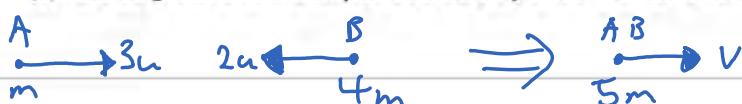


1. A railway truck A of mass m and a second railway truck B of mass $4m$ are moving in opposite directions on a smooth straight horizontal track when they collide directly. Immediately before the collision the speed of truck A is $3u$ and the speed of truck B is $2u$. In the collision the trucks join together. Modelling the trucks as particles, find

(a) the speed of A immediately after the collision. (3)

(b) the direction of motion of A immediately after the collision. (1)

(c) the magnitude of the impulse exerted by A on B in the collision. (3)



a)

Cons. of momentum:

$$\rightarrow 3mu - 2u(4m) = 5mv$$

$$v = -u$$

\therefore Speed of A is u

b)

A moves in the opposite direction

to its initial motion

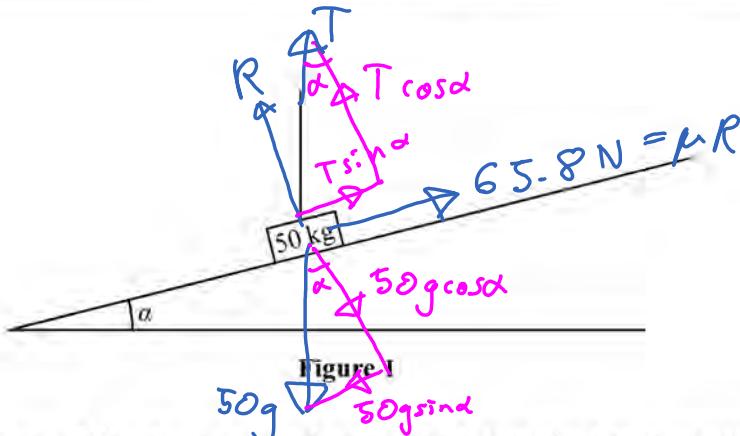
c)

$$\rightarrow I = mv - mu$$

$$= 4m(-u - (-2u))$$

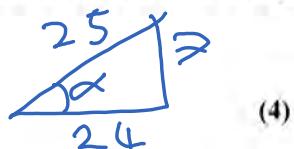
$$= 4mu$$

2.



A block of mass 50 kg lies on a rough plane which is inclined to the horizontal at an angle α , where $\tan \alpha = \frac{7}{24}$. The block is held at rest by a vertical rope, as shown in Figure 1, and is on the point of sliding down the plane. The block is modelled as a particle and the rope is modelled as a light inextensible string. Given that the friction force acting on the block has magnitude 65.8 N, find

- (a) the tension in the rope,



- (b) the coefficient of friction between the block and the plane.

(4)

\therefore equilibrium

a)

$$\uparrow 65.8 + Ts \sin \alpha - 50g s \in \alpha = 0$$

$$T = \left[50g \left(\frac{7}{25} \right) - 65.8 \right] \frac{25}{7}$$

$$= 255 \text{ N}$$

b)

$$\nwarrow R + T c \sin \alpha - 50g c \sin \alpha = 0$$

$$R = 50g \left(\frac{24}{25} \right) - 255 \left(\frac{24}{25} \right)$$

$$= 255.6 \text{ N}$$

$$F = \mu R$$

$$\mu = \frac{F}{R} = \frac{65.8}{255.6} = 0.292$$

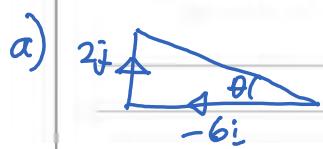
3. [In this question \mathbf{i} and \mathbf{j} are unit vectors directed due east and due north respectively.]

A particle P is moving with constant velocity $(-6\mathbf{i} + 2\mathbf{j}) \text{ m s}^{-1}$. At time $t = 0$, P passes through the point with position vector $(2\mathbf{i} + 5\mathbf{j}) \text{ m}$, relative to a fixed origin O .

- (a) Find the direction of motion of P , giving your answer as a bearing to the nearest degree. (3)

- (b) Write down the position vector of P at time t seconds. (1)

- (c) Find the time at which P is north-west of O . (3)



$$\tan \theta = \frac{2}{6}$$

$$\theta = 18.4^\circ$$

$$\therefore \text{Bearing is } 270 + 18 = 288$$

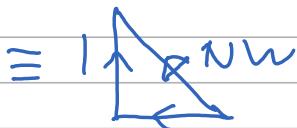
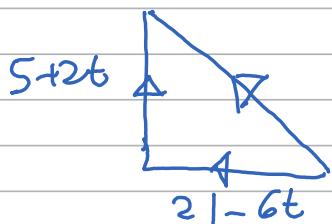
b)

$$\underline{\Sigma} = \underline{\Sigma}_0 + \underline{v} t$$

$$= \begin{pmatrix} 2 \\ 5 \end{pmatrix} + t \begin{pmatrix} -6 \\ 2 \end{pmatrix}$$

$$= (21 - 6t)\mathbf{i} + (5 + 2t)\mathbf{j}$$

c)



$$5 + 2t = -(21 - 6t)$$

$$4t = 26$$

$$t = 6.5$$

4. The points P and Q are at the same height h metres above horizontal ground. A small stone is dropped from rest from P . Half a second later a second small stone is thrown vertically downwards from Q with speed 7.35 m s^{-1} . Given that the stones hit the ground at the same time, find the value of h .

(7)

$$P: s = h$$

$$u = 0$$

$$v = x$$

$$a = g$$

$$t = T$$

$$Q: s = h$$

$$u = -7.35 \text{ m s}^{-1}$$

$$v = x$$

$$a = g$$

$$t = T - 0.5$$

$$\left[s = ut + \frac{1}{2}at^2 \right]$$

$$\left[s = ut + \frac{1}{2}at^2 \right]$$

$$h = \frac{g}{2}T^2 \quad (1)$$

$$h = 7.35(T - 0.5) + \frac{g}{2}(T - 0.5)^2$$

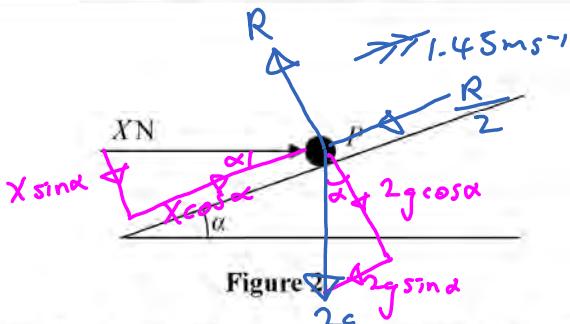
$$\Rightarrow \frac{g}{2}T^2 = 7.35T - \frac{7.35}{2} + \frac{gT^2}{2} - \frac{gT}{2} + \frac{g}{8}$$

$$2.45 = 2.45T$$

$$T = 1$$

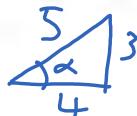
$$\text{In (1): } h = \frac{g}{2}(1)^2 = 4.9 \text{ m}$$

5.



A particle P of mass 2 kg is pushed up a line of greatest slope of a rough plane by a horizontal force of magnitude X newtons, as shown in Figure 2. The force acts in the vertical plane which contains P and a line of greatest slope of the plane. The plane is inclined to the horizontal at an angle α , where $\tan \alpha = \frac{3}{4}$

The coefficient of friction between P and the plane is 0.5



Given that the acceleration of P is 1.45 m s^{-2} , find the value of X .

(10)

$$\uparrow \downarrow \text{ equilibrium: } R - X \sin \alpha - 2g \cos \alpha = 0$$

$$R = \frac{3X}{5} + 2g \left(\frac{4}{5} \right)$$

$$\rightarrow [F = ma]$$

$$X \cos \alpha - \frac{R}{2} - 2g \sin \alpha = 2(1.45)$$

$$\frac{4X}{5} - \frac{3X}{10} - \frac{4g}{5} - \frac{6g}{5} = 2.9$$

$$X = 45 \text{ N}$$

6. A uniform rod AC , of weight W and length $3l$, rests horizontally on two supports, one at A and one at B , where $AB = 2l$. A particle of weight $2W$ is placed on the rod at a distance x from A . The rod remains horizontal and in equilibrium.

(a) Find the greatest possible value of x .

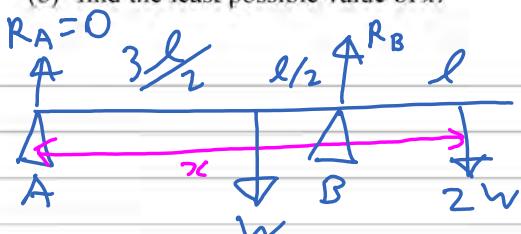
(5)

The magnitude of the reaction of the support at A is R . Due to a weakness in the support at A , the greatest possible value of R is $2W$,

(b) find the least possible value of x .

(5)

a)



At max x , rod is on
limit of tilting
about B
 $\Rightarrow R_A = 0$

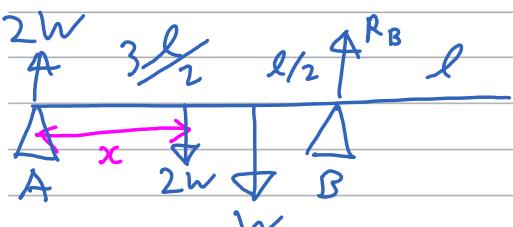
$$A) 2Wx + \frac{3l}{2}W - 2lR_B = 0$$

$$\uparrow R_B = W + 2W$$

$$\Rightarrow 2Wx + \frac{3l}{2}W - 6Wl = 0$$

$$x = \frac{9l}{4}$$

b)



At min x , $R = 2W$

$$B) \frac{lW}{2} + (2l-x)2W = 2l \cdot 2W$$

$$\frac{l}{2} + 4l - 2x = 4l$$

$$x = \frac{l}{4}$$

7. A train travels along a straight horizontal track between two stations A and B . The train starts from rest at A and moves with constant acceleration until it reaches its maximum speed of 108 km h^{-1} . The train then travels at this speed before it moves with constant deceleration coming to rest at B . The journey from A to B takes 8 minutes.

(a) Change 108 km h^{-1} into m s^{-1} .

(2)

(b) Sketch a speed-time graph for the motion of the train between the two stations A and B .

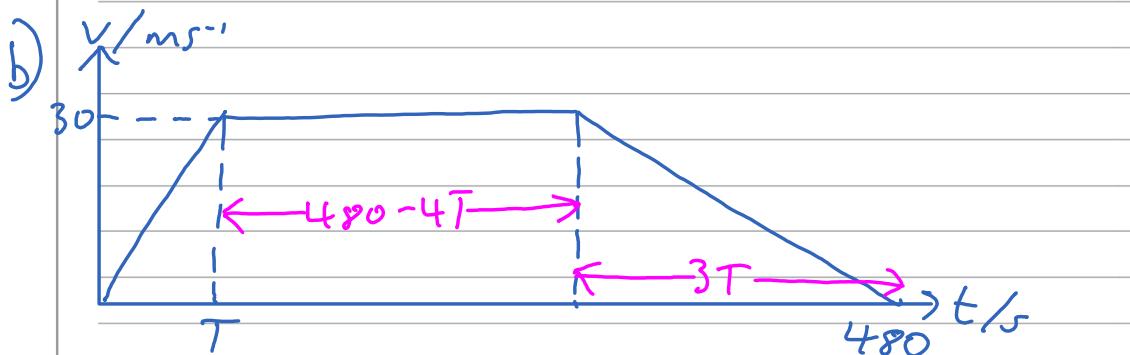
(2)

Given that the distance between the two stations is 12 km and that the time spent decelerating is three times the time spent accelerating,

(c) find the acceleration, in m s^{-2} , of the train.

(6)

a) $108 \text{ km h}^{-1} = \frac{108 \times 1000}{60 \times 60} \text{ ms}^{-1} = 30 \text{ ms}^{-1}$



c) $12000 = \frac{1}{2}(480 - 4T + 4T) 30$

$$800 = 160 - 4T$$

$$T = 40 \text{ s}$$

$$a = \frac{\Delta v}{\Delta t} = \frac{30}{40} = 0.75 \text{ ms}^{-2}$$

8.

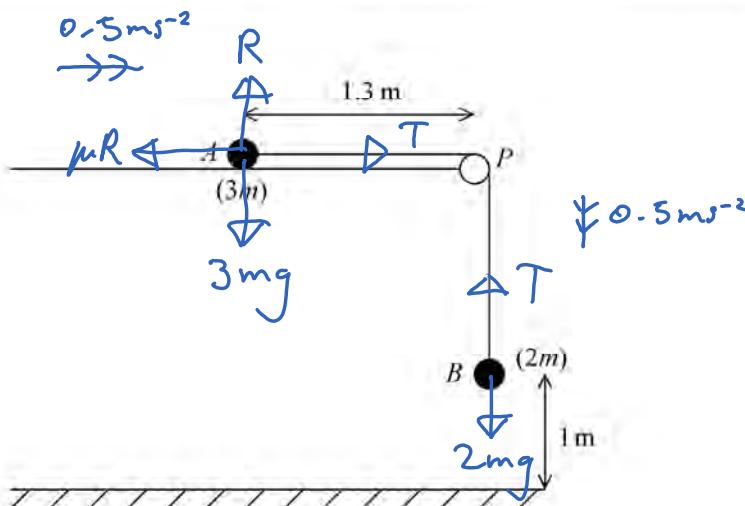


Figure 3

A particle A of mass $3m$ is held at rest on a rough horizontal table. The particle is attached to one end of a light inextensible string. The string passes over a small smooth pulley P which is fixed at the edge of the table. The other end of the string is attached to a particle B of mass $2m$, which hangs freely, vertically below P . The system is released from rest, with the string taut, when A is 1.3 m from P and B is 1 m above the horizontal floor, as shown in Figure 3.

Given that B hits the floor 2 s after release and does not rebound,

- (a) find the acceleration of A during the first two seconds,

(2)

- (b) find the coefficient of friction between A and the table,

(8)

- (c) determine whether A reaches the pulley.

(6)

$$a) \downarrow s = 1\text{ m} \quad [s = ut + \frac{1}{2}at^2]$$

$$u = 0$$

$$v = x \quad l = \frac{1}{2}a(2)^2$$

$$a = ?$$

$$t = 2\text{ s} \quad a = 0.5\text{ ms}^{-2}$$

$$b) B : [F = ma] \downarrow$$

A : $\uparrow \& \text{ equilibrium}$

$$2mg - T = 2m(0.5)$$

$$R = 3mg$$

$$T = m(2g - 1)$$

Question 8 continued

$$A: \rightarrow [F=ma]$$

$$T - \mu R = 3m(0.5)$$

$$m(2g - 1) - \mu(3mg) = 1.5m$$

$$\mu = \frac{2g - 2.5}{3g}$$

$$= 0.582$$

c) When A is 0.3 m from the pulley,
the string becomes slack $\Rightarrow T=0$

$$\uparrow R = 3mg \rightarrow [F=ma]$$

$$- \mu R = 3ma$$

$$a = \frac{-0.582(3mg)}{3m}$$

$$= -5.70 \text{ ms}^{-2}$$

Find speed of A when string becomes slack:

$$s = 1 \text{ m}, u = 0, v = ?, a = 0.5 \text{ ms}^{-2}, t = 2 \text{ s}$$

$$v = u + at = 0.5 \times 2 = 1 \text{ ms}^{-1}$$

Find displacement after string becomes slack:
 $[v^2 = u^2 + 2as] \rightarrow$

$$s = \frac{0 - 1^2}{2(-5.7)} = 0.088 \text{ m}$$

$\therefore A$ does not reach the pulley