

| Question number | Scheme | Marks |
|-----------------|---|---|
| 7 (a) | $16 - 4k(2k - 7) \geq 0$ $2k^2 - 7k - 4 \leq 0$ $(2k + 1)(k - 4) \leq 0$ $-\frac{1}{2} \leq k \leq 4$ Accept $-\frac{1}{2} < k < 4$ | M1 M1 M1 A1 (4) |
| (b) | $\alpha + \beta = \frac{4}{k} \quad \alpha\beta = \frac{2k - 7}{k}$ $\frac{\alpha + 1}{\alpha} + \frac{\beta + 1}{\beta} = \frac{2\alpha\beta + \alpha + \beta}{\alpha\beta} = \frac{2\left(\frac{2k - 7}{k}\right) + \frac{4}{k}}{\frac{2k - 7}{k}}$ $\frac{4k - 10}{k} \times \frac{k}{2k - 7} = \frac{2(2k - 5)}{2k - 7}$ $\frac{\alpha + 1}{\alpha} \times \frac{\beta + 1}{\beta} = \frac{\alpha\beta + \alpha + \beta + 1}{\alpha\beta} = \frac{\frac{2k - 7}{k} + \frac{4}{k} + 1}{\frac{2k - 7}{k}}$ $\frac{3k - 3}{k} \times \frac{k}{2k - 7} = \frac{3(k - 1)}{2k - 7}$ $(2k - 7)x^2 - 2(2k - 5)x + 3(k - 1) = 0$ ALTERNATIVE METHOD Let $w = \frac{x + 1}{x}$ $x = \frac{1}{w - 1}$ Hence $k\left(\frac{1}{w - 1}\right)^2 - 4\left(\frac{1}{w - 1}\right) + 2k - 7 = 0$ $\frac{k}{(w - 1)^2} - \frac{4}{w - 1} + 2k - 7 = 0$ $k - 4(w - 1) + (2k - 7)(w - 1)^2 = 0$ $k - 4w + 4 + (2k - 7)(w^2 - 2w + 1) = 0$ $k - 4w + 4 + 2kw^2 - 4kw + 2k - 7w^2 + 14w - 7 = 0$ $(2k - 7)x^2 - 2(2k - 5)x + 3(k - 1) = 0$ | B1 M1 M1 A1 M1 M1 A1 A1 (8) B1 M1 M1 A1 M1 M1 A1 A1 (8) [12] |

| | Notes |
|------------|---|
| (a) | |
| M1 | For use of $b^2 - 4ac$ (Ignore inequality for this mark) |
| M1 | For a 3 TQ ≤ 0 oe |
| | For solving their 3TQ (Ignore inequality for this mark) May be implied by |
| M1 | $-\frac{1}{2}$ and 4 seen as critical values |
| A1 | For $-\frac{1}{2} \leq k \leq 4$ Allow $<$ instead of \leq Accept $-\frac{1}{2} < k < 4$ |
| (b) | |
| B1 | For $\alpha + \beta = \frac{4}{k}$ and $\alpha\beta = \frac{2k-7}{k}$ |
| M1 | For $\frac{\alpha+1}{\alpha} + \frac{\beta+1}{\beta} = \frac{2\alpha\beta + \alpha + \beta}{\alpha\beta}$ |
| M1 | For substitution into $\frac{2\alpha\beta + \alpha + \beta}{\alpha\beta}$ with some attempt to simplify |
| A1 | For $\frac{2(2k-5)}{2k-7}$ |
| M1 | For $\frac{\alpha+1}{\alpha} \times \frac{\beta+1}{\beta} = \frac{\alpha\beta + \alpha + \beta + 1}{\alpha\beta}$ |
| M1 | For substitution into $\frac{\alpha\beta + \alpha + \beta + 1}{\alpha\beta}$ with some attempt to simplify |
| A1 | For $\frac{3(k-1)}{2k-7}$ |
| A1 | For $(2k-7)x^2 - 2(2k-5)x + 3(k-1) = 0$ |
| | Alternative |
| B1 | For $w = \frac{x+1}{x}$ |
| M1 | For rearranging to make x the subject |
| M1 | For substitution of $x = \frac{1}{w-1}$ into the quadratic |
| A1 | For $\frac{k}{(w-1)^2} - \frac{4}{w-1} + 2k - 7 = 0$ |
| M1 | For multiplying by $(w-1)^2$ |
| M1 | For expanding brackets |
| A1 | For $k - 4w + 4 + 2kw^2 - 4kw + 2k - 7w^2 + 14w - 7 = 0$ |
| A1 | For $(2k-7)x^2 - 2(2k-5)x + 3(k-1) = 0$ |