

Please check the examination details below before entering your candidate information

Candidate surname					Other names				
Centre Number					Candidate Number				


Pearson Edexcel International GCSE

Time 2 hours

Paper reference **4PM1/01R**

Further Pure Mathematics

PAPER 1R



Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ►

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International GCSE in Further Pure Mathematics Formulae sheet

Mensuration**Surface area of sphere** $= 4\pi r^2$ **Curved surface area of cone** $= \pi r \times \text{slant height}$ **Volume of sphere** $= \frac{4}{3}\pi r^3$ **Series****Arithmetic series**Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$ **Geometric series**Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$ Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$ **Binomial series** $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad \text{for } |x| < 1, n \in \mathbb{Q}$ **Calculus****Quotient rule (differentiation)**

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Trigonometry**Cosine rule**In triangle ABC : $a^2 = b^2 + c^2 - 2bc \cos A$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

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Answer all TEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

- 1** Given that $\frac{a - \sqrt{48}}{\sqrt{3} + 1}$ can be written in the form $b\sqrt{3} - 9$ where a and b are integers,

find the value of a and the value of b
Show your working clearly.

(4)

(Total for Question 1 is 4 marks)



P 7 1 8 1 8 A 0 3 2 8

2 In $\triangle ABC$,

$$\angle BAC = 50^\circ \quad AB = 10 \text{ cm} \quad BC = 9 \text{ cm}$$

Given that $\angle BCA = x^\circ$

find the two possible values, to one decimal place, of x

(3)

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Question 2 continued

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(Total for Question 2 is 3 marks)



- 3 An arithmetic series has first term 16 and common difference -5

The sum to n terms of this series is S_n

Given that $S_n < -450$

find the least value of n

(4)

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4 O , A and B are fixed points such that

$$\overrightarrow{OA} = p\mathbf{i} + 2p\mathbf{j} \quad \overrightarrow{OB} = 5\mathbf{i} + 9p\mathbf{j}$$

Given that \overrightarrow{AB} is parallel to $(\mathbf{i} - 2\mathbf{j})$

(a) find the value of p

(6)

(b) Hence find \overrightarrow{AB} as a simplified expression in terms of \mathbf{i} and \mathbf{j}

(2)

(c) Find a unit vector parallel to \overrightarrow{OA}

Give your answer in the form $\frac{\sqrt{a}}{5}(b\mathbf{i} + c\mathbf{j})$ where a , b and c are integers to be found.

(4)

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5

$$f(x) = 2ax^3 + x^2 - bx + 3a \quad \text{where } a \text{ and } b \text{ are integers.}$$

Given that $(x + 2)$ and $(x - 1)$ are both factors of $f(x)$

(a) show that $a = 2$ and find the value of b

(5)

(b) Hence factorise $f(x)$ completely.

(2)

Hence, given that $h(y) = 2^{(3y+2)} + 2^{2y} - 11(2^y) + 6$

(c) solve the equation $h(y) = 0$

Where appropriate give your answers to 3 decimal places.

(5)

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Question 5 continued

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(Total for Question 5 is 12 marks)



6 The curve C has equation $y = \frac{e^{(x^2+1)}}{x^2+1}$

(a) Show that $\frac{dy}{dx} = \frac{Kx^3 e^{(x^2+1)}}{(x^2+1)^2}$ where K is a constant whose value is to be found. (5)

(b) Find an equation of the tangent to C at the point on C where $x = -1$
Simplify your answer. (5)

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- 7 A particle P is moving along the x -axis. At time t seconds ($t \geq 0$) the velocity of P is v m/s where

$$v = t^2 - 10t + 28$$

- (a) Find the velocity of P when $t = 1$

(1)

Given that the distance of P from the origin is 24 m when $t = 3$

- (b) find the distance of P from the origin when $t = 5$

(5)

- (c) Find the acceleration of P when $t = 9$

(2)

- (d) (i) Show that there are no values of t for which P is instantaneously at rest.

- (ii) Find the least magnitude of the velocity of P

(3)

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Question 7 continued

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Question 7 continued

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Question 7 continued

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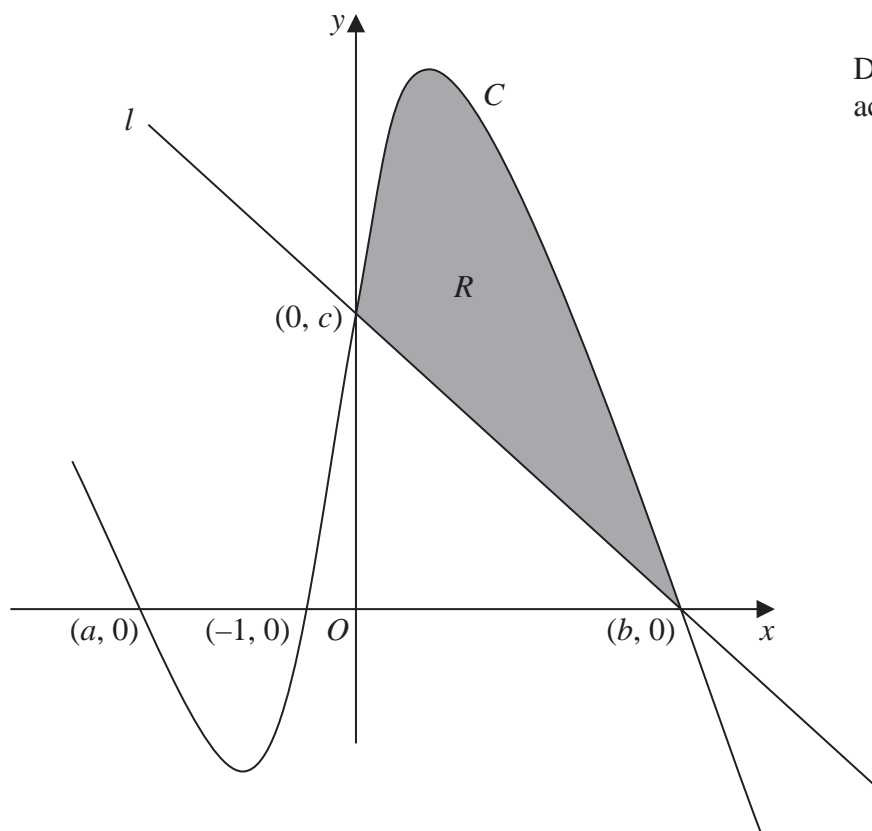
Diagram **NOT**
accurately drawn**Figure 1**

Figure 1 shows part of the curve C with equation $y = f(x)$

The curve C passes through the points with coordinates

$$(a, 0), (-1, 0), (b, 0) \text{ and } (0, c)$$

Given that $f'(x) = 17 + 2x - 3x^2$

(a) show that the equation of C is $y = 15 + 17x + x^2 - x^3$ (4)

(b) Find the value of a , the value of b and the value of c (6)

The straight line l intersects C at the points with coordinates $(b, 0)$ and $(0, c)$

The region R , shown shaded in Figure 1, is bounded by l and C

(c) Use algebraic integration to find the exact area of region R (5)



Question 8 continued

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(Total for Question 8 is 15 marks)



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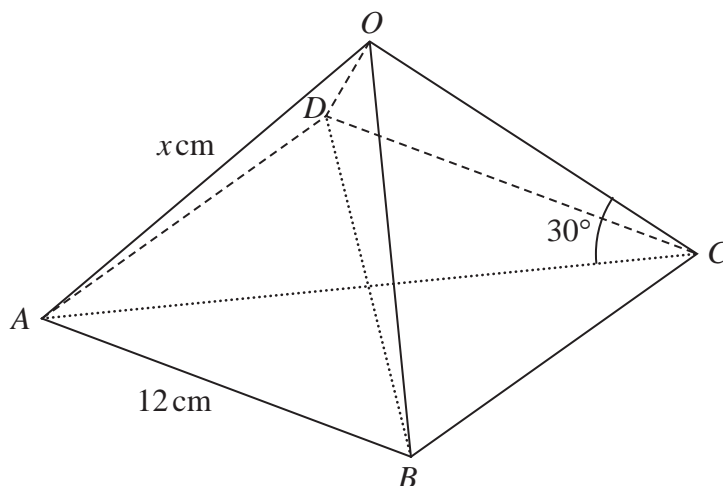
Diagram **NOT**
accurately drawn**Figure 2**

Figure 2 shows the right pyramid $OABCD$ with a square base $ABCD$ of side 12 cm.

$$OA = OB = OC = OD = x \text{ cm} \quad \text{and} \quad \angle OAC = \angle ODB = \angle OCA = \angle OBD = 30^\circ$$

- (a) Find the exact length of AC (2)
- (b) Show that $x = 4\sqrt{6}$ (2)
- (c) Find the total surface area, to the nearest cm^2 , of the pyramid. (5)
- (d) Find the size of the obtuse angle, to the nearest degree, between the plane OAB and the plane OBC (4)

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Question 9 continued

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Question 9 continued

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10 Using formulae from page 2

(a) show that $\cos(A - B) - \cos(A + B) = 2 \sin A \sin B$ (2)

(b) Hence show that $\cos 5\theta - \cos 9\theta = 2 \sin 7\theta \sin 2\theta$ (1)

(c) Solve the equation

$$\cos 5\theta - \cos 9\theta = \sqrt{3} \sin 7\theta \quad \text{for } 0 < \theta \leq \frac{1}{3}\pi$$

Give your solutions in terms of π (7)

(d) Using calculus and showing your working, evaluate

$$\int_0^{\frac{\pi}{7}} 8 \sin 7x \cos 2x \tan 2x \, dx$$

Give your answer to 3 decimal places. (6)

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Question 10 continued

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Question 10 continued

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(Total for Question 10 is 16 marks)

TOTAL FOR PAPER IS 100 MARKS

