



Mark Scheme (Results)

January 2012

International GCSE Mathematics
(4PM0) Paper 01

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Question	Working	Notes
1	$y = -\frac{6}{4}x - \frac{15}{4}$, gradient = $-\frac{3}{2}$ oe $y = \frac{10}{15}x - \frac{9}{15}$, gradient = $\frac{2}{3}$ oe Product of gradients = $-\frac{3}{2} \times \frac{2}{3} = -1 \Rightarrow$ lines perpendicular	M1 A1 A1 A1 4
2	$x(x+2) - (x+1) = 2(x+1)(x+2)$ $x^2 + x - 1 = 2x^2 + 6x + 4$ $x^2 + 5x + 5 = 0$ $x = \frac{-5 \pm \sqrt{25 - 20}}{2} = -3.62, -1.38$	M1 A1 M1 A1 4
3	$(3x+1)(2x-7) < 0$ $-\frac{1}{3} < x < 3\frac{1}{2}$	M1 A1 M1 A1 4
4	$\frac{10!}{7!3!} 1^3 \left(\frac{1}{\sqrt{3}}\right)^7$ $= 120 \frac{1}{27\sqrt{3}}$ $= 120 \frac{1}{27} \frac{\sqrt{3}}{3}$ $= \frac{40}{27} \sqrt{3}$	Allow all marks if x^7 included. M1 A1 M1 rationalise A1 4
5	(a) $\frac{dy}{dx} = x^2 e^x + 2xe^x$ (b) $\frac{dy}{dx} = 5(x^3 + 2x^2 + 3)^4 (3x^2 + 4x)$	M1 two terms with one correct A1 M1 use chain rule A1 $5(x^3 + 2x^2 + 3)^4$ A1 $(3x^2 + 4x)$ 5

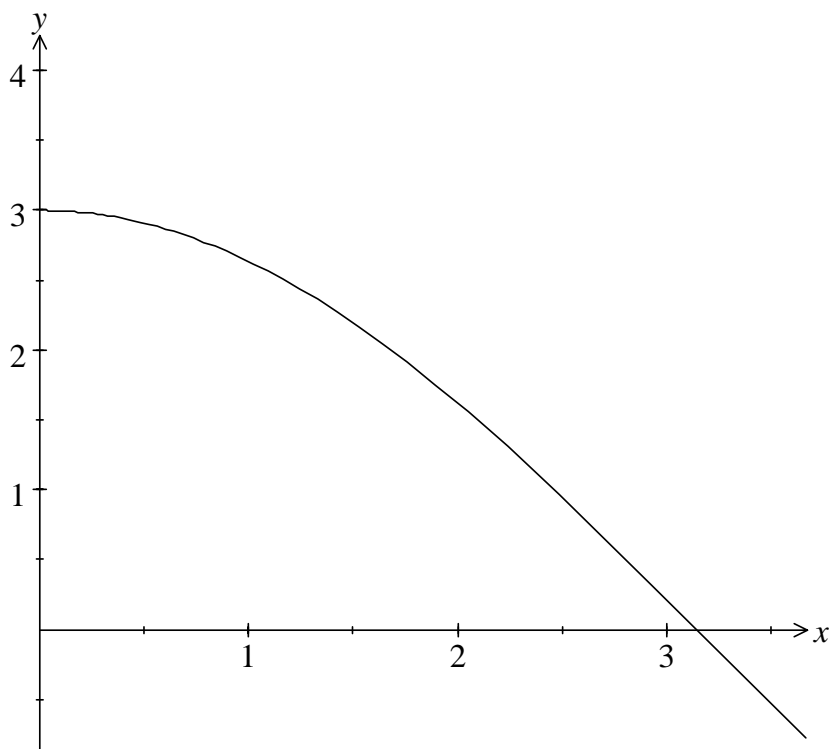
6

(a)

x	0	0.5	1	1.5	2	2.5	3	3.5
y	3	2.91	2.63	2.20	1.62	0.95	0.21	-0.53

B2, 1 (3 correct, 2 correct)

(b)



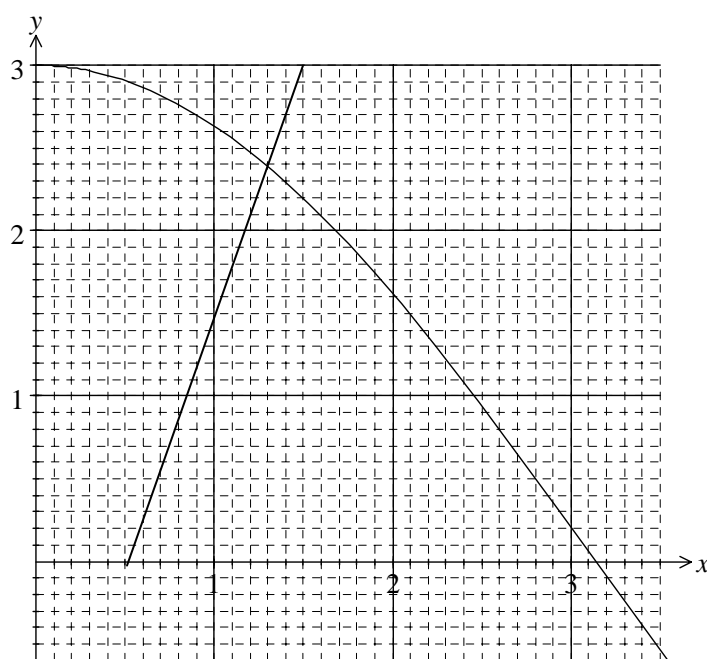
B1 plot points

B1 curve

(c) $2x - 1 = 2 \cos(x/2)$
 $3x - 1\frac{1}{2} = 3 \cos(x/2)$
 $y = 3x - 1\frac{1}{2}$

M1 rearrange

A1

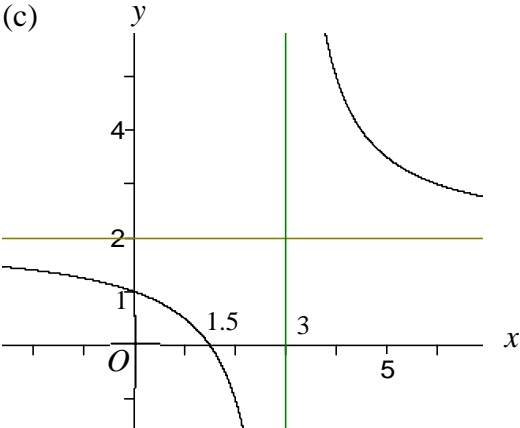


M1 draw their line

A1 1.2, 1.3 or 1.4

 $x = 1.3$

8

7	<p>(a) $A(1\frac{1}{2}, 0), B(0, 1)$</p> <p>(b) (i) $x = 3$ (ii) $y = 2$</p> <p>(c) </p> <p>(d) $\frac{dy}{dx} = \frac{2(x-3) - (2x-3)}{(x-3)^2} = \frac{-3}{(x-3)^2}$ At $B, x = 0$ so $\frac{dy}{dx} = \frac{-3}{(-3)^2} = -\frac{1}{3}$ Grad of normal $= -1/(-1/3) = 3$ Normal $y = 3x + 1$</p> <p>(e) At $D, 3x + 1 = \frac{2x-3}{x-3}$ $3x^2 - 8x - 3 = 2x - 3$ $3x^2 - 10x = 0$ $x(3x - 10) = 0$ $x = 0$ or $x = 10/3$ At $D, x = 3\frac{1}{3}$</p>	<p>B1, B1</p> <p>B1 B1</p> <p>B1 two branches in correct quadrants B1 asymptotes dep on some curve B1 intercepts</p> <p>M1 Quotient rule A1 Result (unsimplified)</p> <p>A1</p> <p>B1ft B1ft</p> <p>M1</p> <p>A1 M1</p> <p>A1 16</p>
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8	<p>(a) $k = \alpha/\beta \times \beta/\alpha = 1$</p> <p>(b) $\alpha\beta = 15$ and $\alpha + \beta = -m$ $-h = \alpha/\beta + \beta/\alpha$ $= \frac{\alpha^2 + \beta^2}{\alpha\beta}$ $= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\beta\alpha}$ $\Rightarrow h = \frac{30 - m^2}{15}$</p> <p>(c) $\alpha\beta = 15 \Rightarrow \alpha(2\alpha + 1) = 15$ $2\alpha^2 + \alpha - 15 = 0$ $(2\alpha - 5)(\alpha + 3) = 0$ $\alpha = 2\frac{1}{2}$ or $\alpha = -3$</p> <p>(d) $\beta = 2 \times 2\frac{1}{2} + 1 = 6$ or $\beta = 2 \times -3 + 1 = -5$ $m = -(\alpha + \beta) = -(2\frac{1}{2} + 6)$ or $-(-3 - 5)$ $m = -8\frac{1}{2}$ or 8</p>	<p>B1</p> <p>M1 A1 M1</p> <p>M1</p> <p>M1</p> <p>A1 oe</p> <p>M1</p> <p>M1 A1</p> <p>M1 M1 A1 13</p>
9	<p>(a) $BD^2 = 5^2 + 6^2 = 61$, $BC^2 = 8^2 + 6^2 = 100$, $CD^2 = 8^2 + 5^2 = 89$ $100 = 61 + 89 - 2\sqrt{61}\sqrt{89}\cos BDC$ $\cos BDC = 25/\sqrt{(61 \times 89)}$ $= 0.3393$ $\angle BDC = 70.2^\circ$</p> <p>(b) Area $BDC = \frac{1}{2}\sqrt{61}\sqrt{89}\sin 70.2^\circ$ $= 34.7 \text{ cm}^2$ (3sf)</p> <p>(c) Area $DAC = \frac{1}{2} \times 5 \times 8 = 20$</p> <p>(d) $20 = \frac{1}{2} \times \sqrt{89} \times AE \Rightarrow AE = 40/\sqrt{89}$</p> <p>(e) Angle is $\angle BEA$ $\tan BEA = 6/AE = 6\sqrt{89}/40$ $= 1.415$ $\Rightarrow \angle BEA = 54.8^\circ$</p>	<p>M1 A2, 1, 0 M1 A1</p> <p>A1</p> <p>M1 A1ft A1 allow 34.6</p> <p>B1</p> <p>M1 A1</p> <p>M1 identify angle M1 A1ft</p> <p>A1 16</p>

10	<p>(a) (i) $\overrightarrow{BC} = -\frac{1}{2}\mathbf{c} - \mathbf{a} + \mathbf{c} = \frac{1}{2}\mathbf{c} - \mathbf{a}$</p> <p>(ii) $\overrightarrow{PQ} = \frac{3}{4}\mathbf{a} + \frac{1}{2}\mathbf{c} + \frac{1}{3}(\frac{1}{2}\mathbf{c} - \mathbf{a}) = \frac{5}{12}\mathbf{a} + \frac{2}{3}\mathbf{c}$</p> <p>(b) (i) $\overrightarrow{AT} = -\frac{3}{4}\mathbf{a} + \lambda(\frac{5}{12}\mathbf{a} + \frac{2}{3}\mathbf{c})$</p> <p>(ii) $\overrightarrow{AT} = \mu(\mathbf{c} - \mathbf{a})$</p> <p>(c) $-\frac{3}{4}\mathbf{a} + \lambda(\frac{5}{12}\mathbf{a} + \frac{2}{3}\mathbf{c}) = \mu(\mathbf{c} - \mathbf{a})$ $\Rightarrow -\frac{3}{4} + \frac{5}{12}\lambda = -\mu$ and $\frac{2}{3}\lambda = \mu$ $\Rightarrow \frac{5}{12}\lambda = \frac{3}{4} - \frac{2}{3}\lambda$ $\Rightarrow 5\lambda = 9 - 8\lambda$ $\Rightarrow \lambda = \frac{9}{13}$ $\Rightarrow PT:TQ = 9:4$</p>	<p>M1 A1</p> <p>M1 $\frac{3}{4}\mathbf{a} + \frac{1}{2}\mathbf{c} + \dots$ M1 $\frac{1}{3}(\frac{1}{2}\mathbf{c} - \mathbf{a})$ A1 B1ft</p> <p>B1</p> <p>M1 M1 A1ft M1</p> <p>A1 A1ft</p> <p>13</p>
11	<p>(a)</p> $V = \pi \int_0^h x^2 dy = \pi \int_0^h (10y - y^2) dy$ $= \pi \left[5y^2 - \frac{1}{3}y^3 \right]_0^h$ $= \pi \left[5h^2 - \frac{1}{3}h^3 \right]$ $= \frac{1}{3}\pi h^2(15 - h)$ <p>(b) $V = \pi(5h^2 - \frac{1}{3}h^3) \Rightarrow \frac{dV}{dh} = \pi(10h - h^2)$</p> <p>(c) $\frac{dV}{dt} = \pi(10h - h^2) \frac{dh}{dt}$ When $h=1.5$, $6 = \pi(15 - 2.25) \frac{dh}{dt}$ $\Rightarrow \frac{dh}{dt} = 6/(12.75\pi) = 0.150 \text{ cm/s (3sf)}$</p> <p>(d) $W = \pi x^2 = \pi(10y - y^2)$ When depth is h, $W = \pi(10h - h^2)$ $\frac{dV}{dt} = \pi(10h - h^2) \frac{dh}{dt} = W \frac{dh}{dt}$ Since $\frac{dV}{dt} = 6$, $\frac{dh}{dt} = 6/W$ so $k = 6$</p>	<p>M1 use of $\int \pi x^2 dy$</p> <p>M1 A1 integration</p> <p>M1 use of correct limits A1 cso</p> <p>B1 oe</p> <p>M1 chain rule</p> <p>M1 A1 substitution A1 cao</p> <p>B1</p> <p>M1 A1</p> <p>13</p>