

| Question number   | Scheme  |  |   |                  | Marks  |
|---|---|--|---|------------------|--|
| 8 (a)(i)  | $e^{3x} - 1 = 9 - 9e^{-3x}$   |  |   |                  | M1   |
|   | $(e^{3x})^2 - e^{3x} = 9e^{3x} - 9$   |  |   |                  | M1   |
|   | $(e^{3x})^2 - 10e^{3x} + 9 = 0 *$   |  |   |                  | A1cso*   |
|   | (ii)  | $(e^{3x} - 1)(e^{3x} - 9) [= 0]$ leading to $e^{3x} =$ |   |                  | M1   |
|   | $e^{3x} = 9 \rightarrow x = \frac{1}{3} \ln 9 *$  |  |   |                  | A1cso*<br>[5]                                      |
| Note: subscripts on marks in (b) indicate which mark is being awarded on open (1=1 <sup>st</sup> etc) |   |  |   |                  |  |
| (b)   | $\left(\int_0^{\frac{1}{3} \ln 9} (9 - 9e^{-3x}) dx =\right)$   |  | $\left(\int_0^{\frac{1}{3} \ln 9} (e^{3x} - 1) dx =\right)$   |                  | M1 <sub>1</sub>                                    |
|   | $9x + 3e^{-3x}$   |  | $\frac{e^{3x}}{3} - x$  |                  | A1 <sub>2</sub> A1 <sub>3</sub>                    |
|   | " $9x + 3e^{-3x}$ " - " $\frac{e^{3x}}{3} - x$ "  | M1 <sub>4</sub>  | $\left(9 \times \frac{1}{3} \ln 9 + 3e^{-3 \times \frac{1}{3} \ln 9}\right) - (9 \times 0 + 3e^0)$<br>and<br>$\left(\frac{e^{3 \times \frac{1}{3} \ln 9}}{3} - \frac{1}{3} \ln 9\right) - \left(\frac{e^0}{3} - 0\right)$ | dM1 <sub>5</sub> |  |
|   | $\left(10 \times \frac{1}{3} \ln 9 - \frac{9}{-3} e^{-3 \times \frac{1}{3} \ln 9} - \frac{e^{3 \times \frac{1}{3} \ln 9}}{3}\right) - \left(10 \times 0 - \frac{9}{-3} e^{-3 \times 0} - \frac{e^{3 \times 0}}{3}\right)$ | dM1 <sub>5</sub>                                       | " $3 \ln 9 - \frac{8}{3}$ " - " $\frac{8}{3} - \frac{1}{3} \ln 9$ "   | M1 <sub>4</sub>  |  |
| $\frac{10}{3} \ln 9 - \frac{16}{3}$   |   |  |   |                  | A1 <sub>6</sub>                                    |
|   | ALT<br>$\int (9 - 9e^{-3x}) - (e^{3x} - 1) dx$  |  |   |                  | M1 <sub>4</sub>                                    |
|   | $(\int (10 - 9e^{-3x} - e^{3x}) dx =) 10x - \frac{9}{-3} e^{-3x} - \frac{e^{3x}}{3}$  |  |   |                  | M1 <sub>1</sub><br>A1 <sub>2</sub> A1 <sub>3</sub> |
|   | $\left(10 \times \frac{1}{3} \ln 9 - \frac{9}{-3} e^{-3 \times \frac{1}{3} \ln 9} - \frac{e^{3 \times \frac{1}{3} \ln 9}}{3}\right) - \left(10 \times 0 - \frac{9}{-3} e^{-3 \times 0} - \frac{e^{3 \times 0}}{3}\right)$ |  |   |                  | dM1 <sub>5</sub>                                   |
|   | $\frac{10}{3} \ln 9 - \frac{16}{3}$ oe  |  |   |                  | A1 <sub>6</sub><br>[6]                             |
| Total 11 marks  |   |  |   |                  |  |

| Part                              | Mark             | Additional Guidance  |
|-----------------------------------|------------------|--|
| Mark parts (i) and (ii) together. |                  |  |
| (a)(i)                            | M1               | For equating the two equations.  |
|                                   | M1               | For multiplying through by $e^{3x}$ , minimum of 2 out of 4 correct terms.<br>(presence of $\pm 10e^{3x}$ indicates 2 correct terms).  |
|                                   | A1*cs0           | Correct solution only, no errors or omissions.   |
|                                   | M1               | Minimally acceptable attempt at solving the equation leading to $e^{3x} =$<br>See general guidance, if the formula is quoted allow up to two slips in substitution, otherwise the substitution must be correct.  |
|                                   | A1*cs0           | Correct solution only, no errors or omissions.<br>If 0 also included then this should be rejected.   |
| (b)                               | M1 <sub>1</sub>  | For attempt to integrate one of:<br>$9 - 9e^{-3x}$ or $e^{3x} - 1$ or $\pm[(9 - 9e^{-3x}) - (e^{3x} - 1)]$<br>Limits may not be present.<br>At least one term correct. Ignore +c if included.  |
|                                   | A1 <sub>2</sub>  | For correct integration of one of the exponential terms<br>$\pm 9e^{-3x} \rightarrow \mp \frac{9}{3}e^{-3x}$ or $\pm e^{3x} \rightarrow \pm \frac{1}{3}e^{3x}$<br>Limits need not be present. Ignore +c if included.   |
|                                   | A1 <sub>3</sub>  | For correct integration of both curves<br>$9x + 3e^{-3x}$ and $\frac{e^{3x}}{3} - x$<br>or for a <b>fully correct</b> integration where the difference between two expressions is found<br>$\pm \left(10x - \frac{9}{-3}e^{-3x} - \frac{e^{3x}}{3}\right)$ or $\pm \left(9x - \frac{9}{-3}e^{-3x} - \frac{e^{3x}}{3} + x\right)$<br><b>Note: this is an M mark in open</b> |
|                                   | M1 <sub>4</sub>  | For the difference between the two expressions either before or after integration.<br>Allow subtraction either way around.<br><b>Note: this is an A mark in open</b>   |
|                                   | dM1 <sub>5</sub> | Substitution of correct limits into their integrated expressions (limits subtracted the correct way around).<br>Dependent on first M scored.<br>If substituting before difference found then must substitute into both integrated expressions.<br>May be implied by awrt 1.99<br>If integration is not correct then substitution must be shown.                            |
|                                   | A1 <sub>6</sub>  | For the correct answer oe.<br>Must be exact value.   |