Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International GCSE

Time 2 hours

Paper reference **4PM1/02R**

Further Pure Mathematics
PAPER 2R

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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Q:1/1/1/1/

P 7 1 6 4 2 A 0 1 3 6

Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\qquad$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\qquad$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$ $\qquad$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

There are no questions on this page.

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 The position vector of the point A is $(3\mathbf{i} - 2\mathbf{j})$, referred to a fixed origin O.

The point B is such that $\overrightarrow{AB} = (6\mathbf{i} + 8\mathbf{j})$

(a) Find the position vector of B as a simplified expression in terms of **i** and **j**

(2)

(b) Find the magnitude of vector $\overrightarrow{AB}$

(1)

(c) Find a unit vector, in terms of **i** and **j**, that is parallel to $\overrightarrow{AB}$

(2)

Question 1 continued

(Total for Question 1 is 5 marks)

2 When poured from a pipe, concrete is formed into the shape of a cuboid with a square base of side x and with a height of $3x$

The volume of the cuboid increases at a constant rate of $8\,m^3/s$

Find the rate of increase, in m/s, of x when $x = 2$ metres.

(6)

Question 2 continued

(Total for Question 2 is 6 marks)

3 A geometric series has first term a and common ratio r, where $r > 0$

Given that the 3rd term of the series is 5 and that the 5th term of the series is $\frac{5}{2}$

(a) find

(i) the exact value of r

(ii) the value of a

(4)

(b) Find the sum to infinity of this series.
Give your answer in the form $p + q\sqrt{2}$ where p and q are integers.

(2)

Question 3 continued

(Total for Question 3 is 6 marks)

4 $f(x) = x^3 + px^2 + qx + 7$ where p and q are integers.

$(x + 1)$ is a factor of $f(x)$
The remainder when $f(x)$ is divided by $(x + 2)$ is -5

(a) Find the value of p and the value of q

(5)

(b) Hence, show that $f(x) = 0$ has only one real root.

(3)

Question 4 continued

(Total for Question 4 is 8 marks)

5 (a) Complete the table of values for $y = e^{3x-2}$ giving your answers to 2 decimal places.

x	0	0.25	0.5	0.75	1
y	0.14				2.72

(2)

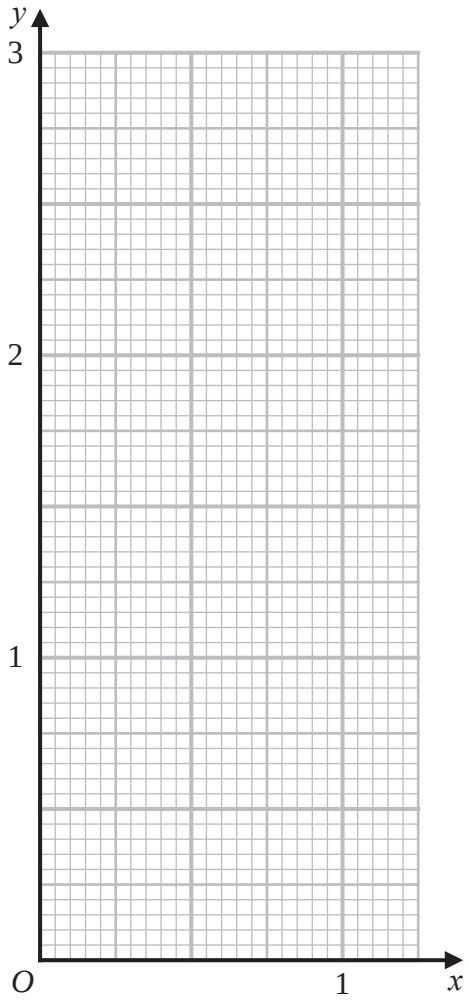
(b) On the grid opposite, draw the graph of $y = e^{3x-2}$ for $0 \leqslant x \leqslant 1$

(2)

(c) By drawing a suitable straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation $3x = 2 + \ln(3 - x)$

(3)

Question 5 continued

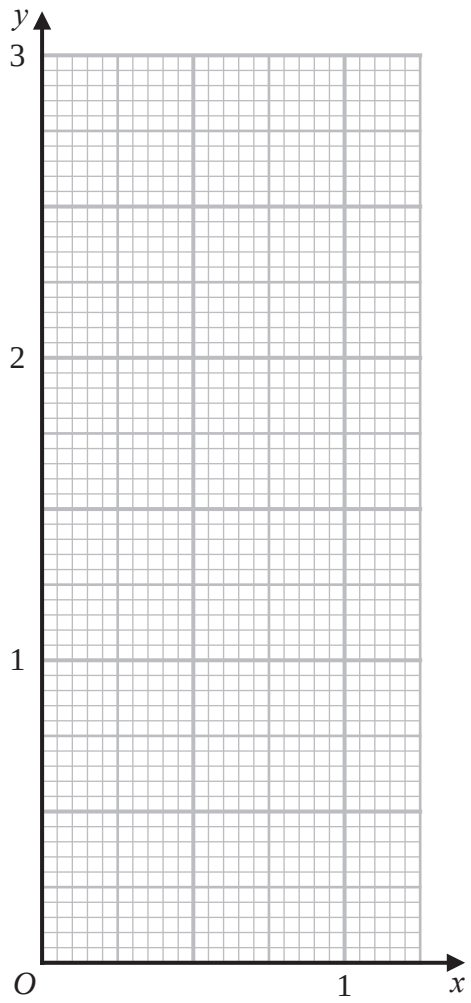


Turn over for a spare grid if you need to redraw your graph.

Question 5 continued

Question 5 continued

Only use this grid if you need to redraw your graph.



(Total for Question 5 is 7 marks)

6

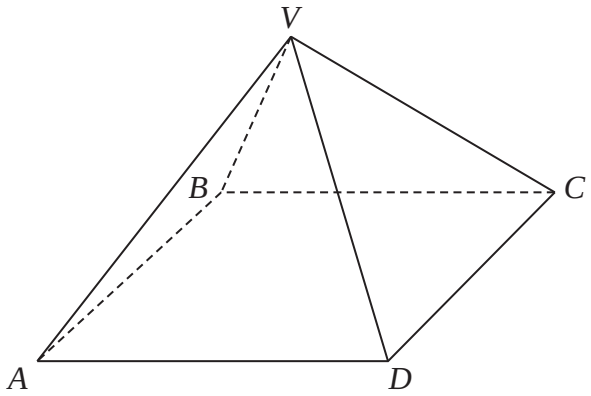


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows a right pyramid *VABCD* with vertex *V* and square base *ABCD*.

Each of the edges of the pyramid has the same length.

Find the size, in degrees to one decimal place, of the angle between the plane *CVD* and the base *ABCD*.

(6)

Question 6 continued

(Total for Question 6 is 6 marks)

7 (a) Solve the equation

$$\cos(3x - 15)° = \frac{\sqrt{3}}{2} \quad \text{for} \quad 0 \leqslant x < 180$$

(4)

(b) Solve, giving your solutions to one decimal place where appropriate,

$$3\tan y° + 4\sin y° = 0 \quad \text{for} \quad -180 \leqslant y < 180$$

(4)

(c) Solve, giving your solutions to one decimal place where appropriate,

$$\cos\theta° = 3\sin^2\theta° - 1 \quad \text{for} \quad -180 \leqslant \theta < 180$$

(4)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 12 marks)

8

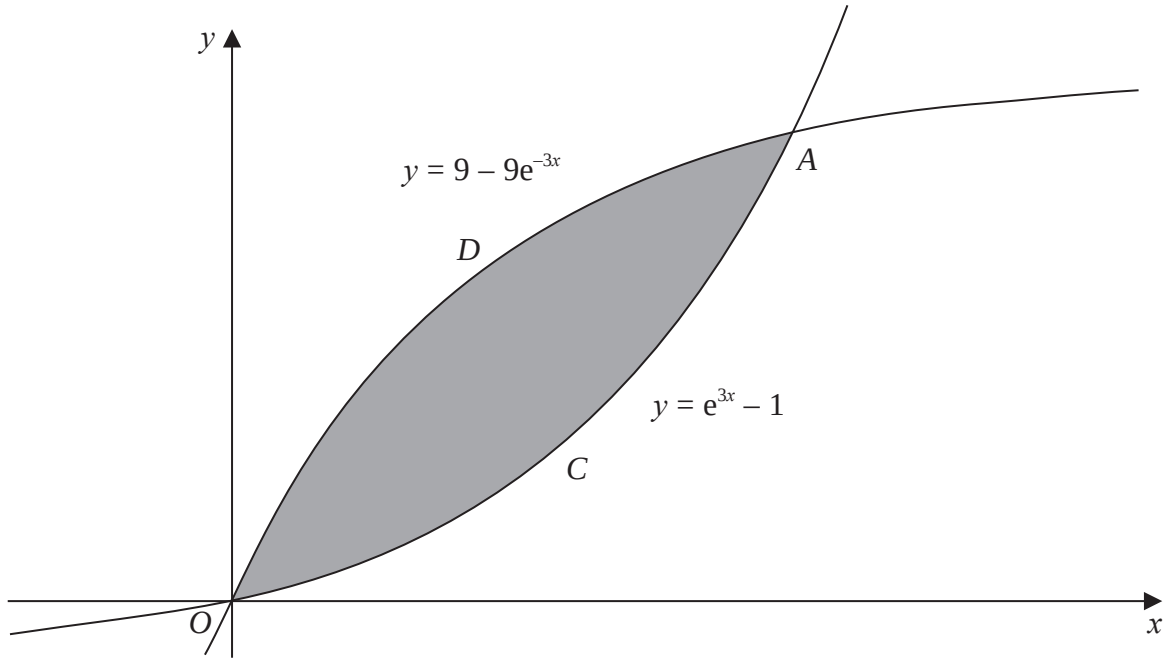


Figure 2

Figure 2 shows part of the curve C with equation $y = e^{3x} - 1$ and part of the curve D with equation $y = 9 - 9e^{-3x}$

The curves intersect at the origin O and the point A.

(a) (i) Show that the x coordinate of the point A satisfies the equation

$$(e^{3x})^2 - 10e^{3x} + 9 = 0$$

(ii) Hence, show that the x coordinate of the point A is $\frac{1}{3}\ln 9$

(5)

The finite region bounded by C and by D is shown shaded in Figure 2.

(b) Use calculus to find the exact area of this region.

(6)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 11 marks)

9 (a) Write $\frac{3}{(3-x)^3}$ in the form $a(1-bx)^{-3}$

where a and b are fractions in their lowest terms.

(2)

(b) Expand $\frac{3}{(3-x)^3}$ in ascending powers of x up to and including the term in x^3

Express each coefficient as a fraction in its lowest terms.

(3)

(c) (i) Use a suitable value of x with your expansion in part (b), to obtain an approximation for $\frac{24}{125}$ to 5 decimal places.

(ii) Find the percentage error, to 2 decimal places, of your approximation from the actual value.

(4)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 9 marks)

10 A curve C has equation

$$y = \frac{7x - 2}{2x - 3} \qquad x \neq \frac{3}{2}$$

(a) Write down an equation of the asymptote to C that is

(i) parallel to the y-axis,

(ii) parallel to the x-axis.

(2)

(b) Find the coordinates of the points of intersection of C with the coordinate axes.

(2)

(c) Using calculus, show that at every point on the curve, the gradient of C is negative.

(4)

(d) Using the axes on the opposite page, sketch C.
Show clearly and label with their equation any asymptotes and the coordinates of the points of intersection of C with the coordinate axes.

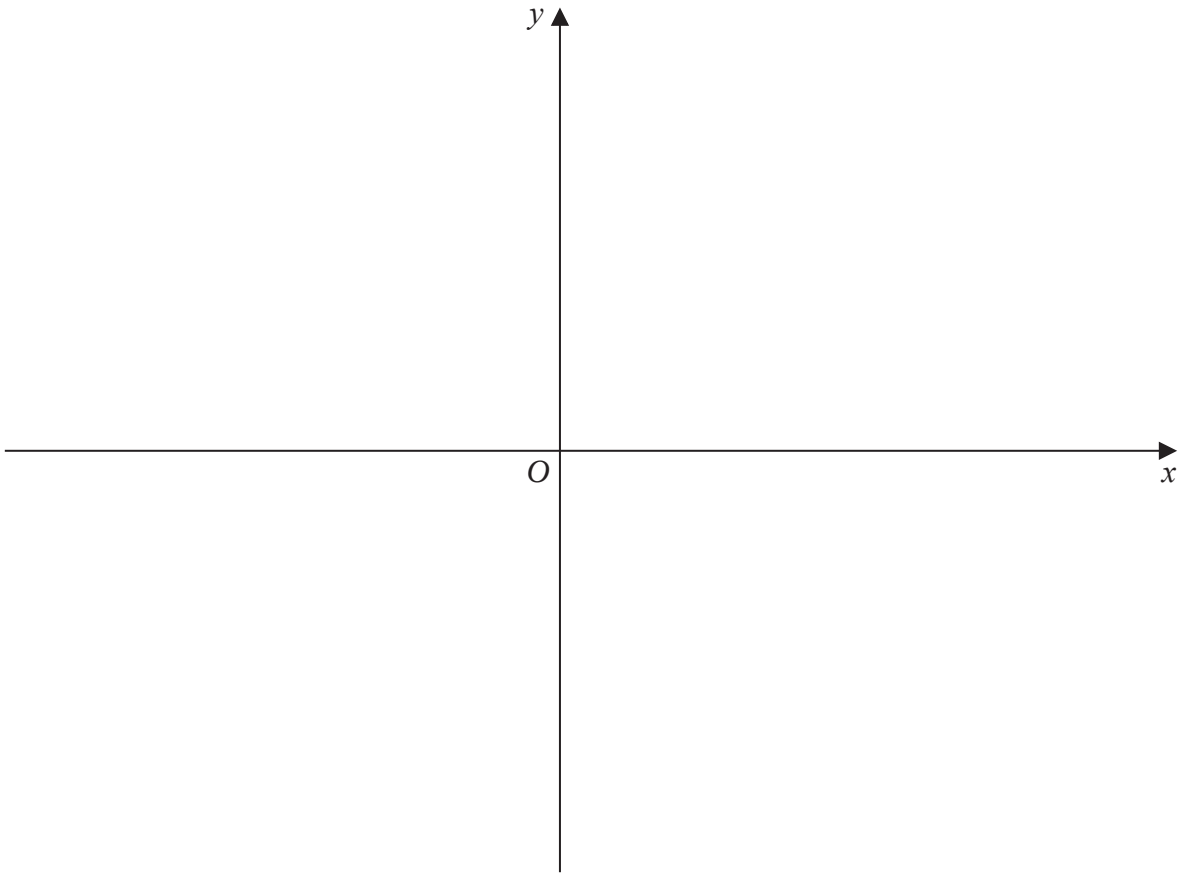
(3)

The straight line l is the normal to C at the point A.
The x coordinate of A is positive and the gradient of l is 17
The line l also intersects C at the point B.

(e) Find the exact coordinates of B.

(7)

Question 10 continued



Question 10 continued

Question 10 continued

(Total for Question 10 is 18 marks)

11

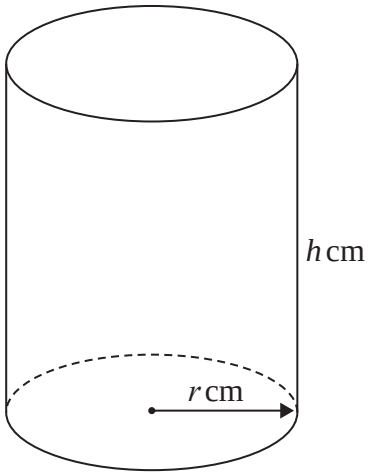


Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows a solid metal right circular cylinder of radius r cm and height h cm.

The total surface area of the cylinder is $600\,\text{cm}^2$

The volume of the cylinder is $V\,\text{cm}^3$

(a) Show that $V = 300r - \pi r^3$

(4)

Given that r can vary,

(b) (i) use calculus to show that the exact value of r for which V is a maximum is

$$r = \sqrt{\frac{100}{\pi}}$$

(ii) justify that this value of r gives a maximum value of V

(5)

The cylinder is melted down and reformed into a sphere of radius p cm.

(c) Find, to one decimal place, the greatest possible value of p

(3)

Question 11 continued

Question 11 continued

(Total for Question 11 is 12 marks)

TOTAL FOR PAPER IS 100 MARKS