Mark Scheme (Results)

Summer 2021

Pearson Edexcel International GCSE

In Mathematics B (4PM1)

Paper 02

Summer 2021
Question Paper Log Number P66025A
Publications Code 4PM1_02_2106_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme - not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is given.
- Crossed out work should be marked **unless** the candidate has replaced it with an alternative response.

- **Types of mark**
    - M marks: method marks
    - A marks: accuracy marks
    - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
    - cao – correct answer only
    - ft – follow through
    - isw – ignore subsequent working
    - SC - special case
    - oe – or equivalent (and appropriate)
    - dep – dependent
    - indep – independent
    - awrt – answer which rounds to
    - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers normally score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**
  If the final answer is wrong, always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
  If there is a choice of methods shown, then award the lowest mark, unless the answer on the answer line makes clear the method that has been used.
  If there is no answer achieved then check the working for any marks appropriate from the mark scheme.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right),\text{ where }|pq|=|c| \quad \text{leading to } x=\ldots.$$

$$\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right)\text{ where }|pq|=|c|\text{ and }|mn|=|a| \quad \text{leading to } x=\ldots.$$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=\ldots.$

3. Completing the square:

$$x^2+bx+c=0:\ \left(x\pm\tfrac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0 \qquad \text{leading to } x=\ldots.$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n\to x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n\to x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication

from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

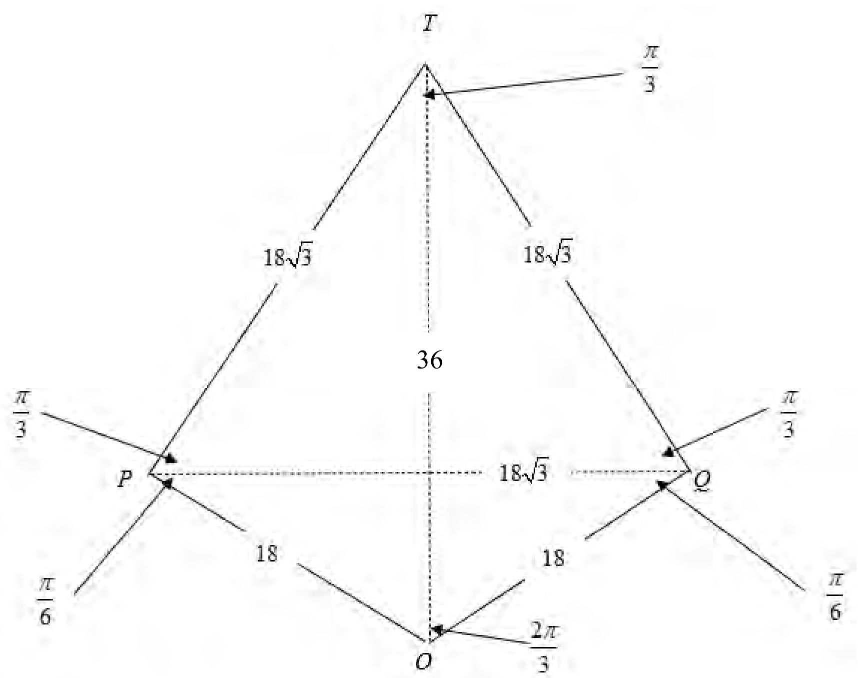
Paper 1		
Question number	Scheme	Marks
1 (a)	$3x < 12$	M1
	$x < 4$	A1
		[2]
(b)	$(2x+1)(x-3) > 0$	M1
	Critical values are $x = -\frac{1}{2}$ and $x = 3$	M1
	$x < -\frac{1}{2}$ $\quad x > 3$	A1
		[3]
(c)	$x < -\frac{1}{2}$ $\quad 3 < x < 4$	B1ft
		[1]
		Total 6 marks

Part	Mark	Guidance
(a)	M1	Attempts to solve the inequality to achieve $3x < 12$ Allow $3x < a$ where a is an integer
	A1	For $x < 4$
(b)	M1	Attempts to solve the inequality by any method to find critical values See General Guidance for acceptable methods. If a calculator is used, the critical values must be fully correct for this mark. Allow = or > for this mark or even no sign at all provided it is clear they are solving a quadratic. $(2x+1)(x-3) > 0 \Rightarrow x = ..., \ ... \quad \left(x = -\frac{1}{2}, \ 3\right)$
	M1	For forming a correct inequality, which must be an open interval, following through their **two** critical values which must have come from the solution of a 3TQ. $x < -\frac{1}{2} \quad x > 3$ Accept any correct notation. E.g., $x < -\frac{1}{2}$ or $x > 3$ Or $\left\{x : x < -\frac{1}{2}\right\} \cup \{x : x > 3\}$ Condone $x < -\frac{1}{2}$ and $x > 3$ for this mark only
	A1	For the correct inequality with the correct critical values using any acceptable notation. Eg, $x < -\frac{1}{2} \quad x > 3$
(c)	B1ft	For $x < -\frac{1}{2} \quad 3 < x < 4$ ft their answers from parts (a) and (b) provided (b) is of the form $x < p$ and/or $x > q$ **Note:** If you have already penalised them for writing $x < -\frac{1}{2}$ **and** $x > 3$ in part (b) then allow $x < -\frac{1}{2}$ **and** $3 < x < 4$ for this mark.

Question number	Scheme	Marks
2 (a)	$2-\frac{1}{25}\left(x^2-20x\right)$	
	$2\mp\frac{1}{25}\left[\left(x\pm10\right)^2-100\right]$	M1
	$6-\frac{1}{25}\left(x-10\right)^2$ So $A=6$ $B=\frac{1}{25}$ $C=-10$	A1 A1 A1 (4)
ALT		
	$A-Bx^2-2BCx-BC^2=2+\frac{4}{5}x-\frac{1}{25}x^2$	{M1}
	$B=\frac{1}{25}$	{A1}
	$-\frac{2}{25}C=\frac{4}{5}$ $C=-10$	{A1}
	$-\frac{1}{25}(100)+A=2$ $A=6$	{A1} (4)
(b)(i)	6	B1 ft
(b)(ii)	10	B1 ft (2)
		Total 6 marks

Part	Mark	Guidance
(a)	**M1**	For a complete method to complete the square to achieve as a minimum $2\mp\frac{1}{25}(x\pm10)^2-p$ or $\mp\frac{1}{25}\left[(x\pm10)^2-q\right]$ where p and q are constants
	A1	For one correct from $A=6$ $B=\frac{1}{25}$ or $C=-10$ whether stated explicitly or embedded
	A1	For two correct from $A=6$ $B=\frac{1}{25}$ or $C=-10$ whether stated explicitly or embedded
	A1	Fully correct $A=6$ $B=\frac{1}{25}$ and $C=-10$ **OR** $6-\frac{1}{25}(x-10)^2$ oe
	ALT – equates coefficients	
	M1	For an attempt to expand $A-B(x+C)^2$ **AND** equate coefficients to the given $\mathrm{f}(x)\Rightarrow A-Bx^2-2BCx-BC^2=2+\frac{4}{5}x-\frac{1}{25}x^2$ Allow $A\pm Bx^2\pm2BCx\pm BC^2$ for the expansion of $A-B(x+C)^2$ There must be an attempt to equate at least one coefficient. $-B=-\frac{1}{25}\Rightarrow B=...$ $-2BC=\frac{4}{5}\Rightarrow C=...$ $A-BC^2=2\Rightarrow A=...$
	A1	For one correct from $A=6$ $B=\frac{1}{25}$ or $C=-10$ whether stated explicitly or embedded
	A1	For two correct from $A=6$ $B=\frac{1}{25}$ or $C=-10$ whether stated explicitly or embedded
	A1	Fully correct $A=6$ $B=\frac{1}{25}$ and $C=-10$ **OR** $6-\frac{1}{25}(x-10)^2$ oe
(b)(i)	**B1ft**	For the value of 6 or ft their A
(ii)	**B1ft**	For the value of 10 or ft their C

Question number	Scheme	Marks
3 (a)	$PQ = 18 \times \frac{2}{3}\pi = 12\pi$	M1 A1 (2)
(b)(i)	$\alpha = \frac{1}{3}\pi$	B1
(b)(ii)	$PT = 18\tan\frac{\pi}{3} = 18\sqrt{3}$	M1 A1
	Area of $OPTQ = 2 \times \frac{1}{2} \times 18 \times 18\sqrt{3}$	M1
	Area of Sector $OPQ = \frac{1}{2} \times 18^2 \times \frac{2\pi}{3}$	M1
	Shaded Area $= 2 \times \frac{1}{2} \times 18 \times 18\sqrt{3} - \frac{1}{2} \times 18^2 \times \frac{2\pi}{3} = 222 \text{ cm}^2$	M1 A1 (7)
		Total 9 marks



Area of triangle $OPQ = 81\sqrt{3}$ or $140.29...$ cm^2

Area of triangle $PQT = 243\sqrt{3}$ or $420.88...$ cm^2

Area of quadrilateral $OTPQ = 324\sqrt{3}$ or $561.18...$ cm^2

Part	Mark	Guidance
(a)	**M1**	Uses the **correct** formula for the length of arc to give $PQ = 18 \times \frac{2}{3}\pi = ...$
	A1	For $PQ = 12\pi$

	ALT – works in degrees (but the angle must be correct at 120°)	
	M1	Uses the **correct** formula for length of arc to give $PQ = \frac{120}{360} \times 2\pi \times 18 = ...$
	A1	For $PQ = 12\pi$
(b)	**(i) B1**	For stating $\alpha = \frac{\pi}{3}$ (Please check the diagram as it may written on there)
	(ii) Method 1 - Allow use of degrees throughout provided the angles are correct. $\left(\angle POQ = 120^\circ,\ \angle PTQ = 60^\circ\right)$	
	Finds length of *PT*	
	M1	For finding length PT: e.g., $\tan\left(\frac{\pi}{3}\right) = \frac{PT}{18} \Rightarrow PT = 18\tan\left(\frac{\pi}{3}\right) = ...$ The given values must be used correctly
	A1	For $PT = 18\sqrt{3}$
	M1	For the area of $OPTQ = 2 \times \frac{1}{2} \times 18 \times '18\sqrt{3}' = (561.18...)$ Their $18\sqrt{3}$ must come from an attempt at using trigonometry.
	Method 2	
	Finds lengths *PQ* and *TO*	
	M1	For finding the lengths PQ **and** TO using any acceptable correct trigonometry. e.g., $PQ = \sqrt{18^2 + 18^2 - 2 \times 18 \times 18\cos\left(\frac{2\pi}{3}\right)} = ...$ and $TO = \frac{\cos\left(\frac{\pi}{3}\right)}{18} = ...$ The given values must be used correctly
	A1	For both correct lengths: $PQ = 18\sqrt{3}$ and $TO = 36$
	M1	For the area of $OPTQ = \frac{1}{2} \times '18\sqrt{3}' \times '36' = \left(561.18... \text{ or } 324\sqrt{3}\right)$ Their $18\sqrt{3}$ and 36 must come from an attempt at using trigonometry
	M1	For the area of sector $OPQ = \frac{1}{2} \times 18^2 \times \frac{2\pi}{3} = (339.29... \text{ or } 108\pi)$ or $\frac{120^\circ}{360^\circ} \times \pi \times 18^2 = (339.29...)$
	M1	For area of $OPTQ$ – area of Sector OPQ $561.18 - 339.29 = 221.887\ldots$
	A1	For 222 cm^2 (must be 3sf) (Units are not required)

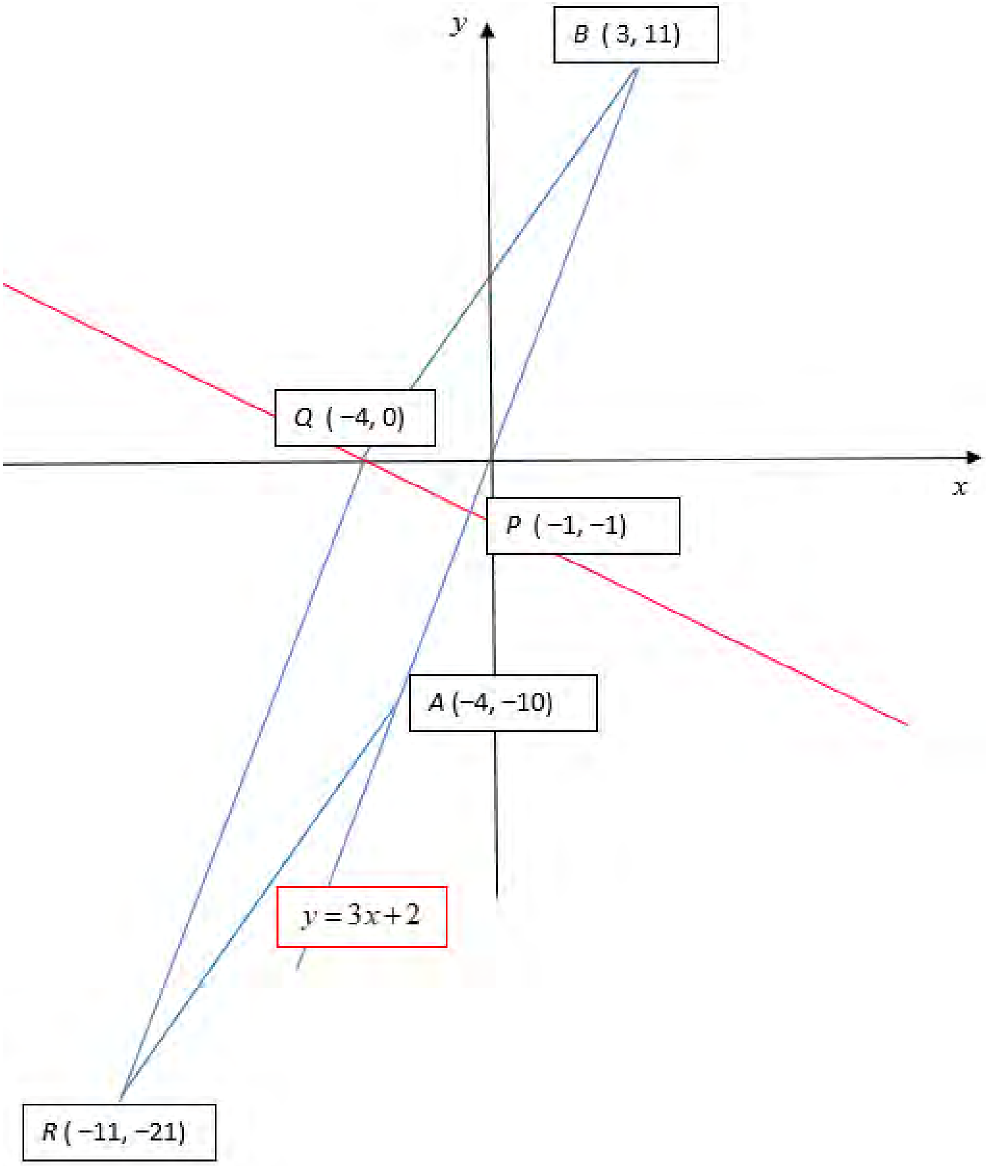
Question number	Scheme	Marks
4 (a)	Gradient = $\dfrac{11+10}{3+4}=3$	
	$y+10=3(x+4)$ or $y-11=3(x-3)$ oe	M1A1 (2)
(b)	e.g. $\left(\dfrac{4\times -4+3\times 3}{3+4},\dfrac{4\times -10+3\times 11}{3+4}\right)=(-1,-1)$	M1 A1 (2)
ALT (b)	Using Vectors $\begin{pmatrix}-4\\-10\end{pmatrix}+\dfrac{3}{7}\begin{pmatrix}7\\21\end{pmatrix}=\begin{pmatrix}-1\\-1\end{pmatrix}$ or $\begin{pmatrix}3\\11\end{pmatrix}-\dfrac{4}{7}\begin{pmatrix}7\\21\end{pmatrix}=\begin{pmatrix}-1\\-1\end{pmatrix}$	{M1} {A1}
(c)	$-\dfrac{1}{3}=\dfrac{n+1}{m+1}\Rightarrow -\dfrac{1}{3}(m+1)=n+1$	M1
	$\left(\sqrt{10}\right)^2=(m+1)^2+(n+1)^2$	M1
	$10=(m+1)^2+\dfrac{1}{9}(m+1)^2$	M1
	$9=(m+1)^2$	M1
	$m=-4 \quad n=0$	A1 A1 (6)
ALT (c)	Using Vectors $\overrightarrow{AB}=\begin{pmatrix}7\\21\end{pmatrix}$ so perpendicular to $\overrightarrow{AB}=\begin{pmatrix}21\\-7\end{pmatrix}$	{M1}
	$\left\lvert\overrightarrow{AB}\right\rvert=7\sqrt{10}\Rightarrow,\left\lvert\overrightarrow{AP}\right\rvert=3\sqrt{10}$	{M1,M1}
	$\overrightarrow{PQ}=\dfrac{\sqrt{10}}{7\sqrt{10}}\times\begin{pmatrix}21\\-7\end{pmatrix}=\begin{pmatrix}3\\-1\end{pmatrix}$	{M1}
	So $Q=(-1-3,-1--1)$	{A1}
	$Q=(-4,0)$	{A1}
(d)(i)	$AB=\sqrt{(3+4)^2+(11+10)^2}=7\sqrt{10}$	M1
	$RQ=\sqrt{(-11+4)^2+(-21)^2}=7\sqrt{10}$	A1
(d)(ii)	Gradient of $RQ=\dfrac{-21-0}{-11+4}=3$	M1
	So Gradient of AB (=3) = Gradient of RQ	A1 (4)

ALT (d)	Using Vectors $\overrightarrow{RQ} = \begin{pmatrix} -4-(-11) \\ 0-(-21) \end{pmatrix} = \begin{pmatrix} 7 \\ 21 \end{pmatrix}$ $\overrightarrow{AB} = \begin{pmatrix} 7 \\ 21 \end{pmatrix} = \overrightarrow{RQ}$ Because the vectors are the same they must be parallel and the same length	{M1} {A1} {M1} {A1}
(e)	Area $= 7\sqrt{10} \times \sqrt{10} = 70$	M1 A1 (2)
ALT (e)	Using Vectors $\frac{1}{2}\begin{vmatrix} 3 & -4 & -11 & -4 & 3 \\ 11 & 10 & -21 & 0 & 11 \end{vmatrix}$ $= 70$	{M1} {A1}
		Total is 16 marks

Part	Mark	Guidance
(a)	**M1**	For a fully correct method of finding an equation of a straight line. $\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}\Rightarrow\frac{y-(-10)}{11-(-10)}=\frac{x-(-4)}{3-(-4)}$ Or finds gradient $\frac{11+10}{3+4}=3$ and uses $y+10=3(x+4)$ or $y-11=3(x-3)$ If $y=mx+c$ is used, they must find a complete equation for this mark. Allow **one** error only for the award of this mark.
	A1	For a correct line in any form. $y+10=3(x+4)$ or $y-11=3(x-3)$ or $y=3x+2$ or even $\frac{y+10}{21}=\frac{x+4}{7}$ but do not allow incomplete processing.
(b)	**M1**	For one correct from $x=-1$ or $y=-1$
	A1	For the correct coordinates of point P $(-1,-1)$ Accept $x=-1$ $y=-1$
(c)	**M1**	Uses the perpendicular gradient to set up an equation in m and n. $-\frac{1}{'3'}=\frac{n-'(-1)'}{m-'(-1)'}\Rightarrow-\frac{1}{'3'}(m+1)=n+1$ or $n=-\frac{1}{3}m-\frac{4}{3}$ Ft their gradient in part (a) and their P from part (b) for this mark.
	M1	Uses Pythagoras theorem to set up an equation in m and n. $\left(\sqrt{10}\right)^2=\left(m-'(-1)'\right)^2+\left(n-'(-1)'\right)^2$ Ft their coordinates of point P form part (b) for this mark.
	M1	Attempts to solve their two equations in n and m simultaneously and forms a quadratic equation in one variable only. $10=(m+1)^2+\frac{1}{9}(m+1)^2\Rightarrow9=(m+1)^2$ or $0=m^2+2m-8$ or $10=9(n+1)^2+(n+1)^2\Rightarrow0=10n^2+20n$
	M1	For solving their either: $9=(m+1)^2\Rightarrow m=...$ or $0=10n^2+20n\Rightarrow n=...$ which must be a quadratic equation.
	A1	For finding **either** $m=-4$ **or** $n=0$ Condone the sight of $m=2$ for this mark.
	A1	For finding **both** $m=-4$ **and** $n=0$ $\Rightarrow(-4,\ 0)$ The final answer must be given as coordinates.
	ALT – using vectors – see main scheme.	
(d)(i)	**M1**	For finding **either** the length $AB=\sqrt{(3+4)^2+(11+10)^2}=7\sqrt{10}$ **Or** $RQ=\sqrt{(-11+4)^2+(-21)^2}=7\sqrt{10}$
	A1	For finding **both** the length $AB=\sqrt{(3+4)^2+(11+10)^2}=7\sqrt{10}$ **And** $RQ=\sqrt{(-11+4)^2+(-21)^2}=7\sqrt{10}$ **and** states they are equal
(d)(ii)	**M1**	The gradient of $RQ=\frac{-21-'0'}{-11-'(-4)'}='3'$ Ft their coordinates from part (c)

	A1	States that the gradient of RQ = gradient of AB [from (a)]
	ALT – Uses vectors, see main scheme. Ft their coordinates of $Q-(m, n)$	
(e)	**M1**	For a correct expression for the area using their length of AB and the given length of PQ $\left(\sqrt{10}\right)$ Area = $'7\sqrt{10}'\times\sqrt{10} = \ldots$
	A1	For the area = 70 [square units]
	ALT – Uses the discriminant	
	M1	For a correct expression of the area in sequential order using their coordinates for Q Area = $\frac{1}{2}\begin{vmatrix} 3 & -4 & -11 & '-4' & 3 \\ 11 & 10 & -21 & '0' & 11 \end{vmatrix}$
	A1	Area = 70 [square units]

Useful sketch



Question number	Scheme	Marks
5 (a)	$u_2 + u_4 = ar + ar^3 = 212.5$ $u_3 + u_4 = ar^2 + ar^3 = 62.5$	M1
	$\dfrac{(1+r^2)}{(r+r^2)} = \dfrac{17}{5}$	M1
	$12r^2 + 17r - 5 = 0$	M1
	$(4r-1)(3r+5) = 0$	dM1
	$r = \dfrac{1}{4} \qquad r = -\dfrac{5}{3}$	A1 (5)
(b)	$r = \dfrac{1}{4} \Rightarrow a = 800$ So $\dfrac{a}{1-r} = \dfrac{800}{\frac{3}{4}} = \dfrac{3200}{3}$	M1 A1 (2)
		Total 7 marks

Part	Mark	Guidance
(a)	M1	For either $ar + ar^3 = 212.5$ or $ar^2 + ar^3 = 62.5$ correct
	M1	For attempting to eliminate a or ar either by division or substitution. e.g. $\dfrac{ar(1+r^2)}{ar(r+r^2)} = \dfrac{212.5}{62.5} \Rightarrow \dfrac{(1+r^2)}{(r+r^2)} = \dfrac{212.5}{62.5} = \left(\dfrac{17}{5}\right)$ An attempt involves some factorisation to eliminate a or ar
	Method 1 – finds a 3TQ	
	M1	For forming a 3TQ in r only using their expressions. $(12r^2 + 17r - 5 = 0 \text{ oe})$ Accept for example $150r^2 + 212.5r - 62.5 = 0$
	dM1	For an attempt to solve their 3TQ to give **two** values of r See General Guidance for the definition of an attempt. For example: $(4r-1)(3r+5) = 0 \Rightarrow r = ..., ...$ This mark is dependent on the **FIRST** M mark being awarded
	Method 2 – finds a cubic equation	
	M1	For forming a cubic with a common factor of r in each term. e.g. $12r^3 + 17r^2 - 5r = 0$
	dM1	For factorising their cubic equation to achieve $r(12r^2 + 17r - 5) = 0$ and for an attempt to solve their 3TQ to give **two** values of r Ignore $r = 0$ if also given. See General Guidance for the definition of an attempt. For example: $(4r-1)(3r+5) = 0 \Rightarrow r = ..., ...$ This mark is dependent on the **FIRST** M mark being awarded
	A1	For the correct values; $r = \dfrac{1}{4}$ and $r = -\dfrac{5}{3}$ (reject $r = 0$ if seen earlier)
(b)	M1	Uses their $r = \dfrac{1}{4}$ [where $\lvert r \rvert < 1$] to find the value of a (800) with the correct formula for the sum of a geometric series to infinity. Condone an incorrect value of a even if they have used $r = \dfrac{1}{4}$ [The formula is given on page 2 of this booklet]. $S_\infty = \dfrac{a}{1-r} = \dfrac{'800'}{1 - '\frac{1}{4}'} = ...$
	A1	For the correct value, $S_\infty = \dfrac{3200}{3}$ or $1066\dfrac{2}{3}$ Do not accept for example 1066.67 unless the stated value is $1066.\dot{6}$

Question number	Scheme	Marks
6 (a)	$\mathrm{f}(3) = 27 + 9p + 9 - 30 + q = 0$	M1 A1
	$9p + q + 6 = 0$ *	A1 cso
		(3)
(b)	$\mathrm{f}(-p) = -p^3 + p^2(p+1) + 10p + q = 0$	M1 A1
	$p^2 + 10p + q = 0$ *	A1 cso
		(3)
(c)	$p^2 + 10p - 9p - 6 = 0$	M1
	$p^2 + p - 6 = 0$	A1
	$(p+3)(p-2) = 0$	M1
	$p = 2 \quad q = -24$	A1 A1
		(5)
(d)	$(x+a)(x-3)(x+2)$	
	So $-3 \times 2 \times a = -24 \quad a = 4$	M1
	$(x+4)(x-3)(x+2)$	A1
		(2)
		Total 13 marks

Part	Mark	Guidance
General guidance for marking parts (a) and (b) • For the award of full marks in parts (a) and/or (b) you **must** see = 0 used in a line of working **before** the final answer. • If a candidate does not use = 0 in either parts (a) or (b) [except in the final line – which is a given answer] deduct the M mark (and the subsequent A marks) in only the **first** occurrence of the absence.		
(a)	**M1**	For using $\mathrm{f}(\pm 3) = 0$ in the given equation set = 0
	A1	For obtaining the correct unsimplified expression: $27 + 9p + 9 - 30 + q = 0$
	A1 cso	For obtaining the given equation $9p + q + 6 = 0$ * **Note: This is a show question. There must be no errors seen.**
(b)	**M1**	For use of $\mathrm{f}(\pm p) = 0$ in the given equation set = 0
	A1	For obtaining the correct unsimplified expression: $-p^3 + p^2(p+1) + 10p + q = 0$

	A1 cso	For obtaining the correct given equation $p^2+10p+q=0$ * **Note: This is a show question. There must be no errors seen.**
(c)	**M1**	For attempting to solve the given two equations simultaneously to achieve a 3TQ in either p or q only. E.g. substitutes $q=\mp 9p\mp 6$ or $\left[p=\dfrac{\mp q\mp 6}{9} \text{ and } p^2=\dfrac{(\mp q\mp 6)^2}{81}\right]$ into $p^2+10p+q=0$ This mark may be implied by the correct 3TQ
	A1	For the correct 3TQ $p^2+p-6=0$ or $q^2+3q-504=0$
	M1	For an attempt to solve their 3TQ in either p or q using factorisation, use of the formula or completing the square. See general guidance for the definition of an attempt. For example: $(p+3)(p-2)=0\Rightarrow p=\ldots$ or $(q+24)(q-21)=0\Rightarrow q=\ldots,\ \ldots$ If a candidate uses their calculator to solve their 3TQ, the final values must be correct for the award of this mark unless a valid method is seen.
	A1	For either the correct value of p OR the correct value of q $p=2$ or $q=-24$ Condone the presence $p=-3$, and/or $q=21$
	A1	For both the correct value of p (= 2) AND the correct value of q (= –24) Must reject $p=-3$, and/or $q=21$ if seen.
(d)	**M1**	[$\text{f}(x) = (x+a)(x-3)(x+'2')$] For attempting to find the value of a $-3\times '2'\times a='-24'\Rightarrow a=\ldots$ OR For an attempt using division with their values of p and q $x^2-x-6\overline{)x^3+3x^2-10x-24}$ giving $x+4$ or $x-3\overline{)x^3+3x^2-10x-24}$ giving x^2+6x+8 Allow a quotient of $x+b$ or x^2+6x+b where b is a constant.
	A1	For the correct factorised expression $(x+4)(x-3)(x+2)$ which must be written out in full on one line.

Question number	Scheme	Marks
7 (a)	2 | 3 | 4 3.73 | 4.28 | 5	B1 B1 (2)
(b)	Points plotted Joined up with a smooth curve	B1ft B1ft (2)
(c)	$\log_3(6-2x)=\dfrac{x}{4}$	M1
	$6-2x=3^{\frac{x}{4}}$	M1
	$8-2x=3^{\frac{x}{4}}+2$	A1
	$y=8-2x$ drawn	M1
	$x=2.1$	A1 (5)
	Total 9 marks	

<table>
<tr><th>Part</th><th>Mark</th><th>Guidance</th></tr>
<tr><td rowspan="2">(a)</td><td>B1</td><td>For two points (rounded correctly) correct from;
<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>3</td><td>3.32</td><td>3.73</td><td>4.28</td><td>5</td><td>5.95</td></tr></table></td></tr>
<tr><td>B1</td><td>All three points correct and correctly rounded.
Penalise rounding only once here. Condone 5.00</td></tr>
<tr><td rowspan="2">(b)</td><td>B1ft</td><td>All points plotted within half of one square. Ft their values of y for $x = 2,3,4$ respectively</td></tr>
<tr><td>B1ft</td><td>All drawn points joined up in a smooth curve
</td></tr>
<tr><td rowspan="5">(c)</td><td>M1</td><td>For use of power law to obtain $\log_3(6-2x) = \frac{\pm x}{4}$</td></tr>
<tr><td>M1</td><td>For removing the $\log_3$ to obtain: $6-2x = 3^{\frac{\pm x}{4}}$
Allow $(6-2x)^4 = 3^{\pm x}$ for this mark.</td></tr>
<tr><td>A1</td><td>For obtaining the equation $8-2x = 3^{\frac{x}{4}}+2$ oe (eg., $-2x+8 = 2+3^{\frac{\pi}{4}}$)</td></tr>
<tr><td>M1</td><td>For drawing their straight line, provided it is of the form $y = k-2x$ where k is a constant and $k \neq 6$

[Check coordinates (1, 6) (2, 4) (3, 2) (4, 0)]</td></tr>
<tr><td>A1</td><td>For the intersection point $(x =)$ 2.1</td></tr>
</table>

Question number	Scheme	Marks
8	$\log_4 a + 2\log_4 b = \frac{5}{2}$	M1
	$\log_4 (ab^2) = \frac{5}{2}$	M1
	$32 = ab^2$	A1
	$2^a = \frac{2^{16}}{2^{2b^2}}$	M1
	$a = 16 - 2b^2$ or $b^2 = 8 - \frac{1}{2}a$	A1
	$32 = a(8 - \frac{1}{2}a)$ or $32 = (16 - 2b^2)b^2$	M1
	$a^2 - 16a + 64 = 0$ or $2b^4 - 16b^2 + 32 = 0$	A1
	$a = 8$ $b = 2$	A1
		Total 8 marks

Mark	Guidance
Log equation	**Method 1 – Works in base 4**
M1	For an attempt to change the base of $3\log_8 b$ to base 4 using $\log_a x = \dfrac{\log_b x}{\log_b a}$ $3\log_8 b = \dfrac{3\log_4 b}{\log_4 8} = \dfrac{3\log_4 b}{\frac{3}{2}} = 2\log_4 b$ [accept $p\log_4 b$ where $p \ne 3$]
M1	Uses $n\log A = \log A^n$ **and** $\log A + \log B = \log AB$ to combine the logs correctly $\log_4(ab^2) = \dfrac{5}{2}$ [ft their p provided $p \ne 1$]
A1	For removing the logs in the equation to obtain $32 = ab^2$ o.e. e.g. $a^2b^4 = 1024$
	Method 2 – Works in base 8
M1	For an attempt to change the base of $\log_4 a$ to base 8 using $\log_a x = \dfrac{\log_b x}{\log_b a}$ $\log_4 a = \dfrac{\log_8 a}{\frac{2}{3}} = \dfrac{3\log_8 a}{2}$ [accept $q\log_8 a$ where $q \ne 1$]
M1	Uses $n\log A = \log A^n$ **and** $\log A + \log B = \log AB$ correctly to combine the logs $\log_8(a^{\frac{3}{2}}b^3) = \dfrac{5}{2}$ [ft their q]
A1	For removing the logs in the equation to obtain $a^{\frac{3}{2}}b^3 = 8^{\frac{5}{2}}$ **and** rearranges (raises both sides to the power of $\dfrac{2}{3}$) to obtain $32 = ab^2$
Second equation	
M1	For attempting to change the second equation to powers of 2 or 4: $2^a = \dfrac{2^{16}}{2^{2b^2}} \Rightarrow \left[2^a = 2^{(16-2b^2)}\right]$ or $4^{\frac{a}{2}} = \dfrac{4^8}{4^{b^2}} = \left(4^{\frac{a}{2}} = 4^{8-b^2}\right)$ At least one correct change of term e.g either 2^{16} or 2^{2b^2} OR either $4^{\frac{a}{2}}$ or 4^8
A1	Combines the powers to achieve $a = 16 - 2b^2$ or $\dfrac{a}{2} = 8 - b^2$ oe
Attempt to solve the simultaneous equations	
M1	For an attempt to solve their equations simultaneously, both of which must be in terms of a and b^2, to obtain a 3TQ in either a or b^2. $32 = a(8 - \frac{1}{2}a) \Rightarrow a^2 - 16a + 64 = 0$ or $32 = (16 - 2b^2)b^2 \Rightarrow 2b^4 - 16b^2 + 32 = 0$
M1	For an attempt to solve their 3TQ in either a or b^2 by any method. See General Guidance for the definition of an attempt For example: $a^2 - 16a + 64 = 0 \Rightarrow (a-8)(a-8) = 0 \Rightarrow a = ...$ $2b^4 - 16b^2 + 32 = 0 \Rightarrow b^4 - 8b^2 + 16 = 0 \Rightarrow (b^2-4)(b^2-4) = 0 \Rightarrow b = ...$
A1	For $a = 8$ and $b = 2$ [If $b = \pm 2$ is given as the final answer, withhold this mark].

Question number	Scheme	Marks
9 (a)	$\frac{\mathrm{d}A}{\mathrm{d}t} = 0.03$	B1
	$A = \frac{1}{2}x^2 \sin 60^\circ = \frac{\sqrt{3}}{4}x^2$	M1
	$\frac{\mathrm{d}A}{\mathrm{d}x} = \frac{\sqrt{3}}{2}x$	A1
	When $x = 2$ $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{\sqrt{3}} \times 0.03 = 0.0173\,\text{cm/s}$	M1 A1 (5)
(b)	$V = \sqrt{3}x^3$ $\quad \frac{\mathrm{d}V}{\mathrm{d}x} = 3\sqrt{3}x^2$	M1
	When $x = 2$ $\frac{\mathrm{d}V}{\mathrm{d}t} = 12\sqrt{3} \times 0.0173 = 0.36$	M1 A1 (3)
		Total 8 marks

Part	Mark	Guidance
(a)	B1	For stating or using correctly in their Chain Rule $\frac{\mathrm{d}A}{\mathrm{d}t}=0.03$
	M1	For using the **correct** formula $\left(\frac{1}{2}ab\sin C\right)$ with the correct lengths and angle of $60°$ or $\frac{\pi}{3}$, for the cross-sectional area of the prism to obtain $A=\frac{1}{2}x^2\sin 60°=\left(\frac{\sqrt{3}}{4}x^2\right)$ and differentiating their expression which must be as a minimum $A=px^2$ to obtain $\frac{\mathrm{d}A}{\mathrm{d}x}=qx$ [where p and q are constants]. [The height of the triangle is $\frac{\sqrt{3}}{2}x$ if they use $\frac{1}{2}\times\text{base}\times\text{height}$]
	A1	For the correct $\frac{\mathrm{d}A}{\mathrm{d}x}=\frac{\sqrt{3}}{2}x$
	M1	For applying a **correct** Chain rule using their $\frac{\mathrm{d}A}{\mathrm{d}x}$ and $x=2$ to obtain $\frac{\mathrm{d}x}{\mathrm{d}t}=\left(\frac{1}{\frac{\mathrm{d}A}{\mathrm{d}x}}\times\frac{\mathrm{d}A}{\mathrm{d}t}\right)=\frac{\mathrm{d}x}{\mathrm{d}A}\times\frac{\mathrm{d}A}{\mathrm{d}t}='\frac{2}{\sqrt{3}}'\times\frac{1}{2}\times 0.03$
	A1	$\frac{\mathrm{d}x}{\mathrm{d}t}=0.0173$
(b)	M1	For a correct expression for the volume using their A from part (a) to obtain $V=\frac{\sqrt{3}}{4}x^2\times 4x=\left(\sqrt{3}x^3\right)$ and differentiating their expression which must be as a minimum $V=mx^3$ to obtain as a minimum $\frac{\mathrm{d}V}{\mathrm{d}x}=nx^2$ [where m and n are constants] $\left(\frac{\mathrm{d}V}{\mathrm{d}x}=3\sqrt{3}x^2\right)$
	M1	For applying a correct Chain rule using their $\frac{\mathrm{d}V}{\mathrm{d}x}$ and $x=2$ to obtain $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}t}=12\sqrt{3}\times 0.0173=\left[0.359...\right]$ $\left(\text{ft their }\frac{\mathrm{d}x}{\mathrm{d}t}\right)$ **Note:** $\frac{\mathrm{d}x}{\mathrm{d}t}=0.0173$ or $\frac{\sqrt{3}}{100}$ $\left(\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}t}=12\sqrt{3}\times\frac{\sqrt{3}}{100}=\frac{9}{25}=0.36\right)$
	A1	For awrt 0.36

Question number	Scheme	Marks
10 (a)	$x = \tan^{-1}(-3) = -72$	M1
	$x = 108 \qquad x = 288$	A1 A1 (3)
(b)	$7\sin^2\theta + \sin\theta\cos\theta = 6(\sin^2\theta + \cos^2\theta)$	M1
	$\sin^2\theta + \sin\theta\cos\theta - 6\cos^2\theta = 0$	
	$\dfrac{\sin^2\theta}{\cos^2\theta} + \dfrac{\sin\theta}{\cos\theta} - 6 = 0$	M1
	$\tan^2\theta + \tan\theta - 6 = 0$	A1 cso (3)
(c)	$(\tan y + 3)(\tan y - 2) = 0$	M1
	$\tan y = -3 \qquad \tan y = 2$	A1
	$y = 108, 288 \qquad y = 63, 243$	A1ft A1 (4)
		Total 10 marks

Part	Mark	Guidance
(a)	**M1**	For using inverse tan to obtain any correct angle $\tan^{-1}(-3) \Rightarrow x = -71.565...^{\circ}$ Accept awrt -72°
	A1	For **either** 108 **or** 288
	A1	For **both** 108 **and** 288
(b)	**M1**	Uses $\sin^2\theta + \cos^2\theta = 1$ on the given equation to obtain $7\sin^2\theta + \sin\theta\cos\theta = 6(\sin^2\theta + \cos^2\theta)$
	M1	For rearranging and dividing through by $\cos^2\theta$ with the $\dfrac{\sin\theta}{\cos\theta} = \tan\theta$ identity to obtain a 3TQ: $\dfrac{\sin^2\theta}{\cos^2\theta} + \dfrac{\sin\theta}{\cos\theta} - 6 = 0 \Rightarrow \left(\tan^2\theta + \tan\theta - 6 = 0\right)$
	ALT 1	
	M1	Divides the given equation through by $\cos^2\theta$ with the $\dfrac{\sin\theta}{\cos\theta} = \tan\theta$ identity to obtain $7\tan^2\theta + \tan\theta = \dfrac{6}{\cos^2\theta}$
	M1	Uses $\sin^2\theta + \cos^2\theta = 1$ to obtain $\tan^2\theta + 1 = \dfrac{1}{\cos^2\theta}$ and uses this result on the given equation and rearranges to achieve a 3TQ to obtain $7\tan^2\theta + \tan\theta = 6\left(1 + \tan^2\theta\right) \Rightarrow \left(\tan^2\theta + \tan\theta - 6 = 0\right)$
	A1	For obtaining the given expression $\tan^2\theta + \tan\theta - 6 = 0$ * in full. **Note: This is a show question, there must be no errors in the solution.**
	ALT 2	
	M1	Uses the identity $\dfrac{\sin\theta}{\cos\theta} = \tan\theta$ with $\tan^2\theta + \tan\theta - 6 = 0$ to achieve $\dfrac{\sin^2\theta}{\cos^2\theta} + \dfrac{\sin\theta}{\cos\theta} - 6 = 0$ and multiplies through by $\cos^2\theta$ to obtain $\sin^2\theta + \sin\theta\cos\theta - 6\cos^2\theta = 0$
	M1	Uses $\sin^2\theta + \cos^2\theta = 1$ to obtain $\sin^2\theta + \sin\theta\cos\theta - 6\left(1 - \sin^2\theta\right) = 0$ and rearranges to obtain $7\sin^2\theta + \sin\theta\cos\theta = 6$
	A1 cso	For obtaining the given expression $7\sin^2\theta + \sin\theta\cos\theta = 6$ in full. **Note: This is a show question, there must be no errors in the solution.**
(c)	**M1**	For changing $7\sin^2 y + \sin y\cos y = 6$ to $\tan^2 y + \tan y - 6 = 0$ [this step must be correct] and then attempting to solve the 3TQ by any method.
	A1	For $\tan y = -3$ and $\tan y = 2$
	A1ft	For **both** $y = 108$ **and** 288 (ft from (a)) **Do not ft angles out of range**
	A1	For **both** $y = 63$ **and** 243
Rounding errors: Penalise rounding only once in this question when first seen provided angles round to 108, 288, 63 or 243		
Extra angles: Deduct one A mark for any extra angles within range. Ignore angles outside of range.		

Question number	Scheme	Marks
11	$e^x = \dfrac{4}{e^x} \Rightarrow e^{2x} = 4$	M1
	$x = \dfrac{1}{2}\ln 4$ or $x = \ln 2$	A1
	$\pi\int_0^{\ln 2} e^{2x}\,dx + \pi\int_{\ln 2}^{a} 16e^{-2x}\,dx$	M1 M1
	$\pi\left[\dfrac{1}{2}e^{2x}\right]_0^{\ln 2} + \pi\left[-8e^{-2x}\right]_{\ln 2}^{a}$	M1
	$\pi\left(2-\dfrac{1}{2}\right)+\pi\left(-8e^{-2a}+2\right)=$	M1
	$\dfrac{7\pi}{2} - 8\pi e^{-2a}$ $\quad a = 2$ and $k = \dfrac{7\pi}{2}$	M1A1
		Total 8 marks

Mark	Guidance
M1	Sets $e^x = \frac{4}{e^x}$ and attempts to make x the subject A minimum of $e^{2x} = 4$ or $e^x = 2$ is required to be seen for this mark.
A1	For obtaining either $x = \frac{1}{2}\ln 4$ or $x = \ln 2$
M1	For stating $\pi\int_0^{\ln 2}\left(e^x\right)^2 dx = \pi\int_0^{\ln 2} e^{2x}dx$ using the limits of their ln 2 (which must be of the form ln k) and 0 correctly **Note:** Condone a missing π here if it is seen at the final M mark.
M1	For stating $\pi\int_{\ln 2}^{a}\left(4e^{-x}\right)^2 dx = \pi\int_{\ln 2}^{a} 16e^{-2x}dx$ using a and the limit of their ln 2 (which must be of the form ln k where k is consistent between the two integrals) correctly **Note:** Condone a missing π if it is seen at the final M mark.
M1	For an attempt to integrate **both** expressions obtaining: Either $\left[\frac{e^{2x}}{2}\right]$ or $\left[\frac{-16e^{-2x}}{2}\right]$ $\left(\text{condone } \left[\frac{16e^{-2x}}{2}\right]\right)$ For this mark ignore the absence of π or incorrect/absent limits [need not be simplified]
dM1	For substituting **their** limits correctly (where their ln 2 must be of the form ln k where k is consistent between the two integrals) into their integrated expression. For this mark ignore the absence of π $\pi\left(\frac{e^{2\ln 2}}{2} - \frac{e^0}{2}\right) + \pi\left(-16e^{-2a} - \frac{\left(-16e^{-2\ln 2}\right)}{2}\right) = \pi\left(2 - \frac{1}{2}\right) + \pi\left(-8e^{-2a} + 2\right)$ This mark is dependent on the previous M mark.
M1	For equating to the given expression for the volume and equating coefficients to find values for a and k. We must see π used here for the award of this mark. $\left(\pi\left(2 - \frac{1}{2}\right) + \pi\left(-8e^{-2a} + 2\right) = \frac{7\pi}{2} - 8\pi e^{-2a}\right)$ $\frac{7\pi}{2} - 8\pi e^{-2a} = k - 8\pi e^{-4} \Rightarrow k = \ldots,\ a = \ldots$
A1	For $a = 2$ and $k = \frac{7\pi}{2}$

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