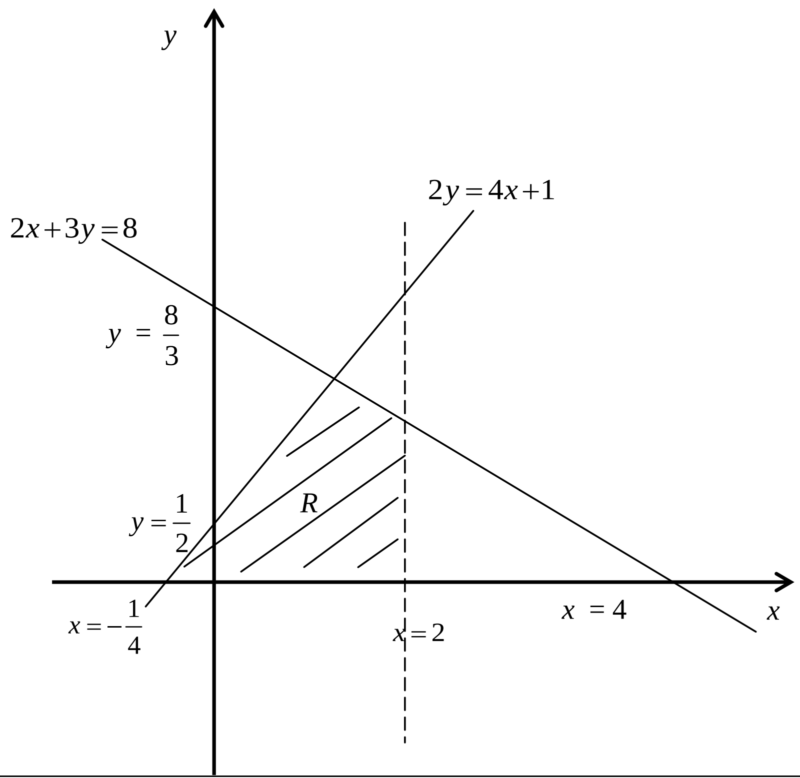
**International GCSE Futher Pure Mathematics – Paper 1 mark scheme**

| **Paper 1** | | | | | |
|---|---|---|---|---|---|
| Question | Working | Answer | Mark | AO | Notes |
| **1 (a)** |  | | B1<br>B1 | 1 | One correct line<br>Both correct lines |
| **(b)** | | | B1<br>B1<br>**(4)** | 1 | Correct line $x$ = 2 drawn<br>Correct region shaded in or out. |

| Question | Working | Answer | Mark | AO | Notes |
|---|---|---|---|---|---|
| **2 (a)** | $\pi = 6\theta \Rightarrow \theta = \frac{\pi}{6} \Rightarrow AOB = \frac{5\pi}{6} = (150^{o})$ | | B1 | 1 | |
| | $AB = \sqrt{10^2 + 6^2 - 2\times10\times6\times\cos\left(\frac{5\pi}{6}\right)} = 15.4894... = 15.5$ cm | 15.5 (cm) | M1A1 | 2 | Accept working in degrees $AOB = 150^{o}$ |
| **(b)** | Area $= \frac{1}{2}\times10\times6\times\sin\frac{5\pi}{6} + \frac{\pi}{6}\times\frac{6^2}{2} = 24.424...$ | 24.4 (cm$^2$) | M1M1A1 | 2 | |
| | **ALTERNATIVE** Area $= \frac{1}{2}\times10\times6\times\sin\frac{5\pi}{6} + \frac{1}{2}\times\pi\times6 = 24.424...$ | | M1M1A1 **(6)** | | |
| **3** | $2\cos(2\theta+30) + \frac{\sin(2\theta+30)}{\cos(2\theta+30)} = 0 \Rightarrow 2\cos^2(2\theta+30) + \sin(2\theta+30) = 0$ | | M1 | 2 | |
| | $\Rightarrow 2 - 2\sin^2(2\theta+30) + \sin(2\theta+30) = 0$ | | M1A1 | | |
| | $\sin(2\theta+30) = \frac{1\pm\sqrt{1-4\times2\times(-2)}}{2\times2} = 1.2807..., -0.7807....$ | | M1 | 3 | Solves 3 TQ |
| | $2\theta + 30 = -51.33167..., 231.33167, 308.66833$ | | A1A1 | | Finds one angle from their 3TQ |
| | $\theta = 100.7,\ 139.3$ | $\theta = 100.7,\ 139.3$ | **(6)** | | |

| Question | Working | Answer | Mark | AO | Notes |
|---|---|---|---|---|---|
| **4** | | | | | |
| **(a)** | $v=0$ so $4t^2-19t+12=0 \Rightarrow (4t-3)(t-4)=0 \Rightarrow t=\frac{3}{4},4$ | $t=\frac{3}{4},4$ | M1A1 | 1 | |
| **(b)** | $s=\int 4t^2-19t+2\,\mathrm{d}t=\frac{4t^3}{3}-\frac{19t^2}{2}+12t+c$ when $t=0$, $s=-4$ $\Rightarrow c=-4$ | | M1M1A1 | 2 | |
| | When $t=6$, $s=\frac{4\times 6^3}{3}-\frac{19\times 6^2}{2}+12\times 6-4=14$ | 14 | A1 | | |
| **(c)** | $a=\frac{\mathrm{d}v}{\mathrm{d}t}=8t-19 \Rightarrow$ $8t-19=0 \Rightarrow t=\frac{19}{8}$ | $t=\frac{19}{8}$ | M1M1A1<br>**(9)** | 3 | |
| **5** | $2x+y=13 \Rightarrow y=13-2x$ | | B1 | 1 | |
| **(a)** | $S=4x^2+(13-2x)^2=4x^2+169-52x+4x^2=8x^2-52x+169$ | $8x^2-52x+169$ | M1A1 | | |
| **(b)** | $\frac{\mathrm{d}S}{\mathrm{d}x}=16x-52$, $\frac{\mathrm{d}S}{\mathrm{d}x}=0 \Rightarrow 16x-52=0 \Rightarrow x=\frac{13}{4}$ | $x=\frac{13}{4}$ | M1M1A1 | 2,3 | |
| | $\frac{\mathrm{d}^2S}{\mathrm{d}x^2}=16$ $16>0$, | Hence minimum | B1 | | |
| **(c)** | $S=8\times\left(\frac{13}{4}\right)^2-52\times\frac{13}{4}+169=\frac{169}{2}=84.5$ | $\frac{169}{2}=84.5$ | M1A1<br>**(9)** | 3 | |

| Question | Working | Answer | Mark | AO | Notes |
|---|---|---|---|---|---|
| **6** | $\frac{dy}{dx} = e^x(x^2-3x)+e^x(2x-3)\left[\Rightarrow e^x(2x-3)=\frac{dy}{dx}-y\right]$ | | M1M1A1 | 4 | |
| | $\frac{d^2y}{dx^2} = e^x(x^2-3x)+e^x(2x-3)+e^x(2x-3)+2e^x = y+2\left(\frac{dy}{dx}-y\right)+2e^x$ | | M1A1 | 4 | |
| | $2e^x = \frac{d^2y}{dx^2}-2\left(\frac{dy}{dx}-y\right)-y \Rightarrow 2e^x = y-2\frac{dy}{dx}+\frac{d^2y}{dx^2}$ * | | M1M1A1<br>**(8)** | | |
| **7** | | | | | |
| **(a)** | $\begin{array}{\|c\|c\|c\|c\|c\|c\|c\|}\hline \boldsymbol{x} & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline \boldsymbol{y} & 3 & \mathbf{3.83} & \mathbf{5} & \mathbf{6.66} & 9 & 12.31 \\ \hline\end{array}$ | | B1B1 | 1 | |
| **(b)** | All points plotted within an accuracy of half of a square.<br>A smooth curve drawn through their points | | B1B1 | 1 | |
| **(c)** | $\log_2(4x-6)^2 - x = 2 \Rightarrow 2\log_2(4x-6) = x+2 \Rightarrow \log_2(4x-6) = \frac{x}{2}+1$ | | M1M1 | 2 | |
| | $\Rightarrow 4x-6 = 2^{\left(\frac{x}{2}+1\right)} \Rightarrow 4x-5 = 2^{\left(\frac{x}{2}+1\right)}+1$ | | | | |
| | Line $y=4x-5$ drawn on graph $\Rightarrow$ so $x$ = 2.8(36) | $x$ = 2.8 | M1A1<br>**(8)** | | |

| Question | Working | Answer | Mark | AO | Notes |
|---|---|---|---|---|---|
| **8** **(a)** | $a = S_1 = 2\times1\times(1+3) = 8$ | $a = 8$ | B1 | 1 | |
| **(b)** | $S_2 = 2\times2\times(2+3) = 20$<br>$S_2 = a + T_2 \Rightarrow T_2 = S_2 - a = 20 - 8 = 12$<br>$d = 12 - 8 = 4$ | $d = 4$ | M1A1 | 1 | |
| **(c)** | Uses given formula for $S_n = 2n(n+3)$ and formula for *nth* term $T_n = a + (n-1)d$ | | | | |
| | $6\left[2(n-4)(n-4+3)\right] = 7\left[8+(n+3-1)4\right]$<br>$\Rightarrow 12n^2 - 88n - 64 = 0 \Rightarrow 3n^2 - 22n - 16 = 0$ | | M1M1<br>M1A1 | 2, 3 | |
| | $(n-8)(3n+2) = 0$<br>$\Rightarrow n = 8, \left(n = -\frac{2}{3}\right)$ | $n = 8$ | M1A1 | | |
| | **ALT**<br>Using formula $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$ | | | | |
| | $6\left[\frac{(n-4)}{2}\left(2\times8+((n-4)-1)4\right)\right] = 7\left[8+(n+3-1)4\right]$<br>$\Rightarrow 12n^2 - 88n - 64 = 0 \Rightarrow 3n^2 - 22n - 16 = 0$ | | M1M1<br>M1A1 | 2, 3 | |
| | $(n-8)(3n+2) = 0$ | | M1A1 | | |
| | $\Rightarrow n = 8, \left(n = -\frac{2}{3}\right)$ | $n = 8$ | **(9)** | | |

| Question | Working | Answer | Mark | AO | Notes |
|---|---|---|---|---|---|
| **9** **(a)** | $\alpha+\beta=-\frac{7}{3}$ $\alpha\beta=-2=-\frac{6}{3}$ so $a=3$ , $b=7$ and $c=-6$ | | B1B1 | 1 | |
| | Hence quadratic equation $\Rightarrow 3x^2+7x-6=0$ oe with integer coefficients | | M1A1 | | |
| **(b)** | $(\alpha-\beta)^2=\alpha^2+\beta^2-2\alpha\beta=(\alpha+\beta)^2-4\alpha\beta=\left(-\frac{7}{3}\right)^2-4\times-2=\frac{121}{9}$ | | | 3 | |
| | $\alpha>\beta$ so $\alpha-\beta=\frac{11}{3}$ * | | M1A1 | | |
| **(c)** | Sum<br>$\frac{\alpha+\beta}{\alpha}+\frac{\alpha-\beta}{\beta}=\frac{\beta(\alpha+\beta)+\alpha(\alpha-\beta)}{\alpha\beta}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{\left(-\frac{7}{3}\right)^2-2\times-2}{-2}=-\frac{85}{18}$ | | M1M1A1 | 3 | |
| | Product $\frac{(\alpha+\beta)}{\alpha}\times\frac{(\alpha-\beta)}{\beta}=\frac{\left(-\frac{7}{3}\right)\times\left(\frac{11}{3}\right)}{-2}=\frac{77}{18}$ | | M1A1 | | |
| | Equation $18y^2+85y+77=0$ oe with integer coefficients | | M1A1 | | |
| | | | **(13)** | | |

| Question | Working | Answer | Mark | AO | Notes |
|---|---|---|---|---|---|
| **10** **(a)** | Let $M$ be the midpoint of diagonals $AC$ or $DB$<br>$AC = \sqrt{8^2 + 8^2} = 8\sqrt{2} \Rightarrow AM = 4\sqrt{2}$<br>$h = \sqrt{12^2 - \left(4\sqrt{2}\right)^2} = \sqrt{112} = \left(4\sqrt{7}\right)$ | $4\sqrt{7}$ | M1M1A1 | 1 | |
| **(b)** | $\tan^{-1}\left(\frac{4\sqrt{7}}{4\sqrt{2}}\right) = 61.87449... \approx 61.9^{\circ}$ | 61.9° | M1A1 | 1 | Or any equivalent trigonometry |
| **(c)** | Let $N$ be the midpoint of $AB$<br>Angle the plane $AOB$ makes with horiz = $\tan^{-1}\left(\frac{4\sqrt{7}}{4}\right) = 69.295... \approx 69.3^{\circ}$ | 69.3° | M1A1 | 3 | Or any equivalent trigonometry |
| **(d)** | **By using the symmetrical properties of the pyramid**<br>Let $S$ be the perpendicular from $P$ to diagonal $AC$<br>Let $R$ be the perpendicular from $S$ to side $BC$<br>In triangle $PSR$ → $PR = \sqrt{\left(2\sqrt{17}\right)^2 - 2^2} = 8$<br>In triangle $PRQ$ → $PQ = \sqrt{8^2 + 4^2} = 4\sqrt{5}$ | $4\sqrt{5}$ | M1A1<br>M1A1 | 3 | Or any equivalent system of right angle triangles |
| **(e)** | Length $AQ = \sqrt{8^2 + 6^2} = 10$<br>Angle of $PQA = \cos^{-1}\left(\frac{10^2 + \left(4\sqrt{5}\right)^2 - 6^2}{2 \times 10 \times 4\sqrt{5}}\right) = 36.39124... \approx 36.4^{\circ}$ | 36.4° | M1<br>M1A1A1<br>**(15)** | 3 | |

| Question | Working | Answer | Mark | AO | Notes |
|---|---|---|---|---|---|
| **(d)** | **ALTERNATIVE without using the symmetrical properties of the pyramid** | | | 3 | |
| | $\cos OAB = \dfrac{12^2+8^2-12^2}{2\times8\times12} = \dfrac{1}{3}$ | | M1 | | |
| | $\Rightarrow PB = \sqrt{6^2+8^2-2\times6\times8\times\dfrac{1}{3}} = 2\sqrt{17}$ | | | | |
| | In triangle *PBC* | | | | |
| | $PC = \sqrt{6^2+\left(8\sqrt{2}\right)^2-2\times6\times\left(8\sqrt{2}\right)\times\cos\left(\tan^{-1}\left(\dfrac{\sqrt{7}}{\sqrt{2}}\right)\right)} = 10$ | | M1 | | |
| | $\Rightarrow$ Angle $PBC = \cos^{-1}\left(\dfrac{8^2+68-10^2}{2\times8\times2\sqrt{17}}\right) = 75.9637.....^{\circ}$ | | | | |
| | In triangle *PBQ;* | | | | |
| | $PQ = \sqrt{6^2+68-2\times6\times2\sqrt{17}\times\cos 75.96375...} = 4\sqrt{5}$ | $4\sqrt{5}$ | M1A1 | | |

| Question | Working | Answer | Mark | AO | Notes |
|---|---|---|---|---|---|
| **11** | Mark parts (i) and (ii) together | | | 1, 3 | |
| **(a)** | $y=\int 6x^2-26x+12\mathrm{d}x=\left[\frac{6x^3}{3}-\frac{26x^2}{2}+12x+c\right]$ | | M1 | | |
| | At the point $(-1,0)$ <br> $0=2(-1)^3-13(-1)^2+12(-1)+C\Rightarrow C=27$ | | M1A1 | | |
| | $\left(2x^3-13x^2+12x+27\right)\div(x+1)=2x^2-15x+27=(2x-9)(x-3)$ | | M1M1A1 | | |
| | $\Rightarrow a=\frac{9}{2}, b=3$ | $a=\frac{9}{2}, b=3$ | B1B1 | | |
| **(b)** | Area $=\int_0^3 2x^3-13x^2+12x+27\mathrm{d}x+\left\lvert\int_3^{\frac{9}{2}} 2x^3-13x^2+12x+27\mathrm{d}x\right\rvert=$ | | M1M1 | 3 | |
| | $\left[\frac{2}{4}x^4-\frac{13}{3}x^3+\frac{12}{2}x^2+27x\right]_0^3+\left\lvert\left[\frac{2}{4}x^4-\frac{13}{3}x^3+\frac{12}{2}x^2+27x\right]_3^{\frac{9}{2}}\right\rvert=\frac{2043}{32}$ | | A1M1A1 | | |
| | So Area $=\frac{2043}{32}$ | Area $=\frac{2043}{32}$ | **(13)** | | |
| | | **Total** | **100** | | |