

Instructions

- ## Information

- ## Advice

- Turn over ►

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Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n - 1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1 - r^n)}{(1 - r)}$

Sum to infinity, $S_\infty = \frac{a}{1 - r} \quad |r| < 1$

Binomial series

$(1 + x)^n = 1 + nx + \frac{n(n - 1)}{2!}x^2 + \dots + \frac{n(n - 1)\dots(n - r + 1)}{r!}x^r + \dots \quad \text{for } |x| < 1, n \in \mathbb{Q}$

Calculus

Quotient rule (differentiation)

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC : $a^2 = b^2 + c^2 - 2bccos A$

$\tan \theta = \frac{\sin \theta}{\cos \theta}$

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

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Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 $f(x) = 2x^2 + (k + 8)x + k$

Show that for all values of k , the equation $f(x) = 0$ has distinct real roots.

(4)

(Total for Question 1 is 4 marks)



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(2)

(3)

(1)

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Question 2 continued

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(Total for Question 2 is 6 marks)



3

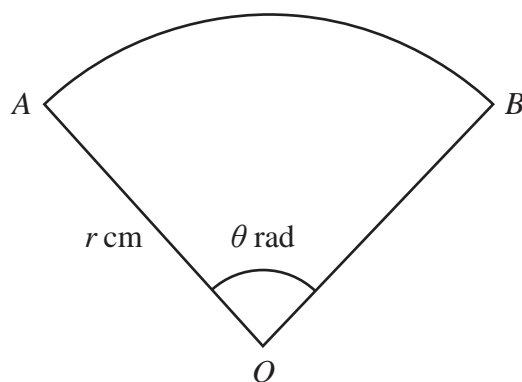
Diagram **NOT**
accurately drawn**Figure 1**

Figure 1 shows the sector OAB of a circle with centre O .

The radius of the circle is r cm and the angle AOB is θ radians.

The area of the sector is 675 cm^2

(a) Show that the perimeter of the sector, P cm, is given by

$$P = 2r + \frac{1350}{r} \quad (3)$$

Given that r can vary,

(b) find, using calculus, the minimum value of P

Give your answer in the form $a\sqrt{b}$ where a is an integer and b is a prime number.

(5)

(c) Justify that the value of P you found in (b) is a minimum.

(2)

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P 7 4 2 8 4 A 0 6 3 2



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Question 3 continued

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(Total for Question 3 is 10 marks)



4 O, A and B are fixed points such that

$$\overrightarrow{OA} = 5\mathbf{i} + 7\mathbf{j} \quad \overrightarrow{AB} = a\mathbf{i} + 16\mathbf{j} \quad \text{and} \quad \left| \overrightarrow{OB} \right| = 5\sqrt{29}$$

- (a) Find the possible values of a

(4)

Given that $a > 0$

- (b) find a unit vector that is parallel to \overrightarrow{AB}

(2)

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Question 4 continued

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(Total for Question 4 is 6 marks)



5 A particle P is moving along the x -axis.

At time t seconds, $t \geq 0$, the velocity, v m/s, of P is given by

$$v = 2t^2 - 19t + 35$$

- (a) Find the acceleration of P when $t = 5$

(2)

The particle comes to instantaneous rest at the points A and B at times t_1 seconds and t_2 seconds respectively, where $t_1 < t_2$

- (b) Find the value of t_1 and the value of t_2

(2)

- (c) Use calculus to find the distance AB

(3)

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Question 5 continued

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(Total for Question 5 is 7 marks)



6

$f(x) = 2x^2 + 5x - p$

The equation $f(x) = 0$ has roots α and β

Given that $\alpha^3 + \beta^3 = -\frac{215}{8}$

(a) find the value of p

(5)

Without solving the equation $f(x) = 0$

(b) form a quadratic equation, with integer coefficients, that has roots

$\frac{\alpha + \beta}{\alpha^2}$ and $\frac{\alpha + \beta}{\beta^2}$

(5)

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Question 6 continued

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Question 6 continued

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Question 6 continued

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(Total for Question 6 is 10 marks)



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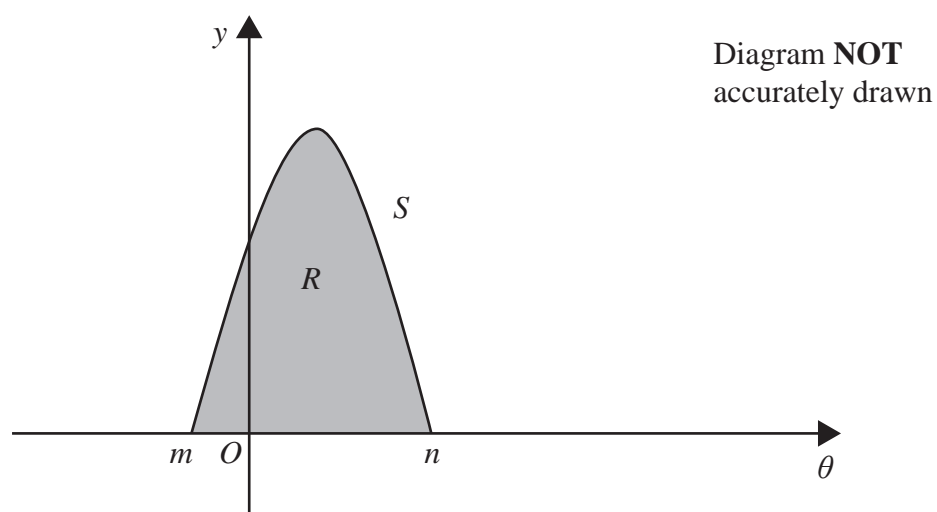


Figure 2

Figure 2 shows part of the curve S with equation $y = (\cos 3\theta + \sqrt{3} \sin 3\theta)^{\frac{1}{2}}$

where $m \leq \theta \leq n$

The curve S meets the x -axis at the point with coordinates $(m, 0)$ and at the point with coordinates $(n, 0)$

- (a) Find the exact value of m and the exact value of n

(3)

The finite region R , shown shaded in Figure 2, is bounded by the curve S , and the x -axis in the region $m \leq \theta \leq n$

The region R is rotated through 2π radians about the θ -axis.

- (b) Use calculus to find the exact volume of the solid generated.

(4)

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Question 7 continued

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Question 7 continued

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Question 7 continued

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(Total for Question 7 is 7 marks)



8 The points A and B have coordinates $(1, 5)$ and $(9, 9)$ respectively.

- (a) Find an equation of line AB , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers to be found. (3)

The line l is perpendicular to AB and passes through the point X which lies on AB such that $AX : XB = 3:1$

- (b) Show that an equation of l is $y = -2x + 22$ (5)

The point C has coordinates $(6, p)$

Given that C lies on l

- (c) find the value of p (1)

$ABCD$ is a parallelogram where the x coordinate of D is negative.

- (d) Find the coordinates of the point D (3)

- (e) Find the area of the parallelogram $ABCD$ (4)

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Question 8 continued

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Question 8 continued

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Question 8 continued

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(Total for Question 8 is 16 marks)



9 A curve C has equation $y = \frac{3-2x}{x+6}$ where $x \neq -6$

- (a) Write down an equation of the asymptote to C that is parallel to the

(i) x -axis (ii) y -axis

(2)

- (b) Find the coordinates of the point where C crosses the

(i) x -axis (ii) y -axis

(2)

- (c) Using the axes opposite, sketch the graph of C , showing clearly its asymptotes and the coordinates of the points where C crosses the coordinate axes.

(3)

- (d) Show that the gradient of the tangent to C is always negative.

(3)

A tangent to C has equation $y = -\frac{3}{5}x + k$ where $k > 0$

- (e) Find the value of k

(5)

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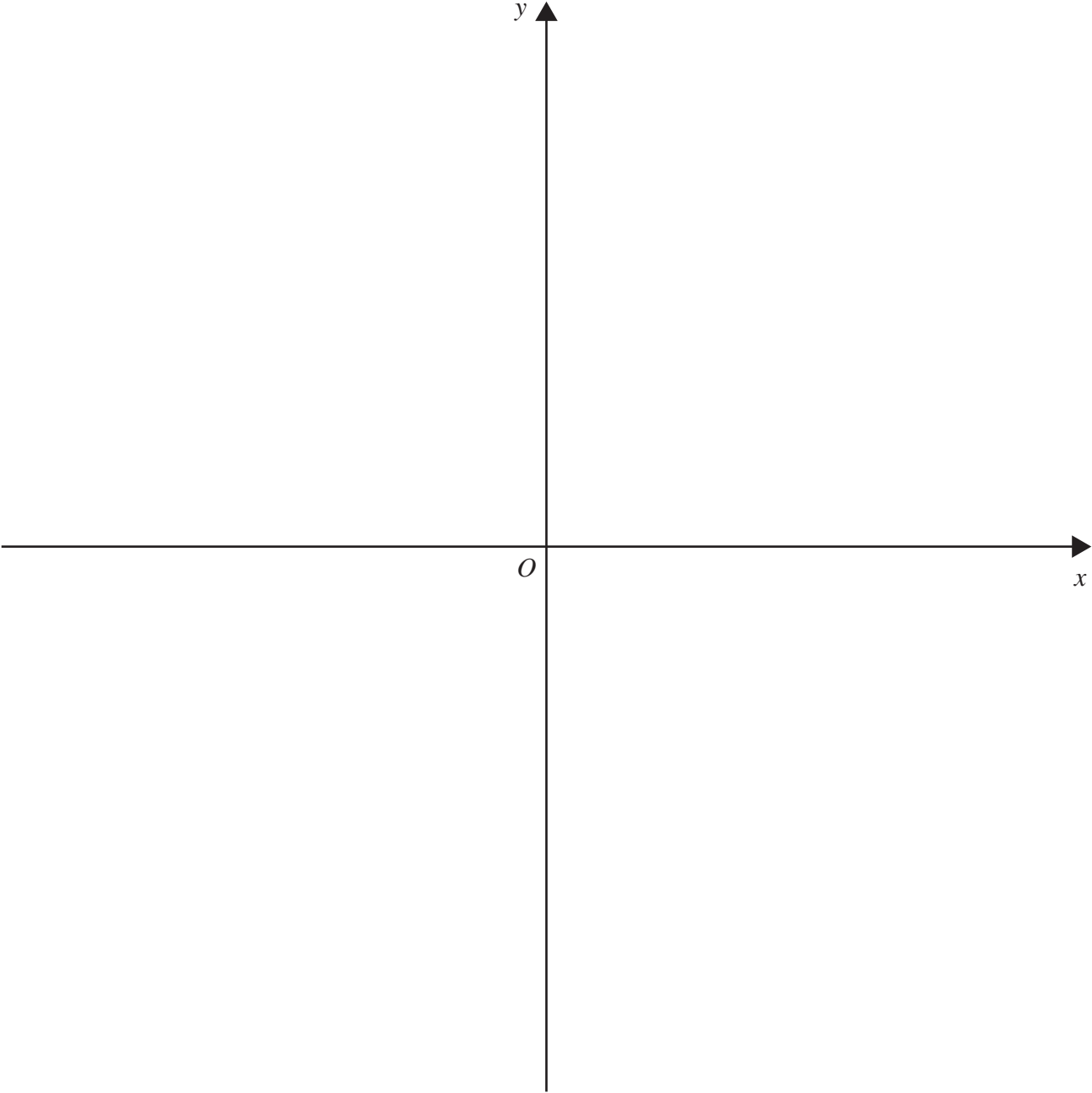
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Question 9 continued



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Question 9 continued

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Question 9 continued

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(Total for Question 9 is 15 marks)



$$\log_4 x^3 + 8\log_x 64 = 22$$

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Question 10 continued

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(Total for Question 10 is 7 marks)



11 (a) Use a formula on page 2 to show that $\sin^2 A = \frac{1}{2}(1 - \cos 2A)$ **(3)**

(b) Show that $\sin^4 x + \cos^4 x = \frac{3 + \cos 4x}{4}$ (5)

(c) Hence solve, in degrees to one decimal place, the equation

$$8\sin^4\left(\frac{\theta}{2}\right) + 8\cos^4\left(\frac{\theta}{2}\right) = 5\sin(2\theta) + 6 \quad \text{for } 0^\circ \leq \theta < 180^\circ \quad (4)$$

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Question 11 continued

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(Total for Question 11 is 12 marks)

TOTAL FOR PAPER IS 100 MARKS

