Please check the examination de	tails below before entering your c	andidate information
Candidate surname	Other nar	mes
Pearson Edexcel International GCSE	Centre Number	Candidate Number
Monday 17 J	une 2019	
Afternoon (Time: 2 hours)	Paper Reference	4PM1/01R
Further Pure N Paper 1R	lathematics	
Calculators may be used.		Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
 - there may be more space than you need.
- You must NOT write anything on the formulae page.
 Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
 - use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶



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International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times \text{slant height}$

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to *n* terms, $S_n = \frac{n}{2} [2a + (n-1)d]$

Geometric series

Sum to *n* terms,
$$S_n = \frac{a(1-r^n)}{(1-r)}$$

Sum to infinity,
$$S_{\infty} = \frac{a}{1-r} |r| < 1$$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots$$
 for $|x| < 1, n \in \mathbb{Q}$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x} \left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)} \right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{\left[\mathrm{g}(x)\right]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc \cos A$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$



Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1

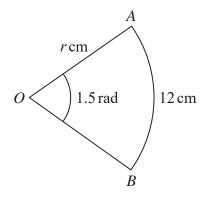


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows sector AOB of a circle with centre O and radius r cm. The angle AOB is 1.5 radians and the length of arc AB is 12 cm.

Calculate

(a) the value of r,

(1)

(b) the area of the sector AOB.

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		(Total for	Ouestion 1 is 3	marks)



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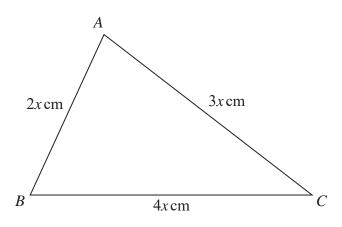


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Figure 2

Figure 2 shows triangle ABC in which

$$AB = 2x \text{ cm}$$
 $AC = 3x \text{ cm}$ $BC = 4x \text{ cm}$

(a) Show that
$$\sin ABC = \frac{3\sqrt{15}}{16}$$

(4)

Given that the area of triangle ABC is $\frac{75\sqrt{15}}{64}$ cm²

(b) find	the val	lue of x .
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(2)



3	(a) Write down the value of $\log_3 9$	(1)
	(b) Solve the equation $\log_3 9t = \log_9 \left(\frac{12}{t}\right)^2 + 2$ where $t > 0$	(-)
	Give your answer in the form $a\sqrt{b}$ where a and b are prime numbers.	(6)

4	$f(x) = e^{3x} \sqrt{1 + 2x}$
(a) Show that	$f'(x) = \frac{2e^{3x}(2+3x)}{x}$

(b) Find an equation of the normal to the curve with equation y = f(x) at the point on the curve where x = 0

Give your answer in the form ax + by + c = 0 where a, b and c are integers. (6)

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Given that the value of r increases by 0.05% use calculus to find an estimate for the percentage increase in the value of A. (5)

Question 5 continued
(Total for Question 5 is 5 marks)



6 (a) Show that $\sum_{r=1}^{n} (4r - 3) = n(2n - 1)$

(3)

(b) Hence, or otherwise, find the least value of *n* such that $\sum_{r=1}^{n} (4r - 3) > 1000$

(3)

Given that $S_n = n(2n - 1)$, $t_n = (4n - 3)$ and that $18 + 3t_{n+7} = S_{n+4}$

(c) find the value of n.

Que	stion 6 continued		



Question 6 continued	

(Total for Question 6 is 10 marks)	Question 6 continued
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7 O, A, B and C are fixed points such that

$$\overrightarrow{OA} = 8\mathbf{i} - 6\mathbf{j}$$
 $\overrightarrow{OB} = 15\mathbf{i} - 6\mathbf{j}$ $\overrightarrow{OC} = 8\mathbf{i} + \mathbf{j}$

(a) Find \overrightarrow{BC} as a simplified expression in terms of **i** and **j**

(2)

(b) Find a unit vector parallel to \overrightarrow{BC}

(2)

The point M is the midpoint of OA and the point N lies on OB such that ON: NB = 1:2

(c) Show that the points M, N and C are collinear.

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Ques	tion 7 continued			



Question 7 continued	

Question 7 continued	
	(Total for Question 7 is 8 marks)



8 (a) Complete the table of values for $y = 2 + \ln(2x + 1)$ giving your answers to 2 decimal places.

(2)

x	0	0.25	0.5	1	1.5	2	3
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(b) On the grid opposite, draw the graph of $y = 2 + \ln(2x + 1)$ for $0 \le x \le 3$

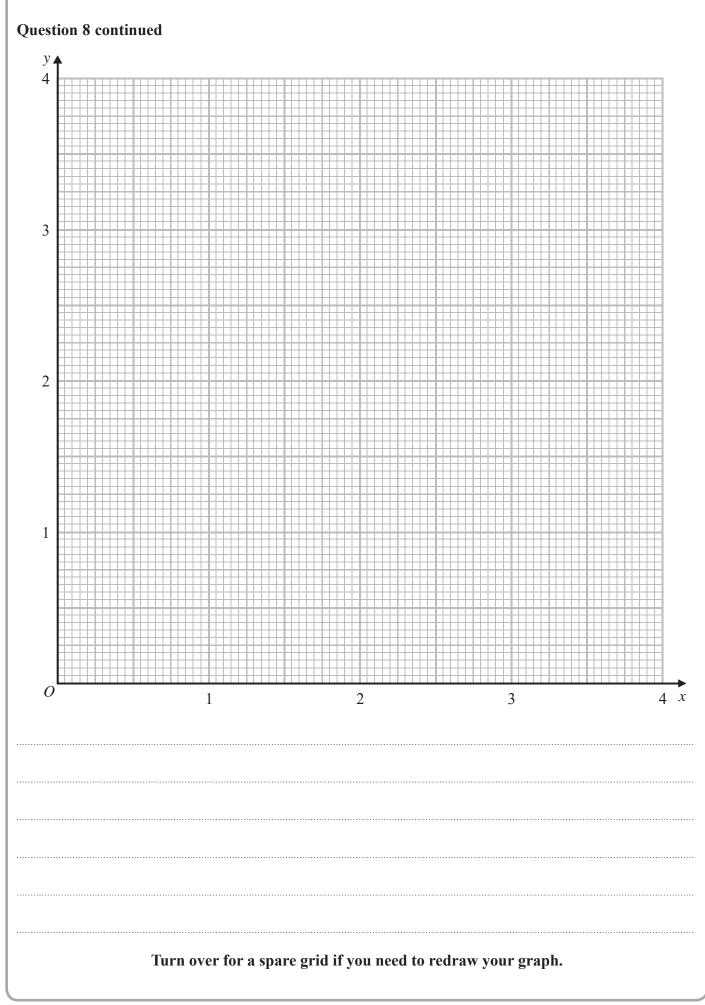
(2)

(c) By drawing an appropriate straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation $\ln(2x+1) = 3x - 4$ in the interval $0 \le x \le 3$

(3)

(d) By drawing an appropriate straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation $e^{(6-x)} - (2x+1)^2 = 0$ in the interval $0 \le x \le 3$

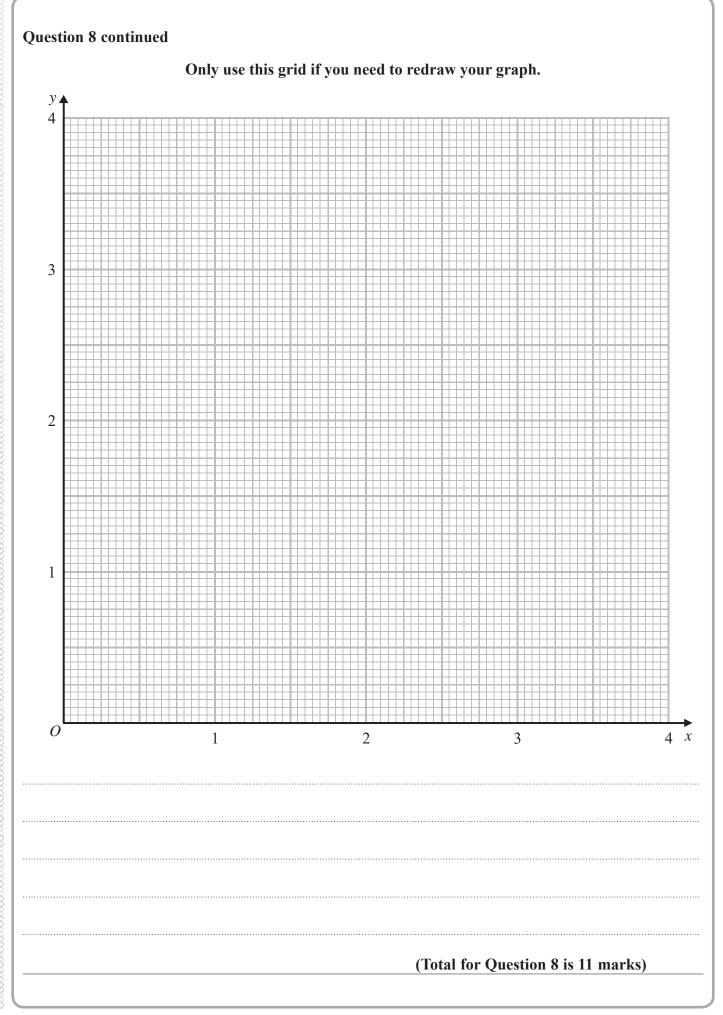






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Question 8 continued	





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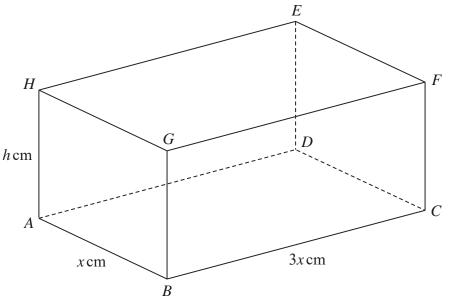


Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows a solid cuboid ABCDEFGH

$$AB = x \text{ cm}$$
 $BC = 3x \text{ cm}$ $AH = h \text{ cm}$

The volume of the cuboid is 540 cm³

The total surface area of the cuboid is Scm²

(a) Show that
$$S = 6x^2 + \frac{1440}{x}$$

(4)

Given that *x* can vary,

(b) use calculus to find, to 3 significant figures, the value of x for which S is a minimum. Justify that this value of x gives a minimum value of x.

(5)

(c) Find, to 3 significant figures, the minimum value of S.

(1)

	Question 9 continued
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Question 9 continued		

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Question 9 continued	
	(Total for Question 9 is 10 marks)



 $f(x) = 6x - x^2 \qquad x \in \mathbb{R}$

Given that f(x) can be written in the form $D(x + E)^2 + F$ where D, E and F are integers,

(a) find the value of D, the value of E and the value of F.

(3)

- (b) Find
 - (i) the maximum value of f(x),
 - (ii) the value of x for which the maximum occurs.

(2)

The curve C has equation y = f(x)

The curve S has equation $y = x^2 - 4x + 8$

The curve S intersects the curve C at two points.

(c) Find the coordinates of each of these two points.

(4)

The finite region R is bounded by the curve C and the curve S.

(d) Use algebraic integration to find the area of R.

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Question 10 continued	

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11	The points A and B have coordinates $(-1, 3)$ and $(5, 6)$ respectively.	
	(a) Find an equation for the line AB.	(2)
	The point P divides AB in the ratio $2:1$	
	(b) Show that the coordinates of P are $(3, 5)$	(2)
	The point C with coordinates (m, n) , where $m > 0$, is such that CP is perpendicular to the line AB.	e
	Given that the radius of the circle which passes through A , P and C is 5	
	(c) find the value of m and the value of n .	(6)
	The point D with coordinates (p, q) is such that the line AD is perpendicular to the line AB and the line DC is parallel to the line AB .	
	(d) Find the value of p and the value of q .	(3)
	(e) Find the area of trapezium ABCD.	
		(4)

Ques	stion 11 continued



Question 11 continued

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	(Total for Question 11 is 17 marks)
	TOTAL FOR PAPER IS 100 MARKS