Question Number	Scheme	Marks
	$9x^2 - 47x + 16 \ (=0)$ oe	A1
	$x = \dfrac{47 \pm \sqrt{47^2 - 4\times 9\times 16}}{18} \ (= 4.8561..., \ 0.36608...)$	dM1
	$BC = 4x - 5 > 0 \ \therefore x = 4.86$	A1 (5)
(b)	$AB = 4.8561, \ BC = 4\times 4.856 - 5 \ (=14.42)$	
	Area $= \dfrac{1}{2}\times 4.8561 \times (4\times 4.856 - 5)\sin 60°$	M1A1ft
	$= 30.33... = 30.3\,(\text{cm}^2)$	A1 (3) [8]

(a)

M1 Use the cosine rule in ΔABC to form a quadratic equation in x

A1 Correct, unsimplified, equation

A1 Correct **simplified** equation, terms in any order. (3TQ; $\cos 60° = \dfrac{1}{2}$ used.)

dM1 Solve their 3TQ by any valid means. Accept the solution of an incorrect equation by formula **only** if the substitution is shown or the formula quoted. (Calculator solutions accepted **only** if the final answer is correct, but not necessarily rounded.)

A1 Use the expression for BC in terms of x to identify the correct value of x. Award if 2 values for x are shown, followed by a clearly identified final single value. Must be 3 significant figures.

(b)

M1 For using any complete method for finding the area of the triangle, including using their value of x to find the lengths of the sides needed.

A1ft Correct numbers used, follow through their value of x.

A1cao Correct area, no ft. Must be 3 significant figures unless penalised in (a)
Use of 4.86 will lose the final A mark for premature approximation as it leads to 30.4.

ALT **For (b)**
Use any other **complete** method to find the area. Must attempt to find all the necessary terms using their value of x

Eg: Heron's formula: Area $= \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{1}{2}(a+b+c)$

Or $\dfrac{1}{2}\times \text{base} \times \text{height}$

M1 method correct and complete; A1ft A1 as main scheme

6 (a)	$p^6 + 6p^5(qx) + \dfrac{6\times 5}{2!}p^4(qx)^2 + \dfrac{6\times 5\times 4}{3!}p^3(qx)^3 + \dfrac{6\times 5\times 4\times 3}{4!}p^2(qx)^4$	M1
	$= p^6 + 6p^5qx + 15p^4q^2x^2 + 20p^3q^3x^3 + 15p^2q^4x^4...$	A1A1 (3)
(b)	$4\times 15p^2q^4 = 9\times 15p^4q^2$	M1

Question Number	Scheme	Marks
	$4q^2 = 9p^2$ oe	A1
	$(p+q)^6 = 15625 \quad (p+q=5)$	M1 (NB A1 on e-PEN)
	$4(5-p)^2 = 9p^2$	
	$10-2p = \pm 3p$	M1
	$p=2\ q=3$ or $p=-10\ q=15$	A1A1 (6) [9]

(a)M1 Apply the binomial expansion to $(p+qx)^6$ or $p^6\left(1+\frac{qx}{p}\right)^6$ or use Pascal's triangle. Must start $p^6+...$ and have qx (or appropriate power of this) in at least one term or start $p^6(1+...)$ and have $\frac{qx}{p}$ (or appropriate power of this) in at least one term. Can have 3!, 4! or 6, 24 (but not 3, 4)

If $\begin{pmatrix} a \\ b \end{pmatrix}$ or C_b^a seen, no marks until coefficients as shown are seen (or final expansion is correct).

A1 Any 3 terms correct.(p^6 can be one of these.)
Allow with $(qx)^2$ etc provided the numerical part has been simplified.

A1 All 5 terms correct. Brackets expanded for this mark.

(b)

M1 Equate 4 times their coeff of x^4 to 9 times their coeff of x^2. Allow if powers of x included or x = 1 substituted in each term. Award on basis of **their coefficients** even if no powers of p included.

A1 Simplified equation as shown. No x seen now. This mark can be gained if x = 1 has been substituted. (Not follow through.) Coefficients can be a multiple of those shown.

M1 (A1 on e-PEN) Obtain a second equation connecting p and q by substituting $x=1$ in f(x). Award for $(p+\text{ their } q)^6 = 15625$ or sub $x=1,\ q=\frac{3}{2}p$ in their expansion

M1 Eliminate either p or q between their 2 equations and obtain a linear equation in one variable.

A1 One pair of values for p and q correct (NB must have previous M mark)

A1 Second pair correct. Pairing must be clear.

NB: If inequality signs used (due to $(p+q)>0$) treat as = but deduct the final A mark if earned.

7(a)	Surface area $= 2\left(5x^2+hx+5xh\right) = 480$	M1
	$480 = 12hx + 10x^2$ oe	A1

Question Number	Scheme	Marks
	$V = 5x^2h = 5x^2 \times \dfrac{480 - 10x^2}{12x}$	dM1
	$V = 200x - \dfrac{25}{6}x^3$ *	A1cso (4)
(b)	$\dfrac{\mathrm{d}V}{\mathrm{d}x} = 200 - \dfrac{25}{2}x^2$	M1
	$\dfrac{\mathrm{d}V}{\mathrm{d}x} = 0 \quad x = 4 \quad (x > 0)$	dM1A1
	$V = 200 \times 4 - \dfrac{25}{6} \times 4^3 = 533\tfrac{1}{3}$ (accept 533 or $\dfrac{1600}{3}$)	dM1A1cao (5) [9]

(a)

M1 Attempt to obtain a **dimensionally correct** expression for the surface area in terms of x and h and equate to 480

A1 Correct equation, as shown or equivalent.

dM1 Use the volume and eliminate h from the expression

A1cso Obtain the given expression for V in terms of x from correct working

(b)

M1 Differentiate the expression for V. $200x \to 200$ or $\dfrac{25x^3}{6} \to kx^2$ must be seen with no integration.

dM1 Equate their derivative to 0 and solve for x

A1 Correct value of x. Must be positive, negative value need not be shown but if seen ignore it.

dM1 Substitute their **positive** value of x in the expression for V and obtain a numerical value for V. Depends on both M marks above.

A1cao For the correct value of V. Can be exact or at least 3 sig figs.

NB If 2 values of x are both **given** and **used**, the correct final answer must be clearly identified or both A marks are lost.

8(a)	$(\alpha + \beta)^2 = p^2 \quad \alpha\beta = +7$	B1

Question Number	Scheme	Marks
(i)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta,\ =p^2-14$	M1,A1
(ii)	$\alpha^2\beta^2=49$	B1ft (4)
(b)	$7\left(p^2-14\right)=5\times49$	
	$p^2=49 \quad p=\pm7$	M1A1 (2)
(c)	$\dfrac{2p}{\alpha^2}+\dfrac{2p}{\beta^2}=\dfrac{2p\left(\alpha^2+\beta^2\right)}{\alpha^2\beta^2}=\dfrac{2p\left(p^2-14\right)}{49}=\dfrac{14\times35}{49}=10$	M1A1
	$\dfrac{2p}{a^2}\times\dfrac{2p}{\beta^2}=\dfrac{4p^2}{\alpha^2\beta^2}=\dfrac{4\times49}{49}=4$	B1
	$x^2-10x+4=0$	M1A1 (5) [11]

(a)

B1 Correct product of the roots (can be implied by use of $2\alpha\beta=14$) and $(\alpha+\beta)^2=p^2$ seen somewhere. (Ignore $\alpha+\beta=p$)

(i)M1 Correct algebraic expression ready for the required substitution.

A1 Correct expression. $\left(p^2-14\right)$

(ii)B1ft Correct numerical value, follow through their product of the roots.

(b)

M1 Substitute their answers from (a) in the given equation and solve for p.

A1 Correct values for p - both required.

(c)

M1 Add the roots of the new equation to obtain a single fraction (denominator to be $\alpha^2\beta^2$ and substitute their positive value of p and values of $(\alpha+\beta)^2$ and $\alpha^2\beta^2$ to obtain a numerical value of this sum.

A1 Correct value of this sum

B1 Correct value of the product of the roots

M1 Use equation x^2- sum of roots$\times x+$product of roots $(=0)$ with their sum and product (numerical values needed) and with or without "= 0"

A1 Completely correct equation as shown or equivalent to the one shown.

9 (a)	$x^3-4x^2-4x+16=(x-2)(x-a)(x-b)$	

Question Number	Scheme	Marks
	$=(x-2)\left(x^2-(a+b)x+ab\right)$	
	$x^3-2x^2-(a+b)x^2+2(a+b)x+abx-2ab$	M1
	$-ab=8,\ -(a+b)-2=-4 \quad a=-2,\ b=4$	M1A1A1 (4)
	ALT: $(x-2)\left(x^2-2x-8\right),=(x-2)(x-4)(x+2)$ M1, M1 $a=-2,\ b=4$ A1A1 corr answers	
(b)	$D:(0,16)$	B1
	$\dfrac{dy}{dx}=3x^2-8x-4$	M1
	At D grad = -4	A1
	$y-16=-4x$ oe	dM1A1 (5)
(c)	$y=0 \Rightarrow x=4$ or $x=4\Rightarrow y=0$ ($\therefore$ passes through B)	B1 (1)
(d)	Area = $\int_0^4\left(16-4x-\left(x^3-4x^2-4x+16\right)\right)dx$	M1
	$=\left[\dfrac{4}{3}x^3-\dfrac{1}{4}x^4\right]_0^4$	dM1A1
	$=\dfrac{4}{3}\times4^3-\dfrac{1}{4}\times4^4\ (-0)$	dM1
	$=21\dfrac{1}{3}$ (21.3 or better) or $\dfrac{64}{3}$	A1 (5) [15]

(a)

M1 Write the cubic as a product of 3 linear brackets and multiply out these 3 brackets. One bracket must be $(x\pm2)$

Question Number	Scheme	Marks
M1	Extract 2 equations in *a* and *b* and solve them	
A1A1	Correct values for *a* and *b*. Coordinates of the points accepted. Award A1A1, A1A0 or A0A0	
ALT 1	**M1** Divide given cubic by $(x \pm 2)$ **M1** Factorise the quadratic obtained. **A1A1** Correct values of *a* and *b* deduced from the resulting brackets. Coordinates of the points accepted. No errors in the working for A1A1 Division by $x+2$ can score M1M1	
ALT 2	Factorise the given cubic **M1**: $(x^2-4)(x-4)\ (=0)$ **M1** $(x-2)(x+2)(x-4)\ (=0)$ **A1A1** Correct values for *a*, *b* A1A1 Coordinates of the points accepted.	
ALT 3	Remainder/factor theorem: **M1** try $x = \pm$ any factor of 16 **M1** Try more factors of 16 until 2 factors found giving no remainder. **A1A1** correct values of *a*, *b* Coordinates of the points accepted.	
OR	**No working shown** and correct answers stated, 4/4	
(b)		
B1	Correct *y* coordinate for *D*.	
M1	Differentiate the given equation for *C*. Minimum 2 terms differentiated and no integration.	
A1	Substitute $x = 0$ to obtain the correct gradient at *D*	
dM1	Use any complete method to obtain an equation of *l* using their gradient and *y* coordinate. If $y = mx + c$ used there must be an attempt to find the value of *c*. Depends on M mark above.	
A1	Correct equation in any form.	
(c)B1	Substitute *y* = 0 or *x* = 4 into the correct equation of *l* to show *l* passes through (4, 0) This correct equation need not have been awarded all the marks in (b) (No conclusion need be given here.)	
(d)		
M1	Use Area = $\int_0^4 (\text{line} - \text{curve})\,dx$ - either way round - or use the **difference** of 2 separate integrals, both with limits 0 and 4.	
dM1	Integrate their single function or both integrals. Depends on the first M mark	
A1	Correct integration for their method.	
dM1	Substitute the correct limits (0 and 4) in their integrated expression(s) and obtain a value for the area. Depends on the both M marks	
A1	Correct area, exact or min 3 significant figures. Must be positive.	
ALT	By splitting the area:	
M1	Reqd area = area $\Delta OBD - \int_0^2 (x^3 - 4x^2 - 4x + 16)\,dx - \int_2^4 (x^3 - 4x^2 - 4x + 16)\,dx$.	
dM1	Triangle area by formula or integration of equation of *l* and curve equation integrated	
A1	Correct integration (and area of triangle = 32 if by formula)	
dM1	Substitute the limits into their integrated expressions and obtain a value for the area. Depends on both M marks.	
A1	Correct area, exact or min 3 significant figures. Must be positive.	
10 (a)	$AC = \sqrt{8^2 + 3^2} = \sqrt{73}$	M1A1
	$\tan 45° = \dfrac{CH}{AC}$, $CH = \sqrt{73} = 8.54$ cm	M1A1 (4)

Question Number	Scheme	Marks
(b)	$\sin 45°$ or $\cos 45° = \dfrac{CH}{AH}$ or $\dfrac{AC}{AH}$ or Pythagoras	M1
	$AH = \sqrt{73} \times \sqrt{2}$, $= 12.1$ cm	A1ft, A1 (3)
(c)	$FN^2 = FH^2 + \left(\dfrac{1}{2}CH\right)^2$	M1
	$= 73 + \dfrac{73}{4}$, $FN = \sqrt{91.25} = 9.55\,\text{cm}$	A1ft, A1 (3)
(d)	$\tan GFB = \dfrac{GB}{FG} = \dfrac{\sqrt{73}}{3}$	M1A1ft
	$\angle GFB = 70.7°$	A1 (3)
(e)	$\sin FNG = \dfrac{FG}{FN} = \dfrac{3}{\sqrt{91.25}}$	M1A1ft
	$\angle FNG = 18.3°$	A1 (3) [16]

NB Penalise failure to round as instructed once for lengths ((a), (b) and (c)) **and** once for angles ((d) and (e)) Use of exact answers for lengths is also only penalised once.

(a)

M1 Use Pythagoras, with a + sign to obtain the length of AC

A1 Correct length AC, seen here or later (or implied by a correct final answer).

M1 Use $\tan 45° = \dfrac{CH}{AC}$ (fraction either way up)

A1 Correct length of CH. Must be 3 significant figures

(b)M1 Use $\sin 45°$ or $\cos 45° = \dfrac{CH}{AH}$ or $\dfrac{AC}{AH}$ or Pythagoras with a + sign (or any other **complete** method)

A1ft $AH =$ their $CH \times \sqrt{2}$ or equivalent. (May be implied by a correct final answer.)

A1 Correct length AH. Must be 3 significant figures unless already penalised in (a).

(c)

M1 Use Pythagoras with a + sign in ΔFHC

A1ft Correct numbers, follow through their AC and CH.

A1 Correct length FN. Must be 3 significant figures unless already penalised above.

(d)

M1 Use any complete method for finding $\angle GFB$ or $\angle HEC$

A1ft Correct numbers used in their method, follow through any previously found lengths used.

A1 Correct answer. Must be in degrees and correct to 1 decimal place. (70.6^{o} from using $CH = 8.54$ scores M1A1A0)

(e)M1 Use any **complete** method for obtaining $\angle FNG$, eg trig as shown, or Pythagoras and cosine rule. (Cosine rule needs $NG = \sqrt{329}/4$)

A1ft Correct numbers in their choice of method, follow through lengths found previously.

A1 Correct answer. Must be in degrees and correct to 1 decimal place unless penalised in (d).

11 (a)	$\log pq^4 - \log pq^2 = \log\left(\dfrac{pq^4}{pq^2}\right) = \log q^2$ or $2\log q$	M1