

Please check the examination details below before entering your candidate information


Candidate surname					Other names				
Centre Number					Candidate Number				

Pearson Edexcel International GCSE

Friday 26 May 2023

Afternoon (Time: 2 hours) **Paper reference 4PM1/01R**

Further Pure Mathematics
PAPER 1R



Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ►

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International GCSE in Further Pure Mathematics Formulae sheet

Mensuration**Surface area of sphere** $= 4\pi r^2$ **Curved surface area of cone** $= \pi r \times \text{slant height}$ **Volume of sphere** $= \frac{4}{3}\pi r^3$ **Series****Arithmetic series**Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$ **Geometric series**Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$ Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$ **Binomial series**

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad \text{for } |x| < 1, n \in \mathbb{Q}$$

Calculus**Quotient rule (differentiation)**

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Trigonometry**Cosine rule**In triangle ABC : $a^2 = b^2 + c^2 - 2bc \cos A$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

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Answer all TEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1

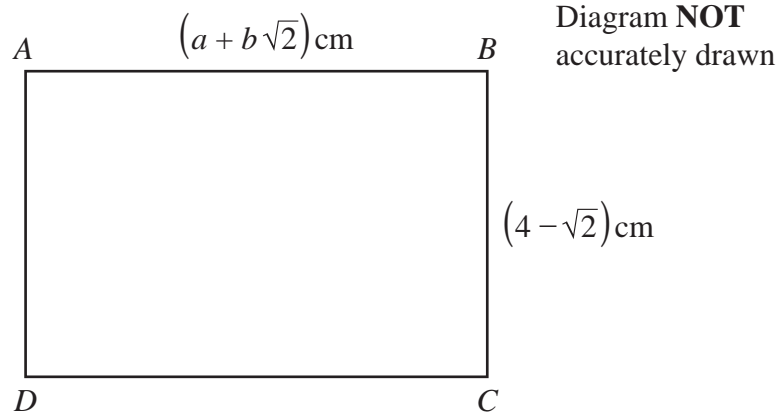


Figure 1

Figure 1 shows the rectangle $ABCD$.

$AD = BC = (4 - \sqrt{2})$ cm and $AB = DC = (a + b\sqrt{2})$ cm where a and b are integers.

The area of the rectangle is $(10 + \sqrt{2})$ cm²

Find the value of a and the value of b

Show your working clearly.

(4)

(Total for Question 1 is 4 marks)



2 One solution to the following simultaneous equations

$$\begin{aligned}y &= px + 9 \\ 6x^2 - xy &= 5\end{aligned}$$

is $\left(-\frac{1}{2}, q\right)$, where p is an integer and q is a prime number.

(a) Find the value of p and the value of q

(4)

(b) Hence find the other solution to the equations.

(4)

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Question 2 continued

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(Total for Question 2 is 8 marks)



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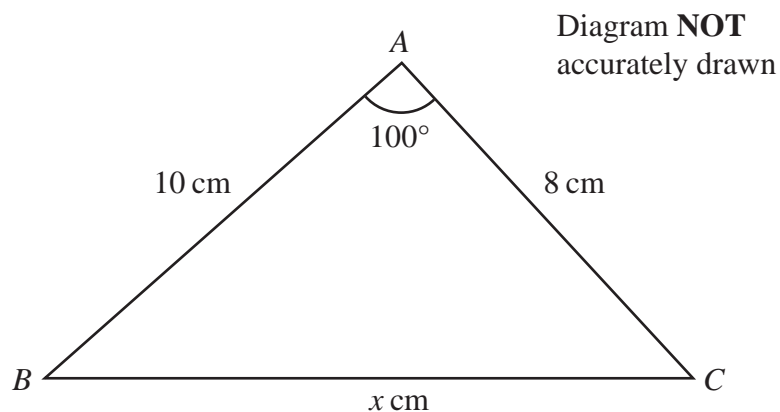


Figure 2

Figure 2 shows triangle ABC where

$$AB = 10 \text{ cm}, \quad AC = 8 \text{ cm}, \quad BC = x \text{ cm} \quad \text{and} \quad \angle BAC = 100^\circ$$

- (a) Find, to 3 significant figures, the value of x (2)
- (b) Find, in degrees to one decimal place, the size of
- (i) angle ABC (ii) angle ACB (3)

The bisector of angle ABC meets AC at the point M

- (c) Find the area, in cm^2 to 3 significant figures, of triangle BMC . (4)



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Question 3 continued

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(Total for Question 3 is 9 marks)



- 4 (a) Complete the table of values for $y = \frac{x}{2} + 6e^{-2x} + 1$
giving your answers to one decimal place.

x	0	1	1.5	2	3	4	5	6
y	7		2.0			3.0		4.0

(2)

- (b) On the grid opposite, draw the graph of $y = \frac{x}{2} + 6e^{-2x} + 1$ for $0 \leq x \leq 6$

(2)

- (c) By drawing a suitable straight line on your graph, obtain estimates, to one decimal place, of the roots of the equation

$$2x + \ln(24 - 5x) = \ln 36$$

(5)

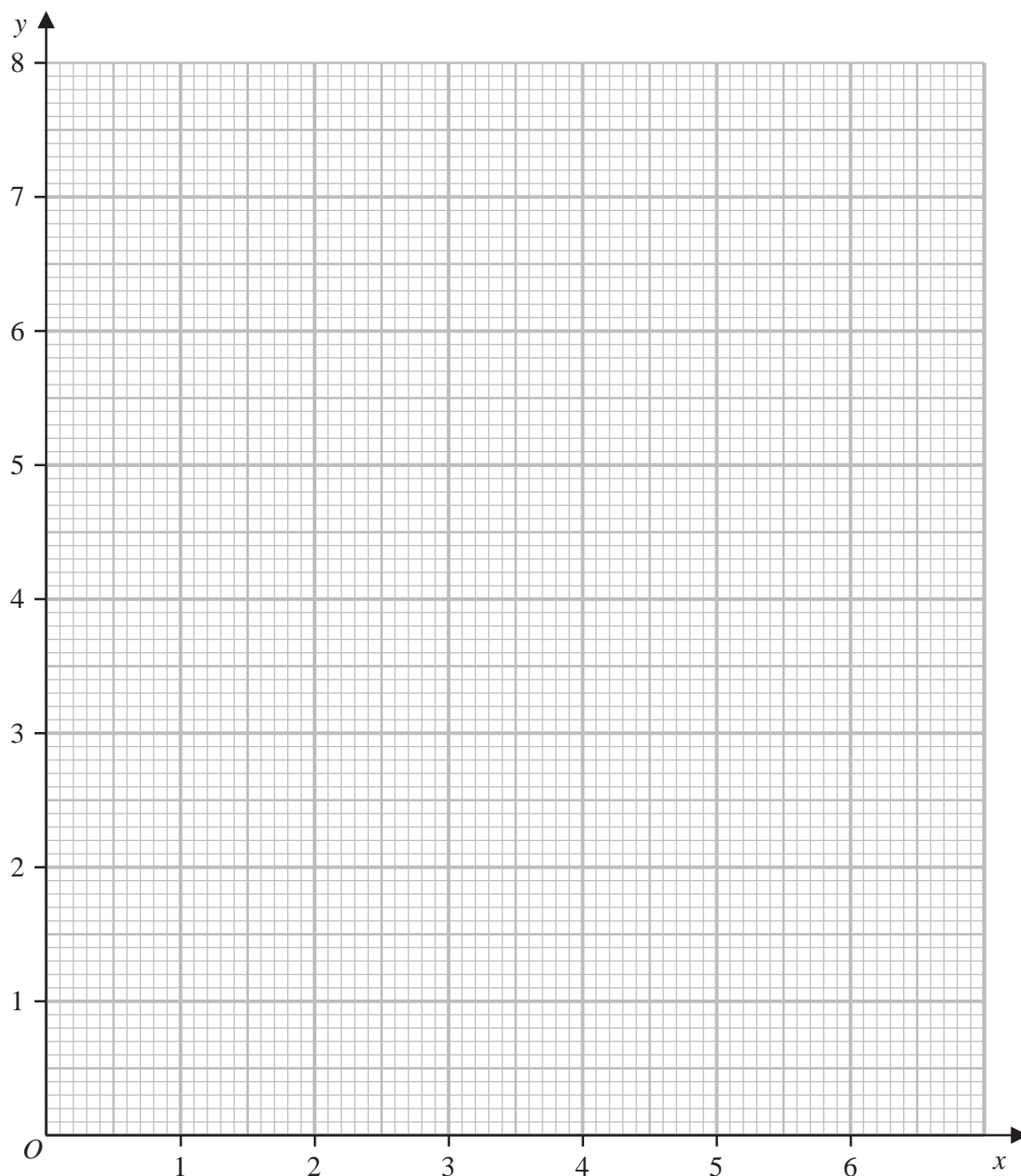
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Question 4 continued



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Turn over for a spare grid if you need to redraw your graph.



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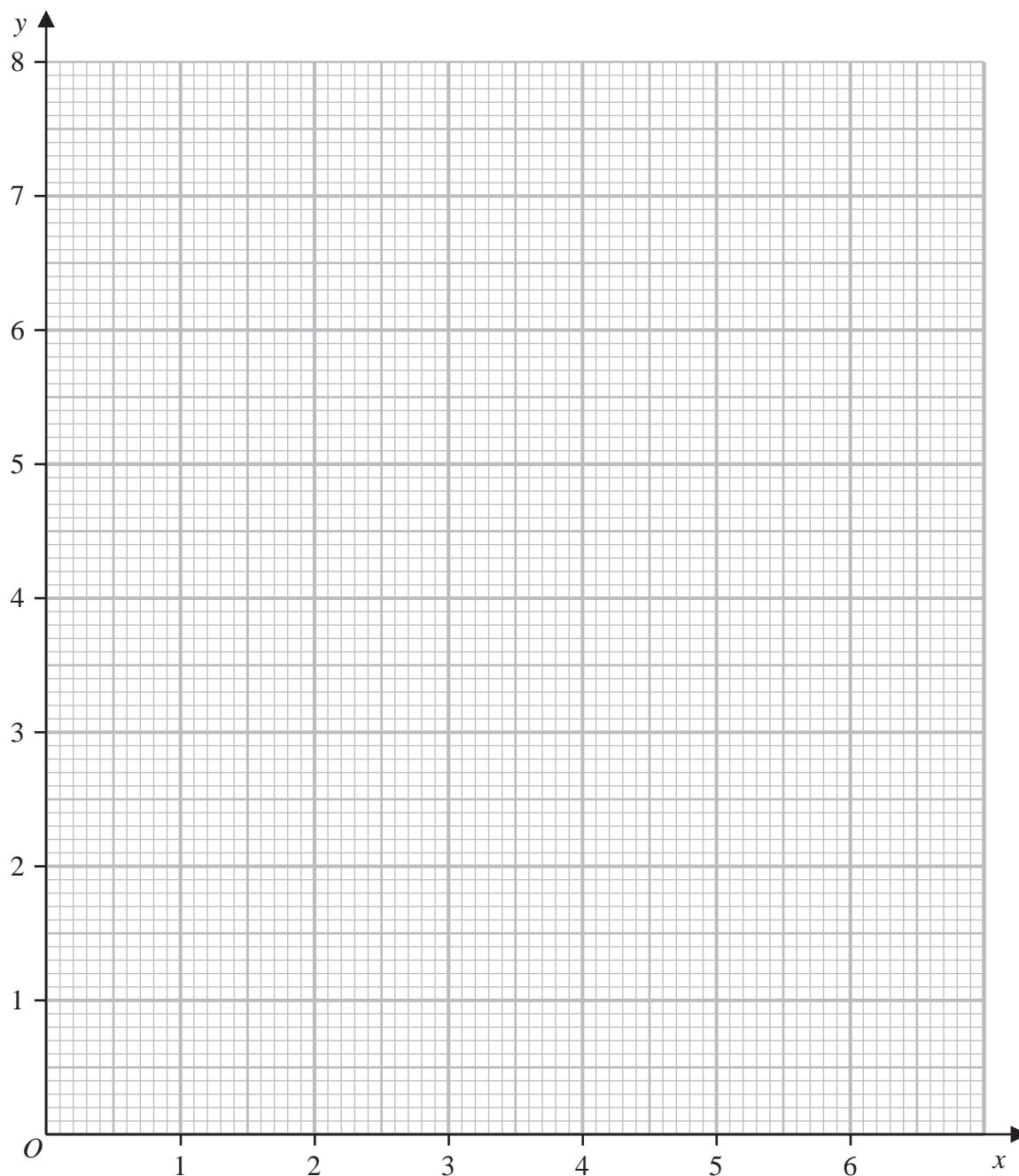
Question 4 continued

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Question 4 continued**Only use this grid if you need to redraw your graph.****(Total for Question 4 is 9 marks)**

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5

$$f(x) = 2x^3 + ax^2 - 14x + b \text{ where } a \text{ and } b \text{ are constants.}$$

When $f(x)$ is divided by $(x - 4)$ the remainder is 39

Given that $(x - 1)$ is a factor of $f(x)$

(a) show that $a = -3$ and find the value of b (5)

(b) Hence factorise $f(x)$ completely. (4)

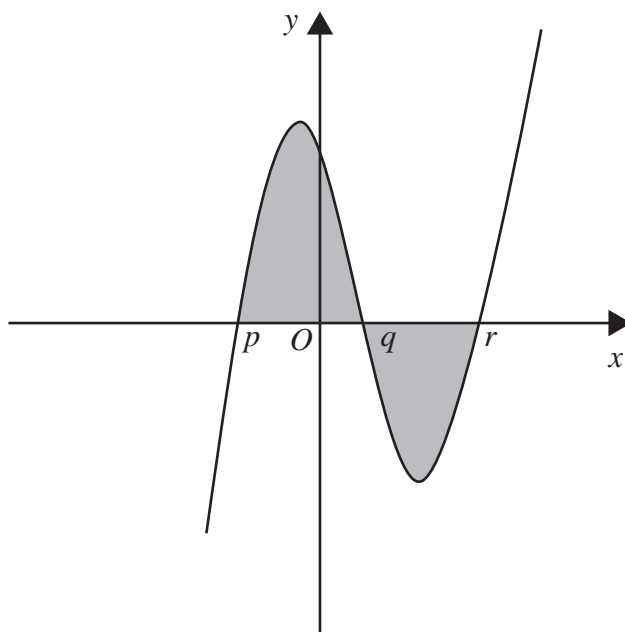


Diagram **NOT**
accurately drawn

Figure 3

Figure 3 shows part of the curve C with equation $y = f(x)$

Given that C crosses the x -axis at the points with coordinates $(p, 0)$, $(q, 0)$ and $(r, 0)$

(c) write down the value of p , the value of q and the value of r (3)

The region shown shaded in Figure 3 is bounded by the curve and the x -axis.

(d) Use algebraic integration to find the exact area of the shaded region. (4)

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Question 5 continued

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Question 5 continued

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Question 5 continued

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(Total for Question 5 is 16 marks)

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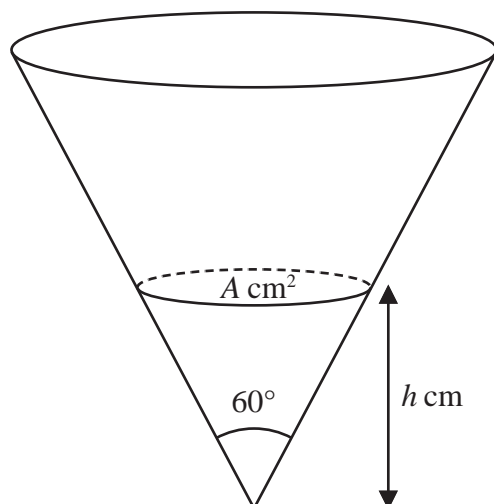
Diagram **NOT**
accurately drawn**Figure 4**

Figure 4 shows a container in the shape of a right circular cone.
The container is fixed with its axis of symmetry vertical.
The vertical angle of the container is 60° as shown in the diagram.

At time t seconds, $t > 0$, the height of oil in the container is h cm and the volume of oil in the container is V cm³

- (a) Show that $V = \frac{1}{9}\pi h^3$ (3)

At time t seconds the surface area of oil in the container is A cm², as shown in Figure 4

Oil is dripping out of the bottom of the container at a constant rate of 4 cm³/s.

- (b) Find the exact rate of change, in cm²/s, of the surface area of oil in the container when $h = 24$ (8)

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Question 6 continued

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Question 6 continued

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(Total for Question 6 is 11 marks)

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- 7 The curve with equation $y = mx^2 + 64\sqrt{x} + 39$ has a stationary point with coordinates $(4, n)$ where m and n are integers.

Using calculus

- (a) find the value of m and the value of n (6)
- (b) determine the nature of the stationary point. (2)

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Question 7 continued

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(Total for Question 7 is 8 marks)

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- 8 The n th term of a geometric series G is U_n and the sum of the first n terms of G is S_n

Given that $U_n = \frac{25}{4} \left(\frac{3}{5} \right)^n$

- (a) find the exact value of U_5

(1)

- (b) Show that $S_n = \sum_{r=1}^n \frac{A}{B} \left(\frac{3}{5} \right)^{r-1}$ where A and B are integers to be found.

(3)

The sum to infinity of G is S

- (c) Find the least value of n such that $S - S_n < 0.045$

(6)

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Question 8 continued

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Question 8 continued

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Question 8 continued

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- 9 (a) Expand $(1 + 2x)^{-\frac{1}{3}}$ in ascending powers of x up to and including the term in x^3 expressing each coefficient as a fraction in its lowest terms. (3)

- (b) Find the range of values of x for which your expansion is valid. (1)

$$f(x) = \frac{2 + kx^2}{(1 + 2x)^{\frac{1}{3}}}$$

- (c) Obtain a series expansion of $f(x)$ in ascending powers of x up to and including the term in x^3 Give your coefficients in terms of k where appropriate. (3)

The coefficient of x^3 in the series expansion of $f(x)$ is $-\frac{8}{3}$

- (d) Find the exact value of k (2)

- (e) Hence, using algebraic integration, estimate the value of

$$\int_{0.1}^{0.2} f(x) \, dx$$

- Give your answer to 4 decimal places. (5)

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Question 9 continued

Handwritten solution for Question 9 continued:

Let $u = \frac{1}{x}$. Then $\frac{du}{dx} = -\frac{1}{x^2}$.

The integral becomes $\int \frac{1}{x^2} \ln x \, dx = \int u \ln u \, du$.

Let $v = \ln u$. Then $\frac{dv}{du} = \frac{1}{u}$.

Using integration by parts, $\int u \ln u \, du = \frac{1}{2} u^2 \ln u - \frac{1}{4} u^2 + C$.

Substituting back, $\frac{1}{2} \frac{1}{x^2} \ln x - \frac{1}{4} \frac{1}{x^2} + C$.

Final answer: $\frac{1}{2x^2} \ln x - \frac{1}{4x^2} + C$.



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Question 9 continued

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Question 9 continued

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(Total for Question 9 is 14 marks)



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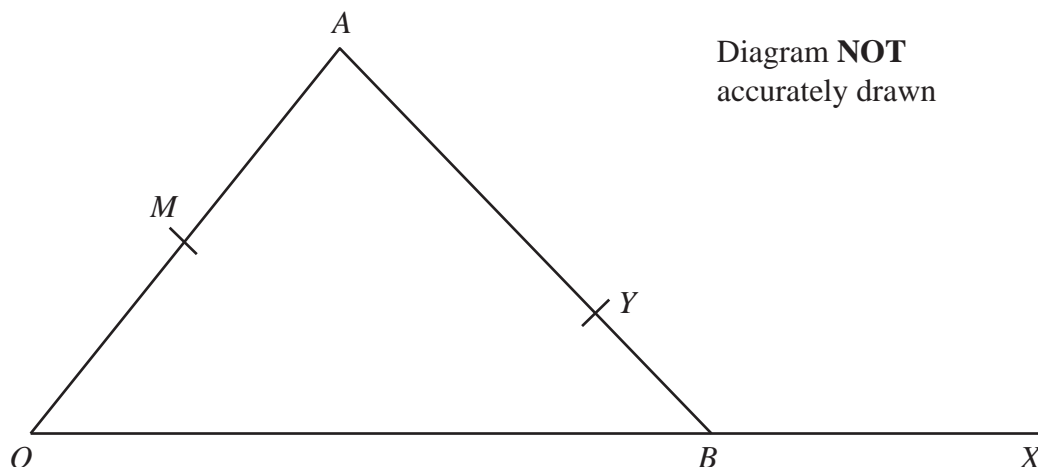


Figure 5

In Figure 5, $\vec{OA} = 2\mathbf{a}$, $\vec{OB} = 4\mathbf{b}$ and M is the midpoint of OA .

The point Y lies on AB such that $AY : YB = 3 : 1$

The point X lies on OB produced.

(a) Find as simplified expressions in terms of \mathbf{a} and \mathbf{b}

- (i) \vec{AB} (ii) \vec{MY}

(3)

The points M , Y and X are collinear.

(b) Find the ratio $OB : OX$

(5)

(c) Find the ratio of $(\text{Area } \triangle YBX) : (\text{Area } \triangle OAX)$

(3)

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Question 10 continued

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Question 10 continued

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(Total for Question 10 is 11 marks)

TOTAL FOR PAPER IS 100 MARKS

