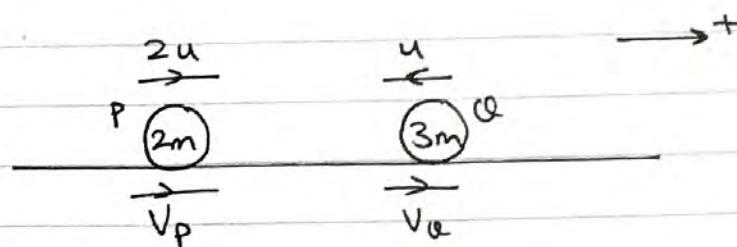


M1 October 2016 (IAL) (MA)

(Q1a)



$$\text{C.L.M} : 2m(2u) + 3m(-u) = 2m(V_p) + 3m(V_Q)$$

$$\Rightarrow u = 2V_p + 3V_Q$$

Impulse on P acts towards the left.

$$\therefore -5mu = 2m(V - u) = -5mu = 2m(V_p - 2u)$$

$$\Rightarrow -5u = 2V_p - 4u$$

$$\Rightarrow 2V_p = -u \quad \therefore V_p = -\frac{u}{2}$$

$$\text{so speed of P} = \boxed{\frac{u}{2}}$$

- b) $V_p < 0$ so P is travelling in the opposite direction to that which we assumed in the diagram.
So yes the dir. of P has been reversed.

c) from (a), $u = 2V_p + 3V_Q$

$$\Rightarrow u = 2\left(-\frac{u}{2}\right) + 3V_Q$$

$$\Rightarrow 3V_Q = 2u \quad \therefore V_Q = \boxed{\frac{2u}{3}} = \text{speed}_Q$$

(Q2a)

$$\sum \vec{F} = m\vec{a} : \underbrace{\begin{pmatrix} -10 \\ a \end{pmatrix} + \begin{pmatrix} b \\ -5 \end{pmatrix} + \begin{pmatrix} 2a \\ 7 \end{pmatrix}}_{m} = 3 \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2a + b - 10 \\ 2 + a \end{pmatrix} = \begin{pmatrix} 9 \\ 12 \end{pmatrix}$$

equating i :

$$2a + b - 10 = 9$$

$$\Rightarrow 2a + b = 19 \sim ①$$

equating j :

$$2 + a = 12$$

$$\Rightarrow \boxed{a = 10} //$$

$$① : 2(10) + b = 19$$

$$\text{so } \boxed{b = -1} //$$

b)

$$a = \frac{v-u}{t} : 3\vec{i} + 4\vec{j} = \frac{(20\vec{i} + 20\vec{j}) - (\vec{u})}{(4)}$$

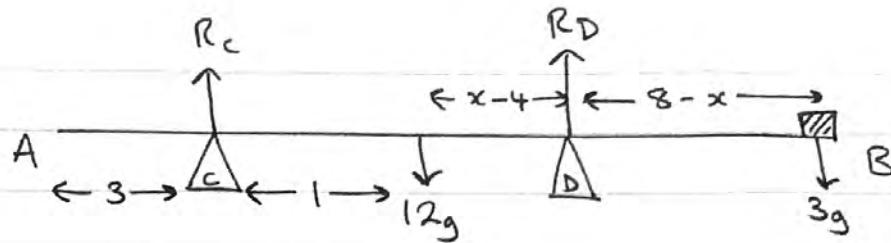
$$\stackrel{\times 4}{\Rightarrow} 12\vec{i} + 16\vec{j} = 20\vec{i} + 20\vec{j} - (\vec{u})$$

$$\Rightarrow \vec{u} = 20\vec{i} - 12\vec{i} + 20\vec{j} - 16\vec{j}$$

$$\Rightarrow \vec{u} = 8\vec{i} + 4\vec{j} //$$

$$\text{hence } u = |\vec{u}| = \sqrt{8^2 + 4^2} = \boxed{4\sqrt{5}} \text{ ms}^{-1}$$

(Q3)



We are told: $R_D = 2R_C$

$$\begin{aligned} R(\uparrow) : \quad R_C + R_D &= 12g + 3g \\ R_C + 2R_C &= 15g \\ \therefore R_C &= \frac{15g}{3} = 5g // \quad \rightarrow R_D = 10g // \end{aligned}$$

$$M(A) : R_C(3) + R_D(x) = 12g(4) + 3g(8)$$

$$5g(3) + 10g(x) = 48g + 24g$$

$$10x + 15 = 72$$

$$10x = 57 \quad \therefore x = \frac{57}{10} = 5.7m$$

(Q4a) $\mathbf{p} = \mathbf{r}_0 + \mathbf{v}t$

$$\mathbf{p} = \left(-\frac{5}{9}\right) \mathbf{i} + t\left(-\frac{1}{2}\right) \mathbf{j} \Rightarrow \boxed{\mathbf{p} = (-5+t)\mathbf{i} + (9-2t)\mathbf{j}}$$

b) $\underline{j} = 2$ when P is due west of A.

$$\therefore 9 - 2t = 2$$

$$\Rightarrow 7 = 2t \quad \therefore t = \frac{7}{2},$$

$$\text{at } t = \frac{7}{2}, \quad \mathbf{p} = \left(-5 + \frac{7}{2}\right) \mathbf{i} + (9 - 7) \mathbf{j}$$

$$\boxed{\mathbf{p} = -\frac{3}{2} \mathbf{i} + 2 \mathbf{j}}$$

c) if they move along parallel lines then their velocity vectors are parallel.

$$\Rightarrow \begin{pmatrix} 2b - 1 \\ 5 - 2b \end{pmatrix} = c \begin{pmatrix} 1 \\ -2 \end{pmatrix} \text{ for some } c.$$

$$\therefore 2b - 1 = c - ① \text{ (equating i)}$$

$$5 - 2b = -2c - ② \text{ (equating ii)}$$

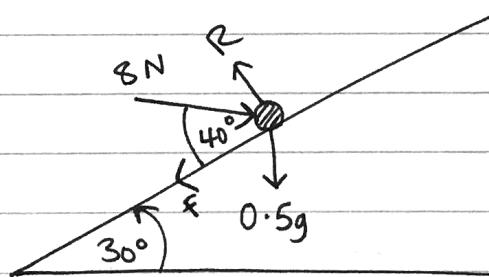
$$\underline{\underline{① + ②}} : 2b - 1 + 5 - 2b = c - 2c$$

$$4 = -c \quad \therefore c = -4 //$$

$$\text{and } 2b - 1 = -4$$

$$\Rightarrow b = \frac{-4 + 1}{2} = \boxed{\frac{-3}{2}}$$

(Q5)



$$R(\uparrow) : R = (8\sin 40 + 0.5g\cos 30)$$

$$R(\leftarrow) : 8\cos 40 = F + 0.5g\sin 30$$

but $F = \mu R$ as P is in the point of sliding up the plane.

$$\therefore 8\cos 40 = \mu R + 0.5g \sin 30$$

$$8\cos 40 = \mu(8\sin 40 + 0.5g \cos 30) + 0.5g \sin 30$$

$$\begin{aligned} \text{so } \mu &= \frac{8\cos 40 - 0.5g \sin 30}{8\sin 40 + 0.5g \cos 30} \\ &= \boxed{0.392} \end{aligned}$$

Q6) For A : $s = s_A$
 $u = 35$
 $v = v_A$
 $a = 0.4$
 $t =$

For B : $s = s_B$
 $u = 44$
 $v = v_B$
 $a = 0.5$
 $t =$

When B overtakes A, $s_B - 200 = s_A$, (since B starts 200m behind)

$$s_A = 35t + 0.2t^2$$

$$\text{and } s_B = 44t + 0.25t^2$$

$$\text{but using } s_B - 200 = s_A,$$

$$\Rightarrow 44t + 0.25t^2 - 200 = 35t + 0.2t^2$$

$$\Rightarrow 0.05t^2 + 9t - 200 = 0$$

$$\times 20 \Rightarrow t^2 + 180t - 4000 = 0$$

By Quadratic formula :

$$t = 20 \quad \text{or} \quad t = -200$$

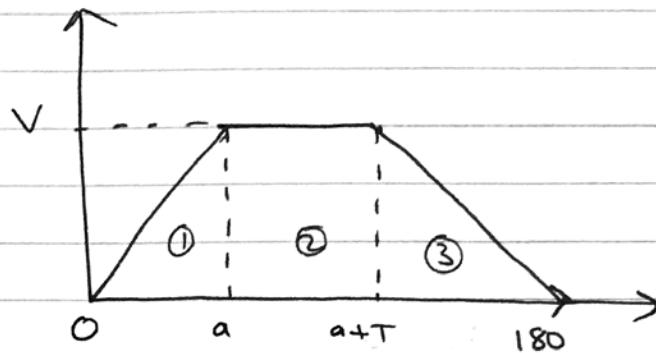
// $\underbrace{t > 0}_{\text{}} \quad \text{.}$

So at $t = 20$ B overtakes A.

$$\text{using } v = u + at \text{ with } B = V_B = 44 + 0.5(20)$$

$$= \boxed{54}$$

(Q7a)



b) $a = \frac{v-u}{t}$ for ① : $t = \frac{V-0}{a}$
 $\therefore V = a$.

$a = \frac{v-u}{t}$ for ③ : $-0.5 = \frac{0-V}{180-(a+T)}$

$$-90 + 0.5(T+V) = -V$$

$$90 - \frac{1}{2}(T+V) = V$$

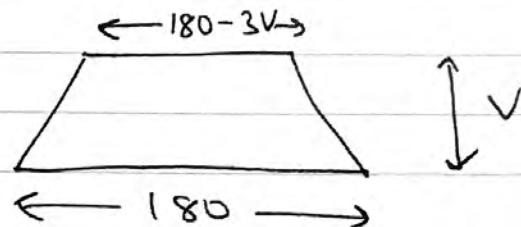
$$180 - T - V = 2V$$

$$\therefore T = 180 - 3V$$



c) Total area under graph = 4800 //

$$\text{Area of trapezium} = \frac{(a+b)h}{2} = 4800$$



$$\therefore 4800 = \frac{1}{2} (180 + 180 - 3v)v$$

$$9600 = 360v - 3v^2$$

$$\div 3 : 3200 = 120v - v^2$$

$$v^2 - 120v + 3200 = 0$$

$$(v-40)(v-80) = 0$$

$$\text{so } \boxed{v=40} \text{ or } v=80$$

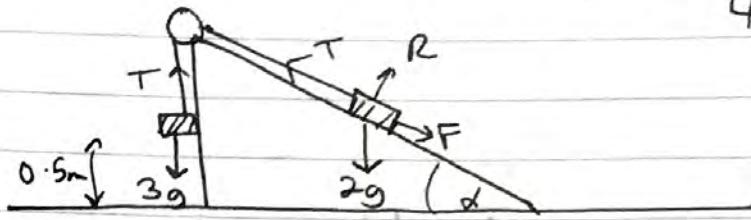
can't be $v=80$ as then area of ① would be greater than 4800m which isn't possible.

(Q8)

$$\cos \alpha = \frac{4}{5}$$

$$\sin \alpha = \frac{3}{5}$$

$$\tan \alpha = \frac{3}{4}$$



N2L(Q): $3g - T = 3a \sim ①$

N2L(P): $T - 2g \sin \alpha - F = 2a \sim ②$

(Q) $\left| \begin{array}{l} S = 0.5 \\ U = 0 \\ V = 1.4 \\ a = a \\ t = \end{array} \right\} \begin{array}{l} V^2 = U^2 + 2as \\ 1.4^2 = 0^2 + 2a(0.5) \\ 1.4^2 = a \quad // \end{array}$

from ①: $T = 3g - 3a = 3g - 3(1.4^2)$
 $= \boxed{23.5\text{N}}$

b) ②: $T - 2g \sin \alpha - \mu R = 2a$

$$23.5 - 2g\left(\frac{3}{5}\right) - \mu(2g \cos \alpha) = 2(1.4^2)$$

$$23.5 - \frac{6g}{5} - \mu(2g \times \frac{4}{5}) = 2(1.4^2)$$

$$\therefore \mu = \frac{23.5 - \frac{6g}{5} - 2(1.4^2)}{2g \times \frac{4}{5}} = \boxed{0.5}$$