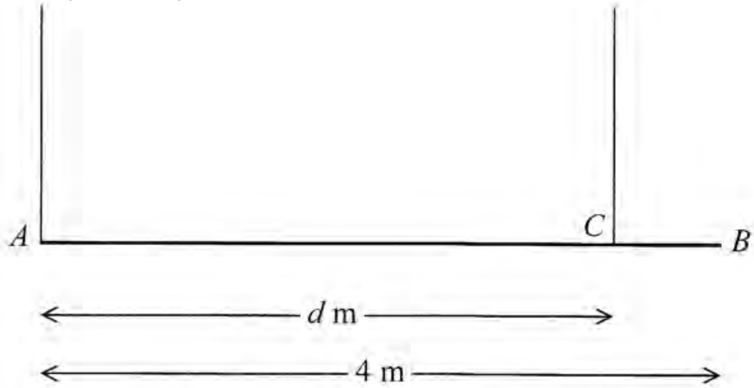


Figure 3

A beam AB has weight W newtons and length 4 m. The beam is held in equilibrium in a horizontal position by two vertical ropes attached to the beam. One rope is attached to A and the other rope is attached to the point C on the beam, where $AC = d$ metres, as shown in Figure 3. The beam is modelled as a uniform rod and the ropes as light inextensible strings. The tension in the rope attached at C is double the tension in the rope attached at A.

(a) Find the value of d.

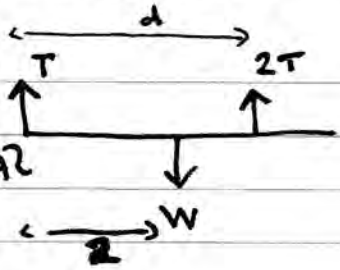
(6)

A small load of weight kW newtons is attached to the beam at B. The beam remains in equilibrium in a horizontal position. The load is modelled as a particle. The tension in the rope attached at C is now four times the tension in the rope attached at A.

(b) Find the value of k.

(6)

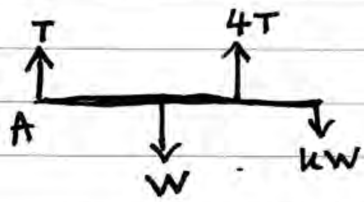
a)



$A\circlearrowright \quad 2W = 2T \times d \quad \therefore \quad d = \frac{W}{T}$

$\uparrow = \downarrow \quad 3T = W \quad \therefore \quad d = \frac{3\cancel{T}}{\cancel{T}} \quad d = 3\text{m}$

b)



$\uparrow = \downarrow \Rightarrow 5T = (k+1)W$

$A\circlearrowright \quad 2W + 4kW = 12T$

$2\cancel{W} + 4k\cancel{W} = \frac{12}{5}(k+1)\cancel{W}$

$2 + 4k = 2.4k + 2.4$

$1.6k = 0.4$

$\therefore k = 0.25$

5. A particle P of mass 0.5 kg is moving under the action of a single force $(3\mathbf{i} - 2\mathbf{j})$ N.

(a) Show that the magnitude of the acceleration of P is $2\sqrt{13}$ m s^{-2}.

(4)

At time $t = 0$, the velocity of P is $(\mathbf{i} + 3\mathbf{j})$ m s^{-1}.

(b) Find the velocity of P at time $t = 2$ seconds.

(3)

Another particle Q moves with constant velocity $\mathbf{v} = (2\mathbf{i} - \mathbf{j})$ m s^{-1}.

(c) Find the distance moved by Q in 2 seconds.

(2)

(d) Show that at time $t = 3.5$ seconds both particles are moving in the same direction.

(3)

a) $F = ma \Rightarrow a = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \div \frac{1}{2} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$ $|a| = \sqrt{6^2 + 4^2}$

$\therefore |a| = \sqrt{52} = 2\sqrt{13}$ #

b) $v = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + 2\begin{pmatrix} 6 \\ -4 \end{pmatrix} = \begin{pmatrix} 13 \\ -5 \end{pmatrix}$ $13i - 5j$

c) $\text{dist} = |\text{Vel} \times \text{time}| = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \times 2 = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ $\therefore \text{dist} = \sqrt{4^2 + 2^2}$

$= 2\sqrt{5} = 4.47\text{m}$

2

d) $V_p = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + 3.5\begin{pmatrix} 6 \\ -4 \end{pmatrix} = \begin{pmatrix} 22 \\ -11 \end{pmatrix}$ $11V_q = \begin{pmatrix} 22 \\ -11 \end{pmatrix}$

$\therefore$ they are moving in the same direction when $t = 3.5$