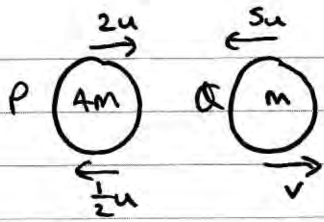
PMT
M1 JAN 13

1. Two particles P and Q have masses $4m$ and m respectively. The particles are moving towards each other on a smooth horizontal plane and collide directly. The speeds of P and Q immediately before the collision are $2u$ and $5u$ respectively. Immediately after the collision, the speed of P is $\frac{1}{2}u$ and its direction of motion is reversed.

(a) Find the speed and direction of motion of Q after the collision.
(4)

(b) Find the magnitude of the impulse exerted on P by Q in the collision.
(3)

a)


CLM ⇒ $8mu - 5mu = -2mu + mv$

$5\cancel{m}u = \cancel{m}v$

$\underline{v = 5u}$

Speed = 5u and the direction of motion is reversed.

b) Mom Q before = $-5mu$ ⇒ Impulse = $\underline{10mu \text{ Ns}}$

Mom Q after = $5mu$

2. A steel girder AB, of mass 200 kg and length 12 m, rests horizontally in equilibrium on two smooth supports at C and at D, where $AC = 2$ m and $DB = 2$ m. A man of mass 80 kg stands on the girder at the point P, where $AP = 4$ m, as shown in Figure 1.

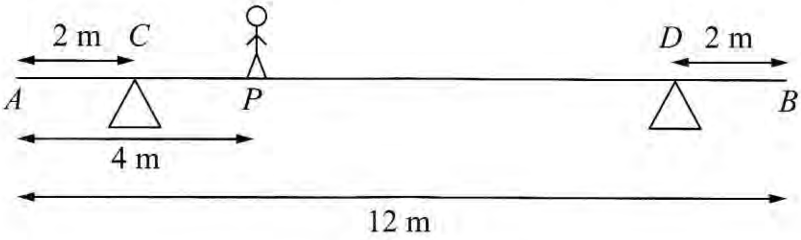


Figure 1

The man is modelled as a particle and the girder is modelled as a uniform rod.

(a) Find the magnitude of the reaction on the girder at the support at C.

(3)

The support at D is now moved to the point X on the girder, where $XB = x$ metres. The man remains on the girder at P, as shown in Figure 2.

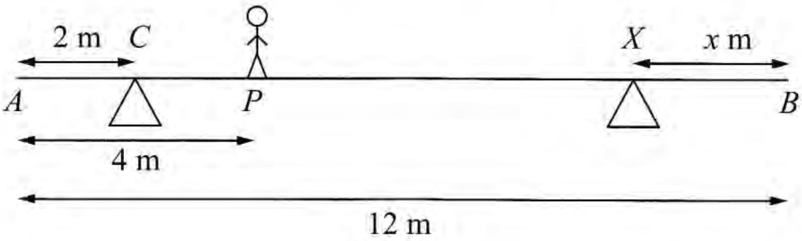


Figure 2

Given that the magnitudes of the reactions at the two supports are now equal and that the girder again rests horizontally in equilibrium, find

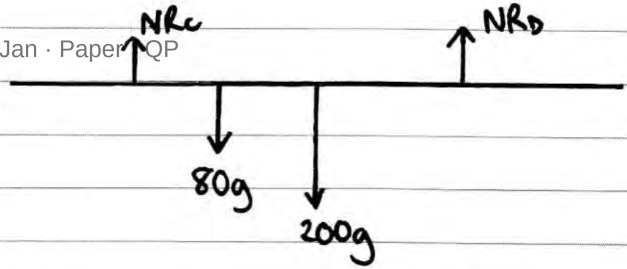
(b) the magnitude of the reaction at the support at X,

(2)

(c) the value of x.

(4)

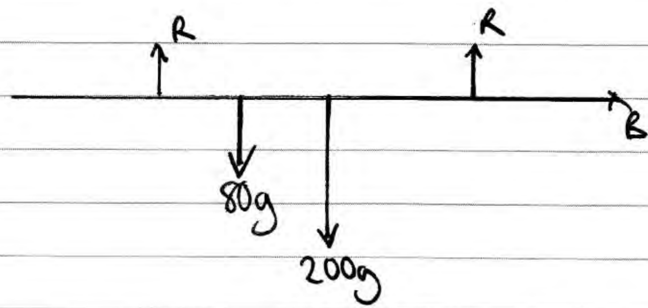
a)



$D \circlearrowright \quad NR_C \times 8 = 200g \times 4 + 80g \times 6$

$8NR_C = 1280g \Rightarrow \underline{NR_C = 160g \text{ N}}$

b)



$\uparrow = \downarrow \quad 2R = 280g \Rightarrow R = 140g.$

$B \circlearrowright \quad R \times x + R \times 10 = 200g \times 6 + 80g \times 8$

$140gx + 1400g = 1840g \Rightarrow 140gx = 440g$

$x = \underline{3.14\text{m}}\ (3sf)$

3. A particle P of mass 2 kg is attached to one end of a light string, the other end of which is attached to a fixed point O. The particle is held in equilibrium, with OP at 30° to the downward vertical, by a force of magnitude F newtons. The force acts in the same vertical plane as the string and acts at an angle of 30° to the horizontal, as shown in Figure 3.

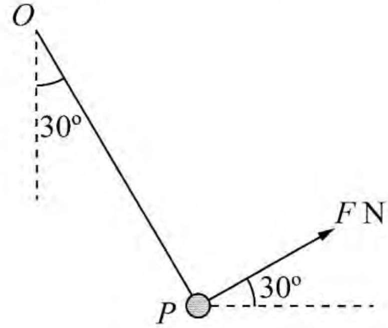


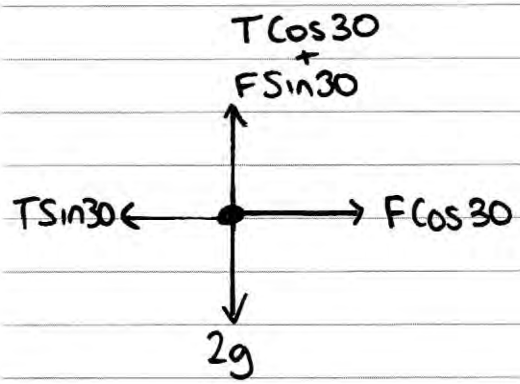
Figure 3

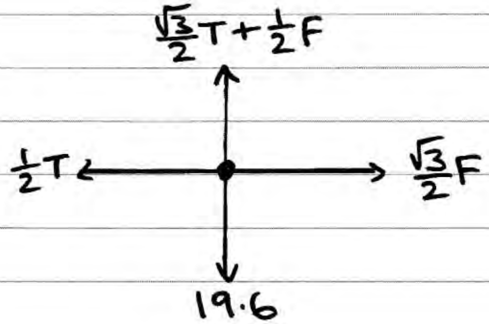
Find

(i) the value of F,

(ii) the tension in the string.

(8)





$\rightarrow = \leftarrow \Rightarrow \quad \frac{1}{2}T = \frac{\sqrt{3}}{2}F \quad \times 2 \quad T = \sqrt{3}F$

$\uparrow = \downarrow \Rightarrow \quad \frac{\sqrt{3}}{2}T + \frac{1}{2}F = 19.6 \quad \times 2 \quad \sqrt{3}T + F = 39.2$

$\Rightarrow 3F + F = 39.2 \Rightarrow 4F = 39.2$

$\therefore \underline{F = 9.8\text{N}}$

$T = \sqrt{3}F \Rightarrow \underline{T = 17\text{N}}$ (2sf)

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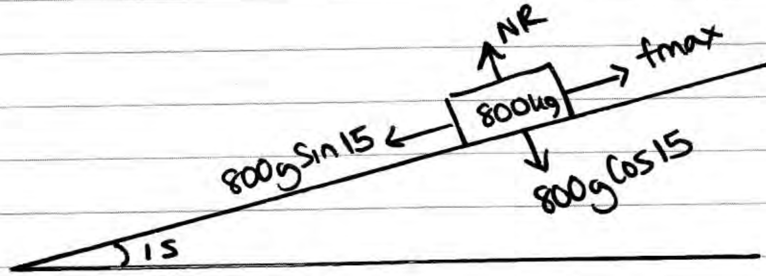
4. A lifeboat slides down a straight ramp inclined at an angle of 15° to the horizontal. The lifeboat has mass 800 kg and the length of the ramp is 50 m. The lifeboat is released from rest at the top of the ramp and is moving with a speed of 12.6 m s^{-1} when it reaches the end of the ramp. By modelling the lifeboat as a particle and the ramp as a rough inclined plane, find the coefficient of friction between the lifeboat and the ramp.

(9)



$s = 50$ $\quad$ $v^2 = u^2 + 2as$

$u = 0$ $\quad$ $12.6^2 = 100a \Rightarrow a = 1.5876.$

$v = 12.6$

a

$\cancel{t}$

$\uparrow = \downarrow \Rightarrow NR = 800g\cos 15 = 7572.858...$

$F_{max} = \mu NR \Rightarrow f_{max} = 7572.858...\mu.$

$\nearrow = \swarrow \Rightarrow 800g\sin 15 - f_{max} = 800a$

$\Rightarrow 2029.1413... - 7572.858..\mu = 1270.08$

$\Rightarrow 759.06... = 7572.858\mu$

$\therefore \underline{\mu = 0.1}$

5.

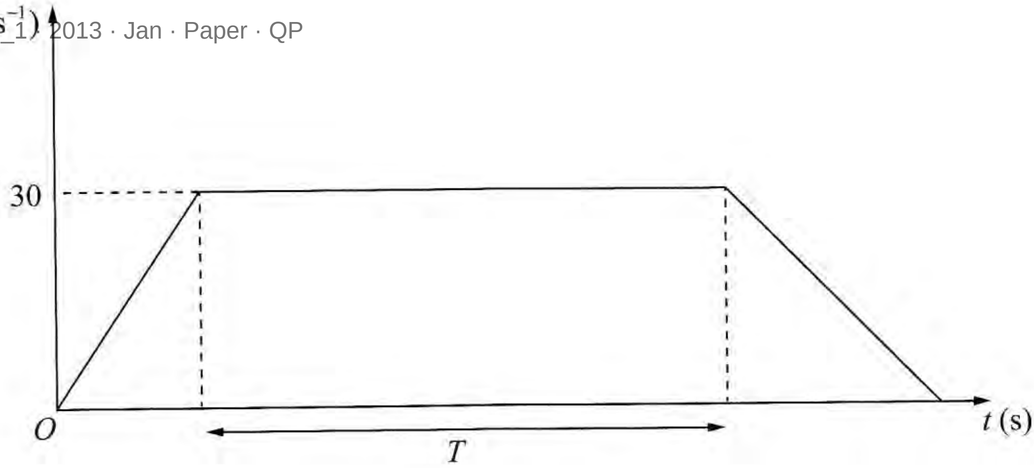


Figure 4

The velocity-time graph in Figure 4 represents the journey of a train P travelling along a straight horizontal track between two stations which are 1.5 km apart. The train P leaves the first station, accelerating uniformly from rest for 300 m until it reaches a speed of 30 m s^{-1}. The train then maintains this speed for T seconds before decelerating uniformly at 1.25 m s^{-2}, coming to rest at the next station.

(a) Find the acceleration of P during the first 300 m of its journey.

(2)

(b) Find the value of T.

(5)

A second train Q completes the same journey in the same total time. The train leaves the first station, accelerating uniformly from rest until it reaches a speed of V m s^{-1} and then immediately decelerates uniformly until it comes to rest at the next station.

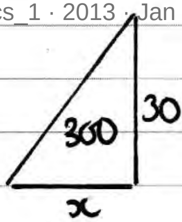
(c) Sketch on the diagram above, a velocity-time graph which represents the journey of train Q.

(2)

(d) Find the value of V.

(6)

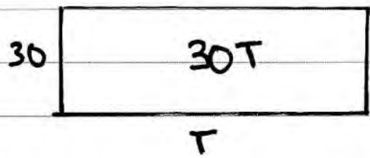
a)

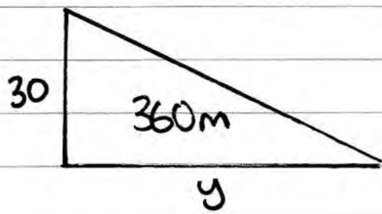


$15x = 300 \Rightarrow x = 20\text{ sec}$

$\therefore \ acc = \dfrac{30}{20} = \underline{1.5\,ms^{-2}}$

b)



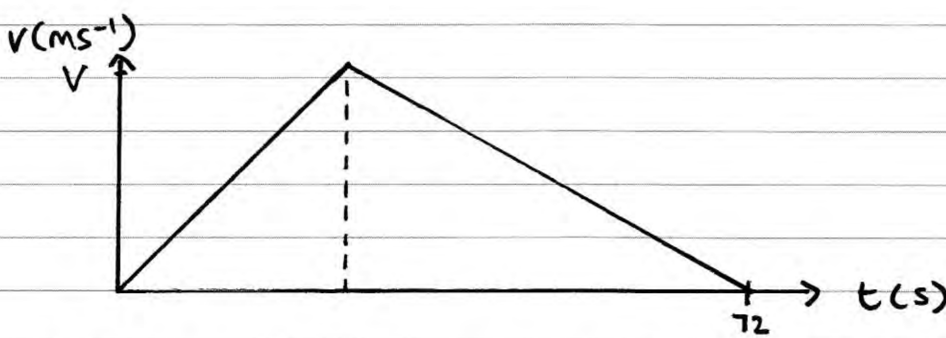


$\dfrac{30}{y} = 1.25 \Rightarrow y = 24\text{ sec}$

Area $= 12 \times 30 = 360\text{m}$

$\therefore \ 30T = 1500 - 360 - 300 \Rightarrow \underline{T = 28\text{ sec}}.$

c) total time $= 72\text{ sec}$



$\therefore \ 36V = 1500 \Rightarrow V = \underline{41.7}\,ms^{-1}$

6. [*In this question,* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.*]

A ship sets sail at 9 am from a port P and moves with constant velocity. The position vector of P is $(4\mathbf{i} - 8\mathbf{j})$ km. At 9.30 am the ship is at the point with position vector $(\mathbf{i} - 4\mathbf{j})$ km.

(a) Find the speed of the ship in km h^{-1}.

(4)

(b) Show that the position vector $\mathbf{r}$ km of the ship, t hours after 9 am, is given by $\mathbf{r} = (4 - 6t)\mathbf{i} + (8t - 8)\mathbf{j}$.

(2)

At 10 am, a passenger on the ship observes that a lighthouse L is due west of the ship. At 10.30 am, the passenger observes that L is now south-west of the ship.

(c) Find the position vector of L.

(5)

a) Speed = $\dfrac{\text{Change in Pos}}{\text{time}} = \dfrac{\begin{pmatrix}-3\\4\end{pmatrix}}{0.5} = \begin{pmatrix}-6\\8\end{pmatrix}$

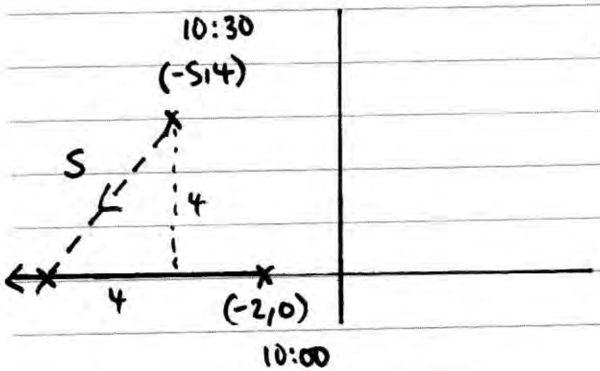
$\therefore$ speed = 10km/h.

b) $r = \begin{pmatrix}4\\-8\end{pmatrix} + t\begin{pmatrix}-6\\8\end{pmatrix} = \begin{pmatrix}4-6t\\-8+8t\end{pmatrix}$ $\therefore r = (4-6t)i + (8t-8)j$

#

c) at 10am $t = 1 \Rightarrow r = \begin{pmatrix}4-6\\-8+0\end{pmatrix} = \begin{pmatrix}-2\\0\end{pmatrix}$

at 10:30am $t = 1.5$ $r = \begin{pmatrix}4-9\\-8+12\end{pmatrix} = \begin{pmatrix}-5\\4\end{pmatrix}$



$L = (-9, 0)$

$\therefore L = \begin{pmatrix}-9\\0\end{pmatrix}$

7.

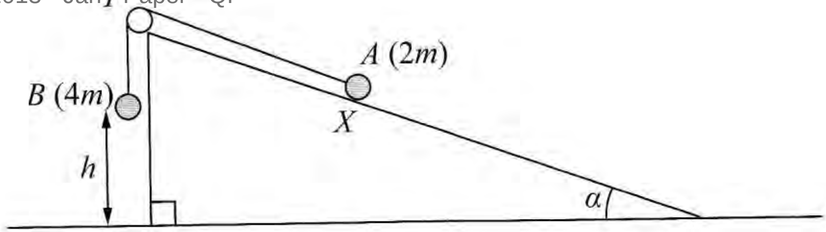


Figure 5

Figure 5 shows two particles A and B, of mass $2m$ and $4m$ respectively, connected by a light inextensible string. Initially A is held at rest on a rough inclined plane which is fixed to horizontal ground. The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$. The coefficient of friction between A and the plane is $\frac{1}{4}$. The string passes over a small smooth pulley P which is fixed at the top of the plane. The part of the string from A to P is parallel to a line of greatest slope of the plane and B hangs vertically below P. The system is released from rest with the string taut, with A at the point X and with B at a height h above the ground.

For the motion until B hits the ground,

(a) give a reason why the magnitudes of the accelerations of the two particles are the same,

(1)

(b) write down an equation of motion for each particle,

(4)

(c) find the acceleration of each particle.

(5)

Particle B does not rebound when it hits the ground and A continues moving up the plane towards P. Given that A comes to rest at the point Y, without reaching P,

(d) find the distance XY in terms of h.

(6)

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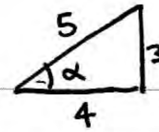
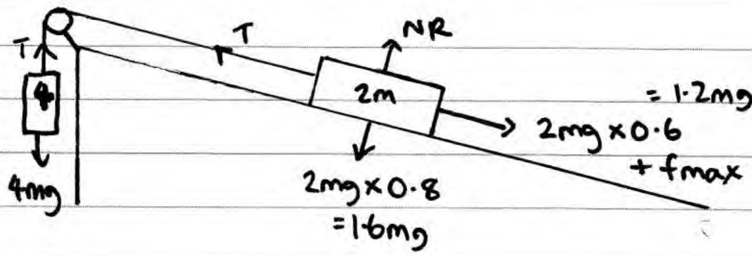
$\tan\alpha = \frac{3}{4}$



$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$



$\mu = \frac{1}{4}$

$NR = 1.6mg$

a) String is inextensible

$f_{max} = \mu NR \Rightarrow f_{max} = 0.4mg$

b) B $\quad 4mg - T = 4ma$

A $\quad T - 1.2mg - 0.4mg = 2ma \Rightarrow T - 1.6mg = 2ma$

$4mg - T = 4ma$

$\underline{T - 1.6mg = 2ma} \; +$

$2.4\cancel{m}g = 6\cancel{m}a$

$\therefore a = 0.4g \quad (3.92\,ms^{-2})$

c) $s = h$ $\quad$ A↖ before B hits the ground

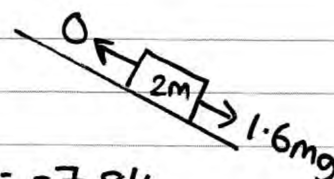
$u = 0$

$v \qquad v^2 = u^2 + 2as \Rightarrow v^2 = 7.84h.$

$a = 3.92$

t

when B hits the ground



R(↖) $\quad 0 - 1.6\cancel{m}g = 2\cancel{m}a \qquad a = -0.8g = -7.84$

$s =$

$u = \sqrt{7.84h}$

$v = 0$

$a = -7.84$

$t =$

$v^2 = u^2 + 2as$

$0 = 7.84h - 15.68s$

$\therefore s = \frac{1}{2}h. \; (+h) \therefore \underline{XY = \frac{3}{2}h}$