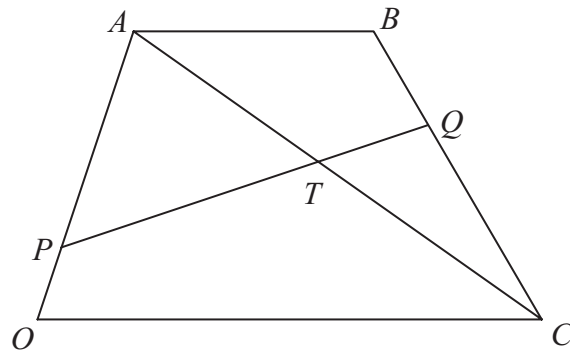**10**

Figure 2

Figure 2 shows a trapezium $OABC$ in which AB is parallel to OC and $AB = \dfrac{1}{2}\,OC$. The point P divides OA in the ratio 1:3 and the point Q divides BC in the ratio 1:2

The line AC intersects the line PQ at the point T.

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$

(a) Find, as simplified expressions in terms of $\mathbf{a}$ and $\mathbf{c}$

   (i) $\overrightarrow{BC}$

   (ii) $\overrightarrow{PQ}$

(5)

(b) (i)   Given that $\overrightarrow{PT} = \lambda\overrightarrow{PQ}$, find an expression for $\overrightarrow{AT}$ in terms of λ, $\mathbf{a}$ and $\mathbf{c}$

   (ii)   Given also that $\overrightarrow{AT} = \mu\overrightarrow{AC}$, find an expression for $\overrightarrow{AT}$ in terms of μ, $\mathbf{a}$ and $\mathbf{c}$

(2)

(c) Use your answers from part (b) to find the value of λ and hence write down the ratio
$PT : TQ$

(6)

Question 10 continued

PMT

Question 10 continued

P 4 0 6 1 6 A 0 2 6 3 2

Question 10 continued
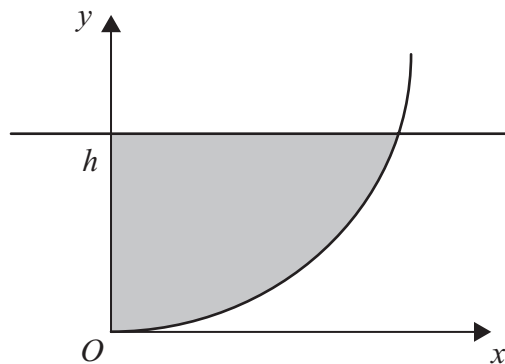
(Total for Question 10 is 13 marks)

11

Figure 3

The centre of the circle C, with equation $x^2 + y^2 - 10y = 0$, has coordinates $(0, 5)$. The circle passes through the origin O. The region bounded by the circle, the positive y-axis and the line $y = h$, where $h < 5$, is shown shaded in Figure 3. The shaded region is rotated through 2π radians about the y-axis.

(a) Show that the volume of the solid formed is $\dfrac{1}{3}\pi h^2(15 - h)$.
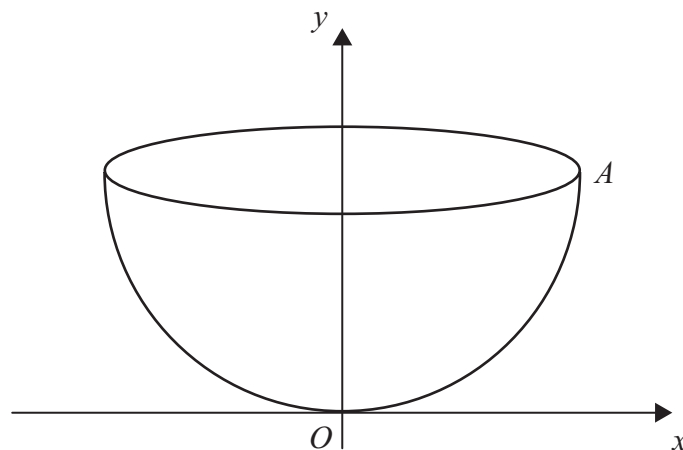
(5)

Figure 4

The point A with coordinates $(5, 5)$ lies on C. A bowl is formed by rotating the arc OA through 2π radians about the y-axis, as shown in Figure 4. Water is poured into the bowl at a constant rate of $6 \text{ cm}^3/\text{s}$. The volume of water in the bowl is $V \text{ cm}^3$ when the depth of water above O is h cm.

(b) Use the formula given in part (a) to find an expression for $\dfrac{\mathrm{d}V}{\mathrm{d}h}$ in terms of h.

(1)

(c) Find, to 3 significant figures, the rate at which h is changing when the water above O is 1.5 cm deep.

(4)

The area of the surface of the water is $W \text{ cm}^2$ when the depth of water above O is h cm.

(d) Show that, for $0 < h < 5$, the rate of change of the depth of water above O is $\dfrac{k}{W}$, stating the value of k.

(3)

Question 11 continued

Question 11 continued

Question 11 continued

Question 11 continued

(Total for Question 11 is 13 marks)

TOTAL FOR PAPER IS 100 MARKS