JAN 11

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1. Two particles B and C have mass m kg and 3 kg respectively. They are moving towards each other in opposite directions on a smooth horizontal table. The two particles collide directly. Immediately before the collision, the speed of B is 4 m s^{-1} and the speed of C is 2 m s^{-1}. In the collision the direction of motion of C is reversed and the direction of motion of B is unchanged. Immediately after the collision, the speed of B is 1 m s^{-1} and the speed of C is 3 m s^{-1}.

a) $\overset{4}{\rightarrow}$ (m) $\overset{2}{\leftarrow}$ (3) | $\overset{1}{\rightarrow}$ (m) $\overset{3}{\rightarrow}$ (3)

$4m-6 = m+9$
$3m = 15$
$\underline{m = 5\text{kg}}$

Find

(a) the value of m, **(3)**

Initial Mom $= -6$
final Mom $= 9$

$\underline{\text{Impulse} = 15\text{Ns}}$

(b) the magnitude of the impulse received by C. **(2)**

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2. A ball is thrown vertically upwards with speed $u \text{ m s}^{-1}$ from a point P at height h metres above the ground. The ball hits the ground 0.75 s later. The speed of the ball immediately before it hits the ground is 6.45 m s^{-1}. The ball is modelled as a particle.

(a) Show that $u = 0.9$ **(3)**

(b) Find the height above P to which the ball rises before it starts to fall towards the ground again. **(2)**

(c) Find the value of h. **(3)**

a) $u\uparrow$ P $\quad \uparrow a$ $\quad v\downarrow = 6.45$

$a\uparrow = -9.8$
$v\uparrow = -6.45$
$t = 0.75$

$v = u + at$
$-6.45 = u - 9.8 \times 0.75$
$\Rightarrow u = \underline{0.9 \text{ ms}^{-1}}$

b) $\uparrow u = 0.9$
$\uparrow v = 0$
$\uparrow a = -9.8$

$v^2 = u^2 + 2as$
$0 = 0.9^2 - 19.6s$
$s = 0.0413\text{m}$ (3sf) above P.

c) $u\downarrow = 0$
$v\downarrow = 6.45$
$a\downarrow = 9.8$

$v^2 = u^2 + 2as$
$6.45^2 = 19.6s$
$s = 2.12\text{m}$ from max h to ground

$h = 2.12... - 0.0413 = \underline{2.08\text{m}}$

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3.

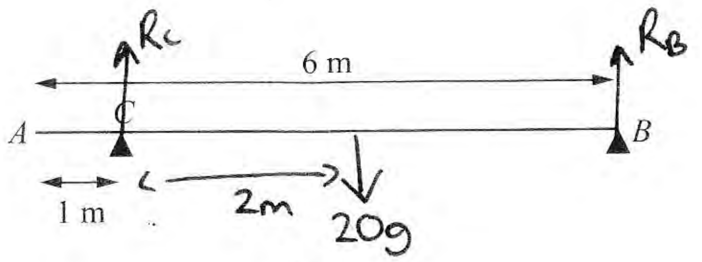


Figure 1

A uniform beam AB has mass 20 kg and length 6 m. The beam rests in equilibrium in a horizontal position on two smooth supports. One support is at C, where $AC = 1$ m, and the other is at the end B, as shown in Figure 1. The beam is modelled as a rod.

(a) Find the magnitudes of the reactions on the beam at B and at C.

(5)

A boy of mass 30 kg stands on the beam at the point D. The beam remains in equilibrium. The magnitudes of the reactions on the beam at B and at C are now equal. The boy is modelled as a particle.

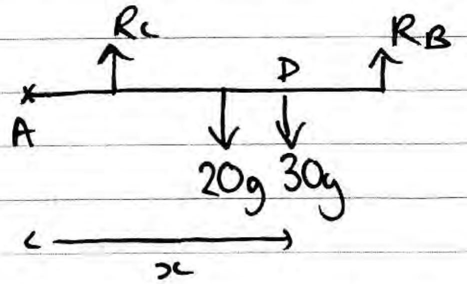
(b) Find the distance AD.

(5)

a) B↺ $20g \times 3 = R_C \times 5 \Rightarrow R_C = \underline{12g\,N}$

$\uparrow = \downarrow \Rightarrow 12g + R_B = 20g \Rightarrow R_B = \underline{8g\,N}$

b)



$R_C = R_B \Rightarrow 2R = 50g$

$R_C = R_B = 25g$

A↺ $R_C \times 1 + R_B \times 6 = 20g \times 3 + 30g \times x$

$25g + 150g = 60g + 30gx$

$115\cancel{g} = 30\cancel{g}x$

$x = \frac{115}{30} = \underline{3.83m}$

4. A particle P of mass 2 kg is moving under the action of a constant force $\mathbf{F}$ newtons. The velocity of P is $(2\mathbf{i} - 5\mathbf{j})$ m s^{-1} at time $t = 0$, and $(7\mathbf{i} + 10\mathbf{j})$ m s^{-1} at time $t = 5$ s.

Find

(a) the speed of P at $t = 0$,

(2)

(b) the vector $\mathbf{F}$ in the form $a\mathbf{i} + b\mathbf{j}$,

(5)

(c) the value of t when P is moving parallel to $\mathbf{i}$.

(4)

a) speed $= \sqrt{2^2+5^2} = \sqrt{29} = 5.39\text{ms}^{-1}$

b) acc $= \dfrac{\text{change in vel}}{\text{time}} = \dfrac{5i+15j}{5} = i+3j\ \text{ms}^{-2}$

$Rf = ma \Rightarrow F = 2(i+3j) = 2i+6j$ N

c) Vel $= (2i-5j)+t(i+3j) = (2+t)i+(-5+3t)j$

Parallel $\Rightarrow$ $j=0$

$-5+3t=0 \Rightarrow 3t=5 \quad t=\dfrac{5}{3}$ sec

5. A car accelerates uniformly from rest for 20 seconds. It moves at constant speed v m s^{-1} for the next 40 seconds and then decelerates uniformly for 10 seconds until it comes to rest.

(a) For the motion of the car, sketch

(i) a speed-time graph,

(ii) an acceleration-time graph.

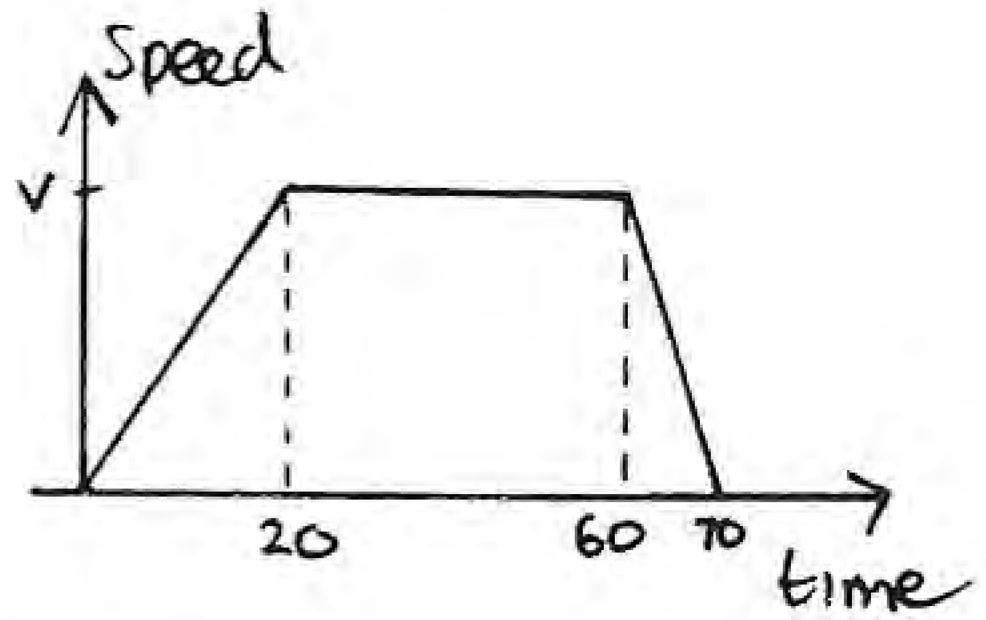
(6)

Given that the total distance moved by the car is 880 m,

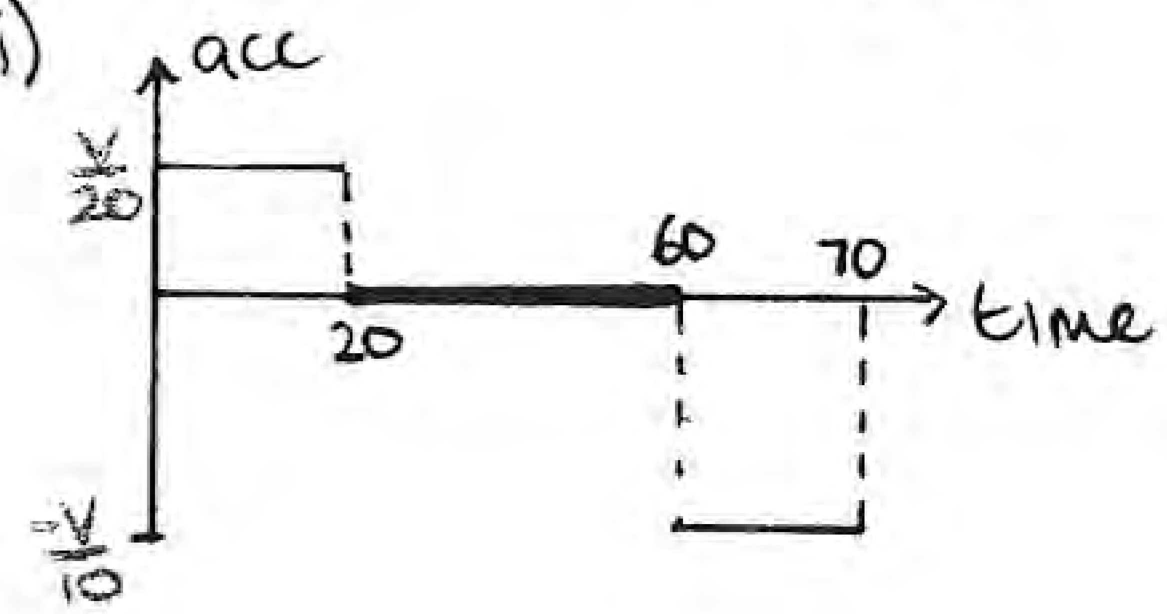
(b) find the value of v.

(4)

i)



ii)



b) $\frac{1}{2}v(40+70)=880$

$55v=880$

$v=16\text{ms}^{-1}$

6.

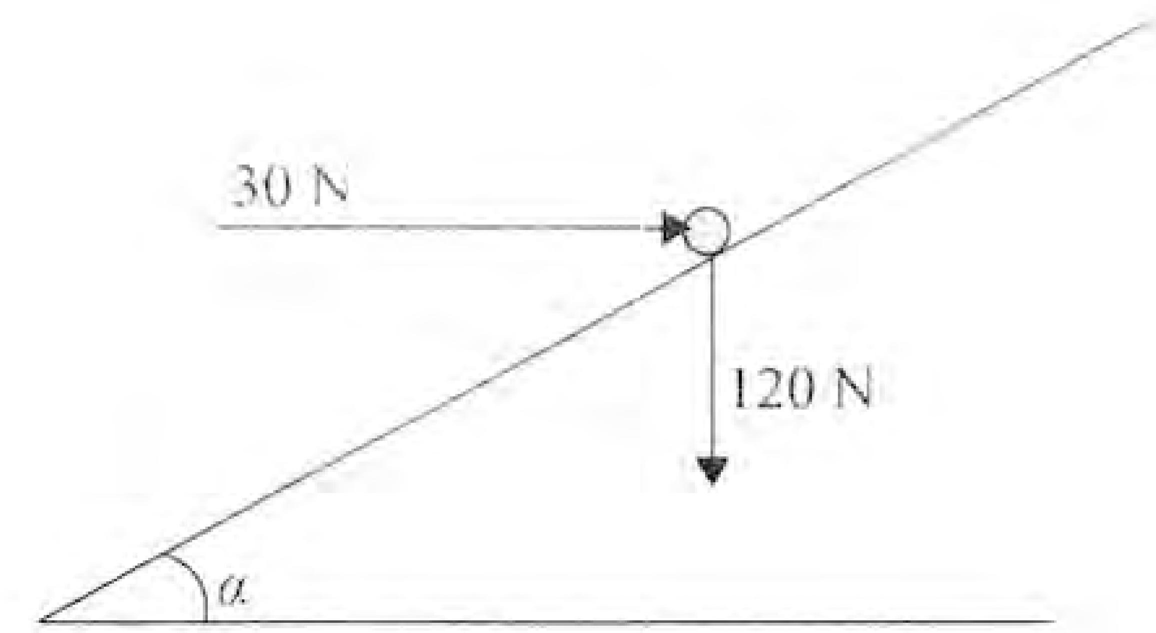


Figure 2

A particle of weight 120 N is placed on a fixed rough plane which is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$.

The coefficient of friction between the particle and the plane is $\frac{1}{2}$.

The particle is held at rest in equilibrium by a horizontal force of magnitude 30 N, which acts in the vertical plane containing the line of greatest slope of the plane through the particle, as shown in Figure 2.

(a) Show that the normal reaction between the particle and the plane has magnitude 114 N. **(4)**

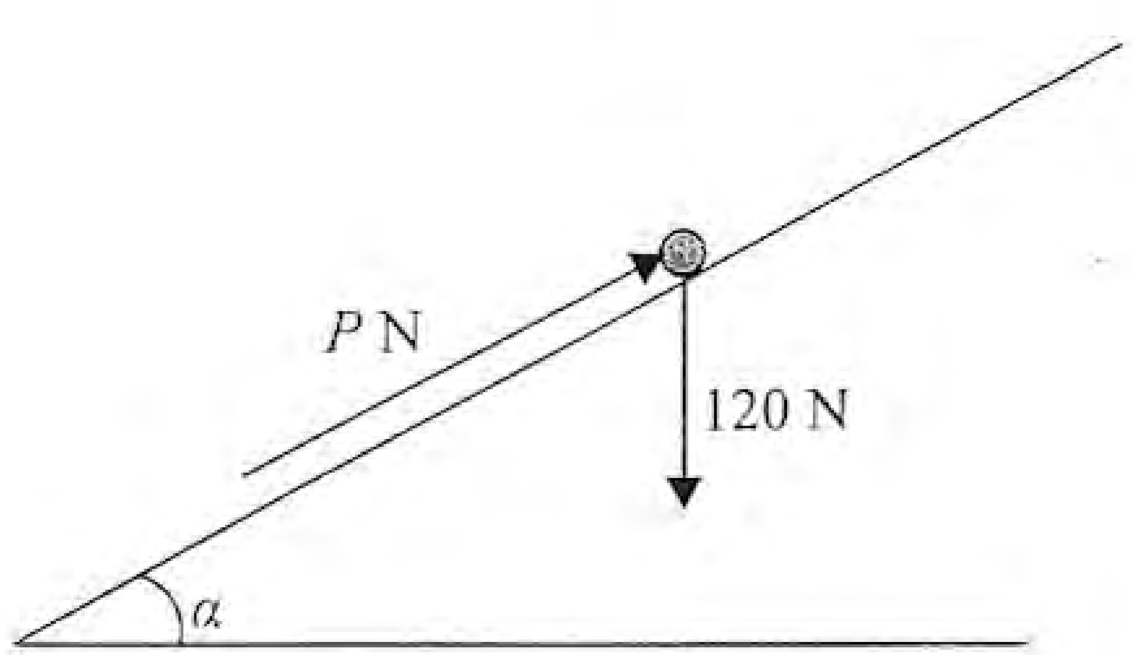


Figure 3

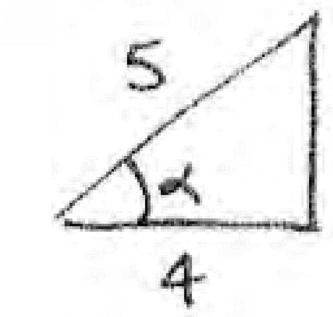
The horizontal force is removed and replaced by a force of magnitude P newtons acting up the slope along the line of greatest slope of the plane through the particle, as shown in Figure 3. The particle remains in equilibrium.

(b) Find the greatest possible value of P. **(8)**

(c) Find the magnitude and direction of the frictional force acting on the particle when $P = 30$. **(3)**

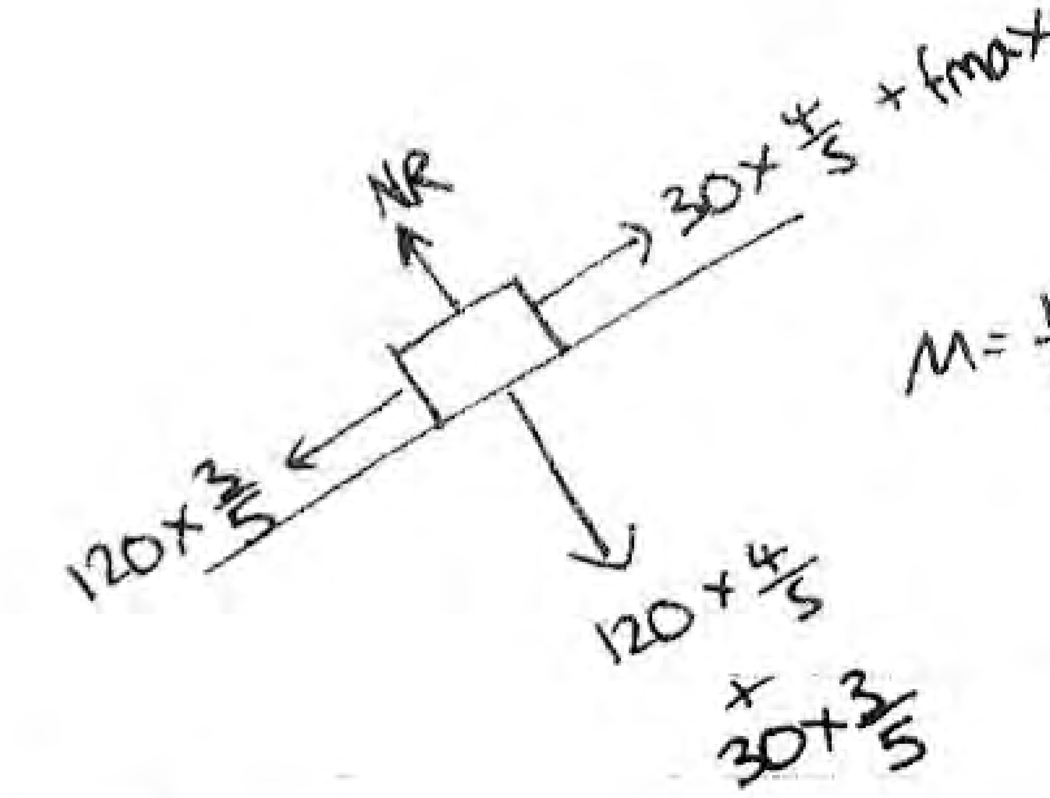
Question 6 continued

a)



$\sin\alpha = \frac{3}{5}$

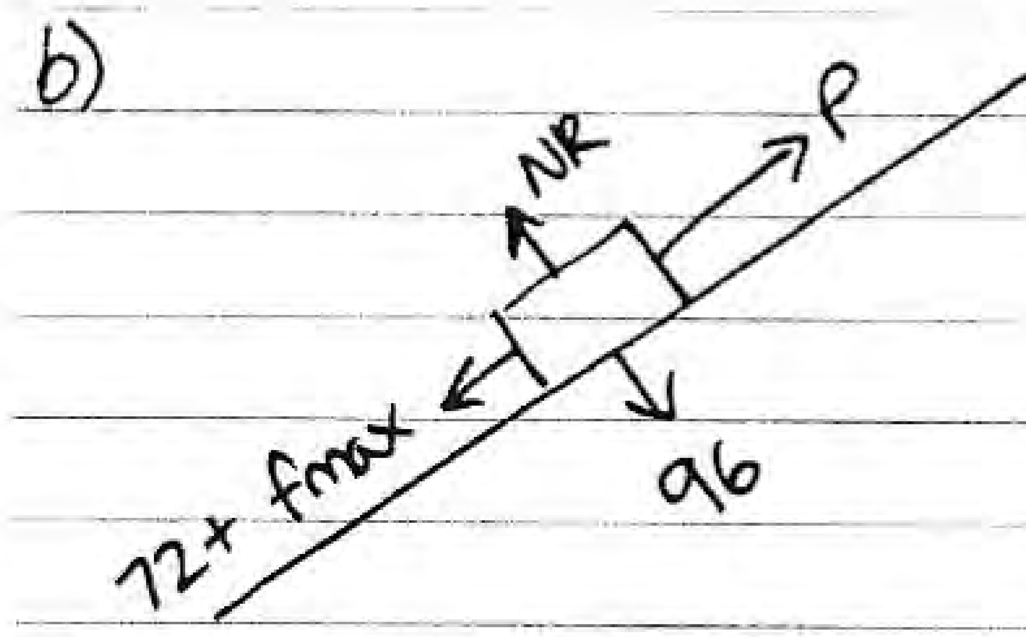
$\cos\alpha = \frac{4}{5}$



$\mu = \frac{1}{2}$

$NR = 120\times\frac{4}{5} + 30\times\frac{3}{5} = \underline{114N}$ #

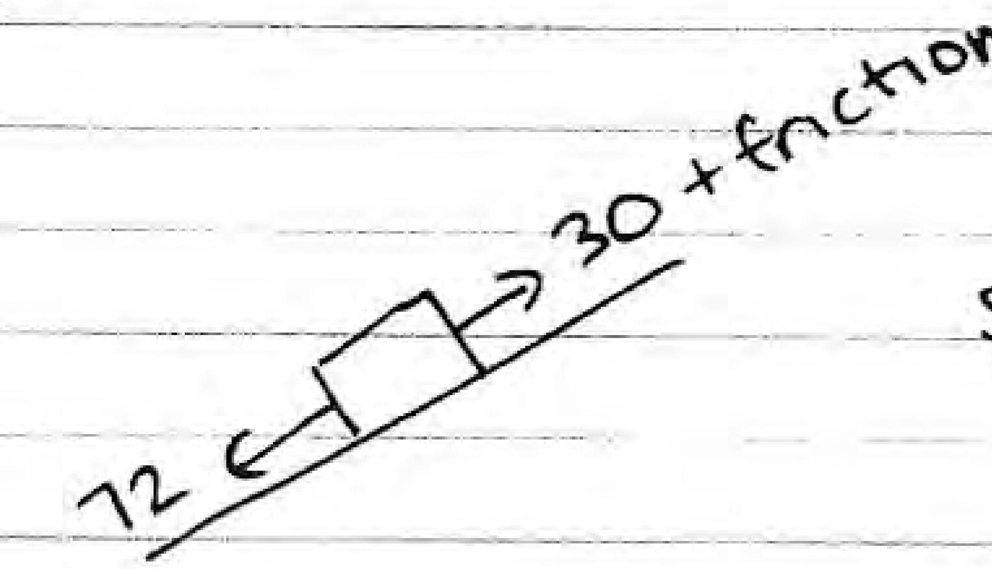
b)



$NR = 96N \Rightarrow f_{max} = 48N$

$P = 72 + 48 = \underline{120N}$

c)



Since $f_{max} = 48N$

friction $= \underline{42N}$

acting up the plane

7.

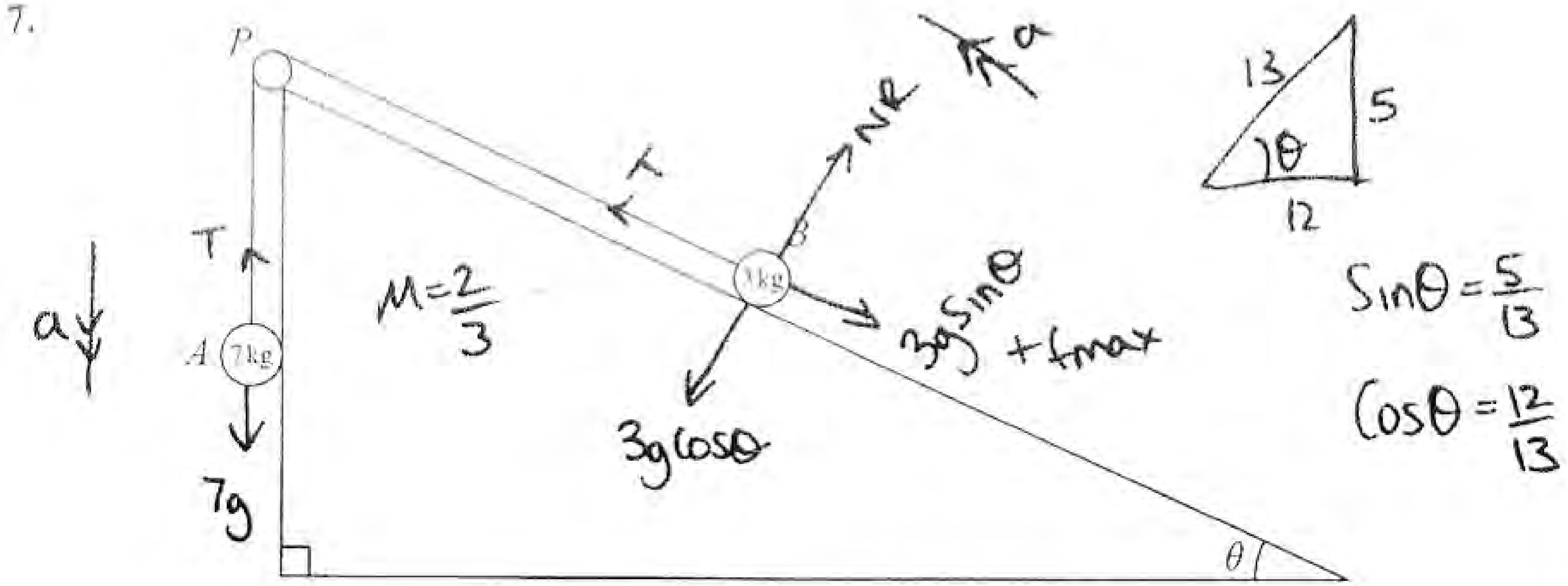


Figure 4

Two particles A and B, of mass 7 kg and 3 kg respectively, are attached to the ends of a light inextensible string. Initially B is held at rest on a rough fixed plane inclined at angle θ to the horizontal, where $\tan\theta = \frac{5}{12}$. The part of the string from B to P is parallel to a line of greatest slope of the plane. The string passes over a small smooth pulley, P, fixed at the top of the plane. The particle A hangs freely below P, as shown in Figure 4. The coefficient of friction between B and the plane is $\frac{2}{3}$. The particles are released from rest with the string taut and B moves up the plane.

(a) Find the magnitude of the acceleration of B immediately after release.

(10)

(b) Find the speed of B when it has moved 1 m up the plane.

(2)

When B has moved 1 m up the plane the string breaks. Given that in the subsequent motion B does not reach P,

(c) find the time between the instants when the string breaks and when B comes to instantaneous rest.

(4)

Question 7 continued

a) $NR = 3g \times \frac{12}{13} = \frac{36}{13}g \qquad f_{max} = \mu NR = \frac{2}{3} \times \frac{36}{13}g = \frac{24}{13}g$

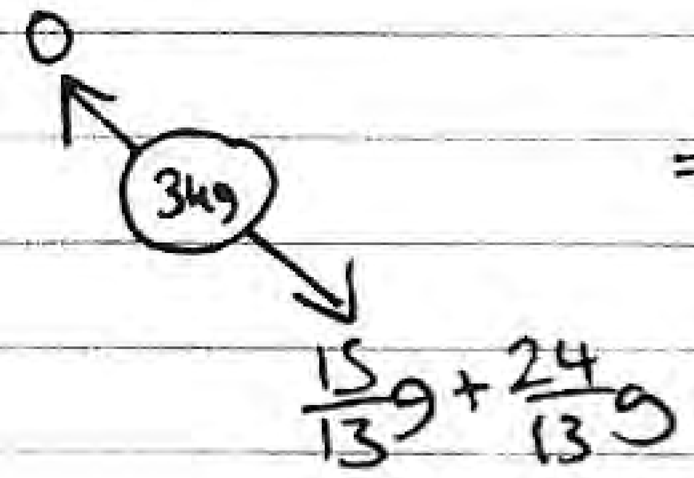
Whole system $\quad 7g - \frac{24}{13}g - 3g \times \frac{5}{13} = 10a \Rightarrow 4g = 10a$

$a = \frac{2}{5}g \text{ ms}^{-2}$

b) $u = 0 \qquad v^2 = u^2 + 2as$

$s = 1 \qquad v^2 = 2(3.92)(1)$

$a = 3.92 \qquad v^2 = 7.84 \quad \Rightarrow \quad v = 2.8 \text{ ms}^{-1}$ when string breaks



$\Rightarrow \quad 0 - 3g = 3a \quad \Rightarrow \quad a = -g = -9.8 \text{ ms}^{-2}$

$u = 2.8 \qquad v = u + at$

$a = -9.8 \qquad 0 = 2.8 - 9.8t \qquad t = \frac{2}{7} \text{ sec}$

$v = 0$