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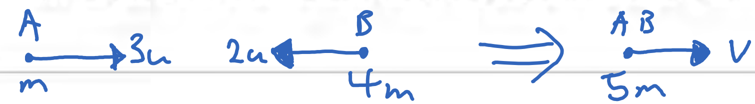
1. A railway truck $A$ of mass $m$ and a second railway truck $B$ of mass $4m$ are moving in opposite directions on a smooth straight horizontal track when they collide directly. Immediately before the collision the speed of truck $A$ is $3u$ and the speed of truck $B$ is $2u$. In the collision the trucks join together. Modelling the trucks as particles, find

(a) the speed of $A$ immediately after the collision. **(3)**

(b) the direction of motion of $A$ immediately after the collision. **(1)**

(c) the magnitude of the impulse exerted by $A$ on $B$ in the collision. **(3)**




a) Cons. of momentum:

$\rightarrow \quad 3mu - 2u(4m) = 5mv$

$v = -u$

$\therefore$ Speed of A is $u$

b) A moves in the opposite direction to its initial motion

c) $\rightarrow \quad I = mv - mu$

$= 4m(-u - (-2u))$

$= 4mu$

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**2.**

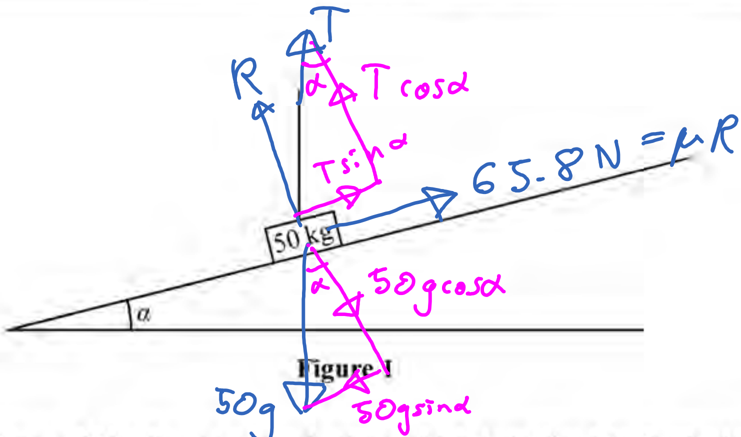
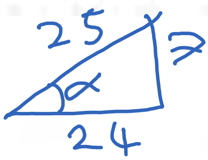



**Figure 1**

A block of mass 50 kg lies on a rough plane which is inclined to the horizontal at an angle $\alpha$, where $\tan\alpha = \frac{7}{24}$. The block is held at rest by a vertical rope, as shown in Figure 1, and is on the point of sliding down the plane. The block is modelled as a particle and the rope is modelled as a light inextensible string. Given that the friction force acting on the block has magnitude 65.8 N, find

(a) the tension in the rope,




**(4)**

(b) the coefficient of friction between the block and the plane.

**(4)**

∴ equilibrium

a) $\nearrow \; 65.8 + T\sin\alpha - 50g\sin\alpha = 0$

$$T = \left[50g\left(\frac{7}{25}\right) - 65.8\right]\frac{25}{7}$$

$$= 255\text{ N}$$

b) $\nwarrow \; R + T\cos\alpha - 50g\cos\alpha = 0$

$$R = 50g\left(\frac{24}{25}\right) - 255\left(\frac{24}{25}\right)$$

$$= 255.6\text{ N}$$

$F = \mu R$

$$\mu = \frac{F}{R} = \frac{65.8}{255.6} = 0.292$$

Leave blank

**3.** [*In this question* **i** *and* **j** *are unit vectors directed due east and due north respectively.*]

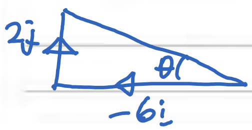
A particle $P$ is moving with constant velocity $(-6\mathbf{i} + 2\mathbf{j})\text{ m s}^{-1}$. At time $t = 0$, $P$ passes through the point with position vector $(21\mathbf{i} + 5\mathbf{j})$ m, relative to a fixed origin $O$.

(a) Find the direction of motion of $P$, giving your answer as a bearing to the nearest degree.
**(3)**

(b) Write down the position vector of $P$ at time $t$ seconds.
**(1)**

(c) Find the time at which $P$ is north-west of $O$.
**(3)**

a)




$\tan\theta = \frac{2}{6}$

$\theta = 18.4°$

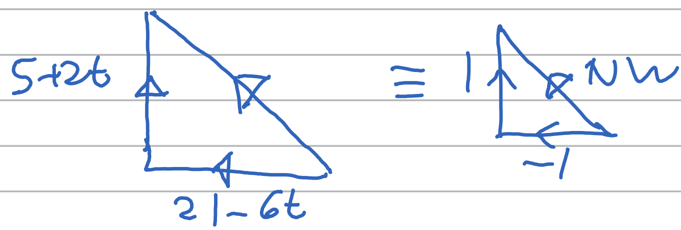
$\therefore$ Bearing is $270 + 18 = 288$

b) $\underline{r} = \underline{r}_0 + \underline{v}t$

$= \begin{pmatrix} 21 \\ 5 \end{pmatrix} + t\begin{pmatrix} -6 \\ 2 \end{pmatrix}$

$= (21 - 6t)\underline{i} + (5 + 2t)\underline{j}$

c)




$5 + 2t = -(21 - 6t)$

$4t = 26$

$t = 6.5$

Leave blank

**4.** The points $P$ and $Q$ are at the same height $h$ metres above horizontal ground. A small stone is dropped from rest from $P$. Half a second later a second small stone is thrown vertically downwards from $Q$ with speed $7.35 \text{ m s}^{-1}$. Given that the stones hit the ground at the same time, find the value of $h$.

**(7)**

$P: s = h \qquad Q: s = h$

$u = 0 \qquad u = 7.35 \text{ ms}^{-1}$

$v = x \qquad v = x$

$a = g \qquad a = g$

$t = T \qquad t = T - 0.5$

$[s = ut + \frac{1}{2}at^2] \qquad [s = ut + \frac{1}{2}at^2]$

$h = \frac{g}{2}T^2 \; (1) \qquad h = 7.35(T - 0.5) + \frac{g}{2}(T - 0.5)^2$

$\Rightarrow \frac{g}{2}T^2 = 7.35T - \frac{7.35}{2} + \frac{gT^2}{2} - \frac{gT}{2} + \frac{g}{8}$

$2.45 = 2.45T$

$T = 1$

In (1): $h = \frac{g}{2}(1)^2 = 4.9 \text{ m}$

Leave blank

**5.**

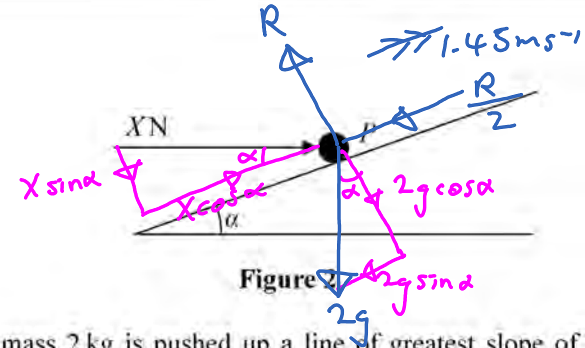
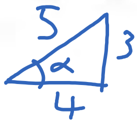



**Figure 2**

A particle $P$ of mass 2 kg is pushed up a line of greatest slope of a rough plane by a horizontal force of magnitude $X$ newtons, as shown in Figure 2. The force acts in the vertical plane which contains $P$ and a line of greatest slope of the plane. The plane is inclined to the horizontal at an angle $\alpha$, where $\tan\alpha = \frac{3}{4}$




The coefficient of friction between $P$ and the plane is 0.5

Given that the acceleration of $P$ is $1.45\ \mathrm{m\ s^{-2}}$, find the value of $X$.

**(10)**

$\uparrow$ ⟂ equilibrium: $R - X\sin\alpha - 2g\cos\alpha = 0$

$$R = \frac{3X}{5} + 2g\left(\frac{4}{5}\right)$$

$\nearrow$ $[F = ma]$

$$X\cos\alpha - \frac{R}{2} - 2g\sin\alpha = 2(1.45)$$

$$\frac{4X}{5} - \frac{3X}{10} - \frac{4g}{5} - \frac{6g}{5} = 2.9$$

$$X = 45\text{ N}$$

**6.** A uniform rod $AC$, of weight $W$ and length $3l$, rests horizontally on two supports, one at $A$ and one at $B$, where $AB = 2l$. A particle of weight $2W$ is placed on the rod at a distance $x$ from $A$. The rod remains horizontal and in equilibrium.

(a) Find the greatest possible value of $x$.

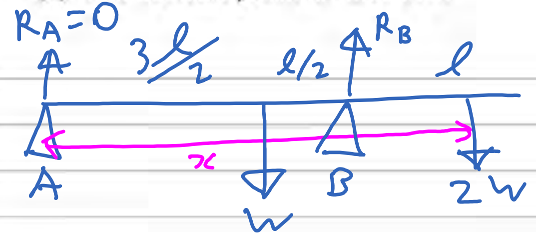
**(5)**

The magnitude of the reaction of the support at $A$ is $R$. Due to a weakness in the support at $A$, the greatest possible value of $R$ is $2W$,

(b) find the least possible value of $x$.

**(5)**

a)




At max $x$, rod is on limit of tilting about B $\Rightarrow R_A = 0$

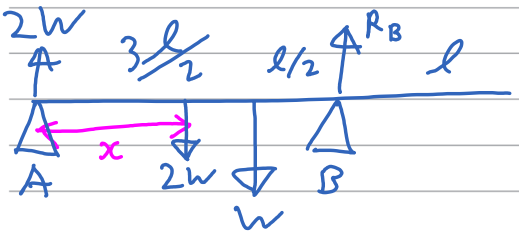
$$A\circlearrowright \quad 2Wx + \frac{3l}{2}W - 2lR_B = 0$$

$$\uparrow \quad R_B = W + 2W$$

$$\Rightarrow 2Wx + \frac{3l}{2}W - 6Wl = 0$$

$$x = \frac{9l}{4}$$

b)




At min $x$, $R = 2W$

$$B\circlearrowright \quad \frac{lW}{2} + (2l - x)2W = 2l \cdot 2W$$

$$\frac{l}{2} + 4l - 2x = 4l$$

$$x = \frac{l}{4}$$

7. A train travels along a straight horizontal track between two stations $A$ and $B$. The train starts from rest at $A$ and moves with constant acceleration until it reaches its maximum speed of 108 km h$^{-1}$. The train then travels at this speed before it moves with constant deceleration coming to rest at $B$. The journey from $A$ to $B$ takes 8 minutes.

(a) Change 108 km h$^{-1}$ into m s$^{-1}$.

**(2)**

(b) Sketch a speed-time graph for the motion of the train between the two stations $A$ and $B$.

**(2)**

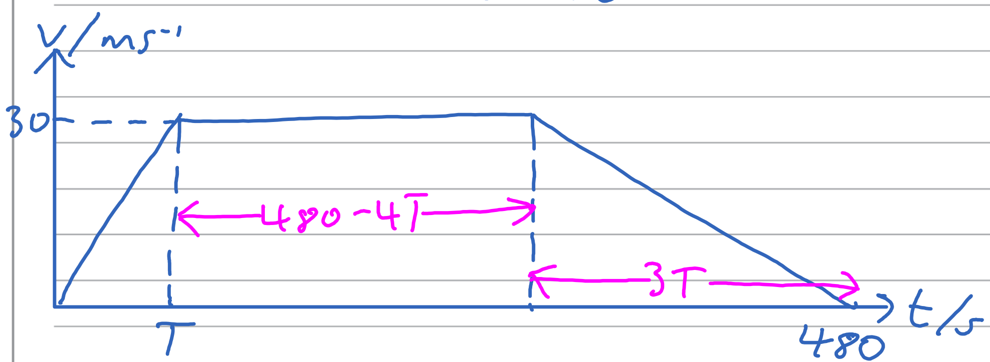
Given that the distance between the two stations is 12 km and that the time spent decelerating is three times the time spent accelerating,

(c) find the acceleration, in m s$^{-2}$, of the train.

**(6)**

a) $108 \text{ kmh}^{-1} = \dfrac{108 \times 1000}{60 \times 60} \text{ ms}^{-1} = 30 \text{ ms}^{-1}$

b)




c) $12000 = \frac{1}{2}(480 - 4T + 480)\,30$

$800 = 960 - 4T$

$T = 40\text{s}$

$a = \dfrac{\Delta v}{\Delta t} = \dfrac{30}{40} = 0.75 \text{ ms}^{-2}$

Leave blank

**8.**

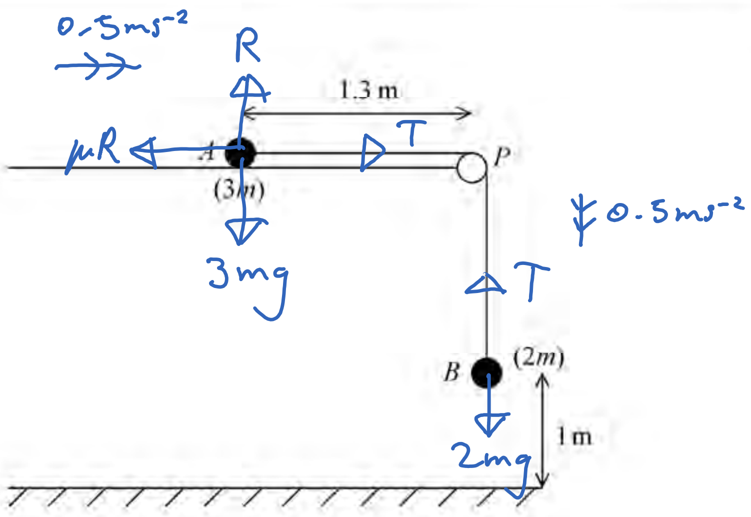



**Figure 3**

A particle $A$ of mass $3m$ is held at rest on a rough horizontal table. The particle is attached to one end of a light inextensible string. The string passes over a small smooth pulley $P$ which is fixed at the edge of the table. The other end of the string is attached to a particle $B$ of mass $2m$, which hangs freely, vertically below $P$. The system is released from rest, with the string taut, when $A$ is 1.3 m from $P$ and $B$ is 1 m above the horizontal floor, as shown in Figure 3.

Given that $B$ hits the floor 2 s after release and does not rebound,

(a) find the acceleration of $A$ during the first two seconds,
**(2)**

(b) find the coefficient of friction between $A$ and the table,
**(8)**

(c) determine whether $A$ reaches the pulley.
**(6)**

a) $\downarrow$ $s = 1\text{m}$ $\quad [s = ut + \frac{1}{2}at^2]$

$u = 0$

$v = x$ $\quad 1 = \frac{1}{2}a(2)^2$

$a = ?$

$t = 2\text{s}$ $\quad a = 0.5\,\text{ms}^{-2}$

b) $B: [F = ma] \downarrow$ $\qquad$ $A: \uparrow \downarrow$ equilibrium

$2mg - T = 2m(0.5)$ $\qquad$ $R = 3mg$

$T = m(2g - 1)$

Leave blank

**Question 8 continued**

A: $\rightarrow [F=ma]$

$T - \mu R = 3m(0.5)$

$m(2g-1) - \mu(3mg) = 1.5m$

$\mu = \dfrac{2g - 2.5}{3g}$

$= 0.582$

c) When A is 0.3m from the pulley, the string becomes slack $\Rightarrow T = 0$

$\uparrow R = 3mg \qquad \rightarrow [F = ma]$

$-\mu R = 3ma$

$a = \dfrac{-0.582(3mg)}{3m}$

$= -5.70 \text{ ms}^{-2}$

Find speed of A when string becomes slack:

$s = 1\text{m}, u = 0, v = ?, a = 0.5\text{ms}^{-2}, t = 2\text{s}$

$v = u + at = 0.5 \times 2 = 1 \text{ms}^{-1}$

Find displacement after string becomes slack:

$[v^2 = u^2 + 2as] \rightarrow$

$s = \dfrac{0 - 1^2}{2(-5.7)} = 0.088\text{m}$ $\quad \therefore$ A does not reach the pulley