# Mark Scheme (Results)

January 2017

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM0) Paper 2

January 2017
Publications Code 4PM0_02_1701_MS

## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- Types of mark
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)
- Abbreviations
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - eeoo – each error or omission
- No working
  If no working is shown then correct answers may score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.
- With working
  Always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.
  Any case of suspected misread loses 2A (or B) marks on that part, but can gain the M marks.
  If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.

- Ignoring subsequent work
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.

- Parts of questions
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded in another.

## General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

**Method mark for solving a 3 term quadratic equation:**

1. Factorisation:

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$ leading to $x=....$

$(ax^2+bx+c)=(mx+p)(nx+q)$ where $|pq|=|c|$ and $|mn|=|a|$ leading to $x=....$

2. Formula:
Attempt to use the **correct** formula (shown explicitly or implied by working) with values for $a$, $b$ and $c$, leading to $x=....$

3. Completing the square:

$x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$ leading to $x=....$

**Method marks for differentiation and integration:**

1. Differentiation
Power of at least one term decreased by 1. $\left(x^n\to x^{n-1}\right)$

2. Integration:
Power of at least one term increased by 1. $\left(x^n\to x^{n+1}\right)$

**Use of a formula:**

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

**or**, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

**Answers without working:**

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required.
(Mark schemes may override this eg in a case of "prove or show....")

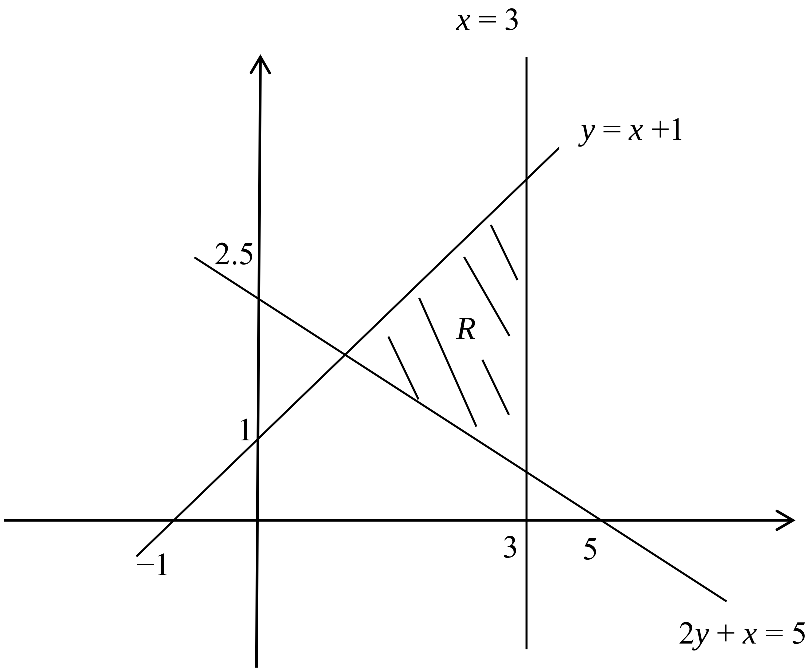
**Exact answers:**

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

**Rounding answers (where accuracy is specified in the question)**

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers.
Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

# Jan 2017
# 4PMO Further Pure Mathematics Paper 2
# Mark Scheme

| Question Number | Scheme | Marks |
|---|---|---|
| **1(a)** |  | B1<br>B1<br>B1 (3) |
| **(b)** | Region $R$ shaded in or out | B1 (1)<br>[4] |

**(a)**

**B1** **B1** **B1** One mark per line. Coordinates of the points where the lines cross the axes needed in each case but equations need not be shown.
"Dashes" on the axes do **not** count.
Award B1B1B1, B1B1B0, B1B0B0 or B0B0B0

**(b)**

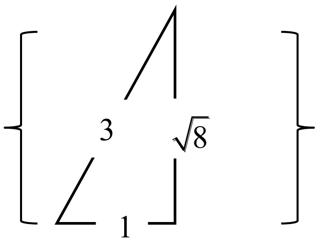
**B1** Correct region shaded, in or out, need not be labelled *R*. (NB Not ft, but lines w/o coordinates on the axes accepted if the lines look to be correctly placed.)

| Question number | Scheme | Marks |
|---|---|---|
| **2. (a)** | $6\left(1-\sin^2\alpha\right)-\sin\alpha=5$ or $6-6\sin^2\alpha-\sin\alpha=5$ | M1 |
| | $6\sin^2\alpha+\sin\alpha-1=0$ * | A1 (2) |
| **(b)** | $\Rightarrow(2\sin\alpha+1)(3\sin\alpha-1)=0\Rightarrow\sin\alpha=\frac{1}{3},-\frac{1}{2}\Rightarrow$ | M1 A1 |
| | $(\alpha=2\theta+40)$ | |
| | $2\theta+40=19.47...,160.5287...,210$ or other correct value | M1 |
| | $\Rightarrow\theta=60.3,\ 85$ | A1A1 (5) [7] |

**(a)**
**M1** Eliminate $\cos^2\alpha$ by using the Pythagorean identity. Working must be shown.
**A1cso** Correct **given** answer reached.
**(b)**
**M1** Factorise the equation given in (a), before or after using a substitution eg $\alpha=2\theta+40$
**A1** Two correct values for $\sin(2\theta+40)°$ or $\sin\alpha$ or $A$ (if substitution used)
**M1** Any one correct value for $(2\theta+40)°$ (Need not lead to $\theta$ in range $0\rightarrow 90$ ) Must be exact or at least 1 dp.
**A1** Either correct value for $\theta$ 60.3 must be 1 dp
**A1** Second correct value
**NB** Ignore additional answers outside the required range. Deduct one A mark ( last 2 A marks only deducted) for each additional answer within the range.

| Question number | Scheme | Marks |
|---|---|---|
| **3** | $\frac{\mathrm{d}r}{\mathrm{d}t}=0.5$ | B1 |
| | $A=\pi r^2\Rightarrow\frac{\mathrm{d}A}{\mathrm{d}r}=2\pi r$ | M1A1 |
| | $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}t}=2\pi r\times 0.5,\ \Rightarrow 2\pi\times 200\times 0.5$ | M1, |
| | $=200\pi=628.3185...\approx 628$ (cm²/s) | A1cao (5) **[5]** |

**B1** Correct statement, seen explicitly or used in chain rule.
**M1** Attempt the differentiation
**A1** Correct derivative
**M1** **USE** the chain rule (ie sub their derivatives, can have $r$)
**A1cao** Must be 3 sf

| Question number | Scheme | Marks |
|---|---|---|
| **4(a)(i)** | $\tan(2x)=\dfrac{2\tan x}{1-\tan^2 x}$ | B1 |
| **(ii)** | $\tan(3x)=\tan(2x+x)=\dfrac{\tan 2x+\tan x}{1-\tan 2x\tan x}$ | M1 |
| | $\tan(3x)=\dfrac{\dfrac{2\tan x}{1-\tan^2 x}+\tan x}{1-\dfrac{2\tan x}{1-\tan^2 x}\times\tan x}=\dfrac{\dfrac{2\tan x}{1-\tan^2 x}+\tan x}{1-\dfrac{2\tan^2 x}{1-\tan^2 x}}$ | M1 |
| | $\tan(3x)=\dfrac{\dfrac{2\tan x+\tan x-\tan^3 x}{1-\tan^2 x}}{\dfrac{1-\tan^2 x-2\tan^2 x}{1-\tan^2 x}}$ or $\dfrac{2\tan x+\tan x-\tan^3 x}{1-\tan^2 x-2\tan^2 x}$ | M1A1 |
| | $\Rightarrow\dfrac{3\tan x-\tan^3 x}{1-3\tan^2 x}$ * | A1 cso (6) |
| **(b)** | $\cos\alpha=\dfrac{1}{3}\Rightarrow$  For perpendicular = $\sqrt{8}$ | M1 |
| | $\Rightarrow\tan\alpha=\sqrt{8}=\left(2\sqrt{2}\right)$ | A1 (2) |
| | **ALT** | |
| | $\sin^2\alpha+\dfrac{1}{9}=1\Rightarrow\sin^2\alpha=\dfrac{8}{9}\Rightarrow\sin\alpha=\dfrac{2\sqrt{2}}{3}\Rightarrow\tan\alpha=\dfrac{\sin\alpha}{\cos\alpha}=2\sqrt{2}$ | |
| **(c)** | $\tan(3x)=\dfrac{2\sqrt{2}\left(3-(2\sqrt{2})^2\right)}{1-3(2\sqrt{2})^2}=\dfrac{2\sqrt{2}(3-8)}{1-24}=\dfrac{10\sqrt{2}}{23}$ | M1A1 (2)[10] |

**(a) (i)B1** Correct expression only but allow $\tan x\tan x$ instead of $\tan^2 x$ and $\tan x\tan x$ instead of $\tan^2 x$

**M1** Use the given formula to change to tan $x$ and tan2$x$

**M1** Use their result from (i) to eliminate tan2$x$. Either numerator or denom. must be correct.

**M1** Write their numerator and denominator as single fractions or multiply numerator and denominator by "$1-\tan^2 x$"

**A1** Correct unsimplified fraction

**(ii)A1cso** Correct result obtained with no errors seen. Work shown must justify each M mark

**(b)**

**M1** Showing a method for finding the perpendicular or find a value for $\sin\alpha$

**A1** Correct **exact** value for tan $\alpha$ .

Answer only given: Correct 2/2 incorrect 0/2

**NB** If no working shown (calculator?) award M1A1 for correct **exact** answer, M0A0 otherwise

**(c)**

**M1** Substitute their value for $\tan\alpha$ (not nec exact) in the identity in (a) (ii)

**A1cao** Correct answer obtained. Must be in the given form. (Calculator solution scores M0A0)

| Question number | Scheme | Marks |
|---|---|---|
| **5(a)** | $\frac{dy}{dx}=(2x-1)^{\frac{1}{2}}\times 3+3x\times(2x-1)^{-\frac{1}{2}}\times\frac{1}{2}\times 2$ | M1A1A1 |
| | $\Rightarrow\frac{3\times(2x-1)+3x}{(2x-1)^{\frac{1}{2}}},\Rightarrow\frac{6x-3+3x}{(2x-1)^{\frac{1}{2}}}\Rightarrow\frac{9x-3}{(2x-1)^{\frac{1}{2}}}=\frac{3(3x-1)}{\sqrt{(2x-1)}}$ * | dM1,A1cso (5) |
| **(b)** | $\frac{dy}{dx}=\frac{3(3\times1-1)}{\sqrt{2\times1-1}}=6$ | B1 |
| | Gradient of normal = $-\frac{1}{6}$ | B1ft |
| | $y=3\times1\times\sqrt{2\times1-1}=3$ | B1 |
| | $y-3=-\frac{1}{6}(x-1),\ \Rightarrow 6y+x-19=0$ oe | M1A1,A1 (6) |
| | | **[11]** |

| | |
|---|---|
| **(a) M1** | Attempt the differentiation using the product rule. Must have two terms added, |
| **A1** | NB M1 on e-PEN Either term correct |
| **A1** | Other term correct Power or square root form for both. |
| | **NB:** $\frac{1}{2}\times 2$ may be missing as = 1 |
| **dM1** | Write their two terms over a common denominator, depends on the first M mark. |
| **A1cso** | Simplify to the GIVEN answer with no errors seen. |
| **(b)** | |
| **B1** | Correct value for d$y$/d$x$ at $x$ = 1 |
| **B1ft** | Correct gradient of normal, follow through their d$y$/d$x$ |
| **B1** | Correct value of $y$ at $x$ = 1 |
| **M1** | Substitute their gradient of normal and coordinates in $y=mx+c$ Use of value of d$y$/d$x$ scores M0 |
| **A1** | Correct values substituted |
| **A1** | Correct equation with integer coefficients, terms in any order (can have 4 terms) |

| Question number | Scheme | Marks |
|---|---|---|
| **6 (a)** | Mark parts (i) and (ii) together | |
| | $987=\frac{21}{2}\left(2\times a+(21-1)d\right)\ \left(\Rightarrow 987=21a+210d\right)$ | M1 |
| | $35=a+(8-1)d\ \ \left(\Rightarrow 35=a+7d\right)$ | A1 (M1 on e-PEN |
| | Solve simultaneous equations, by elimination or substitution<br>$987=21a+210d\ \div 21\Rightarrow 47=a+10d$<br>$735=21a+147d\ \div 21\Rightarrow 35=a+7d\ \Rightarrow a=7,d=4$ | dM1A1A1 (5) |
| **(b)** | $\Rightarrow A=4$ | B1ft |
| | $7=4\times 1+B\Rightarrow B=3$ | M1A1 (3) |
| **(c)** | $S_n=\frac{n}{2}\left(2\times 7+(n-1)4\right)$ or $\frac{n}{2}\left(7+\left(4n+3\right)\right)$ | M1 |
| | $\frac{n}{2}\left(2\times 7+(n-1)4\right)>2000\Rightarrow\frac{n}{2}\left(10+4n\right)>2000\Rightarrow 2n^2+5n-2000>0$ | dM1A1 |
| | $\Rightarrow n=30.39747.....\Rightarrow n=31$ | ddM1A1 (5)<br>[13] |

**(a)M1** Attempt an equation using either piece of information

**A1** NB: M1 on e-PEN. Two correct unsimplified equations

**dM1** Solve the simultaneous equations to obtain a value for either *a* or *d*. Depends on the first M mark.

**A1** One correct answer

**A1** Both answers correct

**(b)**

**B1ft** *A* = their value of *d*

**M1** Use $S_n=\sum_{r=1}^{n}\left(Ar+B\right)$ with $n=1$ and their values of *A* and *a* to obtain a value of *B* or with $n=1$ and $n=2$, their *a* and *d* and solve the simultaneous equations for either *A* or *B* or any other complete method.

**A1** $B=3$ (no ft)

**(c)**

**M1** Use either form of the sum of an arithmetic series to obtain an expression for $S_n$. Formulae used must be correct.

**dM1** Set up an inequality or equation with their sum and 2000 and obtain a 3TQ. Depends on the first M mark

**A1** Correct 3TQ, terms in any order, can have > or = Can be a multiple of the one shown.

**ddM1** Solve by formula or calculator. If formula used, the formula must be correct (can be by implication due to numbers substituted). Negative answer need not be shown. If by calculator award mark by implication if answer is 30.4 or better. Depends on both previous M marks.

**A1cao** $n=31$

**(c)** Solution by trial and error:

**M1** As above

Further marks depend on sight of an inequality(dM1) Correct inequality (A1)

Substitution of at least two values of *n* (M1)

$n=31$ obtained from correct working.

| Question number | Scheme | Marks |
|---|---|---|
| **7(a)** | $\dfrac{27^{(x+2)}-3^{(3x+5)}}{3^x\times 9^{(x+2)}}=\dfrac{3^{3(x+2)}-3^{(3x+5)}}{3^x\times 3^{2(x+2)}}$ | M1A1 |
| | $=\dfrac{3^{(3x+6)}-3^{(3x+5)}}{3^x\times 3^{(2x+4)}}=\dfrac{3^{3x}\times 3^6-3^{3x}\times 3^5}{3^{3x}\times 3^4},=\dfrac{3^{3x}\left(3^6-3^5\right)}{3^{3x}\times 3^4},\left(=\dfrac{486}{81}\right)=6$ | dM1,ddM1,A1 (5) |
| | **ALTs for last 3 marks** | |
| **ALT 1** | $=\dfrac{3^{(3x+6)}-3^{(3x+5)}}{3^x\times 3^{(2x+4)}}=\dfrac{3^{3x}\times 3^6-3^{3x}\times 3^5}{3^{3x}\times 3^4},=\dfrac{3^{3x}\left(3^6-3^5\right)}{3^{3x}\times 3^4}=\dfrac{3^5(3-1)}{3^4},=6$ | |
| **(b)** | $\log_y 2=\dfrac{\log_2 2}{\log_2 y}=\dfrac{1}{\log_2 y}$ or $\log_2 y=\dfrac{\log_y y}{\log_y 2}=\dfrac{1}{\log_y 2}$ | M1 |
| | Forming 3TQ: | |
| | $2\log_2 y+\dfrac{3}{\log_2 y}=7\Rightarrow 2\left(\log_2 y\right)^2+3=7\log_2 y$ | dM1 |
| | $2\left(\log_2 y\right)^2-7\log_2 y+3=0$ OR $2-7\log_y 2+3\left(\log_y 2\right)^2=0$ | |
| | (Let $A=\log_2 y$ ) | |
| | $2A^2-7A+3=0\Rightarrow(2A-1)(A-3)=0\Rightarrow A=\dfrac{1}{2},3$ | ddM1A1 |
| | $\log_2 y=\dfrac{1}{2}\Rightarrow y=2^{\frac{1}{2}}(=\sqrt{2})$ $\log_2 y=3\Rightarrow y=2^3=8$ | A1A1 (6) |
| | | **[11]** |

| | |
|---|---|
| **(a)** | |
| **M1** | Attempt to change power of 9 or 27 to a power of 3 |
| **A1** | Correct unsimplified expression with powers of 3 alone |
| **dM1** | Expand brackets in the powers and write with all powers as single terms, depends on first M mark |
| **ddM1** | Remove common factor in numerator, depends on both previous M marks |
| **A1** | Correct value of *k* obtained (need not be written explicitly as $k = 6$) |
| **(b)** | |
| **M1** | Change base. Can change to base 2 or base *y* or both terms to any other (same) base |
| **dM1** | Obtain a 3TQ Depends on the first M mark. Term can be in any order but must be 3 separate terms. |
| **ddM1** | Solve their 3TQ. Substitution shown not needed. Depends on both previous M marks. |
| **A1** | Correct values for $\log_2 y$ or $A$ OR $\log_y 2$ |
| **A1** | One correct value for *y* |
| **A1** | Second correct value for *y* |

| Question number | Scheme | Marks |
|---|---|---|
| **8(a)** | $\overrightarrow{AB}=\overrightarrow{AO}+\overrightarrow{OB}=-5\mathbf{p}+3\mathbf{q}+11\mathbf{a}=6\mathbf{p}+3\mathbf{q}\ \ (=3(2\mathbf{p}+\mathbf{q}))$ | M1 |
| **(i)** | $\overrightarrow{BC}=\overrightarrow{BO}+\overrightarrow{OC}=-11\mathbf{p}+13\mathbf{p}+\mathbf{q}=2\mathbf{p}+\mathbf{q}$ | M1 |
| | (with conclusion; common direction ,common point etc) | A1 |
| **(ii)** | $\overrightarrow{AB}=3\overrightarrow{BC}\Rightarrow AB:BC=3:1$ oe | B1 (4) |
| **(b)** | $\left(\overrightarrow{MN}=\frac{11}{2}\mathbf{p}-\frac{5}{2}\mathbf{p}+\frac{3}{2}\mathbf{q}=\frac{3}{2}(2\mathbf{p}+\mathbf{q})\right)$ so $\overrightarrow{MN}\parallel\overrightarrow{AB}$ $\left(\text{or } \overrightarrow{MN}=\frac{1}{2}\overrightarrow{AB}\right)$ | B1 |
| | $\Delta OAB:\Delta OAC=3:4$ (ratio of bases) | M1A1 |
| | $\Delta OMN:\Delta OAB=1:4$ (similar $\Delta$s) | |
| | $\Rightarrow$ quad $ABNM:\Delta OAB=3:4$ | M1A1 |
| | $\frac{ABNM}{OAB}=\frac{3}{4}$ and $\frac{OAB}{OAC}=\frac{3}{4}$ | |
| | $\therefore\ \frac{\text{quad } ABNM}{OAC}=\frac{3}{4}\times\frac{3}{4}=\frac{9}{16}\Rightarrow 9:16$ * | dM1A1cso (7) |
| | | [11] |
| **ALT** | By "determinant" method: | |
| | $\overrightarrow{OM}=\frac{5}{2}\mathbf{p}-\frac{3}{2}\mathbf{q}$ and $\overrightarrow{ON}=\frac{11}{2}\mathbf{p}$ | B1 |
| | Area $ABMN=\frac{1}{2}\begin{vmatrix}5 & 11 & 11/2 & 5/2 & 5\\ -3 & 0 & 0 & -3/2 & -3\end{vmatrix}$ | M1 |
| | $=12\frac{3}{8}$ oe | A1 |
| | Area $OAC=\frac{1}{2}\begin{vmatrix}0 & 5 & 13 & 0\\ 0 & -3 & 1 & 0\end{vmatrix}$, $=22$ | M1,A1 |
| | Area $ABMN$ : Area $OAC=12\frac{3}{8}:22=9:16$ | dM1A1 |

**(a)**

**M1** Attempt to find a vector joining any 2 of the points $A$, $B$, $C$

**M1** Attempt a second vector joining any 2 of the points $A$, $B$, $C$
$\overrightarrow{AC}=\overrightarrow{AO}+\overrightarrow{OC}=8\mathbf{p}+4\mathbf{q}$

**(i)A1** Both vectors to be correct and a reason for collinearity given eg $\overrightarrow{AB}=\lambda\overrightarrow{AC}$

**(ii)B1** Correct ratio seen ie 3:1 oe

**(b)**

**B1** Stating that $\overrightarrow{MN}\parallel\overrightarrow{AB}$ B mark so no proof needed either by vectors or use of midpoint theorem. This mark can be given by implication if the similar triangles are used.

**M1** Attempting the ratio $\Delta OAB:\Delta OAC$ using their ratio of bases

**A1** Correct ratio

**M1** Attempting ratio quad $ABNM:\Delta OAB$ using their ratio

**A1** Correct ratio quad $ABNM:\Delta OAB$

**dM1** Multiplying the two ratios to eliminate $\Delta OAB$ Depends on both M marks above

**A1cso** Obtain the GIVEN answer with no errors seen

| | |
|---|---|
| **ALT** | |
| **B1** | Correct vectors |
| **M1** | Use the "determinant" formula with their coefficients. Can start at any point and proceed in order round the quadrilateral (either direction). First and last coefficients must be the same and 1/2 must be included. |
| **A1** | Correct area |
| **M1A1** | Similar for triangle |
| **dM1** | Giving a ratio of the areas with their calculated areas. Depends on both M marks above. |
| **A1cso** | Correct GIVEN result. No errors seen |

There are other methods which may be seen, usually based on $\frac{1}{2}ab\sin C$ formula and using angle *OAC* (and angle *OMN*).

Send to review.

| Question number | Scheme | Marks |
|---|---|---|
| **9 (a)** | $\dfrac{y-(-3)}{5-(-3)}=\dfrac{x-2}{-2-2}\Rightarrow\left(y=-2x+1\right)$ oe | M1A1 (2) |
| **(b) (i)** | $\left(\dfrac{3\times2+1\times-2}{3+1},\dfrac{5\times1+3\times-3}{3+1}\right)=\left(1,-1\right)$ | M1A1 |
| | Gradient of perpendicular $=\dfrac{1}{2}$ | B1 |
| **(ii)** | $y--1=\dfrac{1}{2}\left(x-1\right)\Rightarrow\left(y=\dfrac{x}{2}-\dfrac{3}{2}\right)$ | M1A1 (5) |
| **(c)** | (i) $s=0$ (3, 0)<br>(ii) $t=-1$ $(-1,-2)$ | B1ft<br>B1ft (2) |
| **(d)** | Length $PQ=\sqrt{\left(5--3\right)^2+\left(-2-2\right)^2}=4\sqrt{5}$ | M1 |
| | Length $SN=\sqrt{\left(3-1\right)^2+\left(0-1\right)^2}=\sqrt{5}$ | |
| | Length $TN=\sqrt{\left(1--2\right)^2+\left(-1--2\right)^2}=\sqrt{5}$ | A1 |
| | Area $=\dfrac{1}{2}\left(4\sqrt{5}\times\sqrt{5}+4\sqrt{5}\times\sqrt{5}\right)=20$ | dM1A1 (4) |
| | | [13] |
| **ALT 1** | *PSQT* is a quad with perpendicular diagonals:<br>Length $ST=\sqrt{\left(3--1\right)^2+\left(0--2\right)^2}=2\sqrt{5}$<br>Length $PQ=$<br>$\sqrt{\left(5--3\right)^2+\left(-2-2\right)^2}=4\sqrt{5}$ M1(either)A1(both or $SN$ or $TN$)<br>Area $=\dfrac{1}{2}\times4\sqrt{5}\times2\sqrt{5}=20$ (units$^2$) dM1A1 | |
| **ALT 2** | By "determinant" method<br>Eg Area $=\dfrac{1}{2}\begin{vmatrix}-2 & -1 & 2 & 3 & -2\\ 5 & -2 & -3 & 0 & 5\end{vmatrix}$ M1A1<br>$=\dfrac{1}{2}\left(-2\times-2+-1\times-3...-\left(-2\times0-3\times-3...\right)\right)=20$ dM1A1 | |

**(a)**
**M1** Attempt an equation for *PQ* using any *complete* method
**A1** Correct equation in any form, no simplification needed
**(b)**
**M1** Attempt one of the coordinates of *N* either by using the formula for the coords of a point dividing a line in a given ratio or by diagram. If a diagram is used the method is complete if one of the coords is deduced (and correct).
**A1** Both coords correct.
**NB** If coords are written down without any working shown award M1A1 if both correct or M1A0 if one is correct.
**B1** Correct gradient of perp. May only be seen in the equation of *l*.
**M1** Use their gradient of the perpendicular and their coordinates of *N* to obtain an equation for *l*. If the gradient used is the same as their gradient of *PQ* award M0. Must be a *complete* method.
**A1** Correct equation any form, need not be simplified
**(c)**
**(i)B1ft** $s = 0$ No working needed ft their equation of *l*
**(ii)B1ft** $t = -1$ No working needed ft their equation of *l*
**NB** Award these marks if the coordinates of *S* and/or *T* are given rather than $s = 0$, $t = -1$
**(d)**
**M1** Attempting one of the necessary lengths
**A1** All three lengths correct
**dM1** Attempting the areas of triangles *PSQ* and *PTQ* **and** adding their results
**A1** Correct total area.
**ALT1**
**M1** Attempt length of *PQ* or *ST*
**A1** Correct lengths of *PQ* and one of *SN*, *TN* or *ST*
**dM1** Using their lengths and the formula for the area the quad.
**A1** Correct area obtained
**ALT2**
**M1** Use the "determinant" formula with their coordinates for *S* and *T*. Can start at any point and proceed in order round the diagram (either direction). First and last coordinates must be the same and 1/2 must be included.
**A1** All coordinates correct
**M1** Attempt to evaluate
**A1** Correct area obtained. Must be positive.

| Question number | Scheme | Marks |
|---|---|---|
| **10(a)** | $MD = 6\cos 30 = 3\sqrt{3}$ * | M1A1cso (2) |
| **(b)** | Height of triangle $= 6\sin 30 = 3$ cm $\Rightarrow$ ht $= 8 + 3 = 11$ cm * | M1A1cso (2) |
| **(c)** | $MG = \sqrt{10^2 + \left(3\sqrt{3}\right)^2} \quad \left(= \sqrt{127}\right)$ | M1 |
| | $BG = \sqrt{11^2 + 127} = 2\sqrt{62} = 15.7$ | dM1A1 (3) |
| **(d)** | $AC = 2 \times MD = 6\sqrt{3}$ or $CE = \sqrt{172}$ | B1 |
| | Angle $ECA = \tan^{-1}\left(\dfrac{8}{6\sqrt{3}}\right) = 37.6, \Rightarrow$ Angle required $= 37.6 + 30 = 67.6°$ | M1,A1cao (3) |
| **(e)** | $BE = \sqrt{11^2 + \left(3\sqrt{3}\right)^2} = 2\sqrt{37}$ | M1A1 |
| | (Angle $EAB = 90 + 30 = 120°$) | |
| | $\dfrac{\sin ABE}{8} = \dfrac{\sin 120}{2\sqrt{37}}, \Rightarrow$ angle $ABE = 34.7°$ | M1A1,A1 (5) |
| | | **[15]** |

**(a)**

**M1** Attempt the length of *MD* using sin or cos of 60 or 30 or sin/cosine rule in $\Delta ABC$ to find *AC* and divide by 2

**A1cso** Correct, GIVEN, result from a correct statement

**(b) M1** Attempt the height of the triangle using sin or cos of 60 or 30 or any other complete method **and** add to 8. (Addition must be seen)

**A1cso** 11 (cm) with no errors seen

**(c) M1** Attempt the length of *MG* using Pythagoras with a + sign

**dM1** Attempt the length of *BG* using Pythagoras with a + sign. Depends on the first M mark.

**A1** Correct answer. Must be 3 sf

**(d) B1** Correct (numerical) length of *AC* or *CE* decimals allowed

**M1** Find angle *ECA* using sine, cosine or tangent. Cosine rule may be used in triangle *BCE* to find angle *BCE* but method must be **complete**, so length *BE* must be found.

**A1cao** Obtain the correct answer by adding 30 to 37.6 or from the cosine rule. Must be 1 dp

**(e)M1** Attempt the length of *BE* using Pythagoras with a + sign.

**A1** Correct length. NB: These 2 marks can be awarded for work seen in (d) **provided** used here.

**M1** Any complete method for obtaining angle *ABE* (oe)

**A1** Correct numbers in their choice of method

**A1** Correct answer, must be 1 dp unless already penalised in (d)

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