Pearson Edexcel

# Mark Scheme (Results)

## January 2020

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1)
Paper 01R

**Edexcel and BTEC Qualifications**

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

**Pearson: helping people progress, everywhere**

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

January 2020
Publications Code 4PM1_01R_2001_MS

**General Marking Guidance**

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.
  Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks – can only be awarded when relevant M marks have been gained
  - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
  - cao – correct answer only
  - cso – correct solution only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - awrt – answer which rounds to
  - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers may score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**
  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.

  If a candidate misreads a number from the question: eg. uses 252 instead of 255; follow through their working and deduct 2A marks from any gained provided the work has not been simplified. (Do not deduct any M marks gained.)

  If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes clear the method that has been used

  Examiners should send any instance of a suspected misread to review (but see above for simple misreads).

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. incorrect cancelling of a fraction that would otherwise be correct.

  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

**General Principles for Further Pure Mathematics Marking**

(but note that specific mark schemes may sometimes override these general principles)

**Method mark for solving a 3 term quadratic equation:**

1. Factorisation:

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$ leading to $x=....$

$(ax^2+bx+c)=(mx+p)(nx+q)$ where $|pq|=|c|$ and $|mn|=|a|$ leading to $x=....$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=....$

3. Completing the square:

$x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0$, $q\neq 0$ leading to $x=....$

**Method marks for differentiation and integration:**

1. Differentiation

Power of at least one term decreased by 1. $(x^n\to x^{n-1})$

2. Integration:

Power of at least one term increased by 1. $(x^n\to x^{n+1})$

**Use of a formula:**

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

**or**, where the formula is not quoted, the method mark can be gained by implication
from the substitution of correct values and then proceeding to a solution.

**Answers without working:**

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

**Exact answers:**

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

**Rounding answers (where accuracy is specified in the question)**

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the is rule may allow the mark to be awarded before the final answer is given.

# MARK SCHEME

| Question number | Scheme | Marks |
|---|---|---|
| 1 | $\dfrac{\left(a+\sqrt{3}\right)}{\left(2-\sqrt{3}\right)}\times\dfrac{\left(2+\sqrt{3}\right)}{\left(2+\sqrt{3}\right)}=\dfrac{2a+\sqrt{3}\left(a+2\right)+3}{1}$ | M1 |
| | $2a+\sqrt{3}\left(a+2\right)+3=11+b\sqrt{3}\Rightarrow 11=2a+3,\quad b=a+2$ | M1M1 |
| | Solves the equations in $a$ and $b$ $\Rightarrow a=4,\quad b=6$ | A1<br>[4] |
| | **ALT** | |
| | $\dfrac{a+\sqrt{3}}{2-\sqrt{3}}=11+b\sqrt{3}\Rightarrow a+\sqrt{3}=\left(2-\sqrt{3}\right)\left(11+b\sqrt{3}\right)$ | {M1} |
| | $\Rightarrow a+\sqrt{3}=(22-3b)+(2b-11)\sqrt{3}$<br>$\Rightarrow a=22-3b$ and $1=2b-11$ | {M1}{M1} |
| | Solves the equations in $a$ and $b$ $\Rightarrow a=4,\quad b=6$ | {A1}<br>[4] |
| | | **Total 4 marks** |
| **(a)** | | |
| **M1** | Multiply by $\dfrac{\left(2+\sqrt{3}\right)}{\left(2+\sqrt{3}\right)}$ | |
| **M1** | For either $11=2a+3$ **or** $b=a+2$ | |
| **M1** | For $11=2a+3$ **and** $b=a+2$ | |
| **A1** | $a=4,\quad b=6$ | |
| **ALT** | | |
| **M1** | Multiply by $2-\sqrt{3}$ | |
| **M1** | For either $a=22-3b$ **or** $1=2b-11$ | |
| **M1** | For $a=22-3b$ **and** $1=2b-11$ | |
| **A1** | $a=4,\quad b=6$ | |

| Question number | Scheme | Marks |
|---|---|---|
| 2 (a) | $7+4x-x^2=11-(x-2)^2$<br>[$a$ = 11, $b$ = 1, $c$ = −2]<br>**ALT**<br>$7+4x-x^2=a-b\left(x^2+2cx+c^2\right)$<br>$a-bc^2=7$ $b=1$ $bc=4$ So $a$ = 11, $b$ = 1, $c$ = −2<br>$7+4x-x^2=11-(x-2)^2$ | M1A1A1<br>[3]<br><br>{M1}<br>{A1}{A1}<br>[3] |
| (b) | (i) 11<br>(ii) 2 | B1ft<br>B1ft<br>[2] |
| | | **Total 5 marks** |
| **(a)** | | |
| **M1** | An attempt to factorise to make $x^2$ positive e.g. $-(x\pm p)^2\pm q$ | |
| **A1** | Complete the square to obtain an expression in the form $-(x\pm 2)^2\pm q$ **NB** Any expression in this form will score M1A1 | |
| **A1** | $11-(x-2)^2$ or $a$ = 11, $b$ = 1, $c$ = −2 | |
| **ALT** | | |
| **M1** | Expands $a-b(x+c)$ | |
| **A1** | $a-bc^2=7$ $b=1$ $bc=4$ | |
| **A1** | $11-(x-2)^2$ or $a$ = 11, $b$ = 1, $c$ = −2 | |
| **(b) (i)**<br>**B1ft** | **Mark parts b(i) and b(ii) together**<br>11 follow through their a | |
| **(b) (ii)**<br>**B1ft** | 2 follow through their c<br>**NB** Answer of Max = (2, 11) score B1B1 | |

| Question number | Scheme | Marks |
|---|---|---|
| 3(a) | $\frac{dy}{dx} = 2e^{2x}\left(x^2+1\right)+e^{2x}\left(2x\right)$ | M1A1A1 [3] |
| (b) | When $x = 0$<br>$\frac{dy}{dx} = 2\times1\times1+1\times0 = 2$  $y = e^{2\times0}\left(0+1\right) = 1$<br>$y-1 = 2\left(x-0\right) \Rightarrow y = 2x+1$ | B1B1<br>B1 [3] |
| | | **Total 6 marks** |
| **(a)** | | |
| **M1** | Attempted use of the product rule. Sum of two terms (either way round) with $x^n \to x^{n-1}$ (Condone $e^{2x}$ instead of $2e^{2x}$) Once the correct answer is seen ISW. This mark may be implied by the sum of two terms with one of the two terms correct. | |
| **A1** | Either term correct | |
| **A1** | Both terms correct | |
| **(b)** | | |
| **B1** | When $x = 0$ $\frac{dy}{dx} = 2$ | |
| **B1** | When $x = 0$ $y = 1$ | |
| **B1** | $y = 2x+1$ | |

| Question number | Scheme | Marks |
|---|---|---|
| 4 (a) | $f(2)=2\times2^3+a\times2^2+b\times2+18=0$<br>$f'(x)=6x^2+2ax+b \Rightarrow f'(2)=6\times2^2+2\times a\times2+b=5$<br>$4a+2b+34=0$<br>$4a+b+19=0$<br>$\Rightarrow b=-15,\ a=-1$ | M1<br>M1M1<br>A1<br>M1A1<br>[6] |
| (b) | $x-2\overline{)2x^3-x^2-15x+18}$ with quotient $2x^2+3x-9$<br>$2x^2+3x-9=(x+3)(2x-3)$<br>$\Rightarrow(x-2)(x+3)(2x-3)$ | M1<br>M1A1<br>[3] |
| (c) | $x=2,\ -3,\ \frac{3}{2}$ | B2ft<br>[2] |
| | | **Total 11 marks** |

| | |
|---|---|
| **(a)** | |
| **M1** | $f(2)=0$ leading to an equation in $a$ and $b$ |
| **M1** | Attempt to differentiate |
| **M1** | $f'(2)=5$ leading to an equation in $a$ and $b$ |
| **A1** | $4a+2b+34=0$ and $4a+b+19=0$ |
| **M1** | Solving simultaneously |
| **A1** | $b=-15,\ a=-1$ |
| **(b)** | |
| **M1** | Dividing by $x-2$ to obtain a 3TQ |
| **M1** | Factorising the 3TQ |
| **A1** | All 3 terms correct |
| **(c)** | |
| **B2 ft** | $x=2,\ -3,\ \frac{3}{2}$<br>(**B1** for 2 correct) |

| Question number | Scheme | Marks |
|---|---|---|
| 5 (a) | $\log_4 32 = \dfrac{\log_2 32}{\log_2 4} = \dfrac{5}{2}$* or $\log_4 32 = \log_4 4^{\frac{5}{2}} = \dfrac{5}{2}$*<br>or $\log_4 32 = \log_{2^2} 2^5 = \dfrac{5}{2}$*<br>**ALT**<br>$\log_4 32 = a \quad \Rightarrow 4^a = 32 \quad \Rightarrow a = \dfrac{5}{2}$ * | M1A1cso<br>[2]<br><br>{M1}{A1}<br>cso<br>[2] |
| (b) | $\log_2 x - \log_4 32 + \dfrac{1}{4}\log_x 16 = 0$<br>Let $\log_2 x = y$<br>$y - \dfrac{5}{2} + \dfrac{1}{4}\left(\dfrac{\log_2 16}{\log_2 x}\right) = 0$ or $y - \dfrac{5}{2} + \dfrac{1}{\log_2 x} = 0$<br>$\Rightarrow y - \dfrac{5}{2} + \dfrac{1}{y} = 0$<br>$\Rightarrow 2y^2 - 5y + 2 = 0$<br>$\Rightarrow (2y-1)(y-2) = 0$<br>$\Rightarrow y = \log_2 x = \dfrac{1}{2}$ or $2$<br>$\Rightarrow x = 2^{\frac{1}{2}} = \sqrt{2}$ and $x = 2^2 = 4$ | M1<br><br>M1A1<br>M1<br>M1<br>M1A1<br>[7] |
| | | **Total 9 marks** |
| **(a)**<br>**M1** | For $\log_4 32 = \dfrac{\log_2 32}{\log_2 4}$ or $\log_4 32 = \log_4 4^{\frac{5}{2}}$ or $\log_4 32 = \log_{2^2} 2^5$ | |
| **ALT**<br>**M1** | For $4^a = 32$ | |
| **A1 cso** | Obtains the **given** answer with no errors in the working | |
| **(b)**<br>**M1** | Use of $\log_a x = \dfrac{\log_b x}{\log_b a}$ or $\log_a b = \dfrac{1}{\log_b a}$ | |
| **M1** | Forming a 3TQ | |
| **A1** | $2y^2 - 5y + 2 = 0$ | |
| **M1** | Solving the 3TQ | |
| **M1** | For $y = \log_2 x = \dfrac{1}{2}$ or 2 | |
| **M1** | Either $x = 2^{\frac{1}{2}} = \sqrt{2}$ or $x = 2^2 = 4$ | |
| **A1** | Both $x = 2^{\frac{1}{2}} = \sqrt{2}$ and $x = 2^2 = 4$ | |

<table>
<tr><th>Question number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>6 (a)</td><td>

| $x$ | 0.5 | 1 | 1.5 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|---|
| $y$ | $-11.5$ | **-2** | **0.2** | 1.3 | 2.7 | **3.8** | **4.9** | 5.9 |

</td><td>B2<br>[2]</td></tr>
<tr><td>(b)</td><td>Points plotted within half of a square<br>Points joined together in a smooth curve</td><td>B1ft<br>B1ft<br>[2]</td></tr>
<tr><td>(c)</td><td>$\frac{x^3-3}{x^2}=ax+b \Rightarrow x^3-3=ax^3+bx^2 \Rightarrow 0=x^3(a-1)+bx^2+3$<br>Comparing coefficients<br>$x^3(a-1)-bx^2+3=2x^3+6x^2+3$<br>$\Rightarrow a=3, b=-6$ so line required is $y=3x-6$<br>Draws the line $y=3x-6$ and identifies two intersections with the curve when $x=0.8/0.9$ and $x=2.8/2.9$</td><td>M1<br><br>M1A1<br>M1<br>A1 (both)<br>[5]</td></tr>
<tr><td></td><td>ALT</td><td></td></tr>
<tr><td></td><td>$2x^3-6x^2+3=0 \Rightarrow 2x-6=-\frac{3}{x^2}$</td><td>{M1}</td></tr>
<tr><td></td><td>$3x-6=x-\frac{3}{x^2}$ so line required is $y=3x-6$</td><td>{M1}{A1}</td></tr>
<tr><td></td><td>Draws the line $y=3x-6$ and identifies two intersections with the curve when $x=0.8/0.9$ and $x=2.8/2.9$</td><td>{M1}<br>{A1} both<br>[5]</td></tr>
<tr><td colspan="3" align="right">Total 9 marks</td></tr>
</table>

| | |
|---|---|
| **(a)** | |
| **B2** | All 4 points correct<br>(**B1** for 3 points correct) |
| **(b)** | |
| **B1ft** | Points plotted ft their table allow half a square tolerance |
| **B1ft** | Points joined together with a smooth curve ft their table |
| **(c)** | |
| **M1** | Setting $x-\dfrac{3}{x^2}=ax+b$ and simplifying to $x^3\left(a-1\right)+bx^2+3$ |
| **M1** | Comparing coefficients |
| **A1** | Identifying that the line required is $y=3x-6$ |
| **M1** | $y=3x-6$ drawn intersecting the curve in two places |
| **A1** | $x=0.8/0.9$ and $x=2.8/2.9$ |
| **ALT** | |
| **M1** | Subtracting 3 from both sides and dividing by $x^2$ |
| **M1** | Adding $x$ to both sides |
| **A1** | Identifying that the line required is $y=3x-6$ |
| **M1** | $y=3x-6$ drawn intersecting the curve in two places |
| **A1** | $x=0.8/0.9$ and $x=2.8/2.9$ |

| Question number | Scheme | Marks |
|---|---|---|
| 7 (a)(i) | $a+4d=4x+6$<br>$a+7d=7x+3$<br>$\Rightarrow 3d=3x-3 \Rightarrow d=x-1$* | M1A1cso |
| (ii) | $a+7(x-1)=7x+3 \Rightarrow a=10$ or $a+4(x-1)=4x+6 \Rightarrow a=10$ | M1A1<br>[4] |
| (b) | $42=10+8(x-1) \Rightarrow x=5$ | M1A1<br>[2] |
| (c) | $d=x-1 \Rightarrow d=5-1=4$<br>$S_{n+1}=12U_n+18 \Rightarrow \frac{n+1}{2}\left(2\times 10+\left[(n+1)-1\right]4\right)=12\left[10+(n-1)4\right]+18$<br>$\Rightarrow n^2-18n-40=0$<br>$\Rightarrow (n-20)(n+2)=0 \Rightarrow n=20$ | B1<br>M1<br>M1<br>M1A1<br>[5] |
| | | **Total 11 marks** |

| | |
|---|---|
| **(a) (i)** | |
| **M1** | $a+4d=4x+6$ and $a+7d=7x+3$ |
| **A1 cso** | Obtains the **given** answer with no errors in the working |
| **(a) (ii)** | |
| **M1** | Substitution of $d=x-1$ |
| **A1** | $a=10$ |
| **(b)** | |
| **M1** | Use of $a+8d=42$ |
| **A1** | $x=5$ |
| **(c)** | |
| **B1** | $d=4$ |
| **M1** | Use of $\frac{n}{2}(2a+(n-1)d)$ |
| **M1** | Simplifying to $n^2-18n-40=0$ |
| **M1** | Solving the 3TQ |
| **A1** | $n=20$ if shown must reject $n=-2$ |

| Question number | Scheme | Marks |
|---|---|---|
| 8 | $s=\int\left(3+5t-2t^2\right)\,\mathrm{d}t=3t+\frac{5t^2}{2}-\frac{2t^3}{3}+c$ | M1 |
| | when $t=0\ \ s=5$ | |
| | $5=0+0-0+c$ | B1 |
| | $s=5+3t+\frac{5t^2}{2}-\frac{2t^3}{3}$ | A1 |
| | When $s=x$ | |
| | $\frac{\mathrm{d}x}{\mathrm{d}t}=3+5t-2t^2=0\Rightarrow(2t+1)(t-3)=0\Rightarrow t=3$ | M1A1 |
| | $\Rightarrow x=5+3\times3+\frac{5\times3^2}{2}-\frac{2\times3^3}{3}=\frac{37}{2}$ oe | A1 |
| | $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=5-4t$ when $t=3$ $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=-7\Rightarrow\max$ | M1A1 |
| | | [8] |
| | | **Total 8 marks** |

| | |
|---|---|
| **M1** | Attempt to integrate |
| **B1** | $c=5$ |
| **A1** | $s=5+3t+\frac{5t^2}{2}-\frac{2t^3}{3}$ |
| **M1** | Solving $3+5t-2t^2=0$ |
| **A1** | $t=3$ if shown must reject $t=-\frac{1}{2}$ |
| **A1** | $x=\frac{37}{2}$ oe |
| **M1** | Differentiates to obtain $\left(\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=\right)5-4t$ |
| **A1** | Establish that the maximum has been obtained and give a conclusion |

| Question number | Scheme | Marks |
|---|---|---|
| 9 (a) | $a=-1,\ b=-2$ | B1,B1 [2] |
| (b) | Gradient of $l_1=-2, \Rightarrow$ Gradient of $l_2=\frac{1}{2}$ | B1,B1 |
| | $180=(x+1)^2+(y-6)^2$ | M1 |
| | $\frac{1}{2}=\frac{y-6}{x+1}\Rightarrow x=2y-13$ | M1 |
| | Solves simultaneous equations; | |
| | $180=([2y-13]+1)^2+(y-6)^2\Rightarrow 0=5y^2-60y$ | M1M1 |
| | or $180=(x+1)^2+\left(\frac{1}{2}x+\frac{13}{2}-6\right)^2\Rightarrow 0=x^2+2x-143=0$ | |
| | $y=0,\ y=12\Rightarrow x=-13,\ x=11$ or $x=-13,\ x=11\Rightarrow y=0,\ y=12$ | A1A1 |
| | Coordinates are $(-13,\ 0)$ and $(11,\ 12)$ | [8] |
| (c) | Area of triangle $PQR$ | |
| | $PQ=\sqrt{(6+2)^2+(-1-3)^2}=4\sqrt{5}$ | M1 |
| | Area $=\frac{1}{2}\times 4\sqrt{5}\times 6\sqrt{5}=60$ (units)$^2$ | M1A1 [3] |
| | **ALT** | |
| | Area $=\frac{1}{2}\begin{pmatrix}-13 & -1 & 3 & -13\\ 0 & 6 & -2 & 0\end{pmatrix}=\frac{1}{2}(-78+2+0-0-18-26)=-60$ | {M1}{M1} |
| | $\Rightarrow 60$ (units)$^2$ | {A1} [3] |
| (d) | Coordinates of $R$ required are $(-13,\ 0)$ | |
| | $\angle RPQ=90^\circ$ so $RQ$ is a diameter | |
| | $\left(\frac{-13+3}{2},\frac{0-2}{2}\right)\Rightarrow(-5,\ -1)$ | M1A1 [2] |
| | | **Total 15 marks** |
| **(a)** | | |
| **B1** | $a=-1$ | |
| **B1** | $b=-2$ | |

| | |
|---|---|
| **(b)** | |
| **B1** | Gradient of $l_1 = -2$ |
| **B1** | Gradient of $l_2 = \frac{1}{2}$ |
| **M1** | Use of $PR = 6\sqrt{5}$ to obtain an equation |
| **M1** | Use of gradient of the perpendicular to obtain an equation |
| **M1** | Solves simultaneously |
| **M1** | Simplifies to $5y^2 - 60y = 0$ or $x^2 + 2x - 143 = 0$ |
| **A1** | All 4 values identified 0, 12, –13, 11 |
| **A1** | $(11,\ 12)$ and $(-13,\ 0)$ (must be paired correctly and if written as a coordinate then must be in the correct order) |
| **(c)** | |
| **M1** | $PQ = 4\sqrt{5}$ |
| **M1** | Use of Area $= \frac{1}{2} \times PQ \times PR$ |
| **A1** | 60 (units)$^2$ |
| **ALT** | |
| **M1** | Use of Area $= \frac{1}{2}\begin{pmatrix} -13 & -1 & 3 & -13 \\ 0 & 6 & -2 & 0 \end{pmatrix}$ ft $R$ provided $e < 0$ |
| **M1** | $\frac{1}{2}(-78 + 2 + 0 - 0 - 18 - 26)$ ft $R$ provided $e < 0$ |
| **A1** | 60 (units)$^2$ |
| **(d)** | |
| **M1** | $\left(\frac{-13+3}{2}, \frac{0-2}{2}\right)$ ft $R$ provided $e < 0$ |
| **A1** | $(-5,\ -1)$ |

| Question number | Scheme | Marks |
|---|---|---|
| 10 (a) | $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -(2\mathbf{a}-\mathbf{b}) + 3\mathbf{a} + \mathbf{b} = \mathbf{a} + 2\mathbf{b}$ | M1A1 [2] |
| (b) | $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = 3\mathbf{a} + \mathbf{b} - \mathbf{a} + 3\mathbf{b} = 2\mathbf{a} + 4\mathbf{b} = 2(\mathbf{a} + 2\mathbf{b})$<br>Conclusion required; same direction and $\overrightarrow{OC}$ is a multiple of $\overrightarrow{AB}$ therefore $\overrightarrow{OC}$ is parallel to $\overrightarrow{AB}$. | M1<br>A1 [2] |
| (c) | $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \mathbf{a} + 2\mathbf{b} + (-\mathbf{a} + 3\mathbf{b}) = 5\mathbf{b}$<br>$\overrightarrow{AX} = \mu\overrightarrow{AC} = \mu 5\mathbf{b}$<br>$\overrightarrow{AX} = \overrightarrow{AO} + \overrightarrow{OX} = -(2\mathbf{a}-\mathbf{b}) + \lambda(3\mathbf{a}+\mathbf{b})$<br>$\Rightarrow \mu 5\mathbf{b} = -(2\mathbf{a}-\mathbf{b}) + \lambda(3\mathbf{a}+\mathbf{b})$<br>$\Rightarrow -2 + 3\lambda = 0 \Rightarrow \lambda = \frac{2}{3}$<br>$\Rightarrow 5\mu = 1 + \lambda \Rightarrow \mu = \frac{1}{3}$<br>$\Rightarrow AX : XC = 1:2$ | B1<br>B1<br>M1<br>M1<br>M1<br>A1<br>A1 [7] |
| | | **Total 11 marks** |
| **(a)** | | |
| **M1** | Use of $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$ | |
| **A1** | $\mathbf{a} + 2\mathbf{b}$ | |
| **(b)** | | |
| **M1** | Use of $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC}$ | |
| **A1** | Correct conclusion i.e. $\overrightarrow{OC} = 2\overrightarrow{AB}$ $\therefore \overrightarrow{OC}$ is parallel to $\overrightarrow{AB}$ | |
| **(c)** | | |
| **B1** | $\overrightarrow{AC} = 5\mathbf{b}$ may be implied by 2nd B1 | |
| **B1** | $\overrightarrow{AX} = \mu 5\mathbf{b}$ or $\overrightarrow{XC} = \lambda 5\mathbf{b}$ | |
| **M1** | A correct vector for $\overrightarrow{AX}$ or $\overrightarrow{OX}$ or $\overrightarrow{BX}$ or $\overrightarrow{CX}$ or $\overrightarrow{BC}$ including an unknown multiple of a vector e.g. $\overrightarrow{AX} = -(2\mathbf{a}-\mathbf{b}) + \lambda(3\mathbf{a}+\mathbf{b})$ | |
| **M1** | Equate 2 forms of the same vector e.g. $\mu 5\mathbf{b} = -(2\mathbf{a}-\mathbf{b}) + \lambda(3\mathbf{a}+\mathbf{b})$ | |
| **M1** | Comparing coefficients | |
| **A1** | $\lambda = \frac{2}{3}$ or $\mu = \frac{1}{3}$ | |
| **A1** | $AX : XC = 1:2$ | |

| Question number | Scheme | Marks |
|---|---|---|
| 11 (a) | at $P$ $\quad b=\sqrt{a-2}\Rightarrow b^2=a-2$* | M1A1cso [2] |
| (b) | At $P$ $y=b\Rightarrow y^2=b^2\Rightarrow y^2=a-2$ | B1 |
| | $V=\pi\int_a^{16}\left(\sqrt{x-2}\right)^2\,dx-\pi\int_a^{16}(a-2)\,dx=\pi\int_a^{16}(x-2)\,dx-\pi\int_a^{16}(a-2)\,dx$ | M1 |
| | $\Rightarrow \pi\int_a^{16}(x-a)\,dx$ or $\pi\int_a^{16}\left(\sqrt{x-2}\right)^2\,dx-\pi(a-2)(16-a)$ | |
| | $50\pi=\pi\left[\frac{x^2}{2}-ax\right]_a^{16}$ or $50\pi=\pi\left[\frac{x^2}{2}-2x\right]_a^{16}-\pi(a-2)(16-a)$ | depM1A1 |
| | $50\pi=\pi\left[\left(\frac{256}{2}-16a\right)-\left(\frac{a^2}{2}-a^2\right)\right]$ | depM1 |
| | or $50\pi=\pi\left[\left(96-\frac{a^2}{2}+2a\right)-\left(18a-a^2-32\right)\right]$ | |
| | $\Rightarrow a^2-32a+156=0$ | M1 |
| | $\Rightarrow (a-6)(a-26)=0\Rightarrow a<16$ so $a=6$ | M1A1 |
| | $b^2=a-2\Rightarrow b^2=4\Rightarrow b=2$ | A1 [9] |
| | | **Total 11 marks** |

| | |
|---|---|
| **(a)** | |
| **M1** | $b=\sqrt{a-2}$ |
| **A1 cso** | Obtains the **given** answer with no errors in the working |
| **(b)** | |
| **B1** | $y^2=a-2$ |
| **M1** | Use of $V=\pi\int_a^{16}\left(\sqrt{x-2}\right)^2\,dx-\pi\int_a^{16}(a-2)\,dx$ or $V=\pi\int_a^{16}(x-a)\,dx$ or $\pi\int_a^{16}\left(\sqrt{x-2}\right)^2\,dx-\pi(a-2)(16-a)$ Ignore limits |
| **depM1** | Attempts to integrate. Ignore limits (Dependent on previous M1) |
| **A1** | Correct integration. Ignore limits |
| **depM1** | Correct substitution of limits (Dependent on previous M1) |
| **M1** | Obtaining the 3TQ |
| **M1** | Solving the 3TQ |
| **A1** | $a=6$ |
| **A1** | $b=2$ |

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom