Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International GCSE

Time 2 hours

Paper reference **4PM1/01**

Further Pure Mathematics
PAPER 1

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\quad$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\quad$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B} \qquad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Using calculus find the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos 4\theta \, \mathrm{d}\theta$

Give your answer in the form $-\frac{\sqrt{a}}{b}$ where a is a prime number and b is an integer.

(4)

(Total for Question 1 is 4 marks)

2 $f(x) = 2x^2 - 12x + 5$

Given that $f(x)$ can be written in the form $f(x) = a(x + b)^2 + c$ where a, b and c are integers,

(a) find the value of a, the value of b and the value of c

(3)

(b) Hence find the set of values of x for which $f(x) - 37 > 0$

(4)

Question 2 continued

(Total for Question 2 is 7 marks)

3 A geometric series G has first term a and common ratio r

The 2nd term of G is $\frac{5}{16}$ and the 5th term of G is $\frac{135}{1024}$

(a) Find

(i) the value of r

(ii) the value of a

(5)

Given that G is convergent with sum to infinity S

(b) find the exact value of S

(2)

Question 3 continued

(Total for Question 3 is 7 marks)

4 (a) On the grid opposite, draw the line with equation

(i) $y = 2x - 4$ (ii) $2x + 3y = 12$ (iii) $y + 2x + 2 = 0$

(3)

(b) Show, by shading on the grid, the region R defined by the inequalities

$y \geqslant 2x - 4$ $2x + 3y \leqslant 12$ $y + 2x + 2 \geqslant 0$

(1)

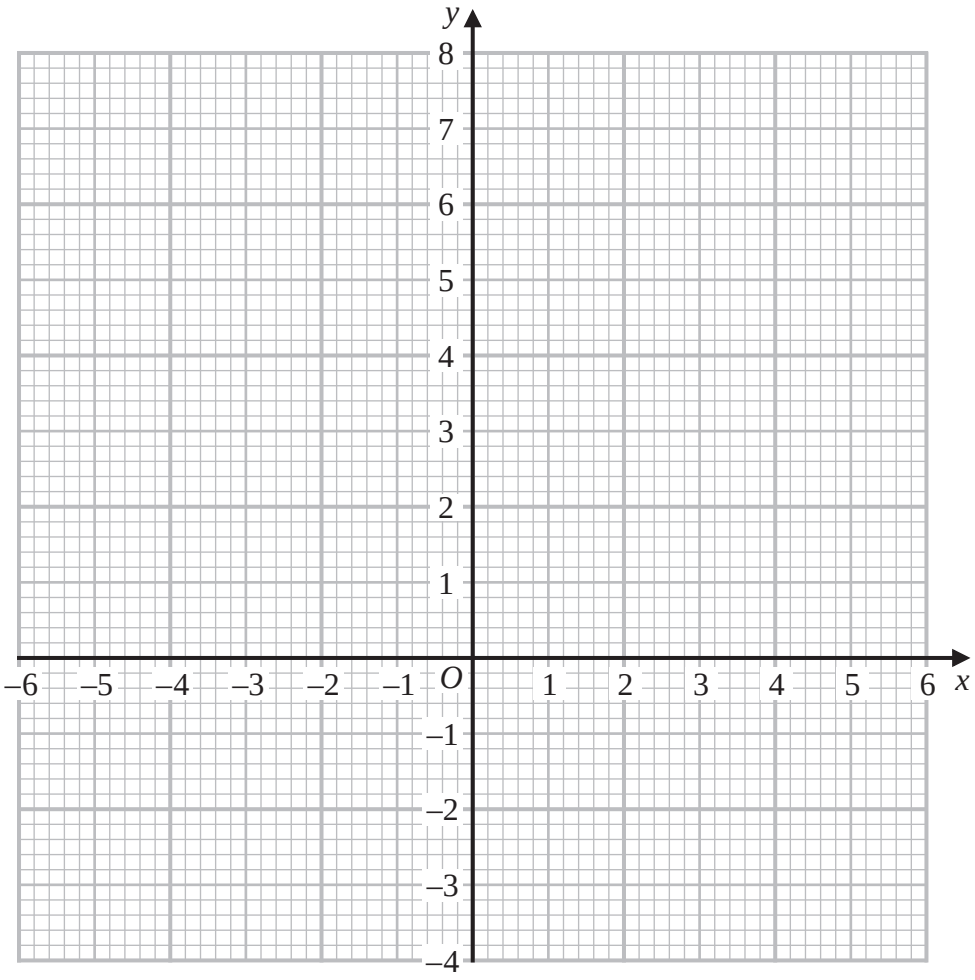
For all points in R, with coordinates (x, y)

$$P = x - 2y$$

(c) find the least value of P

(4)

Question 4 continued

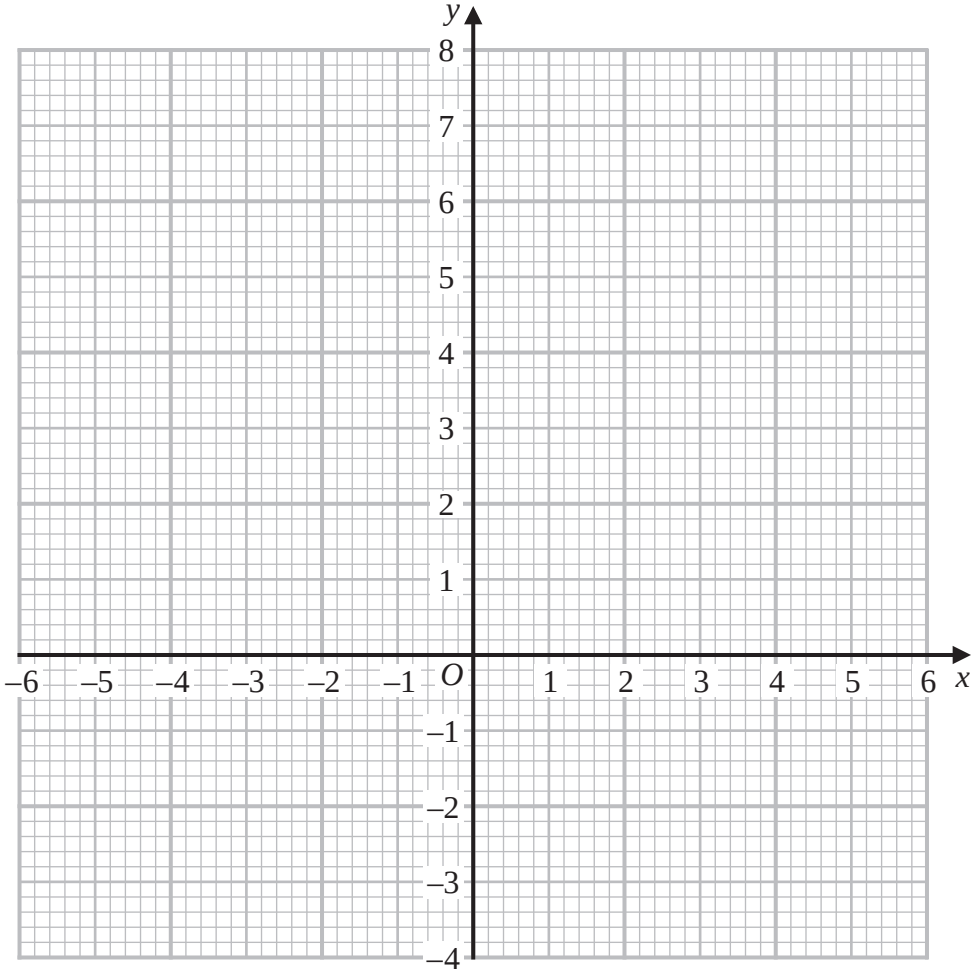


Turn over for a spare grid if you need to redraw your graph.

Question 4 continued

Question 4 continued

Only use this grid if you need to redraw your graph.



(Total for Question 4 is 8 marks)

5 $f(x) = ax^3 + 5bx^2 + 8ax - 4b$ where a and b are integers

Given that

$(x + 2)$ is a factor of $f(x)$

and that

when $f(x)$ is divided by $(x + 3)$ the remainder is 21

(a) show that $a = 2$ and find the value of b

(5)

(b) Use algebra to solve the equation $f(x) = 0$

(4)

Question 5 continued

(Total for Question 5 is 9 marks)

6 In triangle ABC, $AC = x$ cm, $AB = (x + 3)$ cm and $\angle ABC = 30°$

Given that $\angle ACB = \theta°$ where $0 < \theta < 90$

(a) show that

(i) $\sin\theta° = \dfrac{x + 3}{2x}$

(ii) $\cos\theta° = \dfrac{\sqrt{3x^2 - 6x - 9}}{2x}$

(5)

Given that the size of $\angle BAC$: the size of $\angle ABC = 7 : 2$

(b) find the exact value of x

Give your answer in the form $a + a\sqrt{b}$ where a and b are prime numbers.

(5)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 10 marks)

7 (a) Write down the value of $\log_2 16$

(1)

Given that $4 + 2\log_4 x = \log_2 y$

(b) show that $y = 16x$

(4)

(c) Hence solve the equation $4 + 2\log_4 x = \log_2(4x + 5)$

(3)

Question 7 continued

(Total for Question 7 is 8 marks)

8

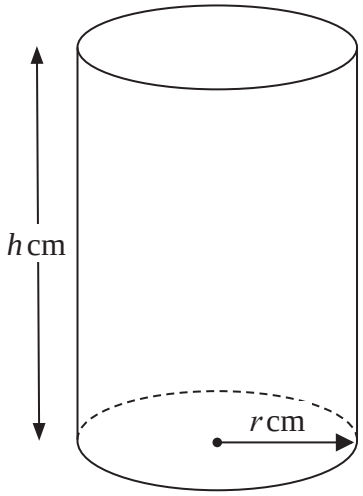


Diagram **NOT** accurately drawn

Figure 1

A solid right circular cylinder has base radius r cm and height h cm as shown in Figure 1.

The cylinder has a volume of 90π cm^3 and a total surface area of S cm^2

(a) Show that $S = 2\pi r^2 + \dfrac{180\pi}{r}$

(3)

Given that r can vary,

(b) use calculus to find, to 3 significant figures, the value of r for which S is a minimum, justifying that this value of r gives a minimum value of S

(5)

(c) Find, to 3 significant figures, the minimum value of S

(2)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 10 marks)

9 (a) Expand $\frac{1}{\sqrt{1-2x}}$ in ascending powers of x up to and including the term in x^3

Simplify each term as far as possible.

(3)

(b) Show that $\frac{1}{\sqrt{0.96}} = \frac{5\sqrt{6}}{12}$

Show all your working clearly.

(2)

(c) Express $\frac{1}{5\sqrt{6}-12}$ in the form $\frac{a\sqrt{6}}{c} + b$ where a, b and c are integers.

Show all your working clearly.

(2)

Using the expansion you found in part (a) with a suitable value of x

(d) find an estimate, to 5 decimal places, of $\frac{9}{5\sqrt{6}-12}$

(4)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 11 marks)

10 The equation of the line L_1 is $y - 2x - 6 = 0$

The point P with coordinates $(2, a)$ lies on L_1

(a) Find the value of a

(1)

The line L_2 is perpendicular to L_1 and passes through P

(b) Show that an equation of L_2 is $x + 2y - 22 = 0$

(4)

Line L_1 crosses the x-axis at the point A and line L_2 crosses the x-axis at the point B

The point C has coordinates (m, n) such that $m > 0$ and $n < 0$

The length of AC is $5\sqrt{2}$ and the gradient of BC is $\frac{1}{4}$

(c) Find the value of m and the value of n

(9)

(d) Find the area of quadrilateral $ACBP$

(3)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 17 marks)

11 Given that

$$y = \frac{e^{4x}}{32}(8x^2 - 4x + 1)$$

(a) show that $\frac{dy}{dx} = x^2e^{4x}$

(5)

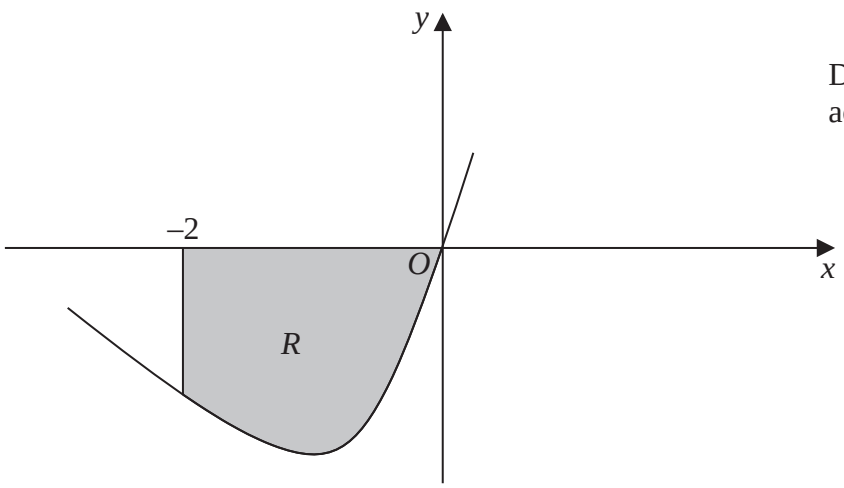


Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows part of the curve C with equation $y = 3xe^{2x}$

The finite region R bounded by C, the straight line with equation $x = -2$ and the x-axis, shown shaded in Figure 2, is rotated though 360° about the x-axis.

(b) Using part (a), find the volume, to 2 significant figures, of the solid formed.

(4)

Question 11 continued

Question 11 continued

(Total for Question 11 is 9 marks)

TOTAL FOR PAPER IS 100 MARKS

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