^\circ\right)$
	M1	For applying the exact values of these sines and cosines and for evaluating the expression.. This step must be explicitly seen.
	A1 cso	For the correct final answer with no errors seen
(b)	M1	For the correct use of sine rule using the exact value for sin 75° and sin 60°
	M1	For rearranging to obtain $BF=\frac{k\sqrt{3}}{\sqrt{6}+\sqrt{2}}$ or $\frac{l\sqrt{3}}{2\sqrt{6}+2\sqrt{2}}$ Where k and l are integers
	A1	For the correct **exact** BF which need not be simplified. $BF=\frac{48\sqrt{3}}{\sqrt{6}+\sqrt{2}}$ or $\frac{96\sqrt{3}}{2\sqrt{6}+2\sqrt{2}}$ or even $\frac{12\sqrt{3}\times 4}{\sqrt{6}+\sqrt{2}}$ but not for example, $\dfrac{12\sqrt{3}}{\frac{\sqrt{6}+\sqrt{2}}{4}}$ unless recovery is seen later
	M1	For multiplying numerator **and** denominator by their conjugate which must of the form e.g $\sqrt{A}\pm\sqrt{B}$ **AND** simplifying to the required form. We need to see at least one intermediate line of working before the final answer here.
	A1 cso	The answer as shown with no errors or omissions seen. This is a given result.
(c)	M1	For finding the length FX using any appropriate trigonometry.
	M1	For using any appropriate trigonometry to find length EF [or XY]
	A1	For awrt 7.1 (cm)
(d)	B1	For identifying the correct angle required.
	M1	For using Pythagoras theorem to find FC $FC=\sqrt{21.51781^2+7.09505^2}=22.65735.....$ Or For using Pythagoras theorem to find CX $FC=\sqrt{15.213^2+7.095^2}=16.786...$ [Note: $BX=15.213...$ and CX = 16.786…]
	M1	For using any appropriate trigonometry to find angle FCX. For example; $\angle FCX=\tan^{-1}\left(\frac{15.2153}{16.786}\right)=42.186...^\circ\approx 42.2^\circ$
	A1	For awrt 42.2°

USEFUL SKETCH



NB:

$AC = 25.03$ cm

$AF = 17.57$ cm

$CF = 22.65$ cm

Angle $FCA = 42.9°$

Question	Scheme	Marks
10(a)	$\overrightarrow{BC}=-\overrightarrow{AB}+\overrightarrow{AC}=-(3\mathbf{a}+4\mathbf{b})+7\mathbf{a}+9\mathbf{b}=4\mathbf{a}+5\mathbf{b}$	B1
	$\overrightarrow{DC}=-\overrightarrow{DA}+\overrightarrow{DC}=-(4\mathbf{a}+5\mathbf{b})+7\mathbf{a}+9\mathbf{b}=3\mathbf{a}+4\mathbf{b}$	B1
	$AB\parallel DC$ and $BC\parallel AD$ hence the quadrilateral is a parallelogram	B1ft [3]
(b)	$\overrightarrow{CF}=-\frac{1}{3}(3\mathbf{a}+4\mathbf{b})$	B1
	$\overrightarrow{AF}=\overrightarrow{AC}+\overrightarrow{CF}=7\mathbf{a}+9\mathbf{b}--\frac{1}{3}(3\mathbf{a}+4\mathbf{b})=\left[6\mathbf{a}+\frac{23}{3}\mathbf{b}\right]$	B1
	$\overrightarrow{AE}=\mu\,\overrightarrow{AF}=\mu\left(6\mathbf{a}+\frac{23}{3}\mathbf{b}\right)$	M1
	$\overrightarrow{AE}=\overrightarrow{AB}+\overrightarrow{BE}=3\mathbf{a}+4\mathbf{b}+\lambda(4\mathbf{a}+5\mathbf{b})=\left[(3+4\lambda)\mathbf{a}+(4+5\lambda)\mathbf{b}\right]$	M1
	$\mu\left(6\mathbf{a}+\frac{23}{3}\mathbf{b}\right)=(3+4\lambda)\mathbf{a}+(4+5\lambda)\mathbf{b}$ $\Rightarrow 6\mu=3+4\lambda \quad \frac{23}{3}\mu=4+5\lambda$	ddM1
	$\Rightarrow \lambda=\frac{3}{2} \quad \left(\mu=\frac{3}{2}\right)$	M1A1
	$\overrightarrow{AE}=\frac{3}{2}\left(6\mathbf{a}+\frac{23}{3}\mathbf{b}\right)=9\mathbf{a}+\frac{23}{2}\mathbf{b}$	B1ft [8]
		Total 11 marks

Part	Mark	Notes
(a)	B1	For the correct vector for $\overrightarrow{BC}$ or $\overrightarrow{CB}$
	B1	For the correct vector for $\overrightarrow{DC}$ or $\overrightarrow{CD}$
	B1ft	For the correct complete conclusion including stating that the quadrilateral is a parallelogram. Ft their vectors for reverse directions.
	ALT	
	B1	For the correct lengths of $\overrightarrow{AD}$ AND $\overrightarrow{BC}$ $\quad \lvert AD\rvert = \lvert BC\rvert = \sqrt{41}$
	B1	For the correct lengths of $\overrightarrow{AB}$ AND $\overrightarrow{DC}$ $\quad \lvert AB\rvert = \lvert DC\rvert = 5$
	B1ft	Opposite sides are of equal length so the quadrilateral is a parallelogram.
(b)	B1	For the **correct vector** for $\overrightarrow{CF}$ or $\overrightarrow{DF}$ or **correct** reverse directions $\left[\overrightarrow{CF} = -\frac{1}{3}(3\mathbf{a}+4\mathbf{b})\right]$ $\left[\overrightarrow{DF} = \frac{2}{3}(3\mathbf{a}+4\mathbf{b})\right]$ this may be embedded in $\overrightarrow{AF}$
	B1	For the **correct vector** for $\overrightarrow{AF}$ using their $\overrightarrow{CF}$ or $\overrightarrow{DF}$ e.g., $\overrightarrow{AF} = \overrightarrow{AC}+\overrightarrow{CF} = 7\mathbf{a}+9\mathbf{b}-\frac{1}{3}(3\mathbf{a}+4\mathbf{b}) = \left[6\mathbf{a}+\frac{23}{3}\mathbf{b}\right]$
	M1	For **one** correct vector for $\overrightarrow{AE}$ using **their** $\overrightarrow{AF}$ and a parameter For example: $\overrightarrow{AE} = \mu\,\overrightarrow{AF} = \mu\left(6\mathbf{a}+\frac{23}{3}\mathbf{b}\right)$
	M1	For a **second** vector for $\overrightarrow{AE}$ using **their** vectors and a second unique parameter For example; $\overrightarrow{AE} = \overrightarrow{AB}+\overrightarrow{BE} = 3\mathbf{a}+4\mathbf{b}+\lambda(4\mathbf{a}+5\mathbf{b}) = [(3+4\lambda)\mathbf{a}+(4+5\lambda)\mathbf{b}]$
	ddM1	For setting their two vectors for $\overrightarrow{BF}$ equal and setting up two equations involving two different parameters. $\mu\left(6\mathbf{a}+\frac{23}{3}\mathbf{b}\right) = (3+4\lambda)\mathbf{a}+(4+5\lambda)\mathbf{b}$ $\Rightarrow 6\mu = 3+4\lambda \quad \frac{23}{3}\mu = 4+5\lambda$ **This mark is dependent on the previous two M marks**
	dddM1	For solving their two simultaneous equations by any method to find a value for λ **This mark is dependent on the previous three M marks**
	A1	For $\mu = \frac{3}{2}$ or $\lambda = \frac{3}{2}$
	B1ft	For writing down a vector for $\overrightarrow{AE}$ using their $\overrightarrow{AF}$ and their λ in the required form, provided $1 < \lambda < 2$

	ALT	
	B1	For the correct vector for $\overrightarrow{CF}$ or $\overrightarrow{DF}$ $\left[\overrightarrow{CF} = -\frac{1}{3}(3\mathbf{a}+4\mathbf{b})\right]$ $\left[\overrightarrow{DF} = \frac{2}{3}(3\mathbf{a}+4\mathbf{b})\right]$
	B1	For the correct vector for $\overrightarrow{AF}$ using their $\overrightarrow{CF}$ or $\overrightarrow{DF}$ e.g., $\overrightarrow{AF} = \overrightarrow{AC} + \overrightarrow{CF} = 7\mathbf{a} + 9\mathbf{b} - \frac{1}{3}(3\mathbf{a}+4\mathbf{b}) = \left[6\mathbf{a} + \frac{23}{3}\mathbf{b}\right]$
	M1	For **one** correct vector for $\overrightarrow{AE}$ using **their** $\overrightarrow{AF}$ and a parameter For example: $\overrightarrow{AE} = \mu\,\overrightarrow{AF} = \mu\left(6\mathbf{a} + \frac{23}{3}\mathbf{b}\right)$
	M1	For a correct vector for $\overrightarrow{BE}$ using their $\overrightarrow{AE}$ $\overrightarrow{BE} = -3\mathbf{a} - 4\mathbf{b} + \mu\left(6\mathbf{a} + \frac{23}{3}\mathbf{b}\right) = \left[(6\mu - 3)\mathbf{a} + \left(\frac{23}{3}\mu - 4\right)\mathbf{b}\right]$
	ddM1	For forming the correct ratio using the vector $\overrightarrow{BC}$ $\frac{(6\mu - 3)}{\left(\frac{23}{3}\mu - 4\right)} = \frac{4}{5}$
	dddM1	For solving the equation to find a value for μ
	A1	For $\mu = \frac{3}{2}$
	B1ft	For writing down a vector for $\overrightarrow{AE}$ using their $\overrightarrow{AF}$ and their λ in the required form For writing down a vector for $\overrightarrow{AE}$ using their $\overrightarrow{AF}$ and their λ in the required form, provided $1 < \lambda < 2$

Question	Scheme	Marks
11(a)(i)	$\cos 2A = \cos^2 A - \sin^2 A, = \cos^2 A - \left(1 - \cos^2 A\right)$	M1,M1
	$\cos 2A = 2\cos^2 A - 1$ *	A1 cso [3]
(ii)	$\sin 2A = \sin A\cos A + \sin A\cos A = 2\sin A\cos A$ *	B1 cso [1]
(b)	$\cos 3A = \cos\left(2A + A\right)$	
	$= \cos 2A\cos A - \sin A\sin 2A$	M1
	$= \left(2\cos^2 A - 1\right)\cos A - 2\sin A\sin A\cos A$	
	$= 2\cos^3 A - \cos A - 2\left(1 - \cos^2 A\right)\cos A$	M1
	$= 2\cos^3 A - \cos A + 2\cos^3 A - 2\cos A$	
	$= 4\cos^3 A - 3\cos A$	M1
	$\Rightarrow \cos^3 A = \dfrac{\cos 3A + 3\cos A}{4}$ *	A1 Cso [4]
	ALT	
	$\cos^3 A = \dfrac{\cos 3A + 3\cos A}{4}$	
	$= \dfrac{\cos 2A\cos A - \sin 2A\sin A + 3\cos A}{4}$	[M1
	$= \dfrac{\left(2\cos^2 A - 1\right)\cos A - 2\sin A\sin A\cos A + 3\cos A}{4}$	
	$= \dfrac{2\cos^3 A - \cos A - 2\left(1 - \cos^2 A\right)\cos A + 3\cos A}{4}$	M1
	$= \dfrac{2\cos^3 A - \cos A + 2\cos^3 A - 2\cos A + 3\cos A}{4}$	M1
	$= \dfrac{4\cos^3 A}{4} = \cos^3 A \quad \left[LHS = RHS\right]$	A1]
(c)	$\left[\cos 3A = 4\cos^3 A - 3\cos A \Rightarrow 2\cos 3A = 8\cos^3 A - 6\cos A\right]$	
	$8\cos^3\left(\dfrac{\theta}{2}\right) - 6\cos\left(\dfrac{\theta}{2}\right) - 1 = 0 \Rightarrow 2\cos 3\left(\dfrac{\theta}{2}\right) - 1 = 0$	M1
	$\cos 3\left(\dfrac{\theta}{2}\right) = \dfrac{1}{2} \Rightarrow 3\left(\dfrac{\theta}{2}\right) = \dfrac{\pi}{3}, \dfrac{5\pi}{3}, \dfrac{7\pi}{3}, \left(\dfrac{11\pi}{3}\right)$	M1
	$\theta = \dfrac{2\pi}{9}, \dfrac{10\pi}{9}, \dfrac{14\pi}{9}$	M1A1 [4]

(d)	$I=\int_0^{\frac{\pi}{6}}\left(4\cos^3\theta-\sin 2\theta\right)\mathrm{d}\theta=\int_0^{\frac{\pi}{6}}\left(\cos 3\theta+3\cos\theta-\sin 2\theta\right)\mathrm{d}\theta$	M1
	$I=\left[\frac{\sin 3\theta}{3}+3\sin\theta+\frac{\cos 2\theta}{2}\right]_0^{\frac{\pi}{6}}$	M1
	$=\left[\frac{\sin 3\left(\frac{\pi}{6}\right)}{3}+3\sin\left(\frac{\pi}{6}\right)+\frac{\cos 2\left(\frac{\pi}{6}\right)}{2}\right]-\left[\frac{\sin 3(0)}{3}+3\sin(0)+\frac{\cos 2(0)}{2}\right]$	M1
	$=\left[\frac{1}{3}+\frac{3}{2}+\frac{1}{4}\right]-\left[0-0+\frac{1}{2}\right]=\frac{19}{12}$	A1 [4]
		Total 16 marks

Part	**Mark**	**Notes**
(a)(i)	M1	For the correct use of the addition formula for cos 2*A*
	M1	For using the Pythagorean identity correctly
	A1 cso	For the correct identity with no errors.
(a)(ii)	B1 cso	$\left[\sin(A+A)\right]=\sin 2A=\sin A\cos A+\cos A\sin A=2\sin A\cos A$
(b)	**Method 1 – Starting with cos 3*A***	
	M1	For using the correct addition formula for cos 3*A* to obtain the correct expansion.
	M1	For applying at least **TWO** of the following seen anywhere in this part of the question: • cos 2*A* identity from (a) correctly • the sin 2*A* identity from (a) applied correctly • the identity $\sin^2 A=1-\cos^2 A$ applied correctly
	M1	For simplifying the expression to be in terms of $\cos^3 A, \cos 3A$ and $\cos A$ only
	A1 cso	For the correct identity with no errors
	ALT – Method 2 – starting with the RHS to show = LHS	
	M1	For using the correct addition formula for cos 3*A* to obtain the correct expansion. You can ignore everything else for this and the next mark.
	M1	For applying at least **TWO** of the following seen anywhere in this part of the question: [again, ignore anything other than the cos 3*A* here] • cos 2*A* identity from (a) correctly • the sin 2*A* identity form (a) applied correctly the identity $\sin^2 A=1-\cos^2 A$ applied correctly
	M1	Collects up like terms and simplifies to obtain $\frac{4\cos^3 A}{4}$
	A1	For showing LHS = RHS with no errors

(c)	M1	Substitutes $2\cos 3\left(\frac{\theta}{2}\right)$ in place of $8\cos^3\left(\frac{\theta}{2}\right)-6\cos\left(\frac{\theta}{2}\right)$ correctly to obtain at least $k\cos 3\left(\frac{\theta}{2}\right)-1=0$ where *k* is a constant
	M1	Rearranges to obtain $\cos 3\left(\frac{\theta}{2}\right)=\frac{1}{k}$ and takes the inverse cosine to find at least any one **correct** value for $3\left(\frac{\theta}{2}\right)$
	M1	Finds at least one correct value for θ
	A1	Finds all three values with no extra values within range. Ignore any values given outside of the range.
(d)	M1	For a **correct** substitution as shown. $4\cos^3\theta=\cos 3\theta+3\cos\theta$
	M1	For an acceptable attempt to integrate either $\cos 3\theta \to \pm\frac{\sin 3\theta}{3}$ or $-\sin 2\theta \to \pm\frac{\cos 2\theta}{2}$
	M1	For substituting both given values the correct way around. This mark can be awarded for a complete substitution SEEN into any changed expression. Award this mark for **correct** integration **and** a value of $\frac{19}{12}$ seen without explicit substitution.
	A1	For $\frac{19}{12}$

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