Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International GCSE

Centre Number | | | | | |

Candidate Number | | | | |

Thursday 18 June 2020

Morning (Time: 2 hours) | Paper Reference **4PM1/02**

Further Pure Mathematics

Paper 2

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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P 6 2 2 8 5 A 0 1 3 2

Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\quad$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\quad$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B} \qquad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Answer all TEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 A particle P is moving in a straight line. At time t seconds, $t \geqslant 0$, the displacement, s metres, of P from a fixed point O of the line is given by

$$s = 3 + 8t + t^2 - \frac{1}{3}t^3$$

Find the distance of P from O when P is instantaneously at rest.

(4)

(Total for Question 1 is 4 marks)

2 The region enclosed by the curve with equation $y = e^{3x}$, the x-axis, the y-axis and the line with equation $x = 3$ is rotated through 360° about the x-axis.

Use algebraic integration to find, in terms of π and e, the volume of the solid generated.

(4)

Question 2 continued

(Total for Question 2 is 4 marks)

3 (a) Expand

$$(1 + px)^{-5} \quad p \neq 0$$

in ascending powers of x, up to and including the term in x^4
Give each term in its simplest form.

(3)

The coefficient of x^r in the expansion is c_r

Given that $c_4 = 2c_3$

(b) find the value of p.

(2)

Question 3 continued

(Total for Question 3 is 5 marks)

4 (i) Solve the equation $16\log_r 4 = \log_4 r$

(2)

(ii) Solve the equation $\log_5 9 + \log_5 12 + \log_5 15 + \log_5 18 = 1 + \log_5 x + \log_5 x^2$

(5)

Question 4 continued

(Total for Question 4 is 7 marks)

5 (a) Show that $\sum_{r=1}^{n}(3r+5) = \frac{1}{2}n(3n+13)$

(3)

(b) Hence evaluate $\sum_{r=35}^{50}(3r+5)$

(2)

Given that $\sum_{r=1}^{n}(3r+5) = 385$

(c) find the value of n.

(3)

Question 5 continued

(Total for Question 5 is 8 marks)

6 $f(x) = 4x^2 - 3x - 5$

The equation $f(x) = 0$ has roots α and β

Without solving the equation $f(x) = 0$

(a) form an equation, with integer coefficients, that has roots $\frac{2\alpha}{\beta}$ and $\frac{2\beta}{\alpha}$

(6)

$g(x) = 4x^2 + px + q$ where p and q are constants

Given that the equation $g(x) = 0$ has roots $3\alpha + \beta$ and $\alpha + 3\beta$

(b) find the value of p and the value of q

(5)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 11 marks)

7 A geometric series has first term $(x - 3)$, second term $(x + 1)$ and third term $(4x - 2)$.

(a) Find the two possible values of x.

(5)

Given that $x < 1$,

(b) show that the series is convergent.

(2)

The sum to infinity of the series is S.

(c) Find the value of S.

(2)

The sum of the first n terms of the series is S_n

Given that $\frac{S}{S_n} = \frac{256}{255}$

(d) find the value of n.

(3)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 12 marks)

8 The curve C_1 has equation $y = 5e^{-2x} + 4$

The curve C_2 has equation $y = e^{2x}$

The curves C_1 and C_2 intersect at the point A.

(a) Find the exact coordinates of A.

(4)

The tangent at A to C_1 intersects the x-axis at the point B.

(b) Show that the x coordinate of B is $\frac{1}{2}(5 + \ln 5)$

(5)

The tangent at A to C_2 intersects the x-axis at the point D.

(c) Find the area of ΔABD.

(6)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 15 marks)

9 A curve C has equation

$$y = \frac{2 + 4x - x^2}{2x + 1} \qquad x \neq -\frac{1}{2}$$

(a) Write the equation of C in the form $ax^2 + (by - 4)x + (y - c) = 0$, where a, b and c are integers whose values are to be found.

(3)

(b) Hence show that x is real when $y \leqslant 2$ and when $y \geqslant 3$

(4)

(c) Find the coordinates of the stationary points on C.

(6)

(d) Sketch C showing clearly

(i) the exact coordinates of the points where C crosses the x-axis,

(ii) the asymptote to C that is parallel to the y-axis,

(iii) the coordinates of the stationary points.

(5)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 18 marks)

10 (a) Show that

$$\cos(A + B) + \cos(A - B) = 2\cos A\cos B$$

(2)

(b) Hence show that

$$\cos P + \cos Q = 2\cos\frac{P + Q}{2}\cos\frac{P - Q}{2}$$

(3)

(c) Solve, for $0 \leqslant \theta \leqslant \frac{\pi}{2}$, the equation

$$\cos 5\theta + \cos 7\theta = 0$$

Give each solution in terms of π

(4)

(d) Show that

$$\cos 8x + 2\cos 6x + \cos 4x = 4\cos 6x\cos^2 x$$

(3)

(e) Use calculus to find the exact value of

$$\int_0^{\frac{\pi}{3}} \cos 6x\cos^2 x\,\mathrm{d}x$$

(4)

Question 10 continued

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 16 marks)

TOTAL FOR PAPER IS 100 MARKS