Question number	Scheme	Marks
10 (a)(i)	$a+3ar=8$	B1
	$ar\times ar^2=4ar^4 \Rightarrow (a=4r)$	B1
	Solves simultaneous equations $4r(1+3r)=8 \Rightarrow 12r^2+4r-8=0 \Rightarrow (3r-2)(r+1)=0$	M1
	$\Rightarrow r=\frac{2}{3}\ (r=-1)$	A1
(ii)	$a=4\times\frac{2}{3}=\frac{8}{3}$	A1 (5)
(b)	$U_n=\frac{8}{3}\times\left(\frac{2}{3}\right)^{n-1} \Rightarrow U_n=\frac{2^3\times 2^{n-1}}{3\times 3^{n-1}}=\frac{2^{n+2}}{3^n}$ *	M1A1cso (2)
(c)	$U_n<0.05 \Rightarrow \frac{2^{n+2}}{3^n}<0.05 \quad \left(\Rightarrow \left(\frac{2}{3}\right)^n\times 4<0.05\right)$	M1
	$\Rightarrow n>\log_{\left(\frac{2}{3}\right)}\frac{0.05}{4} \Rightarrow n>10.807... \Rightarrow n=11$	dM1A1cao (3) [10]
	ALT $\frac{8}{3}\left(\frac{2}{3}\right)^{n-1}=\frac{2^{n+2}}{3^n}=\left(\frac{2}{3}\right)^n\times 4<\frac{1}{20}$	{ M1
	So $\left(\frac{2}{3}\right)^n<\frac{1}{80}$ or $\left(\frac{3}{2}\right)^n=(1.5)^n>80$	dM1
	Leading to $n>\frac{\log 80}{\log 1.5}=10.8...\ \ n=11$	A1cao}(3)

(a)	
(i) B1	For $a+3ar=8$
B1	For $ar\times ar^2=4ar^4$
M1	Solving the simultaneous equations by any valid method. Must get to $r=...$ or $a=...$ Must solve a 3TQ by the usual rules
A1	Correct value for r. $r=-1$ need not be seen, but if shown it must be eliminated or made clear that $r=\frac{2}{3}$ is the only correct answer by eg underlining
(ii) A1	$a=\frac{8}{3}$
(b)	
M1	Use the correct formula for the nth term with their r and a
A1cso	Simplify to the correct **given** result, no errors in the work. Must see 8 changed to 2^3

(c)	
M1	Use the result in (b) to form an inequality or equation ALT: use the formula for the *n*th term
dM1	Attempt to solve their inequality, using logs (any base) or trial and error. Log work must be correct for their inequality or equation. If an equation is used the values of *n* either side of their answer must be tested before this mark can be awarded. Depends on first M mark of (c)
A1cao	Correct answer ($n = 11$) from correct working. Trial and error can be done on a calculator, so correct answer may get M1dM1A1

Question number	Scheme	Marks
11 (a) (i)	$\cos 2x = \cos^2 x - \sin^2 x$ $\cos 2x = 1 - \sin^2 x - \sin^2 x = 1 - 2\sin^2 x$ *	M1 M1A1cso
(ii)	$\dfrac{13\sin x - 2\cos 2x - 10}{(4\sin x - 3)} = \dfrac{13\sin x - 2(1 - 2\sin^2 x) - 12}{(4\sin x - 3)}$	M1
	$\Rightarrow \dfrac{4\sin^2 x + 13\sin x - 12}{(4\sin x - 3)} = \dfrac{(4\sin x - 3)(\sin x + 4)}{(4\sin x - 3)} = \sin x + 4$	M1M1A1 cso (7)
(b)	Let $A = \left(\theta + \dfrac{\pi}{6}\right)$ in either method:	
ALT 1	Uses (a) (i): $10 + 2\cos 2A - 13\sin A = 2\sin A + 8$ $2(1 - 2\sin^2 A) - 15\sin A + 2 = 0$	
	$4\sin^2 A + 15\sin A - 4 = 0$	M1
	$(4\sin A - 1)(\sin A + 4) = 0$	
	$\sin A = \dfrac{1}{4}$ $(\sin A = -4$ not poss$)$	dM1
	$\left(\theta + \dfrac{\pi}{6}\right) = 0.252680..., 2.888912..., 6.535865...$	ddM1A1
	$\theta = 6.01$	A1 (5)
ALT 2	Uses (a) (ii): $10 + 2\cos 2A - 13\sin A = 2(\sin A + 4) \Rightarrow \dfrac{13\sin A - 2\cos 2A - 10}{\sin A + 4} = -2$ M1 $\Rightarrow 4\sin A - 3 = -2 \Rightarrow \sin A = \dfrac{1}{4}$ dM1 $\sin\left(\theta + \dfrac{\pi}{6}\right) = \dfrac{1}{4} \Rightarrow \left(\theta + \dfrac{\pi}{6}\right) = 0.252680..., 2.888912..., 6.535865...$ ddM1A1 $\theta = 6.01$ A1cao (5)	

(c)	$\int_0^{\frac{\pi}{2}} \frac{13\sin x - 2\cos 2x - 10 + 4x\sin x - 3x}{4\sin x - 3}\,\mathrm{d}x = \int_0^{\frac{\pi}{2}} \frac{13\sin x - 2\cos 2x - 10}{4\sin x - 3} + \frac{x(4\sin x - 3)}{4\sin x - 3}\,\mathrm{d}x$ $\Rightarrow \int_0^{\frac{\pi}{2}} \frac{13\sin x - 2\cos 2x - 10}{4\sin x - 3} + x\,\mathrm{d}x = \int_0^{\frac{\pi}{2}} \sin x + 4 + x\,\mathrm{d}x$ $\int_0^{\frac{\pi}{2}} \sin x + 4 + x\,\mathrm{d}x = \left[-\cos x + 4x + \frac{x^2}{2}\right]_0^{\frac{\pi}{2}}$ $\int_0^{\frac{\pi}{2}} \sin x + 4 + x\,\mathrm{d}x = \left(0 + 2\pi + \frac{\pi^2}{8}\right) - (-1) = 2\pi + \frac{\pi^2}{8} + 1$ oe	M1A1 dM1 ddM1A1cao (5) [17]

(a)	
(i)M1	Set $A = B = x$ in the given identity Allow with x or any other single variable. Can have $x + x$ or $2x$
M1	Use $\cos^2 x + \sin^2 x = 1$ to eliminate $\cos^2 x$ Allow with x or any other single variable and $x + x$ or $2x$
A1cso	Obtain the **given** result with no errors in the working. The variable must be x now and $x + x$ must have become $2x$
(ii)M1	Use the result given in (a) to eliminate $\cos 2x$
M1	Simplify the numerator to a 3TQ
M1	Factorise the numerator. Correct factorisation implies the previous M mark.
	These M marks can be awarded for work on the numerator alone – award if denominator not seen yet.
A1cso	Obtain the **given** result with no errors in the working. **Must** have seen the denominator for evidence of the cancellation.
(b)	
ALT 1	
M1	Use (a) (i) to obtain a quadratic in sin A. Terms in any order. (Can be done w/o the substitution.)
dM1	Solve their 3TQ and reach sin A = ... sin A = - 4 need not be seen. Depends on the first M mark.
ddM1	Obtain at least one value for $\left(\theta + \frac{\pi}{6}\right)$ or $(\theta + 30°)$ (Need not be the one to give a final answer in the required range). Depends on both M marks above.
A1	For $\theta + \frac{\pi}{6} = 6.535\ldots$
A1	For $\theta = 6.01$ Ignore answers outside the range, extras inside score A0. If final answer is in degrees, both A marks are lost. If degrees are changed to radians both A marks are available even if penultimate answer is in degrees.

ALT 2	
M1	Set $\theta+\frac{\pi}{6}=A$ in the given equation and rearrange to the expression shown. (Can be done w/o the substitution.)
dM1	Use the identity from (a) (ii) to obtain a value for $\sin A$ or $\sin\left(\theta+\frac{\pi}{6}\right)$ or $(\theta+30°)$ Depends on the first M mark.
ddM1	Obtain at least one value for $\left(\theta+\frac{\pi}{6}\right)$ (Need not be the one to give a final answer in the required range)
A1	For $\theta+\frac{\pi}{6}=6.535...$
A1cao	For $\theta=6.01$ Ignore answers outside the range, extras inside score A0. If final answer is in degrees, both A marks are lost. If degrees are changed to radians both A marks are available even if penultimate answer is in degrees.
NB	If compound angle formulae used – send to review.
(c)	
M1	Use the identity from (a) (ii) to simplify the integrand from the given function. Must **not** ignore $4x\sin x-3x$ so $\int(4+\sin x)\mathrm{d}x$ scores M0
A1	Correct changed integrand.
dM1	Attempt the integration. $x\to\frac{x^2}{k}$, $k=1$ or 2 **and** $\sin x\to\pm\cos x$ Depends on first M mark of (c)
ddM1	Substitute the given limits. Depends on both M marks of (c)
A1cao	For $2\pi+\frac{\pi^2}{8}+1$ Must be exact but any equivalent accepted provided the trig functions have been replaced with their numerical values.
	Decimal answer, 8.516...may score 4/5 but w/o working implies from a calculator and scores 0/5

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