

		<b>This mark is dependent on BOTH previous M marks.</b>
	A1	For a conclusion. A simple # sign, 'shown', 'QED' or underlining is sufficient.

Question number	Scheme	Marks
8	$\alpha + \beta = 4k\sqrt{2}$ and $\alpha\beta = 2k^4 - 1$ $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta \Rightarrow (4k\sqrt{2})^2 = 66 + 2(2k^4 - 1)$ $k^4 - 8k^2 + 16 = 0$ $(k^2 - 4)^2 = 0 \Rightarrow k = 2$ $(\alpha + \beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$ <b>or</b> $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$ $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ <b>or</b> $\alpha^3 + \beta^3 = (\alpha + \beta)(66 - \alpha\beta)$ $\alpha^3 + \beta^3 = (8\sqrt{2})^3 - 3 \times 31 \times 8\sqrt{2} = 280\sqrt{2}$ <b>or</b> $8\sqrt{2}(66 - 31) = 280\sqrt{2}$ $p = 280$	B1 B1 M1 A1 M1 M1 A1 M1 A1 M1 A1 (11)
<b>Total 11 marks</b>		

Mark	Notes
B1	For <b>either</b> $\alpha + \beta = 4k\sqrt{2}$ <b>or</b> $\alpha\beta = 2k^4 - 1$
B1	For <b>both</b> $\alpha + \beta = 4k\sqrt{2}$ <b>and</b> $\alpha\beta = 2k^4 - 1$
M1	For the correct algebra on $\alpha^2 + \beta^2$ (in any order) <b>and</b> substitution of their values of $\alpha\beta$ and $\alpha + \beta$ providing both sum and product are in terms of $k$ . $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta \Rightarrow \alpha^2 + \beta^2 = ('4k\sqrt{2}')^2 - 2('2k^4 - 1')$
A1	For obtaining $(4k\sqrt{2})^2 = 66 + 2(2k^4 - 1)$ in any order.
M1	For simplifying to form a 3TQ in $k^4$ i.e., $4k^4 - 32k^2 + 64 = 0$ oe Accept as a minimum $4k^4 - 32k^2 \pm Q = (0)$ $Q \neq 0$
M1	For factorising or solving the 3TQ using any valid method. See General Guidance.
A1	For $k = 2$ If they also give $k = -2$ withhold this mark.

<b>Method 1</b>	
M1	For expanding $(\alpha + \beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$ or $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$
A1	For obtaining $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ or $\alpha^3 + \beta^3 = (\alpha + \beta)(66 - \alpha\beta)$ This must be such that $\alpha\beta$ and $\alpha + \beta$ can be substituted in directly.
M1	For substitution of $\alpha + \beta$ and $\alpha\beta$ for their positive value of $k$ into a <b>correct</b> expansion of $\alpha^3 + \beta^3$ <b>NOTE:</b> If they do not obtain $k = 2$ , then full substitution of their numerical value for $k$ into $\alpha + \beta$ <b>and</b> $\alpha\beta$ must be seen for the award of this mark. For example: $\alpha^3 + \beta^3 = (4 \times \text{'their } k \text{' } \sqrt{2})^3 - 3 \times (2 \times \text{'their } k^4 - 1) \times (4 \times \text{'their } k \text{' } \sqrt{2})$
A1	For $p = 280$
<b>Method 2</b>	
M1	Finds the exact value of $\alpha$ and $\beta$ Solves the equations $(\alpha\beta = 2k^4 - 1$ and $\alpha + \beta = 4k\sqrt{2})$ simultaneously to give a value for $\alpha$ and $\beta$ $\alpha = \frac{31}{\beta} \Rightarrow \alpha + \beta = \frac{31}{\beta} + \beta = 8\sqrt{2} \Rightarrow \beta^2 - 8\sqrt{2}\beta + 31 = 0$ $\Rightarrow \beta = \dots \quad \alpha = \dots$
A1	For $\alpha = 1 + 4\sqrt{2} \quad \beta = -1 + 4\sqrt{2}$ OR $\beta = 1 + 4\sqrt{2} \quad \alpha = -1 + 4\sqrt{2}$
M1	Substitutes these values into $\alpha^3 + \beta^3 = p\sqrt{2}$ to find a value for $p$ $(1 + 4\sqrt{2})^3 + (-1 + 4\sqrt{2})^3 = 24\sqrt{2} + 256\sqrt{2} = 280\sqrt{2} \Rightarrow p = \dots$
A1	$p = 280$