Mark Scheme (Results)

January 2016

International GCSE Further Pure Mathematics 4PM0/01

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications come from Pearson, the world's leading learning company. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information, please visit our website at www.edexcel.com.

Our website subject pages hold useful resources, support material and live feeds from our subject advisors giving you access to a portal of information. If you have any subject specific questions about this specification that require the help of a subject specialist, you may find our Ask The Expert email service helpful.

www.edexcel.com/contactus

Pearson: helping people progress, everywhere
Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

January 2016
Publications Code UG043226

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)
- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers may score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.
- **With working**
  Always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.
  Any case of suspected misread loses 2A (or B) marks on that part, but can gain the M marks.
  If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.
- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded in another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right), \text{ where } |pq|=|c| \quad \text{leading to } x=....$$

$$\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right) \text{ where } |pq|=|c| \text{ and } |mn|=|a| \quad \text{leading to } x=....$$

2. Formula:
Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=....$

3. Completing the square:

$$x^2+bx+c=0: \left(x\pm\tfrac{b}{2}\right)^2\pm q\pm c=0, \quad q\neq 0 \qquad \text{leading to } x=....$$

Method marks for differentiation and integration:

1. Differentiation
Power of at least one term decreased by 1. $\left(x^n \to x^{n-1}\right)$

2. Integration:
Power of at least one term increased by 1. $\left(x^n \to x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)
Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Question Number	Scheme	Marks
1.	$\left(\mathrm{f}(x) = 3x^3 + 2\sin x - 4x^{-2}\right)$	
(a)	$\mathrm{f}'(x) = 9x^2 + 2\cos x + 8x^{-3}$	M1A1A1 (3)
(b)	$\int \mathrm{f}(x)\,\mathrm{d}x = \dfrac{3x^4}{4} - 2\cos x - \dfrac{4x^{-1}}{-1} + c$	M1A1A1B1 (4) **(7)**

Notes

(a)

M1 for an attempt at differentiating any of the terms, (see General Guidance) ,

Note; If you need to base your decision for the M mark on the 2sin x term,

then $\left(\dfrac{\mathrm{d}(\sin x)}{\mathrm{d}x} \to +\cos x\right)$

A1 for at least 2 terms correct (need not be simplified)

A1 all three terms fully correct (need not be simplified)

(b)

M1 for an attempt at integrating any of the terms, (see General Guidance)

Note; If you need to base your decision for the M mark on the 2sin x term,

then $\left(\int \sin x \ \mathrm{d}x \to -\cos x\right)$

A1 for at least 2 terms correct (need not be simplified)

A1 for all three terms correct (need not be simplified)

B1 for + c

Note: Any attempt to integrate their f'(x) is M0

Question Number	Scheme	Marks
2	expand and re-arrange to achieve 3TQ	
	$\Rightarrow 4x^2 - 19x + 12 > 0$	M1
	$\Rightarrow (4x-3)(x-4) > 0 \quad \Rightarrow$ correct cvs $x = \frac{3}{4}$, $x = 4$	M1A1
	$\Rightarrow x < \frac{3}{4}, x > 4 \quad \Rightarrow x < \frac{3}{4}$ OR $x > 4$ (Outside region)	M1A1
		(5)
	Or any equivalent notation eg., $\left(-\infty, \frac{3}{4}\right) \cup (4, \infty)$	

Notes

M1 for attempting to expand the bracket and collecting up like terms to achieve a 3TQ

Can have > 0 , = 0 < 0 or even the expression on its own.

An acceptable attempt is to expand the bracket to 3 or 4 terms to give $4x^2 \pm kx \pm 9$

M1 for attempting to **solve** their 3TQ (see General Guidance for an acceptable attempt to factorise, complete square or use formula). For the award of this mark, their 3TQ must be either > 0, = 0 or < 0, and they must achieve their critical values.

(*It is not enough just to factorise without leading to roots or a solution)*

A1 for the correct critical values of x =4 and $x = \frac{3}{4}$

M1 for selecting the outside region for their critical values, ft their values.

A1 for the correct inequality as shown

We will accept; a comma, a space, the word **or**, between $x < \frac{3}{4} \quad x > 4$

$x < \frac{3}{4}$ and $x > 4$ is M1A0

Use of $\geq$ and $\leq$ in an otherwise correct region is M1A0

Question Number	Scheme	Marks
3	$V=\frac{4}{3}\pi r^3 \rightarrow \frac{\mathrm{d}V}{\mathrm{d}r}=4\pi r^2$	M1A1
	$36000\pi=\frac{4}{3}\pi r^3 \quad \Rightarrow r=30$	M1A1
	$\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}\times\frac{\mathrm{d}r}{\mathrm{d}V}$ oe	M1
	$\frac{\mathrm{d}r}{\mathrm{d}t}=60\times\frac{1}{4\pi\times 30^2} \quad \Rightarrow \frac{\mathrm{d}r}{\mathrm{d}t}=\frac{1}{60\pi}=0.0053\,\mathrm{cm/s}$	M1A1
		(7)

Notes

M1 for attempting to differentiate the expression for the volume of a sphere. If their formula is incorrect allow $V = a\pi r^3$ where a is a constant as a minimum. (see General Guidance for an attempt)

A1 for a fully correct $\frac{\mathrm{d}V}{\mathrm{d}r} = 4\pi r^2$

M1 for equating the given volume to the correct formula for the volume of a sphere **AND** attempting to find the value for r. **Just equating the given volume to the formula is not enough for the award of this mark. They must reach *r* =………**

A1 $r = 30$

M1 for a correct expression of chain rule (any way around) there will be some variations so please check anything unusual.

M1 for substituting their $\frac{\mathrm{d}V}{\mathrm{d}r}$, and using the given value of $\frac{\mathrm{d}V}{\mathrm{d}t}$ to find $\frac{\mathrm{d}r}{\mathrm{d}t}$.

A1 for a correct value of 0.0053 (cm/s) (Units not required)

ALT

M1 for re-arranging the formula for the volume of a sphere to make r the subject

An acceptable attempt is $r = \left(\frac{aV}{b}\right)^{\frac{1}{3}}$ where a and b are constants

A1 for $r = \left(\frac{3V}{4\pi}\right)^{\frac{1}{3}}$

M1 for attempting to differentiate a rearranged formula for the volume of a sphere

$$r = \left(\frac{3V}{4\pi}\right)^{\frac{1}{3}} \Rightarrow \frac{\mathrm{d}r}{\mathrm{d}V} = \frac{V^{-\frac{2}{3}}}{3}\left(\frac{3}{4\pi}\right)^{\frac{1}{3}}$$

A1 for a fully correct $\frac{\mathrm{d}r}{\mathrm{d}V}$

M1 for a correct expression of chain rule

M1 for substituting the given V into their differentiated $\frac{\mathrm{d}r}{\mathrm{d}V}$, using the given $\frac{\mathrm{d}V}{\mathrm{d}t}$ and using a correct chain rule.

A1 for a correct value of 0.0053 (cm/s)

Question Number	Scheme	Marks
4	Arithmetic series $S_3 = 6p$	B1
	Geometric series $p + pr + pr^2 = p(1+r+r^2)$	B1
	$6p = p(1+r+r^2), \Rightarrow 6 = 1+r+r^2, \Rightarrow r^2 + r - 5 = 0$ 3TQ	M1
	$r = \dfrac{-1 \pm \sqrt{1^2 - 4\times 1 \times -5}}{2\times 1}, \Rightarrow r = \dfrac{-1+\sqrt{21}}{2}$ *	M1d,A1
		(5)

Notes

B1 for a correct S_3 of $6p$ or $(p+2p+3p)$ for the arithmetic series

B1 for a correct S_3 for the geometric series

M1 for equating the two series **AND** attempting to form a 3TQ in any order.

M1d for using the formula or completing the square (see General Guidance for an acceptable attempt) to solve the 3TQ

A1 for the given answer only

Note: $r = \dfrac{-1 \pm \sqrt{21}}{2}$ **is A0**

Note: this is a show question. Sufficient working must be seen to award marks.

ALT

B1 uses summation formula for arithmetic series $S_3 = \dfrac{3}{2}\left(2p + (3-1)p\right)$

B1 uses summation formula for geometric series $S_3 = \dfrac{p(r^3-1)}{(r-1)}$ or $\dfrac{p(1-r^3)}{(1-r)}$

M1 for equating the two series, and attempting to form a cubic in r

$$\frac{3}{2}\left(2p+(3-1)p\right) = \frac{p(1-r^3)}{(1-r)} \Rightarrow r^3 - 6r + 5 = 0$$

M1d for solving the cubic by;

- establishing that $(r-1)$ is a factor
- dividing their cubic by $(r-1)$ to achieve $r^2 + r - 5 = 0$
- for using the formula or completing the square

A1 for the given answer only

Note: $r = \dfrac{-1 \pm \sqrt{21}}{2}$ **is A0**

Note: this is a show question. Sufficient working must be seen to award marks.

Question Number	Scheme	Marks
5. (a)	$\frac{1}{\sqrt{4-x}} = \frac{1}{2}(1-\frac{x}{4})^{-\frac{1}{2}}$ $\quad p = \frac{1}{2}\ , q = \frac{1}{4}$	B1B1 (2)
(b)	(i) $\left(1-\frac{x}{4}\right)^{-\frac{1}{2}} = \left\{1+\left(\frac{-1}{2}\right)\left(\frac{-x}{4}\right)+\left(\frac{\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)\left(\frac{-x}{4}\right)^2}{2!}\right)+\frac{\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)\left(\frac{-5}{2}\right)\left(\frac{-x}{4}\right)^3}{3!}+\ldots\right.$	M1A1
	$\frac{1}{2}(1-\frac{x}{4})^{-\frac{1}{2}} = \frac{1}{2}+\frac{x}{16}+\frac{3x^2}{256}+\frac{5x^3}{2048}$	A1
	(ii) Range $\quad -4 < x < 4$ or $\|x\| < 4$	B1 (4)
(c)	$2(1+x)(\frac{1}{2}+\frac{x}{16}+\frac{3x^2}{256}), = 1+\frac{9x}{8}+\frac{19x^2}{128}+\ldots$	M1,M1A1
	(i) $a=1$, (ii) $b=\frac{9}{8}$, (iii) $c=\frac{19}{128}$	(3)
		(9)

Notes

(a)

B1 for either $p=\frac{1}{2}$ or $q=\frac{1}{4}$

B1 for both $p=\frac{1}{2}$ and $q=\frac{1}{4}$

(b) (i)

The M1 and first A1 in this part are for the binomial expansion. Ignore *p* for the first 2 marks

M1 for using a **binomial** expansion at least up to the term in x^3. If there are errors in substitution, withhold this mark if the formula is not seen. Each term, must have at least, the correct power of their $\left(\frac{-x}{4}\right)$. The expansion must start with 1.

A1 for a fully correct **binomial** expansion. Need not be simplified for this mark.

A1 for a fully correct simplified expansion with correct p and q.

(ii)

B1 for the correct validity range of x

(c)

M1 for replacing the fraction with their binomial expansion

$$2(1+x)\left(\frac{1}{2}+\frac{x}{16}+\frac{3x^2}{256}+...\right)=2\left[\frac{1}{2}+\frac{x}{16}+\frac{3x^2}{256}+\frac{x}{2}+\frac{x^2}{16}+\frac{3x^3}{256}+...\right]$$

Up to the term in x^2 there will be 5 terms un-simplified.

M1 **This is A1 in Epen**

for attempting to multiply their binomial expansion by $2(1 + x)$

We need to see 5 terms here minimum; ignore powers of x^3 and above.

A1 for a correct expansion with correct values of a, b and c. The values of a, b and c need not be shown explicitly, they can be embedded in the expansion.

Question Number	Scheme	Marks
6. (a)	$x = \cos^{-1} 0.4, \Rightarrow x = 1.159, x = -1.159$	M1A1
		(2)
(b)	$2\theta + \frac{\pi}{4} = 0.98279..., 4.1244..., 7.26597...$	M1A1
	$\theta = 0.099, \theta = 1.669$	M1A1
		(4)
		(6)

Notes

(a)

M1 for one correct value of x **in radians** within the given range

A1 for **both** correct values of x

Special Case; Answers in terms of π (0.369 π and −0.369 π) M1A0
66.422º and −66.422º M1A0
1.16 OR −1.16 M1A0
1.15 OR −1.15 M1A0

(b)

M1 for any correct value of $\left(2\theta+\frac{\pi}{4}\right)$ **in radians** within the given range

A1 for two correct values of $\left(2\theta+\frac{\pi}{4}\right)$.

M1 for at least one correct value of θ. If they have worked in degrees so far, allow this mark for either $\theta =$ 5.655º or 95.655º .

A1 for the 2 correct values as shown.

No working: For both correct answers with no working award M1A1M1A1
For one correct answer with no working seen award M1A0M1A0

Special Case: Working in degrees throughout; 5.655º and 96.655º award M1A0M1A0

Additional notes regarding extra angles and rounding

If they find extra angles **OUTSIDE** of the given range, ignore them.
If they find extra angles **WITHIN** the range deduct one A mark for each angle up to a maximum of 2 A marks for the question as a whole.

Penalised rounding **ONLY ONCE** for the first angle incorrectly rounded. However, an angle given with insufficient decimal places is deemed incorrect and is penalised every time.

Question Number	Scheme	Marks
7. (a)	$\frac{\sin A}{5} = \frac{\sin 30}{4}, \Rightarrow \sin A = \frac{5\sin 30}{4}, \Rightarrow A = 38.68(2.....)$	M1A1
	$BDC = 180 - 36.682187... = 141.3178.. \Rightarrow 141.3$	A1 (3)
(b)	$AD^2 = 4^2 + 4^2 - 2\times 4\times 4\times \cos 102.6356, \Rightarrow AD = 6.24$ (3sf)	M1M1A1 (3)
(c)	Area $= \frac{1}{2}\times 4\times 4\times \sin 102.6636 = 7.81$ Area = 7.81 cm^2	M1A1 (2) **(8)**

Notes

(a)

M1 for the correct use of Sine Rule (either way around)

A1 for angle of 38.7º seen.

A1 for the correct angle *BDC*

(b)

There are several ways of finding *AD*.

Apply marks on the following principle:

The first mark (M mark - is a B mark in Epen) is for using a method with correct trigonometry.

The second mark (M mark) is for substituting the correct values.

The third mark (A mark) is for 6.24 or 6.25

For example;

M1 for the correct use of Cosine Rule to find the length *AD*

M1 for substituting the correct values

A1 for the correct length of *AD*

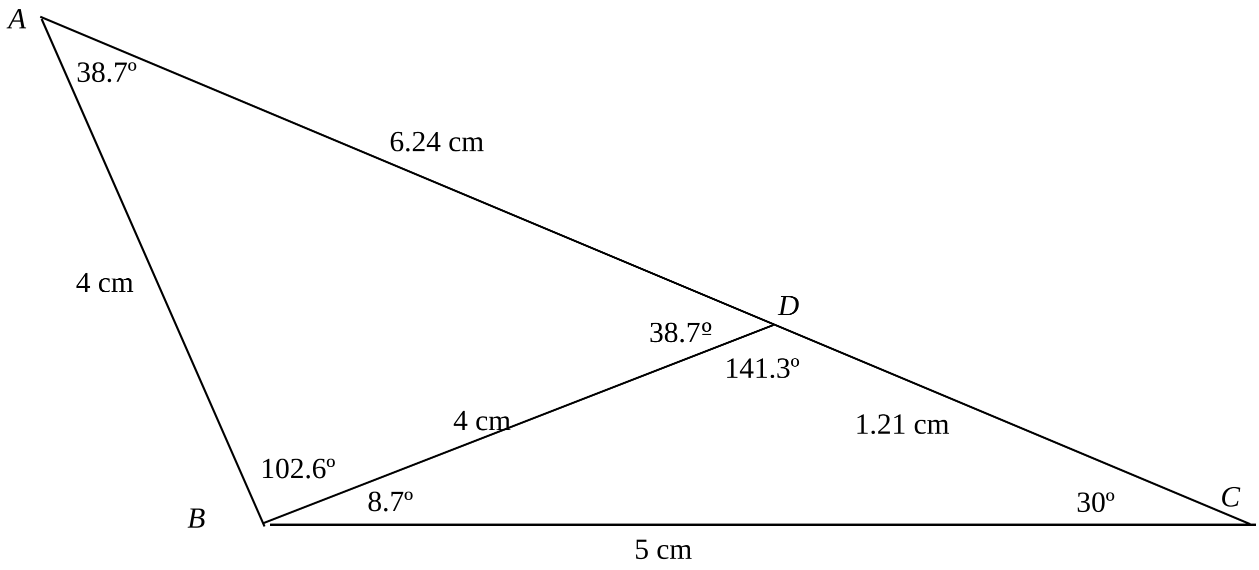
(c)

M1 for the correct use of the formula or method for the area of a triangle. Ft their values

A1 for the correct area = 7.81 allow 7.8

<u>Please see Q6 for additional notes regarding extra angles and rounding</u>

<u>Useful Sketch</u>



Question Number	Scheme	Marks
8. (a)	$v=\int a\,\mathrm{d}t=6t-\frac{4}{2}t^2(+c)$	M1A1
	(i) $v=0,$ (when $t=0$) so $c=0\Rightarrow v=6t-2t^2$	A1
	(ii) $s=\int v\,\mathrm{d}t=3t^2-\frac{2t^3}{3}(+d)$	M1
	$s=5, t=0,\Rightarrow d=5,\Rightarrow s=3t^2-\frac{2t^3}{3}+5$	M1A1
		(6)
(b)	$6t-2t^2=0\Rightarrow 2t(3-t)=0,\Rightarrow (t=0),\Rightarrow t=3$	M1M1A1
	$s=3\times3^2-\frac{2\times3^3}{3}+5=14\text{ m}$	M1A1
		(5)
		(11)

Notes

(a) (i)

M1 for attempting to integrate the given expression for a (see General Guidance for the definition of an attempt)

A1 for the correct expression for v (with or without c)

A1 for the correct expression for v **AFTER** using $t = 0$ when $v = 0$. For the award of this mark they must have had ($+c$) following integration. **They cannot just assume c = 0.**

(ii)

M1 for an attempt to integrate their expression for v (with or without d)

M1d for substituting in $s=5, t=0$ to attempt to find d. Note; they cannot earn this mark without $+d$.

A1 for a fully correct expression for s

(b)

M1 setting their $v = 0$ which must be a quadratic expression.

M1 attempting to solve a two term quadratic – take t out as a common factor.

A1 for finding $t = 3$

M1 for substituting their $t = 3$ into their expression for s **AND** attempting to evaluate

A1 for s =14 (m)

Note: **If they give an answer of 14m + 5 m = 19 m after 14 m is seen, award A0**

Question Number	Scheme	Marks
9 (a)	$8-2x-x^2=0,\Rightarrow(4+x)(2-x),\Rightarrow x=2,x=-4$	M1A1 A1 (3)
(b)	$\text{Area}=\int_{-4}^{2}8-2x-x^2\text{d}x=\left[8x-x^2-\frac{x^3}{3}\right]_{-4}^{2}$	M1A1
	$\text{Area}=\left[8\times2-2^2-\frac{2^3}{3}\right]-\left[8\times-4-(-4)^2-\frac{(-4)^3}{3}\right]=36$	M1A1 (4)
(c)	$8-2x-x^2=x^2+x+6\quad\Rightarrow 2x^2+3x-2=0$	
	$\Rightarrow(2x-1)(x+2)+0,\Rightarrow x=\frac{1}{2},-2$	M1A1 (2)
(d)	$\text{Area}=\int_{-2}^{0.5}(8-2x-x^2)-(x^2+x+6)\text{d}x$	
	$\text{Area}=\int_{-2}^{0.5}(2-3x-2x^2)\text{d}x=\left[2x-\frac{3x^2}{2}-\frac{2x^3}{3}\right]_{-2}^{0.5}$	M1A1ft
	$\text{Area}=\left[2\times0.5-\frac{3(0.5)^2}{2}-\frac{2(0.5)^3}{3}\right]-\left[2\times(-2)-\frac{3(-2)^2}{2}-\frac{2(-2)^3}{3}\right]=\frac{125}{24}$	M1dA1 (4)
		(13)

Notes

(a)

M1 for setting the equation for the curve = 0 and attempting to solve the 3TQ

A1 for correct factorisation, or completion of square or correct use of the formula

A1 for the correct values of x

(b)

M1 for an attempt at integrating ± the equation of the curve. For this mark, they do not need limits.

A1 for ± the correct integrated expression (with or without limits)

M1 for substitution of their values from (a) into their **integrated** expression

A1 for the correct area = 36 **Note:** −36 is AO but allow recovery to 36.

NOTE: Answers without calculus get 0000

(c)

M1 for equating the equations of the curves, achieving a 3TQ, and attempting to solve

A1 for both correct x coordinates

(d)

M1 for a correct expression for the area **INCLUDING** both of their limits from part (c) with an attempt to integrate. This must be the difference of two expressions, either way around, or they can work with two separate integrals and subtract at the end.

A1ft for a fully correct integrated expression for the area, including their limits from (c). Need not be simplified. Ft their limits from (c).

M1d for substituting their limits the correct way around

A1 for the correct area of $\frac{125}{24}$, or 5.208. Accept awrt 5.21. This must be positive, but allow recovery if they subtract the other way around to give a negative, but write their final answer as a positive.

Do not isw if they go on to further processing, for example $36-\frac{125}{24}=\frac{739}{24}$ written after $\frac{125}{24}$ is found as the answer to their subtracted integrals.

Question Number	Scheme	Marks
10. (a)	$2\log_y x + \dfrac{2\log_y y}{\log_y x} = 5$	M1
	For forming 3TQ $\quad 2(\log_y x)^2 + 2 = 5\log_y x$ oe	M1dA1
	$(2\log_y x - 1)(\log_y x - 2) = 0 \Rightarrow \log_y x = \dfrac{1}{2}, \Rightarrow \log_y x = 2$ *	M1ddA1 (5)
	$\log_y x = \dfrac{1}{2} \Rightarrow x = y^{\frac{1}{2}}, \log_y x = 2 \Rightarrow x = y^2$	M1A1
(b)	$xy = 27 \Rightarrow y^{\frac{1}{2}}.y = 27 \Rightarrow y^{\frac{3}{2}} = 27 \Rightarrow y = 9$ so $x = \sqrt{9} = 3$	M1dA1
	$xy = 27 \Rightarrow y^2.y = 27 \Rightarrow y^3 = 27 \Rightarrow y = 3$ so $x = 3^2 = 9$	M1dA1 (6)
	ALT	
	$\log_y xy = \log_y 27 \quad \Rightarrow \log_y x + \log_y y = \log_y 27$	M1A1
	$\log_y x + 1 = \log_y 27 \Rightarrow 2 + 1 = \log_y 27$	
	$\Rightarrow 3 = \log_y 27 \Rightarrow y^3 = 27 \Rightarrow y = 3$	M1dA1
	$\Rightarrow 1 + \dfrac{1}{2} = \log_y 27 \Rightarrow y^{\frac{3}{2}} = 27 \Rightarrow y = 9$	M1dA1 (6) **(11)**
	$\Rightarrow y = 3, x = 9$, $\Rightarrow x = 3, y = 9$	

Notes

(a)

M1 for using change of base formula for $2\log_x y = \dfrac{2\log_y y}{\log_y x}$

M1d for using their result above in an attempt to form 3TQ in $\log_y x$

A1 for correct 3TQ (allow a substitution for $\log_y x$)

M1dd for an attempt to solve their 3TQ. They must arrive at a solution for the award of this mark. This mark is dependent on both previous M marks being awarded.

A1 for the correct values of $\log_y x$ (Note: This is a show question)

(b)

M1 for converting at least one logarithm correctly to a power of y $y = x^{\frac{1}{2}}$ or $y = x^2$

A1 for both correct values $y = x^{\frac{1}{2}}$ and $y = x^2$

M1d for substituting either $y = x^2$ **or** $y = x^{\frac{1}{2}}$ into the equation $xy = 27$ and attempting to find a value for y or x.

A1 for either $y = 9, x = 3$ **OR** $x = 9, y = 3$

M1d for substituting both $y = x^2$ **and** $y = x^{\frac{1}{2}}$ into the equation $xy = 27$ and attempting to find a value for y or x.

A1 for both $x = 9, y = 3$ **AND** $y = 9, x = 3$

ALT

M1 for taking logs of both sides and using properties of logs to attempt to form the equation $\log_y x + \log_y y = \log_y 27$

A1 for the correct equation

M1d for using one of the values of $\log_y x$, and using the fact that $\log_y y = 1$ leading to $\log_y 27 = ...$

A1 for either $y = 9, x = 3$ **OR** $x = 9, y = 3$

M1d for using the other value of $\log_y x$ to find the second value of y

A1 for both $x = 9, y = 3$ **AND** $y = 9, x = 3$

Question Number	Scheme	Marks
11 (a)	$4+3x-x^2=\frac{25}{4}-\left(x-\frac{3}{2}\right)^2$	M1A1 (2)
(b)	$\left(\frac{3}{2},\frac{25}{4}\right)$	B1ft (1)
(c)	$f'(x)=3-2x \Rightarrow f'(1)=3-2\times1=1$ $y=6$	M1A1 B1
	$(y-6)=1(x-1)\Rightarrow(y=x+5)$	M1A1 (5)
(d)	$-1=3-2x\Rightarrow x=2$	B1
	$\{(y-6)=-1(x-2),\Rightarrow y=-x+8\}$	M1A1
	$\Rightarrow -x+8=x+5\Rightarrow x=\frac{3}{2},\Rightarrow y=\frac{13}{2}\ \Rightarrow(\frac{3}{2},\frac{13}{2})$	M1A1 (5)
(e)	$AB=\sqrt{(\frac{3}{2}--3)^2+(\frac{13}{2}-2)^2}=\frac{9\sqrt{2}}{2}$	M1A1 (2)
(f)	$\left\{AD=\sqrt{(\frac{3}{2}-8)^2+(\frac{13}{2}-0)^2}=\frac{13\sqrt{2}}{2}\right\}$	M1A1ftA1
	Area = $\frac{9\sqrt{2}}{2}\times\frac{13\sqrt{2}}{2}\times\frac{1}{2}=\frac{117}{4}$ units^2	(3)
	ALT $\frac{1}{2}\begin{vmatrix}1.5 & -3 & 8 & 1.5\\ 6.5 & 2 & 0 & 6.5\end{vmatrix}=\frac{117}{4}$	{M1A1ftA1} **(18)**

Notes

(a)
M1 for attempting to complete the square (see General Guidance)
A1 for the correctly completed expression.
(b)
B1ft for the correct coordinates $\left(\frac{3}{2}, \frac{25}{4}\right)$ or accept $x = \frac{3}{2}, y = \frac{25}{4}$

(c)
M1 for an attempt at differentiating f (x) **AND** substituting $x = 1$ to find a value for the gradient
A1 for gradient = 1
B1 for $y = 6$
M1d for substituting $x = 1$, their y, and m into a correct formula for the equation of a straight line, or by using $y = mx + c$. Award the mark when they have found the value of c using their y and m.
A1 for the correct equation of the line as shown in any form
(d)
B1 for stating $x = 2$ when $m = -1$

Alternative for this mark

B1 they use the coordinate (8, 0) and find $m = -1$
M1 for substituting their x, y, and m into a correct formula for the equation of a straight line, or by using $y = mx + c$. Award the mark when they have found the value of c using their y, x and m.
A1 for the correct equation of the line as shown in any form
M1 for equating the lines to give a value of x or y and attempting to find a value for **BOTH** x and y
A1 for the correct coordinates or values
(e)
M1 for using formula or Pythagoras to find the length AB
A1 for $AB = \frac{9\sqrt{2}}{2}$ or any equivalent surd. For example, $\sqrt{\frac{162}{4}}$, $\frac{9}{\sqrt{2}}$
(f)

M1 for attempting to find the length AD
A1ft For using their values of AB and AD to find the area of the triangle ABD
A1 for the area of the triangle = 29.25 $(\text{units})^2$ oe

ALT

M1 using determinants correctly with(8,0), (−3,2) and their coordinates for the point A of the triangle
A1ft for their values in a correct calculation (either way around)
A1 for the area of the triangle = 29.25 $(\text{units})^2$ oe

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London WC2R 0RL