Question number	Scheme	Marks
9. **(a) (i)**	$\alpha+\beta=\left(\frac{4}{3}\right)$	B1
(ii)	$\alpha\beta=\frac{6}{3}=2$	B1 (2)
(b)	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)\Rightarrow\left(\frac{4}{3}\right)^3-3\times2\times\left(\frac{4}{3}\right)=-\frac{152}{27}$ *	M1M1A1 (3)
(c)	$\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}=\frac{\alpha^3+\beta^3}{\alpha^2\beta^2}=\frac{-\frac{152}{27}}{4}=-\frac{38}{27}$	M1A1
	$\frac{\alpha}{\beta^2}\times\frac{\beta}{\alpha^2}=\frac{1}{\alpha\beta}=\frac{1}{2}$	B1
	$x^2+\frac{38}{27}x+\frac{1}{2}=0\Rightarrow54x^2+76x+27=0$ oe (integer multiples)	M1A1 (5) **(10)**

Notes		
(a) (i)	B1	For the sum $\alpha+\beta=\left(\frac{4}{3}\right)$
(ii)	B1	For the product $\alpha\beta=\frac{6}{3}$ oe
(b)	M1	For the **correct** algebra to find $\alpha^3+\beta^3$ e.g., $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$ $\alpha^3+\beta^3=(\alpha+\beta)\left((\alpha+\beta)^2-3\alpha\beta\right)$ Their final expansion must be given in a form such that they can substitute their sum and product directly.
	M1	For substituting their values for the sum and product into their $\alpha^3+\beta^3$ Note $\alpha^2+\beta^2=-\frac{20}{9}$
	A1	For $-\frac{152}{27}$ **Note:** This is a 'show' question. Every step must be correct for the award of this mark.

(c)	M1	For the correct algebra on the sum $\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}=\frac{\alpha^3+\beta^3}{\alpha^2\beta^2}$ **and** substitution of their $\alpha+\beta$ and $\alpha\beta$.
	A1	For the correct sum of $-\frac{38}{27}$ allow $\frac{-\frac{152}{27}}{4}$
	B1	For the correct product of $\frac{1}{2}$
	M1	For using their sum and their product correctly to form an equation. $\left(x^2+(-\text{sum})\times x+\text{product}\right)=0$ (condone missing $=0$)
	A1	For the correct equation as shown. Accept any integer multiples. e.g $108x^2+152x+54=0$ etc
ALT (c)	M1	Attempts to form the equation as follows. Must be $-$ve sum, + ve product $\left(x-\frac{\alpha}{\beta^2}\right)\left(x-\frac{\beta}{\alpha^2}\right)=x^2-\left(-x\left(\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}\right)\right)+\frac{\alpha\beta}{(\alpha\beta)^2}\ (=0)$
	M1	$\left(x-\frac{\alpha}{\beta^2}\right)\left(x-\frac{\beta}{\alpha^2}\right)=x^2-\left(-x\left(\frac{\alpha^3+\beta^3}{\alpha^2\beta^2}\right)\right)+\frac{\alpha\beta}{(\alpha\beta)^2}$ Correct algebra only
	First A1	$\left(x-\frac{\alpha}{\beta^2}\right)\left(x-\frac{\beta}{\alpha^2}\right)=x^2+x\left(\frac{\mathbf{\frac{152}{27}}}{\mathbf{4}}\right)+\frac{\alpha\beta}{(\alpha\beta)^2}=x^2+x\left(\mathbf{\frac{38}{27}}\right)+\frac{\alpha\beta}{(\alpha\beta)^2}$
	B1	$\left(x-\frac{\alpha}{\beta^2}\right)\left(x-\frac{\beta}{\alpha^2}\right)=x^2+x\left(\frac{38}{27}\right)+\mathbf{\frac{2}{4}}$
	Final A1	$x^2+\frac{38}{27}x+\frac{1}{2}=0\Rightarrow 54x^2+76x+27=0$ oe with integer multiples