Question number	Scheme	Marks
6 (a)	$\left(\alpha-\beta\right)^2=\alpha^2-2\alpha\beta+\beta^2$	M1
	$\left(\alpha-\beta\right)^2=(\alpha+\beta)^2-2\alpha\beta-2\alpha\beta$	M1
	$(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$ *	A1 cso (3)
(b)	$\alpha+\beta=7k$ and $\alpha\beta=k^2$	B1
	$\alpha-\beta=\sqrt{49k^2-4k^2}$	M1
	$=\sqrt{45}k=3k\sqrt{5}$ *	A1 cso (3)
(c)	Sum = $\alpha+\beta$ $(=7k)$	B1ft
	Product $(\alpha+1)(\beta-1)=\alpha\beta-(\alpha-\beta)-1\Rightarrow(\alpha+1)(\beta-1)=k^2-3k\sqrt{5}-1$	M1
	So $x^2-7kx+k^2-3k\sqrt{5}-1=0$	M1A1 (4)
		Total 10 marks

Note: You may see a method based on the difference of two squares for part (a)
i.e. $\left(\alpha+\beta\right)^2-\left(\alpha-\beta\right)^2=4\alpha\beta$

<u>Solution</u>

$$\left(\alpha-\beta\right)^2=\left(\alpha+\beta\right)^2-4\alpha\beta\Rightarrow\left(\alpha+\beta\right)^2-\left(\alpha-\beta\right)^2=4\alpha\beta$$

$$\begin{aligned}\left(\alpha+\beta\right)^2-\left(\alpha-\beta\right)^2&=\left([\alpha+\beta]+[\alpha-\beta]\right)\left([\alpha+\beta]-[\alpha-\beta]\right)\\&=(2\alpha)(2\beta)\\&=4\alpha\beta\end{aligned}$$

$$\text{LHS} = \text{ RHS (hence shown)}$$

If this is a full and correct solution as shown (no errors) – award full marks – otherwise, please send to Review.

Part	Mark	Notes
(a)	**M1**	For expanding $(\alpha-\beta)^2=\alpha^2-2\alpha\beta+\beta^2 \Rightarrow (\alpha^2+\beta^2-2\alpha\beta)$ or expanding $(\alpha+\beta)^2=\alpha^2+\beta^2+2\alpha\beta$ This must be correct for this mark.
	M1	Replaces $\alpha^2+\beta^2$ with $(\alpha+\beta)^2-2\alpha\beta$ in their expansion of $(\alpha-\beta)^2$. And attempts to collect terms $(\alpha-\beta)^2=(\alpha+\beta)^2-2\alpha\beta-2\alpha\beta$
	A1 cso	For the correct given expression **with no errors seen.** $(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$ *
	ALT	
	M1	For expanding $(\alpha-\beta)^2=\alpha^2-2\alpha\beta+\beta^2 \Rightarrow (\alpha^2+\beta^2-2\alpha\beta)$ or expanding $(\alpha+\beta)^2=\alpha^2+\beta^2+2\alpha\beta$ This must be correct for this mark.
	M1	For expanding the RHS and equates to their expansion of $(\alpha-\beta)^2$ and attempting to simplify. $\alpha^2+\beta^2-2\alpha\beta=(\alpha^2+\beta^2+2\alpha\beta)-4\alpha\beta$
	A1 cso	Both sides of the equivalence are shown to be equal **with no errors seen.** $(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$ *
(b)	**B1**	For both $\alpha+\beta=7k$ and $\alpha\beta=k^2$ This may be implied in later work e.g. by use of $49k^2$ and $4k^2$
	M1	For substituting their values for the sum and product into the **given** expression for $(\alpha-\beta)^2$, simplifying and square rooting both sides. $(\alpha-\beta)^2=(7k)^2-4k^2 \Rightarrow \alpha-\beta=\sqrt{(7k)^2-4k^2}=\sqrt{45k^2}$ Condone $\pm\sqrt{45k^2}$ for this mark.
	A1 cso	For the correct value of $\alpha-\beta=3k\sqrt{5}$ *
(c)	**B1ft**	For the sum $(\alpha+1+\beta-1)=\alpha+\beta=7k$ Ft their $\alpha+\beta$
	M1	For the product in terms of k. Correctly multiplying out $(\alpha+1)(\beta-1)$ and substituting in their value $\alpha\beta$ and the correct value $\alpha-\beta=3k\sqrt{5}$ $(\alpha+1)(\beta-1)=\alpha\beta-(\alpha-\beta)-1 \Rightarrow (\alpha+1)(\beta-1)=k^2-3k\sqrt{5}-1$
	M1	For correctly forming an equation with their sum and product $x^2-{}'7k'x+{}'k^2-3k\sqrt{5}-1'(=0)$ Condone the absence of = 0 for this mark
	A1	For the correct equation $x^2-7kx+k^2-3k\sqrt{5}-1=0$ Allow $p=-7k \quad q=k^2-3k\sqrt{5}-1$