Question	Scheme	Marks
10(a)	$\overrightarrow{BC} = -\overrightarrow{AB} + \overrightarrow{AC} = -(3\mathbf{a}+4\mathbf{b}) + 7\mathbf{a}+9\mathbf{b} = 4\mathbf{a}+5\mathbf{b}$	B1
	$\overrightarrow{DC} = -\overrightarrow{DA} + \overrightarrow{DC} = -(4\mathbf{a}+5\mathbf{b}) + 7\mathbf{a}+9\mathbf{b} = 3\mathbf{a}+4\mathbf{b}$	B1
	$AB \parallel DC$ and $BC \parallel AD$ hence the quadrilateral is a parallelogram	B1ft [3]
(b)	$\overrightarrow{CF} = -\frac{1}{3}(3\mathbf{a}+4\mathbf{b})$	B1
	$\overrightarrow{AF} = \overrightarrow{AC} + \overrightarrow{CF} = 7\mathbf{a}+9\mathbf{b} - -\frac{1}{3}(3\mathbf{a}+4\mathbf{b}) = \left[6\mathbf{a}+\frac{23}{3}\mathbf{b}\right]$	B1
	$\overrightarrow{AE} = \mu\,\overrightarrow{AF} = \mu\left(6\mathbf{a}+\frac{23}{3}\mathbf{b}\right)$	M1
	$\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BE} = 3\mathbf{a}+4\mathbf{b}+\lambda(4\mathbf{a}+5\mathbf{b}) = \left[(3+4\lambda)\mathbf{a}+(4+5\lambda)\mathbf{b}\right]$	M1
	$\mu\left(6\mathbf{a}+\frac{23}{3}\mathbf{b}\right) = (3+4\lambda)\mathbf{a}+(4+5\lambda)\mathbf{b}$	
	$\Rightarrow 6\mu = 3+4\lambda \quad \frac{23}{3}\mu = 4+5\lambda$	ddM1
	$\Rightarrow \lambda = \frac{3}{2} \quad \left(\mu = \frac{3}{2}\right)$	M1A1
	$\overrightarrow{AE} = \frac{3}{2}\left(6\mathbf{a}+\frac{23}{3}\mathbf{b}\right) = 9\mathbf{a}+\frac{23}{2}\mathbf{b}$	B1ft [8]
		Total 11 marks

Part	Mark	Notes
(a)	B1	For the correct vector for $\overrightarrow{BC}$ or $\overrightarrow{CB}$
	B1	For the correct vector for $\overrightarrow{DC}$ or $\overrightarrow{CD}$
	B1ft	For the correct complete conclusion including stating that the quadrilateral is a parallelogram. Ft their vectors for reverse directions.
	ALT	
	B1	For the correct lengths of $\overrightarrow{AD}$ AND $\overrightarrow{BC}$ $\quad |AD| = |BC| = \sqrt{41}$
	B1	For the correct lengths of $\overrightarrow{AB}$ AND $\overrightarrow{DC}$ $\quad |AB| = |DC| = 5$
	B1ft	Opposite sides are of equal length so the quadrilateral is a parallelogram.
(b)	B1	For the **correct vector** for $\overrightarrow{CF}$ or $\overrightarrow{DF}$ or **correct** reverse directions $\left[\overrightarrow{CF} = -\frac{1}{3}(3\mathbf{a}+4\mathbf{b})\right]$ $\left[\overrightarrow{DF} = \frac{2}{3}(3\mathbf{a}+4\mathbf{b})\right]$ this may be embedded in $\overrightarrow{AF}$
	B1	For the **correct vector** for $\overrightarrow{AF}$ using their $\overrightarrow{CF}$ or $\overrightarrow{DF}$ e.g., $\overrightarrow{AF} = \overrightarrow{AC} + \overrightarrow{CF} = 7\mathbf{a}+9\mathbf{b} - \frac{1}{3}(3\mathbf{a}+4\mathbf{b}) = \left[6\mathbf{a} + \frac{23}{3}\mathbf{b}\right]$
	M1	For **one** correct vector for $\overrightarrow{AE}$ using **their** $\overrightarrow{AF}$ and a parameter For example: $\overrightarrow{AE} = \mu\,\overrightarrow{AF} = \mu\left(6\mathbf{a} + \frac{23}{3}\mathbf{b}\right)$
	M1	For a **second** vector for $\overrightarrow{AE}$ using **their** vectors and a second unique parameter For example; $\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BE} = 3\mathbf{a}+4\mathbf{b}+\lambda(4\mathbf{a}+5\mathbf{b}) = \left[(3+4\lambda)\mathbf{a} + (4+5\lambda)\mathbf{b}\right]$
	ddM1	For setting their two vectors for $\overrightarrow{BF}$ equal and setting up two equations involving two different parameters. $\mu\left(6\mathbf{a} + \frac{23}{3}\mathbf{b}\right) = (3+4\lambda)\mathbf{a} + (4+5\lambda)\mathbf{b}$ $\Rightarrow 6\mu = 3+4\lambda \quad \frac{23}{3}\mu = 4+5\lambda$ **This mark is dependent on the previous two M marks**
	dddM1	For solving their two simultaneous equations by any method to find a value for λ **This mark is dependent on the previous three M marks**
	A1	For $\mu = \frac{3}{2}$ or $\lambda = \frac{3}{2}$
	B1ft	For writing down a vector for $\overrightarrow{AE}$ using their $\overrightarrow{AF}$ and their λ in the required form, provided $1 < \lambda < 2$

	ALT	
	B1	For the correct vector for $\overrightarrow{CF}$ or $\overrightarrow{DF}$ $\left[\overrightarrow{CF} = -\frac{1}{3}(3\mathbf{a}+4\mathbf{b})\right]$ $\left[\overrightarrow{DF} = \frac{2}{3}(3\mathbf{a}+4\mathbf{b})\right]$
	B1	For the correct vector for $\overrightarrow{AF}$ using their $\overrightarrow{CF}$ or $\overrightarrow{DF}$ e.g., $\overrightarrow{AF} = \overrightarrow{AC} + \overrightarrow{CF} = 7\mathbf{a}+9\mathbf{b}-\frac{1}{3}(3\mathbf{a}+4\mathbf{b}) = \left[6\mathbf{a}+\frac{23}{3}\mathbf{b}\right]$
	M1	For **one** correct vector for $\overrightarrow{AE}$ using **their** $\overrightarrow{AF}$ and a parameter For example: $\overrightarrow{AE} = \mu\,\overrightarrow{AF} = \mu\left(6\mathbf{a}+\frac{23}{3}\mathbf{b}\right)$
	M1	For a correct vector for $\overrightarrow{BE}$ using their $\overrightarrow{AE}$ $\overrightarrow{BE} = -3\mathbf{a}-4\mathbf{b}+\mu\left(6\mathbf{a}+\frac{23}{3}\mathbf{b}\right) = \left[(6\mu-3)\mathbf{a}+\left(\frac{23}{3}\mu-4\right)\mathbf{b}\right]$
	ddM1	For forming the correct ratio using the vector $\overrightarrow{BC}$ $\frac{(6\mu-3)}{\left(\frac{23}{3}\mu-4\right)} = \frac{4}{5}$
	dddM1	For solving the equation to find a value for μ
	A1	For $\mu = \frac{3}{2}$
	B1ft	For writing down a vector for $\overrightarrow{AE}$ using their $\overrightarrow{AF}$ and their λ in the required form For writing down a vector for $\overrightarrow{AE}$ using their $\overrightarrow{AF}$ and their λ in the required form, provided $1<\lambda<2$

Question	Scheme	Marks
11(a)(i)	$\cos 2A = \cos^2 A - \sin^2 A, = \cos^2 A - \left(1-\cos^2 A\right)$	M1,M1
	$\cos 2A = 2\cos^2 A - 1$ *	A1 cso [3]
(ii)	$\sin 2A = \sin A\cos A + \sin A\cos A = 2\sin A\cos A$ *	B1 cso [1]
(b)	$\cos 3A = \cos(2A+A)$	
	$= \cos 2A\cos A - \sin A\sin 2A$	M1
	$= \left(2\cos^2 A - 1\right)\cos A - 2\sin A\sin A\cos A$	
	$= 2\cos^3 A - \cos A - 2\left(1-\cos^2 A\right)\cos A$	M1
	$= 2\cos^3 A - \cos A + 2\cos^3 A - 2\cos A$	
	$= 4\cos^3 A - 3\cos A$	M1
	$\Rightarrow \cos^3 A = \dfrac{\cos 3A + 3\cos A}{4}$ *	A1 Cso [4]
	ALT	
	$\cos^3 A = \dfrac{\cos 3A + 3\cos A}{4}$	
	$= \dfrac{\cos 2A\cos A - \sin 2A\sin A + 3\cos A}{4}$	[M1
	$= \dfrac{\left(2\cos^2 A - 1\right)\cos A - 2\sin A\sin A\cos A + 3\cos A}{4}$	
	$= \dfrac{2\cos^3 A - \cos A - 2\left(1-\cos^2 A\right)\cos A + 3\cos A}{4}$	M1
	$= \dfrac{2\cos^3 A - \cos A + 2\cos^3 A - 2\cos A + 3\cos A}{4}$	M1
	$= \dfrac{4\cos^3 A}{4} = \cos^3 A \quad [LHS = RHS]$	A1]
(c)	$\left[\cos 3A = 4\cos^3 A - 3\cos A \Rightarrow 2\cos 3A = 8\cos^3 A - 6\cos A\right]$	
	$8\cos^3\left(\dfrac{\theta}{2}\right) - 6\cos\left(\dfrac{\theta}{2}\right) - 1 = 0 \Rightarrow 2\cos 3\left(\dfrac{\theta}{2}\right) - 1 = 0$	M1
	$\cos 3\left(\dfrac{\theta}{2}\right) = \dfrac{1}{2} \Rightarrow 3\left(\dfrac{\theta}{2}\right) = \dfrac{\pi}{3}, \dfrac{5\pi}{3}, \dfrac{7\pi}{3}, \left(\dfrac{11\pi}{3}\right)$	M1
	$\theta = \dfrac{2\pi}{9}, \dfrac{10\pi}{9}, \dfrac{14\pi}{9}$	M1A1 [4]

(d)	$I=\int_0^{\frac{\pi}{6}}\left(4\cos^3\theta-\sin 2\theta\right)\,\mathrm{d}\theta=\int_0^{\frac{\pi}{6}}\left(\cos 3\theta+3\cos\theta-\sin 2\theta\right)\,\mathrm{d}\theta$	M1
	$I=\left[\frac{\sin 3\theta}{3}+3\sin\theta+\frac{\cos 2\theta}{2}\right]_0^{\frac{\pi}{6}}$	M1
	$=\left[\frac{\sin 3\left(\frac{\pi}{6}\right)}{3}+3\sin\left(\frac{\pi}{6}\right)+\frac{\cos 2\left(\frac{\pi}{6}\right)}{2}\right]-\left[\frac{\sin 3(0)}{3}+3\sin(0)+\frac{\cos 2(0)}{2}\right]$	M1
	$=\left[\frac{1}{3}+\frac{3}{2}+\frac{1}{4}\right]-\left[0-0+\frac{1}{2}\right]=\frac{19}{12}$	A1 [4]
		Total 16 marks

Part	Mark	Notes
(a)(i)	M1	For the correct use of the addition formula for cos 2*A*
	M1	For using the Pythagorean identity correctly
	A1 cso	For the correct identity with no errors.
(a)(ii)	B1 cso	$\left[\sin(A+A)\right]=\sin 2A=\sin A\cos A+\cos A\sin A=2\sin A\cos A$
(b)	**Method 1 – Starting with cos 3*A***	
	M1	For using the correct addition formula for cos 3*A* to obtain the correct expansion.
	M1	For applying at least **TWO** of the following seen anywhere in this part of the question: • cos 2*A* identity from (a) correctly • the sin 2*A* identity from (a) applied correctly • the identity $\sin^2 A=1-\cos^2 A$ applied correctly
	M1	For simplifying the expression to be in terms of $\cos^3 A, \cos 3A$ and $\cos A$ only
	A1 cso	For the correct identity with no errors
	ALT – Method 2 – starting with the RHS to show = LHS	
	M1	For using the correct addition formula for cos 3*A* to obtain the correct expansion. You can ignore everything else for this and the next mark.
	M1	For applying at least **TWO** of the following seen anywhere in this part of the question: [again, ignore anything other than the cos 3*A* here] • cos 2*A* identity from (a) correctly • the sin 2*A* identity form (a) applied correctly the identity $\sin^2 A=1-\cos^2 A$ applied correctly
	M1	Collects up like terms and simplifies to obtain $\frac{4\cos^3 A}{4}$
	A1	For showing LHS = RHS with no errors

(c)	M1	Substitutes $2\cos 3\left(\frac{\theta}{2}\right)$ in place of $8\cos^3\left(\frac{\theta}{2}\right)-6\cos\left(\frac{\theta}{2}\right)$ correctly to obtain at least $k\cos 3\left(\frac{\theta}{2}\right)-1=0$ where k is a constant
	M1	Rearranges to obtain $\cos 3\left(\frac{\theta}{2}\right)=\frac{1}{k}$ and takes the inverse cosine to find at least any one **correct** value for $3\left(\frac{\theta}{2}\right)$
	M1	Finds at least one correct value for θ
	A1	Finds all three values with no extra values within range. Ignore any values given outside of the range.
(d)	M1	For a **correct** substitution as shown. $4\cos^3\theta=\cos 3\theta+3\cos\theta$
	M1	For an acceptable attempt to integrate either $\cos 3\theta \to \pm\frac{\sin 3\theta}{3}$ or $-\sin 2\theta \to \pm\frac{\cos 2\theta}{2}$
	M1	For substituting both given values the correct way around. This mark can be awarded for a complete substitution SEEN into any changed expression. Award this mark for **correct** integration **and** a value of $\frac{19}{12}$ seen without explicit substitution.
	A1	For $\frac{19}{12}$

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