Please check the examination d	etails below before enter	ring your candidate information
Candidate surname		Other names
Pearson Edexcel International GCSE	Centre Number	Candidate Number
Monday 20 J	lanuary	2020
Morning (Time: 2 hours)	Paper Re	eference 4PM1/02
Further Pure N Level 2 Paper 2	/lathema	tics
Calculators may be used.		Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
 - there may be more space than you need.
- You must NOT write anything on the formulae page.
 Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for each question are shown in brackets
 use this as a quide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶



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International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times \text{slant height}$

Volume of sphere =
$$\frac{4}{3}\pi r^3$$

Series

Arithmetic series

Sum to *n* terms,
$$S_n = \frac{n}{2} [2a + (n-1)d]$$

Geometric series

Sum to *n* terms,
$$S_n = \frac{a(1-r^n)}{(1-r)}$$

Sum to infinity,
$$S_{\infty} = \frac{a}{1-r} |r| < 1$$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots$$
 for $|x| < 1, n \in \mathbb{Q}$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x} \left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)} \right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{\left[\mathrm{g}(x)\right]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc \cos A$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$



(4)

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

A particle *P* is moving along a straight line that passes through the fixed point *O*. At time t seconds, $t \ge 0$, the displacement, s metres, of P from O is given by

$$s = t^3 + 4t^2 - 27t + 4$$

Find the value of t at the instant when the velocity of P is 8 m/s .	

(Total for Organian 1 is 4 months)
(Total for Question 1 is 4 marks)



2 Find the set of values of x fo	r which
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(a)
$$3 + 2x \le x + 2$$

(1)

(b)
$$8x^2 + 10x < 3$$

(4)

(c) **both**
$$3 + 2x \le x + 2$$
 and $8x^2 + 10x < 3$

(1)





3

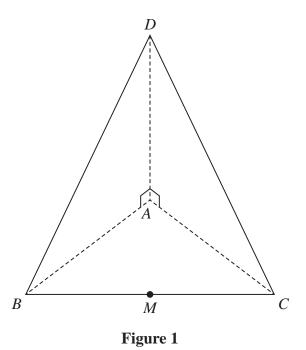


Diagram **NOT** accurately drawn

Figure 1 shows a triangular pyramid ABCD.

The base, ABC, of the pyramid is a horizontal isosceles triangle with $AB = AC = 10 \,\text{cm}$ and $BC = 16 \,\text{cm}$. The midpoint of BC is M.

The face BCD of the pyramid is an isosceles triangle with $BD = CD = 26 \,\mathrm{cm}$ and D is vertically above A.

$$\angle BAD = \angle CAD = 90^{\circ}$$

(a) Calculate the length, in cm, of AM.

(2)

Calculate, in degrees to the nearest degree,

(b) the size of $\angle BCD$,

(3)

(c) the size of the angle between the planes BCA and BCD.

(4)

Question 3 continued		





4 The points A, B, C and D are the vertices of a quadrilateral ABCD such that

$$\overrightarrow{AB} = 7\mathbf{i} + p\mathbf{j}$$

$$\overrightarrow{AC} = 11\mathbf{i} - p\mathbf{j}$$

$$\overrightarrow{AD} = 4\mathbf{i} - 2p\mathbf{j}$$

(a) Show that, for all values of p, ABCD is a parallelogram.

(3)

Given that $|\overrightarrow{BD}| = 3\sqrt{10}$

(b) find the possible values of p.

(3)

Given that p > 0

(c) find a unit vector which is parallel to \overrightarrow{BD} .

(1)

Question 4 continued		



Question 4 continued



5	Given that α and β are such that $\alpha + \beta = \frac{7}{2}$ and $\alpha\beta = 2$	
	(a) form a quadratic equation with integer coefficients that has roots α and β ,	(2)
	(b) form a quadratic equation with integer coefficients that has roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.	(6)
		(6)



6

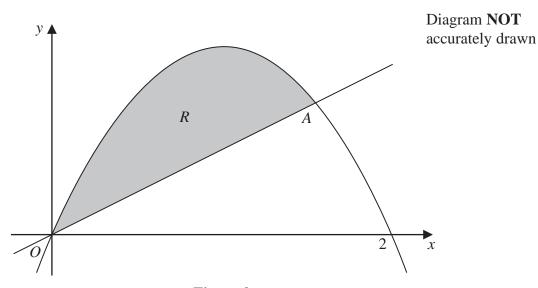


Figure 2

The region R, shown shaded in Figure 2, is bounded by the curve with equation $y = 2x - x^2$ and the line with equation 2y - x = 0

The curve and the line intersect at the origin O and the point A.

(a) Show that the point A has coordinates $\left(\frac{3}{2}, \frac{3}{4}\right)$.

(2)

The region R is rotated through 360° about the x-axis.

(b) Use algebraic integration to find, in terms of π , the volume of the solid formed.

(6)

Question 6 continued		



Question 6 continued	



7	The 7th term of a geometric series is 192 and the 8th term of this geometric series is 115	52
	(a) Find, as a fraction in its simplest form, the 4th term of this geometric series.	(3)
	A different geometric series G has a common ratio r and n th term t_n	
	Given that $t_3 = 24$ and $t_2 + t_3 + t_4 = -36$	
	(b) show that r satisfies the equation	
	$2r^2 + 5r + 2 = 0$	(5)
	Given further that G is convergent with sum to infinity S ,	
	(c) find the value of <i>S</i> .	(4)

	Question 7 continued
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Question 7 continued	



8	Given that $y = e^{3x} \sin 2x$	
	show that $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = 0$	
		(8)

	Question 8 continued
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Question 8 continued	



**9** A curve *C* has equation

$$y = \frac{qx - 2}{x - p} \qquad x \neq p$$

The curve crosses the y-axis at the point A.

The line l with equation y = x + 2 is the normal to C at A.

- (a) (i) Show that p = 1
  - (ii) Find the value of q.

(7)

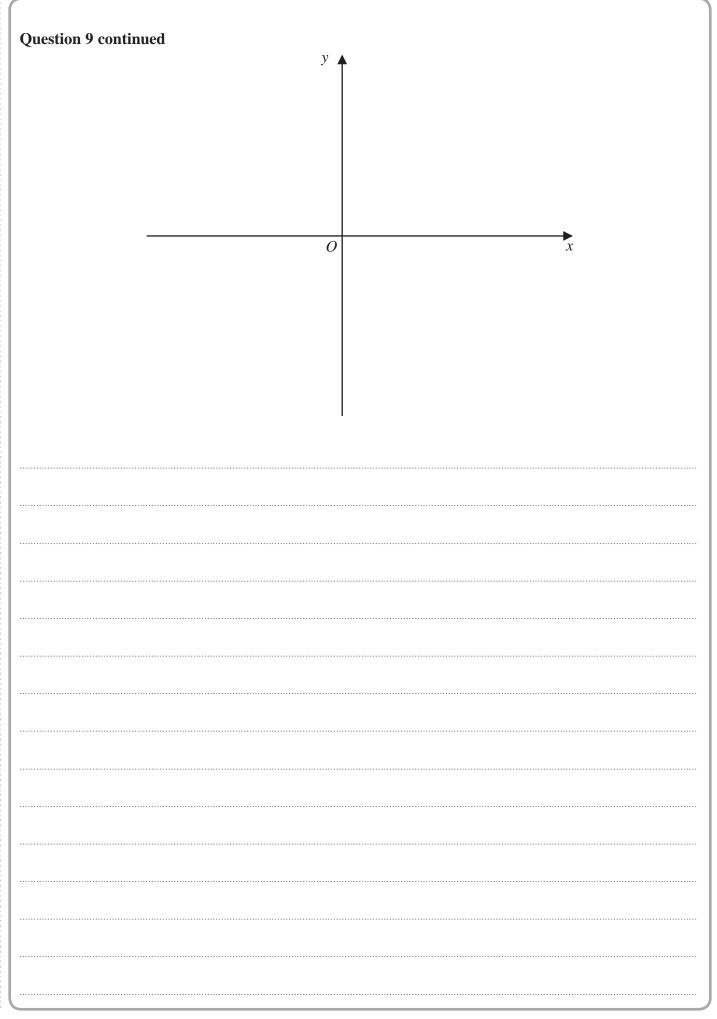
(b) Using the axes on the opposite page, sketch C, showing clearly the asymptotes and the coordinates of the points where C crosses the coordinate axes.

(5)

The line l meets C again at the point D.

(c) Find the x coordinate of D.

(4)





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Question 9 continued	

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10	10 The volume of a sphere is increasing at a constant rate of $40 \mathrm{cm}^3/\mathrm{s}$ .  Find the rate of increase, in cm²/s, of the surface area of the sphere at the instant when the radius is $4 \mathrm{cm}$ .		
		(9)	
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# 11 (a) Express the equation

$$3\sin(A-B) = \sin(A+B)$$

in the form  $\tan A = k \tan B$ , giving the value of the integer k.

(4)

(b) Given that  $\theta \neq \frac{(2n+1)\pi}{2}$  where  $n \in \mathbb{Z}$ ,

show that 
$$\frac{\cos^4 \theta - \sin^4 \theta}{\cos^2 \theta} = 1 - \tan^2 \theta$$

(3)

(c) Using the exact values of  $\sin x^{\circ}$ ,  $\cos x^{\circ}$  and  $\tan x^{\circ}$  for x = 30, 45, 60 show that

(i) 
$$\cos 15^\circ = \frac{\sqrt{6} + \sqrt{2}}{4}$$

(2)

(ii) 
$$\tan 255^\circ = \frac{3 + \sqrt{3}}{3 - \sqrt{3}}$$

**(4)** 


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Question 11 continued	
	(Total for Question 11 is 13 marks)
	TOTAL FOR PAPER IS 100 MARKS

