Question	Scheme	Marks
10	$8\log_x 64 = \dfrac{8\log_4 64}{\log_4 x}$	M1
	$\log_4 x^3 = 3\log_4 x$	M1
	$\log_4 x^3 + 8\log_x 64 = 22 \Rightarrow 3\log_4 x + \dfrac{8\log_4 64}{\log_4 x} = 22$	
	$\Rightarrow 3(\log_4 x)^2 + 8\log_4 64 = 22\log_4 x \Rightarrow 3(\log_4 x)^2 - 22\log_4 x + 24 = 0$	M1
	$3(\log_4 x)^2 - 22\log_4 x + 24 = 0 \Rightarrow (3\log_4 x - 4)(\log_4 x - 6) = 0$	M1
	$\Rightarrow \log_4 x = \dfrac{4}{3},\quad 6$	A1
	$x = 4^{\frac{4}{3}}$ or awrt 6.35 and $x = 4096$	M1A1 [7]
		Total 7 marks

NOTE WELL! This can be solved using a modern calculator. No working = no marks. Award marks only for work explicitly seen.

Mark	Notes
Works in base 4	
M1	For changing the base of the log correctly to base 4 $3\log_4 x + 8\log_x 64 = 22 \Rightarrow 3\log_4 x + \dfrac{8\log_4 4^3}{\log_4 x} = 22$
M1	For applying the power law correctly seen anywhere in their work. This mark can also be awarded for explicit application of the power law on $\log_4 64 = 3\log_4 4$
M1	For multiplying through by $\log_4 x$ and forming a 3TQ in log base 4
M1	For solving their 3TQ by any valid and correct method. If there is no method seen with an incorrect 3TQ or with incorrect solutions following a correct 3TQ this is M0 They must obtain two values for their log.
A1	For both correct values of log x $\left[\dfrac{4}{3},\quad 6\right]$
M1	For undoing either log correctly. Allow this mark for any erroneous log they find, but undo correctly.
A1	For **both** values of x Accept $4^{\frac{4}{3}}$ or awrt 6.35 and 4^6 or 4096

Works in base *x*	
M1	For changing the base of the log correctly to base x or vice versa. $\log_4 x^3 + 8\log_x 64 = 22 \Rightarrow \dfrac{3\log_x x}{\log_x 4} + 8\log_x 4^3 = 22$
M1	For applying the power law correctly seen anywhere in their work. This mark can also be awarded for explicit application of the power law on $\log_x 64 = 3\log_x 4$
M1	For multiplying through by $\log_x 4$ and forming a 3TQ in log base x $\dfrac{3}{\log_x 4} + 24\log_x 4 = 22 \Rightarrow 24(\log_x 4)^2 - 22\log_x 4 + 3 = 0$
M1	For solving their 3TQ by any valid and correct method. If there is no method seen with an incorrect 3TQ or with incorrect solutions following a correct 3TQ this is M0 $24(\log_x 4)^2 - 22\log_x 4 + 3 = 0 \Rightarrow (4\log_x 4 - 3)(6\log_x 4 - 1) = 0 \Rightarrow \log_x 4 = ..., ...$ They must obtain two values for their log.
A1	For both correct values. $\log_x 4 = \dfrac{3}{4}, \dfrac{1}{6}$
M1	For undoing either log correctly, but they **must** obtain a value for $x =$ ….. $x = 4^{\frac{4}{3}}$ or $x = 4^6$ Allow this mark for any erroneous log they find, but undo correctly.
A1	Accept $4^{\frac{4}{3}}$ or awrt 6.35 and 4^6 or 4096
Works in base 2	
M1	Changes the base of **at least one** log to base 2. $\dfrac{3}{2}\log_2 x$, $\dfrac{8\log_x x^6}{\log_2 x}$
M1	For applying the power law correctly seen anywhere in their work. This mark can also be awarded for explicit application of the power law on $\log_2 64 = 6\log_2 2$
M1	For multiplying through by $\log_2 x$ and forming a 3TQ in base 2 $\dfrac{3}{2}(\log_2 x)^2 - 22\log_2 x + 48 = 0 \Rightarrow \left[3(\log_2 x)^2 - 44\log_2 x + 96 = 0\right]$
M1	For solving their 3TQ by any valid and correct method. If there is no method seen with an incorrect 3TQ or with incorrect solutions following a correct 3TQ this is M0 $(12\log_2 x - 1)(8\log_2 x - 3) = 0 \Rightarrow \log_2 x = ..., ...$
A1	For both correct values. $\log_2 x = \dfrac{8}{3}, 12$
M1	For undoing either log correctly. $x = 2^{\frac{8}{3}}$ or $x = 2^{12}$ Allow this mark for any erroneous log they find, but undo correctly.
A1	Accept $2^{\frac{8}{3}}$ or awrt 6.35 and 2^{12} or 4096

Question	Scheme	Marks
11(a)	$\cos 2A = \cos^2 A - \sin^2 A \Rightarrow \cos 2A = \left(1-\sin^2 A\right) - \sin^2 A = 1 - 2\sin^2 A$	M1M1
	$2\sin^2 A = 1 - \cos 2A \Rightarrow \sin^2 A = \frac{1}{2}\left(1-\cos 2A\right)$ *	A1cso [3]
(b)	$\cos^2 A = \frac{1}{2}\left(1+\cos 2A\right)$	B1
	$\sin^4 x + \cos^4 x = \left(\left(\frac{1-\cos 2x}{2}\right)^2 + \left(\frac{1+\cos 2x}{2}\right)^2\right)$	
	$= \frac{1}{4}\left(\left(1-\cos 2x\right)^2 + \left(1+\cos 2x\right)^2\right)$	
	$= \frac{1}{4}\left(1 - 2\cos 2x + \cos^2 2x + 1 + 2\cos 2x + \cos^2 2x\right)$	M1
	$= \frac{1}{4}\left(2 + 2\cos^2 2x\right) = \left\{\frac{1}{2}\left(1+\cos^2 2x\right)\right\}$	A1
	$\left\{\cos^2 2x = \frac{1+\cos 4x}{2}\right\}$	
	$\sin^4 x + \cos^4 x = \frac{1}{2}\left(1 + \frac{1+\cos 4x}{2}\right) = \frac{3+\cos 4x}{4}$ *	M1A1 cso [5]
(c)	$5\sin 2\theta + 6 = 8\left(\frac{3+\cos 2\theta}{4}\right) \Rightarrow 5\sin 2\theta + 6 = 6 + 2\cos 2\theta$	M1
	$\Rightarrow 5\sin 2\theta = 2\cos 2\theta \Rightarrow \frac{\sin 2\theta}{\cos 2\theta} = \frac{2}{5} \Rightarrow \tan 2\theta = \frac{2}{5}$	M1
	$2\theta = 21.801...^\circ, 201.801...^\circ, 381.801...^\circ$	A1
	$\Rightarrow \theta = 10.9^\circ, 100.9^\circ$	A1
	Penalise extra angles in range by withholding the final A mark. Extra angles out of range – ignore.	[4]
		Total 12 marks

Part	Mark	Notes
(a)	M1	For using the summation formula for cos 2*A* They must start with either $\cos 2A = \cos^2 A - \sin^2 A$ or $\cos\left(A+A\right) = \cos A\cos A - \sin A\sin A$
	M1	For eliminating $\cos^2 A$ using $\cos^2 A + \sin^2 A = 1$ and attempting to rearrange to the required result. This is not dependent on the first M mark, so if they start with another identity for cos2*A* they can still get this mark.
	A1 cso	For the correct identity with no errors seen. This is a given result.
	NB Some candidates work backwards – that is fine, please follow their working.	

Working with a different variable. If they work in this part with a different variable (eg A) then award all the marks as appropriate up to the last mark. If they leave their final answer in terms of another variable, withhold the final A mark only. If they however, change to x on the final line award all the marks [provided everything is correct]		
(b)	**Main method**	
	B1	For use of the correct identity for $\cos^2 x$ $\left[\cos^2 x = \frac{1}{2}(1+\cos 2x)\right]$ or if they convert $\cos^2 x$ to $\sin^2 x$ use the Pythagorean identity $\cos^2 x = 1-\sin^2 x$ and apply the given identity.
	M1	For squaring both identities. This must be a correct expansion. Ft their identity for $\left[\cos^2 x = \frac{1}{2}(1+\cos 2x)\right]$
	A1	For collecting like terms and obtaining $\frac{1}{2}\left(1+\cos^2 2x\right)$ oe. For example, $\frac{1}{4}\left(2+2\cos^2 2x\right)$
	M1	For applying the identity: $\cos^2 A = \frac{1}{2}(1+\cos 2A) \Rightarrow \left[\cos^2 2A = \frac{1}{2}(1+\cos 4A)\right]$ on $\cos^2 2x$ **only** again to achieve an expression in cos $4x$ **only**
	A1 cso	For the correct identity with no errors seen. This is a given result. You must check every line of their work carefully.
	ALT 1	
	B1	For use of the correct identity for $\sin 2A = 2\sin A\cos A$ [seen later in their working].
	M1	For using the expansion of $\sin^2 x+\cos^2 x$ as follows $\sin^4 x+\cos^4 x = \left(\sin^2 x+\cos^2 x\right)^2 - 2\sin^2 x\cos^2 x$ This must be correct
	A1	For obtaining $\sin^4 x+\cos^4 x = 1-\frac{1}{2}\sin^2 2x$ oe. For example; $\frac{1}{2}\left(2-\sin^2 2x\right)$
	M1	For applying the given identity on $\sin^2 2x$ **only** to achieve an expression in cos $4x$ **only** $\sin^2 A = \frac{1}{2}(1-\cos 2A) \Rightarrow \left[\sin^2 2A = \frac{1}{2}(1-\cos 4A)\right]$ $\sin^4 x+\cos^4 x = 1-\frac{1}{2}\left(\frac{1-\cos 4x}{2}\right) \Rightarrow \left(\sin^4 x+\cos^4 x = \frac{3+\cos 4x}{4}\right)$
	A1 cso	For the correct identity with no errors seen. This is a given result. You must check every line of their work carefully.

	ALT 2 – Works backwards from the given result	
	B1	For use of the correct identity for $\sin 2A = 2\sin A\cos A$ [seen later in their working].
	M1	Applies the cos2*A* identity and converts 3 into $3\left(\sin^2 2x + \cos^2 2x\right)$ $\frac{3+\cos 4x}{4} = \frac{3\sin^2 2x + 3\cos^2 2x + \left(\cos^2 2x - \sin^2 2x\right)}{4}$
	A1	Obtains $\frac{3+\cos 4x}{4} = \frac{4\cos^2 2x + 2\sin^2 2x}{4}$
	M1	Applies the cos2*A* identity and expands the bracket. $\frac{3+\cos 4x}{4} = \frac{4\left(\cos^2 x - \sin^2 x\right)^2 + 8\sin^2 x\cos^2 x}{4}$ $= \frac{4\cos^4 x - 8\sin^2 x\cos^2 x + 4\sin^4 x + 8\sin^2 x\cos^2 x}{4}$
	A1 cso	Simplifies to the required result with no errors seen $\frac{3+\cos 4x}{4} = \sin^4 x + \cos^4 x$ *

(c)	M1	For obtaining the correct equation in terms of $\sin 2\theta$ and $\cos 2\theta$ $6 + 2\cos 2\theta = 5\sin 2\theta + 6$ Accept unsimplified, accept even: $8\left(\frac{3+\cos 2\theta}{4}\right) = 5\sin 2\theta + 6$
	M1	For using the $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta}$ identity correctly on their expression following an expression in the form $A\cos 2\theta = B\sin 2\theta$ and must be in terms of 2θ
	A1	For achieving at least one correct angle for 2θ **NB This is an M mark in Epen**
	A1	For awrt both 10.9° and 100.9° Penalise extra angles in range by withholding the final A mark. Extra angles out of range – ignore.

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