Question Number	Scheme	Marks
6.	(a) (i) $\alpha + \beta = -p$	B1
	(ii) $ or \begin{cases} \alpha^2 + p\alpha + 1 = 0 \\ \beta^2 + p\beta + 1 = 0 \end{cases} $	
	$\alpha^{2} + \beta^{2} = (\alpha + \beta)^{2} - 2\alpha\beta$ $\alpha^{2} + \beta^{2} + p(\alpha + \beta) + 2 = 0$	M1
	$= p^2 - 2 \qquad \qquad \alpha^2 + \beta^2 = p^2 - 2$	A1
	(iii) $(\alpha + \beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$	M1
	$\Rightarrow \alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = (-p)^3 - 3(-p)$	M1 A1
	$=3p-p^3$	
	alternatives $\begin{cases} \alpha^3 + p\alpha^2 + \alpha = 0 \\ \beta^3 + p\beta^2 + \beta = 0 \end{cases}$	
	$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2) \qquad \alpha^3 + \beta^3 + p(\alpha^2 + \beta^2) + (\alpha + \beta) = 0 \text{ M1}$	
	$=-p(p^2-2-1) \alpha^3+\beta^3+p(p^2-2)-p=0 M1$	
	$=3p-p^3 \qquad \qquad \alpha^3+\beta^3=3p-p^3 \qquad \qquad \text{A1}$	
	(b) $x^2 - (3p - p^3)x + 1 = 0$	M1ft A1ft (8)

Notes

Question 6

(a) (i) B1 for
$$\alpha + \beta = -p$$
 or $\left(-\frac{p}{1}\right)$

(Note $\alpha\beta = 1$)

(ii) M1 for
$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$
 and substituting in values for $\alpha + \beta$, and $\alpha\beta$

Or for
$$\begin{cases} \alpha^2 + p\alpha + 1 = 0 \\ \beta^2 + p\beta + 1 = 0 \end{cases}$$
$$\Rightarrow \alpha^2 + \beta^2 + p(\alpha + \beta) + 2 = 0$$

A1 for $\alpha^2 + \beta^2 = p^2 - 2$ oe (Simplification is not required for this mark)

(iii) M1 for expanding $(\alpha + \beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$ (allow some slips in algebra for this mark). Do **NOT** accept $(\alpha + \beta)^3 = \alpha^3 + \beta^3$ for this mark

M1 leading to $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ fully correct

A1 for $\alpha^3 + \beta^3 = 3p - p^3$ oe (Simplification is not required for this mark)

Please refer to ms for alternative methods

(b) M1 for using
$$x^2$$
 – their sum $\times x$ + product (= 0 not needed for this mark)

`A1ft for
$$x^2 - (3p - p^3)x + 1 = 0$$
 (follow through their values for this mark)

Note: = 0 must be seen with a correct equation for this mark Simplification is not required for this mark