Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International GCSE

Friday 17 November 2023

Morning (Time: 2 hours)

Paper reference **4PM1/02**

Further Pure Mathematics

PAPER 2

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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P 7 3 5 8 6 A 0 1 3 2

Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\quad$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\quad$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$ $\quad$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 The equation $kx^2 + 8x + 3k = 0$ where k is a constant, has real unequal roots.

Find the set of values of k giving your answer in an exact simplified form.

(5)

(Total for Question 1 is 5 marks)

2 In triangle ABC, $AB = 3x$ cm, $BC = 5x$ cm and $\angle ABC = 110°$

(a) Find, in degrees to one decimal place, the size of $\angle BCA$

(4)

The area of triangle ABC is 24 cm^2

(b) Find, to 3 significant figures, the value of x

(3)

Question 2 continued

(Total for Question 2 is 7 marks)

3 A particle P moves along the x-axis.

At time t seconds $(t \geqslant 0)$ the acceleration, a m/s^2, of P is given by $a = 6t - 16$

When $t = 0$, P is at the origin and is moving with velocity 12 m/s.

(a) Find an expression in terms of t for

(i) the velocity of P at time t seconds

(ii) the displacement of P at time t seconds.

(4)

(b) Hence find the time at which P first returns to the origin.

(3)

Question 3 continued

(Total for Question 3 is 7 marks)

4 (a) On the axes opposite, draw the line with equation

(i) $y = -x - 1$ (ii) $y - 3x + 8 = 0$ (iii) $2y = x + 8$

(3)

(b) Show, by shading on your graph, the region R defined by the inequalities

$y \geqslant -x - 1$ and $y \geqslant 3x - 8$ and $2y \leqslant x + 8$

(1)

For all points in R, with coordinates (x, y)

$$P = 2y - 3x$$

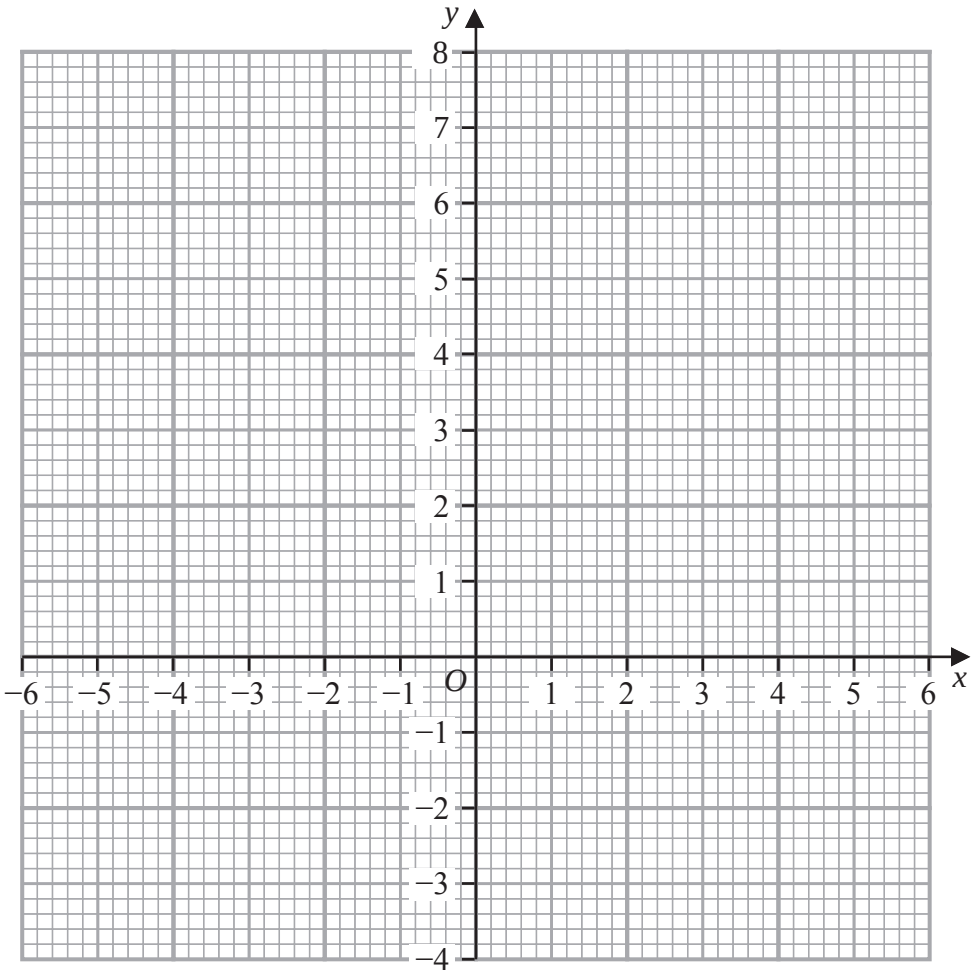
(c) Find

(i) the greatest value of P

(ii) the least value of P

(4)

Question 4 continued

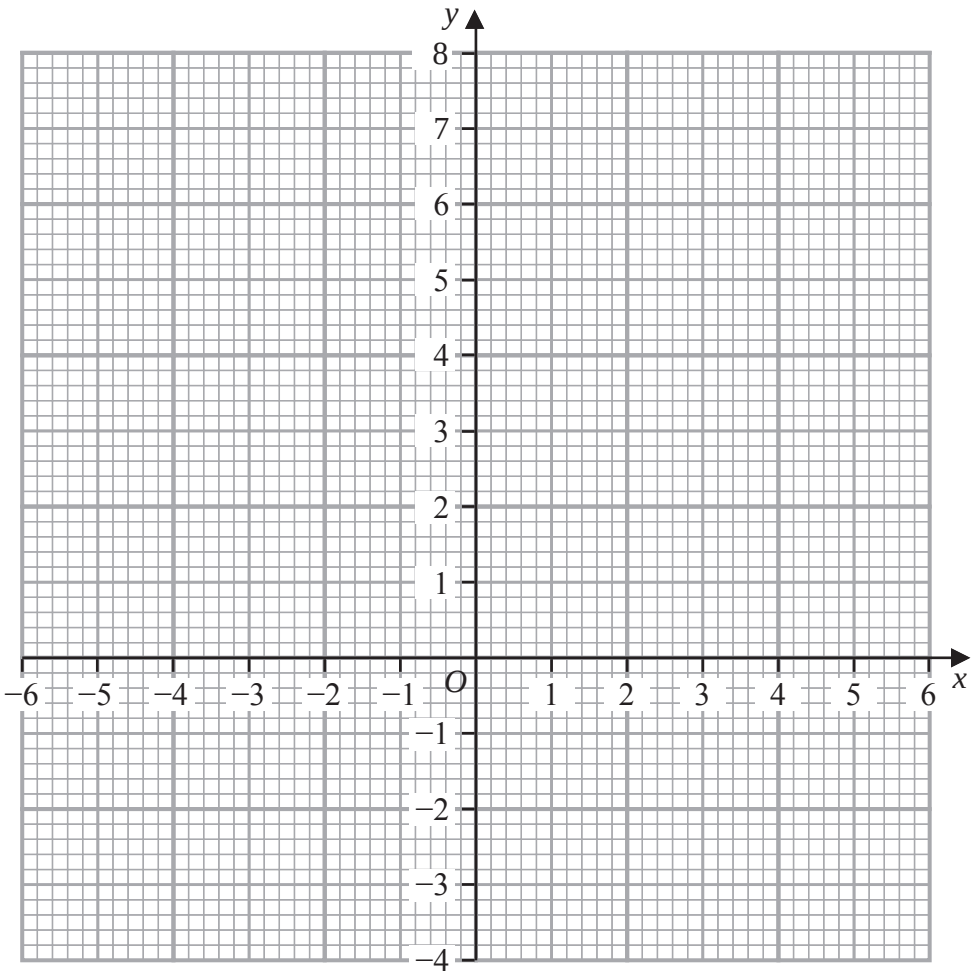


Turn over for a spare grid if you need to redraw your graph.

Question 4 continued

Question 4 continued

Only use this grid if you need to redraw your graph.



(Total for Question 4 is 8 marks)

5

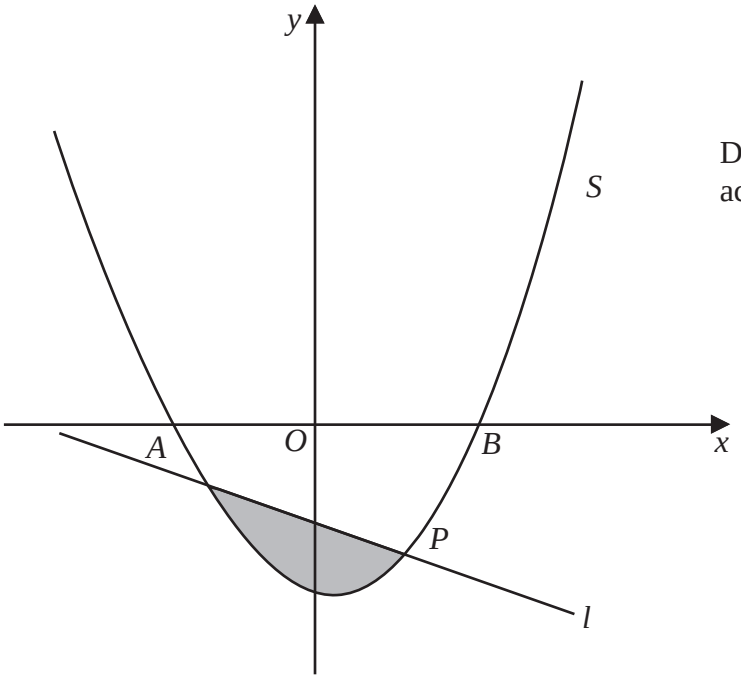


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows part of the curve S with equation $y = px^2 + qx + r$ where p, q and r are constants.

The points A, B and P with coordinates $(-2, 0)$, $(6, 0)$ and $(4, -6)$ respectively lie on S

(a) Show that an equation of S is $y = \frac{x^2}{2} - 2x - 6$

(3)

The line l is the normal to S at the point P

(b) Show that an equation of l is $2y + x + 8 = 0$

(5)

The finite region shown shaded in Figure 1 is bounded by S and l

(c) Use algebraic integration to find the exact area of the shaded region.

(7)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 15 marks)

6 The volume of oil in a container is V cm^3 when the height of the oil is h cm.
Oil is pouring into the container at a constant rate of 12 cm^3/s.
Given that $V = 3h^3$

find the exact rate, in cm/s, at which the height of the oil is increasing when $V = 1536$ cm^3

(7)

Question 6 continued

(Total for Question 6 is 7 marks)

7 Two numbers x and y are such that $3x - y = 4$

$S = 5x^3 + y^2$

(a) Show that $S = 5x^3 + 9x^2 - 24x + 16$

(2)

Given that x can vary,

(b) use calculus to find the value of x for which S is a minimum, justifying that this value of x gives a minimum value of S

(5)

(c) Find the minimum value of S

(2)

Question 7 continued

(Total for Question 7 is 9 marks)

8 The sum to n terms of an arithmetic series A is S_n

The sum of the first four terms of A is 42 and the fifth term of A is 23

(a) Show that $S_n = \sum_{r=1}^{n} (Pr - Q)$ where P and Q are prime numbers.

(6)

$S_{2n} - 3U_n = 1062$ where U_n is the nth term of A

(b) Find the value of n

(4)

Question 8 continued

(Total for Question 8 is 10 marks)

9

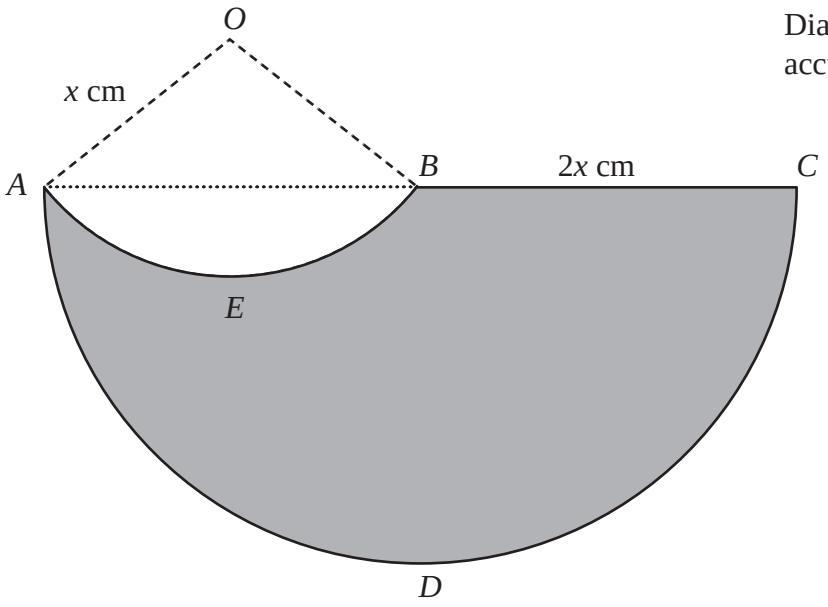


Diagram **NOT** accurately drawn

Figure 2

A logo, *AEBCD*, is shown shaded in Figure 2.

The straight line *ABC* is the diameter of the semicircle *ADC*
AEB is an arc of a circle with centre *O*
All angles are measured in radians.

- $BC = 2x$ cm
- $OA = OB = x$ cm
- length of arc $AEB = 1.8x$ cm

The perimeter of the logo is *P*

(a) Show that $P = ax(\pi + \pi \sin 0.9 + b)$ where *a* and *b* are constants to be found.

(7)

Given that $x = 10$ cm,

(b) find, in cm^2 to 3 significant figures, the area of the logo.

(6)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 13 marks)

10 The roots of a quadratic equation are α and β where

$$\alpha + \beta = -\frac{5}{2} \text{ and } \alpha^3 + \beta^3 = \frac{115}{8}$$

(a) Show that $\alpha\beta = 4$

(3)

(b) Form a quadratic equation with integer coefficients, that has roots

$$\frac{\alpha^2 + 1}{\beta} \text{ and } \frac{\beta^2 + 1}{\alpha}$$

(7)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 10 marks)

11

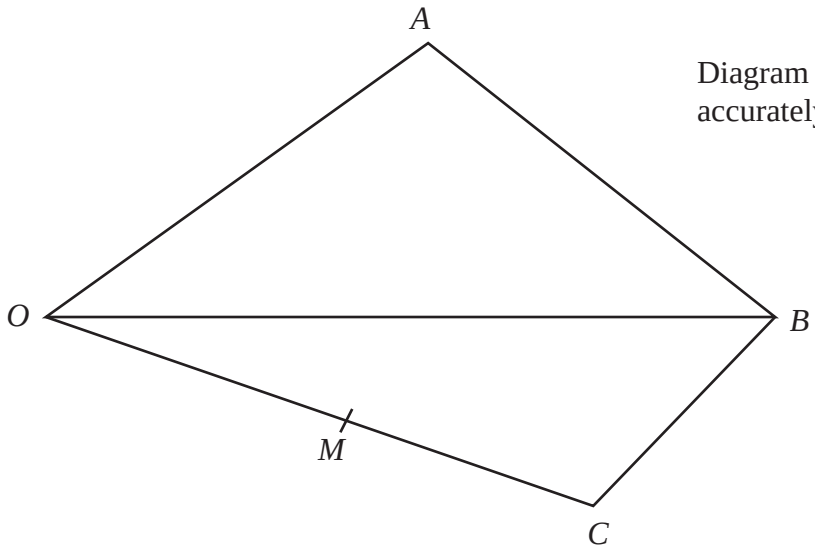


Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows quadrilateral $OABC$ where

$$\overrightarrow{OA} = 4\mathbf{p} + 5\mathbf{q} \qquad \overrightarrow{OB} = 3\mathbf{p} + \mathbf{q} \qquad \overrightarrow{OC} = 2\mathbf{p} - 4\mathbf{q}$$

The point M is the midpoint of OC

(a) Find $\overrightarrow{MA}$ as a simplified expression in terms of **p** and **q**

(3)

The point N lies on OB such that M, N and A are collinear.

(b) Find the ratio $MN : NA$

(6)

Question 11 continued

Question 11 continued

(Total for Question 11 is 9 marks)

TOTAL FOR PAPER IS 100 MARKS