

Question number	Scheme	Marks
9 (a)	$x^2 - \text{sum} \times x + \text{product} = 0$ $x^2 + \frac{5}{2}x - 5 = 0$ $2x^2 + 5x - 10 = 0$ or integer multiples	M1A1 (2)
(b) (i)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \left(\frac{25}{4}\right) + 10 = \frac{65}{4}$	M1A1
(ii)	$(\alpha + \beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$ $\Rightarrow \alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = -\frac{125}{8} + 15\left(-\frac{5}{2}\right) = -\frac{425}{8}$ ALT $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2) = \left(-\frac{5}{2}\right)\left(\frac{73}{4} + 5\right) = -\frac{425}{8}$	M1A1A1 (5) {M1A1A1}
(c)	Product $\left(\alpha - \frac{1}{\alpha^2}\right) \times \left(\beta - \frac{1}{\beta^2}\right) = \left(\frac{\alpha^3 - 1}{\alpha^2}\right) \left(\frac{\beta^3 - 1}{\beta^2}\right) = \frac{\alpha^3\beta^3 - (\alpha^3 + \beta^3) + 1}{\alpha^2\beta^2}$ $= \frac{-125 - \frac{425}{8} + 1}{36} = -\frac{567}{200}$ Sum $\left(\alpha - \frac{1}{\alpha^2}\right) + \left(\beta - \frac{1}{\beta^2}\right) = \left(\frac{\alpha^3 - 1}{\alpha^2}\right) + \left(\frac{\beta^3 - 1}{\beta^2}\right)$ $= \frac{\alpha^3\beta^2 - \beta^2 + \alpha^2\beta^3 - \alpha^2}{\alpha^2\beta^2} = \frac{\alpha^2\beta^2(\alpha + \beta) - (\alpha^2 + \beta^2)}{\alpha^2\beta^2}$ $= \frac{25\left(-\frac{5}{2}\right) - \frac{65}{4}}{25} = -\frac{63}{20}$ oe Equation	M1 A1 M1 A1

	$\text{Sum} = -\frac{63}{20}, \text{ Product} = -\frac{567}{200}$ $\Rightarrow x^2 + \frac{63}{20}x - \frac{567}{200} (= 0)$ $x^2 + \frac{314}{100}x - \frac{567}{200} (= 0) \quad \text{M1}$ $200x^2 + 630x - 567 = 0 \quad \text{A1}$	M1A1 (6) [13]
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Additional Notes		
Part	M	Guidance
(a)	M1	Forms a quadratic equation with the given product and sum $\left(x^2 + \frac{5}{2}x - 5\right) = 0$ not required for this mark. Allow $y = \dots$ for this mark
	A1	For $2x^2 + 5x - 10 = 0$ or equivalent equation with integer coefficients only. Look out for $= 0$ which must be present.
(b) (i)	M1	Uses the correct algebra to form $\alpha^2 + \beta^2$ and substitutes the given values of the sum and product.
	A1	For $\alpha^2 + \beta^2 = \frac{65}{4}$
(ii)	M1	Uses the correct algebra to form an expression for $\alpha^3 + \beta^3$ For example; $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ or $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$
		Their algebraic expansion must be sufficiently arranged to allow substitution of $\alpha^2 + \beta^2$, $\alpha + \beta$ and $\alpha\beta$
	A1	Substitutes the given values for the sum and product into their form of $\alpha^3 + \beta^3$
	A1	For $\alpha^3 + \beta^3 = -\frac{425}{8}$ oe
(c)	M1	Product For the correct algebra to achieve $\frac{\alpha^3\beta^3 - (\alpha^3 + \beta^3) + 1}{\alpha^2\beta^2}$ or $\alpha\beta - \left(\frac{\alpha^3 + \beta^3}{\alpha^2\beta^2}\right) + \frac{1}{\alpha^2\beta^2}$ and substitutes their $(\text{product})^3$, $(\text{product})^2$ and their $\alpha^3 + \beta^3$ Their algebra must be sufficient to substitute $\alpha\beta$, $\alpha^3 + \beta^3$ and $\alpha^2\beta^2$ in directly.
	A1	Product $= -\frac{567}{200}$ oe
	M1	Sum For the correct algebra to achieve $\frac{\alpha^2\beta^2(\alpha + \beta) - (\alpha^2 + \beta^2)}{\alpha^2\beta^2}$ or $\alpha + \beta - \left(\frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}\right)$ (but in a form where the sum and product can be substituted) and substitutes their $(\text{product})^2$ and their $\alpha^2 + \beta^2$