

Question number	Scheme	Marks
10 (a)	$f\left(\frac{3}{4}\right) = 16\left(\frac{3}{4}\right)^3 + 11\left(\frac{3}{4}\right) - 15$ $\left(\frac{27}{4} + \frac{33}{4} - 15\right) = 0$	M1 A1cso [2]
(b)(i)	$[\sin 2\theta = \sin(\theta + \theta)] = \sin \theta \cos \theta + \sin \theta \cos \theta$	M1
	$(\sin 2\theta =) 2 \sin \theta \cos \theta$ *	A1cso [2]
(b)(ii)	$[\cos 2\theta = \cos(\theta + \theta)] = \cos \theta \cos \theta - \sin \theta \sin \theta = \cos^2 \theta - \sin^2 \theta$	M1
	$= \cos^2 \theta - (1 - \cos^2 \theta) = 2 \cos^2 \theta - 1$ *	M1 A1cso [3]
(c)	$27 \cos \theta (2 \cos^2 \theta - 1) + 19 \sin \theta (2 \sin \theta \cos \theta) - 15 (= 0)$	M1
	$54 \cos^3 \theta - 27 \cos \theta + 38(1 - \cos^2 \theta) \cos \theta - 15 (= 0)$	M1
	$54 \cos^3 \theta - 27 \cos \theta + 38 \cos \theta - 38 \cos^3 \theta - 15 (= 0)$ $(16 \cos^3 \theta + 11 \cos \theta - 15 = 0)$ oe	A1
	(When $x = \cos \theta$ ) $16x^3 + 11x - 15 = 0$ *	A1cso [4]
(d)	$(16x^3 + 11x - 15 = 0 \Rightarrow (4x - 3)(4x^2 + 3x + 5) = 0) \rightarrow 4x^2 + 3x + 5$	M1
	$b^2 - 4ac = 9 - 80 < 0$ (So no real roots)	M1 A1
	(Only solution is) $4x - 3 = 0$ or $4 \cos \theta - 3 = 0$ so $\cos \theta = \frac{3}{4}$ *	A1cso [4]
<b>Total 15 marks</b>		

Part	Marks	Notes
(a)	<b>M1</b>	For $f\left(\frac{3}{4}\right) = 16\left(\frac{3}{4}\right)^3 + 11\left(\frac{3}{4}\right) - 15$
	<b>A1*cso</b>	For a fully correct solution, with $= 0$ stated, with no errors seen
(b)(i)	<b>M1</b>	For sight of $\sin \theta \cos \theta + \sin \theta \cos \theta$
	<b>A1*cso</b>	For obtaining the given expression, with no errors seen
(b)(ii)	<b>M1</b>	For $\cos \theta \cos \theta - \sin \theta \sin \theta = \cos^2 \theta - \sin^2 \theta$
	<b>M1</b>	For correctly using $\sin^2 \theta = 1 - \cos^2 \theta$
	<b>A1*cso</b>	For obtaining the given expression with no errors seen
(c)	<b>M1</b>	For correctly substituting $\sin 2\theta = 2 \sin \theta \cos \theta$ and $\cos 2\theta = 2 \cos^2 \theta - 1$
	<b>M1</b>	For expanding the first set of brackets (allow one error only in expansion) and correct use of $\sin^2 \theta = 1 - \cos^2 \theta$ ,
	<b>A1</b>	$54 \cos^3 \theta - 27 \cos \theta + 38 \cos \theta - 38 \cos^3 \theta - 15 (= 0)$ oe all brackets expanded
	<b>A1*cso</b>	For obtaining the given equation
(d)	<b>M1</b>	For any valid, complete method to find the quadratic factor which must be of the form $4x^2 + ax \pm 5$
	<b>M1</b>	For use of the discriminant from their quadratic factor
	<b>A1</b>	For $b^2 - 4ac = 9 - 80 < 0$

Question number	Scheme	Marks
11 (a)(i)	$(a =) - 8$	B1 [1]
(a)(ii)	$e^{2x} = 9 \Rightarrow 2x = \ln 9$	M1
	$x = \frac{1}{2} \ln 9 = \ln 3$ *	A1cso [2]
ALT	$e^{2x} = 9 \Rightarrow \left[ (e^x)^2 = 3^2 \right] \Rightarrow e^x = 3$	M1
	$x = \ln 3$ *	A1cso
(b)	$\pi \int_0^{\ln 3} (e^{2x} - 9)^2 dx$	M1
	$\pi \int_0^{\ln 3} (e^{4x} - 18e^{2x} + 81) dx$	A1
	$(\pi) \left[ \frac{1}{4} e^{4x} - 9e^{2x} + 81x \right]_{(0)}^{(\ln 3)}$ oe	M1 A1ft
	$(\pi) \left[ \left( \frac{81}{4} - 81 + 81 \ln 3 \right) - \left( \frac{1}{4} - 9 \right) \right]$	M1
	$\pi(81 \ln 3 - 52)$	A1 [6]
<b>Total 9 marks</b>		
	<b>A1*cso</b>	For a fully correct solution.

Part	Mark	Notes
(a)(i)	<b>B1</b>	For $(a =) - 8$
(a)(ii)	<b>M1</b>	For substituting $y = 0$ and rearranging to $2x = \ln 9$ or $x = \frac{1}{2} \ln 9$
	<b>A1*cso</b>	For a fully correct solution with no errors – must see $x = \frac{1}{2} \ln 9$ or $2x = 2 \ln 3$
ALT	<b>M1</b>	For substituting $y = 0$ and manipulating to give $e^x = 3$
	<b>A1*cso</b>	For a fully correct solution with no errors
(b)	<b>M1</b>	For $\pi \int_0^{\ln 3} (e^{2x} - 9)^2 dx$
	<b>A1</b>	For $\pi \int_0^{\ln 3} (e^{4x} - 18e^{2x} + 81) dx$ oe. If $\pi$ seen for first M1, can be omitted.
	<b>M1</b>	For minimally acceptable attempt to integrate two terms of their expression (at least one term integrated correctly). $\pi$ and limits can be omitted.
	<b>A1ft</b>	For $\frac{1}{4} e^{4x} - 9e^{2x} + 81x$ Ft their expression of the form $e^{4x} + ce^{2x} + d$ $\pi$ and limits can be omitted
	<b>M1</b>	For correct substitution of 0 and their $b$ into a changed expression. The substitution must be fully shown for this mark. $\pi$ can be omitted. This mark can be implied by a correct final answer.
	<b>A1</b>	For obtaining the correct expression

$\pi$  must only be present for the final A1 mark and the first M1 mark (can be implied by first or final A1)

