

Question Number	Scheme	Marks
8(a)	<p>(i) $\frac{2}{3}\mathbf{b} - \mathbf{a}$</p> <p>(ii) $\overrightarrow{OE} = \overrightarrow{OA} + \frac{2}{5}\overrightarrow{AD} = \mathbf{a} + \frac{2}{5}\left(\frac{2}{3}\mathbf{b} - \mathbf{a}\right) = \frac{3}{5}\mathbf{a} + \frac{4}{15}\mathbf{b}$</p> <p>(iii) $\overrightarrow{BE} = \overrightarrow{OE} - \overrightarrow{OB} = \frac{3}{5}\mathbf{a} + \frac{4}{15}\mathbf{b} - \mathbf{b} = \frac{3}{5}\mathbf{a} - \frac{11}{15}\mathbf{b}$</p>	<p>B1</p> <p>M1A1</p> <p>M1A1 (5)</p>
(b)	<p>$\overrightarrow{FE} = \overrightarrow{OE} - \overrightarrow{OF} = \frac{3}{5}\mathbf{a} + \frac{4}{15}\mathbf{b} - \lambda\mathbf{a}$</p> <p>$F, E, B$ collinear $\frac{\frac{3}{5} - \lambda}{\frac{4}{15}} = \frac{\frac{3}{5}}{-\frac{11}{15}}$</p> <p>$\frac{3 - 5\lambda}{4} = \frac{3}{-11}$</p> <p>$\lambda = \frac{9}{11}$</p>	<p>M1A1</p> <p>M1A1</p> <p>A1 (5)</p>
	<p>ALT</p> <p>$\overrightarrow{OF} + \overrightarrow{FB} = \overrightarrow{OB}$</p> <p>$\lambda\mathbf{a} + \mu\left(-\frac{3}{5}\mathbf{a} + \frac{11}{15}\mathbf{b}\right) = \mathbf{b}$</p> <p>$\mu = \frac{15}{11} \quad \lambda = \frac{3}{5}\mu$</p> <p>$\lambda = \frac{9}{11}$</p>	<p>M1</p> <p>A1</p> <p>M1A1</p> <p>A1 (5)</p>
(c)	<p>$\Delta OFB = 5 \text{ units}^2 \Rightarrow \Delta OAB = \frac{11}{9} \times 5 \text{ units}^2$</p> <p>$\Delta OAD = \frac{2}{3} \Delta OAB = \frac{2}{3} \times \frac{55}{9} = \frac{110}{27} \text{ units}^2$</p>	<p>M1</p> <p>M1A1</p>
	<p>ALT</p> <p>$\frac{\text{area } \Delta OFB}{\text{area } \Delta OAD} = \frac{9/11}{2/3} = \frac{27}{22}$</p> <p>$\text{area } \Delta OAD = \frac{22}{27} \times 5 = \frac{110}{27}$</p>	<p>M1</p> <p>M1A1 (3) [13]</p>

Notes

(a) (i)

B1: for $\frac{2}{3}\mathbf{b} - \mathbf{a}$

(ii)

M1: for $\overrightarrow{OE} = \overrightarrow{OA} + \frac{2}{5}\overrightarrow{AD}$ (for the vector statement)

(or for any other valid path)

A1: $\overrightarrow{OE} = \frac{3}{5}\mathbf{a} + \frac{4}{15}\mathbf{b}$

(iii)

M1: for $\overrightarrow{BE} = \overrightarrow{OE} - \overrightarrow{OB}$ (for the vector statement)
(again for any other valid path)

A1: $\overrightarrow{BE} = \frac{3}{5}\mathbf{a} - \frac{11}{15}\mathbf{b}$

(b)

M1: for $\overrightarrow{FE} = \overrightarrow{OE} - \overrightarrow{OF}$

A1: for $\overrightarrow{FE} = \frac{3}{5}\mathbf{a} + \frac{4}{15}\mathbf{b} - \lambda\mathbf{a} \quad \left(= \mathbf{a}\left(\frac{3}{5} - \lambda\right) + \frac{4}{15}\mathbf{b} \right)$

M1: for using their \overrightarrow{FE} and \overrightarrow{BE} to form;

$$\frac{\frac{3}{5} - \lambda}{\frac{4}{15}} = \frac{\frac{3}{5}}{-\frac{11}{15}} \quad \text{or} \quad \frac{\frac{3}{5} - \lambda}{\frac{3}{5}} = \frac{\frac{4}{15}}{-\frac{11}{15}}$$

A1: for the correct equation in λ

A1: $\lambda = \frac{9}{11}$

ALT

M1: for $\overrightarrow{OF} + \overrightarrow{FB} = \overrightarrow{OB}$ oe

A1: for the correct expression in terms of λ and μ (or any other letter for the second constant)M1: for comparing coefficients of λ and their μ A1: for achieving μ and an expression for λ in terms of μ

A1: $\lambda = \frac{9}{11}$

(c)

M1: for stating and using that area of triangle $\triangle OAB = \frac{11}{9} \times \text{area of } \triangle OFB \Rightarrow \triangle OAB = \frac{11}{9} \times 5$

Note: area of triangle OAB = the reciprocal of their $\lambda \times 5$

M1: for stating and using that area of $\triangle OAD = \frac{2}{3} \times \text{area of } \triangle OAB$

A1: area of triangle $OAD = \frac{110}{27}$

ALT 1

M1: for the ratio of areas of triangle OFB and triangle OAD as follows;

$$\frac{\text{area } \triangle OAB}{\text{area } \triangle OFB} = \frac{11}{9} \quad \text{and} \quad \frac{\text{area } \triangle OAD}{\text{area } \triangle OAB} = \frac{2}{3} \Rightarrow$$

$$\frac{\text{area } \triangle OAD}{\text{area } \triangle OFB} = \frac{11}{9} \times \frac{2}{3} = \frac{22}{27}$$

M1: for $\frac{\triangle OAD}{5} = \frac{22}{27}$

A1: area of triangle $OAD = \frac{110}{27}$

ALT 2

M1: for using $\frac{1}{2}ab \sin C$ on triangles OAD and OFB

$$\text{Triangle } OFB: \frac{1}{2} \times \frac{9}{11} |\mathbf{a}| \times |\mathbf{b}| \times \sin \theta = 5 \quad \text{AND} \quad \text{Area } OAD = \frac{1}{2} \times |\mathbf{a}| \times \frac{2}{3} |\mathbf{b}| \times \sin \theta$$

M1: for substituting $\sin \theta = \frac{110}{9|\mathbf{a}||\mathbf{b}|}$ into $\Rightarrow \text{Area } OAD = \frac{|\mathbf{a}||\mathbf{b}|}{3} \times \frac{110}{9|\mathbf{a}||\mathbf{b}|} \left(= \frac{110}{27} \right)$

A1: area of triangle $OAD = \frac{110}{27}$