8 Using the identities $\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

(a) (i) show that $\cos 2A = 1 - 2\sin^2 A$

(3)

(ii) express $\sin 2A$ in terms of $\sin A$ and $\cos A$, simplifying your answer.

(1)

(b) Hence show that $\sin 3A = 3\sin A - 4\sin^3 A$

(4)

(c) Solve, for $-90° \leqslant A \leqslant 90°$, the equation

$$8\sin^3 A - 6\sin A = 1$$

(4)

(d) (i) Find $\int \sin^3\theta \, d\theta$

(ii) Evaluate $\int_0^{\frac{\pi}{4}} \sin^3\theta \, d\theta$, giving your answer in the form $\frac{a - b\sqrt{2}}{c}$, where a, b, and c are integers.

(5)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 17 marks)