Please check the examination details below I	before entering your candidate information
Candidate surname	Other names
Pearson Edexcel Interna	
Friday 7 June 2024	
	Paper 4PM1/02
Further Pure Mather Paper 2	ematics
Calculators may be used.	Total Marks

## **Instructions**

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  - there may be more space than you need.
- You must NOT write anything on the formulae page.
   Anything you write on the formulae page will gain NO credit.

## Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  - use this as a guide as to how much time to spend on each question.

## **Advice**

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶





## **International GCSE in Further Pure Mathematics Formulae sheet**

#### Mensuration

**Surface area of sphere** =  $4\pi r^2$ 

**Curved surface area of cone** =  $\pi r \times \text{slant height}$ 

**Volume of sphere** = 
$$\frac{4}{3}\pi r^3$$

#### **Series**

## **Arithmetic series**

Sum to *n* terms, 
$$S_n = \frac{n}{2} [2a + (n-1)d]$$

## Geometric series

Sum to *n* terms, 
$$S_n = \frac{a(1-r^n)}{(1-r)}$$

Sum to infinity, 
$$S_{\infty} = \frac{a}{1-r} |r| < 1$$

## **Binomial series**

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots$$
 for  $|x| < 1, n \in \mathbb{Q}$ 

#### **Calculus**

## **Quotient rule (differentiation)**

$$\frac{\mathrm{d}}{\mathrm{d}x} \left( \frac{\mathrm{f}(x)}{\mathrm{g}(x)} \right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{\left[\mathrm{g}(x)\right]^2}$$

## **Trigonometry**

#### Cosine rule

In triangle *ABC*:  $a^2 = b^2 + c^2 - 2bc \cos A$ 

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

## Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$



# Answer all ELEVEN questions.

# Write your answers in the spaces provided.

# You must write down all the stages in your working.

 $f(x) = 6x^3 - 13x^2 + ax - 10 \text{ where } a \text{ is a constant}$ 1

Given that (3x - 2) is a factor of f(x)

(a) show that a = 21

**(2)** 

(b)	Hence show	algebraically t	that the curve	y = f(x)	has only	one intersection	n with the
	x-axis.						

**(4)** 



(Total for Question 1 is 6 marks)

2 The quadratic equation $3x^2 - 5x + 1 = 0$ has roots $\alpha$ and $\beta$									
	Without solving the equation,								
	form a quadratic equation with integer coefficients, that has roots $\frac{\alpha}{2\beta}$ and $\frac{\beta}{2\alpha}$	(8)							

Question 2 continued
(Total for Oreation 2 in 9 months)
(Total for Question 2 is 8 marks)



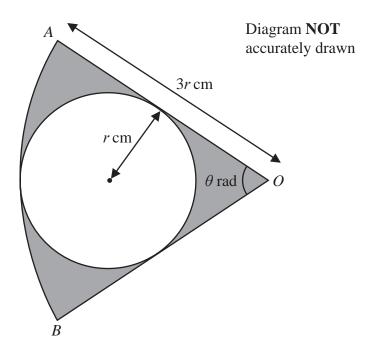


Figure 1

Figure 1 shows the sector AOB of a circle with centre O and radius 3r cm

A circle with radius r cm touches OA and OB and the arc AB

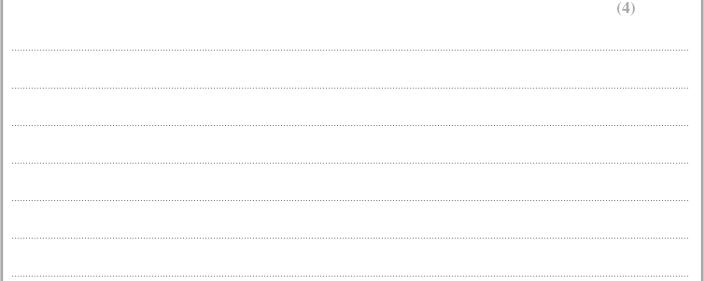
Angle *AOB* is  $\theta$  radians, where  $0 < \theta < \frac{\pi}{2}$ 

(a) Find the exact value of  $\theta$ 

(2)

The area of the region shown shaded in Figure 1 is  $\,8\pi$  cm<sup>2</sup>

(b) Find the value of r



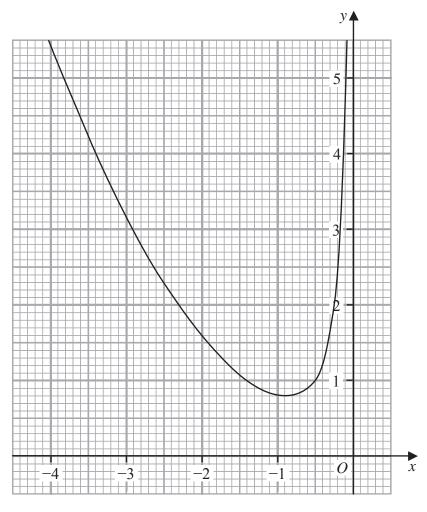


Figure 2

Figure 2 shows part of the curve with equation  $y = \frac{x^2}{3} - \frac{1}{2x}$  for -4 < x < 0By drawing a suitable straight line on the grid, obtain estimates, to one decimal place, of the roots of the equation  $4x^3 + 3x^2 - 36x - 6 = 0$  in the interval -4 < x < 0

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Question 4 continued	
(Total for Questi	ion 4 is 4 marks)



$$y = e^{2x} \left( x^2 - 5x \right)$$

Show that 
$$2e^{2x} = \frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y$$

(7)


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Question 5 continued
(Total for Question 5 is 7 marks)



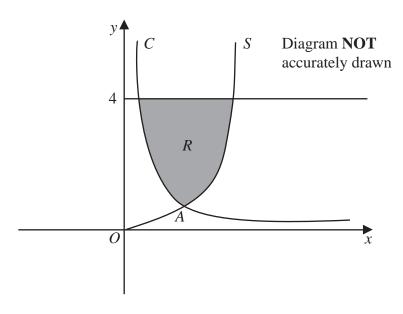


Figure 3

Figure 3 shows part of the curve C with equation  $y = \frac{1}{4x}$ , x > 0 and part of the curve S with equation  $y = 2x^2$ ,  $x \ge 0$ 

The curve C and the curve S intersect at the point A

(a) Find the coordinates of point A

(3)

The finite region R, shown shaded in Figure 3, bounded by the curve C, the curve S and the straight line y = 4 is rotated through 360° about the y-axis.

(b) Find, using algebraic integration, the exact volume of the solid formed.

(7)


Question 6 continued
(Total for Question 6 is 10 marks)



7 (a) Expand  $(1+2x^2)^{-\frac{3}{4}}$  in ascending powers of x up to and including the term in  $x^6$  Express each coefficient as an exact fraction in its lowest terms.

(3)

$$f(x) = \frac{(2+kx)}{(1+2x^2)^{\frac{3}{4}}}$$
 where  $k \neq 0$ 

(b) Obtain a series expansion for f(x) in ascending powers of x up to and including the term in  $x^5$ 

Give each coefficient in terms of k where appropriate.

(2)

The coefficient of the term in  $x^5$  is fourteen times the coefficient of the term in  $x^2$ 

(c) Find the value of k

(2)






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Question 7 continued	

Question 7 continued
(Total for Question 7 is 7 marks)



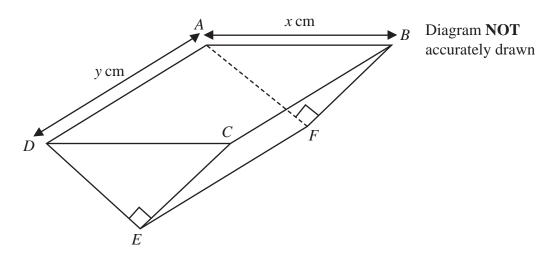


Figure 4

Figure 4 shows a solid right triangular prism ABCDEF

The cross section of the prism is an isosceles triangle.

• 
$$\angle DEC = \angle AFB = 90^{\circ}$$

• 
$$AB = DC = x \text{ cm}$$

• 
$$AD = BC = FE = y \text{ cm}$$

• 
$$AF = BF = DE = CE$$

The triangular faces of the prism are vertical and the edges AD, BC and FE are horizontal.

The volume of the prism is 3.6 cm<sup>3</sup>

The total external surface area of the prism is S cm<sup>2</sup>

(a) Show that S satisfies the equation

$$S = \frac{x^2}{2} + \frac{72\left(\sqrt{2} + 1\right)}{5x} \tag{4}$$

Given that x can vary,

(b) use calculus, to find to 3 significant figures, the value of *x* for which *S* is a minimum.

Justify that this value of x gives a minimum value of S

(4)

(c) Hence find, to 2 significant figures, the minimum value of S

(2)





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Question 8 continued	

Question 8 continued			
(Total for Question 8 is 10 marks)			



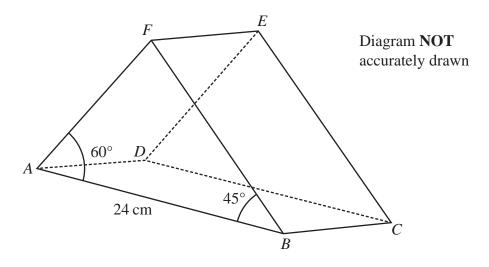


Figure 5

Figure 5 shows a right triangular prism ABCDEF where ABCD is a rectangle.

$$AF = DE$$
  $BF = CE$   $AD = FE = BC$   $AB = DC = 24 \text{ cm}$   
 $\angle ABF = \angle DCE = 45^{\circ}$   $\angle BAF = \angle CDE = 60^{\circ}$ 

Using a formula from page 2,

(a) show that 
$$\sin AFB = \frac{\sqrt{2} + \sqrt{6}}{4}$$

(3)

Without using a calculator,

(b) show that 
$$BF = 12(3\sqrt{2} - \sqrt{6})$$
 cm

(5)

The angle between the plane AEB and the plane ABCD is  $65^{\circ}$ 

(c) Find, in cm to 2 significant figures, the length of EF

(3)

(d) Find, in degrees to one decimal place, the size of the angle between the line CF and the plane ABCD

**(4)** 



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Question 9 continued	

Question 9 continued	
(Total for	r Question 9 is 15 marks)
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10 The points A, B, C and D are the vertices of a quadrilateral such that

$$\overrightarrow{AB} = 3\mathbf{a} + 4\mathbf{b}$$
  $\overrightarrow{AC} = 7\mathbf{a} + 9\mathbf{b}$   $\overrightarrow{AD} = 4\mathbf{a} + 5\mathbf{b}$ 

(a) Show that ABCD is a parallelogram.

(3)

BC is extended to the point E such that BCE is a straight line.

Point F lies on CD such that CF : FD = 1 : 2

Given that A, F and E are collinear,

(b) find the vector  $\overrightarrow{AE}$  in the form  $X\mathbf{a} + Y\mathbf{b}$  where X and Y are rational numbers to be found.

(8)


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Question 10 continued	



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Question 10 continued	

(Total for Question 10 is 11 marks)



- 11 Using formulae from page 2, show that
  - (a) (i)  $\cos 2A = 2\cos^2 A 1$

(3)

(ii)  $\sin 2A = 2\sin A\cos A$ 

(1)

(b) Show that  $\cos^3 A = \frac{\cos 3A + 3\cos A}{4}$ 

(4)

Hence, or otherwise,

(c) solve, giving exact values in terms of  $\pi$ 

$$8\cos^{3}\left(\frac{\theta}{2}\right) - 6\cos\left(\frac{\theta}{2}\right) - 1 = 0 \quad \text{for } 0 \leqslant \theta \leqslant 2\pi$$
(4)

(d) use algebraic integration to find the exact value of

$$\int_0^{\frac{\pi}{6}} \left( 4\cos^3\theta - \sin 2\theta \right) d\theta \tag{4}$$





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Question 11 continued	
	(Total for Question 11 is 16 marks)
	TOTAL FOR PAPER IS 100 MARKS

