Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International GCSE

Thursday 8 June 2023

Morning (Time: 2 hours)

Paper reference **4PM1/02**

Further Pure Mathematics
PAPER 2

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\qquad$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\qquad$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$ $\qquad$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

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Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Given that $\frac{a + 2\sqrt{5}}{3 - \sqrt{5}} = \frac{11 + b\sqrt{5}}{2}$ where a is an integer and b is prime,

find the value of a and the value of b
Show your working clearly.

(5)

(Total for Question 1 is 5 marks)

2 The nth term of a convergent geometric series is $8^{(1-2n)}$

Find the sum to infinity of the series.

Give your answer in the form $\frac{p}{q}$ where p and q are integers to be found.

(6)

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Question 2 continued

(Total for Question 2 is 6 marks)

3

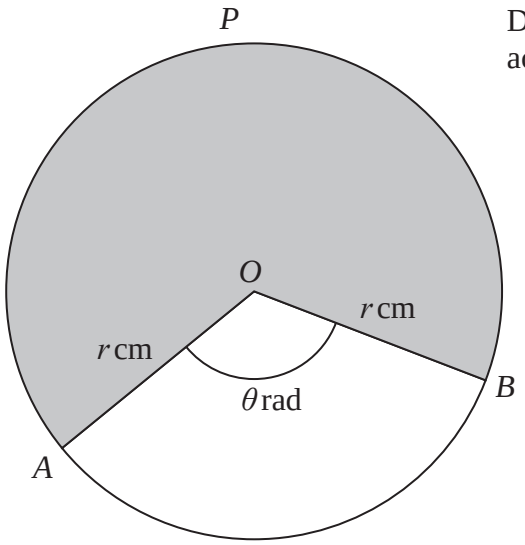

Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows a circle, centre O, with radius r cm.

The points A, P and B lie on the circle.

The obtuse angle $AOB = \theta$ radians.

The area of the sector $APBO$, shown shaded, is 372.4 cm^2 and the length of the arc APB is 53.2 cm.

Find, to 3 significant figures where appropriate, the value of

(i) r

(ii) θ

(6)

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Question 3 continued

(Total for Question 3 is 6 marks)

4

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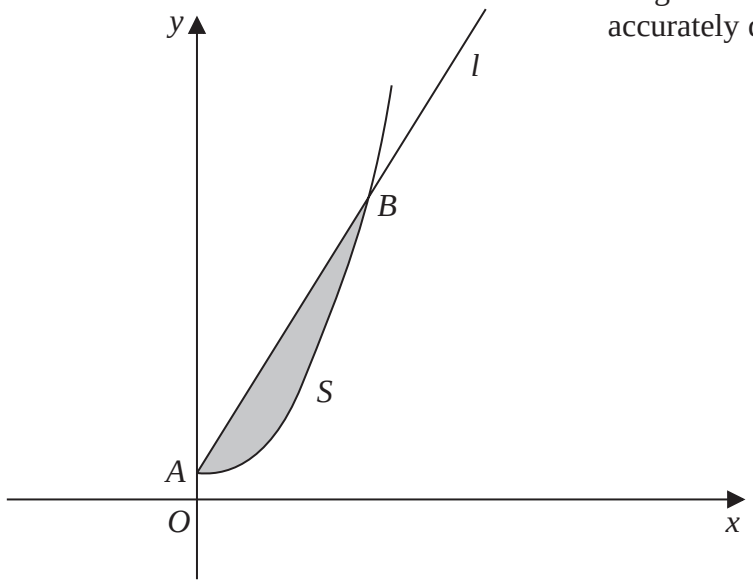

Figure 2

The curve S with equation $y = \frac{x^2}{4} + 2$ where $x \geqslant 0$ and the line l with equation $2y - x - 4 = 0$ where $x \geqslant 0$ intersect at the points A and B, as shown in Figure 2.

(a) (i) Show that the coordinates of point A are $(0, 2)$

(ii) Find the coordinates of the point B

(4)

The finite region bounded by S and l, shown shaded in Figure 2, is rotated through 2π radians about the y-axis.

(b) Use algebraic integration to find the volume of the solid generated. Give your answer in terms of π

(4)

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Question 4 continued

Question 4 continued

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Question 4 continued

(Total for Question 4 is 8 marks)

5 (a) On the grid opposite draw the line with equation

(i) $y = 2x + 5$ (ii) $4y = x - 8$ (iii) $5y + 3x = 30$

(3)

(b) Show, by shading, the region R defined by the inequalities

$y \leqslant 2x + 5$ $4y \geqslant x - 8$ $5y + 3x \leqslant 30$

(1)

For all points in R with coordinates (x, y)

$$P = 2x - 5y$$

(c) Using your graph, find the least value of P

(3)

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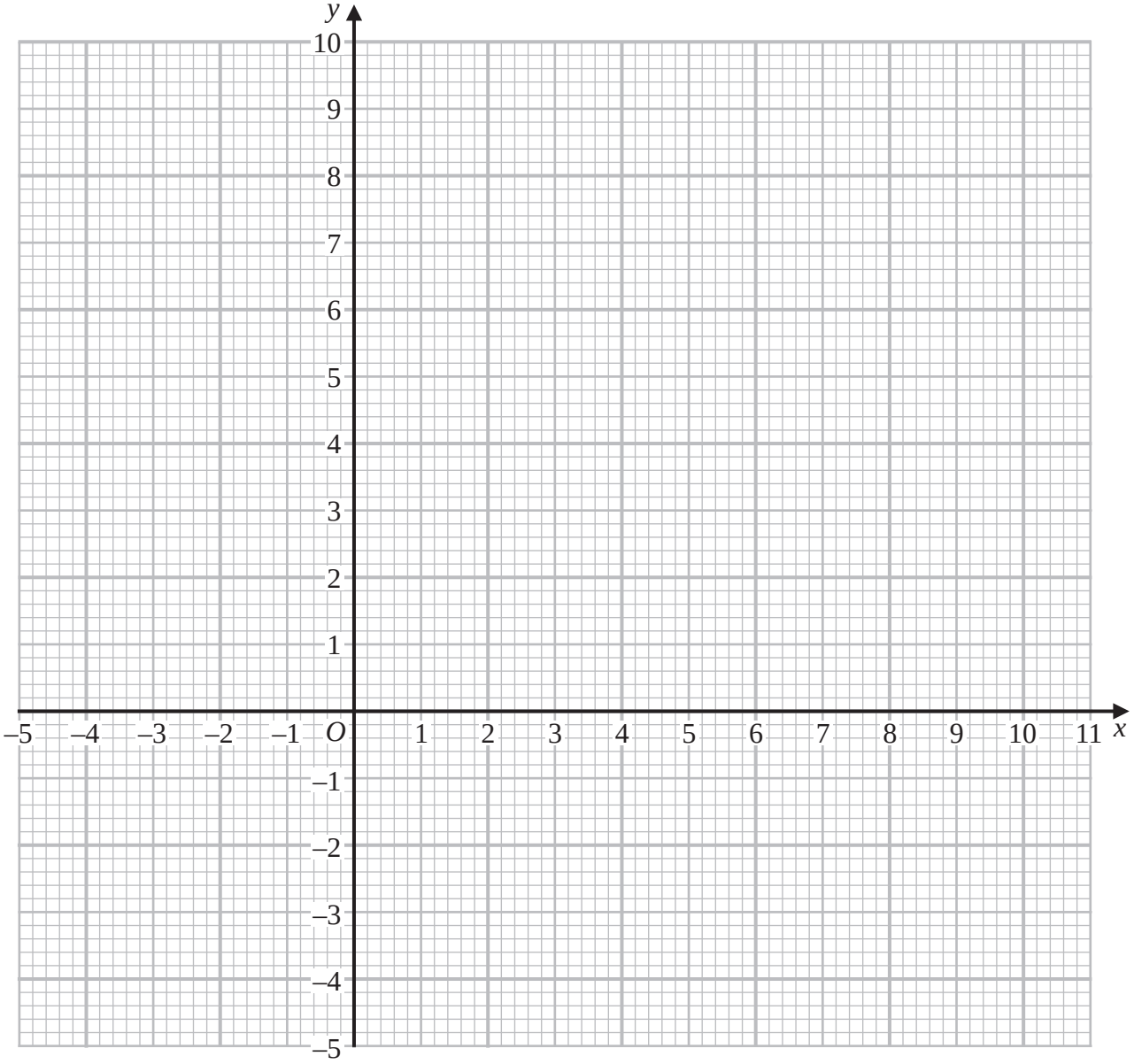
Question 5 continued



Turn over for a spare grid if you need to redraw your graph.

Question 5 continued

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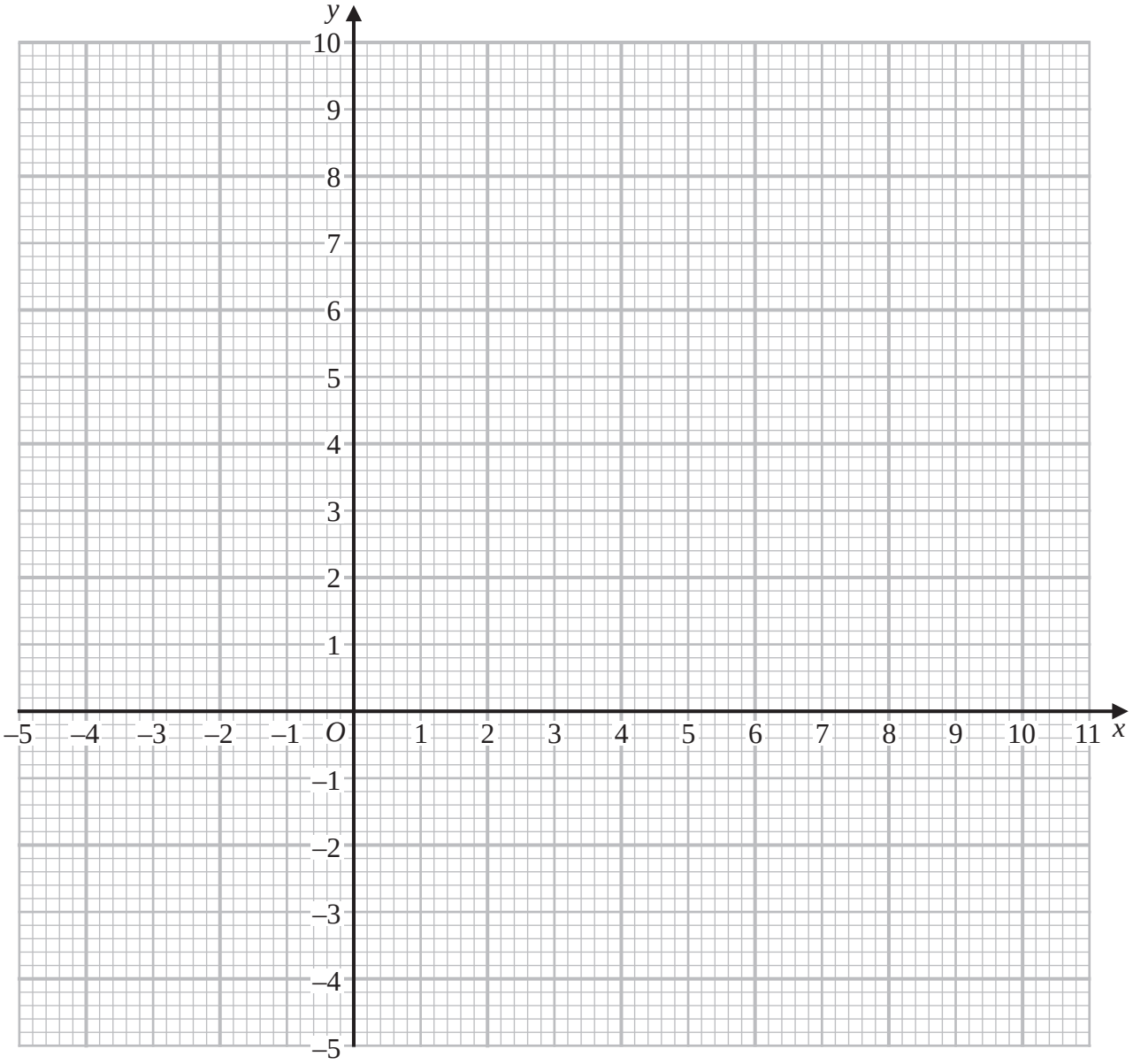
Question 5 continued

Only use this grid if you need to redraw your graph.



(Total for Question 5 is 7 marks)

6

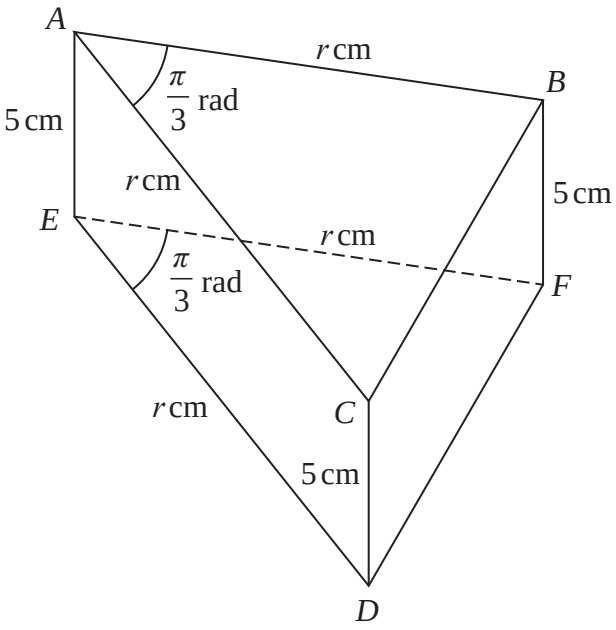


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Figure 3

Figure 3 shows a right triangular prism *ABCDEF*. A cross section *ABC* of the prism is a triangle in which $AB = AC = r$ cm and $\angle CAB = \frac{\pi}{3}$ radians.

In the prism

$$AE = BF = CD = 5\,\text{cm} \qquad ED = EF = r\,\text{cm} \text{ and } \angle DEF = \frac{\pi}{3} \text{ radians}$$

(a) Show that the volume of the prism is $\frac{5\sqrt{3}}{4} r^2$ cm^3

(1)

The volume of the prism is increasing in such a way that the size of $\angle CAB$ and the size of $\angle DEF$ remain constant and the length of *AE*, the length of *BF* and the length of *CD* remain constant.
The lengths of *AB*, *AC*, *ED* and *EF* are each increasing at a constant rate of 0.2 cm/s

(b) Find the exact rate of increase, in cm^3/s, of the volume of the prism when the area of the rectangular face *BCDF* is 60 cm^2

(5)

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Question 6 continued

Question 6 continued

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Question 6 continued

(Total for Question 6 is 6 marks)

7 (a) Expand $\left(1+\frac{x}{3}\right)^{-3}$ in ascending powers of x up to and including the term in x^3

Where appropriate express each coefficient as an exact fraction in its lowest terms.

(3)

(b) Write down the range of values of x for which your expression is valid.

(1)

(c) Express $(3+x)^{-3}$ in the form $P(1+Qx)^{-3}$ where P and Q are rational numbers whose values should be stated.

(2)

$$f(x) = \frac{(1+4x)}{(3+x)^3}$$

(d) Obtain a series expansion for $f(x)$ in ascending powers of x up to and including the term in x^2

(2)

(e) Hence, using algebraic integration, obtain an estimate of $\int_0^{0.2} f(x)\,dx$

Give your answer to 5 significant figures.

(3)

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Question 7 continued

Question 7 continued

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Question 7 continued

(Total for Question 7 is 11 marks)

8 The points A and B have coordinates $(-6, 8)$ and $(12, 2)$ respectively.

(a) Find an equation of the straight line passing through A and B in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

(b) Find the exact length of AB

(2)

The point X with coordinates (m, n) lies on AB such that $AX : XB = 1 : 2$

(c) Find the value of m and the value of n

(2)

The line L passes through the point X and is perpendicular to AB
The point C with coordinates (p, q) lies on L where $p > 0$ and $q > 0$

Given that AB is a diameter of a circle and C also lies on the circumference of the circle,

(d) find

(i) the exact value of p

(ii) the exact value of q

(7)

(e) Find the exact area of triangle ABC

(3)

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Question 8 continued

Question 8 continued

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Question 8 continued

(Total for Question 8 is 17 marks)

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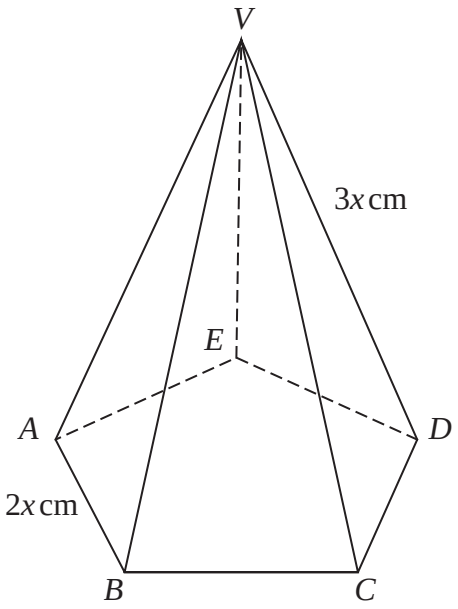

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Figure 4

Figure 4 shows a right pyramid with vertex V and base $ABCDE$ which is a regular pentagon.

$$AB = BC = CD = DE = EA = 2x\,\text{cm}$$
$$VA = VB = VC = VD = VE = 3x\,\text{cm}$$

Find, in degrees to one decimal place, the size of the angle between the plane VBC and the base $ABCDE$

(6)

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Question 9 continued

(Total for Question 9 is 6 marks)

10 The curve C with equation $y = \dfrac{6 - 3x}{x - 4}$ where $x \neq 4$, crosses the x-axis at the point P and the y-axis at the point Q

(a) Find the coordinates of

(i) P (ii) Q

(2)

(b) Write down an equation of the asymptote to C which is

(i) parallel to the y-axis (ii) parallel to the x-axis

(2)

(c) Sketch C showing clearly the asymptotes and the coordinates of the points P and Q

(3)

The line L is the normal to C at the point on C where $x = 2$

(d) Find an equation of L

(6)

The line L intersects C again at the point R

(e) Find the x coordinate of R

(3)

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Question 10 continued

Question 10 continued

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Question 10 continued

(Total for Question 10 is 16 marks)

11 The roots of a quadratic equation E are α and β where $\alpha > \beta > 0$

Given that $\alpha - \beta = 2\sqrt{6}$ and $\alpha^2 + \beta^2 = 30$

(a) show that

(i) $\alpha\beta = 3$

(4)

(ii) $\alpha + \beta = 6$

(2)

(b) Without solving E

(i) find the value of $\alpha^4 + \beta^4$

(2)

(ii) find the exact value of $\alpha^4 - \beta^4$

(2)

Given that $\alpha^4 = P + Q\sqrt{6}$ where P and Q are positive integers,

(c) find the value of P and the value of Q

(2)

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Question 11 continued

Question 11 continued

(Total for Question 11 is 12 marks)

TOTAL FOR PAPER IS 100 MARKS

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