# Mark Scheme (Results)

# Summer 2019

## Pearson Edexcel International GCSE In Further Pure Mathematics (4PM1) Paper 01R

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**General Marking Guidance**

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.
  Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working

- SC - special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- awrt – answer which rounds to
- eeoo – each error or omission

- **No working**
  If no working is shown then correct answers normally score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**
  If there is a wrong answer indicated on the answer line always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
  If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes clear the method that has been used.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

## General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

**Method mark for solving a 3 term quadratic equation:**

1. Factorisation:

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$ leading to $x=\ldots.$

$(ax^2+bx+c)=(mx+p)(nx+q)$ where $|pq|=|c|$ and $|mn|=|a|$ leading to $x=\ldots.$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=\ldots.$

3. Completing the square:

$x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$ leading to $x=\ldots.$

**Method marks for differentiation and integration:**

1. Differentiation

Power of at least one term decreased by 1. $(x^n\to x^{n-1})$

2. Integration:

Power of at least one term increased by 1. $(x^n\to x^{n+1})$

**Use of a formula:**

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

**or**, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

**Answers without working:**

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."
**Exact answers:**

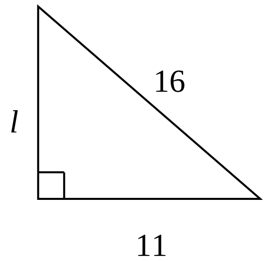
When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

**Rounding answers (where accuracy is specified in the question)**

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

## June 2019
## 4PM1 Further Pure Mathematics Paper 1

| Question number | Scheme | Marks |
|---|---|---|
| 1 (a) | $l = r\theta \Rightarrow r = \frac{12}{1.5} = 8$ | B1 [1] |
| (b) | $A = \frac{1.5}{2} \times 8^2 = 48 \text{ (cm}^2\text{)}$ | M1A1 [2] |
| | **ALT 1** | |
| | $A = \frac{l^2}{2\theta} = \frac{12^2}{2 \times 1.5} = 48 \text{ (cm}^2\text{)}$ | {M1A1} [2] |
| | **ALT 2** | |
| | $A = \frac{1}{2}rl = \frac{1}{2} \times 8 \times 12 = 48 \text{ (cm}^2\text{)}$ | {M1A1} [2] |
| | | Total 3 marks |
| (a) | | |
| B1 | $r = 8$ | |
| (b) | $A = 48 \text{ (cm}^2\text{)}$ units not required | |
| M1 | Use of $A = \frac{1}{2}r^2\theta$ | |
| A1 | $A = 48 \text{ (cm}^2\text{)}$ units not required | |
| ALT 1: M1 | Use of $A = \frac{l^2}{2\theta}$ | |
| A1 | $A = 48 \text{ (cm}^2\text{)}$ units not required | |
| ALT 2: M1 | Use of $A = \frac{1}{2}rl$ | |
| A1 | $A = 48 \text{ (cm}^2\text{)}$ units not required | |

| Question number | Scheme | Marks |
|---|---|---|
| 2 (a) | $\cos ABC = \dfrac{(2x)^2+(4x)^2-(3x)^2}{2\times 2x\times 4x} = \dfrac{x^2(4+16-9)}{x^2(16)} = \dfrac{11}{16}$ | M1A1 |
| |  $l=\sqrt{16^2-11^2}=3\sqrt{15}$ | M1 |
| | $\sin ABC = \dfrac{3\sqrt{15}}{16}$ * | A1<br>[4] |
| | **ALT**<br>$\sin^2 ABC = 1-\dfrac{121}{256}=\dfrac{135}{256}$ Þ $\sin ABC = \dfrac{3\sqrt{15}}{16}$* | {M1A1} |
| (b) | $\dfrac{75\sqrt{15}}{64}=\dfrac{1}{2}\times 2x\times 4x\times\dfrac{3\sqrt{15}}{16}\Rightarrow x^2=\dfrac{25}{16}\Rightarrow x=\dfrac{5}{4}$ oe<br>(positive root only) | M1A1<br>[2] |
| | | Total 6 marks |
| (a)<br>M1 | Use the cosine rule, either form. If not for angle *ABC* there must be a complete method shown for obtaining *ABC* | |
| A1 | Correct expression for $\cos ABC$ | |
| M1 | Use of Pythagoras' leading to $l = \ldots$ | |
| A1 | Obtains the **given** expression for $\sin ABC$ | |
| ALT:<br>M1 | Use of $\sin^2\theta+\cos^2\theta=1$ leading to $\sin^2\theta=\ldots$ | |
| A1 | Obtains the **given** expression for $\sin ABC$ | |
| (b)<br>M1 | Use of $\dfrac{1}{2}ab\sin C=\dfrac{75\sqrt{15}}{64}$ Need not be simplified. | |
| A1 | $x=\dfrac{5}{4}$ oe | |

| Question number | Scheme | Marks |
|---|---|---|
| 3 (a) | $\log_3 9 = 2$ | B1<br>[1] |
| (b) | $\log_3 9t = \log_9\left(\frac{12}{t}\right)^2 + 2 \Rightarrow \log_3 9 + \log_3 t = 2\left(\log_9 12 - \log_9 t\right) + 2$ | M1M1 |
| | $\log_3 9 + \log_3 t = 2\left(\frac{\log_3 12}{\log_3 9} - \frac{\log_3 t}{\log_3 9}\right) + 2$ | M1 |
| | $\Rightarrow \log_3 9 + \log_3 t = \log_3 12 - \log_3 t + 2$ | |
| | $\Rightarrow 2\log_3 t = \log_3 12 \Rightarrow \log_3 t^2 = \log_3 12$ | A1 |
| | $\Rightarrow t^2 = 12 \Rightarrow t = 2\sqrt{3}$ | M1A1<br>[6] |
| | | Total 7 marks |
| (a)<br>B1 | $(\log_3 9 =)2$ | |
| (b) | **The M marks can be seen anywhere in the solution** | |
| M1 | Use of $\log AB = \log A + \log B$ or $\log \frac{A}{B} = \log A - \log B$ | |
| M1 | Use of $\log A^n = n\log A$ | |
| M1 | Use of $\log_a x = \frac{\log_b x}{\log_b a}$ | |
| A1 | Simplifying to $2\log_3 t = \log_3 12$ oe or $\log_3\left(\frac{9t^2}{12}\right) = 2$ oe | |
| M1 | Simplify to $t^2 = \ldots$ | |
| A1 | $t = 2\sqrt{3}$ | |

| Question number | Scheme | Marks |
|---|---|---|
| 4 (a) | $\text{f}'(x) = 3\text{e}^{3x}(1+2x)^{\frac{1}{2}} + \text{e}^{3x} \times \frac{1}{2} \times 2(1+2x)^{-\frac{1}{2}}$ | M1A1 |
| | $\Rightarrow \text{f}'(x) = \dfrac{3\text{e}^{3x}(1+2x)+\text{e}^{3x}}{\sqrt{1+2x}} \Rightarrow \text{f}'(x) = \dfrac{2\text{e}^{3x}(2+3x)}{\sqrt{1+2x}}$* | M1A1 [4] |
| (b) | When $x = 0$ | |
| | $\text{f}'(0) = \dfrac{2\text{e}^0(2+0)}{\sqrt{1+0}} = 4$ Gradient of Normal $= -\dfrac{1}{4}$ | B1B1 |
| | $\text{f}(0) = \text{e}^0\sqrt{1+2\times 0} = 1$ | B1 |
| | Equation of Normal to curve $y = \text{f}(x)$ when $x = 0$ | |
| | $y - 1 = -\dfrac{1}{4}(x-0)$ | M1A1 |
| | $\Rightarrow x + 4y - 4 = 0$ | A1 [6] |
| | | Total 10 marks |
| (a) | | |
| M1 | Use of the product rule. Sum of two terms (either way round) with $x^n \to x^{n-1}$ (Condone $e^{3x}$ instead of $3e^{3x}$) | |
| A1 | Both terms correct | |
| M1 | Simplifying their product rule expression to a single expression with denominator $a\sqrt{1+2x}$ where $a$ is a constant | |
| A1 | Obtains the **given** expression | |
| (b) | | |
| B1 | $\text{f}'(0) = 4$ | |
| B1 | Gradient of Normal $= -\dfrac{1}{4}$ | |
| B1 | $\text{f}(0) = 1$ | |
| M1 | Substitution of (0, ‘1’) and ‘gradient of normal’ (but not 4) into the formula for a line | |
| A1 | $y - 1 = -\dfrac{1}{4}(x-0)$ oe | |
| A1 | $x + 4y - 4 = 0$ | |

| Question number | Scheme | Marks |
|---|---|---|
| 5 | | |
| | $A=\pi(3r)^2=9\pi r^2 \Rightarrow \dfrac{\mathrm{d}A}{\mathrm{d}r}=18\pi r$ | M1 |
| | $\delta A \approx \dfrac{\mathrm{d}A}{\mathrm{d}r}\times\delta r=18\pi r(\delta r)$ | B1 |
| | $\dfrac{\delta A}{A}\approx\dfrac{18\pi r}{A}\delta r=\dfrac{18\pi r}{9\pi r^2}\delta r=2\dfrac{\delta r}{r}$ | M1 |
| | So when $\dfrac{\delta r}{r}=0.05\% \Rightarrow \dfrac{\delta A}{A}\approx 0.1\%$ so the area increases by about 0.1% | M1A1 |
| | **ALT** | |
| | Radius (after increase) = $3r\times\left(1+\dfrac{0.05}{100}\right)$ | {M1} |
| | = 3.0015$r$ | {B1} |
| | Area before increase = $\pi(3r)^2=9\pi r^2$ Area after increase = | |
| | $A=\pi(3.0015r)^2=9.00900225\pi r^2$ | {M1} |
| | Percentage increase = $\dfrac{9.00900225\pi r^2-9\pi r^2}{9\pi r^2}\times 100=0.100025\approx 0.1\%$ so the area increases by about 0.1% | {M1} {A1} |
| | | Total 5 marks |
| M1 | Differentiate $A$ wrt $r$ | |
| B1 | Use of $\delta A\approx\dfrac{\mathrm{d}A}{\mathrm{d}r}\times\delta r$ | |
| M1 | Use of $\dfrac{\delta A}{A}$ | |
| M1 | Use of $\dfrac{\delta r}{r}=0.05\%$ | |
| A1 | Area increases by about 0.1% | |
| ALT: | | |
| M1 | Finding the radius after the increase (may be implied by 3.0015$r$) | |
| B1 | 3.0015$r$ (may be implied by a correct area after the increase) | |
| M1 | Finding the area after the increase | |
| M1 | Use of $\dfrac{\text{Area (new)-Area (original)}}{\text{Area (original)}}\times 100$ | |
| A1 | Area increases by about 0.1% | |

| Question number | Scheme | Marks |
|---|---|---|
| 6 (a) | $a = 4\times 1 - 3 = 1, \qquad (d = 4)$ | B1 |
| | $\sum_{r=1}^{n} 4r - 3 = \frac{n}{2}\left(2\times 1 + (n-1)4\right) = n(2n-1)$ * | M1A1 |
| | | [3] |
| (b) | $n(2n-1) > 1000 \Rightarrow 2n^2 - n - 1000 > 0$ | M1 |
| | $\frac{-(-1)\pm\sqrt{(-1)^2 - 4\times 2\times(-1000)}}{2\times 2} \Rightarrow n > 22.612... \Rightarrow n = 23$ | M1A1 |
| | | [3] |
| (c) | $3t_{(n+7)} + 18 = S_{(n+4)}$ | |
| | $\Rightarrow 3\left[4(n+7)-3\right] + 18 = (n+4)\left[2(n+4)-1\right]$ | M1 |
| | $\Rightarrow 2n^2 + 3n - 65 = 0$ | A1 |
| | $2n^2 + 3n - 65 = (2n+13)(n-5) = 0 \Rightarrow n = 5$ | depM1A1 |
| | | [4] |
| | | Total 10 marks |
| (a) | | |
| B1 | $a = 1$ | |
| M1 | Use of $S = \frac{n}{2}(2a + (n-1)d)$ or $S = \frac{n}{2}(a + L)$ | |
| A1 | Obtains the **given** expression | |
| (b) | | |
| M1 | Sets up a 3 term quadratic from the given information (Condone = rather than >) | |
| M1 | Solve their 3 term quadratic (May be implied by 22.6 …) | |
| A1 | $n$ = 23 | |
| (c) | | |
| M1 | Substitution of $n$ + 7 and $n$ + 4 | |
| A1 | A correct 3 term quadratic | |
| depM1 | Solve their 3 term quadratic (Dependent on previous M mark) | |
| A1 | $n$ = 5 (must reject other answer if offered) | |

| Question number | Scheme | Marks |
|---|---|---|
| 7 (a) | $\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$ | M1 |
| | $\overrightarrow{BC} = -(15\mathbf{i} - 6\mathbf{j}) + 8\mathbf{i} + \mathbf{j} = -7\mathbf{i} + 7\mathbf{j}$ | A1<br>[2] |
| (b) | $\lvert\overrightarrow{BC}\rvert = \sqrt{98} = \left(7\sqrt{2}\right)$ | B1 |
| | Unit vector is $\frac{1}{\sqrt{98}}(-7\mathbf{i} + 7\mathbf{j})$ oe | B1<br>[2] |
| (c) | $\left(\overrightarrow{OM} = 4\mathbf{i} - 3\mathbf{j}\right)$ $\overrightarrow{ON} = 5\mathbf{i} - 2\mathbf{j}$ | B1 |
| | $\Rightarrow \overrightarrow{MN} = -(4\mathbf{i} - 3\mathbf{j}) + 5\mathbf{i} - 2\mathbf{j}(= \mathbf{i} + \mathbf{j})$ | M1 |
| | $\Rightarrow \overrightarrow{MC} = -(4\mathbf{i} - 3\mathbf{j}) + 8\mathbf{i} + \mathbf{j}(= 4\mathbf{i} + 4\mathbf{j})$ | M1 |
| | Conclusion: $\overrightarrow{MN}$ and $\overrightarrow{MC}$ are parallel oe (and have same point of origin ($M$)) hence they are collinear. | A1<br>[4] |
| | **ALT 1** | |
| | $\left(\overrightarrow{OM} = 4\mathbf{i} - 3\mathbf{j}\right)$ $\overrightarrow{ON} = 5\mathbf{i} - 2\mathbf{j}$ or $\overrightarrow{NB} = 10\mathbf{i} - 4\mathbf{j}$ | |
| | $\Rightarrow \overrightarrow{MN} = -(4\mathbf{i} - 3\mathbf{j}) + 5\mathbf{i} - 2\mathbf{j}(= \mathbf{i} + \mathbf{j})$ | {B1} |
| | $\Rightarrow \overrightarrow{NC} = 10\mathbf{i} - 4\mathbf{j} - 7\mathbf{i} + 7\mathbf{j}(= 3\mathbf{i} + 3\mathbf{j})$ or | {M1} |
| | $\Rightarrow \overrightarrow{NC} = -(5\mathbf{i} - 2\mathbf{j}) + 8\mathbf{i} + \mathbf{j}(= 3\mathbf{i} + 3\mathbf{j})$ | {M1} |
| | Conclusion: $\overrightarrow{MN}$ and $\overrightarrow{NC}$ are parallel oe (and share the same point ($N$)) hence they are collinear. | {A1}<br>[4] |
| | **ALT 2** | |
| | $\left(\overrightarrow{OM} = 4\mathbf{i} - 3\mathbf{j}\right)$ $\overrightarrow{ON} = 5\mathbf{i} - 2\mathbf{j}$ or $\overrightarrow{NB} = 10\mathbf{i} - 4\mathbf{j}$ | |
| | $\Rightarrow \overrightarrow{MC} = -(4\mathbf{i} - 3\mathbf{j}) + 8\mathbf{i} + \mathbf{j}(= 4\mathbf{i} + 4\mathbf{j})$ | {B1} |
| | $\Rightarrow \overrightarrow{NC} = 10\mathbf{i} - 4\mathbf{j} - 7\mathbf{i} + 7\mathbf{j}(= 3\mathbf{i} + 3\mathbf{j})$ | {M1} |
| | Conclusion: $\overrightarrow{MC}$ and $\overrightarrow{NC}$ are parallel oe(and share the same point (C)) hence they are collinear. | {M1}<br>{A1}<br>[4] |
| | | Total 8 marks |

| | |
|---|---|
| (a)<br>M1<br>A1 | $\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$<br>$\overrightarrow{BC} = -7\mathbf{i} + 7\mathbf{j}$ |
| (b)<br>B1<br>B1 | $\sqrt{98}$ oe<br>$\frac{1}{\sqrt{98}}(-7\mathbf{i} + 7\mathbf{j})$ oe |
| (c)<br>B1<br>M1<br>M1<br>A1 | $\overrightarrow{ON} = 5\mathbf{i} - 2\mathbf{j}$ (may be implied by $\overrightarrow{MN}$)<br>$\overrightarrow{MN} = -(4\mathbf{i} - 3\mathbf{j}) + 5\mathbf{i} - 2\mathbf{j}(= \mathbf{i} + \mathbf{j})$<br>$\overrightarrow{MC} = -(4\mathbf{i} - 3\mathbf{j}) + 8\mathbf{i} + \mathbf{j}(= 4\mathbf{i} + 4\mathbf{j})$<br>Correct conclusion from correct working e.g. $\overrightarrow{MC} = 4\overrightarrow{MN}$ |
| ALT 1<br>B1<br>M1<br>M1<br>A1 | $\overrightarrow{ON} = 5\mathbf{i} - 2\mathbf{j}$ or $\overrightarrow{NB} = 10\mathbf{i} - 4\mathbf{j}$ (may be implied by $\overrightarrow{MN}$ or $\overrightarrow{NC}$)<br>$\overrightarrow{MN} = -(4\mathbf{i} - 3\mathbf{j}) + 5\mathbf{i} - 2\mathbf{j}(= \mathbf{i} + \mathbf{j})$<br>$\overrightarrow{NC} = 10\mathbf{i} - 4\mathbf{j} - 7\mathbf{i} + 7\mathbf{j}(= 3\mathbf{i} + 3\mathbf{j})$ or $-(5\mathbf{i} - 2\mathbf{j}) + 8\mathbf{i} + \mathbf{j}(= 3\mathbf{i} + 3\mathbf{j})$<br>Correct conclusion from correct working e.g. $\overrightarrow{NC} = 3\overrightarrow{MN}$ |
| ALT 2<br>B1<br>M1<br>M1<br>A1 | $\overrightarrow{NB} = 10\mathbf{i} - 4\mathbf{j}$ (may be implied by $\overrightarrow{NC}$)<br>$\overrightarrow{MC} = -(4\mathbf{i} - 3\mathbf{j}) + 8\mathbf{i} + \mathbf{j}(= 4\mathbf{i} + 4\mathbf{j})$<br>$\overrightarrow{NC} = 10\mathbf{i} - 4\mathbf{j} - 7\mathbf{i} + 7\mathbf{j}(= 3\mathbf{i} + 3\mathbf{j})$<br>Correct conclusion from correct working e.g. $\overrightarrow{NC} = \frac{3}{4}\overrightarrow{MC}$<br><br>**For part c: Send any geometrical solutions to review** |

| Question number | Scheme | Marks |
|---|---|---|
| 8 (a) | <table><tr><td>$x$</td><td>0</td><td>0.25</td><td>0.5</td><td>1</td><td>1.5</td><td>2</td><td>3</td></tr><tr><td>$y$</td><td>2</td><td>**2.41**</td><td>**2.69**</td><td>3.10</td><td>3.39</td><td>3.61</td><td>**3.95**</td></tr></table> | B2 [2] |
| (b) | Points plotted within half of a square<br>Points joined together in a smooth curve | B1ft<br>B1ft<br>[2] |
| (c) | $\ln(2x+1)=3x-4 \Rightarrow \ln(2x+1)+2=3x-2$<br>Graph of $y=3x-2$ drawn. Intersection point is at $x$ = 1.8 or 1.9<br>(Accept either) | M1<br>M1A1<br>[3] |
| (d) | $e^{(6-x)}=(2x+1)^2 \Rightarrow 6-x=\ln(2x+1)^2 \Rightarrow 6-x=2\ln(2x+1)$<br>$\Rightarrow \ln(2x+1)+2=5-\frac{x}{2}$<br>Graph of $y=5-\frac{x}{2}$ drawn. Intersection point is at $x$ = 2.4 or 2.5<br>(Accept either) | M1<br>M1<br>M1A1<br>[4] |
| | | Total 11 marks |
| (a)<br>B2 | All 3 points correct<br>(B1 for 2 points correct) | |
| (b)<br>B1ft<br>B1ft | Points plotted ft their table allow half a square tolerance<br>Points joined together with a smooth curve ft their table | |
| (c)<br>M1<br>M1<br>A1 | $\ln(2x+1)+2=3x-2$<br>$y=3x-2$ drawn.<br>Intersection point is at ($x$ =) 1.8 or 1.9 Accept either | |
| (d)<br>M1<br>M1<br>M1<br>A1 | $6-x=2\ln(2x+1)$<br>$\ln(2x+1)+2=5-\frac{x}{2}$<br>Graph of $y=5-\frac{x}{2}$ drawn<br>Intersection point is at ($x$ =) 2.4 or 2.5 Accept either | |

| Question number | Scheme | Marks |
|---|---|---|
| 9 (a) | $540 = 3x^2h \Rightarrow h = \dfrac{180}{x^2}$<br>$S = 2\left(3x^2 + 3xh + xh\right) = 6x^2 + 8xh$<br>$\Rightarrow S = 6x^2 + 8x \times \dfrac{180}{x^2} = 6x^2 + \dfrac{1440}{x}$ * | M1<br>M1<br>depM1A1<br>[4] |
| (b) | $S = 6x^2 + 1440x^{-1}$<br>$\dfrac{dS}{dx} = 12x - 1440x^{-2}$<br>At min/max $\dfrac{dS}{dx} = 0$<br>$12x - 1440x^{-2} = 0 \Rightarrow x^3 = 120 \Rightarrow x = 4.93242....$<br>$x \approx 4.93$ (3sf)<br>$\dfrac{d^2S}{dx^2} = 12 + \dfrac{2880}{x^3} \Rightarrow$ Always positive for positive values of $x$, hence minimum | M1<br>M1A1<br>M1A1ft<br>[5] |
| (c) | $S = 6 \times 4.93242^2 + \dfrac{1440}{4.93242} = 437.9185 \approx 438$ | B1<br>[1] |
| | | Total 10 marks |
| (a) | | |
| M1 | Rearrange the equation for volume to make $h$ the subject | |
| M1 | Obtains an expression for $S$ in terms of $x$ and $h$. | |
| depM1 | Dependent on previous M1. Use the equation to eliminate $h$ to give an expression for $S$ in terms of $x$ only. | |
| A1 | Obtains the **given** expression for $S$. | |
| (b) | | |
| M1 | Attempts to differentiates $S$ wrt $x$ with $x^n \to x^{n-1}$ | |
| M1 | Equate their derivative to zero and solve for $x$ | |
| A1 | Correct value of $x$, min 3 sf (Do not accept $\sqrt[3]{120}$ ) | |
| M1 | Obtains a correct second derivative from their first derivative.<br>(If signs of $\dfrac{dS}{dx}$ on either side of their $x$ are considered, numerical calculations must be shown.) | |
| A1 ft | Establish that the minimum has been obtained and give a conclusion. No need to calculate the value of the second derivative.<br>Follow through their $x$ provided $x > 0$ and the second derivative is algebraically correct or if signs of $\dfrac{dS}{dx}$ on either side of their $x$ were considered these need to be calculated and correct | |
| (c) | | |
| B1 | Correct value of $S$. Must be 3 sf | |

| Question number | Scheme | Marks |
|---|---|---|
| 10 (a) | $6x - x^2 = -\left(x^2 - 6x\right)$<br>$-\left(x^2 - 6x\right) = -\left\{\left(x-3\right)^2 - 9\right\} \Rightarrow \mathrm{f}\left(x\right) = -\left(x-3\right)^2 + 9$<br>$D = -1,\ \ E = -3$ and $F = 9$ | M1A1A1<br>[3] |
| (b) | (i) $\mathrm{f}\left(x\right)_{\max} = 9$<br>(ii) $x = 3$ | B1ft<br>B1ft<br>[2] |
| (c) | $6x - x^2 = x^2 - 4x + 8 \Rightarrow 2x^2 - 10x + 8 = 0$<br>$2x^2 - 10x + 8 = \left(2x-2\right)\left(x-4\right) \Rightarrow x = 1,\ \ x = 4$<br>$y = 5,\ \ y = 8$<br>Coordinates are (1, 5) and (4, 8) | M1<br><br>M1A1<br><br>A1<br>[4] |
| (d) | Area = $\int_1^4 \left(6x - x^2\right)\,\mathrm{d}x - \int_1^4 \left(x^2 - 4x + 8\right)\,\mathrm{d}x = \int_1^4 \left[-2x^2 + 10x - 8\right]\,\mathrm{d}x$<br>$= \left[\dfrac{-2x^3}{3} + \dfrac{10x^2}{2} - 8x\right]_1^4$<br>$= \left(\dfrac{-2\times 4^3}{3} + \dfrac{10\times 4^2}{2} - 8\times 4\right) - \left(\dfrac{-2\times 1^3}{3} + \dfrac{10\times 1^2}{2} - 8\times 1\right) = 9$ (units$^2$) | M1<br><br>M1<br><br>M1A1<br>[4] |
| | | Total 13 marks |
| (a)<br>M1<br>A1<br><br>A1 | An attempt to factorise to make $x^2$ positive e.g. $-\left(x \pm a\right)^2 \pm b$<br>Complete the square to obtain an expression in the form $-\left(x \pm 3\right)^2 \pm q$ **NB** Any expression in this form will score M1A1<br>$D = -1,\ \ E = -3$ and $F = 9$ | |
| (b)<br>B1 ft<br>B1 ft | $\left(\mathrm{f}\left(x\right)_{\max} =\right)9$ or follow through their value for $F$.<br>$\left(x =\right)3$ or follow through their value for $E$. | |
| (c)<br>M1<br>M1<br>A1<br>A1 | Equating the two curves and simplifying to a 3 term quadratic<br>Solve their 3 term quadratic<br>$x = 1,\ \ x = 4$<br>(1, 5) and (4, 8) | |
| (d)<br>M1<br><br>M1<br>M1<br>A1 | Use of $\int_a^b \left(\mathrm{f}(x) - \mathrm{g}(x)\right)\mathrm{d}x$ or $\int_a^b \mathrm{f}(x)\mathrm{d}x - \int_a^b \mathrm{g}(x)\mathrm{d}x$ Ignore limits<br>( $\mathrm{f}(x)$ and $\mathrm{g}(x)$ can be either way round)<br>Attempt the integration. Limits not needed.<br>Substitute the correct limits.<br>9 (units$^2$)<br>**NB** A correct answer with no working will score 4 out of 4 | |

| Question number | Scheme | Marks |
|---|---|---|
| 11 (a) | $\dfrac{y-6}{x-5}=\dfrac{6-3}{5--1}\Rightarrow 2y-12=x-5$ | M1A1 [2] |
| (b) | $\left(\dfrac{2\times5+1\times-1}{2+1},\dfrac{2\times6+1\times3}{3}\right)\Rightarrow(3,\ 5)*$ | M1A1 [2] |
| (a) M1 | A fully correct method for finding the equation of a straight line e.g. $\dfrac{y-y_1}{x-x_1}=\dfrac{y_2-y_1}{x_2-x_1}$ | |
| A1 | $2y-12=x-5$ oe | |
| (b) M1 | Use of $\left(\dfrac{qx_1+px_2}{p+q},\dfrac{qy_1+py_2}{p+q}\right)$ or $\begin{pmatrix}-1\\3\end{pmatrix}+\dfrac{2}{3}\times\begin{pmatrix}6\\3\end{pmatrix}$ | |
| A1 | Obtains the **given** coordinates | |

| | | |
|---|---|---|
| (c) | Gradient of perpendicular to $AB = -2$ | B1 |
| | $\frac{5-n}{3-m} = -2 \Rightarrow (11 = 2m + n)$ | M1 |
| | Radius = 5, so the length of $AC = 10$ | |
| | $100 = (3-n)^2 + (-1-m)^2 \Rightarrow (90 = n^2 - 6n + m^2 + 2m)$ | M1 |
| | Solves simultaneous equations<br>$90 = 121 - 44m + 4m^2 - 6(11-2m) + m^2 + 2m$ | depM1 |
| | $35 = 5m^2 - 30m \Rightarrow m^2 - 6m - 7 = 0$<br>$(m-7)(m+1) = 0 \Rightarrow m = 7,\quad n = -3$ | depM1<br>A1<br>[6] |
| **ALT (c) using vectors** | | |
| (c) | $\overrightarrow{AB} = \begin{pmatrix}5\\6\end{pmatrix} - \begin{pmatrix}-1\\3\end{pmatrix} = \begin{pmatrix}6\\3\end{pmatrix} \Rightarrow$ Perpendicular vector to $\overrightarrow{AB} = \begin{pmatrix}3\\-6\end{pmatrix}$ | B1 |
| | $\lvert\overrightarrow{AB}\rvert = \sqrt{45} = 3\sqrt{5} \Rightarrow \lvert AP\rvert = 2\sqrt{5} \qquad \lvert\overrightarrow{AC}\rvert = 10$ | M1M1 |
| | Using Pythagoras $\lvert\overrightarrow{PC}\rvert = \sqrt{100-20} = \sqrt{80} = 4\sqrt{5}$ | depM1 |
| | $\overrightarrow{PC} = \frac{4\sqrt{5}}{3\sqrt{5}} \times \begin{pmatrix}3\\-6\end{pmatrix} = \begin{pmatrix}4\\-8\end{pmatrix}$ | depM1 |
| | $\Rightarrow$ Coordinates of $C$ are $(3+(4),\ 5+(-8)) \Rightarrow (7,-3)$ | A1<br>[6] |
| **(c)** | | |
| B1 | Gradient of perpendicular to $AB = -2$ (Can be implied by M1) | |
| M1 | Obtains an equation using the gradient of the perpendicular and the points $(m, n)$ and $P$ (Condone if given in terms of $x$ and $y$) | |
| M1 | Obtains a second equation using $AC = 10$ and the points $A$ and $C$ | |
| depM1 | Solve simultaneously to obtain a 3 term quadratic | |
| depM1 | Solve their 3 term quadratic | |
| A1 | $m = 7,\quad n = -3$ | |
| ALT: | | |
| B1 | Perpendicular vector to $\overrightarrow{AB} = \begin{pmatrix}3\\-6\end{pmatrix}$ | |
| M1 | Finds $\lvert\overrightarrow{AB}\rvert$ or $\lvert\overrightarrow{AP}\rvert$ or $\overrightarrow{PC} = \begin{pmatrix}3\\5\end{pmatrix} + \lambda\begin{pmatrix}3\\-6\end{pmatrix}$ | |
| M1 | States $\lvert\overrightarrow{AC}\rvert = 10$ or $\lambda = \frac{4}{3}$ | |
| depM1 | Use of Pythagoras to find $\lvert\overrightarrow{PC}\rvert$ or $\overrightarrow{PC} = \begin{pmatrix}3\\5\end{pmatrix} + \frac{4}{3} \times \begin{pmatrix}3\\-6\end{pmatrix}$ | |
| depM1 | Finds $\overrightarrow{PC}$ | |
| A1 | $m = 7,\quad n = -3$ | |

| | | |
|---|---|---|
| (d) | $\frac{y-'-3'}{x-'7'}=\frac{1}{2}\Rightarrow\left\{y=\frac{x-13}{2}\right\},\quad \frac{y-3}{x--1}=-2\Rightarrow\{y=-2x+1\}$<br>Solving simultaneous equations by any method<br>$\frac{x-13}{2}=-2x+1\Rightarrow p=3$ and $q=-5$ | M1<br><br>M1A1<br>[3] |
| **ALT (d) using vectors** | | |
| (d) | $\overrightarrow{AD}=\overrightarrow{PC}=\begin{pmatrix}4\\-8\end{pmatrix}$<br>Coordinates of point D $\Rightarrow(-1+(4),\ 3+(-8))=(3,-5)$ | M1<br><br>M1A1<br>[3] |
| (e) | Length of $AB$ $\sqrt{(6-3)^2+(5--1)^2}=3\sqrt{5}$<br>Length of $CD$ $\sqrt{(-3--5)^2+(7-3)^2}=\sqrt{20}=2\sqrt{5}$<br>Area of trapezium = $\frac{1}{2}\left(3\sqrt{5}+2\sqrt{5}\right)\times4\sqrt{5}=50$ (units$^2$) | M1<br>A1<br><br>M1A1<br>[4] |
| **ALT (e) using vectors** | | |
| | Area = $\frac{1}{2}\begin{vmatrix}-1&5&7&3&-1\\3&6&-3&-5&3\end{vmatrix}$<br>$=\frac{1}{2}\left[(-6-15-35+9)-(15+42-9+5)\right]=50$ | <br>M1A1<br><br>M1A1<br>[4] |

| |
|---|
| Total 17 marks |

| | |
|---|---|
| **(d)** | |
| M1 | Obtains a linear equation using the given information |
| M1 | Obtains a 2nd linear equation using the given information and solves simultaneously |
| A1 | $p=3$ and $q=-5$ **NB** A correct answer, no incorrect working scores 3 out of 3 |
| ALT: | |
| M1 | Use $\overrightarrow{AD}=\overrightarrow{PC}$ |
| M1 | Substitution of the point (–1, 3) to find the coordinates of point $D$ |
| A1 | $p=3$ and $q=-5$ |
| **(e)** | |
| M1 | Use Pythagoras to find either the length of $AB$ or $CD$ |
| A1 | Both lengths correct |
| M1 | Use of area of trapezium formula using their lengths |
| A1 | 50 (units not required) |
| ALT: | |
| M1 | Use of Area = $\frac{1}{2}\begin{vmatrix}a&c&e&g&a\\b&d&f&h&b\end{vmatrix}$ with the coordinates of $A$, $B$, $C$ and $D$ |
| A1 | Area = $\frac{1}{2}\begin{vmatrix}-1&5&7&3&-1\\3&6&-3&-5&3\end{vmatrix}$ |
| M1 | Attempt to evaluate the Area |
| A1 | 50 (units not required) |

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