


Please check the examination details below before entering your candidate information

Candidate surname					Other names				
Centre Number					Candidate Number				

Pearson Edexcel International GCSE

Time 2 hours Paper reference **4PM1/01R**

Further Pure Mathematics
PAPER 1R



Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ►

P66309A

©2022 Pearson Education Ltd.

L:1/1/1/1/




Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration**Surface area of sphere** $= 4\pi r^2$ **Curved surface area of cone** $= \pi r \times \text{slant height}$ **Volume of sphere** $= \frac{4}{3}\pi r^3$ **Series****Arithmetic series**Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$ **Geometric series**Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$ Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$ **Binomial series**

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad \text{for } |x| < 1, n \in \mathbb{Q}$$

Calculus**Quotient rule (differentiation)**

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Trigonometry**Cosine rule**In triangle ABC : $a^2 = b^2 + c^2 - 2bc \cos A$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

- 1** (a) On the grid below, draw the graph of the line with equation

(i) $4x + 5y = 20$ (ii) $3y - 4x = -12$

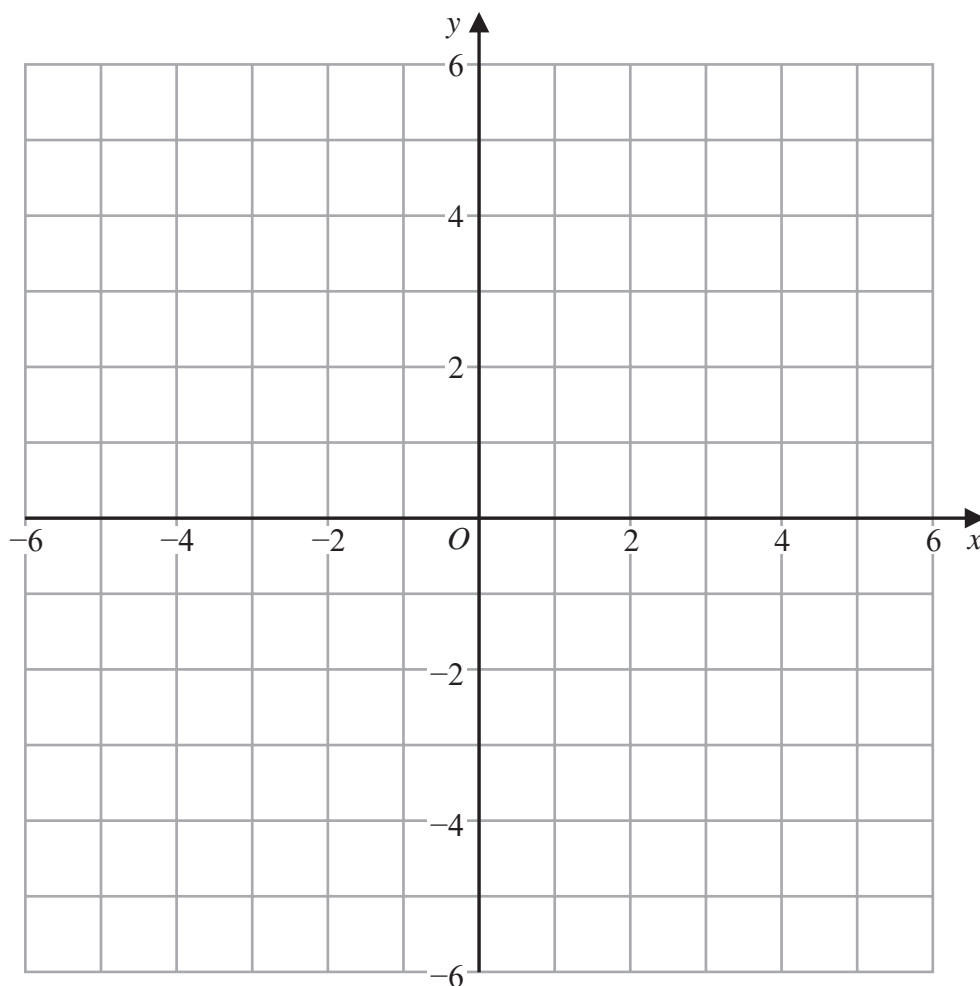
(2)

- (b) Show, by shading on the grid, the region R defined by the inequalities

$4x + 5y \leq 20$ $3y - 4x \geq -12$ $y \leq 3$ $x \geq 1$

Label the region R .

(2)



Turn over for a spare grid if you need to redraw your graph.



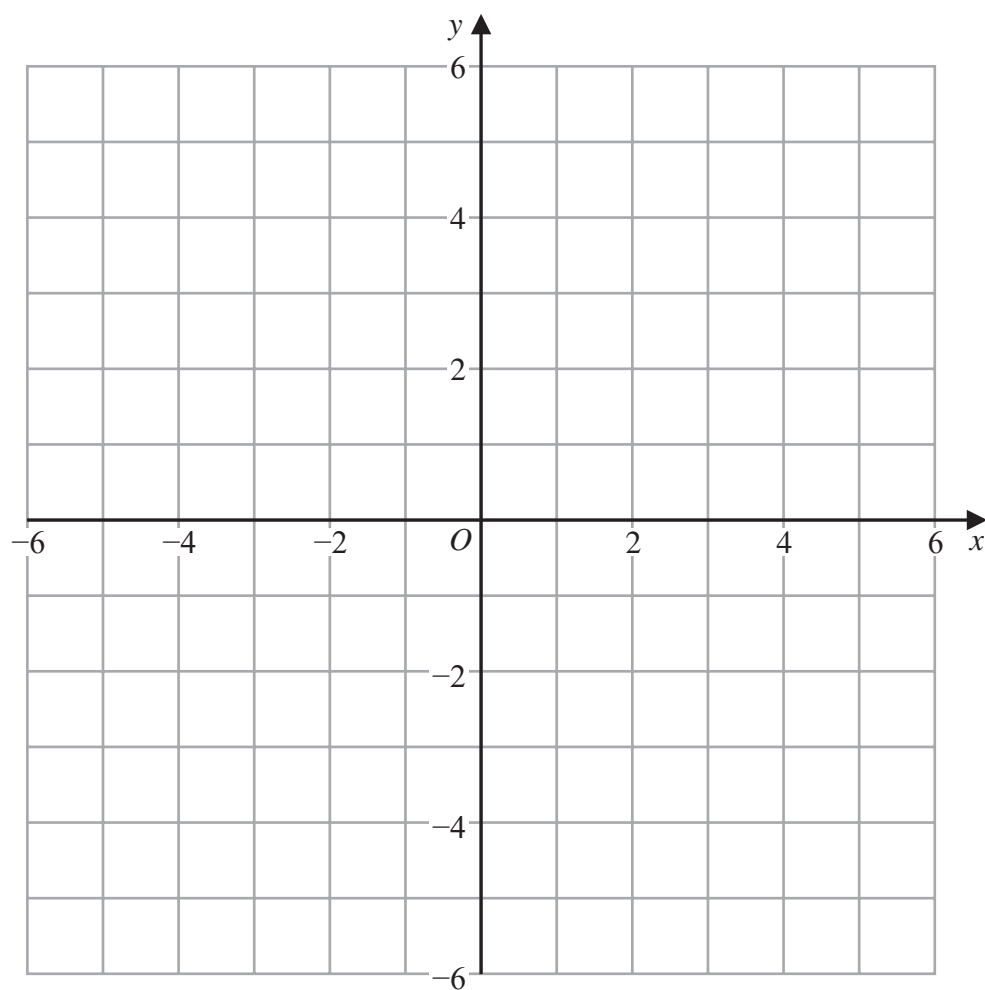
Question 1 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 1 continued**Only use this grid if you need to redraw your graph.**

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(Total for Question 1 is 4 marks)

2 The n th term of an arithmetic series is u_n

Given that $u_5 = 46$ and that $u_{20} = 181$

(a) find

(i) the common difference of this series,

(ii) the first term of this series.

(4)

(b) Evaluate $\sum_{n=21}^{50} u_n$

(4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 2 continued

Handwriting practice area with horizontal dotted lines.

(Total for Question 2 is 8 marks)



- 3 The point A has coordinates $(1, 7)$ and the point B has coordinates $(9, 3)$

The line l is the perpendicular bisector of AB

- (a) Find an equation of l

(5)

The line l crosses the x -axis at the point C

- (b) Find the area of the triangle ABC

(5)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 3 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 3 is 10 marks)



4

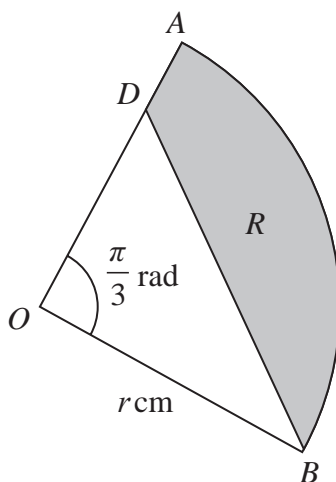
Diagram **NOT**
accurately drawn**Figure 1**

Figure 1 shows sector OAB of a circle with centre O and radius r cm.

The angle $AOB = \frac{\pi}{3}$ radians.

The point D divides OA in the ratio 3 : 1

The area of the region R , shown shaded in Figure 1, is 79.5 cm^2

- (a) Calculate the value, to 2 significant figures, of r (4)
- (b) Calculate the perimeter, in cm to 2 significant figures, of the region R (4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 4 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 4 is 8 marks)

5

$$f(x) = (1 + ax)^n \text{ where } n > 2$$

- (a) Write down the expansion of $f(x)$ in ascending powers of x up to and including the term in x^3

Give each coefficient in terms of n and a

(2)

In the expansion of $f(x)$, the coefficient of x is 8 and the coefficient of x^2 is 30

- (b) Find the value of n and the value of a

(4)

- (c) Calculate the coefficient of x^3 in $f(x)$

(2)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 5 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 5 is 8 marks)

- 6 The points P , Q , R and S are the vertices of a quadrilateral $PQRS$ such that

$$\overrightarrow{PQ} = 2\mathbf{i} + 3\mathbf{j} \quad \overrightarrow{PR} = -\mathbf{i} + 18\mathbf{j} \quad \overrightarrow{PS} = -3\mathbf{i} + 15\mathbf{j}$$

- (a) Show that $PQRS$ is a parallelogram.

(4)

- (b) Find a unit vector parallel to \overrightarrow{QS} as a simplified expression in terms of \mathbf{i} and \mathbf{j}

(4)

The point T lies on QS such that $QT:TS = 5:8$

- (c) Find \overrightarrow{PT} as a simplified expression in terms of \mathbf{i} and \mathbf{j}

(2)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Area for writing answers, consisting of multiple horizontal dotted lines.



P 6 6 3 0 9 A 0 1 5 3 6

Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 6 is 10 marks)



7

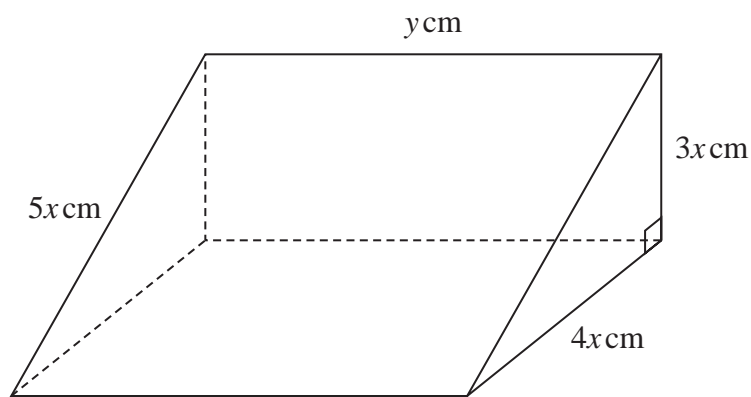
Diagram **NOT**
accurately drawn**Figure 2**

Figure 2 shows a block of wood in the shape of a right triangular prism.

The cross section of the prism is a right-angled triangle with sides of length $3x\text{ cm}$, $4x\text{ cm}$ and $5x\text{ cm}$.

The length of the prism is $y\text{ cm}$.

The total surface area of the five faces of the prism is 144 cm^2

The volume of the prism is $V\text{ cm}^3$

(a) Show that

$$V = 72x - 6x^3 \quad (5)$$

Given that x can vary,

(b) use calculus to find the value of x for which V is a maximum, justifying that this value gives a maximum value of V

(4)

(c) Find the maximum value of V

(2)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Handwriting practice area with horizontal dotted lines.



P 6 6 3 0 9 A 0 1 9 3 6

Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 7 is 11 marks)

P 6 6 3 0 9 A 0 2 1 3 6

8 The curve C has equation $y = 2x^2 - \sin x$

The point A on C has x coordinate π

Show that an equation of the normal to C at the point A is

$$x + (4\pi + 1)y - \pi(8\pi^2 + 2\pi + 1) = 0 \quad (8)$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 8 is 8 marks)



9 (a) Complete the table of values for

$$y = \frac{x^3 + 4}{5 - x}$$

giving your answers to 2 decimal places.

\mathbf{x}	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
\mathbf{y}	-0.57	0.10	0.5		0.8		1.25		4

(2)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(b) On the grid opposite, draw the graph of

$$y = \frac{x^3 + 4}{5 - x} \quad \text{for } -2 \leq x \leq 2$$

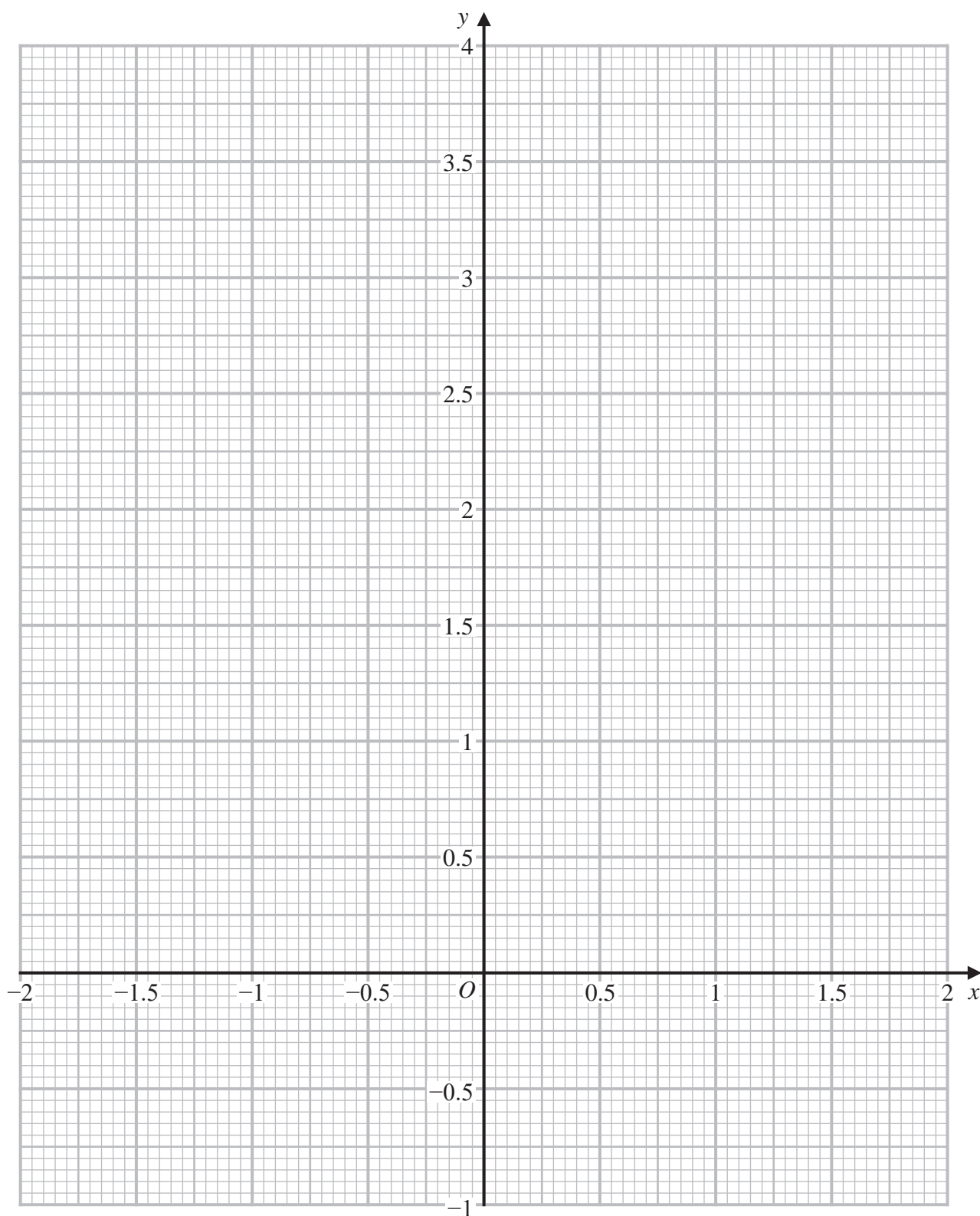
(2)

(c) By drawing a suitable straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation $x^3 - x^2 + 8x - 11 = 0$ in the interval $-2 \leq x \leq 2$

(5)



Question 9 continued



Turn over for a spare grid if you need to redraw your graph.



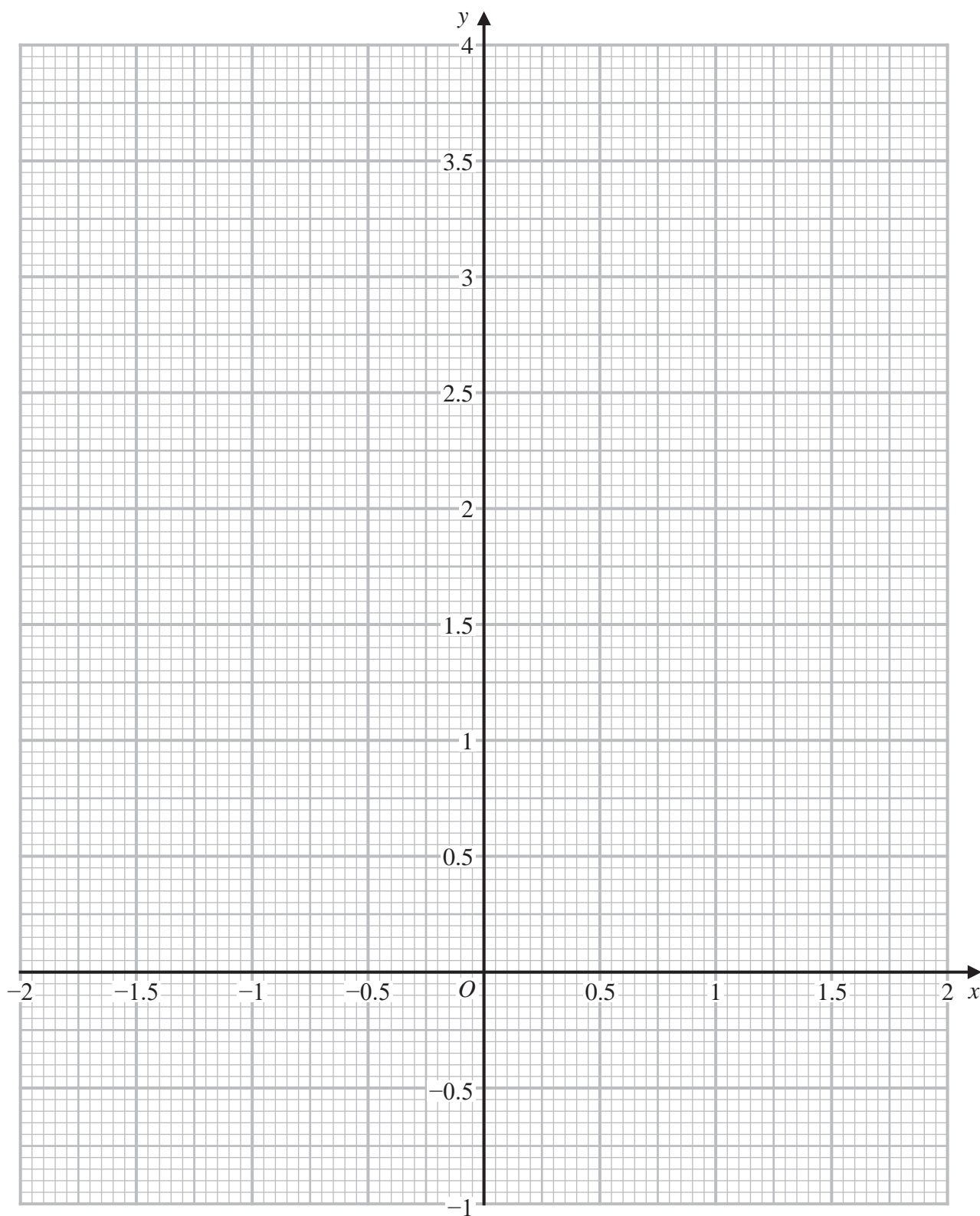
Question 9 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 9 continued**Only use this grid if you need to redraw your graph.****(Total for Question 9 is 9 marks)**

- 10 (a) Use the factor theorem to show that $(4x - 3)$ is a factor of

$$16x^3 + 11x - 15 \quad (2)$$

- (b) Using formulae given on page 2, show that

(i) $\sin 2\theta = 2 \sin \theta \cos \theta$

(ii) $\cos 2\theta = 2 \cos^2 \theta - 1 \quad (5)$

- (c) Show that the equation

$$27 \cos \theta \cos 2\theta + 19 \sin \theta \sin 2\theta - 15 = 0$$

becomes the equation

$$16x^3 + 11x - 15 = 0$$

by using the substitution $x = \cos \theta$

(4)

- (d) Hence show that any solution of the equation

$$27 \cos \theta \cos 2\theta + 19 \sin \theta \sin 2\theta - 15 = 0$$

is given by $\cos \theta = \frac{3}{4}$

(4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 10 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Area for writing answers, consisting of multiple horizontal dotted lines.



P 6 6 3 0 9 A 0 2 9 3 6

Question 10 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 10 continued

DO NOT WRITE IN THIS AREA

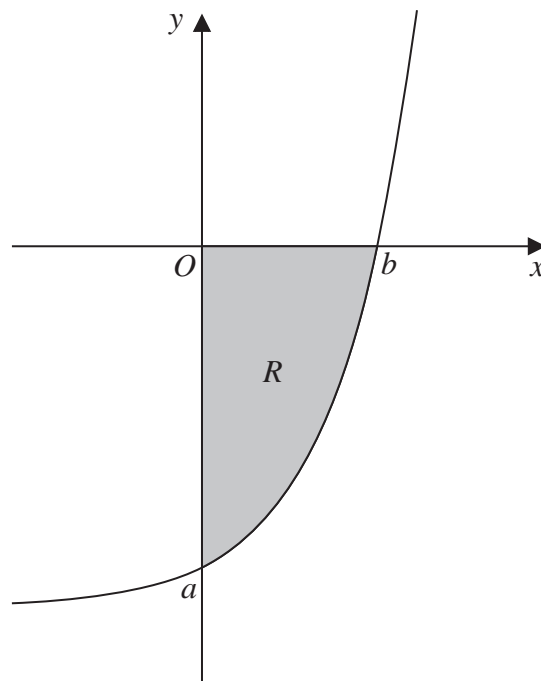
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 10 is 15 marks)

P 6 6 3 0 9 A 0 3 1 3 6

11

Diagram **NOT**
accurately drawn**Figure 3**

The finite region R , shown shaded in Figure 3, is bounded by the curve with equation $y = e^{2x} - 9$ and the coordinate axes.

The curve crosses the coordinate axes at the points with coordinates $(0, a)$ and $(b, 0)$

(a) (i) Find the value of a

(ii) Show that $b = \ln 3$

(3)

The region R is rotated through 360° about the x -axis.

(b) Use calculus to find the volume of the solid generated.

Give your answer in the form $\pi(p \ln 3 + q)$, where p and q are integers.

(6)

.....

.....

.....

.....

.....

.....

.....

.....



Question 11 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Handwriting practice area with horizontal dotted lines.



P 6 6 3 0 9 A 0 3 3 3 6

Question 11 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 11 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Area for writing answers, consisting of multiple horizontal dotted lines.



P 6 6 3 0 9 A 0 3 5 3 6

Question 11 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 11 is 9 marks)

TOTAL FOR PAPER IS 100 MARKS