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| Please check the examination details below before entering your candidate information | | | |
| Candidate surname | | Other names | |
| Pearson Edexcel International GCSE | Centre Number | | Candidate Number |
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| <div style="border: 1px solid black; padding: 5px; font-size: 1.5em; font-weight: bold;">Thursday 20 June 2019</div> | | | |
| Morning (Time: 2 hours) | | Paper Reference 4PM1/02R | |
| <div style="font-size: 1.2em; font-weight: bold;">Further Pure Mathematics</div> <div style="font-weight: bold;">Paper 2R</div> | | |  |
| <div style="border: 1px solid black; padding: 5px; font-weight: bold;">Calculators may be used.</div> | | | <div style="border: 1px solid black; padding: 5px; font-weight: bold;">Total Marks</div> |

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ►

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International GCSE in Further Pure Mathematics Formulae sheet

Mensuration**Surface area of sphere** $= 4\pi r^2$ **Curved surface area of cone** $= \pi r \times \text{slant height}$ **Volume of sphere** $= \frac{4}{3}\pi r^3$ **Series****Arithmetic series**Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$ **Geometric series**Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$ Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$ **Binomial series** $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad \text{for } |x| < 1, n \in \mathbb{Q}$ **Calculus****Quotient rule (differentiation)**

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Trigonometry**Cosine rule**In triangle ABC : $a^2 = b^2 + c^2 - 2bc \cos A$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

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Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1

$$f(x) = (x - 3)[x^2 + (p - 2)x + q]$$

Given that $f(0) = -12$

(a) find the value of q .

(2)

(b) Find the range of values of p for which the cubic equation $f(x) = 0$ has only one real root.

(5)

(Total for Question 1 is 7 marks)



2

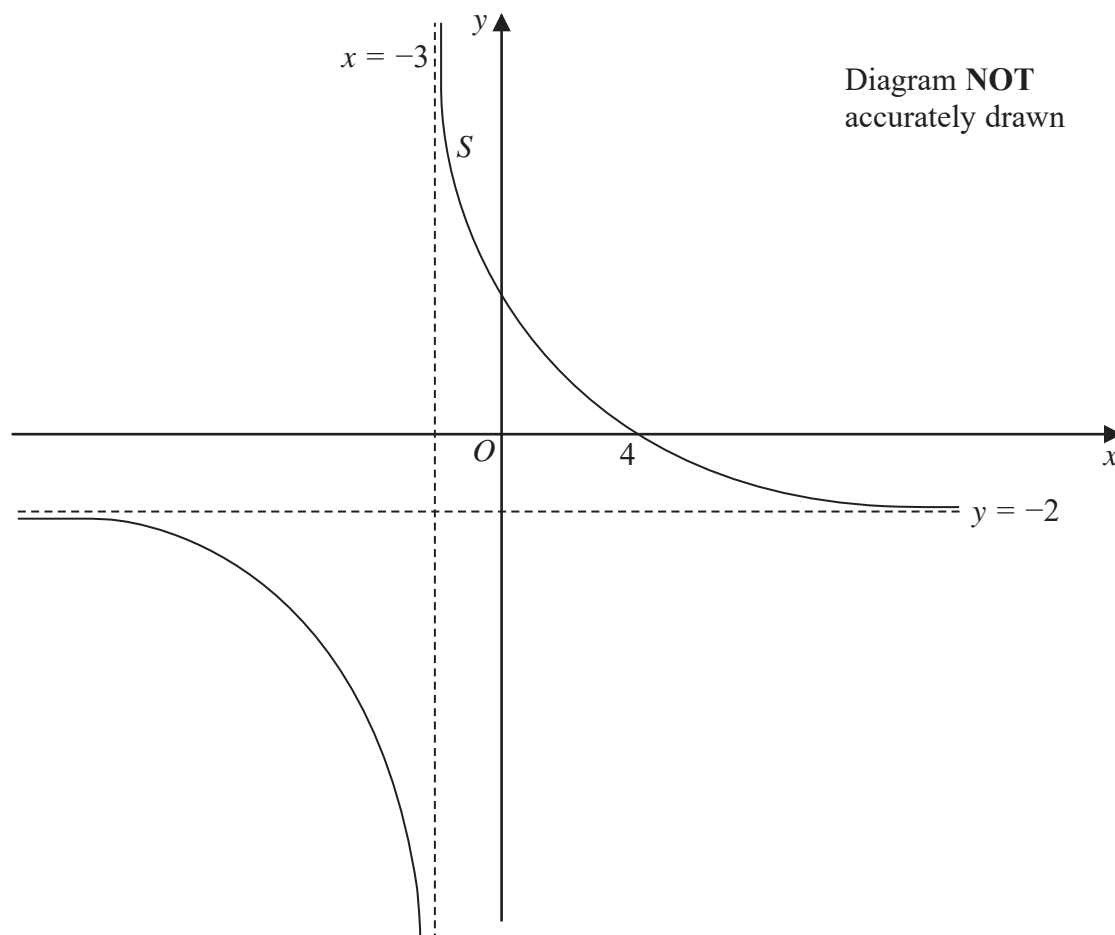


Figure 1

Figure 1 shows part of the curve S with equation $y = \frac{ax + b}{x + c}$ where a , b and c are integers.

The asymptote to S that is parallel to the x -axis has equation $y = -2$

The asymptote to S that is parallel to the y -axis has equation $x = -3$

The curve crosses the x -axis at the point with coordinates $(4, 0)$

The curve crosses the y -axis at the point with coordinates $(0, p)$ where p is a rational number.

Find

- the value of a ,
- the value of b ,
- the value of c ,
- the value of p .

(4)

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Question 2 continued

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(Total for Question 2 is 4 marks)



3

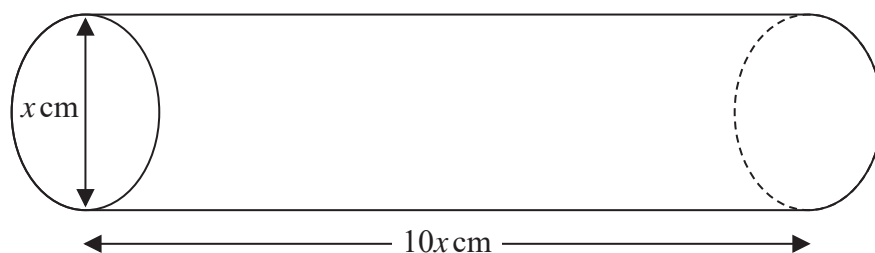
Diagram **NOT**
accurately drawn**Figure 2**

Figure 2 shows a solid right circular cylindrical metal rod.

The diameter of the rod is x cm and the length of the rod is $10x$ cm.

The rod is being heated so that the length of the rod is increasing at a rate of 0.005 cm/s.

Find the rate of increase, in cm^3/s to 2 significant figures, of the volume of the rod when $x = 3$

(6)

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Question 3 continued

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(Total for Question 3 is 6 marks)



- 4 A particle P moves along the x -axis. At time t seconds ($t \geq 0$) the acceleration, $a \text{ m/s}^2$, of P is given by $a = 6t - 12$

When $t = 0$, P is at rest at the origin.

- (a) Find the velocity of P when $t = 2$

(3)

At time T seconds, $T > 0$, P is instantaneously at rest.

- (b) Find the value of T .

(2)

- (c) Find the distance travelled by P in the first 8 seconds of its motion.

(3)

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Question 4 continued

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(Total for Question 4 is 8 marks)



- 5 (a) On the grid opposite, draw the graphs of the lines with equations

$$2x + 3y = 24 \quad y = 2x \quad 3y = 2x - 12 \quad (3)$$

- (b) Show, by shading on the grid, the region R defined by the inequalities

$$2x + 3y \leq 24 \quad y \leq 2x \quad 3y \geq 2x - 12 \quad y \geq 0 \quad (1)$$

For all points in R , with coordinates (x, y)

$$F = 2x + 5y$$

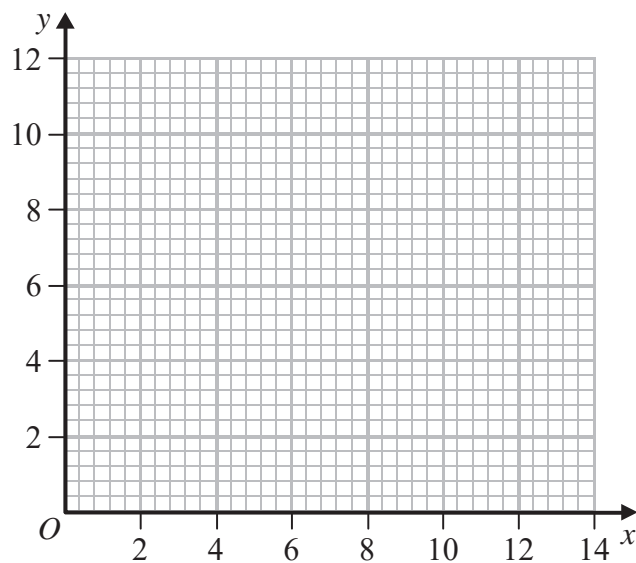
- (c) Find the greatest value of F . (3)

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Question 5 continued

Turn over for a spare grid if you need to redraw your graphs.



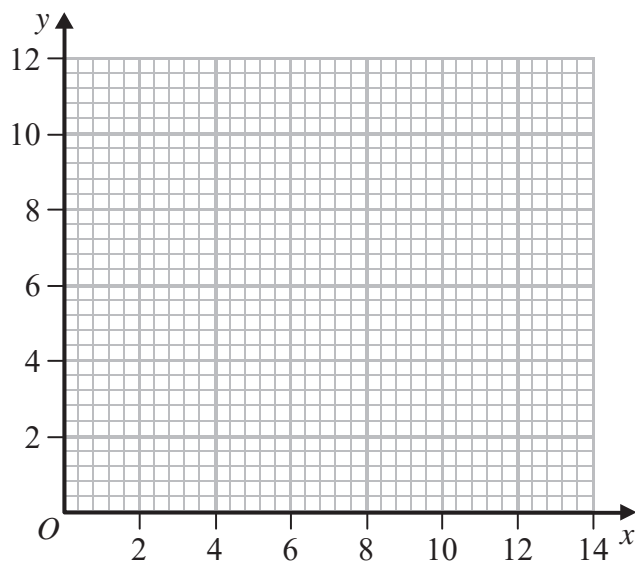
Question 5 continued

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Question 5 continued**Only use this grid if you need to redraw your graphs.****(Total for Question 5 is 7 marks)**

6 Given that $\sqrt{9-x}$ can be expressed in the form $p(1+qx)^{\frac{1}{2}}$ where p and q are constants

(a) find the value of p and the value of q . (2)

(b) Hence expand $\sqrt{9-x}$ in ascending powers of x up to and including the term in x^3 expressing each coefficient as an exact fraction in its lowest terms. (3)

Using the expansion you found in part (b) with a suitable value of x ,

(c) find an estimate to 5 decimal places for the value of $\sqrt{\frac{31}{4}}$ (3)

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Question 6 continued

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Question 6 continued

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Question 6 continued

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(Total for Question 6 is 8 marks)



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7 The n th term of a geometric series G is u_n

The first term of G is a and the common ratio of G is r , where $r > 0$

Given that $u_3 = 4$ and that $u_7 = 16$

(a) (i) show that $r = \sqrt{2}$

(ii) find the value of a .

(3)

(b) Find the least value of n for which $u_n > 500$

(4)

The sum of the first n terms of G is S_n

(c) Find S_{20}

Give your answer in the form $p(1 + \sqrt{2})$ where p is an integer.

(4)

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Question 7 continued

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Question 7 continued

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(Total for Question 7 is 11 marks)



8

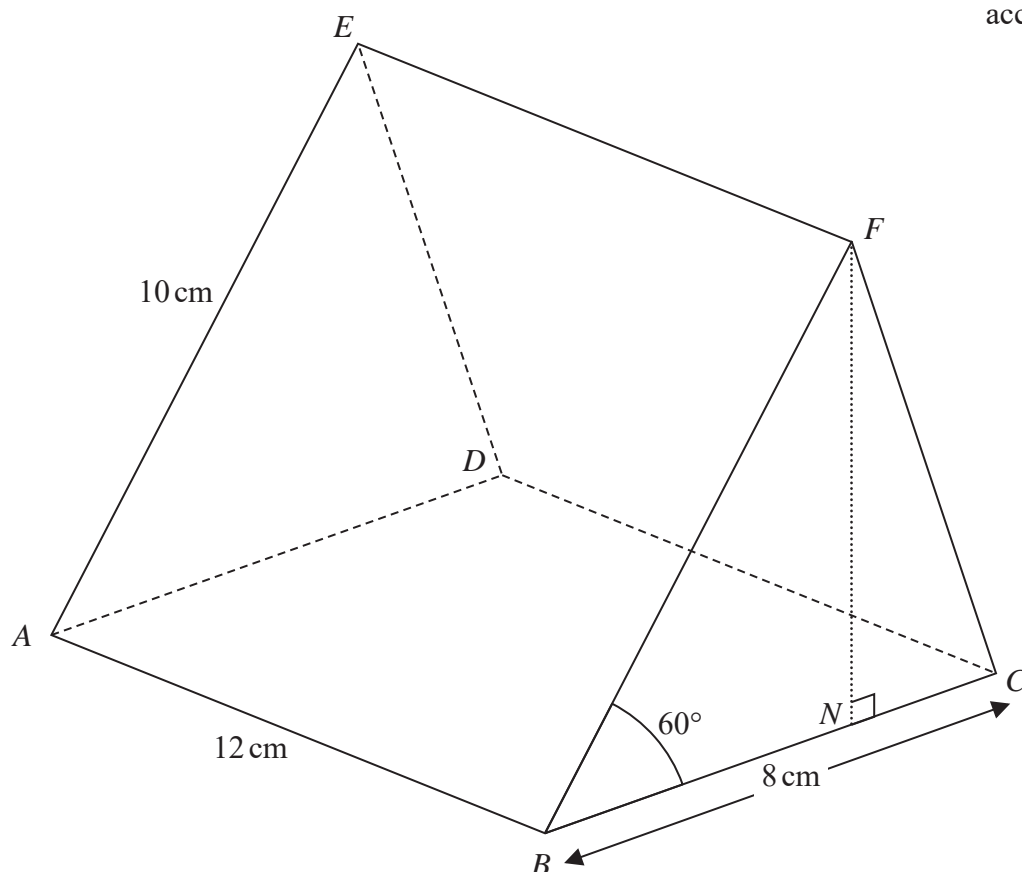
Diagram **NOT**
accurately drawn**Figure 3**

Figure 3 shows a right prism $ABCDEF$. The cross section BCF of the prism is a triangle.

$$AB = DC = 12 \text{ cm} \quad BC = AD = 8 \text{ cm} \quad BF = AE = 10 \text{ cm} \quad \angle FBC = \angle EAD = 60^\circ$$

The point N lies on BC such that FN is perpendicular to BC .

(a) Show that $BN = 5 \text{ cm}$. (2)

(b) Find, in cm to 3 significant figures, the length of EN . (3)

The midpoint of BF is X and the midpoint of FC is Y .

(c) Find, in degrees to one decimal place, the size of the angle between the plane $ABCD$ and the plane $AXYD$. (2)

(d) Find, in degrees to one decimal place, the size of the angle AYE . (6)



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Question 8 continued

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Question 8 continued

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Question 8 continued

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(Total for Question 8 is 13 marks)

P 6 1 8 8 3 A 0 2 5 3 6

- 9 The finite region R enclosed by the y -axis, the straight line with equation $y + 2x = 13$ and the curve with equation $y = x^2 - 2$, is defined for points with coordinates (x, y) with $x \geq 0$

The region R is rotated through 360° about the y -axis.

Use algebraic integration to find the volume of the solid generated.
Give your answer in terms of π .

(9)

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Question 9 continued

Area for writing answers to Question 9 continued.



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Question 9 continued

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Question 9 continued

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(Total for Question 9 is 9 marks)

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10 (a) Use the formula for $\cos(A + B)$ to show that $\cos 2A = 2\cos^2 A - 1$ (2)

(b) Show that $\cos 4A = 8\cos^4 A - 8\cos^2 A + 1$ (4)

(c) Solve the equation $\cos^2\left(\frac{\theta}{4} + \frac{\pi}{24}\right)\left[\cos^2\left(\frac{\theta}{4} + \frac{\pi}{24}\right) - 1\right] = -\frac{1}{16}$ $0 \leq \theta < 2\pi$

Give your answers in terms of π . (5)

$$f(A) = 4\cos^4 A - 4\cos^2 A + 1$$

(d) Using calculus, find the exact value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} f(A) dA$

Give your answer in the form $a\pi - b\sqrt{c}$ where a and b are fractions in their lowest terms and c is a prime number. (4)

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Question 10 continued

Area for writing answers to Question 10 continued.



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Question 10 continued

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Question 10 continued

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(Total for Question 10 is 15 marks)



11 The quadratic equation $x^2 - px + q = 0$ where $p > 0$, has roots α and β .

Given that $2\alpha\beta = 3$ and that $4(\alpha^2 + \beta^2) = k^2 - 6k - 3$ where $k > 3$

(a) (i) write down the value of q ,

(ii) find an expression, in terms of k , for p .

(5)

Given also that $7\alpha\beta = 3(\alpha + \beta)$

(b) find the value of k .

(2)

(c) Hence form an equation, with integer coefficients, which has roots

$$\frac{\alpha}{\alpha + \beta} \text{ and } \frac{\beta}{\alpha + \beta}$$

(5)

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Question 11 continued

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(Total for Question 11 is 12 marks)**TOTAL FOR PAPER IS 100 MARKS**