

Question Number	Scheme	Marks
10(a)	$(V =) 50 = \pi r^2 h$ $A = 2\pi r^2 + 2\pi r h = 2\pi r^2 + 2\pi r \times \frac{50}{\pi r^2}$ $A = 2\pi r^2 + \frac{100}{r} *$	B1 M1 A1 (3)
(b)	$\frac{dA}{dr} = 4\pi r - \frac{100}{r^2}$ $\frac{dA}{dr} = 0 \Rightarrow 4\pi r = \frac{100}{r^2}$ $r = \sqrt[3]{\frac{100}{4\pi}} = \sqrt[3]{7.9577...} = 1.996$	M1 M1 A1 (3)
(c)	$\frac{d^2 A}{dr^2} = 4\pi + \frac{200}{r^3}$ $r = 1.996 \Rightarrow \frac{d^2 A}{dr^2} > 0 \therefore \text{min}$	M1 M1A1 cso (3)
(d)	$A_{\min} = 2\pi \times 1.996^2 + \frac{100}{1.996}$ $A_{\min} = 75$	M1 A1cso (2) [11]

Notes

(a)

B1 for equating the formula for the volume of a cylinder to the given volume of 50

M1 for substituting their $50 = \pi r^2 h$ into the formula for the surface area of a cylinder

$$S = 2\pi r^2 + 2\pi r h$$

A1 for the answer as shown (this is a show question; beware of 'fudging' answers)

(b)

M1 for an attempt to differentiate the **GIVEN** expression only for S M1 for setting their $\frac{dA}{dr} = 0$

$$\text{Accept } 4\pi r = \frac{100}{r} \Rightarrow r = \dots$$

A1 for the answer as shown $r = 1.996$

(c)

M1 for attempting to find the second derivative of their $\frac{dA}{dr}$ (see General Guidance)M1 for substituting their value of r into their $\frac{d^2 A}{dr^2}$, but r must be positive.A1 for $\frac{d^2 A}{dr^2} > 0$ hence minimum cso Their value for $\frac{d^2 A}{dr^2}$ must be correct (37.716.. when $r = 1.9966$.)

Also accept a conclusion by inspection. ie., 4π and $\frac{100}{r^3}$ are both positive hence $\frac{d^2 A}{dr^2} > 0$

ALTM1 for substituting a value for $r < '1.996'$ **or** $r > '1.996'$ to test gradient $\frac{dA}{dr}$ around $r = '1.996'$.

When $r < 1.996$ $\frac{dA}{dr} < 0$, when $r > 1.996$, $\frac{dA}{dr} > 0$.

M1 for substituting a value for $r < '1.996'$ **and** $r > '1.996'$ to test $\frac{dA}{dr}$.A1 for conclusion; that as r increases the gradient goes from negative to positive hence, minimum.

(d)

M1 for substituting their r into the **GIVEN** expression for A and evaluating, but their r must be a positive value.A1 for $A_{\min} = 75$ cso

