Question	Scheme	Marks
10(a)	$\alpha+\beta=-\dfrac{k}{2} \qquad \alpha\beta=2$	B1
	$(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta, \quad \alpha^2-\beta^2=(\alpha+\beta)(\alpha-\beta)$	B1B1
	$\Rightarrow \alpha^2-\beta^2=(\alpha+\beta)\sqrt{(\alpha+\beta)^2-4\alpha\beta}$	
	$\Rightarrow \alpha^2-\beta^2=\left(-\dfrac{k}{2}\right)\sqrt{\left(-\dfrac{k}{2}\right)^2-4\times 2}=\dfrac{7\sqrt{17}}{4}$	M1
	$\Rightarrow \dfrac{k^4}{4}-8k^2-\dfrac{833}{4}=0 \Rightarrow k^4-32k^2-833=0$ or any equivalent	M1A1
	$\Rightarrow (k^2-49)(k^2+17)=0 \Rightarrow k=-7$ *	M1A1 cso [8]
(b)	Product: $\left[(\alpha-\beta)(\alpha+\beta)=\alpha^2-\beta^2=\dfrac{7\sqrt{17}}{4}\right]$ from (a)	B1
	Sum: $(\alpha-\beta)+(\alpha+\beta)=\dfrac{\sqrt{17}}{2}+\dfrac{7}{2}=\left(\dfrac{\sqrt{17}+7}{2}\right)$	M1
	$x^2-\left(\dfrac{\sqrt{17}+7}{2}\right)x+\dfrac{7\sqrt{17}}{4}=0 \Rightarrow 4x^2-2\left(\sqrt{17}+7\right)x+7\sqrt{17}=0$	M1A1 [4]
		Total 12 marks

There is an alternative solution using the roots of the equation.

Roots are $\dfrac{-k\pm\sqrt{k^2-4\times 2\times 4}}{2\times 2}$

$$\Rightarrow \left(\frac{-k+\sqrt{k^2-32}}{4}\right)^2-\left(\frac{-k-\sqrt{k^2-32}}{4}\right)^2=\frac{7\sqrt{17}}{4}$$

$$\Rightarrow \left(-k+\sqrt{k^2-32}\right)^2-\left(-k-\sqrt{k^2-32}\right)^2=28\sqrt{17}$$

$$\Rightarrow k^2-2k\sqrt{k^2-32}+k^2-32-k^2-2k\sqrt{k^2-32}-k^2+32=28\sqrt{17}$$

$$\Rightarrow -4k\sqrt{k^2-32}=28\sqrt{17} \Rightarrow k\sqrt{k^2-32}=7\sqrt{17}$$

$$\Rightarrow k^2\left(k^2-32\right)=833 \Rightarrow k^4-32k^2-833=0$$

$$\Rightarrow \left(k^2-49\right)\left(k^2+17\right)=0 \Rightarrow k=-7\ *$$

Part	Mark	Notes
(a)	B1	For the correct sum in terms of k and the correct product. Look out for these embedded in their work.
	B1	For the correct algebra on $(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$ Substitution not required for this mark, although it can be implied from correct substitution without the actual algebra seen.
	B1	For the correct algebra on $\alpha^2-\beta^2$ Award also for $(\alpha+\beta)(\alpha-\beta)=\frac{7\sqrt{17}}{4}$ Substitution not required for this mark, although it can be implied from correct substitution without the actual algebra seen.
	M1	For substituting in the values of their sum and product into their $\alpha^2-\beta^2$ This can be implied from sight of $\left(-\frac{k}{2}\right)\sqrt{\left(-\frac{k}{2}\right)^2-4\times 2}=\frac{7\sqrt{17}}{4}$
	M1	For forming a 3TQ in terms of k^2 in any form
	A1	For the correct 3TQ Accept it in any form as long as it is 3 terms. For example, accept $\frac{k^4}{16}-2k^2-\frac{833}{16}=0$ etc
	M1	For any acceptable attempt seen to solve their 3TQ [see General Guidance] If there is no working, just $k=-7$ following a 3TQ then award M0 However, accept evidence of $k^2=49$ $\left[\text{with } k^2=-17\right]$
	A1 cso	For the correct value of k * **Note: This is a given value**
(b)	B1	For a value for the product using their results from (a) or just writes it down.
	M1	For a correct method to find the value of the sum using their values for $(\alpha-\beta)$ and $(\alpha+\beta)$
	M1	For forming a 3TQ with their sum and product. = 0 is not required for the award of this mark
	A1	For a correct equation simplified or unsimplified. For example, accept, $x^2-\left(\frac{\sqrt{17}+7}{2}\right)x+\frac{7\sqrt{17}}{4}=0$ as well as $4x^2-2\left(\sqrt{17}+7\right)x+7\sqrt{17}=0$ o.e.

Question	Scheme	Marks
11(a)	$\mathrm{f}(\theta)=(2\cos\theta-\sin\theta)(2\sin\theta+\cos\theta)$ $=4\sin\theta\cos\theta+2\cos^2\theta-2\sin^2\theta-\sin\theta\cos\theta$ $=3\sin\theta\cos\theta+2(\cos^2\theta-\sin^2\theta)\Rightarrow\frac{3}{2}\sin 2\theta,+2\cos 2\theta$ $=\frac{3}{2}\sin 2\theta+2\cos 2\theta$ *	M1 M1A1 cso [3]
(b)	$\frac{3}{2}\sin 2\theta+2\cos 2\theta+2=0\Rightarrow 3\sin 2\theta+4\cos 2\theta+4=0$ $\Rightarrow 3\sin 2\theta+4(\cos^2\theta-\sin^2\theta)+4(\sin^2\theta+\cos^2\theta)=0$ $\Rightarrow 6\sin\theta\cos\theta+8\cos^2\theta=0$ $\Rightarrow\cos\theta(6\sin\theta+8\cos\theta)=0$ $\Rightarrow\cos\theta=0\Rightarrow\theta=\frac{\pi}{2}$ * $\left[\Rightarrow\tan\theta=-\frac{4}{3}\right]$	M1M1 M1 M1 A1 cso [5]
(c)	Area $=\int_0^{\frac{\pi}{2}}\left(\frac{3}{2}\sin 2\theta+2\cos 2\theta+2\right)\mathrm{d}\theta=\left[-\frac{3}{4}\cos 2\theta+\frac{2\sin 2\theta}{2}+2\theta\right]_0^{\frac{\pi}{2}}$ $=\left(-\frac{3}{4}\cos 2\left(\frac{\pi}{2}\right)+\frac{2\sin 2\left(\frac{\pi}{2}\right)}{2}+2\left(\frac{\pi}{2}\right)\right)-\left(-\frac{3}{4}\cos 0+\frac{2\sin 0}{2}+2\times 0\right)$ $=\left(\frac{3}{4}+\pi\right)-\left(-\frac{3}{4}\right)=\frac{3}{2}+\pi$	M1 M1 A1 [3]
		Total 11 marks

Part	Mark	Notes
(a)	M1	For multiplying out the brackets **correctly** and simplifying to $k\sin\theta\cos\theta,\ \ l(\cos^2\theta-\sin^2\theta)$ where k and l are integers**.** Condone invisible brackets if the working is clear.
	M1	For using the identity for $\cos 2\theta$ on their $2(\cos^2\theta-\sin^2\theta)$ **OR** For using the identity for $\sin 2\theta$ on their $3\sin\theta\cos\theta$
	A1 cso	For the correct identity as shown with no errors. You must check every line of working carefully – this is a given answer.
	Solutions based on RHS = LHS are acceptable. Apply the above marks – if you are not sure, please send to REVIEW	

(b)	M1	For correctly using the identity $\cos 2\theta = \cos^2\theta - \sin^2\theta$
	M1	For correctly using the identity $\sin^2\theta + \cos^2\theta = 1$
	M1	For correctly using the identity $\sin 2\theta = 2\sin\theta\cos\theta$
	M1	For factorising the resulting expression. You must see this step
	A1 cso	For $\cos\theta = 0 \Rightarrow \theta = \dfrac{\pi}{2}$
	ALT	
	M1	For correctly using $\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\dfrac{3}{2}\sin 2\theta + 2\cos 2\theta + 2 = \dfrac{3}{2}\sin 2\theta + 2\left(\cos^2\theta - \sin^2\theta\right) = 0$
	M1	For correctly using the identity $\sin^2\theta + \cos^2\theta = 1$ $\dfrac{3}{2}\sin 2\theta + 2\cos 2\theta + 2 = \dfrac{3}{2}\sin 2\theta + 2\left(\cos^2\theta - 1 + \cos^2\theta\right) + 2 = 0$ $\Rightarrow \left[\dfrac{3}{2}\sin 2\theta + 4\cos^2\theta = 0\right]$
	M1	For correctly using the identity $\sin 2\theta = 2\sin\theta\cos\theta$ $\dfrac{3}{2}\sin 2\theta + 4\cos^2\theta = 3\sin\theta\cos\theta + 4\cos^2\theta = 0$
	M1	For factorising the resulting expression. You must see this step $3\sin\theta\cos\theta + 4\cos^2\theta = \cos\theta\left(3\sin\theta + 4\cos\theta\right) = 0$
	A1	$\cos\theta = 0 \Rightarrow \theta = \dfrac{\pi}{2}$ *
(c)	M1	For integrating the given expression. Minimally acceptable integration as shown below: $\dfrac{3}{2}\sin 2\theta \Rightarrow \pm\dfrac{3}{4}\cos 2\theta$ $2\theta\cos 2\theta \Rightarrow \pm\left(\dfrac{2}{2}\right)\sin 2\theta$ $2 \Rightarrow 2\theta$
	M1	For substituting **BOTH** of the given limits the correct way around into **their changed expression** and subtracting.. There must be a minimum of two terms to substitute limits into. This must be explicitly seen
	A1	For the correct exact area as shown. [Approximate area = 4.641….]

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