June 2016 M1 UK Solutions Kprime2

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1. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two cars P and Q are moving on straight horizontal roads with constant velocities. The velocity of P is $(15\mathbf{i} + 20\mathbf{j})\ \text{m s}^{-1}$ and the velocity of Q is $(20\mathbf{i} - 5\mathbf{j})\ \text{m s}^{-1}$

(a) Find the direction of motion of Q, giving your answer as a bearing to the nearest degree.

(3)

At time $t = 0$, the position vector of P is $400\mathbf{i}$ metres and the position vector of Q is $800\mathbf{j}$ metres. At time t seconds, the position vectors of P and Q are $\mathbf{p}$ metres and $\mathbf{q}$ metres respectively.

(b) Find an expression for

(i) $\mathbf{p}$ in terms of t,

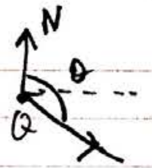
(ii) $\mathbf{q}$ in terms of t.

(3)

(c) Find the position vector of Q when Q is due west of P.

(4)

1. (a)



$\begin{pmatrix} 20 \\ -5 \end{pmatrix}$

$$\theta = 90^\circ + \arctan\left(\frac{5}{20}\right) = 104.03\ldots$$

$$\therefore\ \theta = 104^\circ \text{ (nearest degree)}$$

(b) Consider P:

$\underset{\sim}{r_0} = 400\underset{\sim}{i} \qquad \underset{\sim}{v_P} = \begin{pmatrix} 15 \\ 20 \end{pmatrix}$

@ time t, $\quad \underset{\sim}{r} = \begin{pmatrix} 400 \\ 0 \end{pmatrix} + t\begin{pmatrix} 15 \\ 20 \end{pmatrix}$

$$\Rightarrow \underset{\sim}{p} = \begin{pmatrix} 400 + 15t \\ 20t \end{pmatrix}$$

P 4 6 7 0 8 A 0 2 2 8

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Question 1 continued

(i) $\underset{\sim}{p} = (400+15t)\underset{\sim}{i} + 20t\,\underset{\sim}{j}$

(ii) Consider Q:

$\underset{\sim}{r}_Q = \begin{pmatrix} 0 \\ 800 \end{pmatrix} \quad \underset{\sim}{v}_Q = \begin{pmatrix} 20 \\ -5 \end{pmatrix}$

$\Rightarrow$ ~~$\underset{\sim}{r} = 20t$~~

$\underset{\sim}{r} = \begin{pmatrix} 0 \\ 800 \end{pmatrix} + t\begin{pmatrix} 20 \\ -5 \end{pmatrix}$

$\therefore \underset{\sim}{q} = \begin{pmatrix} 20t \\ 800-5t \end{pmatrix} \Rightarrow \underset{\sim}{q} = 20t\underset{\sim}{i} + (800-5t)\underset{\sim}{j}$

(c) $\begin{pmatrix} 20t \\ 800-5t \end{pmatrix}$ $\quad \cdot p \begin{pmatrix} 400+15t \\ 20t \end{pmatrix}$

$\Rightarrow 800 - 5t = 20t$

$\therefore 25t = 800$

$\therefore t = 32$

$t = 32 \Rightarrow \underset{\sim}{q} = 640\underset{\sim}{i} + 640\underset{\sim}{j}$

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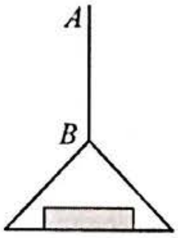


Figure 1

A vertical rope AB has its end B attached to the top of a scale pan. The scale pan has mass 0.5 kg and carries a brick of mass 1.5 kg, as shown in Figure 1. The scale pan is raised vertically upwards with constant acceleration 0.5 m s^{-2} using the rope AB. The rope is modelled as a light inextensible string.

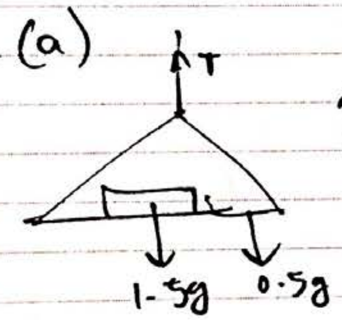
(a) Find the tension in the rope AB.

(3)

(b) Find the magnitude of the force exerted on the scale pan by the brick.

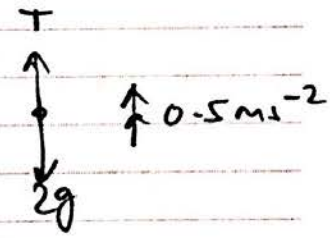
(3)

2 (a)



$\uparrow 0.5 \text{ ms}^{-2}$

$\therefore$



$\uparrow$: $F = ma$

$\Rightarrow T - 2g = \cancel{2a}\ 2 \times 0.5$

$\Rightarrow T = 1 + 2g$ N

$T = 20.6$ N
(3sf)

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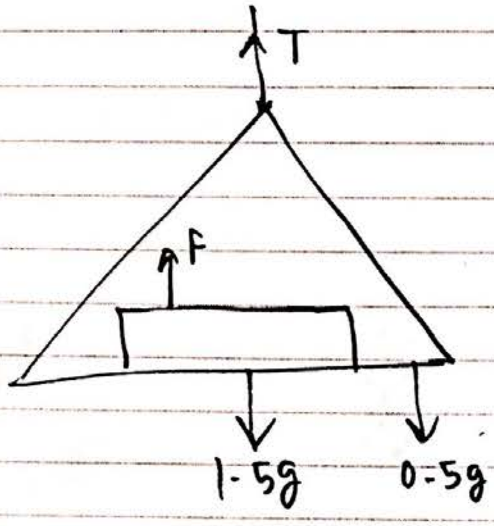
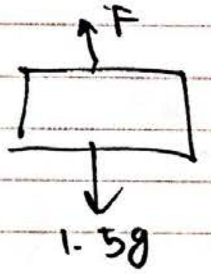
Question 2 continued

(b)



F is the contact force.

Consider just the brick:



$\uparrow 0.5\ \mathrm{ms}^{-2}$

$\uparrow$: $F - 1.5g = 1.5 \times 0.5$

$\therefore F = \frac{3}{4} + 1.5g$

$\Rightarrow F = 15.5\ \mathrm{N}$ (3SF)

Q2

(Total 6 marks)

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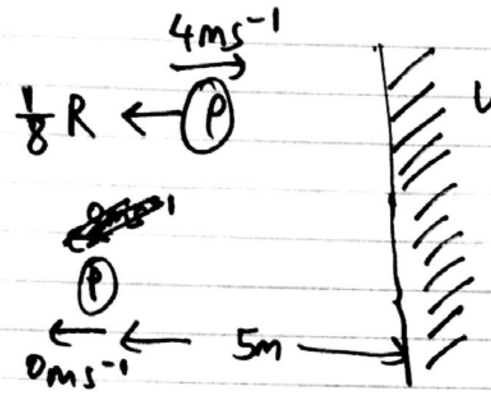
3. A particle P of mass 0.4 kg is moving on rough horizontal ground when it hits a fixed vertical plane wall. Immediately before hitting the wall, P is moving with speed 4 m s^{-1} in a direction perpendicular to the wall. The particle rebounds from the wall and comes to rest at a distance of 5 m from the wall. The coefficient of friction between P and the ground is $\frac{1}{8}$.

Find the magnitude of the impulse exerted on P by the wall.

(7)

3.

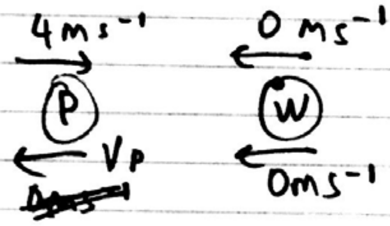


F = friction

$F = \frac{1}{8}R$

$R = 0.4g$

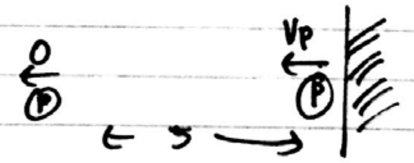
$\therefore F = \underline{\frac{1}{20}g}$ N



W=wall

We need v_p to calculate Impulse.

$s = 5$
$u = V_p$
$v = 0$
$a = -\frac{g}{8}$
$t =$



$\overset{+}{\leftarrow}$ $F = ma$

$\Rightarrow -F = 0.4a$

$\Rightarrow -\frac{g}{20} = 0.4a$

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Question 3 continued

$\therefore a = -\frac{1}{8}g \text{ ms}^{-2}$

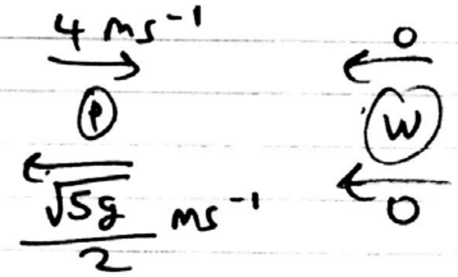
$\therefore v^2 = u^2 + 2as$

$\Rightarrow \quad 0 = (v_p)^2 - 2\left(\frac{g}{8}\right)(5)$

$\therefore \quad 0 = (v_p)^2 - \frac{5}{4}g$

$(v_p)^2 = \frac{5}{4}g$

$\therefore \quad v_p = \frac{\sqrt{5g}}{2} \text{ ms}^{-1}$



$\overset{+}{\leftarrow} \quad I = m(\cancel{v} - u)$

$\therefore \quad I = 0.4\left(\frac{\sqrt{5g}}{2} + 4\right)$

$\Rightarrow \quad I = 3 \text{ NS}$

Q3

(Total 7 marks)

P 4 6 7 0 8 A 0 9 2 8

Turn over

4. Two trains M and N are moving in the same direction along parallel straight horizontal tracks. At time $t = 0$, M overtakes N whilst they are travelling with speeds 40 m s^{-1} and 30 m s^{-1} respectively. Train M overtakes train N as they pass a point X at the side of the tracks.

After overtaking N, train M maintains its speed of 40 m s^{-1} for T seconds and then decelerates uniformly, coming to rest next to a point Y at the side of the tracks.

After being overtaken, train N maintains its speed of 30 m s^{-1} for 25 s and then decelerates uniformly, also coming to rest next to the point Y.

The times taken by the trains to travel between X and Y are the same.

(a) Sketch, on the same diagram, the speed-time graphs for the motions of the two trains between X and Y.

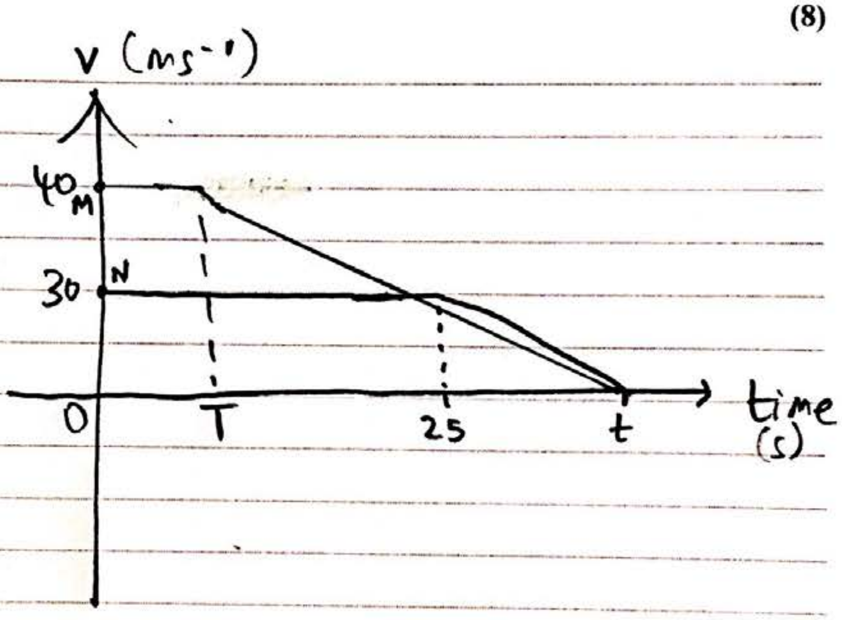
(4)

Given that $XY = 975$ m,

(b) find the value of T.

(8)

(a)



(b) $XY = 975\text{m}$ let $t = \Sigma\text{time}$

Consider Area train N:

$$\frac{30}{2}(t+25) = 15(t+25)$$

$$\therefore 15(t+25) = 975 \Rightarrow t = 40$$

~~t=40~~

Question 4 continued

Consider area train M:

$$\frac{40}{2}(T+40) = 20(T+40)$$

$$\therefore 20(T+40) = 975$$

$$\Rightarrow T = 8.75 \text{ seconds}$$

Q4

(Total 12 marks)

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5.

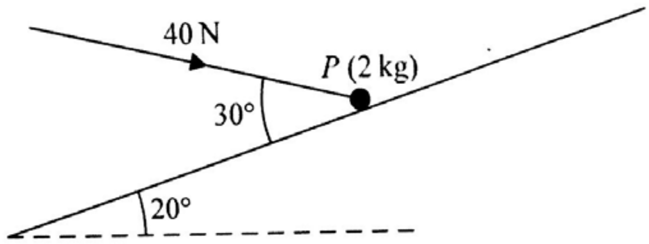
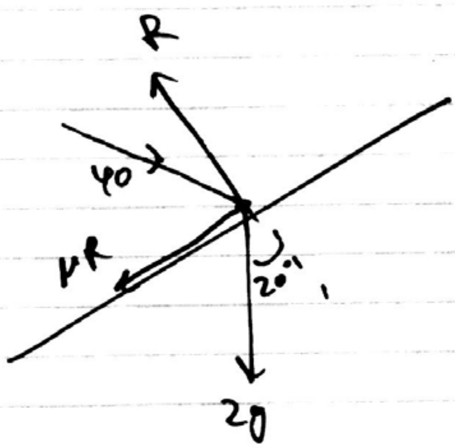


Figure 2

A particle P of mass 2 kg is held at rest in equilibrium on a rough plane by a constant force of magnitude 40 N. The direction of the force is inclined to the plane at an angle of 30°. The plane is inclined to the horizontal at an angle of 20°, as shown in Figure 2. The line of action of the force lies in the vertical plane containing P and a line of greatest slope of the plane. The coefficient of friction between P and the plane is μ.

Given that P is on the point of sliding up the plane, find the value of μ.

(10)



$\nearrow$: $40\cos 30 = \mu R + 2g\sin 20$

$\nwarrow$: $R = 2g\cos 20 + 40\sin 30 = 38.417\ldots$

$\mu = \dfrac{40\cos 30 - 2g\sin 20}{38.417\ldots} = 0.7271\ldots$

$\Rightarrow \mu = \underline{\underline{0.727}}$ (3sf)

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6. A non-uniform plank *AB* has length 6 m and mass 30 kg. The plank rests in equilibrium in a horizontal position on supports at the points *S* and *T* of the plank where $AS = 0.5$ m and $TB = 2$ m.

When a block of mass *M* kg is placed on the plank at *A*, the plank remains horizontal and in equilibrium and the plank is on the point of tilting about *S*.

When the block is moved to *B*, the plank remains horizontal and in equilibrium and the plank is on the point of tilting about *T*.

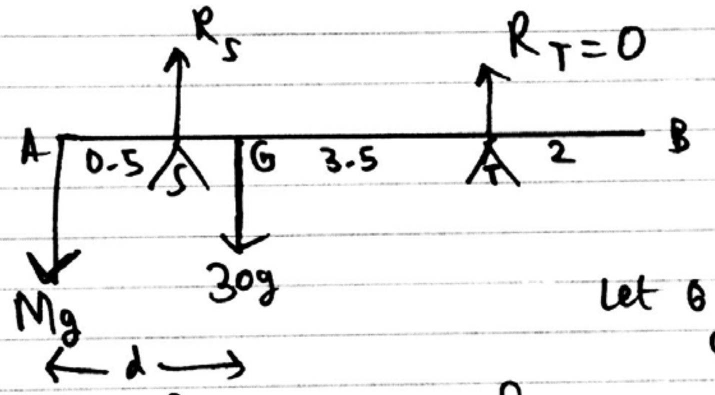
The distance of the centre of mass of the plank from *A* is *d* metres. The block is modelled as a particle and the plank is modelled as a non-uniform rod. Find

(i) the value of *d*,

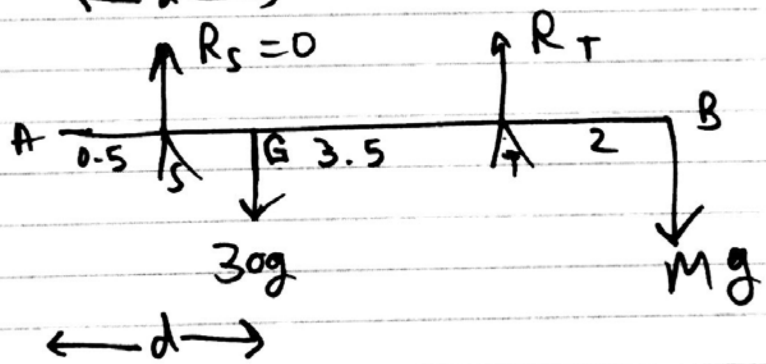
(ii) the value of *M*.

(7)

6



Let G be centre of mass



Consider when block is placed at A:

$$M(A): \quad 30g(d) = 0.5 R_S \qquad (1)$$

Question 6 continued

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Consider when block is placed at B:

$M(B)$: $30g(6-d) = 2R_T$

$\Rightarrow 180g - 30gd = 2R_T$ ②

Sub in ① in ②:

$180g - \frac{1}{2}R_S = 2R_T$

Also: $(M+30)g = R_S = R_T$

$\therefore 180g - \frac{1}{2}(M+30)g = 2(M+30)g$

$\therefore 180g - \frac{g}{2}M - 15g = 2gM + 60g$

$\therefore$ ~~$120g =$~~ $105g = \left(2g + \frac{g}{2}\right)M$

$\therefore 105 = \left(2 + \frac{1}{2}\right)M = \frac{5}{2}M$

$\Rightarrow M = 42\text{ kg}$

$M = 42 \Rightarrow R_S = 72g$

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Question 6 continued

Sub in ①

$30g(d) = 0.5 \times 72g$

$30\cancel{g}d = 36\cancel{g}$

$30d = 36 \Rightarrow d = 1.2$

$\therefore$ (i) $d = 1.2$ m

(ii) $M = 42$ kg

Leave blank

7. Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on a particle P.

The force $\mathbf{F}_1$ is given by $\mathbf{F}_1 = (-\mathbf{i} + 2\mathbf{j})$ N and $\mathbf{F}_2$ acts in the direction of the vector $(\mathbf{i} + \mathbf{j})$.

Given that the resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ acts in the direction of the vector $(\mathbf{i} + 3\mathbf{j})$,

(a) find $\mathbf{F}_2$

(7)

The acceleration of P is $(3\mathbf{i} + 9\mathbf{j})$ m s^{-2}. At time $t = 0$, the velocity of P is $(3\mathbf{i} - 22\mathbf{j})$ m s^{-1}

(b) Find the speed of P when $t = 3$ seconds.

(4)

7. (a) $\underset{\sim}{F_1} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ $\quad \underset{\sim}{F_2} = k\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Let $\underset{\sim}{R}$ = resultant, $\quad \underset{\sim}{R} = \lambda\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$\underset{\sim}{R} = \underset{\sim}{F_1} + \underset{\sim}{F_2}$

$\Rightarrow \underset{\sim}{R} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} + k\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

~~$\Rightarrow \underset{\sim}{F_2} = \underset{\sim}{R} - F_1$~~

$\therefore \underset{\sim}{R} = \begin{pmatrix} k-1 \\ 2+k \end{pmatrix} = \lambda\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$\Rightarrow \quad k - 1 = \lambda \quad \Rightarrow k = \lambda + 1$

& $\quad k + 2 = 3\lambda$

$\Rightarrow \quad \lambda + 1 + 2 = 3\lambda$

$\therefore \quad 2\lambda = 3 \Rightarrow \lambda = \frac{3}{2}$

$\therefore \quad k = \frac{3}{2} + 1 = \underline{\underline{\frac{5}{2}}}$

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Question 7 continued

$\therefore \underset{\sim}{F_2} = \frac{5}{2}\begin{pmatrix}1\\1\end{pmatrix}$

$\therefore \underline{F_2} = \frac{5}{2}\underset{\sim}{i} + \frac{5}{2}\underset{\sim}{j}$

(b) $\underset{\sim}{a} = \begin{pmatrix}3\\9\end{pmatrix}$ $\underset{\sim}{u} = \begin{pmatrix}3\\-22\end{pmatrix}$ $t = 3$

$\underset{\sim}{v} = \underset{\sim}{u} + t\underset{\sim}{a}$

$\therefore \underset{\sim}{v} = \begin{pmatrix}3\\-22\end{pmatrix} + 3\begin{pmatrix}3\\9\end{pmatrix}$

$\therefore \underset{\sim}{v} = \begin{pmatrix}12\\5\end{pmatrix}$

$\therefore |\underset{\sim}{v}| = \sqrt{12^2+5^2} = 13\,ms^{-1}$

Speed $= 13\,ms^{-1}$

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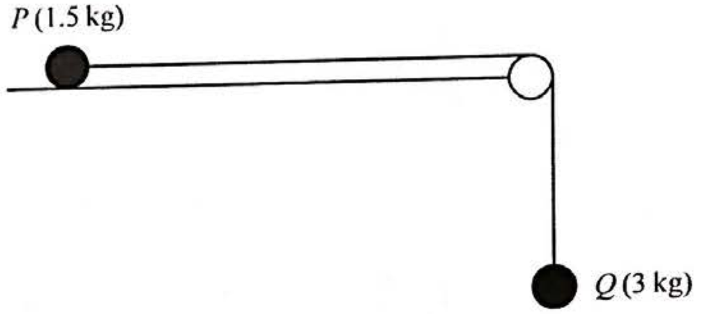


Figure 3

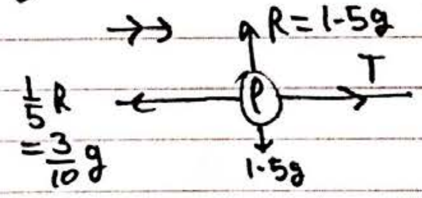
Two particles P and Q have masses 1.5 kg and 3 kg respectively. The particles are attached to the ends of a light inextensible string. Particle P is held at rest on a fixed rough horizontal table. The coefficient of friction between P and the table is $\frac{1}{5}$. The string is parallel to the table and passes over a small smooth light pulley which is fixed at the edge of the table. Particle Q hangs freely at rest vertically below the pulley, as shown in Figure 3. Particle P is released from rest with the string taut and slides along the table.

Assuming that P has not reached the pulley, find

(a) the tension in the string during the motion,
(8)

(b) the magnitude and direction of the resultant force exerted on the pulley by the string.
(4)

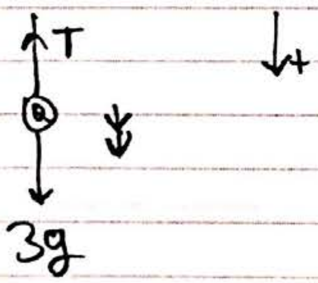
8 (a) Consider P:



$\xrightarrow{+}$ $F = ma$

$\therefore T - \frac{3}{10}g = 1.5a$

Consider Q:



$\downarrow +$ $F = ma$

$3g - T = 3a$

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Question 8 continued

$$T - \frac{3}{10}g = \frac{3}{2}a \Rightarrow 2T - \frac{3}{5}g = 3a$$

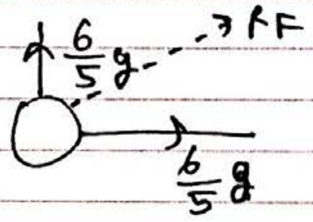
$$\& \quad 3g - T = 3a$$

$$\therefore 3g - T = 2T - \frac{3}{5}g$$

$$\therefore 3T = 3g + \frac{3}{5}g = \frac{18}{5}g$$

$$\Rightarrow T = \frac{6}{5}g \text{ N}$$

(b)



$$RF = \sqrt{\left(\tfrac{6}{5}g\right) + \left(\tfrac{6}{5}g\right)^2} = 16.63\ldots$$

$$\therefore RF = 16.6 \text{ N} \quad (3\text{sf})$$

Direction: 45° below pulley (south west)

Q8

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END