

Question Number	Scheme	Marks
10(a)	$V = \frac{1}{3} \pi h r^2 = \frac{1}{3} \pi h \times (h \tan 30)^2 \quad \left(= \frac{1}{3} \pi h^3 \times \left(\frac{1}{\sqrt{3}} \right)^2 = \frac{1}{9} \pi h^3 \right)$	B1
	$V = 0.4t$	B1
	$0.4t = \frac{2}{5} t = \frac{1}{9} \pi h^3$	M1
	$h^3 = \frac{18t}{5\pi} \quad *$	A1cso (4)
(b)	Area of top = $\pi (h \tan 30)^2 = \frac{1}{3} \pi h^2$	B1
	$\frac{dA}{dh} = \frac{2}{3} \pi h$	M1
	$\frac{dA}{dt} = \frac{dA}{dh} \times \frac{dh}{dt}$	M1
	$h^3 = \frac{18t}{5\pi}$	
	$3h^2 = \frac{18}{5\pi} \frac{dt}{dh}$	M1
	$\frac{dh}{dt} = \frac{6}{5\pi h^2}$	A1
	$\frac{dA}{dt} = \frac{2}{3} \pi h \times \frac{6}{5\pi h^2} = \frac{4}{5h} \quad *$	A1cao (6)
(c)	$t = 10 \quad h = \sqrt[3]{\frac{180}{5\pi}} \quad \frac{dA}{dt} = \frac{4}{5h} = 0.355 \text{ cm}^2/\text{s}$	M1A1cao (2)
		[12]

10(a)B1	$V = \frac{1}{3}\pi h \times (h \tan 30)^2$ (or $V = \frac{1}{9}\pi h^3$) (ie replace r)
B1	$V = 0.4t$
M1	Equating their 2 expressions for V to obtain an equation without r
A1cso	Re-arrange to $h^3 = \frac{18t}{5\pi}$ with no errors seen
(b)	
B1	Area of top $= \frac{1}{3}\pi h^2$
M1	Differentiate their expression for the area of the top wrt h
M1	Chain rule connecting $\frac{dA}{dt}$, $\frac{dA}{dh}$ and $\frac{dh}{dt}$, any equivalent form or a useful chain rule with more derivatives eg $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dh} \times \frac{dh}{dt}$ see alt solution below
M1	Differentiate the given expression from (a) wrt h or t
A1	$\frac{dh}{dt} = \frac{6}{5\pi h^2}$ or $\frac{dt}{dh} = \frac{5\pi h^2}{6}$ or any equivalent expression in terms of t .
A1cao	Substitute for $\frac{dA}{dh}$ and $\frac{dh}{dt}$ in the chain rule to obtain the given expression for $\frac{dA}{dt}$ No errors seen.
ALTs:	
1	Using $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dh} \times \frac{dh}{dt}$ B1 $\frac{dA}{dr} = 2\pi r$ M1 Find $\frac{dr}{dh} = \frac{1}{\sqrt{3}}$ M1 Chain rule $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dh} \times \frac{dh}{dt}$ M1A1A1 As main scheme
2	Using $\frac{dA}{dt} = \frac{dA}{dh} \times \frac{dh}{dV} \times \frac{dV}{dt}$ B1M1 As main scheme M1 Chain rule $\frac{dA}{dt} = \frac{dA}{dh} \times \frac{dh}{dV} \times \frac{dV}{dt}$ M1 Attempt $\frac{dV}{dh}$ using their expression for V in terms of h found in (a) A1 $\frac{dV}{dt} = 0.4$ A1 Complete to required result.
3	Using $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$ B1 $\frac{dA}{dr} = 2\pi r$ M1 $t = \frac{5}{6}\pi r^2 h$ (Obtained from $0.4t = \frac{1}{3}\pi r^2 h$ (in (a)) M1 $\frac{dt}{dr} = \frac{5\sqrt{3}}{2}\pi r^2$ M1 $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt} \left(= 2\pi r \times \frac{2}{5\sqrt{3}\pi r} = \frac{4}{5\sqrt{3}r} \right)$ A1 Use $h = \sqrt{3}r$ in their $\frac{dA}{dt}$ A1 Correct final result, no errors seen

(c)	
M1	Use $t = 10$ to obtain the corresponding value of h $h = \left(\sqrt[3]{\frac{180}{5\pi}} \text{ or } 2.2545... \right)$ and substitute
	their value of h in the expression from (b) to obtain $\frac{dA}{dt} = \dots$
A1cao	$\frac{dA}{dt} = 0.355 \text{ (cm}^2\text{/s)}$ Must be 3sf.

