Question Number	Scheme	Marks
9(a)	$\cos ADB = \dfrac{6^2 + x^2 - 12^2}{2 \times 6x} = \dfrac{x^2 - 108}{12x}$	M1A1 (2)
(b)	$\cos BDC = \dfrac{6^2 + x^2 - 6^2}{2 \times 6x} = \dfrac{x^2}{12x}$	B1
	$ADB = 180 - BDC \Rightarrow -\dfrac{x^2}{12x} = \dfrac{x^2 - 108}{12x}$	M1A1
	$2x^2 = 108 \Rightarrow x = 3\sqrt{6}$	
	$AC = 6\sqrt{6}$	A1 (4)
(c)	$\dfrac{\sin(\theta° + \phi°)}{2x} = \dfrac{\sin BCD}{12} \Rightarrow \dfrac{\sin(\theta° + \phi°)}{x} = \dfrac{\sin BCD}{6}$	M1A1
	$\dfrac{\sin \phi°}{x} = \dfrac{\sin BCD}{6}$	M1
	$\therefore \sin \phi° = \sin(\theta° + \phi°)$	A1 (4)
(d)	$\sin(\theta° + \phi°) = \sin \phi° \Rightarrow (\theta + \phi) = 180 - \phi$ $\left(\text{or } \phi \text{ or } 360 + \phi \text{ (not possible)}\right)$	M1
	$\therefore \theta = 180 - 2\phi$	A1 (2)
		[12]

Part	Mark	Notes
(a)	M1	For using a correct cosine rule $\cos ADB = \dfrac{6^2 + x^2 - 12^2}{2 \times 6x} = \dfrac{x^2 - 108}{12x}$ or $12^2 = x^2 + 6^2 - 2 \times 6 \times x \cos ADB$
	A1	Simplifies to $\cos ADB = \dfrac{x^2 - 108}{12x}$
(b)	B1	For the correct expression $\cos BDC = \dfrac{6^2 + x^2 - 6^2}{2 \times 6x} = \dfrac{x^2}{12x}$
	M1	$\cos BDC = -\cos ADB$ and $\cos BDC = -\dfrac{x^2}{12x}$ so $-\dfrac{x^2}{12x} = \dfrac{x^2 - 108}{12x}$ and attempts to solve
	ALT 1 – uses triangles *BAD* and *BAC*	
	B1	For **both** of the following correct expressions for cos *BAD* and cos *BAC*: $\cos BAD = \dfrac{12^2 + x^2 - 6^2}{2 \times 12 \times x}$ and $\cos BAC = \dfrac{12^2 + (2x)^2 - 6^2}{2 \times 12 \times 2x}$
	M1	$\angle BAC = \angle BAD$ so equates their two expressions $\dfrac{12^2 + x^2 - 6^2}{2 \times 12 \times x} = \dfrac{12^2 + (2x)^2 - 6^2}{2 \times 12 \times 2x} \Rightarrow \dfrac{108 + x^2}{24x} = \dfrac{108 + 4x^2}{48x}$ and attempts to solve
	ALT 2 – uses triangles *BCD* and *BCA*	

	B1	For **both** of the following correct expressions for cos *BAD* and cos *BAC*: $\cos BCD = \dfrac{6^2+x^2-6^2}{2\times6\times x}$ and $\cos BCA = \dfrac{6^2+(2x)^2-12^2}{2\times6\times2x}$
	M1	$\dfrac{6^2+x^2-6^2}{2\times6\times x} = \dfrac{6^2+(2x)^2-12^2}{2\times6\times2x} \Rightarrow x^2 = 2x^2-54$ and attempts to solve
	Final A marks for all three methods	
	A1	For the correct value of $x = 3\sqrt{6}$
	A1	For $AC = 6\sqrt{6}$
(c)	M1	Uses sine rule on triangle *ABC*: $\dfrac{\sin(\theta°+\phi°)}{2x} = \dfrac{\sin BCD}{12} \Rightarrow \dfrac{\sin(\theta°+\phi°)}{x} = \dfrac{\sin BCD}{6}$
	A1	Achieves the correct expression for $\sin(\theta°+\phi°) = \dfrac{x\sin BCD}{6}$
	M1	Uses sine rule on triangle *BDC* : $\dfrac{\sin\phi°}{x} = \dfrac{\sin BCD}{6} \Rightarrow \left(\sin\phi° = \dfrac{x\sin BCD}{6}\right)$
	A1	Shows that $\sin\phi° = \sin(\theta°+\phi°)$ with no errors
	ALT 1 – Uses exact values for the trigonometric ratios and the expansion for sin (*A* + *B*)	
	M1	Finds $\cos\theta = \dfrac{7}{8} \Rightarrow \sin\theta = \dfrac{\sqrt{15}}{8}$ **or** $\cos\phi = \dfrac{1}{4} \Rightarrow \sin\phi = \dfrac{\sqrt{15}}{4}$ Accept $\theta = 28.95...°$ **or** $\phi = 75.52...° \Rightarrow \sin\phi = 0.968...$
	A1	Finds $\cos\theta = \dfrac{7}{8} \Rightarrow \sin\theta = \dfrac{\sqrt{15}}{8}$ **and** $\cos\phi = \dfrac{1}{4} \Rightarrow \sin\phi = \dfrac{\sqrt{15}}{4}$ Accept $\theta = 28.95...°$ **and** $\phi = 75.52...° \Rightarrow \sin\phi = 0.968...$
	M1	Expands $\sin(\theta+\phi) = \dfrac{\sqrt{15}}{8}\times\dfrac{1}{4} + \dfrac{\sqrt{15}}{4}\times\dfrac{7}{8} = \left(\dfrac{\sqrt{15}}{4}\right)$ Or $\sin(\theta+\phi)° = \sin(28.95°)\cos(75.52°) + \sin(75.52°)\cos(28.95°) = 0.968... = \sin\phi°$
	A1	Shows that $\sin(\theta+\phi) = \dfrac{\sqrt{15}}{4}$ and $\sin\phi = \dfrac{\sqrt{15}}{4}$ so $\sin(\theta+\phi) = \sin\phi$ with no errors. If they use approximate values for sin θ and ϕ withhold this final mark so A0
	ALT 2 – Uses $\angle BCD = 52.2...°$	
	M1	Finds $\angle BCD = 52.2...°$ using cosine rule and applies sine rule on triangle *ABC* $\dfrac{\sin(\theta°+\phi°)}{6\sqrt{6}} = \dfrac{\sin 52.2°}{12}$
	A1	Shows that $\sin(\theta°+\phi°) = 0.968...$
	M1	Uses sine rule on triangle *BD*: $\dfrac{\sin\phi°}{3\sqrt{6}} = \dfrac{\sin 52.2°}{6} \Rightarrow \sin\phi° = 0.968... = \sin(\theta°+\phi°)$
	A1	If they use an approximate value for angle *BCD* withhold this final mark so A0
(d)	M1	For writing $\sin\phi° = \sin(180-\phi)° \Rightarrow \theta°+\phi° = 180°-\phi°$
	A1	For rearranging $\theta°+\phi° = 180°-\phi°$ to achieve $\therefore \theta = 180-2\phi$ This is a show question and there must be no errors here.