Question	Scheme	Marks
5	$\sqrt{2x+6}=\frac{x}{3}+1\Rightarrow 2x+6=\frac{x^2}{9}+\frac{2x}{3}+1\Rightarrow\frac{x^2}{9}-\frac{4x}{3}-5=0$	M1
	$\Rightarrow x^2-12x-45=0\Rightarrow(x+3)(x-15)=0\Rightarrow x=-3,15$	M1A1
	$\left[\text{When } x=15,\quad y=6\right]$	
	$(V=)\pi\int_{'-3'}^{'15'}(2x+6)\ \mathrm{d}x-\frac{1}{3}\times\pi\times'6'^2\times('15'-'-3')$	M1M1
	$(V=)(\pi)\left[\frac{2x^2}{2}+6x\right]_{'-3'}^{'15'}-216\pi$	M1
	$(V=)\pi\left[\left(\frac{2\times'15'^2}{2}+6\times'15'\right)-\left(\frac{2\times('-3')^2}{2}+6\times('-3)'\right)\right]-216\pi(=324\pi-216\pi)=108\pi$	M1A1 [8]
ALT last 5 marks	OR	
	$(V=)\pi\int_{'-3'}^{'15'}(2x+6)\ \mathrm{d}x-\pi\int_{'-3'}^{'15'}\left(\frac{x^2}{9}+\frac{2x}{3}+1\right)\mathrm{d}x$ or $\pi\int_{'-3'}^{'15'}(2x+6)\ \mathrm{d}x-\pi\int_{'-3'}^{'15'}\left(\frac{x}{3}+1\right)^2\ \mathrm{d}x$	M1M1
	$(V=)(\pi)\left[\frac{2x^2}{2}+6x\right]_{'-3'}^{'15'}-\pi\left[\frac{x^3}{9\times3}+\frac{2x^2}{3\times2}+x\right]_{'-3'}^{'15'}$	M1
	$V=\pi\left[\left(\frac{2\times'15'^2}{2}+6\times'15'\right)-\left(\frac{2\times('-3')^2}{2}+6\times('-3')\right)\right]$ $-\pi\left[\left(\frac{'15'^3}{9\times3}+\frac{2\times'15'^2}{3\times2}+15\right)-\left(\frac{('-3')^3}{9\times3}+\frac{2\times('-3')^2}{3\times2}'-3'\right)\right]=108\pi$	M1A1
	Total 8 marks	

Mark	Notes
M1	Sets the equation of the curve = equation of line and attempts to form a 3TQ, which must include an attempt to expand $\left(\frac{1}{3}x+1\right)\left(\frac{1}{3}x+1\right)$. Allow only 1 error in processing overall. Accept as a minimum for this mark: $\frac{x^2}{9} \pm Jx - K = 0$ or $x^2 \pm Lx - M = 0$ $J, K, L, M > 0$
M1	Solves their 3TQ using a full and minimally acceptable method (see general guidance) to find two values. They must show their method if using an incorrect 3TQ. Note this is not a dependent mark, so the candidate can solve a 3TQ not of the form required for the first M mark.
A1	$x = -3, 15$ M1 M1 A1 may be awarded for both correct values given without working.
M1	For the correct expression for the volume of rotation under the curve including π and ft their $x = -3, 15$.
M1	For the correct expression for the volume of the cone. Ft their $x = -3, 15$ and their value of y only (must be clear they are using a value of y. '6' cannot be 15 or – 3). $\frac{1}{3} \times \pi \times '6'^2 \times ('15' - '-3')$
M1	For an attempt to integrate their expression for the curve. Minimally acceptable attempt (see general guidance) – no power of x to decrease. π and limits don't need to be present.
M1	For substitution of their limits into any changed expression representing their volume of the curve and evaluation of the final volume using their volume of the cone. Not a dependent mark but substitution **must** be into their changed expression. This mark may be awarded for the work being done in parts and subtracted in any order at any point, but **there must be a subtraction.** This M mark may be implied by a correct final answer.
A1	For $V = 108\pi$ **This mark may not be awarded if the subtraction is the incorrect way around and the sign of the answer is simply changed at the end.**
ALT last 5 marks	
M1	For the correct expression for the volume of rotation under the curve including π and ft their $x = -3, 15$
M1	For the correct expression for the volume of rotation under the straight line including π and ft their $x = -3, 15$
Where candidates have combined the 2 expressions, M1 M1 can be awarded for $\pm\pi\int_{-3}^{15}\left(\frac{x^2}{9} - \frac{4x}{3} - 5\right)\mathrm{d}x$ **or** for a clear attempt to combine the correct two expressions, even if the following simplification is incorrect. [Note for final A mark correct integral is $\pi\int_{-3}^{15}\left(-\frac{x^2}{9} + \frac{4x}{3} + 5\right)\mathrm{d}x$	
M1	For an attempt to integrate the expression for **either** the curve **or** the line **or** what they intend to be **<u>their</u>** combined expression. Minimally acceptable attempt (see general guidance) – no power of x to decrease. π and limits do not need to be present for this mark.
M1	For substitution of their limits into any changed expressions representing their volume of the line and the curve and evaluation of the final volume or their combined expression. There must be at least one correct substitution of both limits into each expression. Not a dependent mark but substitution **must** be into changed expressions. This mark may be awarded for the work being done in parts and subtracted in any order at any point, but **there must be a subtraction.** This M mark may be implied by a correct final answer.
A1	For $V = 108\pi$ **This mark may not be awarded if the subtraction is the incorrect way around and the sign of the answer is simply changed at the end.**

Question	Scheme	Marks
PLEASE REMEMBER TO CHECK DIAGRAM FOR RELEVANT LENGTHS AND WORKING **Throughout the question where lengths are in brackets eg (AC =), this means you don't have to see AC =.** **But the mark cannot be awarded for a clearly incorrect method eg VO = $\sqrt{x^2+x^2}$ for the first method mark in (a) scores M0**		
6(a)	$(AC=)\sqrt{x^2+x^2}\left(=\sqrt{2}x\right)$ **or** $(AC=)\dfrac{x}{\sin 45}$ **or** $(AC=)\dfrac{x}{\cos 45}$ $(AO=)\sqrt{\left(\dfrac{x}{2}\right)^2+\left(\dfrac{x}{2}\right)^2}\left(=\sqrt{\dfrac{x^2}{2}}=\dfrac{x}{\sqrt{2}}\right)$ **or** $(AO=)\dfrac{\frac{x}{2}}{\sin 45}$ **or** $(AO=)\dfrac{\frac{x}{2}}{\cos 45}$	M1
	$(VO=)\sqrt{x^2-\left(\dfrac{"\sqrt{2}x"}{2}\right)^2}$ **or** $\sqrt{x^2-\left(\dfrac{1}{2}\sqrt{x^2+x^2}\right)^2}$ **or** $\sqrt{x^2-\dfrac{x^2}{2}}\left(=\sqrt{\dfrac{1}{2}x^2}\right)$ **or** $(VO=)\dfrac{\sqrt{2}}{2}\tan 45$	M1
	$=\dfrac{\sqrt{2}}{2}x$ (cm)*	A1*cso [3]
ALT	Any indication the candidate realises triangle AVC is right angled and isosceles.	M1
	$\sin 45=\dfrac{VO}{x}$ **or** $x\sin 45=VO$	M1
	$VO=\dfrac{\sqrt{2}}{2}x$ *	A1*cso [3]
(b)	$(\angle AVC=)\cos^{-1}\left(\dfrac{x^2+x^2-\left(\sqrt{2}x\right)^2}{2\times x\times x}\right)$ **or** $(\angle AVC=)2\times\sin^{-1}\left(\dfrac{\frac{\sqrt{2}}{2}x}{x}\right)$ **or** $(\angle AVC=)2\times\cos^{-1}\left(\dfrac{\frac{\sqrt{2}}{2}x}{x}\right)$ **or** $(\angle AVC=)2\times\tan^{-1}\left(\dfrac{\frac{\sqrt{2}}{2}x}{\frac{\sqrt{2}}{2}x}\right)$	M1
	$=90^\circ$	A1 [2]

(c)	Let M be the midpoint of AB and let N be the midpoint of DC	
	$(VM = VN =)\sqrt{x^2-\left(\frac{x}{2}\right)^2} = \left(\frac{\sqrt{3}}{2}x\right)$	M1
	$\cos MVN = \dfrac{\left("\frac{\sqrt{3}}{2}x"\right)^2+\left("\frac{\sqrt{3}}{2}x"\right)^2-x^2}{2\times\left("\frac{\sqrt{3}}{2}x"\right)\times\left("\frac{\sqrt{3}}{2}x"\right)}$ **or** $(\angle MVN =)\cos^{-1}\left(\dfrac{\left("\frac{\sqrt{3}}{2}x"\right)^2+\left("\frac{\sqrt{3}}{2}x"\right)^2-x^2}{2\times\left("\frac{\sqrt{3}}{2}x"\right)\times\left("\frac{\sqrt{3}}{2}x"\right)}\right)$ **or** $(\angle MVN =)2\times\cos^{-1}\left(\dfrac{\frac{\sqrt{2}}{2}x}{"\frac{\sqrt{3}}{2}x"}\right)$ **or** $(\angle MVN =)2\times\sin^{-1}\left(\dfrac{\frac{1}{2}x}{"\frac{\sqrt{3}}{2}x"}\right)$ **or** $(\angle MVN =)2\times(\angle MVO =)$ **and** $\cos MVO = \dfrac{\frac{\sqrt{2}}{2}x}{"\frac{\sqrt{3}}{2}x"}$	M1
	$70.528...^\circ \approx 70.5^\circ$	A1 [3]
	ALT $OM = \frac{x}{2}$	M1
	$(\angle MVN =)2\tan^{-1}\left[\dfrac{\frac{x}{2}}{\frac{\sqrt{2}x}{2}}\right] = \left(2\tan^{-1}\left[\frac{1}{\sqrt{2}}\right] =\right)70.528...^\circ \approx 70.5^\circ$	M1A1 [3]
(d)	Volume $= \frac{1}{3}\times\left(\frac{\sqrt{2}}{2}x\right)\times x^2\left(=\frac{\sqrt{2}}{6}x^3\right)$ oe	M1
	$200 = \frac{1}{3}\times\left(\frac{\sqrt{2}}{2}x\right)\times x^2 \Rightarrow x =$	dM1
	$x = 9.4672... \approx 9.47(\text{cm})$	A1 [3]
		Total 11 marks

Part	Mark	Notes
For part (a), we must see the algebraic proof, including *x* throughout.		
(a)	M1	For correct use of Pythagoras theorem or any correct appropriate trigonometry and a complete method to find the length *AC* or *AO* ie candidates must rearrange correctly – minimum steps as shown in mark scheme and no errors must be present.
	M1	For a full, correct and complete method to find *VO*, using their *AC/AO*.
	A1 cso	For $\frac{\sqrt{2}}{2}x$ (cm) – correct solution only, no errors or omissions, minimum steps as shown.
ALT	M1	Any indication the candidate realises triangle *AVC* is right angled and isosceles. This mark may be implied by correct working for the next method mark or present on the diagram.
	M1	For the correct trig minimum steps as shown and no errors.
	A1 cso	For $\frac{\sqrt{2}}{2}x$ (cm) – correct solution only, no errors or omissions.
(b)	M1	For any correct appropriate trigonometry to find angle *AVC*. $(\angle AVC =)$ If finding $\angle AVO$, must be doubled for this mark. *AC* must be $\sqrt{2}x$ oe, *AO* must be $\frac{\sqrt{2}}{2}x$ oe.
	A1	For $90°$
	Allow M1A1 for candidates writing down the correct answer by inference.	
Consistent omission of *x* is permissible for all marks in part (b).		
(c)	M1	For correct Pythagoras theorem to find the lengths of *VM* or *VN*
	M1	For any correct trigonometry to find angle *MVN*, such as the statements shown in the mark scheme. The candidate can state, for example, cos(*MVN*) = fully correct expression (using their *VN* and *VM*) or, for example, $\cos^{-1}$(fully correct expression), where they do not have to write angle *MVN* = i.e – this can involve any fully correct method to reach an equation which would allow them to use inverse trig to find *MVN* or if they use inverse trig, a fully correct expression must be seen with the correct inverse trig function. Send any items where allocation of marks is unclear (potentially using other methods to the main and ALT) to Review. or fully correct **method** to find angle *MVO* which must then be doubled for this mark.
	A1	For any answer stated which rounds to 70.5° (ie more decimal places is condoned).
Consistent omission of *x* is permissible for all marks in part (c).		
ALT	M1	States or uses $OM = \frac{x}{2}$
	M1	For fully correct **method** to find angle *MVO* which must then be doubled for this mark.
	A1	For any answer stated which rounds to 70.5° (ie more decimal places is condoned).
(d)	M1	For a correct unsimplified expression for the volume of the pyramid as shown.
	dM1	For placing their expression = 200, which must involve a term in x^3, and a fully correct method/rearrangement to solve. Note this mark is dependent on the first method mark, so they must have got the correct unsimplified expression even if then simplified incorrectly.
	A1	For any final answer stated which rounds to 9.47 (ie more decimal places is condoned).