

Question Number	Scheme	Marks
1.(a)	$S_n = \frac{n}{2}(2a + (n-1)d)$ $\sum = \frac{n}{2}(2 + (n-1) \times 1) \quad \text{or} \quad S_n = \frac{n}{2}(a + l)$ $\sum = \frac{n}{2}(1+n) \quad * \quad \frac{n}{2}(1+n)$	M1 A1 cso (2)
(b)	$\sum_{1}^{100} r = 50(100+1) = 5050$ <p>Multiples of 7 $S_{14} = \frac{14}{2}(7+98) = 735$</p> <p>Reqd sum = $5050 - 735 = 4315$</p>	M1(either sum using result in (a)) A1(both sums correct) A1cso (3) [5]

(a) M1 This is a "show" question, so the formula used must be quoted first if using $S_n = \frac{n}{2}(a + l)$.

For $S_n = \frac{n}{2}(2a + (n-1)d)$, the unsimplified substitution is sufficient.

A1cso Obtain the GIVEN result

(b) M1 Attempt the sum of all integers up to 100 (100 terms) **or** the multiples of 7 (14 terms)

A1 **Both** sums correct

A1cso Subtract to obtain 4315

ALT List the terms needed and add: Question says "Hence", so the result must be used at least once, probably seen in the sum of all the integers up to 100. The multiples of 7 may be listed and added before subtracting from the sum up to 100.

2. (a)	Missing values: 7, 5.09, 3.5, 3.46	B1(any 2) B1(all corr.) (2)
(b)	Plot points Smooth curve through points plotted	B1ft B1 (2)
(c)	$x - 3 + \frac{3}{x^2} = 0$ $x + \frac{6}{x^2} = 6 - x$ <p>Draw $y = 6 - x$</p> <p>$x = 1.3$ or 1.4, 2.5 or 2.6</p>	M1A1 dM1 A1 (4) [8]

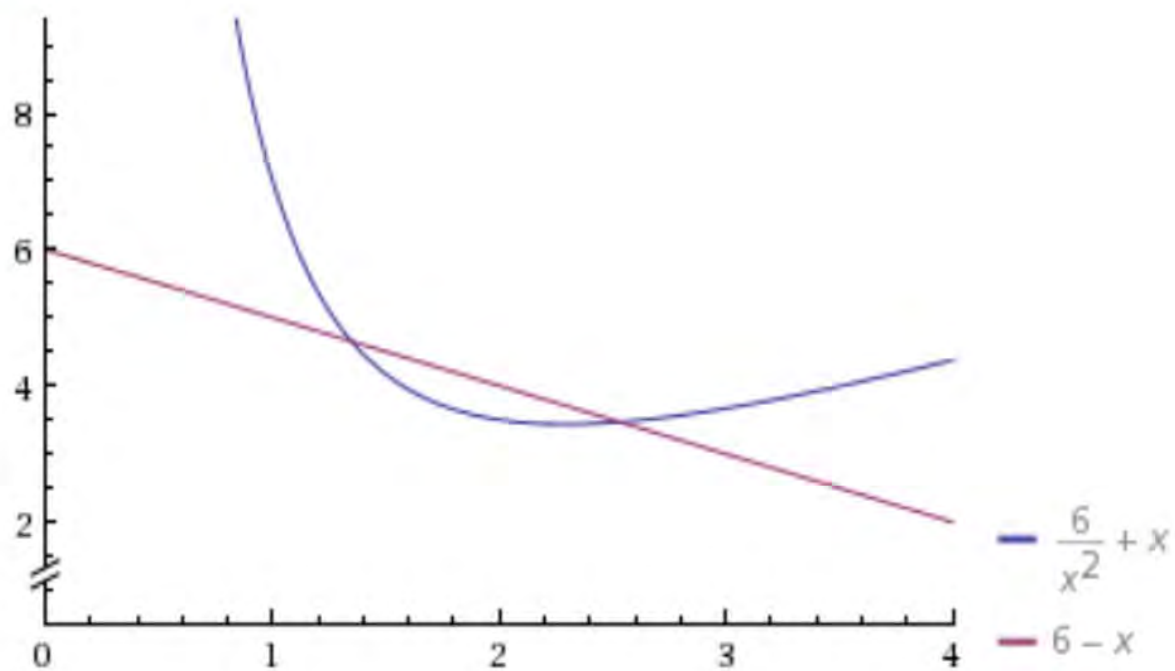
(a) B1 Any 2 values correct
B1 Remaining 2 values correct

(b) B1ft Plot *their* points correctly
B1 Draw a smooth curve through *their* plotted points.

(c) M1 Attempt to re-arrange the given equation to the form $x + \frac{6}{x^2} = ax + b$ (a, b to be numbers now or later)
A1 Correct RHS
dM1 Draw *their* line Must be of form $y = ax + b$ with $a \neq 0$
A1 $x = 1.3$ or 1.4 , 2.5 or 2.6 Must be 1 dp unless more figures are shown and a mark has been lost in (a) because of failing to round

4PM0/02: Q2 GRAPH PLOT

Plot:

**Points of intersection:**

$$x = 1.347 \quad y = 4.653 \quad (4\text{sf})$$

$$x = 2.532 \quad y = 3.468 \quad (4\text{sf})$$

3. (a)	$ar^2 - ar^3 = 2ar^4 \quad (\text{or } ar^3 - ar^2 = 2ar^4)$ $ar^2(2r^2 + r - 1) = 0$ $ar^2(2r - 1)(r + 1) = 0$ $r = \frac{1}{2} \quad *$	M1 dM1 A1 cso (3)
(b)	$400 = \frac{a}{1 - \frac{1}{2}}$ $a = 200$	M1 A1 (2)
(c)	$S_{10} = \frac{a(1 - r^{10})}{1 - r} = 400 \times \left(1 - \left(\frac{1}{2} \right)^{10} \right)$ $= 399.609375 \quad \text{or} \quad \frac{25575}{64} \quad \text{or} \quad 399\frac{39}{64}$	M1 A1cao (2) [7]

- (a) M1** Attempt to form an equation connecting the 3rd, 4th and 5th terms
depM1 Attempt to factorise their quadratic (usual rules). May divide by ar^2 first.
A1cso Obtain the GIVEN answer from correct working. Both solutions for the quadratic must be shown and $r = -1$ rejected
- (b) M1** Use the formula for the sum to infinity with $r = \frac{1}{2}$
A1 Obtain the correct value of a
- (c) M1** Use the formula for the sum of the first 10 terms, $r = \frac{1}{2}$ and their a
A1cao Obtain the correct sum (min 3 decimal places or exact answer)
- (a)** If done by verification: Scores M1M0A0 (max) as $r = \frac{1}{2}$ not shown to be the **only** solution.

4(a)	$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$ $= \mathbf{i} - 8\mathbf{j}$	M1 A1 (2)
(b)	$ \overrightarrow{PQ} = \sqrt{1^2 + 8^2} = \sqrt{65}$ Unit vector $= \pm \frac{1}{\sqrt{65}}(\mathbf{i} - 8\mathbf{j})$ oe	M1 A1ft(on \overrightarrow{PQ}) (2)
(c)	$ \overrightarrow{OP} = \sqrt{3^2 + 6^2} = \sqrt{45}$ $ \overrightarrow{OQ} = \sqrt{4^2 + 2^2} = \sqrt{20}$ $ \overrightarrow{OP} ^2 + \overrightarrow{OQ} ^2 = \overrightarrow{PQ} ^2$ \therefore right angle at O ALT: Use gradients	M1 either length A1 (both corr.) M1 A1 cso (4) [8]

Vector arrows may be missing and \mathbf{i} and \mathbf{j} need not be made bold/underlined. Column vectors allowed apart from (a)

(a) **M1** A correct statement as shown, or equivalent to that

A1 For $\mathbf{i} - 8\mathbf{j}$

(b) **M1** Attempting the modulus of *their* \overrightarrow{PQ} Pythagoras statement must have a + sign

A1ft Giving a unit vector, follow through their \overrightarrow{PQ} . Can be in direction P to Q or reverse.

(c) **M1** Attempting the length of either \overrightarrow{OP} or \overrightarrow{OQ} numbers to be added in the Pythagoras statement

A1 **Both** lengths correct, as given or equivalent exact answers

M1 Attempt to show their lengths fit Pythagoras

A1cso A correct solution and a concluding statement. (Accept eg "shown" or an appropriate symbol)

ALT using gradients:

M1 Attempting the gradient of either vector (numbers needed)

A1 Both gradients correct $\text{grad } OP = 2$; $\text{grad } OQ = -\frac{1}{2}$

M1 Forming the product of their gradients

A1cso Product to be -1 **and** a concluding statement given

Special case: If both gradients are found as $\frac{"x"}{"y"}$, award M1A1M1A0 (max)

If one gradient found as $\frac{"x"}{"y"}$ and the other as $\frac{"y"}{"x"}$ only M marks are available

By Scalar Product: M1A1Correct product M1 Multiply out A1 All correct and a conclusion

5(a)	$t^3 - 5t^2 + 6t = 0 \quad t^2 - 5t + 6 = 0$ $(t-3)(t-2) = 0$ $t = 2, t = 3$	M1 dM1 A1 (3)
(b)	$v = \frac{ds}{dt} = 3t^2 - 10t + 6$ $t = 1 \quad v = 3 \times 1^2 - 10 \times 1 + 6 = -1, \quad v \text{ or speed} = 1 \text{ m/s}$	M1A1 A1, A1ft (4)
(c)	$\frac{dv}{dt} = 6t - 10$ $t = 2 \quad a = 6 \times 2 - 10 = 2 \text{ m/s}^2$ $t = 3 \quad a = 6 \times 3 - 10 = 8 \text{ m/s}^2$	M1 M1 (A1 on ePEN) ft on values of t A1 (3) [10]

- (a) M1** Equate the given expression to zero and divide by (or take out) the common factor t
dM1 Factorise *their* resulting quadratic or use another method to solve (usual rules).
A1 Both (correct) values of t . Ignore $t = 0$ if included.
- (b) M1** Attempt to differentiate the given expression
A1 Correct differentiation
A1 Obtain $v = -1$ by substituting $t = 1$ in a correct expression for v
A1ft Change a negative answer to a *positive* one. Follow through their v
- (c) M1** Differentiate again to obtain an expression for the acceleration
M1 Substitute one of *their* answers from (a) in their expression for the acceleration
A1 For 2 and 8 (Ignore $a = \pm 10$ from $t = 0$)

6 (a)	$S = \frac{1}{2} \times 10^2 \theta - \frac{1}{2} \times 6^2 \theta$ $= 32\theta \quad *$	M1A1 A1cso (3)
(b)	$\frac{dS}{dt} = 32 \frac{d\theta}{dt}$ $\frac{dS}{dt} = 32 \times 0.2 = 6.4$	M1 A1 (2)
(c)	$20 = 32\theta$ $\theta = \frac{20}{32} \quad \text{oe inc } 0.625$ $\text{Perim} = 10 \times \frac{20}{32} + 6 \times \frac{20}{32} + 2 \times 4$ $= 18 \text{ cm}$ <p>ALT: $\text{Perim} = 10\theta + 6\theta + 8 = 16\theta + 8 = \frac{1}{2}S + 8 = 18$</p>	M1 A1 M1A1ft (their θ) A1 (5) [10]

Can be worked in degrees but final answers must be exact and correct.

(a) M1 Using Area = $\frac{1}{2} r^2 \theta$ to obtain an expression for S by subtracting 2 areas

A1 Correct numbers in the expression

A1cso Obtaining the GIVEN answer from correct working

(b) M1 Differentiate $S = 32\theta$ with respect to t (inc use of chain rule) **Evidence of differentiation needed.** $32 \times 0.2 = 6.4$ alone is **not** sufficient

A1 Substitute $\frac{d\theta}{dt} = 0.2$ and obtain $\frac{dS}{dt} = 6.4$

(c) M1 Form an equation for θ

A1 Solve the equation to obtain the value of θ (any equivalent fraction)

M1 Forming an expression for the perimeter of the shaded area. Must have 2 arcs and 2 straight lines all added. Award with θ or their value.

A1ft All numbers correct, follow through their value for θ

A1 Correct perimeter

ALT:

M1 Form an expression for the perimeter in terms of θ Must be evidence of all 4 parts.

A1 Collect terms (may be awarded by implication)

M1 Substitute $\theta = \frac{S}{32}$ to obtain an expression for the perimeter in terms of S

A1 Correct expression

A1 Substitute $S = 20$ to obtain the correct perimeter.

7(a)	$\frac{1}{3} \times h \times 48 + 48h = 256$ $h = 4 \quad *$	M1 A1 (2)
(b)	$FH = 10 \quad \text{or} \quad \frac{1}{2} FH = 5$ $VF^2 = 8^2 + \left(\frac{1}{2} \text{ their } FH \right)^2$ $VF = 9.43 \text{ cm}$	B1 M1 A1 (3)
(c)	$\tan A = \frac{4}{5}$ $A = 38.7^\circ$	M1A1 A1 (3)
(d)	$\tan \phi = \frac{4}{3} \quad \tan \theta = \frac{4}{6}$ $\phi = 53.13$ $\theta = 33.69$ $\text{Reqd angle} = 86.8^\circ$	M1 (either) A1 A1 A1 cao (4) [12]

- (a) M1** Forming an equation for the volume of the solid by adding the volumes of the two parts and equating to 256
A1 Solving to get the GIVEN answer
ALT Form an expression for the volume and substitute $h = 4$ (M1) obtain 256 (A1)
- (b) B1** Correct length of the diagonal or half-diagonal of the base (or top) of the cuboid
M1 Using Pythagoras (with height 8 and their $\frac{1}{2} FH$ and a + sign) to find VF^2
A1 Correct value for VF **must** be 3 sf
- (c) M1** Finding an expression for the tan of the **required** angle (or cos or sin)
A1 Correct numbers in their chosen trig function
A1 Obtain $A = 38.7^\circ$ **must** be to nearest 0.1°
- (d) M1** Obtaining a trig function for either of the two angles needed
A1 Correct value for either angle
A1 Correct value for the other angle
 These two marks can be awarded by implication if all work done on a calculator and final answer is correct
A1cao Correct result when their angles are added. **must** be to nearest 0.1° unless penalised in (c)
ALT M1 Use cosine rule either form in the correct triangle. Lengths must be found
 A1,A1 Deduct 1 per error ie A1A0 for 1 error seen
 A1 correct answer
 Correct cosine rule: $\cos \theta = \frac{5^2 + 52 - 73}{2 \times 5\sqrt{52}}$ oe

8(a)	$x=0 \Rightarrow y=0 \therefore 0=0+0+0+c$	B1 (1)
(b)	$\lfloor f(2) \rfloor 0 = 8 + 4a + 2b \quad \lfloor f(4) \rfloor 0 = 64 + 16a + 4b$ $a = -6, \quad b = 8$ <i>ALT:</i> Use factors and multiply out $a = -6, \quad b = 8$	M1 A1 A1 (3) M1 A1, A1
(c)	$x=3 \quad y=3^3 - 6 \times 3^2 + 8 \times 3 = -3$ $\text{Grad } l = -\frac{3}{3} = -1$ $\frac{dy}{dx} = 3x^2 - 12x + 8$ $x=3 \quad \frac{dy}{dx} = 3 \times 9 - 12 \times 3 + 8 = -1 \quad \therefore l \text{ is the tangent at } P$	B1 M1 M1 A1 cso (4)
(d)	Eqn of $l: \quad y = -x$ $\text{Area} = \int (f(x) - (-x)) dx = \int (x^3 - 6x^2 + 8x + x) dx$ $= \left[\frac{1}{4}x^4 - 2x^3 + \frac{9}{2}x^2 \right]_0^3$ $= \frac{1}{4} \times 3^4 - 2 \times 3^3 + \frac{9}{2} \times 3^2 - (-0)$ $= 6\frac{3}{4} \quad \left(\text{accept } 6.75 \text{ or } \frac{27}{4} \right)$	M1 M1A1ft dM1 A1cao (5)[13]

(a) B1 Show $c=0$ by substituting $x=0$ in the curve equation or state $x=0 \Rightarrow y=0$

Or $x(x-2)(x-4)=0$ (Must be evidence for $c=0$)

(b) M1 **Either** use the factor theorem with $x=2$ **and** $x=4$ and attempt to solve the resulting equations **Or** use factors $(x-2)(x-4)$ and multiply out

A1 Either a or b correct

A1 Other value correct If factor method used values need not be shown explicitly.

$y = x^3 - 6x^2 + 8x$ scores A1A1

(c) B1 Correct value for y coordinate of P obtained using the equation of C

M1 Attempt the gradient or equation of l or equation of tangent

M1 Differentiate *their* curve equation

A1cso Substitute $x=3$ in the derivative to obtain the correct gradient of the tangent at P **and** give a concluding statement - "shown" is sufficient.

ALT B1 y coordinate M1 Equation of l M1 Solve with C A1 $x(x-3)^2=0$ and conclusion

(d) M1 Attempting the area integral, $\pm(C-l)$, with *their* equation of l Correct limits are needed.

M1 Attempting the integration. Limits not needed. Integrand need not be simplified

A1ft Correct integration, ignore limits. No simplification needed. ft their equation of l

dM1 Correct use of limits for their method. Dependent on both M marks above.

A1 Correct answer, any form. Must be positive.

NB: If the area is split into parts, 3 integrals are needed (C 0 to 2, 2 to 3 and l) or 2 integrals and a triangle. Award M1 if an appropriate expression using these 3 parts is seen Correct limits are needed. M1 Integrate **both** functions in separate integrals or integrate C and find the area of the triangle. Rest as above

9 (a)	(6,6)	B1B1 (2)
(b)	$AB^2 = 8^2 + 6^2$ $AB = \sqrt{100} = 10$	M1 A1 (2)
(c)	$\text{Grad } AB = -\frac{6}{8}$ $\text{Grad } \perp = \frac{8}{6}$ oe Eqn \perp bisector: $y - 6 = \frac{4}{3}(x - 6)$ $3y = 4x - 6$ or any integer multiple of this	M1 A1 M1 A1 (4)
(d)	$y = 2$ $6 = 4d - 6$ $d = 3$	M1 A1 (2)
(e)	(12,14)	B1B1 (2)
(f)	$DE = \sqrt{9^2 + 12^2} = 15$ $\text{Area} = \frac{1}{2} \times AB \times DE = \frac{1}{2} \times 10 \times 15$ $= 75 \text{ (units}^2\text{)}$	M1 M1A1ft (their lengths) A1 (4) [16]
ALT	$\text{Area} = \frac{1}{2} \begin{vmatrix} 2 & 3 & 10 & 12 & 2 \\ 9 & 2 & 3 & 14 & 9 \end{vmatrix} = 75 \text{ units}^2$	M2A1ft(their coords) A1

(a) B1B1 B1B1 Both coords correct; B1B0 One correct, or recognisable method but neither correct

(b) M1 Attempt Pythagoras with + sign

A1 Correct length

(c) M1 Attempt the gradient of AB Must be $\frac{y}{x}$.

A1 Correct gradient of the perpendicular

M1 Attempt the equation of the perp bisector with their coordinates of M . Gradient must be $\frac{-1}{\text{their grad } AB}$.

A1 Correct equation, **must** be in the required form

(d) M1 Substitute $y = 2$ in *their* equation for l

A1 Obtain $d = 3$ or $x = 3$

(e) B1B1 B1B1 Both coords correct; B1B0 One correct.

(f) M1 Attempt the length of DE . Numbers to be added in Pythagoras. (Or DM and ME if using triangles ABE and ADE)

M1 Any complete method for the area of the kite - kite formula or by triangles

A1ft Correct lengths substituted follow through their lengths

A1cao Area = 75 (units²)

ALT: M2 for a correct "determinant" with $\frac{1}{2}$ and 4 different pairs of coordinates, first repeated as fifth;

A1ft *their* coords used correctly (ie clockwise or anticlockwise round the kite from any vertex) A1 correct area

10(a)	$\log_3 9 = 2$	B1 (1)
(b)	$\log_9 4 = \frac{\log_3 4}{\log_3 9}$ $= \frac{1}{2} \log_3 4 \quad k = \frac{1}{2}$	M1 A1 (2)
(c)	$2x \log_3 x - 3 \log_3 x - 4x \log_9 4 + 6 \log_9 4 = (2x - 3) \log_3 x - 2(2x - 3) \log_9 4$ $= (2x - 3)(\log_3 x - 2 \log_9 4)$ $= (2x - 3)(\log_3 x - \log_3 4)$ $= (2x - 3) \log_3 \left(\frac{x}{4} \right)$ $= \log_3 \left(\frac{x}{4} \right)^{(2x-3)} *$	M1 A1 M1 A1 cso (5)
(d)	$\log_3 \left(\frac{x}{4} \right)^{(2x-3)} = 0$ $2x - 3 = 0 \quad \text{or} \quad \frac{x}{4} = 1$ $x = \frac{3}{2}, \quad x = 4$	M1 (either) A1A1 (3) [11]

(a) B1 Correct answer

(b) M1 Use change of base formula. (Can change both sides to any consistent base)

A1 Obtain $k = \frac{1}{2}$

(c) M1 Attempt to move to two log terms $\pm 2(2x - 3)$ or $\pm(4x - 6)$ with $\log_9 4$

A1 Complete to two correct brackets

M1 Change base of $\log_9 4$ to 3 Use their value of k from (b) for this

M1 Combine the two logs to form a single log

A1 Obtain the GIVEN answer - brackets not needed in the index

May work in reverse:

M1 Bring down power; **A1** obtain 2 logs from the single log, must be a difference;

M1 change $\log_3 4$ to integer $\times \log_3 4$; **M1** Multiply out brackets; **A1** GIVEN answer

There are variations on this - look for the 3 M marks and award the first A mark after first M mark and second A mark at end if all correct.

(d) M1 Obtain either of the two linear equations shown

A1 One correct answer

A1 Second correct answer

