Question number	Scheme	Marks
10 (a) (i)	$\mathbf{a}+\mathbf{c}$	B1
(ii)	$\frac{1}{2}(\mathbf{c}-\mathbf{a})$	B1 (2)
(b)	$\overrightarrow{OX}=OA+AM+\lambda MN$ $\mathbf{a}+\frac{1}{2}\mathbf{c}+\lambda(\frac{1}{2}\mathbf{c}-\frac{1}{2}\mathbf{a})$	M1 A1
	$\mu(\mathbf{a}+\mathbf{c})$	B1
	$\mathbf{a}+\frac{1}{2}\mathbf{c}+\lambda\left(\frac{1}{2}\mathbf{c}-\frac{1}{2}\mathbf{a}\right)=\mu(\mathbf{a}+\mathbf{c})$	M1
	$1-\frac{1}{2}\lambda=\mu \qquad \frac{1}{2}+\frac{1}{2}\lambda=\mu$	M1
	$1-\frac{1}{2}\lambda=\frac{1}{2}+\frac{1}{2}\lambda$	M1
	$\lambda=\frac{1}{2} \qquad \mu=\frac{3}{4}$	A1 A1 (8)
(c)	Triangle $XBN=\frac{1}{8}$ of $\frac{1}{2}$ the parallelogram	M1
	Quadrilateral $OXNC=\frac{7}{8}$ of $\frac{1}{2}$ the parallelogram	M1
	So Quadrilateral $OXNC=\frac{7}{16}$ of the parallelogram $\therefore$ 7 : 16	A1 (3) **[13]**

Part	Mark	Additional Guidance
(a)(i)	B1	For the correct vector **a** + **c**
(ii)	B1	For the correct vector $\frac{1}{2}(\mathbf{c}-\mathbf{a})$
(b)	M1	For the correct vector statement $\overrightarrow{OX}=\overrightarrow{OA}+\overrightarrow{AM}+\lambda\overrightarrow{MN}$
	A1	For the correct vector (need not be simplified) $\overrightarrow{OX}=\mathbf{a}+\frac{1}{2}\mathbf{c}+\frac{\lambda}{2}(\mathbf{c}-\mathbf{a})$ or $\overrightarrow{OX}=\mathbf{a}+\frac{1}{2}\mathbf{c}+\lambda\left(\frac{\mathbf{c}}{2}-\frac{\mathbf{a}}{2}\right)$
	B1ft	For $\overrightarrow{OX}=\mu(\mathbf{a}+\mathbf{c})$ ft their $\overrightarrow{OB}=\text{'}\mathbf{a}+\mathbf{c}\text{'}$
	M1	For equating their two vector statements for $\overrightarrow{OX}$ $\mathbf{a}+\frac{1}{2}\mathbf{c}+\frac{\lambda}{2}(\mathbf{c}-\mathbf{a})=\mu(\mathbf{a}+\mathbf{c})$
	M1	For equating coefficients of **a** and **c** $\mathbf{a}+\frac{1}{2}\mathbf{c}+\frac{\lambda}{2}(\mathbf{c}-\mathbf{a})=\mu(\mathbf{a}-\mathbf{c})\Rightarrow\mathbf{a}\left(1-\frac{\lambda}{2}\right)+\mathbf{c}\left(\frac{1}{2}+\frac{\lambda}{2}\right)=\mu\mathbf{a}+\mu\mathbf{c}$ $\Rightarrow\mu=1-\frac{\lambda}{2},\quad\mu=\frac{1}{2}+\frac{\lambda}{2}$
	M1	For attempting to solve their two simultaneous equations in terms of λ and μ. $1-\frac{\lambda}{2}=\frac{1}{2}+\frac{\lambda}{2}\Rightarrow\lambda=\ldots\Rightarrow\mu=\ldots$ | $1-\mu=\mu-\frac{1}{2}\Rightarrow\mu=\ldots\Rightarrow\lambda=\ldots$

	A1	For **either** $\lambda = \frac{1}{2}$ **or** $\mu = \frac{3}{4}$
	A1	For **both** $\lambda = \frac{1}{2}$ **and** $\mu = \frac{3}{4}$
	ALT	
	M1	For the correct vector statement $\overrightarrow{MX} = \overrightarrow{MO} + \overrightarrow{OX}$
	A1	For the correct vector (need not be simplified) $\overrightarrow{MX} = -\frac{\mathbf{c}}{2} - \mathbf{a} + \mu(\mathbf{a}+\mathbf{c})$
	B1ft	$\overrightarrow{MX} = \frac{\lambda}{2}(\mathbf{c}-\mathbf{a})$ ft their $\overrightarrow{MN} = '\frac{1}{2}(\mathbf{c}-\mathbf{a})'$
	M1	For equating the two vector statements for $\overrightarrow{MX}$ $-\frac{\mathbf{c}}{2} - \mathbf{a} + \mu(\mathbf{a}+\mathbf{c}) = \frac{\lambda}{2}(\mathbf{c}-\mathbf{a})$
	M1	For equating coefficients of $\mathbf{a}$ and $\mathbf{c}$ $-\frac{\mathbf{c}}{2} - \mathbf{a} + \mu(\mathbf{a}+\mathbf{c}) = \frac{\lambda}{2}(\mathbf{c}-\mathbf{a}) \Rightarrow \mathbf{c}\left(-\frac{1}{2}+\mu\right) + \mathbf{a}(\mu-1) = \mathbf{c}\frac{\lambda}{2} - \mathbf{a}\frac{\lambda}{2}$ $\Rightarrow \frac{\lambda}{2} = \mu - \frac{1}{2}$ and $-\frac{\lambda}{2} = \mu - 1$
	M1	For attempting to solve their two simultaneous equations in terms of λ and μ. $\mu - \frac{1}{2} = -(\mu-1) \Rightarrow \mu = \left(\frac{3}{4}\right)$ $\quad \frac{\lambda}{2} = 1 - '\frac{3}{4}' \Rightarrow \lambda = \left(\frac{1}{2}\right)$
	A1	For **either** $\lambda = \frac{1}{2}$ **or** $\mu = \frac{3}{4}$
	A1	For **both** $\lambda = \frac{1}{2}$ **and** $\mu = \frac{3}{4}$
(c)	M1	For area of $\Delta XBN = \frac{1}{8}\Delta OBC$ so $\frac{1}{8}$ of $\frac{1}{2}$ of the area of parallelogram *OABC* $\left(\begin{array}{l} \Delta OBC = \frac{1}{2}\times OB \times BC \times \sin\angle XBN \\ \Delta XBN = \frac{1}{2}\times\frac{1}{4}OB\times\frac{1}{2}BC\times\sin\angle XBN = \frac{1}{8}\Delta OBC \end{array}\right)$
	M1	Therefore Quadrilateral *OXNC* = $\frac{7}{8}$ of $\frac{1}{2}$ of the area of parallelogram *OABC* ft their fraction from the first M mark provided it is $< \frac{1}{2}$
	A1	Quadrilateral *OXNC* $= \frac{7}{16}$ of the area of parallelogram *OABC* so ratio is 7:16

Question number	Scheme	Marks
11 (a)	Let x = the length of the side of the triangle and h = the length of the prism	
	$\frac{1}{2}x^2 \sin 60\, h = 72$ or $\frac{1}{2}\left(\sqrt{\left(x^2 - \left(\frac{1}{2}x\right)^2\right)}\right)xh = 72$	M1
	$\frac{\sqrt{3}x^2h}{4} = 72$	M1
	$h = \frac{288}{\sqrt{3}x^2}$	A1
	$S = 2 \times \frac{1}{2}x^2 \sin 60 + 3xh$ or $2 \times \frac{1}{2}\left(\sqrt{\left(x^2 - \left(\frac{1}{2}x\right)^2\right)}\right)x + 3xh$	M1
	$S = \frac{\sqrt{3}x^2}{2} + 3x\left(\frac{288}{\sqrt{3}x^2}\right)$	M1
	$S = \frac{\sqrt{3}x^2}{2} + \frac{288\sqrt{3}}{x}$ *	A1 cso (6)
(b)	$\frac{dS}{dx} = \sqrt{3}x - \frac{288\sqrt{3}}{x^2} (= 0)$	M1
	$x^3 = 288$	dM1
	$x = \sqrt[3]{288} = 6.604$	A1
	$\frac{d^2S}{dx^2} = \sqrt{3} + \frac{576\sqrt{3}}{x^3}$	ddM1
	$\frac{d^2S}{dx^2} > 0$ (when x =6.6) ∴ value is a minimum	A1 (5)
(c)	Substitutes their x into $S = \frac{\sqrt{3}x^2}{2} + \frac{288\sqrt{3}}{x}$	M1
	$S = 113$	A1 (2)
		[13]

Part	**Mark**	**Additional Guidance**
(a)	M1	For the correct expression for the volume of the prism in terms of x and h (or other letter for the length, e.g. l) Simplification not required for this mark $72 = \left(\frac{1}{2} \times x \times x \times \sin 60^\circ\right) \times h$ or $72 = \left(\frac{1}{2} \times x \times \sqrt{x^2 - \frac{x^2}{4}}\right) \times h$ or $72 = \left(\frac{\sqrt{3}}{4}x^2\right) \times h$
	M1	For an attempt to find an expression for h in terms of x Accept as a minimum $h = \frac{k}{x^2}$ where k is a positive integer
	A1	For $h = \frac{288}{\left(\sqrt{3}\right)x^2}$ or $h = \frac{96\sqrt{3}}{x^2}$

	M1	For an expression for S in terms of x and h (ft their area of the triangle) $S=2\left(\frac{1}{2}\times x\times x\times\sin 60^{\circ}\right)+3xh$ or $S=2\left(\frac{1}{2}\times x\times\sqrt{x^2-\frac{x^2}{4}}\right)+3xh$ $\left(S=\frac{\sqrt{3}}{2}x+3xh\right)$
	M1	For substituting their h into their S $S=2\left(\frac{1}{2}\times x\times x\times\sin 60^{\circ}\right)+\left(3x\times\frac{288}{\left(\sqrt{3}\right)x^2}\right)$ or $S=2\left(\frac{1}{2}\times x\times\sqrt{x^2-\frac{x^2}{4}}\right)+\left(3x\times\frac{288}{\left(\sqrt{3}\right)x^2}\right)$
	A1	For the correct expression for S as given. **The expression must be set equal to S.** $S=\frac{\sqrt{3}x^2}{2}+\frac{288\sqrt{3}}{x}$ exactly as seen here. * This is a given result so full working must be seen.
(b)	M1	For an attempt to differentiate the **given** expression for S $\frac{dS}{dx}=\sqrt{3}x-\frac{288\sqrt{3}}{x^2}$ or $\frac{dS}{dx}=\sqrt{3}x-288\sqrt{3}x^{-2}$ (See General Guidance for the definition of an attempt)
	dM1	For setting their differentiated expression = 0 and attempting to solve for x $\sqrt{3}x-\frac{288\sqrt{3}}{x^2}=0\Rightarrow x^3=288\Rightarrow x=(6.604)$ (rounded correctly) This mark is dependent on the first M mark in (b)
	A1	For $x = 6.604$ rounded correctly
	dM1	For attempting the second derivative (usual definition of an attempt) $\frac{d^2S}{dx^2}=\sqrt{3}+\frac{576\sqrt{3}}{x^3}$ This mark is dependent on first M mark in (b)
	A1ft	Concludes either that $\frac{d^2S}{dx^2}>0$ for all **positive** values of x **or** substitutes in their value of x to show that $\frac{d^2S}{dx^2}=5.19.....$ hence positive so must be a minimum. Only ft if the final conclusion is a minimum provided their $\frac{d^2S}{dx^2}$ is algebraically correct
	ALT – for justifying the minimum using their derivative	
	dM1	Chooses a value either side of their value of x and substituting them into their $\frac{dS}{dx}$ e.g. $x = 6$ and 7 $\frac{dS}{dx}=\sqrt{3}\times 6-\frac{288\sqrt{3}}{6^2}=-3.46...$ and $\frac{dS}{dx}=\sqrt{3}\times 7-\frac{288\sqrt{3}}{7^2}=1.944...$
	A1ft	Concludes that the gradient function moves from negative to positive hence must be a minimum.
(c)	M1	Substitutes their value of x into the given expression for S $S=\frac{\sqrt{3}\,'6.604'^2}{2}+\frac{288\sqrt{3}}{'6.604'}=...$
	A1	For $S = 113$ rounded correctly