

Question Number	Answer	Notes	Marks
10	<p>(a)</p> $\alpha + \beta = -(k-3) \quad \alpha\beta = 4$ $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $= (k-3)^2 - 8$ <p>(b) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}, \quad = \frac{7}{4}$</p> $\frac{1}{\alpha^2\beta^2} = \frac{1}{16}$ <p>Eqn: $x^2 - \frac{28}{16}x + \frac{1}{16} (=0)$</p> $16x^2 - 28x + 1 = 0$ <p>(c)</p> $4(k^2 - 6k + 1) = 7 \times 16$ $k^2 - 6k - 27 = 0$ $(k-9)(k+3) = 0$ $k = 9 \quad k = -3$	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1,A1</p> <p>B1</p> <p>M1</p> <p>A1ft</p> <p>M1A1</p> <p>M1d</p> <p>A1A1</p>	(13)

Notes

(a)

B1 for BOTH $\alpha + \beta = -(k-3)$ oe, AND $\alpha\beta = 4$ M1 for the correct algebra on $(\alpha + \beta)^2$ to give $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ oeA1 for $\alpha^2 + \beta^2 = (k-3)^2 - 8$ or $\alpha^2 + \beta^2 = k^2 - 6k + 1$
Simplification not required

(b)

M1 for an attempt at a sum of roots $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$ A1 for sum $= \frac{7}{4}$ oe

B1 for correct product $= \frac{1}{16}$

M1 for using x^2 – their sum $\times x$ + their product ($= 0$ not required for this mark)
The sum and product must be numerical values only.

A1ft for $16x^2 - 28x + 1 = 0$ oe integer values
(follow through their values for this mark) Note: $= 0$ must be seen, but simplification is not required for this mark.

(c)

M1 for using $4 \times (\text{their } \alpha^2 + \beta^2) = 7 \times \text{their } \alpha^2 \beta^2$

A1 for $k^2 - 6k - 27 = 0$

M1d for attempting to solve their 3TQ (usual rules) this is dependant on first M mark being awarded

A1 for **either** $k = 9$ or $k = -3$

A1 for **both** $k = 9$ or $k = -3$

Question Number	Answer	Notes	Marks
11	<p>(a)</p> <p>Through $(p, 8)$ $40 = 4(p^2 + 1)$ $10 = p^2 + 1$ $p = 3$ At $P(3, 8)$ $40 - 72 + q = 0$ $q = 32$</p> <p>(b) $5 \frac{dy}{dx} = 8x$, $p = 3$ $\frac{dy}{dx} = \frac{24}{5}$ grad normal $= -\frac{5}{24}$ Eqn. normal: $y - 8 = -\frac{5}{24}(x - 3)$ $24y + 5x = 207$ o.e.</p> <p>(c)</p> <p>normal meets x-axis at $\frac{207}{5}$ tangent meets x-axis at $\frac{32}{24} \left(= \frac{4}{3} \right)$ Area of $\Delta = \frac{1}{2} \left(\frac{207}{5} - \frac{4}{3} \right) \times 8 = 160 \frac{4}{15}$</p> <p>(d)</p> <p>Vol of curve $= \pi \int_0^3 \left[\frac{4}{5}(x^2 + 1) \right]^2 dx$ $= \frac{16}{25} \pi \int_0^3 (x^4 + 2x^2 + 1) dx = \frac{16}{25} \pi \left[\frac{x^5}{5} + \frac{2}{3}x^3 + x \right]_0^3$ $= \frac{16}{25} \pi \left[\frac{3^5}{5} + \frac{2}{3} \times 3^3 + 3 \right]$ Cone $= \frac{1}{3} \pi \times 8^2 \left(3 - \frac{4}{3} \right)$ Reqd. vol $= \frac{16}{25} \pi \left[\frac{3^5}{5} + \frac{2}{3} \times 3^3 + 3 \right] - \frac{1}{3} \pi \times 8^2 \times \frac{5}{3}, = 28$</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1,A1</p> <p>A1ft</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>M1,A1</p>	(18)

Notes

(a)

(i)

M1 for substituting coordinates $(p, 8)$ to get $40 = 4(p^2 + 1)$ (or $40 = 4(x^2 + 1)$)A1 for solving $40 = 4(p^2 + 1)$ to give $p = 3$ Note: this is a show question, all working must be seen.

(ii)

M1 for using $(3, 8)$ in line l to give $40 - 72 + q = 0$ A1 for $q = 32$

(b)

Either Method 1M1 for an attempt at differentiating $5y = 4(x^2 + 1)$ to give $5 \frac{dy}{dx} = 8x$ oe.A1 for $\frac{24}{5}$ (accept $\frac{24}{5}$ embedded in the equation of the line provided it is used correctlylater for the gradient of the normal $= -\frac{5}{24}$ A1ft for the gradient of the normal $-\frac{5}{24}$ or their negative inverted gradient of tangent.

M1 for an attempt at the equation of the normal using their gradient of the normal, which must be a numerical value, and which must be a changed value from the gradient of the tangent. The formula must be seen first if there are errors in substitution.

Or, as above for the gradient using a complete method for $y = mx + c$ to achieve a value for c .A1 for $24y + 5x = 207$ oe.**Or Method 2**

M1 for dividing through by 5 and extracting the gradient

A1 for $m = \frac{24}{5}$ (accept $\frac{24}{5}$ embedded in the equation of the line provided it is usedcorrectly later for the gradient of the normal $= -\frac{5}{24}$ A1ft for the gradient of the normal $-\frac{5}{24}$ or their negative inverted gradient of tangent.

M1 for an attempt at the equation of the normal using their gradient of the normal, which must be a numerical value, and which must be a changed value from the gradient of the tangent. The formula must be seen first if there are errors.

Or, as above for the gradient using a complete method for $y = mx + c$ to achieve a value for c .A1 for $24y + 5x = 207$ oe.

(c)

M1 for using their equations for line l and their normal to substitute $y = 0$ to find the intersections with the x axis at $\frac{32}{24}$ and $\frac{207}{5}$ respectively.

M1ft for using Area of $\Delta = \frac{1}{2} \left(\frac{207}{5} - \frac{4}{3} \right) \times 8 = 160 \frac{4}{15}$ follow through their values, but the value of 8 for the height must be used.

A1 for area = $160 \frac{4}{15}$ or $\frac{2404}{15}$ oe

(d)

M1 for the correct expression of the Volume of revolution of the given curve.

Volume = $\int \pi y^2 dx$. Limits must be seen (not necessarily correct) for this mark.

M1 for an attempt at squaring and integrating the given curve. Ignore missing π

A1 for a correct integration and attempt at evaluation, (simplification not required) of

$$= \frac{16}{25} \pi \left[\frac{3^5}{5} + \frac{2}{3} \times 3^3 + 3 \right] \text{ oe } \pi \text{ may be missing.}$$

$$(\text{Volume of revolution of curve} = \frac{5568}{125} \pi)$$

B1 for using a correct formula for the volume of a cone and values for x of $x = 3$ and their intersection of the tangent with the x axis, with a height of 8.

$$\text{M1 for the required volume} = \frac{16}{25} \pi \left[\frac{3^5}{5} + \frac{2}{3} \times 3^3 + 3 \right] - \frac{1}{3} \pi \times 8^2 \times \frac{5}{3}$$

A1 for Volume = 28 (2sf) 28.23803103..

Alternative

M1 for the expression of the Volume of revolution of the region using the given equation

$$\text{in Volume} = \int_0^3 \pi \left[\frac{4}{5} (x^2 + 1) \right]^2 dx - \int_{\frac{4}{3}}^3 \pi \left[\frac{1}{5} (24x - 32) \right]^2 dx$$

Limits must be seen (not necessarily correct) for this mark, **although the limits for the curve and line must be different**. The correct expression for volume must be used. ie., Volume of revolution = $\pi \int y^2 dx$

M1 for an attempt at squaring and integrating the **given curve**. A combined expression for the curve and line gets M0 even if there is some correct integration. Ignore missing π

A1 for a fully correct integration of **either** the curve or the line.

$$\text{Vol} = \frac{16}{25} \pi \left[\frac{x^5}{5} + \frac{2}{3} x^3 + x \right]_0^3 - \frac{\pi}{25} [192x^3 - 768x^2 + 1024x]_{\frac{4}{3}}^3$$

B1 For a fully correct integration of the volume of revolution of **both** the curve and line.

M1d for substitution of x values (into integrated expressions) **of the curve and line separately** and an attempt at evaluation.

$$(\text{Volume} = \frac{5568}{125}\pi - \frac{320}{9}\pi)$$

A1 for Volume = 28 (2sf) 28.23803103..

