



S15 M1

1. Particle  $P$  of mass  $m$  and particle  $Q$  of mass  $km$  are moving in opposite directions on a smooth horizontal plane when they collide directly. Immediately before the collision the speed of  $P$  is  $5u$  and the speed of  $Q$  is  $u$ . Immediately after the collision the speed of each particle is halved and the direction of motion of each particle is reversed.

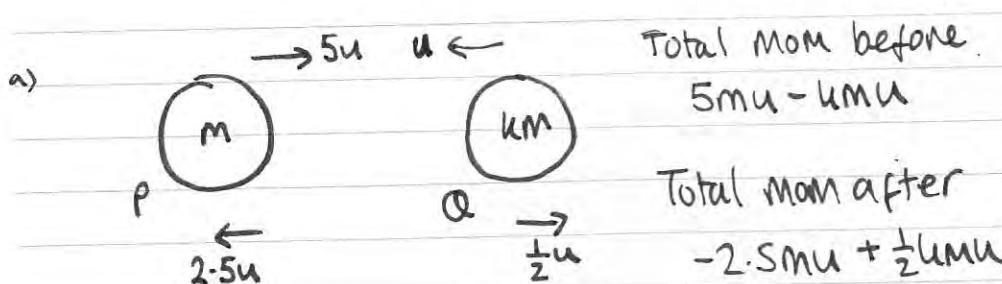
Find

(a) the value of  $k$ ,

(3)

(b) the magnitude of the impulse exerted on  $P$  by  $Q$  in the collision.

(3)



$$\text{CLM} \Rightarrow 5mu - kmu = -2.5mu + \frac{1}{2}kmu$$

$$7.5mu = 1.5mu \quad \therefore u = \frac{7.5}{1.5} = 5$$

b) Mom  $P$  before =  $5mu$      $\therefore$  Impulse = change in mom  
 Mom  $P$  after =  $-2.5mu$   
 $= 7.5mu$

2

Modelling the stone as a particle moving freely under gravity,

- (a) find the greatest height above  $O$  reached by the stone,

(2)

- (b) find the length of time for which the stone is more than  $14.7 \text{ m}$  above  $O$ .

(5)

a)

$$0 \uparrow \cdot v=0 \text{ at gh.}$$

$$g = -9.8$$

$$19.6 \uparrow$$

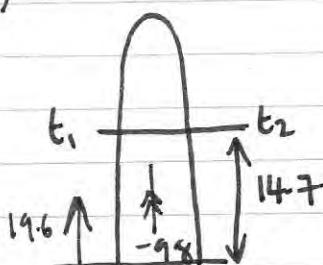
$$\begin{array}{l} s \\ u = 19.6 \\ v = 0 \\ a = -9.8 \\ t \end{array}$$

$$\begin{aligned} v^2 &= u^2 + 2as \\ 0 &= 19.6^2 - 19.6s \end{aligned}$$

$$s = \frac{19.6^2}{19.6} = 19.6 \text{ m}$$

b)

$$\text{total time above } 14.7 = t_2 - t_1$$



$$\begin{array}{l} s = 14.7 \\ u = 19.6 \\ v \\ a = -9.8 \\ t \end{array}$$

$$\begin{aligned} s &= ut + \frac{1}{2}at^2 \\ 14.7 &= 19.6t - 4.9t^2 \end{aligned}$$

$$4.9t^2 - 19.6t + 14.7 = 0$$

$$\begin{aligned} \div 4.9 \\ t^2 - 4t + 3 = 0 \end{aligned}$$

$$(t-3)(t-1) = 0$$

$$t_1 = 1 \quad t_2 = 3$$

$\therefore$  total time above

= 2 seconds

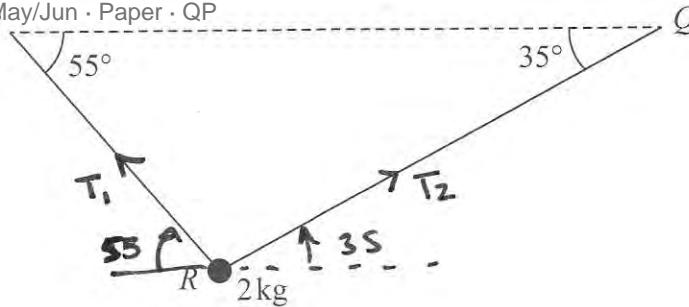


Figure 1

A particle of mass 2 kg is suspended from a horizontal ceiling by two light inextensible strings,  $PR$  and  $QR$ . The particle hangs at  $R$  in equilibrium, with the strings in a vertical plane. The string  $PR$  is inclined at  $55^\circ$  to the horizontal and the string  $QR$  is inclined at  $35^\circ$  to the horizontal, as shown in Figure 1.

Find

- the tension in the string  $PR$ ,
- the tension in the string  $QR$ .

$$\begin{aligned} & T_1 \sin 55 + T_2 \sin 35 \\ & T_1 \cos 55 \leftarrow \quad \rightarrow T_2 \cos 35 \\ & 2g \end{aligned} \quad \begin{aligned} \vec{R}\vec{F} = 0 \quad (7) \\ \therefore T_2 \cos 35 = T_1 \cos 55 \\ T_2 = \frac{\cos 55}{\cos 35} T_1 \end{aligned}$$

$$R\vec{F} = 0 \quad \therefore T_1 \sin 55 + T_2 \sin 35 = 19.6$$

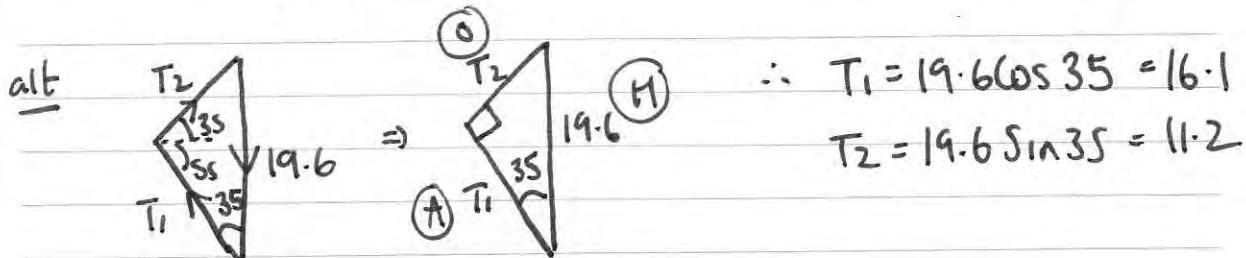
$$T_1 \sin 55 + T_1 \frac{\cos 55 \times \sin 35}{\cos 35} = 19.6$$

$$T_1 (\sin 55 + \cos 55 \tan 35) = 19.6$$

$$T_1 = 16.055 \dots$$

$$\underline{T_1 = 16.1 \text{ N}}$$

$$b) T_2 = \frac{\cos 55}{\cos 35} \times T_1 \quad \therefore T_2 = 11.24 \dots \quad \underline{T_2 = 11.2 \text{ N}}$$



$$\therefore T_1 = 19.6 \cos 35 = 16.1$$

$$T_2 = 19.6 \sin 35 = 11.2$$

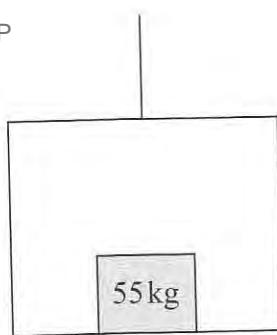


Figure 2

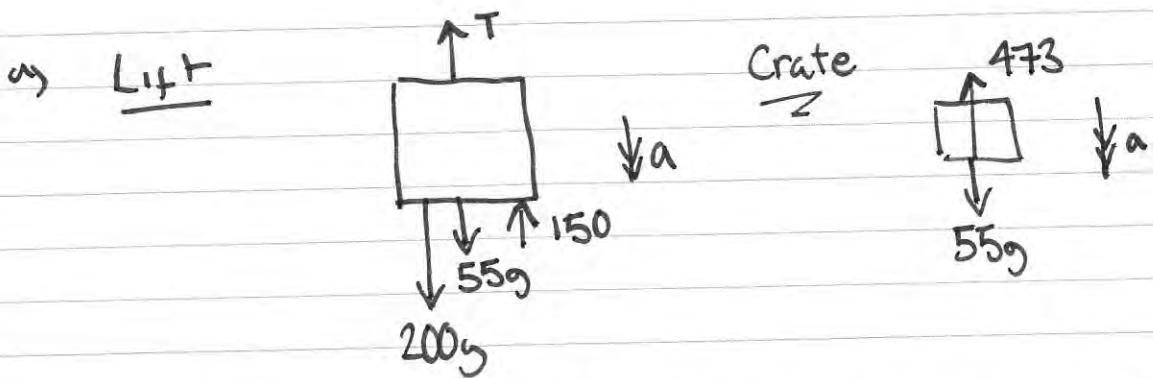
A lift of mass 200 kg is being lowered into a mineshaft by a vertical cable attached to the top of the lift. A crate of mass 55 kg is on the floor inside the lift, as shown in Figure 2. The lift descends vertically with constant acceleration. There is a constant upwards resistance of magnitude 150 N on the lift. The crate experiences a constant normal reaction of magnitude 473 N from the floor of the lift.

- (a) Find the acceleration of the lift.

(3)

- (b) Find the magnitude of the force exerted on the lift by the cable.

(4)



$$\text{from the crate } 55g - 473 = 55a \quad \therefore a = 1.2 \text{ ms}^{-2}$$

$$\text{from the lift } 200g + 55g - 150 - T = 255a$$

$$2349 - T = 306 \quad \therefore T = 2043 \text{ N}$$

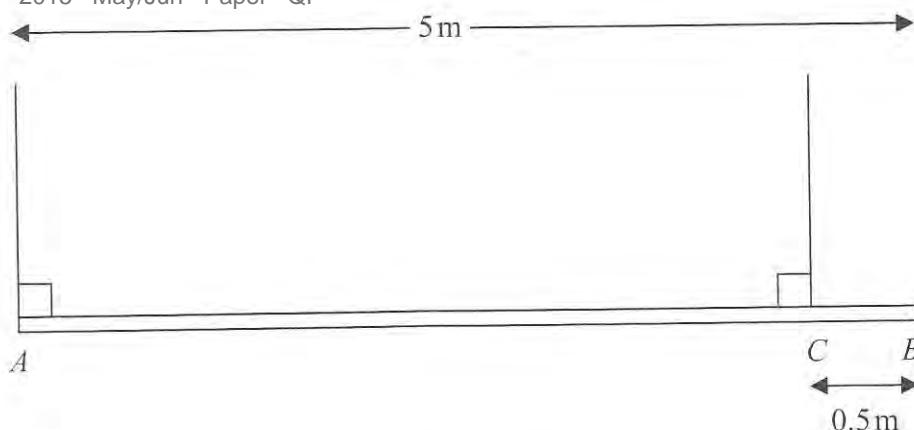


Figure 3

A beam  $AB$  has length 5 m and mass 25 kg. The beam is suspended in equilibrium in a horizontal position by two vertical ropes. One rope is attached to the beam at  $A$  and the other rope is attached to the point  $C$  on the beam where  $CB = 0.5$  m, as shown in Figure 3. A particle  $P$  of mass 60 kg is attached to the beam at  $B$  and the beam remains in equilibrium in a horizontal position. The beam is modelled as a uniform rod and the ropes are modelled as light strings.

(a) Find

- (i) the tension in the rope attached to the beam at  $A$ ,
- (ii) the tension in the rope attached to the beam at  $C$ .

(6)

Particle  $P$  is removed and replaced by a particle  $Q$  of mass  $M$  kg at  $B$ . Given that the beam remains in equilibrium in a horizontal position,

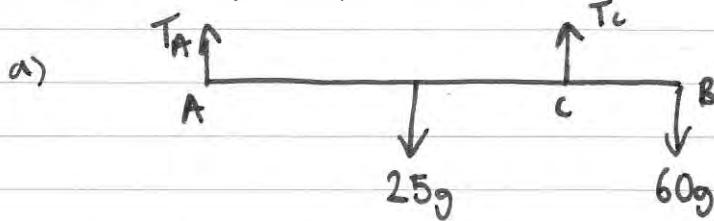
(b) find

- (i) the greatest possible value of  $M$ ,
- (ii) the greatest possible tension in the rope attached to the beam at  $C$ .

(6)

**Question 5 continued**

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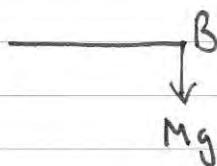
$$\text{At } C: 25g \times 2.5 + 60g \times 5 = T_C \times 4.5$$

$$\frac{725}{2}g = \frac{a}{2} T_C \quad \therefore T_C = 80.5g$$

$$R_f \uparrow = 0 \quad T_A + T_C = 25g + 60g \quad \therefore T_A = 85g - 80.5g \\ = 4.5g$$

$$\therefore T_A = \frac{43.6N}{2} \quad T_C = \frac{789.4N}{2}$$

b)

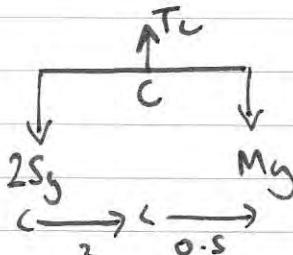


greatest value of M would result in  $T_A = 0$

$$\text{At } C: 25g \times 2 = Mg \times \frac{1}{2}$$

$$100g = Mg$$

$$\therefore \text{Max } M = 100kg$$



$$R_f \uparrow = 0 \Rightarrow T_C = 125g = 1225N$$

6. A particle  $P$  is moving with constant velocity. The position vector of  $P$  at time  $t$  seconds  
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 $(t \geq 0)$  is  $\mathbf{r}$  metres, relative to a fixed origin  $O$ , and is given by

$$\mathbf{r} = (2t - 3)\mathbf{i} + (4 - 5t)\mathbf{j}$$

- (a) Find the initial position vector of  $P$ .

(1)

The particle  $P$  passes through the point with position vector  $(3.4\mathbf{i} - 12\mathbf{j})$  m at time  $T$  seconds.

- (b) Find the value of  $T$ .

(3)

- (c) Find the speed of  $P$ .

(4)

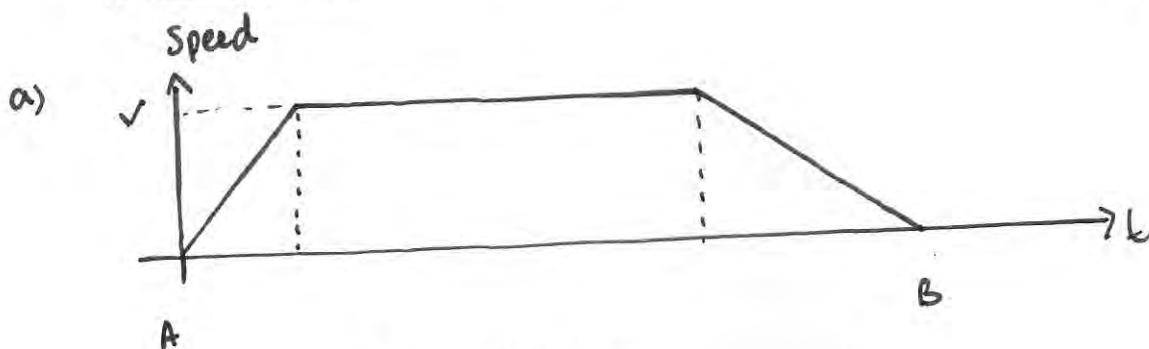
$$a) \quad \mathbf{r} = \begin{pmatrix} -3+2t \\ 4-5t \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} + t \begin{pmatrix} 2 \\ -5 \end{pmatrix} \quad \therefore \text{original pos vector} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

$$b) \quad \begin{pmatrix} -3+2T \\ 4-5T \end{pmatrix} = \begin{pmatrix} 3.4 \\ -12 \end{pmatrix} \quad \therefore 2T = 6.4 \quad \therefore T = \frac{3.2}{2} \text{ sec}$$

$$c) \quad |\mathbf{v}| = \sqrt{\left(\frac{2}{5}\right)^2 + (-5)^2} = \sqrt{29} = 5.39 \text{ ms}^{-1}$$

7. A train travels along a straight horizontal track between two stations,  $A$  and  $B$ . The train starts from rest at  $A$  and moves with constant acceleration  $0.5 \text{ m s}^{-2}$  until it reaches a speed of  $V \text{ m s}^{-1}$ , ( $V < 50$ ). The train then travels at this constant speed before it moves with constant deceleration  $0.25 \text{ m s}^{-2}$  until it comes to rest at  $B$ .

- (a) Sketch in the space below a speed-time graph for the motion of the train between the two stations  $A$  and  $B$ . (2)



b) total time =  $5\text{min} = 300\text{sec}$

i)  $\text{acc} = \text{gradient} = \frac{1}{2}$      $\frac{V}{t_1} = \frac{1}{2}$      $\therefore t_1 = 2V$

ii)  $\frac{V}{t_2} = \frac{1}{4}$      $\therefore t_2 = 4V$

iii)  $300 - 2V - 4V = 300 - 6V$

c)  $\frac{(300-6V+300)}{2} \times V = 6300$

The total time for the journey from  $A$  to  $B$  is 5 minutes.

- (b) Find, in terms of  $V$ , the length of time, in seconds, for which the train is

- (i) accelerating,
- (ii) decelerating,
- (iii) moving with constant speed.

(5)

Given that the distance between the two stations  $A$  and  $B$  is 6.3 km,

- (c) find the value of  $V$ .

(6)

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$$\left( \frac{650}{2} \right) V = 6300$$

$$300V - 3V^2 = 6300 \Rightarrow 3V^2 - 300V + 6300 = 0$$

$$\div 3 \quad V^2 - 100V + 210 = 0$$

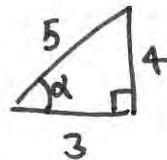
$$(V - 30)(V - 70) = 0$$

$$\therefore V = \frac{30}{2}$$

8.

Mechanics 1 · May/Jun · Paper: QP ·  $\tan \alpha = \frac{4}{3}$ 

$$\alpha = \tan^{-1} \left( \frac{4}{3} \right) = 53.1^\circ$$



$$\sin \alpha = 0.8$$

$$\cos \alpha = 0.6$$

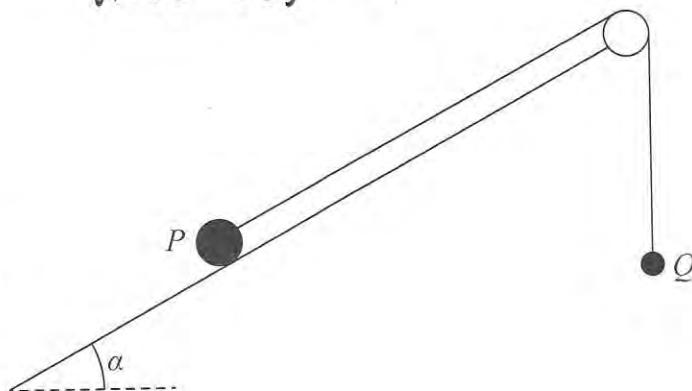


Figure 4

Two particles  $P$  and  $Q$  have mass 4 kg and 0.5 kg respectively. The particles are attached to the ends of a light inextensible string. Particle  $P$  is held at rest on a fixed rough plane, which is inclined to the horizontal at an angle  $\alpha$  where  $\tan \alpha = \frac{4}{3}$ . The coefficient of friction between  $P$  and the plane is 0.5. The string lies along the plane and passes over a small smooth light pulley which is fixed at the top of the plane. Particle  $Q$  hangs freely at rest vertically below the pulley. The string lies in the vertical plane which contains the pulley and a line of greatest slope of the inclined plane, as shown in Figure 4. Particle  $P$  is released from rest with the string taut and slides down the plane.

Given that  $Q$  has not hit the pulley, find

- (a) the tension in the string during the motion,

(11)

- (b) the magnitude of the resultant force exerted by the string on the pulley.

(4)

$m = \frac{1}{2}$

$$4g \cos 53.1^\circ = 3.2g$$

$$4g \sin 53.1^\circ = 2.4g$$

$$N = 4g \cos 53.1^\circ = 3.2g$$

$$R + f_{\max} = ma$$

$$T - \frac{1}{2}g = \frac{1}{2}a$$

$$f_{\max} = \mu N R = \frac{1}{2} \times 2.4g = 1.2g$$

$$R f = m a \quad 3.2g - T - f_{\max} = 4a$$

$$3.2g - T - 1.2g = 4a$$

$$2g - T = 4a$$

$$T - \frac{1}{2}g = \frac{1}{2}a$$

$$\frac{3}{2}g = \frac{9}{2}a \quad \therefore a = \frac{1}{3}g$$

b)  $2T(\omega \frac{90-\alpha}{2}) \quad \underline{\underline{101}}$

$$= 2\left(\frac{2}{3}g\right)(0)\left(\frac{90-53.1}{2}\right)$$

$$= 12.4 \text{ N}$$

$$\therefore T = \frac{1}{2}a + \frac{1}{2}g = \frac{1}{6}g + \frac{1}{2}g$$

$$\therefore T = \frac{4}{6}g \quad \therefore T = \frac{2}{3}g = 6.53 \text{ N}$$