Question	Scheme	Marks
10(a)	$AC = \sqrt{(2x)^2 + x^2} = (\sqrt{5}x)$	B1
	$VA = '\sqrt{5}x' \sin 45^o = \frac{\sqrt{10}}{2}x$ *	M1A1 cso [3]
(b)	Height $= \sqrt{\left(\frac{\sqrt{10}}{2}x\right)^2 - \left(\frac{\sqrt{5}}{2}x\right)^2} = \frac{\sqrt{5}}{2}x$	M1A1 [2]
(c)	$\cos\angle VBA = \frac{1}{\frac{\sqrt{10}}{2}}$	M1
	$\Rightarrow \angle VBA = 50.768\ldots^o \approx 50.8^o$ awrt	A1 [2]
(d)	Let M be the point at which the diagonals AC and BD meet.	
	The required angle is either AMB or DMC	B1
	$\text{AM} = \text{MB} = \frac{AC}{2} = \frac{\sqrt{5}x}{2}$ or $\text{DM}=\text{MC}=\frac{AC}{2} = \frac{\sqrt{5}x}{2}$	B1
	$\cos\angle AMB = \frac{\left(\frac{\sqrt{5}}{2}\right)^2 + \left(\frac{\sqrt{5}}{2}\right)^2 - 2^2}{2 \times \frac{\sqrt{5}}{2} \times \frac{\sqrt{5}}{2}}$ oe	M1
		A1 [4]
	$\left(\cos\angle AMB = -\frac{3}{5} \Rightarrow \angle AMB = 126.869\ldots^o\right) \approx 126.9^o$ awrt	
(e)	$9\sqrt{5} = \frac{1}{3} \times 2x \times x \times "\frac{\sqrt{5}}{2}x" \Rightarrow x = \cdots$	M1
	$x = 3$	A1 [2]
		Total 13 marks

Part	Mark	Notes
(a)	**B1**	For using correct Pythagoras theorem to find length AC or ½ AC, explicit substitution must be seen.
	M1	For correct trigonometry using their AC in triangle VAC or half of their AC in triangle VAM or correct Pythagoras theorem (let M be the centre of the base at which the diagonals AC and BD meet) to find length VA. *e.g.* $$VA = \sqrt{5}x\cos(45°) = \frac{\sqrt{10}}{2}x \quad or$$ $$VA = \frac{\sqrt{5}}{2}x \div \cos(45°) = \frac{\sqrt{10}}{2}x \quad or \quad VA = \frac{\sqrt{5}}{2}x \div \sin(45°) = \frac{\sqrt{10}}{2}x$$
	A1 cso	Achieves printed answer with no errors. Condone AV instead of VA Note: $\frac{\sqrt{10}}{2}x$ *written as* $\frac{\sqrt{10x}}{2}$ *is A0*, $\frac{\sqrt{10}}{2}x$ *written as* $\frac{\sqrt{10}x}{2}$ *is A1*
(b)	**M1**	Uses the correct Pythagoras theorem or correct trigonometry with the given length VA=$\frac{\sqrt{10}}{2}x$ to find the height of the pyramid.
	A1	For $h = \frac{\sqrt{5}}{2}x$ Correct exact answer only scores M1A1. This can be written down as the triangle is isosceles.
(c)	**M1**	**Ignore missing *x* consistently throughout their solution.** For using any appropriate trigonometry to find the angle VBA If attempts cosine rule, look for correct application of the cosine rule for the required angle: $$\left(\frac{\sqrt{10}}{2}x\right)^2 = \left(\frac{\sqrt{10}}{2}x\right)^2 + (2x)^2 - 2\times\frac{\sqrt{10}}{2}x\times 2x\cos(VBA) \text{ oe}$$
	A1	For the correct angle. awrt 50.8 º (with or without degree sign)
(d)	**B1**	Identifies the angle required.(can be implied/embedded in their working)
	B1	For the correct lengths AM and MB **or** DM and MC **or** BE and ME Let E to be mid point of AB, correct length BE=x, ME=0.5x scores B1, this mark can be implied/embedded in their working)
	M1	**Ignore missing *x* consistently throughout their solution.** For using any appropriate trigonometry to find the required angle or half of the required angle.
	A1	For the correct angle. awrt 126.9º (with or without degree sign)
(e)	**M1**	For using the correct formula for the volume of a pyramid with their height of the pyramid and attempts to find a value of x.
	A1	For $x = 3$ with or without unit, ignore incorrect unit

Question	Scheme	Marks
11(a)	$\cos(A+A)=\cos A\cos A-\sin A\sin A$ $\Rightarrow \cos 2A=\cos^2 A-(1-\cos^2 A)$	M1
	$\cos 2A=2\cos^2 A-1$ *	A1 cso [2]
(b)	$(2\cos^2 A-1)^2=(\cos 2A)^2$	M1
	$\cos 4A=2\cos^2 2A-1\Rightarrow \cos^2 2A=\dfrac{\cos 4A+1}{2}$	M1
	$(2\cos^2 2A-1)^2=\dfrac{\cos 4A+1}{2}$ *	A1 cso [3]
(b) ALT	$\cos 4A=2\cos^2 2A-1$	M1
	$=2(2\cos^2 A-1)^2-1$	M1
	$(2\cos^2 A-1)^2=\dfrac{\cos 4A+1}{2}$ *	A1 cso [3]
(c)	$y=\dfrac{\sin 2x}{2}+\dfrac{(2\cos^2 x-1)^2}{2}+\dfrac{1}{8}\Rightarrow y=\dfrac{\sin 2x}{2}+\dfrac{\cos 4x}{4}+\dfrac{3}{8}$	M1
	$\dfrac{dy}{dx}=\cos 2x-\sin 4x$	dM1
	$\dfrac{dy}{dx}=\cos 2x-2\sin 2x\cos 2x\{=0\}$	B1
	$\{\cos 2x(1-2\sin 2x)=0\}\Rightarrow \sin 2x=\dfrac{1}{2}$	
	$\Rightarrow x=\dfrac{\sin^{-1}\left(\frac{1}{2}\right)}{2}=\cdots$	ddM
	$x=\frac{\pi}{12}$	A1
	$y=\dfrac{\sin 2\left("\frac{\pi}{12}"\right)}{2}+\dfrac{\cos 4\left("\frac{\pi}{12}"\right)}{4}+\dfrac{3}{8}=\dfrac{3}{4}\{\Rightarrow\left(\dfrac{\pi}{12},\ \dfrac{3}{4}\right)\}$	M1A1 [7]
		Total 12 marks

Part	Mark	Notes
(a)	M1	For correct use of $cos(A+B) = cos\,A\,cos\,B - sin\,A\,sin\,B \Rightarrow$ $cos(A+A) = cos\,A\,cos\,A - sin\,A\,sin\,A$ ***OR*** $cos\,2\,A = cos^2\,A - sin^2\,A$ **with** the Pythagorean identity to eliminate sine from the identity.
	A1 cso	Achieves printed answer with no errors seen. Left hand side must be cos2A not cos(A+A)
(b)	M1	Uses the given result from (a), replaces $(2\cos^2 A - 1)^2$with $(\cos 2A)^2$
	M1	Uses the given result from (a) or a correct identity to replace $\cos^2 2A$ with $\frac{\cos 4A+1}{2}$
	A1 cso	Fully correct proof showing all necessary steps. (If they meet in the middle, they must have a conclusion, accept e.g. shown, proved, L=R, etc.)
(b) ALT	M1	Uses the given result from (a) on cos 4A
	M1	Uses the given result from (a) or a correct identity on cos 2A
	A1 cso	Fully correct proof showing all necessary steps. (If they meet in the middle, they must have a conclusion, accept e.g. shown, proved, L=R, etc.)
(c)	M1	Rearranges the given y into the form $\frac{sin\,2x}{2} + \frac{cos\,4x+1}{k} + l$, oe, where k and l are nonzero constants Or $\frac{sin\,2x}{2} + \frac{cos\,4x}{P} + Q$, oe, where P and Q are nonzero constants (Allow mixed variables x and A for this method mark only)
	dM1	For an attempt to differentiate their y into the form $P\cos(2x) \pm Q\sin(4x)\ oe, where\ P, Q\ are\ nonzero\ constants$ **Depends on the previous method mark.**
	B1	Uses $\sin 4x = 2\sin 2x \cos 2x$ or $\sin 4x = \sqrt{1-\cos^2 4x}$ oe in their dy/dx (may be implied by correct working)
	ddM1	Sets their differentiated expression = 0 (can be implied) and attempts a correct method to solve for x=… **Depends on both previous method marks.**
	A1	For $x = \frac{\pi}{12}$, ignore extra x values are outside of given range. Withhold A mark if extra solution seen in the given range.
	M1	For attempting to find the value of y using their x, found by solving their $\frac{dy}{dx} = 0$, provided x is in the range $0 \leqslant x \leqslant \frac{\pi}{6}$ Substitutes into the given y **or** their y, correct answer implies this mark, but for incorrect answer, must see explicit substitution
	A1	For the correct exact coordinates of P, $\left(\frac{\pi}{12}, \frac{3}{4}\right)$ or $\left(\frac{\pi}{12}, 0.75\right)$, accept $x = \frac{\pi}{12}$, $y = \frac{3}{4}$ or $x = \frac{\pi}{12}$, $y = 0.75$

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