

Question Number	Scheme	Marks
<b>5.</b>	<p>(a) <math>30 = 2y + 2x + \pi x</math>  <math>2y = 30 - 2x - \pi x</math>  <math>y = 15 - x - \frac{1}{2}\pi x</math>      oe</p> <p>(b) <math>A = 2xy + \frac{1}{2}\pi x^2</math>  <math>= x(30 - 2x - \pi x) + \frac{1}{2}\pi x^2</math>  <math>= 30x - 2x^2 - \frac{1}{2}\pi x^2</math>      *</p> <p>(c) <math>\frac{dA}{dx} = 30 - 4x - \pi x</math>  At maximum, <math>\frac{dA}{dx} = 0 \Rightarrow 30 - 4x - \pi x = 0</math>  <math>\Rightarrow x = \frac{30}{4 + \pi}</math> [= 4.201]  <math>\frac{d^2A}{dx^2} = -4 - \pi &lt; 0 \Rightarrow</math> maximum  Maximum area = <math>30\left(\frac{30}{4 + \pi}\right) - 2\left(\frac{30}{4 + \pi}\right)^2 - \frac{1}{2}\pi\left(\frac{30}{4 + \pi}\right)^2 = 63</math> to 2SF</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1 A1</p> <p>M1 A1 (11)</p>

**Notes for Question 5**

(a) M1 for setting up the equation  $30 = 2y + 2x + \pi x$  ...Allow with a circle (ie  $2\pi x$ ) and the missing side ( $4x$  instead of  $2x$ )

A1cao for re-arranging to get  $y = 15 - x - \frac{1}{2}\pi x$  oe

(b) M1 for  $A = 2xy + \frac{1}{2}\pi x^2$  and sub **their** expression for  $y$  (semicircle or circle  $\pi x^2$  for M1)

A1cso for  $A = 30x - 2x^2 - \frac{1}{2}\pi x^2$  \*

**NB: Watch for double sign errors in (a) and (b) and deduct A marks as appropriate.**

(c)

M1 for differentiating the **given** expression for  $A$  wrt  $x$

M1 for equating **their** differential to 0 (NB this is not a dependent M mark)

A1 for obtaining  $x = \frac{30}{4 + \pi}$

M1 for attempting the second differential

A1 for stating this differential is negative, so a maximum, **providing** the second differential is correct. A **correct** differential with no mention of  $\frac{d^2A}{dx^2}$  can get M1A1

*Alternatives for the last two marks:*

M1 for testing the sign of  $\frac{dA}{dx}$  on either side of **their**  $x$ . Numerical calculations must be seen. Only one turning point so values chosen need not be close to their  $x$ .

A1 for correct results and the conclusion, **providing**  $\frac{dA}{dx}$  is correct

OR: M1 Graph of  $A = 30x - 2x^2 - \frac{1}{2}\pi x^2$  is a parabola/quadratic opening downwards (must be stated or a sketch shown)

A1 turning point is therefore a maximum.

**Then:**

M1 for substituting **their positive**  $x$  in the **given** expression for  $A$ .

A1cao for  $A_{\max} = 63$  (cm<sup>2</sup>) **Must be 2 sf.**