# Mark Scheme (Results)

## Summer 2024

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1) Paper 02R

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

**Method mark for solving a 3 term quadratic equation:**

1. Factorisation:

$(x^2 + bx + c) = (x + p)(x + q)$,where $|pq| = |c|$ leading to $x =\ldots$.

$(ax^2 + bx + c) = (mx + p)(nx + q)$ where$|pq| = |c|$ and $|mn| = |a|$leading to $x =\ldots$.

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading ccto $x =\ldots$.

3. Completing the square:

$x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$ leading to $x =\ldots$.

**Method marks for differentiation and integration:**

1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. Integration:

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

**Use of a formula:**

Generally, the method mark is gained by **either** quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values **or**, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

**Answers without working:**

The rubric states "Without sufficient working, correct answers may be awarded no marks". General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

**Exact answers**

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

**Rounding answers (where accuracy is specified in the question)**

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the is rule may allow the mark to be awarded before the final answer is give

| Question | Scheme | Marks |
|---|---|---|
| 1 | $[\pm(2k+6)]^2 - 4\times k\times 16$<br>$4k^2 - 40k + 36 = 0$<br>$\rightarrow \{4\}(k-1)(k-9)\{=0\} \rightarrow k = ...$<br>$k = 1, 9$ | M1<br>dM1<br>M1<br>A1<br>[4] |
| | | **Total 4 marks** |

| Mark | Notes |
|---|---|
| **M1** | **Allow working on expression, equation or any inequality for all M marks**<br>Applies $b^2 - 4ac = [\pm(2k+6)]^2 - 4\times k\times 16$ and might be seen embedded in an attempt at the quadratic formula. |
| **dM1** | Forms a 3TQ in $k$ by attempting to expand and collect the like terms. |
| **M1** | Attempts to solve their 3TQ in $k$ to find at least one real value of $k$ by any correct method (See General Principles). Condone labelling their $\boldsymbol{k}$ as $\boldsymbol{x}$ for this method mark.<br>If there is no method shown, (use of a calculator) then the 3TQ must be correct and both $k$ values must be fully correct ($k$ = 1, $k$ = 9) for evidence for this mark. |
| **A1** | For $k$ = 1, 9 selected as their final answers. (Extra inequality answers for $k$ must be rejected) |

| Question | Scheme | Marks |
|---|---|---|
| **1 ALT I (Via completing Square)** | $k(x - \frac{k+3}{k})^2 - \frac{(k+3)^2}{k} + 16$<br>$16 - \frac{(k+3)^2}{k} = 0 \rightarrow k^2 - 10k + 9 = 0$<br>$(k-1)(k-9)\{=0\} \rightarrow k = ...$<br>$k = 1, 9$ | M1<br>dM1<br>M1<br>A1<br>[4] |
| | | **Total 4 marks** |

| Mark | Notes |
|---|---|
| **M1** | Attempts to complete the square for $kx^2 - (2k+6)x + 16$ (See General Principles)<br><br>Similarly, divides both sides of the given quadratic equation by $k$ first, then attempts to complete the square: $(x - \frac{k+3}{k})^2 - \frac{(k+3)^2}{k^2} + \frac{16}{k}$ |
| **dM1** | Sets the $y$ coordinate of their turning point to be zero ( = 0 can be implied) attempts to expand and collect like terms. |
| **M1** | Solves their 3TQ or polynomial to find at least one real value of $k$ by any correct method (See General Principles)<br>If there is no method shown, (use of a calculator) then their 3TQ must be correct and both $k$ values must be fully correct ($k$ =1, $k$ = 9)for evidence for this mark. |
| **A1** | For $k$ = 1, 9 only |

| Question | Scheme | Marks |
|---|---|---|
| **1 ALT II (Via sum and product of roots)** | $\alpha+\beta=\pm\frac{2k+6}{k},\alpha\times\beta=\frac{16}{k}\to\left\{\alpha+\alpha=\pm\frac{2k+6}{k},\alpha\times\alpha=\frac{16}{k}\right\}$ | M1 |
| | $(\frac{k+3}{k})^2=\frac{16}{k}\to k^2-10k+9=0$ | dM1 |
| | $(k-1)(k-9)\{=0\}\to k=...$ | M1 |
| | $k=1\ ,\ 9$ | A1 |
| | | [4] |
| | | **Total 4 marks** |

| Mark | Notes |
|---|---|
| **M1** | For: $\alpha+\beta=\pm\frac{2k+6}{k},\alpha\times\beta=\frac{16}{k}\to\left\{\alpha+\alpha=\pm\frac{2k+6}{k},\alpha\times\alpha=\frac{16}{k}\right\}$ |
| **dM1** | Uses the fact that the quadratic equation has equal roots $\alpha=\beta,$ sets up a correct equation in $k$, expands, collects like terms to form a 3TQ (or a Cubic equation)<br>e.g. $\left(\frac{2k+6}{2k}\right)^2=\frac{16}{k}\to 4k^3-40k^2+36k=0$ or $4k^2-40k+36=0$ |
| **M1** | Solves their 3TQ or Cubic equation to find at least one real value of $k$ by any correct method (See General Principles)<br>If there is no method shown, (use of a calculator) then their 3TQ or Cubic must be correct and both $k$ values must be fully correct ($k=1, k=9$) for evidence for this mark. |
| **A1** | For $k=1,\ 9$ only |

**ALT III**: (**Via differentiation**)

**M1**: Differentiates $kx^2-(2k+6)x+16$ with respect to $x$ to achieve a linear expression

**dM1**: Sets their derivative to 0, solves for $x$ and substitutes $x$ back to $kx^2-(2k+6)x+16=0$ to form a 3TQ.
**M1A1** same as ALT II

e.g. $kx^2-(2k+6)x+16\to 2kx-2k-6=0\to x=1+\frac{3}{k}$

$\to k\left(1+\frac{3}{k}\right)^2-(2k+6)(1+\frac{3}{k})+16=0$

$\to k=...$

**Send to review if not sure**

| Question | Scheme | Marks |
|---|---|---|
| **2(a)** | $\dfrac{2}{\sqrt{1+3x}} = 2(1+3x)^{\left(-\frac{1}{2}\right)}$ | B1 |
| | $\{2\}\left(1+\left(-\frac{1}{2}\right)(3x)+\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(3x)^2}{2!}+\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)(3x)^3}{3!}+\ldots\right)$ | M1A1 |
| | $2-3x+\dfrac{27x^2}{4}-\dfrac{135x^3}{8}+\ldots$ | A1 [4] |
| **(b)** | $-\dfrac{1}{3}<x<\dfrac{1}{3}$ (Accept $\lvert x\rvert<\dfrac{1}{3}$) | B1 [1] |
| | | **Total 5 marks** |

| Part | Mark | Notes |
|---|---|---|
| **(a)** | **B1** | Correct simplification of the given expression.<br>This mark may be implied by a correct expansion. |
| | **M1** | Attempts at the binomial expansion $(1+3x)^{\pm\frac{1}{2}}$ or $2(1+3x)^{\pm\frac{1}{2}}$<br>For an attempt at the binomial expansion.<br>• The first term is 1 or 2<br>• The powers of $3x$ are correct in all terms, e.g. $(3x)^2$<br>• The correct denominators are used, 2! and 3! oe |
| | **A1** | Allow this mark for at least 1 correct algebraic term, correctly simplified, from<br>$-3x+\dfrac{27x^2}{4}-\dfrac{135x^3}{8}$ or $-\dfrac{3}{2}x+\dfrac{27x^2}{8}-\dfrac{135x^3}{16}$ |
| | **A1** | Correct simplified expansion in ascending order, coefficients must be in simplest **fractions**. (Ignore extra terms with powers > 3) Do not isw |
| **(b)** | **B1** | For the correct range of values of $x$. $-\dfrac{1}{3}<x<\dfrac{1}{3}$ or $\lvert x\rvert<\dfrac{1}{3}$ , do not accept $\lvert 3x\rvert<1$ |

**ALT** for (a) Uses **Maclaurin's expansion (If seen send to review)**

$$f(x) = f(0) + f'(0)x + \frac{f''(x)}{2!}x^2 + \frac{f'''(x)}{3!}x^3 + \ldots$$

**B1** Correct simplification of the given expression.

**M1** Achieves $f'(x) = P(1+3x)^{-\frac{3}{2}}, f''(x) = Q(1+3x)^{-\frac{5}{2}}, f'''(x) = R(1+3x)^{-\frac{7}{2}}, P, Q, R \neq 0$ and attempts to find the values of *f'(0), f''(0)* and *f'''(0)*

**A1** Correct unsimplified expansion OR at least 2 correct simplified terms

**A1** Fully correct simplified expansion. (Ignore extra terms with powers higher than 3)

For reference, the correct derivatives are:

$$f'(x) = -3(1+3x)^{-\frac{3}{2}}, f''(x) = \frac{27}{2}(1+3x)^{-\frac{5}{2}}, f'''(x) = \frac{-405}{4}(1+3x)^{-\frac{7}{2}}$$

| Question | Scheme | Marks |
|---|---|---|
| 3(a) | $\overrightarrow{OA} = \overrightarrow{OB} - \overrightarrow{AB} = 7i + 2aj - (i + 3aj) = 6i - aj$ | M1A1 |
| | $\left\lvert\overrightarrow{OA}\right\rvert = \sqrt{6^2 + a^2} = 3\sqrt{5} \Rightarrow 6^2 + a^2 = 45 \Rightarrow a^2 = 9 \Rightarrow a = 3$ | M1A1 [4] |
| (b) | $\overrightarrow{OA} = 6i - '3'\,j$ | B1ft |
| | {Unit vector parallel to $\overrightarrow{OA}$ is} $\{\pm\}\frac{1}{3\sqrt{5}}(6i - 3j)$ oe ,isw | B1 [2] |
| | | **Total 6 marks** |

| Part | Mark | Notes |
|---|---|---|
| **(a)** | **M1** | For the correct vector statement with vector $\overrightarrow{OA}$ or $\overrightarrow{AO}$<br>e.g. $\overrightarrow{OA} = \overrightarrow{OB} - \overrightarrow{AB}$ or $\left\{\overrightarrow{OA} =\right\} 7i + 2aj - (i + 3aj)$ oe |
| | **A1** | For the correct simplified or unsimplified vector $\overrightarrow{OA}$ **or** $\overrightarrow{AO}$ |
| | **M1** | For using the correct Pythagoras theorem on their $\overrightarrow{OA}$ or $\overrightarrow{AO}$ with $3\sqrt{5}$, leading to a value for $a^2$ or $a$ |
| | **A1** | For the correct value of $a$=3 only (must reject $a$=−3 if present)<br><br>**NB:A value of $a$=3 must come from a correct vector of OA with no incorrect work seen.** |
| **(b)** | **B1ft** | For the correct vector for $\overrightarrow{OA}$ or $\overrightarrow{AO}$ using their $a$ (which $a$>0), (Substitutes their a value to $\overrightarrow{OA} = 6i - aj$ or $\overrightarrow{AO} = -6i + aj$ |
| | **B1** | Correct unit vector $\frac{1}{3\sqrt{5}}(6i - 3j)$ $oe$, can be in either direction, so accept + or − isw |

**Extra notes:**

1. Allow working in column vectors throughout.
2. Condone missing arrow on their vectors.
3. **Special Case for part a:** Correct answer with minimum work, no incorrect working shown e.g. $\sqrt{6^2 + a^2} = 3\sqrt{5}$ M1A1M1 $\rightarrow 6^2 + a^2 = 45 \rightarrow a^2 = 9 \rightarrow a = 3$ A1

| Question | Scheme | Marks |
|---|---|---|
| 4(a) | $f'(x) = 3px^2 + 2qx - 37$<br>$f'(-2) = 3p(-2)^2 + 2q(-2) - 37 = -33$<br>$f(-5) = p(-5)^3 + q(-5)^2 - 37(-5) - 12q = 0$<br>$\begin{cases} 12p - 4q = 4 \\ -125p + 13q = -185 \end{cases} \Rightarrow p = \cdots\ q = \cdots$<br>$p = 2, q = 5$ | M1<br>M1<br>M1<br>dM1<br>A1A1<br>[6] |
| (b) | $x+5\overline{)2x^3 + "5"x^2 - 37x - "60"}$ with quotient $2x^2 - "5"x - "12"$<br>$OR$<br>$2x^3 + "5"x^2 - 37x - "60" = (x+5)(Ax^2 + Bx + C)$<br>$\Rightarrow A = 2, B = \cdots, C = \cdots$<br>$2x^2 - 5x - 12 = (2x+3)(x-4)$<br>$\{f(x) =\}(x+5)(2x+3)(x-4)$ | M1<br>M1<br>A1<br>[3] |
| (c) | $(x+5)(2x+3)(x-4) = 0, \Rightarrow\ x = -5,\ 4,\ -\frac{3}{2}$ | M1A1<br>[2] |
| | | **Total 11 marks** |

| Part | Mark | Notes |
|---|---|---|
| **(a) Mark i and ii together** | **M1** | For differentiating the given expression in terms of $p$ and $q$ with respect to $x$, at least one term correct and no terms integrated. |
| | **M1** | Attempts $f'(\pm 2) = -33$ |
| | **M1** | Attempts $f(\pm 5) = 0$ |
| | **dM1** | For solving the two resulting simultaneous equations by any correct method (allow slips) providing there are two variables $p$ and $q$ in both, leading a value of $p$ or $q$. Explicit method to solve must been seen.<br>**Depends on all previous method marks** |
| | **A1** | For $p = 2$ or $q = 5$ from correct working in two correct equations |
| | **A1** | For both $p = 2$ and $q = 5$ from correct working |
| **(b)** | **M1** | For using polynomial division (or any other **algebraic** method) to find the quadratic factor.<br>**Division:** Look for divisor of $(x+5)$, quotient of a quadratic $2x^2 -$ their $qx + c$ $(c \neq 0)$<br>**Compare coefficients**: minimum required is an identity of the correct form followed by values of $A = 2, B = -$their $q, C = \ldots (C \neq 0)$ |
| | **M1** | Correct method of factorising their quadratic factor. (See General Principles) |
| | **A1** | Correct factorised expression written in full on one line. |
| **(c)** | **M1** | Correct method of solving their $f(x) = 0$. Their $f(x)$ must have 3 solutions, one of which must be $x = -5$, "=0" or their method may be implied by their $x$ values. They must use their result in part b which $f(x)$ reaches 2 or 3 factors, if part b is not attempted, a full algebraic method of solving $f(x)=0$ must be seen in this part. |
| | **A1** | For all three correct values of $x$, no extra solutions. |

(a) **ALT I Algebraic Division**

**M1** Same as main scheme
**M1** Their f'*(x)* is in the form of a 3TQ, look for divisor of ($x$+2), quotient of a linear expression leading to a constant remainder in $\boldsymbol{p}$ and $\boldsymbol{q}$, sets = -33
**M1** For dividing f($x$)*,* look for divisor of ($x$+5), quotient of a 3TQ, leading to a constant remainder in $\boldsymbol{p}$ and $\boldsymbol{q}$**,** sets=0
**dM1A1** Same as main scheme

You may also see variations for their division, marking is similar as above, (if not sure, send to review) e.g.
For f'*(x)*

$$\begin{array}{r|cccc} -2 & 3p & & 2q & -37 \\ & & & -6p & 12p-4q \\ \hline & 3p & & -7p+2q & 12p-4q-37=-33 \end{array}$$

For f*(x)*

$$\begin{array}{r|cccc} -5 & p & q & -37 & -12q \\ & & -5p & -5q+25p & 25q-125p+185 \\ \hline & p & q-5p & -5+25p-37 & 13q-125p+185=0 \end{array}$$

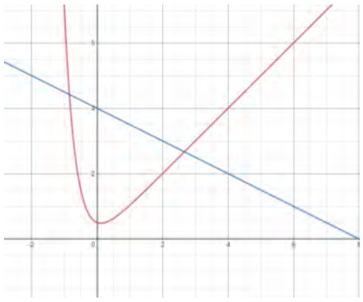
(a) **ALT II Compare coefficients**
**M1** Same as main scheme
**M1** Sets their f'*(x)* which is in the form of a 3TQ, equal to ($x$+2)(K$x$+C)-33 (K$\neq 0, C \neq 0$) , then attempts to equate the coefficients of $x$ to form an equation in $\boldsymbol{p}$ and $\boldsymbol{q}$
**M1** Sets f($x$)=($x$+5) $[\boldsymbol{p}x^2 + Rx + S\,]$ $(R \neq 0, S \neq 0)$, then attempts to equate their coefficients of $x$ to form an equation in $\boldsymbol{p}$ and $\boldsymbol{q}$.
**dM1A1** Same as main scheme

For reference, the correct working for this method:

$$\mathrm{f}'(x) = 3px^2 + 2qx - 37$$
$$= (x+2)(3px-2) - 33$$
$$\Rightarrow 6px - 2x = 2qx$$
$$6p - 2 = 2q$$
$$\mathrm{f}(x) = px^3 + qx^2 - 37x - 12q = (x+5)\left[px^2 + (q-5p)x - \frac{12q}{5}\right]$$
$$\Rightarrow 5(q-5p)x - \frac{12q}{5}x = -37x$$
$$5(q-5p) - \frac{12q}{5} = -37$$
$$\frac{13q}{5} - 25p = -37$$

| Question | Scheme | Marks |
|---|---|---|
| 5 | $\frac{dr}{dt} = 0.7$ | B1 |
| | $\{\frac{dF}{dr}\} = -2 \times \frac{3}{20} r^{-3}$ oe | M1A1 |
| | $\{\frac{dF}{dt} = \frac{dF}{dr} \times \frac{dr}{dt} =\} - \frac{0.3}{2.8^3} \times 0.7 = -0.0095663 \ldots$ (newtons/s) | M1dM1A1 [6] |
| | Accept **awrt** $-0.00957$ or $-9.57 \times 10^{-3}$ | |
| | | **Total 6 marks** |

| Mark | Notes |
|---|---|
| **B1** | For writing $\frac{dr}{dt} = 0.7$ can be implied or embedded in later work. Condone using letter ***d*** or ***D*** or ***R*** only instead of ***r***. |
| **M1** | For an attempt to differentiate the formula for *F*, for $\ldots r^{-2} \to \ldots r^{-3}$ oe<br>Attempts quotient rule, look for $\frac{-Pr}{Qr^4}$ $(P > 0,\ Q > 0)$ oe, if the quotient rule is quoted, it must be correct, if clearly they use incorrect quotient rule M0 |
| **A1** | For a correct differentiated expression simplified or unsimplified |
| **M1** | For a **correct** chain rule, accept in any order. (correct chain rule stated or implied by their working) Condone using letter d instead of r.<br><br>For example, accept<br><br>$\frac{dF}{dr} = \frac{dF}{dt} \times \frac{dt}{dr}$ *or* $\frac{dF}{dr} = \frac{dF}{dt} \div \frac{dr}{dt}$ $\left(\frac{dF}{dt} \text{ does not need to be the subject}\right)$ |
| **dM1** | For substituting the value of $r=2.8$ into their dF/dr in a **correct** chain rule, leading to<br><br>$\frac{dF}{dt} = \ldots$( a value, not a numerical expression) , the value can be a fraction or decimal.<br>If their final answer is not correct, explicit substitution must be seen.<br>**Depends on the previous method mark only.** |
| **A1** | For the correct value of $\frac{dF}{dt}$ (Condone lack of units or incorrect units) isw |

| Question | Scheme | Marks |
|---|---|---|
| 6 | $\log_2(8-3x)-1=-4x-1 \Rightarrow \log_2(8-3x)-\log_2 2=-4x-1$ | M1 |
| | $\Rightarrow \log_2\left(\frac{8-3x}{2}\right)=-4x-1$ | |
| | $\Rightarrow \frac{8-3x}{2}=2^{-4x-1}$ | M1 |
| | $\Rightarrow 4-\frac{3}{2}x+x=x+2^{-(4x+1)}$ | M1 |
| | $\Rightarrow 4-\frac{1}{2}x=x+2^{-(4x+1)}$ | M1A1 |
| | Draws the line $y=4-\frac{x}{2}$ | M1 |
| | For the values $x=-0.8/0.9$ and $x=2.6/2.7$ | A1 |
| |  | [7] |
| | | **Total 7 marks** |

| Mark | Notes |
|---|---|
| | **General principles of marking this question for the first 5 marks**<br>**Award the first two marks when you see them.** |
| **M1** | Award the first M mark for dealing with the log wherever you see it<br>For example: $\log_2(8-3x)=-4x \Rightarrow 8-3x=2^{-4x}$ |
| **M1** | Award the second M mark for either adding – 1 to **both** sides and converting $-1=\log_2 2$ , **or** by dividing throughout by 2 and dealing with the power of – 1.<br>For example: $\frac{8-3x}{2}=\frac{2^{-4x}}{2} \Rightarrow 4-\frac{3x}{2}=2^{-4x-1}$ |
| **M1** | Award the third M mark for adding $x$ to **both** sides |
| **M1** | For simplifying and obtaining $(y)=x+2^{-4x-1}$ or $(y)=x+2^{-(4x+1)}$ on one side and a linear equation of the form $y=K\pm\frac{1}{2}x$ (may be unsimplified) where $K$ is a constant, on the other side. |
| **A1** | For obtaining the correct equation $y=4-\frac{1}{2}x$ (may be unsimplified) |
| **M1** | For drawing their line correctly<br>• Their line is in the form $y=k-\frac{x}{2}$ , $k$ nonzero constant (may be unsimplified)<br>• There must be at least one intersection point with the given curve. |
| **A1** | Correct $x$ values (both rounded to 1dp), ignore if $y=$…is present, accept also given as coordinates, even if their $y$ coordinates are incorrect. |

**ALT I** for first 5 marks

**M1:** Correctly removes log, achieves $2^{-4x} = 8 - 3x$

**M1:** For dividing by 2 throughout $\frac{2^{-4x}}{2} = \frac{8-3x}{2} \Rightarrow 2^{-(4x+1)} = 4 - \frac{3}{2}x$

**M1:** For adding $x$ to both sides

**M1:** Rearranges leading to $x + 2^{-4x-1} = 4 - \frac{x}{2}$

**A1**: For obtaining the correct equation $y = 4 - \frac{1}{2}x$ (may be unsimplified)

**ALT II** for first 5 marks

**M1**: Correctly removes log, achieves $2^{-4x} = 8 - 3x$

**M1**: Rearrange the given curve to $y - x = 2^{-4x} \times 2^{-1}$

**M1**: Attempts to make $2^{-4x}$ the subject from $y - x = 2^{-4x} \times 2^{-1} \Rightarrow 2^{-4x} = \ldots$

**M1**: Equates the two resulting equations, then attempts to make y=…

**A1**: For obtaining the correct equation $y = 4 - \frac{1}{2}x$ (may be unsimplified)

For reference the two correct equations are: $\begin{cases} 2^{-4x} = 8 - 3x \\ 2^{-4x} = 2y - 2x \end{cases} \Rightarrow 8 - 3x = 2y - 2x \Rightarrow y = 4 - \frac{x}{2}$

**ALT III** for first 5 marks

**M1:** Correctly removes log, achieves $2^{-4x} = 8 - 3x$

**M1:** For $2^{-(4x+1)} = 2^{-4x} \times 2^{-1}$ (may be embedded)

**M1:** Replaces $2^{-4x}$ with their $8 - 3x$

**M1:** Rearranges leading to y=…

**A1**: For obtaining the correct equation $y = 4 - \frac{1}{2}x$ (may be unsimplified)

| Question | Scheme | Marks |
|---|---|---|
| 7(a) | $x = -\frac{5}{4}$ | B1 [1] |
| (b) | $\{\frac{\mathrm{d}y}{\mathrm{d}x}\} = \frac{2x(4x+5) - 4(x^2-1)}{(4x+5)^2}$ | M1A1A1 |
| | $y = 0$ | B1 |
| | $\{\frac{\mathrm{d}y}{\mathrm{d}x}\} = \frac{2(-1)(4(-1)+5) - 4((-1)^2-1)}{(4(-1)+5)^2} = -2$ | |
| | $\rightarrow \{gradient\ of\ normal =\} \frac{1}{2}$ | M1 |
| | $y - 0 = \frac{1}{2}\big(x - (-1)\big) oe$ | M1A1cso |
| | | [7] |
| (c) | $\frac{x+1}{2} = \frac{x^2-1}{4x+5}$ | M1 |
| | $2x^2 + 9x + 7 = 0$ ***OR*** $(x+1)(4x+5) = 2(x+1)(x-1)$ | dM1 |
| | $(2x+7)(x+1) = 0 \Rightarrow x = \cdots$ ***OR*** $(4x+5) = 2(x+1) \Rightarrow x = \cdots$ | ddM1 |
| | $x = -\frac{7}{2}, \{x = -1\}$ | A1 |
| | $y = \frac{"-\frac{7}{2}" + 1}{2} = \cdots$ | M1 |
| | $\left(-\frac{7}{2}, -\frac{5}{4}\right)$ | A1 [6] |
| | | **Total 14 marks** |

| Part | Mark | Notes |
|---|---|---|
| (a) | B1 | $x = -\frac{5}{4}$ $or$ $x = -1.25$, do not accept $4x+5=0$ |
| (b) | M1 | Attempts to differentiate $y$.<br>**Quotient Rule:** Look for<br>$$\frac{Px(4x+5) - Q(x^2-1)}{(4x+5)^2}, where\ P > 0, Q > 0$$<br>If the quotient rule is quoted, it must be correct. Condone invisible bracket for this mark.<br><br>**Product Rule:** Look for $Px(4x+5)^{-1} \pm Q(x^2-1)(4x+5)^{-2}$, $where\ P > 0, Q > 0$ |
| | A1 | Quotient rule: Correct denominator $(4x+5)^2$ and one correct term in the numerator, $2x(4x+5)$ or $-4(x^2-1)$.<br>Product rule: One term correct |
| | A1 | Fully correct. $\left\{\frac{dy}{dx} =\right\} \frac{2x(4x+5)-4(x^2-1)}{(4x+5)^2}$<br>or $\left\{\frac{dy}{dx} =\right\} 2x(4x+5)^{-1} - 4(x^2-1)(4x+5)^{-2}$ oe |
| | B1 | $y = 0$ seen or implied |
| | M1 | Correct method finding gradient of normal to the curve where $x = -1$ |
| | M1 | Correct method of forming the equation of a straight line using the point **(-1,their y value)** with a "**changed**" gradient.( "changed" means a different gradient from their tangent gradient, their normal could be found by an incorrect method)<br><br>If using $y=mx+c$, a value for $c$ must be found and the equation formed. |
| | A1cso | For the correct line in any form, simplified or unsimplified. (isw) Once correct equation is seen, award the mark, ignore later incorrect simplification.<br>e.g. $2y = x+1$ or $2y-x-1=0$ or $y=0.5x+0.5$ oe or even $y - 0 = \frac{1}{2}(x-(-1))$ |
| (c) | M1 | For equating the equation of their line with the equation of the curve. |
| | dM1 | For forming an equation with all like terms collected.<br>**OR** cross multiplies then factorises e.g. $2(x^2-1) = 2(x+1)(x-1)$ |
| | ddM1 | Solves their equation with a correct method and finds at least one real value of $x$ which is not -1, method of solving could be implied by correct answers if calculator used.<br><br>**OR** cancels $(x+1)$ and solves their linear equation. |
| | A1 | $x = -\frac{7}{2}, \quad \{x = -1\}$ |
| | M1 | For substituting their $x$ value (not -1) to either the equation of their normal or the equation of the given curve to find a value for $y$ |
| | A1 | For the correct exact coordinates of $D\left(-\frac{7}{2}, -\frac{5}{4}\right)$ $oe$<br>Allow $x = -3.5$, $y = -1.25$ instead of given as coordinates. |

| Question | Scheme | Marks |
|---|---|---|
| **8(a)** | $a + ar = 360$ oe e.g. $\frac{a(1-r^2)}{1-r} = 360$<br>$ar + ar^2 = 288$ oe | M1M1 (B1B1on ePen |
| | $r = \left\{\frac{a(1+r)}{ar(1+r)} =\right\}\frac{288}{360} = \frac{4}{5}$ | dM1A1 |
| | $a = \frac{360}{1 + "\frac{4}{5}"} = 200$ | M1A1 |
| | $U_n = 200\left(\frac{4}{5}\right)^{n-1}$ * | A1cso(B1 on ePen)<br>[7] |
| **8(a) ALT** | $a + ar = 360$ oe e.g. $\frac{a(1-r^2)}{1-r} = 360$<br>$ar + ar^2 = 288$ oe | M1M1 (B1B1 on ePen) |
| | $a\left(\frac{360}{a} - 1\right) + a\left(\frac{360}{a} - 1\right)^2 = 288$<br>$\Rightarrow a = 200$ | dM1A1 |
| | $r = \frac{360}{"200"} - 1 = \frac{4}{5}$ | M1A1 |
| | $U_n = 200\left(\frac{4}{5}\right)^{n-1}$ * | A1cso(B1 on ePen)<br>[7] |
| **(b)** | {$S$ is convergent because} $\left\|\frac{4}{5}\right\| < 1$ | B1<br>[1] |
| **(c)** | $S_\infty = \frac{'200'}{1-'\left(\frac{4}{5}\right)'} = 1000$ | M1A1cao<br>[2] |
| **(d)** | $\frac{200\left(1-\left(\frac{4}{5}\right)^n\right)}{1-\left(\frac{4}{5}\right)} > 978$ | M1 |
| | $\left(\frac{4}{5}\right)^n < \frac{11}{500}$ | dM1 |
| | $n\,log\left(\frac{4}{5}\right) < log\left(\frac{11}{500}\right)$<br>$\Rightarrow n > 17.104 \ldots$ | ddM1 |
| | $n = 18$ | A1<br>[4] |
| | | **Total 14 marks** |

| Part | Mark | Notes |
|---|---|---|
| (a) | M1 | For either correct equation. Allow use of $U_1$ as the first term instead of a |
| | M1 | For both correct equations. Allow use of $U_1$ as the first term instead of a |
| | dM1 | For eliminating $a$ from their equations, reaches $r$ =... (Depends on previous mark) |
| | A1 | For the correct value of $r$ |
| | M1 | Uses their value of $r$ to find a value for $a$ |
| | A1 | For $a = 200$ |
| | A1 cso | For the correct required expression of $\boldsymbol{U_n} = 200\left(\frac{4}{5}\right)^{n-1}$ ( must have $\boldsymbol{U_n}$**)** |
| (a)ALT | M1 | For either correct equation. |
| | M1 | For both correct equations |
| | dM1 | For eliminating $r$ from their equations, reaches $a$ =... (Depends on previous mark) |
| | A1 | For the correct value of a |
| | M1 | Uses their value of a to find a value for $r$ |
| | A1 | For the correct value of r |
| | A1 cso | For the correct required expression of $U_n = 200\left(\frac{4}{5}\right)^{n-1}$ (must have $\boldsymbol{U_n}$**)** |
| (b) | B1 | For stating the correct reason, $\lvert r\rvert < 1$ or $-1 < r < 1$ or "0.8" < 1 or $\left\lvert "\frac{4}{5}" \right\rvert < 1$<br>**Do not accept** just $r < 1$ without a correct reason |
| (c) | M1 | For using their value of $A$ /$a$ /first term of their geometric sequence and their r (provided their $\lvert r\rvert < 1$)in a correct formula for the sum to infinity.<br>$S_\infty = \frac{'their\ A'}{1-'(their\ r)'}$ |
| | A1cao | For the correct value of 1000 |
| (d) | M1 | Uses their a and r (their r can be $\lvert r\rvert < 1$ or $\lvert r\rvert > 1$) to set the correct formula for the sum of a geometric series > 978 or =978 or ≥ 978<br><br>(>978 or =978 or ≥ 978 may be implied by later work.) |
| | dM1 | Attempts to rearrange and achieves<br>$(their\ r)^n$ " < "$k$ ($k$ is non zero positive constant)<br>**Allow strict or non-strict inequalities or equation**:<br><,>, ≥ , ≤ or = |
| | ddM1 | For correctly takes logs (any base)/ln, and achieves a positive value for $n$, it maybe implied by 17.1…but not 18<br><br>**ALT**: trial and error to find a value of $n$ (at least tries $n$=17 and $n$=18) |
| | A1 | For $n = 18$<br>Must come from **correct** working, achieves $n$=18 with no incorrect inequalities in their working. |

| |
|---|
| If a candidate does not score the dM1 mark but arrives at an answer of 17.1...<br>this scores a maximum of M1 as clear use of calculator is not condoned. |

| Question | Scheme | Marks |
| --- | --- | --- |
| 9 | **Area under the curve** | |
| | $A_c = \int_{-1}^{0}\left(-2e^{3x}+4\right)dx, = \left[\frac{-2e^{3x}}{3}+4x\right]_{-1}^{0}$ | M1M1 |
| | $\Rightarrow\left(\frac{-2e^{3\times 0}}{3}+4\times 0\right)-\left(\frac{-2e^{3\times -1}}{3}+4\times -1\right), = \frac{10}{3}+\frac{2e^{-3}}{3}$ | M1A1 |
| | **Area under trapezium** | |
| | Method 1 [uses formula for the area of a trapezium] | |
| | $y=2 \quad y=-2e^{-3}+4$ | |
| | $A_T = \frac{1}{2}\times 1\times\left('2'+'-2e^{-3}+4'\right)=\left[3-e^{-3}\right]$ | B1 |
| | Method 2 [finds equation of the line and integrates] | M1 |
| | Equation of the line: $y=\left(2e^{-3}-2\right)x+2 \quad \left(\text{accept } y=-1.9x+2\right)$ | |
| | $A_T = \int_{-1}^{0}'\left(\left(2e^{-3}-2\right)x+2\right)'\,dx = \left[\frac{'\left(2e^{-3}-2\right)'x^2}{2}+\frac{'2'x}{2}\right]_{-1}^{0} = \left[3-e^{-3}\right]$ | |
| | **Shaded Area** | |
| | $A_{shaded} = \left(\frac{10}{3}+\frac{2e^{-3}}{3}\right)-\left(3-e^{-3}\right)=\frac{1+5e^{-3}}{3}$ | M1A1<br>[8] |
| | | **Total 8 marks** |

| Mark | Notes |
|---|---|
| M1 | For a correct statement for the area under the curve with **correct limits**. This mark can be implied/embedded by later correct work. **Condone missing dx** |
| M1 | Attempts to integrate the given curve to a form of $Ae^{3x}+Bx$<br>**Ignore limits for this mark**. |
| M1 | For substituting in both of their limits into their integrated expression in the form $Ae^{3x}+Bx$ and subtracts. ($A$, $B$ are nonzero constants)<br>Correct exact answer $\frac{10}{3}+\frac{2e^{-3}}{3}$ implies the correct substitution.<br>If their answer is not correct and exact, explicit substitution needs to be seen. |
| A1 | For the correct **exact** area under the curve. If not seen explicitly, this can be implied by correct exact final answer. |
| **Method 1 - trapezium** | |
| B1 | For both correct **exact** values of $y$ |
| M1 | For the correct method of finding area of the trapezium using their values for $y$<br>Look for $\frac{1}{2}\times 1\times(\textit{sum of their y values})$ |
| **Method 2 – equation of line** | |
| B1 | For the correct equation of the line $L$, must reach $y = \ldots$<br>$y=\left(2e^{-3}-2\right)x+2$ (accept $y=-1.9x+2$) |
| M1 | For the correct area under the line $L$ using their line. Check for correct integration of their line and full substitution of – 1 if their area is incorrect. (If the terms are equal to zero, substitution of 0 does not need to be seen.)<br>Allow also $A_T=\int_{-1}^{0}{'}(-1.9x+2){'}\,dx=\left[\frac{'-1.9'x^2}{2}+\frac{'2'x}{2}\right]_{-1}^{0}$ |
| **Combines the areas** | |
| M1 | Correct strategy to find the value of shaded area<br>The integrated area of the curve - the area of trapezium |
| A1 | For the correct **exact** area of the shaded region in the required form |
| **Some candidates are combining the curve – line in one calculation – mark this as follows.** | |
| | • Wherever you see the **correct** equation of the line, score the 5th Mark [B mark]<br>• Mark the work for the curve (which may be embedded in a combined integral) using the first 4 marks.<br>• Mark the area under the line using the 6th mark [M mark – using their equation in the form $y = mx + c$]<br>• When you see them subtracting the areas – award the penultimate M mark<br>If they add the areas, this is M0<br>• Final A mark is for the correct answer only |

| Question | Scheme | Marks |
|---|---|---|
| **10(a)** | $AC = \sqrt{(2x)^2 + x^2} = (\sqrt{5}x)$ | B1 |
| | $VA = '\sqrt{5}x' \sin 45^o = \frac{\sqrt{10}}{2}x$ * | M1A1 cso [3] |
| **(b)** | Height $= \sqrt{\left(\frac{\sqrt{10}}{2}x\right)^2 - \left(\frac{\sqrt{5}}{2}x\right)^2} = \frac{\sqrt{5}}{2}x$ | M1A1 [2] |
| **(c)** | $\cos\angle VBA = \dfrac{1}{\frac{\sqrt{10}}{2}}$ | M1 |
| | $\Rightarrow \angle VBA = 50.768\ldots^o \approx 50.8^o$ awrt | A1 [2] |
| **(d)** | Let *M* be the point at which the diagonals *AC* and *BD* meet.<br>The required angle is either *AMB* or *DMC* | B1 |
| | AM = MB $= \frac{AC}{2} = \frac{\sqrt{5}x}{2}$ or DM=MC=$\frac{AC}{2} = \frac{\sqrt{5}x}{2}$ | B1 |
| | $\cos\angle AMB = \dfrac{\left(\frac{\sqrt{5}}{2}\right)^2 + \left(\frac{\sqrt{5}}{2}\right)^2 - 2^2}{2 \times \frac{\sqrt{5}}{2} \times \frac{\sqrt{5}}{2}}$ *oe* | M1 |
| | $\left(\cos\angle AMB = -\frac{3}{5} \Rightarrow \angle AMB = 126.869\ldots^o\right) \approx 126.9^o$ awrt | A1 [4] |
| **(e)** | $9\sqrt{5} = \frac{1}{3} \times 2x \times x \times "\frac{\sqrt{5}}{2}x" \Rightarrow x = \cdots$ | M1 |
| | $x = 3$ | A1 [2] |
| | | **Total 13 marks** |

| Part | Mark | Notes |
|---|---|---|
| **(a)** | **B1** | For using correct Pythagoras theorem to find length *AC or ½ AC*, explicit substitution must be seen. |
| | **M1** | For correct trigonometry using their AC in triangle VAC or half of their AC in triangle VAM or correct Pythagoras theorem (let M be the centre of the base at which the diagonals *AC* and *BD* meet) to find length *VA*. *e.g.* $VA = \sqrt{5}x\cos(45°) = \frac{\sqrt{10}}{2}x$ *or* $VA = \frac{\sqrt{5}}{2}x \div \cos(45°) = \frac{\sqrt{10}}{2}x$ *or* $VA = \frac{\sqrt{5}}{2}x \div \sin(45°) = \frac{\sqrt{10}}{2}x$ |
| | **A1 cso** | Achieves printed answer with no errors. Condone AV instead of VA<br>Note:<br>$\frac{\sqrt{10}}{2}x$ *written as* $\frac{\sqrt{10x}}{2}$ *is A0*, $\frac{\sqrt{10}}{2}x$ *written as* $\frac{\sqrt{10}x}{2}$ *is A1* |
| **(b)** | **M1** | Uses the correct Pythagoras theorem or correct trigonometry with the given length VA=$\frac{\sqrt{10}}{2}x$ to find the height of the pyramid. |
| | **A1** | For $h = \frac{\sqrt{5}}{2}x$<br>Correct exact answer only scores M1A1. This can be written down as the triangle is isosceles. |
| **(c)** | **M1** | **Ignore missing *x* consistently throughout their solution.**<br>For using any appropriate trigonometry to find the angle *VBA*<br>If attempts cosine rule, look for correct application of the cosine rule for the required angle:<br>$\left(\frac{\sqrt{10}}{2}x\right)^2 = \left(\frac{\sqrt{10}}{2}x\right)^2 + (2x)^2 - 2\times\frac{\sqrt{10}}{2}x\times 2x\cos(VBA)$ oe |
| | **A1** | For the correct angle. awrt 50.8 º (with or without degree sign) |
| **(d)** | **B1** | Identifies the angle required.(can be implied/embedded in their working) |
| | **B1** | For the correct lengths AM and MB **or** DM and MC **or** BE and ME<br>Let E to be mid point of AB, correct length BE=*x,* ME=0.5*x* scores B1, this mark can be implied/embedded in their working) |
| | **M1** | **Ignore missing *x* consistently throughout their solution.**<br>For using any appropriate trigonometry to find the required angle or half of the required angle. |
| | **A1** | For the correct angle. awrt 126.9º (with or without degree sign) |
| **(e)** | **M1** | For using the correct formula for the volume of a pyramid with their height of the pyramid and attempts to find a value of *x*. |
| | **A1** | For $x = 3$ with or without unit, ignore incorrect unit |

| Question | Scheme | Marks |
|---|---|---|
| 11(a) | $cos(A+A) = cos\,A\,cos\,A - sin\,A\,sin\,A$<br>$\Rightarrow cos\,2\,A = cos^2\,A - (1 - cos^2\,A)$ | M1 |
| | $cos\,2\,A = 2\,cos^2\,A - 1\,*$ | A1 cso<br>[2] |
| (b) | $(2\cos^2 A - 1)^2 = (\cos 2A)^2$ | M1 |
| | $\cos 4A = 2\cos^2 2A - 1 \Rightarrow \cos^2 2A = \dfrac{\cos 4A + 1}{2}$ | M1 |
| | $(2\cos^2 2A - 1)^2 = \dfrac{\cos 4A + 1}{2}\,*$ | A1 cso<br>[3] |
| (b) ALT | $cos\,4\,A = 2\,cos^2\,2\,A - 1$ | M1 |
| | $= 2(2\,cos^2\,A - 1)^2 - 1$ | M1 |
| | $(2\,cos^2\,A - 1)^2 = \dfrac{cos\,4\,A + 1}{2}\,*$ | A1 cso<br>[3] |
| (c) | $y = \dfrac{sin\,2\,x}{2} + \dfrac{(2\,cos^2\,x - 1)^2}{2} + \dfrac{1}{8} \Rightarrow y = \dfrac{sin\,2\,x}{2} + \dfrac{cos\,4\,x}{4} + \dfrac{3}{8}$ | M1 |
| | $\dfrac{dy}{dx} = cos\,2\,x - sin\,4\,x$ | dM1 |
| | $\dfrac{dy}{dx} = cos\,2\,x - 2\,sin\,2\,x\,cos\,2\,x\{= 0\}$ | B1 |
| | $\{cos\,2\,x(1 - 2\,sin\,2\,x) = 0\} \Rightarrow sin\,2\,x = \dfrac{1}{2}$ | |
| | $\Rightarrow x = \dfrac{\sin^{-1}\left(\frac{1}{2}\right)}{2} = \cdots$ | ddM |
| | $x = \dfrac{\pi}{12}$ | A1 |
| | $y = \dfrac{sin2\left("\frac{\pi}{12}"\right)}{2} + \dfrac{cos4\left("\frac{\pi}{12}"\right)}{4} + \dfrac{3}{8} = \dfrac{3}{4}\{\Rightarrow \left(\dfrac{\pi}{12}, \dfrac{3}{4}\right)\}$ | M1A1<br>[7] |
| | | **Total 12 marks** |

| Part | Mark | Notes |
|---|---|---|
| (a) | M1 | For correct use of $cos(A+B) = cos\,A\,cos\,B - sin\,A\,sin\,B \Rightarrow$ $cos(A+A) = cos\,A\,cos\,A - sin\,A\,sin\,A$ ***OR*** $cos\,2\,A = cos^2\,A - sin^2\,A$ **with** the Pythagorean identity to eliminate sine from the identity. |
| | A1 cso | Achieves printed answer with no errors seen. Left hand side must be cos2A not cos(A+A) |
| (b) | M1 | Uses the given result from (a), replaces $(2\cos^2 A - 1)^2$with $(\cos 2A)^2$ |
| | M1 | Uses the given result from (a) or a correct identity to replace $\cos^2 2A$ with $\frac{\cos 4A+1}{2}$ |
| | A1 cso | Fully correct proof showing all necessary steps. (If they meet in the middle, they must have a conclusion, accept e.g. shown, proved, L=R, etc.) |
| (b) ALT | M1 | Uses the given result from (a) on cos 4A |
| | M1 | Uses the given result from (a) or a correct identity on cos 2A |
| | A1 cso | Fully correct proof showing all necessary steps. (If they meet in the middle, they must have a conclusion, accept e.g. shown, proved, L=R, etc.) |
| (c) | M1 | Rearranges the given $y$ into the form $\frac{sin\,2x}{2} + \frac{cos\,4x+1}{k} + l$ , oe, where $k$ and $l$ are nonzero constants Or $\frac{sin\,2x}{2} + \frac{cos\,4x}{P} + Q$, oe, where P and Q are nonzero constants (Allow mixed variables $x$ and A for this method mark only) |
| | dM1 | For an attempt to differentiate their $y$ into the form $P\cos(2x) \pm Q\sin(4x)\ oe, where\ P, Q\ are\ nonzero\ constants$ **Depends on the previous method mark.** |
| | B1 | Uses $\sin 4x = 2\sin 2x \cos 2x$ or $\sin 4x = \sqrt{1-\cos^2 4x}$ oe in their dy/dx (may be implied by correct working) |
| | ddM1 | Sets their differentiated expression = 0 (can be implied) and attempts a correct method to solve for $x=$... **Depends on both previous method marks.** |
| | A1 | For $x = \frac{\pi}{12}$, ignore extra $x$ values are outside of given range. Withhold A mark if extra solution seen in the given range. |
| | M1 | For attempting to find the value of $y$ using their $x$, found by solving their $\frac{dy}{dx} = 0$, provided $x$ is in the range $0 \leqslant x \leqslant \frac{\pi}{6}$ Substitutes into the given $y$ **or** their $y$, correct answer implies this mark, but for incorrect answer, must see explicit substitution |
| | A1 | For the correct exact coordinates of P, $\left(\frac{\pi}{12}, \frac{3}{4}\right)$ $or$ $\left(\frac{\pi}{12}, 0.75\right)$, accept $x = \frac{\pi}{12},\ y = \frac{3}{4}\ or\ x = \frac{\pi}{12},\ y = 0.75$ |

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