

Question	Scheme	Marks
10(a)	$\cos(A - B) = \cos A \cos B + \sin A \sin B$ $\cos(A + B) = \cos A \cos B - \sin A \sin B$ $\cos(A - B) - \cos(A + B) = \sin A \sin B - (-\sin A \sin B) = 2 \sin A \sin B$ *	M1A1cso [2]
(b)	$[A - B = 5\theta, A + B = 9\theta \Rightarrow A = 7\theta, B = 2\theta]$ $\cos 5\theta - \cos 9\theta = 2 \sin 7\theta \sin 2\theta$ *	B1 cso [1]
(c)	$\sqrt{3} \sin 7\theta = 2 \sin 7\theta \sin 2\theta \Rightarrow 0 = 2 \sin 7\theta \sin 2\theta - \sqrt{3} \sin 7\theta$ $0 = \sin 7\theta (2 \sin 2\theta - \sqrt{3}) \Rightarrow \sin 7\theta = 0, 2 \sin 2\theta - \sqrt{3} = 0$ $\sin 7\theta = 0 \Rightarrow 7\theta = 0, \pi, 2\pi \Rightarrow \theta = \frac{\pi}{7}, \frac{2\pi}{7}$ $2 \sin 2\theta - \sqrt{3} = 0 \Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2} \Rightarrow 2\theta = \frac{\pi}{3}, \frac{2\pi}{3} \Rightarrow \theta = \frac{\pi}{6}, \frac{\pi}{3}$	M1A1 M1 M1A1 M1A1 [7]
(d)	$\tan 2x = \frac{\sin 2x}{\cos 2x} \Rightarrow \tan 2x \cos 2x = \sin 2x$ $\int_0^{\frac{\pi}{7}} 8 \sin 7x \sin 2x \, dx = \left[\int_0^{\frac{\pi}{7}} 4 \times (2 \sin 7x \sin 2x) \, dx \right]$ $\int_0^{\frac{\pi}{7}} 4 (\cos 5x - \cos 9x) \, dx = 4 \left[\frac{\sin 5x}{5} - \frac{\sin 9x}{9} \right]_0^{\frac{\pi}{7}}$ $4 \left[\frac{\sin 5x}{5} - \frac{\sin 9x}{9} \right]_0^{\frac{\pi}{7}} = 4 \left[\left(\frac{\sin 5 \times \frac{\pi}{7}}{5} - \frac{\sin 9 \times \frac{\pi}{7}}{9} \right) - \left(\frac{\sin 0}{5} - \frac{\sin 0}{9} \right) \right] \#$ $= 0.9729... \approx 0.973$	M1 M1 M1M1 M1A1 [6]
Total 16 marks		

Question	Notes	Marks
10(a)	From the formula sheet $\cos(A - B) = \cos A \cos B + \sin A \sin B$ $\cos(A + B) = \cos A \cos B - \sin A \sin B$ Subtracts the two equations to give: $\cos(A - B) - \cos(A + B) = \sin A \sin B - (-\sin A \sin B)$	M1
	For the correct identity as shown with no errors, $\cos(A - B) - \cos(A + B) = 2 \sin A \sin B$ *	A1 cso [2]
(b)	For finding the value of A and the value of B $A - B = 5\theta, \quad A + B = 9\theta$ $\Rightarrow A = 7\theta, \quad B = 2\theta$ Or as a minimum: $\cos(7\theta - 2\theta) - \cos(7\theta + 2\theta) = 2 \sin 7\theta \sin 2\theta$ *	B1cso [1]
(c)	Sets $\sqrt{3} \sin 7\theta = 2 \sin 7\theta \sin 2\theta$	M1
	Achieves the correct equation allow the terms in any order. $0 = 2 \sin 7\theta \sin 2\theta - \sqrt{3} \sin 7\theta$	A1
	Factorises their equation $0 = \sin 7\theta (2 \sin 2\theta - \sqrt{3}) \Rightarrow \sin 7\theta = 0, \quad 2 \sin 2\theta - \sqrt{3} = 0$	M1
	For finding at least one correct value for θ using $\sin 7\theta = 0$ $\sin 7\theta = 0 \Rightarrow 7\theta = 0, \pi, 2\pi \Rightarrow \theta = \frac{\pi}{7}, \frac{2\pi}{7}$	M1
	For both correct values $\theta = \frac{\pi}{7}, \frac{2\pi}{7}$	A1
	For finding one correct value of θ using $2 \sin 2\theta - \sqrt{3} = 0$ $2 \sin 2\theta - \sqrt{3} = 0 \Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2} \Rightarrow 2\theta = \frac{\pi}{3}, \frac{2\pi}{3} \Rightarrow \theta = \frac{\pi}{6}, \frac{\pi}{3}$	M1
	For both correct values $\theta = \frac{\pi}{6}, \frac{\pi}{3}$	A1 [7]
(d)	Uses the identity for $\tan 2x = \frac{\sin 2x}{\cos 2x} \Rightarrow \tan 2x \cos 2x = \sin 2x$	M1
	Substitutes the above into $8 \sin 7x \cos 2x \tan 2x$ to give $\int_0^{\frac{\pi}{7}} 8 \sin 7x \sin 2x \, dx = \left[\int_0^{\frac{\pi}{7}} 4 \times (2 \sin 7x \sin 2x) \, dx \right]$	M1
	Ignore integral sign and limits for this mark.	
	For substituting $\cos 5x - \cos 9x$ for $2 \sin 7x \sin 2x$ to give $\int_0^{\frac{\pi}{7}} 4 (\cos 5x - \cos 9x) \, dx$ Ignore integral sign and limits for this mark.	M1

	<p>Integrates $\int_0^{\frac{\pi}{7}} 4(\cos 5x - \cos 9x) \, dx = 4 \left[\frac{\sin 5x}{5} - \frac{\sin 9x}{9} \right]_0^{\frac{\pi}{7}}$</p> <p>Ignore limits for this mark.</p> <p>As a minimum they must obtain: $\left(\pm \frac{\sin 5x}{5} \pm \frac{\sin 9x}{9} \right)$ for the integration.</p>	M1
	<p>Substitutes the limits the correct way around</p> $4 \left[\frac{\sin 5x}{5} - \frac{\sin 9x}{9} \right]_0^{\frac{\pi}{7}} = 4 \left[\left(\frac{\sin 5 \times \frac{\pi}{7}}{5} - \frac{\sin 9 \times \frac{\pi}{7}}{9} \right) - \left(\frac{\sin 0}{5} - \frac{\sin 0}{9} \right) \right]$ $= (0.9729\dots)$	M1
	<p>For the correct value of $\int_0^{\frac{\pi}{7}} 8 \sin 7x \cos 2x \tan 2x \, dx = 0.973$</p>	A1 [6]
Total 16 marks		

