Question Number	Scheme	Marks
9		
(a)	$\cos(\theta+\theta)=\cos\theta\cos\theta-\sin\theta\sin\theta$	M1
	$\cos 2\theta=\cos^2\theta-(1-\cos^2\theta)$	M1
	$\cos 2\theta = 2\cos^2\theta-1$ *	A1cso (3)
(b)	$\sin 2\theta = 2\sin\theta\cos\theta$	B1 (1)
(c)	$\cos 3\theta=\cos 2\theta\cos\theta-\sin 2\theta\sin\theta$	M1
	$=(2\cos^2\theta-1)\cos\theta-2\sin\theta\cos\theta\sin\theta$	M1
	$=2\cos^3\theta-\cos\theta-2(1-\cos^2\theta)\cos\theta$	M1
	$=4\cos^3\theta-3\cos\theta$ *	A1cso (4)
(d)	$1=8\cos^3\theta-6\cos\theta=2\cos 3\theta$	
	$\cos 3\theta=\dfrac{1}{2}$	M1
	$3\theta=\dfrac{\pi}{3},\ \dfrac{5\pi}{3},\ \dfrac{7\pi}{3}$	M1
	$\theta=\dfrac{\pi}{9},\ \dfrac{5\pi}{9},\ \dfrac{7\pi}{9}$	A1A1 (4)
(e)	$\int(8\cos^3\theta+4\sin\theta)\,\mathrm{d}\theta=\int(2\cos 3\theta+6\cos\theta+4\sin\theta)\,\mathrm{d}x$	M1
	$=\dfrac{2}{3}\sin 3\theta+6\sin\theta-4\cos\theta\ (+c)$	A1
(ii)	$=\dfrac{2}{3}\sin\pi+6\sin\dfrac{\pi}{3}-4\cos\dfrac{\pi}{3}-(-4\cos 0)$	dM1
	$=6\times\dfrac{\sqrt{3}}{2}-2+4\ \ =3\sqrt{3}+2$	A1cao cso (4) [16]

9	If c, s used for cos and sin allow for all marks except final A marks in each section. For these marks the candidate must return to cos, sin as appropriate.
(a)	
M1	Replace A and B with θ in $\cos(A+B)=\cos A\cos B-\sin A\sin B$
M1	Use $\sin^2\theta=1-\cos^2\theta$ to eliminate $\sin^2\theta$
A1cso	Obtain the **given** identity with no errors seen
(b)	
B1	$\sin 2\theta=2\sin\theta\cos\theta$ Must be simplified
(c)	
M1	Use $\cos(A+B)=\cos A\cos B-\sin A\sin B$ with $A=2\theta,\ B=\theta$ to eliminate 3θ
M1	Use $\cos 2\theta=2\cos^2\theta-1$ **and** $\sin 2\theta=2\sin\theta\cos\theta$ to obtain an expression with powers of $\sin\theta$ and $\cos\theta$
M1	Use $\sin^2\theta=1-\cos^2\theta$ to eliminate $\sin^2\theta$ leaving powers of cos only
A1cso	Obtain the **given identity** with no errors seen
(d)	
M1	Use the identity given in (c) to change the equation to the form $\cos 3\theta=k$ where $-1<k<1$
M1	Obtain 1 value in range $0\leqslant 3\theta<3\pi$ in terms of π for 3θ
A1	Any 2 correct values for θ
A1	Third correct value for θ Ignore any answers outside the range, deduct one A mark if extras within the range are included.
ALT:	Can work in degrees for the M marks; A marks only available if answers changed to radians without loss of accuracy.
(e)	
(i) M1	Change the given integrand to one which can be integrated, either by using the identity in (c) or by obtaining $\int\left(8\cos^3\theta+4\sin\theta\right)\mathrm{d}\theta=\int\left(8\left(1-\sin^2\theta\right)\cos\theta+4\sin\theta\right)\mathrm{d}x$
A1	Correct integration $\frac{2}{3}\sin 3\theta+6\sin\theta-4\cos\theta\ (+c)$ or $8\sin\theta-\frac{8}{3}\sin^3\theta-4\cos\theta\ (+c)$ Constant of integration may be missing.
(ii) dM1	Substitute both limits into their answer from (i)- evidence needed of substitution of 0 Substitution of upper limit followed by - 0 qualifies
A1cao cso	$3\sqrt{3}+2$ oe two terms only.
	Watch for: $\int\left(8\cos^3\theta+4\sin\theta\right)\mathrm{d}\theta=\left[8\sin^3\theta-4\cos\theta\right]_0^{\frac{\pi}{3}}$ $=3\sqrt{3}-2-(-4)$ $=$ correct answer!! But from completely INCORRECT working.