Write your name here

Surname	Other names

Pearson Edexcel
International GCSE

Centre Number	Candidate Number

Further Pure Mathematics

Paper 1

Friday 12 January 2018 – Morning
Time: 2 hours

Paper Reference
4PM0/01

Calculators may be used.

Total Marks

Instructions

- Use black ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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P 5 3 2 9 1 A 0 1 3 6

Pearson

Answer all TEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 $f(x) = 6 + 5x - 2x^2$

Given that $f(x)$ can be written in the form $p(x + q)^2 + r$, where p, q and r are rational numbers,

(a) find the value of p, the value of q and the value of r.

(3)

(b) Hence, or otherwise, find

(i) the maximum value of $f(x)$,

(ii) the value of x for which this maximum occurs.

(2)

$$g(x) = 6 + 5x^3 - 2x^6$$

(c) Write down

(i) the maximum value of $g(x)$,

(ii) the exact value of x for which this maximum occurs.

(3)

Question 1 continued

(Total for Question 1 is 8 marks)

2 (a) On the grid opposite, draw

(i) the line with equation $y = 3x - 3$

(ii) the line with equation $3x + 2y = 12$

(2)

(b) Show, by shading, the region R defined by the inequalities

$$y \leqslant 3x - 3 \qquad 3x + 2y \leqslant 12 \qquad y \geqslant -1$$

(2)

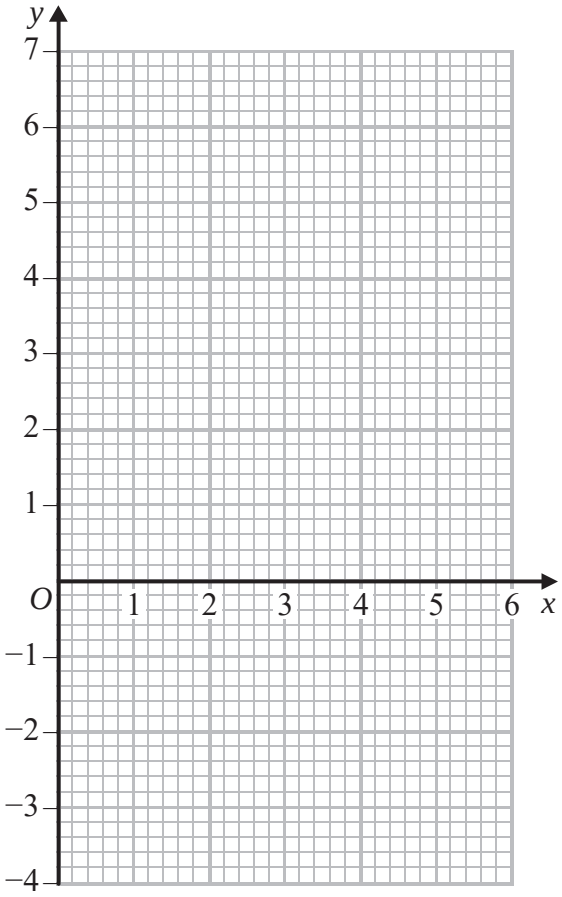
For all points in R with coordinates (x, y)

$$P = 4x - y$$

(c) Find the greatest value of P.

(4)

Question 2 continued

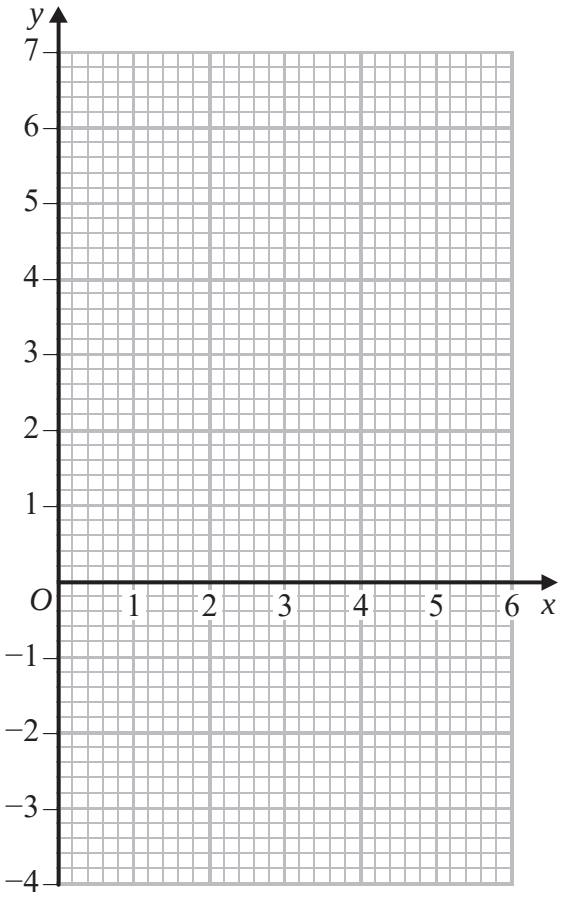


Turn over for a spare grid if you need to redraw your graph

Question 2 continued

Question 2 continued

Only use this grid if you need to redraw your graph



(Total for Question 2 is 8 marks)

3 The volume of a right circular cone is increasing at a constant rate of $27\,\text{cm}^3/\text{s}$. The radius of the base of the cone is always 1.5 times the height of the cone.

Calculate the rate of change of the height of the cone, in cm/s to 3 significant figures, when the height of the cone is 4 cm.

(6)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

(Total for Question 3 is 6 marks)

4 A particle P moves along the x-axis. At time t seconds ($t \geqslant 0$), the displacement of P from the origin is x metres and the velocity, v m/s, of P is given by $v = 2t^2 - 16t + 30$

(a) Find the times at which P is instantaneously at rest.

(2)

(b) Find the acceleration of P at each of these times.

(3)

When $t = 0$, P is at the point where $x = -4$

(c) Find the distance of P from the origin when P first comes to instantaneous rest.

(3)

Question 4 continued

(Total for Question 4 is 8 marks)

5 (a) Complete the table of values for $y = \dfrac{x^3 + 2}{x + 1}$ giving your answers to 2 decimal places where appropriate.

x	0	0.5	1	1.5	2	3	4
y		1.42		2.15		7.25	

(2)

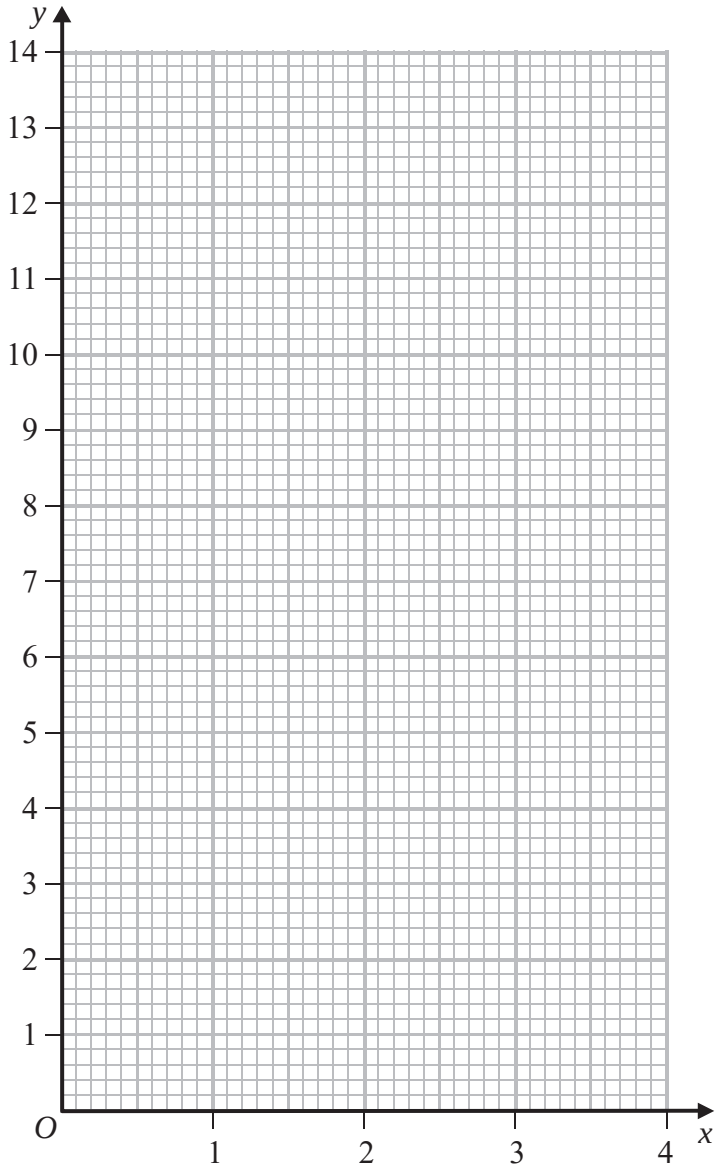
(b) On the grid opposite draw the graph of $y = \dfrac{x^3 + 2}{x + 1}$ for $0 \leqslant x \leqslant 4$

(2)

(c) By drawing a suitable straight line on your graph obtain an estimate, to 1 decimal place, of the root of the equation $x^3 + x^2 - 3x - 2 = 0$ in the interval $0 \leqslant x \leqslant 4$

(5)

Question 5 continued

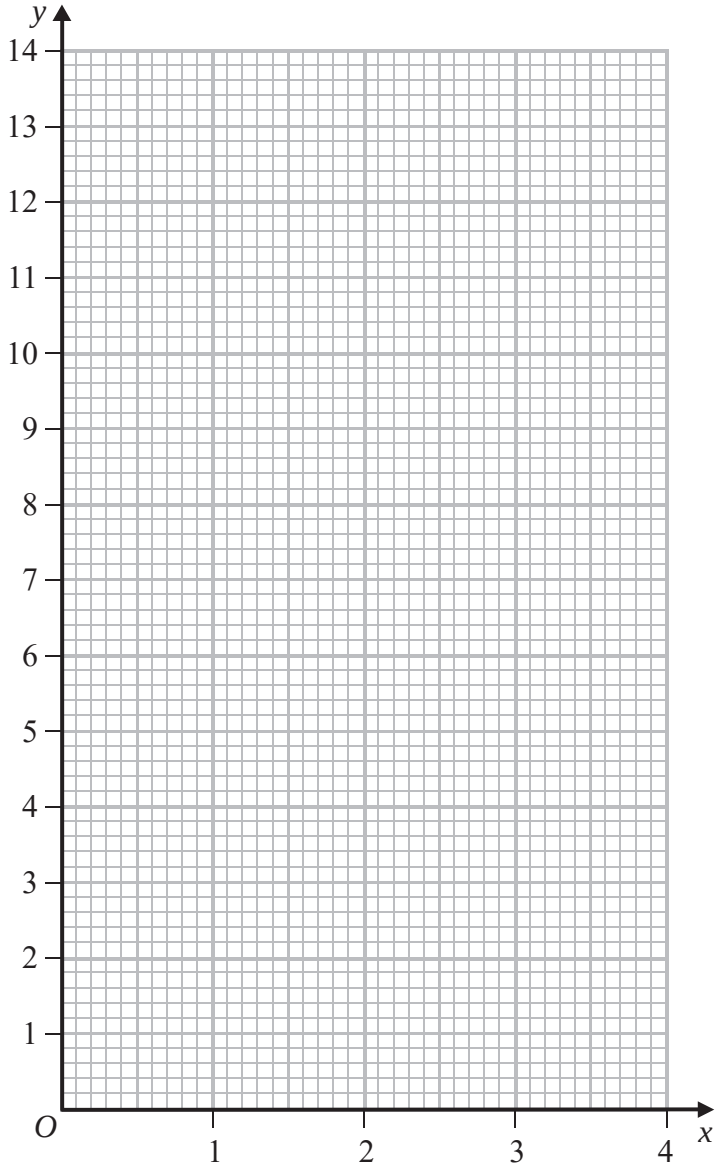


Turn over for a spare grid if you need to redraw your graph

Question 5 continued

Question 5 continued

Only use this grid if you need to redraw your graph



(Total for Question 5 is 9 marks)

6

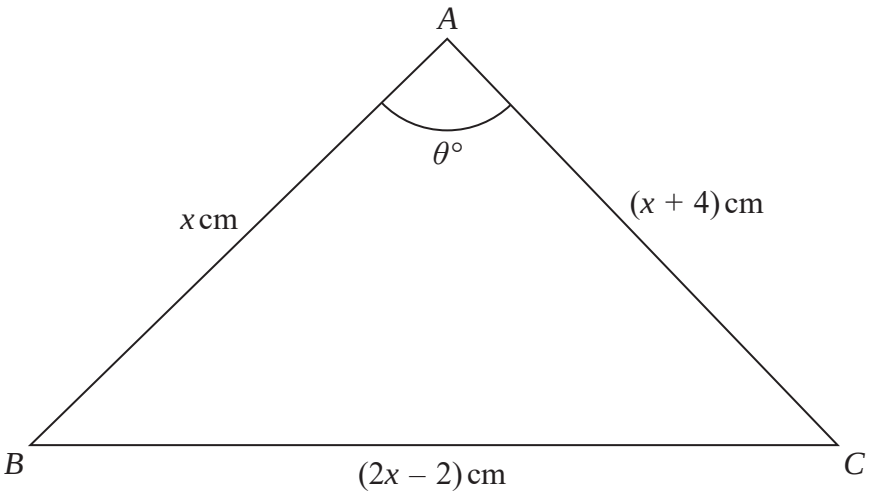


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows the triangle ABC with $AB = x$ cm, $BC = (2x - 2)$ cm, $AC = (x + 4)$ cm and $\angle BAC = \theta°$

Given that $\tan\theta° = \sqrt{255}$ and without finding the value of θ,

(a) show that $\cos\theta° = \frac{1}{16}$

(2)

Hence find

(b) the value of x,

(5)

(c) the size, in degrees to 1 decimal place, of $\angle ABC$,

(2)

(d) the area, in cm^2 to 3 significant figures, of triangle ABC.

(2)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 11 marks)

7 (a) Expand $(1 - 4x^2)^{-\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^6, giving each coefficient as an integer.

(3)

(b) Write down the range of values of x for which your expansion is valid.

(1)

(c) Expand $\dfrac{3 + x}{\sqrt{(1 - 4x^2)}}$ in ascending powers of x up to and including the term in x^4, giving each coefficient as an integer.

(3)

(d) Hence, use algebraic integration to obtain an estimate, to 3 significant figures, of

$$\int_0^{0.3} \frac{3 + x}{\sqrt{(1 - 4x^2)}}\,\mathrm{d}x$$

(4)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 11 marks)

8 The sixth term of a geometric series G, with common ratio r ($r \neq 0$), is four times the second term.

(a) Find the two possible exact values of r.

(2)

The sum of the third and seventh terms of G is 30

(b) Find the first term of the series.

(3)

Given that $r > 0$

(c) find the sum of the first 10 terms of G.

(2)

Given that t_n is the nth term of G,

(d) find the least value of n for which $t_n > 2400$

(3)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 10 marks)

9 It is given that α and β are such that $\alpha + \beta = -\frac{5}{2}$ and $\alpha\beta = -5$

(a) Form a quadratic equation with integer coefficients that has roots α and β

(2)

Without solving the equation found in part (a)

(b) find the value of

(i) $\alpha^2 + \beta^2$

(ii) $\alpha^3 + \beta^3$

(5)

(c) Hence form a quadratic equation with integer coefficients that has roots $\left(\alpha - \frac{1}{\alpha^2}\right)$ and $\left(\beta - \frac{1}{\beta^2}\right)$

(6)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 13 marks)

10 $\cos(A + B) = \cos A \cos B - \sin A \sin B$

(a) Show that $\cos^2\theta = \frac{1}{2}(\cos 2\theta + 1)$

(3)

Given that $f(\theta) = 8\cos^4\theta + 8\sin^2\theta - 7$

(b) show that $f(\theta) = \cos 4\theta$

(5)

(c) Solve, for $0 \leqslant \theta \leqslant \frac{\pi}{2}$, the equation

$$16\cos^4\left(\theta - \frac{\pi}{6}\right) + 16\sin^2\left(\theta - \frac{\pi}{6}\right) - 15 = 0$$

(4)

(d) Using calculus, find the exact value of

$$\int_0^{\frac{\pi}{2}} (8\cos^4\theta + 8\sin^2\theta + 2\sin 2\theta)\, d\theta$$

(4)

Question 10 continued

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 16 marks)

TOTAL FOR PAPER IS 100 MARKS