Question Number	Answer	Marks
10		
(a)	$ar + ar^2 = 7.5$	M1
	$S = \dfrac{a}{1-r} = 20$	A1
	$\dfrac{7.5}{r+r^2} = 20(1-r)$	M1dep
	$3 = 8(1-r)(r+r^2) = 8(r-r^3)$	
	$8r^3 - 8r + 3 = 0$ *	A1 (4)
(b)	$8\times\dfrac{1}{8} - 8\times\dfrac{1}{2} + 3 = 0$	B1 (1)
(c)	$(2r-1)(4r^2+2r-3) = 0$ (or by division)	M1
	$\left(r = \dfrac{1}{2}\right)$ $r = \dfrac{-2\pm\sqrt{4-4\times4\times(-3)}}{8} = \dfrac{-2\pm\sqrt{52}}{8}, = 0.65\ldots\ -1.15$	M1,A1
	$r = 0.65$ too big $r = -1.15$ not convergent	
	$\therefore$ only possible value for r is $\dfrac{1}{2}$	A1 (4)
(d)	$\dfrac{a}{1-\frac{1}{2}} = 20$ $\left(\text{or } a = \dfrac{7.5}{\frac{1}{2}\times\frac{3}{2}}\right)$	M1
	$a = 10$	A1 (2)
(e)	99% of 20 or 0.99×20 or 19.8 seen	B1
	$\dfrac{10(1-0.5^n)}{1-0.5} > 19.8$	M1A1
	$1-0.5^n > \dfrac{19.8}{20}$ $(=0.99)$	
	$0.01 > 0.5^n$	M1
	Solve by logs to obtain $n > 6.6$ (or by trial and error)	M1 dep
	$n = 7$	A1 (6) [17]

(e)	**Alt:** $S_n = \dfrac{10\left(1-\left(\frac{1}{2}\right)^n\right)}{\frac{1}{2}}$	
	$= 20 - 20\left(\frac{1}{2}\right)^n > 0.99 \times 20$	M1A1 B1 (0.99x20)
	$20\left(\frac{1}{2}\right)^n < 0.01 \times 20$	
	$\left(\frac{1}{2}\right)^n < \frac{1}{100}$	M1
	$2^n > 100$	
	$\Rightarrow n = 7$ is least value (Award M1 A0 if $n = 6.6$ seen)	M1depA1..(6)

Notes

(a)

M1 for forming an equation using the given information - award for either equation. Formulae used must be correct

A1 for forming a second equation and both equations fully correct

M1dep for eliminating *a* between the two equations. The two equations do not need to be correct but the first M mark must have been gained.

A1cso for $8r^3 - 8r + 3 = 0$ *

(b)

B1 for substituting $r = \frac{1}{2}$ in the **given** equation and showing that this gives lhs = 0

There are longer methods. Provided the work shows that $r = \frac{1}{2}$ is a root of the equation, award B1.

(c)

M1 for using the factor $(2r-1)$ to factorise the equation either by inspection or division.

This work may have been done in (b). If seen in (b) award this mark.

M1 for solving the quadratic by the formula or completing the square (see general principles for further information)

A1 for **both** values of *r* from the quadratic. One sf or surd form is sufficient here

A1ft for deducing that $r=\frac{1}{2}$ is the only possible value. Award this mark even if the values obtained from the quadratic are incorrect, providing they are **both** outside the range -1 < *r* < 0.6. If the range is stated to be 0 < *r* < 0.6 award A0.

(d)

M1 for using either of the equations formed in (a) with $r=\frac{1}{2}$ to obtain a value for *a*

A1cao for $a=10$

(e)

B1 for 99% of 20 (or 0.99×20 or 19.8 seen)

M1 for using the formula for the sum of the first *n* terms (formula must be correct) and setting up an inequality or equation with $r=\frac{1}{2}$, their *a* and their evaluated 99% of 20

A1 for a completely correct inequality or equation

M1 for solving to a 2 term inequality or equation with $\left(\frac{1}{2}\right)^n$ oe included

M1dep for solving *their* inequality or equation, logs can be used or trial and error. If logs used, with a correct inequality, expect to see *n* > 6.6 oe; if trial and error used expect to see indication that 6 is too small and 7 works (or too large if solving an equation). Dependent on **both** previous M marks.

A1cso for $n=7$. (Some candidates make two sign errors in their working. Such work can gain the M marks but scores A0 here as their solution is incorrect.)