Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
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**Pearson Edexcel International GCSE**

| Centre Number | Candidate Number |
|---|---|
| | |

# Monday 20 January 2020

Morning (Time: 2 hours) | Paper Reference **4PM1/02R**

# Further Pure Mathematics

## Paper 2R

**Calculators may be used.**

Total Marks

## Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

## Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

*Turn over* ▶

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## International GCSE in Further Pure Mathematics Formulae sheet

### Mensuration

**Surface area of sphere** = $4\pi r^2$

**Curved surface area of cone** = $\pi r \times$ slant height

**Volume of sphere** = $\frac{4}{3}\pi r^3$

### Series

**Arithmetic series**

Sum to $n$ terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

**Geometric series**

Sum to $n$ terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

**Binomial series**

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

### Calculus

**Quotient rule (differentiation)**

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

### Trigonometry

**Cosine rule**

In triangle $ABC$: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

### Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

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**Answer all ELEVEN questions.**

**Write your answers in the spaces provided.**

**You must write down all the stages in your working.**

1

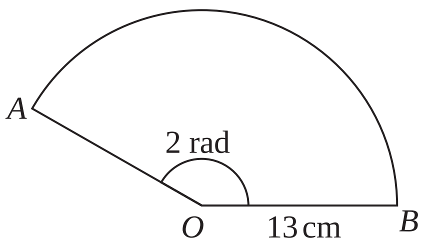


Diagram **NOT** accurately drawn

**Figure 1**

Figure 1 shows the sector *AOB* of a circle with centre *O*.
The radius of the circle is 13 cm and angle *AOB* = 2 radians.

(a) Find the length of the arc *AB*.

**(1)**

(b) Find the area of the sector *AOB*.

**(2)**

**(Total for Question 1 is 3 marks)**

**2**

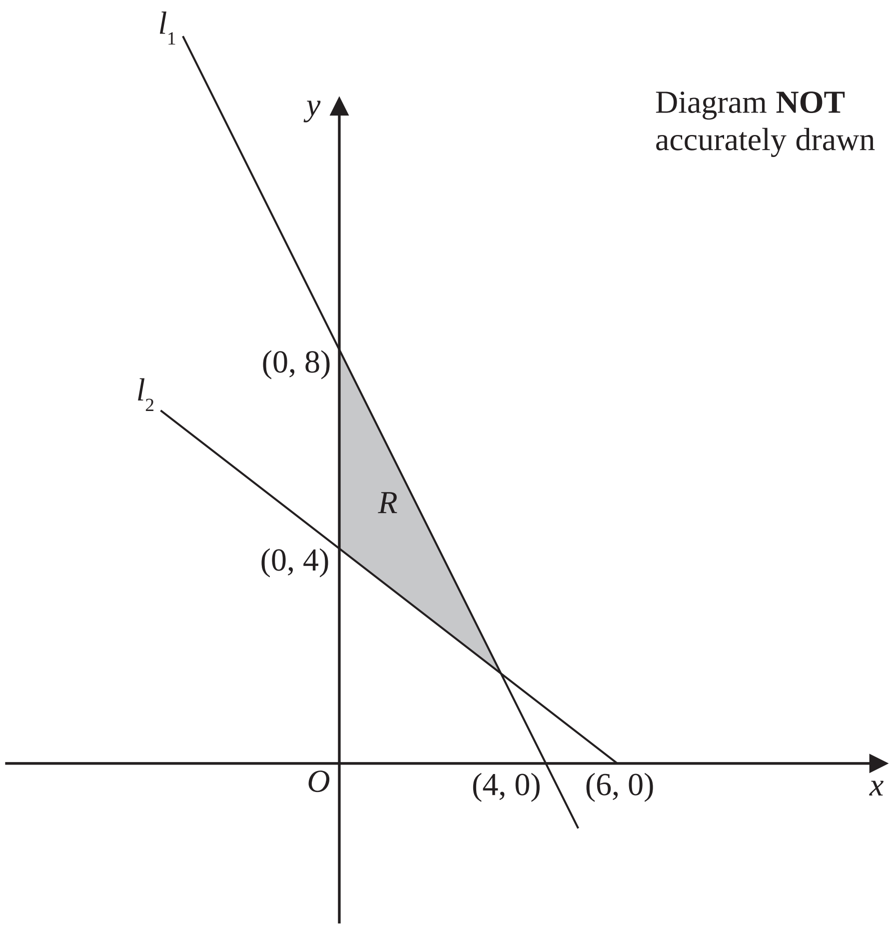


**Figure 2**

Figure 2 shows the shaded region $R$ bounded by the line $l_1$, the line $l_2$ and the $y$-axis.

The points with coordinates (0, 8) and (4, 0) lie on $l_1$

The points with coordinates (0, 4) and (6, 0) lie on $l_2$

(a) Find, in the form $ax + by = c$, where $a$, $b$ and $c$ integers, an equation of

(i) $l_1$

(ii) $l_2$

**(3)**

(b) Hence write down three inequalities that define the region $R$.

**(3)**

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**Question 2 continued**

**(Total for Question 2 is 6 marks)**

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**3** In triangle $ABC$, $AB = 11$ cm and $BC = 12$ cm.

The area of triangle $ABC = 33\,\text{cm}^2$

Find, in cm to 3 significant figures, the two possible lengths of $AC$.

**(5)**

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**Question 3 continued**

**(Total for Question 3 is 5 marks)**

**4**

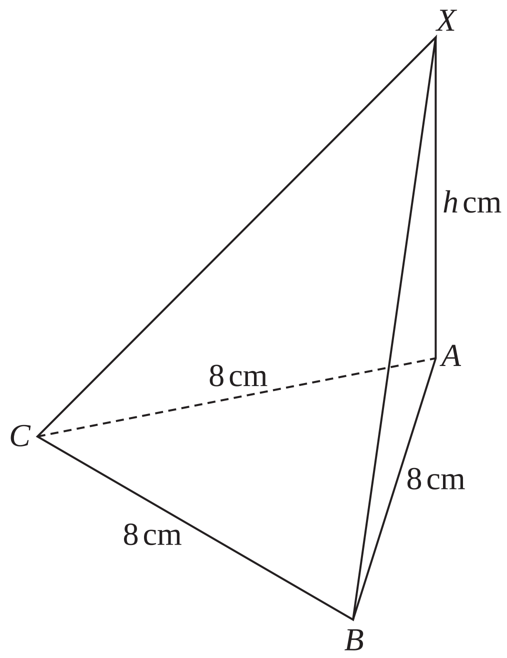


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**Figure 3**

Figure 3 shows a triangular pyramid *ABCX*.
The base *ABC* of the pyramid is an equilateral triangle where $AB = BC = CA = 8$ cm.
The vertex *X* of the pyramid is such that *AX* is perpendicular to the base of the pyramid and $AX = h$ cm.

The volume of the pyramid is $48\sqrt{3}$ cm$^3$

(a) Show that $h = 9$

**(3)**

(b) Find, in degrees to one decimal place, the size of angle *BXC*.

**(3)**

(c) Find, in degrees to one decimal place, the size of the angle between the plane *BCX* and the base *ABC* of the pyramid.

**(3)**

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**Question 4 continued**





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**Question 4 continued**

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**Question 4 continued**

**(Total for Question 4 is 9 marks)**

**5** (a) Show that $(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = \alpha^3 + \beta^3$

**(2)**

The roots of the equation $2x^2 + 3x + 6 = 0$ are $\alpha$ and $\beta$

Without solving the equation,

(b) find the value of $\alpha^3 + \beta^3$

**(2)**

(c) Show that $(\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = \alpha^4 + \beta^4$

**(2)**

(d) Form a quadratic equation with integer coefficients that has roots $(\alpha^3 - \beta)$ and $(\beta^3 - \alpha)$

**(6)**

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**Question 5 continued**

**Question 5 continued**

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**Question 5 continued**

**(Total for Question 5 is 12 marks)**

**6**

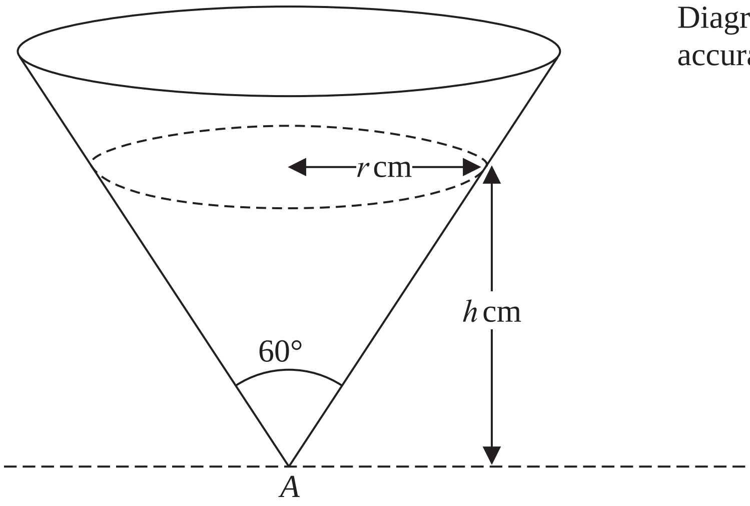


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**Figure 4**

Figure 4 shows a hollow right circular cone fixed with its axis of symmetry vertical.

The cone is inverted and contains liquid, which is dripping out of a small hole at the vertex $A$ of the cone at a constant rate of $0.9\,\text{cm}^3/\text{s}$.

At time $t$ seconds after the liquid starts to drip from the cone, the height of the liquid is $h$ cm above $A$. The volume of liquid in the cone at time $t$ seconds is $V\,\text{cm}^3$

The vertical angle of the cone is 60°

(a) Show that $V = \dfrac{1}{9}\pi h^3$

**(2)**

(b) Find, in cm/s to 3 significant figures, the rate at which the height of the liquid is decreasing when the height of the liquid in the cone above the vertex is 1.2 cm.

**(4)**

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**Question 6 continued**

**Question 6 continued**

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**Question 6 continued**

**(Total for Question 6 is 6 marks)**

**7** The geometric series $G$ has first term $a$, common ratio $r$ and $n$th term $u_n$

Given that $u_4 = \mathrm{e}^{x+2}$ and that $u_7 = \mathrm{e}^{\frac{2x+1}{2}}$

(a) show that $r = \mathrm{e}^{-\frac{1}{2}}$

**(3)**

(b) Hence find $a$ in terms of e and $x$.

**(3)**

Given that the sum to infinity of $G$ can be written as $\dfrac{\mathrm{e}^p}{\mathrm{e}^{\frac{1}{2}} - 1}$

(c) find an expression for $p$ in terms of $x$.

**(3)**

Given that $u_{18} > 1.6$ and that $x$ is an integer,

(d) find the least value of $x$.

**(4)**

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**Question 7 continued**

**Question 7 continued**

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**Question 7 continued**

**(Total for Question 7 is 13 marks)**

**8** (a) Write down the value of $k$ such that $\sin 2A = k\sin A\cos A$

**(1)**

$$g(A) = 2 + 3\cos A - \sin A - 3\sin 2A - 2\cos^2 A$$

Given that $g(A)$ can be written in the form $(p\cos A - \sin A)(q - r\sin A)$ where $p$, $q$ and $r$ are integers,

(b) find the value of $p$, the value of $q$ and the value of $r$.

**(3)**

(c) Hence solve, in radians to 3 significant figures where appropriate, the equation

$$g(2\theta) = 0 \quad \text{for} \quad 0 \leqslant \theta < \pi$$

**(6)**

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**Question 8 continued**



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**Question 8 continued**

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**Question 8 continued**

**(Total for Question 8 is 10 marks)**

**9** Given that $\dfrac{1}{(2-x)^3}$ can be written as $p(1-qx)^{-3}$

(a) find the value of $p$ and the value of $q$.

**(2)**

(b) Expand $\dfrac{1}{(2-x)^3}$ in ascending powers of $x$ up to and including the term in $x^3$ and express each coefficient as an exact fraction in its lowest terms.

**(3)**

$$f(x) = \frac{a+bx}{(2-x)^3} \text{ where } a \text{ and } b \text{ are integers}$$

The first three terms of the expansion of $f(x)$ are $\dfrac{3}{8} - \dfrac{43}{16}x + cx^2$

(c) Find the value of $a$ and the value of $b$.

**(3)**

(d) Find the exact value of $c$.

**(2)**

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**Question 9 continued**





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**Question 9 continued**

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**Question 9 continued**

**(Total for Question 9 is 10 marks)**

**10** The equation of a curve $C$ is $y = \mathrm{f}(x)$ where $\mathrm{f}'(x) = 3x^2 - 4x - p$ and $p \neq 0$

The points with coordinates (2, 0) and (−1, 9) lie on $C$.

(a) Show that $C$ has equation $y = x^3 - 2x^2 - 4x + 8$

**(6)**

The straight line $l$ has equation $y = 8 - 4x$

(b) Use algebraic integration to find the exact area of the finite region bounded by $C$ and $l$.

**(6)**

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**Question 10 continued**

**(Total for Question 10 is 12 marks)**

**11** The curve $C$ has equation $y = \dfrac{3x-2}{x+1}$

(a) Write down an equation of the asymptote to $C$ which is parallel to the

(i) $x$-axis (ii) $y$-axis

**(2)**

(b) Find the coordinates of the point where $C$ crosses the

(i) $x$-axis (ii) $y$-axis

**(2)**

(c) Sketch $C$, showing clearly the asymptotes and the coordinates of the points where $C$ crosses the coordinate axes.

**(3)**

The straight line $l$ has equation $y = mx + 4$

Given that there are **no** points of intersection between $l$ and $C$,

(d) show algebraically that the range of possible values of $m$ can be written as

$$a - 2\sqrt{b} < m < a + 2\sqrt{b}$$

where $a$ and $b$ are integers whose values need to be found.

**(7)**

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**Question 11 continued**

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**Question 11 continued**

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**Question 11 continued**

**Question 11 continued**

**(Total for Question 11 is 14 marks)**

**TOTAL FOR PAPER IS 100 MARKS**

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