Mark Scheme (Results)

January 2022

Pearson Edexcel International GCSE

In Further Pure Mathematics (4PM1)

Paper 01R

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme - not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is given.
- Crossed out work should be marked **unless** the candidate has replaced it with an alternative response.

- **Types of mark**
    - M marks: method marks
    - A marks: accuracy marks
    - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
    - cao – correct answer only
    - ft – follow through
    - isw – ignore subsequent working
    - SC - special case
    - oe – or equivalent (and appropriate)
    - dep – dependent
    - indep – independent
    - awrt – answer which rounds to
    - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers normally score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**
  If the final answer is wrong, always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
  If there is a choice of methods shown, then award the lowest mark, unless the answer on the answer line makes clear the method that has been used.
  If there is no answer achieved then check the working for any marks appropriate from the mark scheme.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right),\text{ where }|pq|=|c|$$ leading to $x=\ldots.$

$$\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right)\text{ where }|pq|=|c|\text{ and }|mn|=|a|$$ leading to $x=\ldots.$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=\ldots.$

3. Completing the square:

$$x^2+bx+c=0:\ \left(x\pm\tfrac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$$ leading to $x=\ldots.$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n\rightarrow x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n\rightarrow x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication

from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

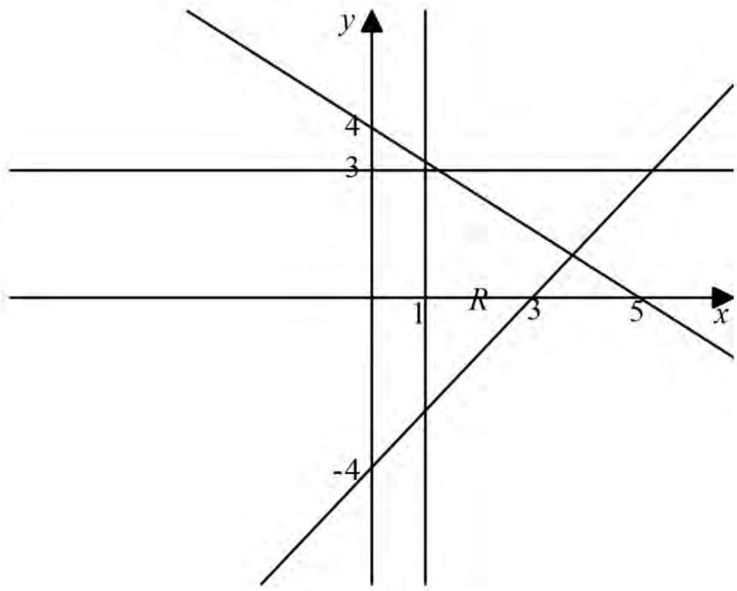
General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Question number	Scheme	Marks
1 (a)(i)	$4x+5y=20$ drawn	B1 (1)
(a)(ii)	$3y-4x=-12$ drawn	B1 (1)
(b)	$y=3$ and $x=1$ drawn	B1
	Correct region defined (see below) 	B1ft (2)
		Total 4 marks

Part	Mark	Notes
(a) (i)	**B1**	Correct line
(ii)	**B1**	Correct line
(b)	**B1**	Both lines correct
	B1ft	Correctly shaded region ft correct $y = 3$ and $x = 1$, any line drawn with a positive gradient and negative y-intercept and any line drawn with a negative gradient and positive y-intercept where the region shaded is correct for the lines drawn.

Question number	Scheme	Marks
2 (a)(i)	$a+4d=46$ $\quad a+19d=181$ oe	B1
	$"15d=135" \Rightarrow d=9$	M1 A1 [3]
(a)(ii)	$a=10$	B1 [1]
(b)	$\sum_{n=1}^{50} u_n = 25(2\times"10"+(50-1)\times"9")=11525$	M1
	$\sum_{n=1}^{20} u_n = 10(2\times"10"+(20-1)\times"9")=1910$	M1
	$"11525"-"1910"=9615$	ddM1 A1 [4]
ALT	(First term =) $"10"+"9"\times(21-1)=190$	{M1}
	(Last term =) $"10"+"9"\times(50-1)=451$	{M1}
	$\frac{30}{2}("190"+"451")=9615$	{ddM1}{A1} [4]
		Total 8 marks

Part	Marks	Notes
(a) (i)	**B1**	For $a+4d=46$ and $a+19d=181$, both correct oe.
	M1	For solving their equations simultaneously, valid correct method with one slip only (algebraic or sign error), leading to $d=$
	A1	For $d=9$
(a) (ii)	**B1**	For $a=10$
(b)	**M1**	For use of their a and d in the correct formula for the sum to n terms $\frac{n}{2}(2"a"+(n-1)"d")$ with $n=50$
	M1	For use of their a and d in the correct formula for the sum to n terms $\frac{n}{2}(2"a"+(n-1)"d")$ with $n=20$
	ddM1	For their $"\sum_{n=1}^{50} u_n"-"\sum_{n=1}^{20} u_n"$ Dependent on both previous method marks
	A1	For 9615
	ALT	
	M1	For correct use of $"a"+(n-1)"d"$ with $n=21$ with their a and their d
	M1	For correct use of $"a"+(n-1)"d"$ with $n=50$ with their a and their d
	ddM1	For correct use of the formula $\frac{n}{2}(a_{21}+a_{50})$ Dependent on both previous method marks
	A1	For 9615

Question number	Scheme	Marks
3 (a)	(Midpoint of AB =) $(5,5)$	B1
	(Gradient of AB =) $\frac{3-7}{9-1} = \left(-\frac{1}{2}\right)$ oe	M1
	(Gradient of l =) $-\left(\frac{1}{\text{"}-\frac{1}{2}\text{"}}\right)$ (= 2)	A1ft
	$y-\text{"5"}=\text{"2"}(x-\text{"5"})$ oe	M1 A1 (5)
ALT	Let D (x, y) be any point on the line	
	$(x-1)^2+(y-7)^2=(x-9)^2+(y-3)^2$	{B1}
	$x^2-2x+1+y^2-14y+49=x^2-18x+81+y^2-6y+9$	{M1} {A1ft}
	$16x-40=8y \rightarrow (y=2x-5)$	M1 A1
(b)	$x=\frac{5}{2}$	B1ft
	(length of C to midpoint of AB =) $\sqrt{(\text{"5"}-0)^2+(\text{"5"}-2.5)^2}\left(=\frac{5\sqrt{5}}{2}\right)$	M1
	(length of AB =) $\sqrt{8^2+4^2}\left(=4\sqrt{5}\right)$	B1 (A1 on ePen)
	$\frac{1}{2}\times 4\sqrt{5}\times\text{"}\frac{5\sqrt{5}}{2}\text{"}$	M1
	25	A1 [5]
ALT1	Use of determinant	
	$x=\frac{5}{2}$	B1ft
	$\frac{1}{2}\begin{vmatrix}1 & 9 & \text{"2.5"} & 1\\ 7 & 3 & 0 & 7\end{vmatrix}$	M1 A1
	$\left(\frac{1}{2}\right)\left\|(1\times3+9\times0+\text{"2.5"}\times7)-(1\times0+\text{"2.5"}\times3+9\times7)\right\|$ oe	M1
	25	A1 [5]

ALT2	Area of rectangle – (sum of areas of 3 triangles)	
	$x=\frac{5}{2}$	B1ft
	$\frac{1}{2}\times\left("\frac{5}{2}"-1\right)\times 7+\frac{1}{2}\times\left(9-"\frac{5}{2}"\right)\times 3+\frac{1}{2}(9-1)(7-3)$ $\left(=\frac{21}{4}+\frac{39}{4}+16=31\right)$	M1
	$(9-1)\times 7=56$	A1
	56 – "31"	M1
	35	A1 [5]
		Total 10 marks

Part	Marks	Notes
(a)	**B1**	For midpoint of $AB = (5,5)$
	M1	For correctly finding the unsimplified gradient of $AB = \dfrac{3-7}{9-1}$ oe
	A1ft	For correctly finding the unsimplified gradient of line l, use of their gradient for AB to find the negative reciprocal. $-\left(\dfrac{1}{\text{"}-\frac{1}{2}\text{"}}\right)$
	M1	For a correct method to find the equation of l using their midpoint and their gradient, which must have involved finding the negative reciprocal of the gradient of AB
	A1	For $y-5=2(x-5)$ ft oe
ALT	**B1**	For correctly equating the two expressions for AD^2 and AB^2 to give the equation shown
	M1	For an expansion of either their LHS or their RHS provided their equation is in the form. $x-a\,^2+\,y-b\,^2=\,x-c\,^2+\,y-d\,^2$ Allow one numerical or algebraic slip.
	A1ft	For a correct expansion of both their LHS and their RHS provided their equation is in the form. $x-a\,^2+\,y-b\,^2=\,x-c\,^2+\,y-d\,^2$
	M1	For simplifying leading to a linear equation
	A1	For $y=2x-5$ oe
(b)	**B1ft**	For $x=\dfrac{5}{2}$ ft their equation from part (a) with $y=0$
	M1	For finding the unsimplified length of C to midpoint of AB = $\sqrt{(\text{"}5\text{"}-0)^2+(\text{"}5\text{"}-\text{"}2.5\text{"})^2}\left(=\dfrac{5\sqrt{5}}{2}\right)$ Allow use of their (5, 5) and their C
	B1 (A1 on ePen)	For finding the unsimplified length of $AB=\sqrt{8^2+4^2}\ (=4\sqrt{5})$
	M1	For area of triangle = $\dfrac{1}{2}\times 4\sqrt{5}\times\text{"}\dfrac{5\sqrt{5}}{2}\text{"}$
	A1	For 25
ALT1	**B1ft**	As main
	M1	For the determinant set up with their C
	A1	For the determinant set up fully correctly
	M1	For correctly evaluating their determinant, with their C, full working must be shown.
	A1	25
ALT2	**B1ft**	As main
	M1	For the correct calculation of the three areas using their C
	A1	For the correct rectangle area

	M1	For the correct calculation, using their values for the three areas
	A1	25

Question number	Scheme	Marks
4 (a)	$\frac{1}{2}.\frac{\pi}{3}r^2 - \frac{1}{2}.r.\frac{3r}{4}\sin\left(\frac{\pi}{3}\right)(=79.5)$	M1 M1
	$\frac{\pi}{6}r^2 - \frac{3\sqrt{3}}{16}r^2 = 79.5$ leading to $r =$	ddM1
	$r = \sqrt{\frac{79.5}{\left(\frac{\pi}{6} - \frac{3\sqrt{3}}{16}\right)}} = 20$	A1 [4]
(b)	$(BD^2) = "20"^2 + \left(\frac{3}{4}\times"20"\right)^2 - 2\times 20\times\left(\frac{3}{4}\times"20"\right)\cos\frac{\pi}{3}$	M1
	$= 5\sqrt{13}$ or awrt 18.02	M1
	Perimeter = $("20"-15) + \frac{"20"\pi}{3} + "5\sqrt{13}" = 44$	M1 A1 [4]
	Total 8 marks	

Part	Marks	Notes
(a)	**M1**	For $\frac{1}{2}.\frac{\pi}{3}r^2$
	M1	For $\frac{1}{2}.r.\frac{3r}{4}\sin\left(\frac{\pi}{3}\right)$
	ddM1	For the correct unsimplified equation and attempting to rearrange (allow two slips) to make r the subject (dep on both previous M marks)
	A1	For 20
(b)	**M1**	For correct use of the cosine rule, as shown (ft their r)
	M1 (A1 on ePen)	For $5\sqrt{13}$ or awrt 18.02 – allow the candidate show they are finding the square root of their BD^2 to give an answer for BD
	M1	For correctly finding the perimeter of ABD with correct use of arc length $= r\theta$ (ft their r and their BD)
	A1	For 44

Question number	Scheme	Marks
5 (a)	$1+anx+\frac{n(n-1)}{2}a^2x^2+\frac{n(n-1)(n-2)}{3!}a^3x^3$	M1 A1 [2]
(b)	$an=8 \qquad \frac{n(n-1)}{2}a^2=30$	
	$\frac{n(n-1)}{2}\left(\frac{8}{n}\right)^2=30$ or $\frac{\left(\frac{8}{a}\right)\left(\frac{8}{a}-1\right)}{2}a^2=30$	M1
	$32n-32=30n \rightarrow n=$ or $64-8a=60 \rightarrow a=$	dM1
	$n=16 \qquad a=\frac{1}{2}$	A1 A1 [4]
(c)	$\frac{"16"\times("16"-1)("16"-2)}{3!}\times\left("\frac{1}{2}"\right)^3=70$	M1 A1 [2]
		Total 8 marks

Part	Mark	Notes
(a)	**M1**	For an attempt to find the binomial expansion • The expansion must begin with 1 or 1^n • The denominators must be correct (ie. 2! And 3!) on the third and fourth terms. The power of x must be correct (ie. Must see x, x^2 and x^3, with the correct corresponding denominators). Simplification not necessary – may see $(ax)^2$ and $(ax)^3$
	A1	For a fully correct expression (Allow 1^n for 1) - must see a^2x^2 and a^3x^3, but 2! And 3! is acceptable
(b)	**M1**	For correct substitution of either $\frac{8}{n}$ or $\frac{8}{a}$ into their coefficient of x^2
	dM1	For rearranging and forming a linear equation, leading to $a=$ or $n=$
	A1	For $n=16$
	A1	For $a=\frac{1}{2}$
(c)	**M1**	For correct substitution seen of their n and a into their coefficient of x^3
	A1	For 70

Question number	Scheme	Marks
6 (a)	eg $\overrightarrow{SR} = \overrightarrow{SP} + \overrightarrow{PR} = (3\mathbf{i} - 15\mathbf{j} - \mathbf{i} + 18\mathbf{j})$ $= 2\mathbf{i} + 3\mathbf{j}$ or eg $\overrightarrow{QR} = \overrightarrow{QP} + \overrightarrow{QR} = (-2\mathbf{i} - 3\mathbf{j} - \mathbf{i} + 18\mathbf{j})$ $= -3\mathbf{i} + 15\mathbf{j}$	M1 A1 M1 A1
	eg $\overrightarrow{PQ} = \overrightarrow{SR}$ or $\overrightarrow{PS} = \overrightarrow{QR}$ oe	M1
	(As opposite sides are equal in length and parallel) *PQRS* is a parallelogram *	A1cso [4]
(b)	$\overrightarrow{QS} = \overrightarrow{QP} + \overrightarrow{PS} = (-2\mathbf{i} - 3\mathbf{j} - 3\mathbf{i} + 15\mathbf{j})$ $= -5\mathbf{i} + 12\mathbf{j}$	M1 A1
	Unit vector $= (\pm)\frac{1}{13}(-5\mathbf{i} + 12\mathbf{j})$	M1 A1 [4]
(c)	$\overrightarrow{PT} = 2\mathbf{i} + 3\mathbf{j} + \frac{5}{13}(\text{"}-5\mathbf{i} + 12\mathbf{j}\text{"}) = \frac{1}{13}\mathbf{i} + \frac{99}{13}\mathbf{j}$	M1 A1 [2]
		Total 10 marks

Part	Mark	Notes
(a)	**M1**	For stating a correct valid vector path, e.g. $\overrightarrow{SR} = \overrightarrow{SP} + \overrightarrow{PR}$
	A1	For $\pm(2\mathbf{i} + 3\mathbf{j})$ or $\pm(-3\mathbf{i} + 15\mathbf{j})$
	M1	For a correct vector statement leading to a conclusion eg $\overrightarrow{PQ} = \overrightarrow{SR}$
	A1*cso	For some form of conclusion.
(b)	**M1**	For using e.g. $\overrightarrow{QS} = \overrightarrow{QP} + \overrightarrow{PS}$ or $\overrightarrow{SQ}$
	A1	For $-5\mathbf{i} + 12\mathbf{j}$
	M1	For an attempt to use Pythagoras with a plus sign, allow use of their vector $\pm\overrightarrow{QS}$
	A1	For $(\pm)\frac{1}{13}(-5\mathbf{i} + 12\mathbf{j})$ oe Must be one vector only ie can't list as ±
(c)	**M1**	For using e.g. $\overrightarrow{PT} = \overrightarrow{PQ} + \frac{5}{13}\left(\text{their } \overrightarrow{QS}\right)$
	A1	For $\frac{1}{13}\mathbf{i} + \frac{99}{13}\mathbf{j}$ oe

Question number	Scheme	Marks
7 (a)	(Surface area =) $12x^2+4xy+3xy+5xy=144$ oe	M1
	$12x^2+12xy=144$ oe leading to $\dfrac{144-12x^2}{12x}$ oe	A1
	$V=\dfrac{1}{2}\times3x\times4x\times y\ (=6x^2y)$	M1
	$=6x^2\times\dfrac{144-12x^2}{12x}=6x(12-x^2)$	M1
	$=72x-6x^3$ *	A1 cso [5]
(b)	$\left(\dfrac{\mathrm{d}V}{\mathrm{d}x}\right)=72-18x^2$	M1
	$\left(\dfrac{\mathrm{d}V}{\mathrm{d}x}=\right)$"$72-18x^2$"$=0$ $\Rightarrow x=$	M1 (A1 on ePen)
	$x=2$	A1 (M1 on ePen)
	$\left(\dfrac{\mathrm{d}^2V}{\mathrm{d}x^2}\right)=-36x$ $x=2$ $\dfrac{\mathrm{d}^2V}{\mathrm{d}x^2}<0$ Therefore maximum	B1 (A1 on ePen) [4]
(c)	$V=72\times2-6\times2^3=96$	M1 A1 [2]
		Total 11 marks

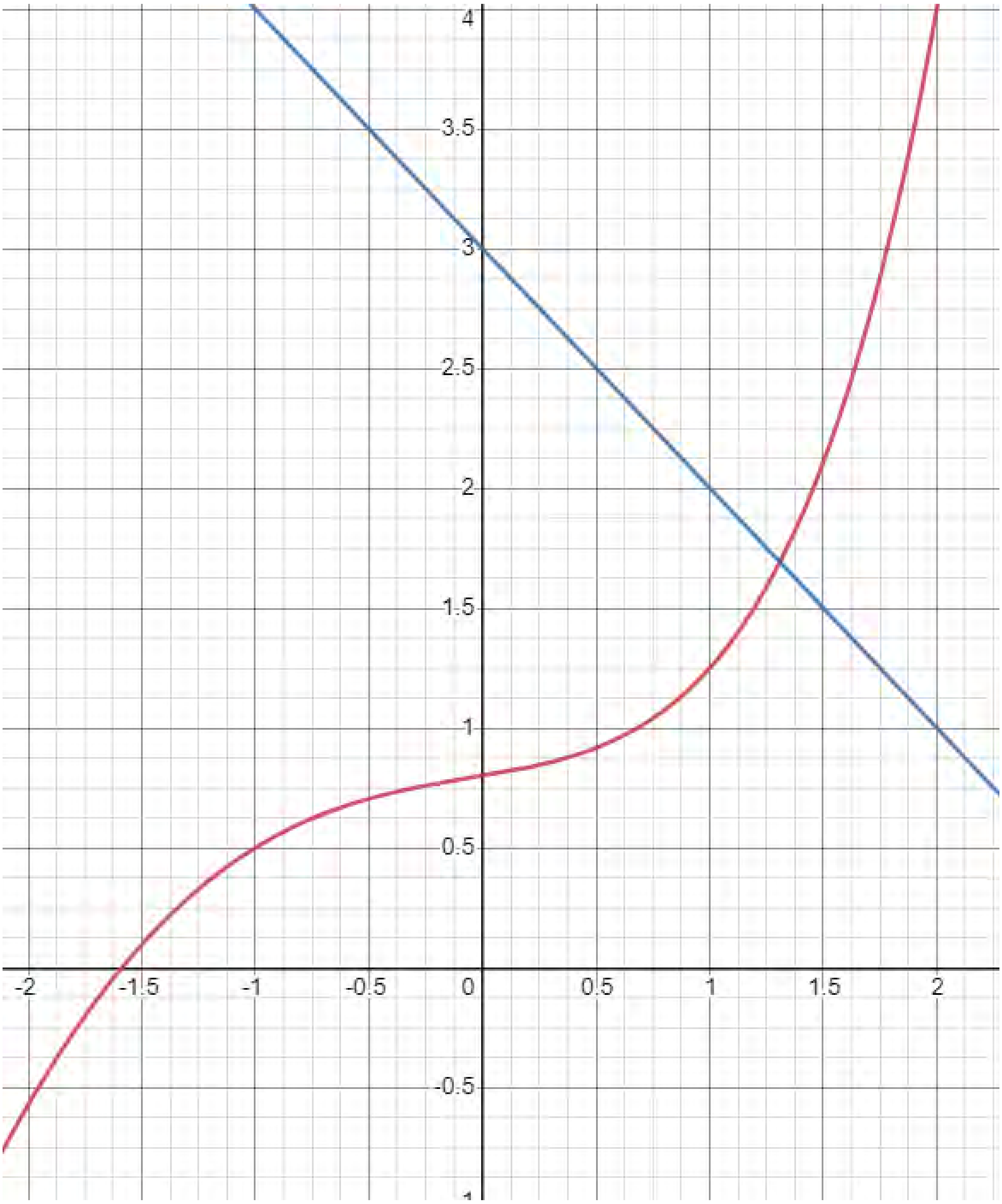
Part	Mark	Notes
(a)	**M1**	For surface area as an equation in x and y (correct and unsimplified)
	A1	For $12x^2 + 12xy = 144$ oe and correctly making y the subject $\frac{144-12x^2}{12x}$ (oe)
	M1	$V = \frac{1}{2} \times 3x \times 4x \times y \ (= 6x^2 y)$
	M1	For correct substitution of their expression for y, which must be in terms of x only, into their formula for V.
	A1*cso	Obtains the given expression
(b)	**M1**	For attempting to differentiate V wrt x At least one term must be fully correct, the other – follow general guidance
	M1 (A1 on ePen)	For setting their derivative (which must involve a changed expression) = 0 and a complete solution to find the value of x
	A1 (M1 on ePen)	For $x = 2$ only
	B1 (A1 on ePen)	For a correct second derivative and correct justification that this is a maximum, with some form of conclusion. All work to be correct for this mark. i.e it is acceptable, as this is a simple substitution to state if $x = 2$, $\frac{d^2V}{dx^2} < 0$, but if a substitution of $x = 2$ is made, the value for the second derivative must be given as – 72.
ALT final B1	**B1 (A1 on ePen)**	Testing and substituting into the correct second derivative with appropriate values either side of $x = 2$. For correct justification that this is a maximum, with some form of conclusion. All work to be correct for this mark
(c)	**M1**	For correct substitution of their x into V
	A1	96

Question number	Scheme	Marks
8	When $(x=\pi \quad y=)2\pi^2$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=4x-\cos x$	M1
	When $x=\pi$ $\frac{\mathrm{d}y}{\mathrm{d}x}=4\pi-\cos\pi \quad =4\pi+1$	M1 A1
	Gradient of the normal $=-\frac{1}{"4\pi+1"}$	M1
	$y-2\pi^2=-\frac{1}{4\pi+1}(x-\pi)$ oe	M1
	$\left[(4\pi+1)(y-2\pi^2)=-x+\pi\right]$	
	$4\pi y+y-8\pi^3-2\pi^2-\pi+x=0$	dM1
	$x+(4\pi+1)y-\pi(8\pi^2+2\pi+1)=0$ *	A1 cso
		Total 8 marks

Mark	Notes
B1	For $(y=)2\pi^2$
M1	For differentiating *y* wrt *x*, to give an expression of the form $ax - \cos x$
M1	For correct substitution of $x=\pi$ into their derivative
A1	For $\frac{\mathrm{d}y}{\mathrm{d}x}=4\pi+1$
M1	For gradient of the normal $=-\frac{1}{"(4\pi+1)"}$
M1	For a fully correct method to find the equation of the line, using the correct *x* coordinate and their *y* coordinate and their gradient, which must be $=-\frac{1}{their\,(4\pi+1)}$ and must have come from differentiation. If $y = mx + c$ is the method used, *c* must be found and the line written as *y* =. If this method is used, this mark can be implied by the next method mark.
dM1	For expanding and rearranging their equation (allow one sign or algebraic error) to form an unsimplified equation = 0 Dependent on previous method mark.
A1cso	For obtaining the given expression

Question number	Scheme	Marks
9 (a)	<table><tr><td>–0.5</td><td>0.5</td><td>1.5</td></tr><tr><td>0.70</td><td>0.92</td><td>2.11</td></tr></table>	B1 B1 [2]
(b)	Points plotted	B1ft
	Joined with a smooth curve	B1ft [2]
(c)	$x^3+4=x^2-8x+15$	M1
	$x^3+4=(x-5)(x-3)$ oe	M1
	$\dfrac{x^3+4}{5-x}=3-x$	A1
	$y=3-x$ drawn	M1
	$x=1.3/1.4$	A1 [5]
ALT	$\dfrac{x^3+4}{5-x}=ax+b \Leftrightarrow x^3+4=5ax-ax^2+5b-bx$	{M1}
	$\Leftrightarrow x^3+ax^2+\ b-5a\ \ x+4-5b\equiv x^3-x^2+8x-11$ $\Leftrightarrow a=-1 \quad b=3$	{M1} {A1}
	$y=3-x$ drawn	{M1}
	$x=1.3/1.4$	{A1} [5]
		Total 9 marks

Part	**Marks**	**Notes**
(a)	**First B1**	1 point correct
	Second B1	For all 3 points correct
(b)	**B1ft**	For points plotted within half a square ft their points
	B1ft	For points joined with a smooth curve ft their table
(c)	**M1**	For correctly rearranging the given equation to give $x^3+4=x^2-8x+15$
	M1	For factorising their quadratic, minimum attempt, see general guidance.
	A1	For $\dfrac{x^3+4}{5-x}=3-x$
	M1	For $y=3-x$ drawn
	A1	$x=1.3/1.4$
	ALT	
ALT	**M1**	For $\dfrac{x^3+4}{5-x}=ax+b$ and an attempt to multiply both sides by $(5-x)$, allow two errors only (sign or algebraic).
	M1	For correctly stating $x^3+ax^2+\ b-5a\ \ x+4-5b\equiv x^3-x^2+8x-11$
	A1	For $a=-1 \quad b=3$
	M1	For $y=3-x$ drawn
	A1	$x=1.3/1.4$


4
3.5
3
2.5
2
1.5
1
0.5
-0.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

Question number	Scheme	Marks
10 (a)	$\text{f}\left(\frac{3}{4}\right)=16\left(\frac{3}{4}\right)^3+11\left(\frac{3}{4}\right)-15$	M1
	$\left(\frac{27}{4}+\frac{33}{4}-15\right)=0$ *	A1cso [2]
(b)(i)	$\left[\sin 2\theta=\sin(\theta+\theta)\right]=\sin\theta\cos\theta+\sin\theta\cos\theta$	M1
	$(\sin 2\theta =)2\sin\theta\cos\theta$ *	A1cso [2]
(b)(ii)	$\left[\cos 2\theta=\cos(\theta+\theta)=\right]\cos\theta\cos\theta-\sin\theta\sin\theta=\cos^2\theta-\sin^2\theta$	M1
	$=\cos^2\theta-(1-\cos^2\theta)=2\cos^2\theta-1$ *	M1 A1cso [3]
(c)	$27\cos\theta(2\cos^2\theta-1)+19\sin\theta(2\sin\theta\cos\theta)-15(=0)$	M1
	$54\cos^3\theta-27\cos\theta+38(1-\cos^2\theta)\cos\theta-15(=0)$	M1
	$54\cos^3\theta-27\cos\theta+38\cos\theta-38\cos^3\theta-15(=0)$ $\left(16\cos^3\theta+11\cos\theta-15=0\right)$ oe	A1
	(When $x=\cos\theta$) $16x^3+11x-15=0$ *	A1cso [4]
(d)	$\left(16x^3+11x-15=0\Rightarrow(4x-3)(4x^2+3x+5)=0\right)\rightarrow 4x^2+3x+5$	M1
	$b^2-4ac=9-80<0$ (So no real roots)	M1 A1
	(Only solution is) $4x-3=0$ or $4\cos\theta-3=0$ so $\cos\theta=\frac{3}{4}$ *	A1cso [4]
		Total 15 marks

Part	Marks	Notes
(a)	**M1**	For $\text{f}\left(\frac{3}{4}\right)=16\left(\frac{3}{4}\right)^3+11\left(\frac{3}{4}\right)-15$
	A1*cso	For a fully correct solution, with = 0 stated, with no errors seen
(b)(i)	**M1**	For sight of $\sin\theta\cos\theta+\sin\theta\cos\theta$
	A1*cso	For obtaining the given expression, with no errors seen
(b)(ii)	**M1**	For $\cos\theta\cos\theta-\sin\theta\sin\theta=\cos^2\theta-\sin^2\theta$
	M1	For correctly using $\sin^2\theta=1-\cos^2\theta$
	A1*cso	For obtaining the given expression with no errors seen
(c)	**M1**	For correctly substituting $\sin 2\theta=2\sin\theta\cos\theta$ and $\cos 2\theta=2\cos^2\theta-1$
	M1	For expanding the first set of brackets (allow one error only in expansion) and correct use of $\sin^2\theta=1-\cos^2\theta$,
	A1	$54\cos^3\theta-27\cos\theta+38\cos\theta-38\cos^3\theta-15(=0)$ oe all brackets expanded
	A1*cso	For obtaining the given equation
(d)	**M1**	For any valid, complete method to find the quadratic factor which must be of the form $4x^2+ax\pm 5$
	M1	For use of the discriminant from their quadratic factor
	A1	For $b^2-4ac=9-80<0$

Question number	Scheme	Marks
11 (a)(i)	$(a=)-8$	B1 [1]
(a)(ii)	$e^{2x}=9\Rightarrow 2x=\ln 9$	M1
	$x=\frac{1}{2}\ln 9=\ln 3$ *	A1cso [2]
ALT	$e^{2x}=9\Rightarrow\left[\left(e^x\right)^2=3^2\right]\Rightarrow e^x=3$	M1
	$x=\ln 3$ *	A1cso
(b)	$\pi\int_0^{\ln 3}\left(e^{2x}-9\right)^2\,dx$	M1
	$\pi\int_0^{\ln 3}\left(e^{4x}-18e^{2x}+81\right)dx$	A1
	$(\pi)\left[\frac{1}{4}e^{4x}-9e^{2x}+81x\right]_{(0)}^{(\text{"}\ln 3\text{"})}$ oe	M1 A1ft
	$(\pi)\left[\left(\frac{81}{4}-81+81\ln 3\right)-\left(\frac{1}{4}-9\right)\right]$	M1
	$\pi\left(81\ln 3-52\right)$	A1 [6]
		Total 9 marks
	A1*cso	For a fully correct solution.

Part	Mark	Notes
(a)(i)	**B1**	For $(a=)-8$
(a)(ii)	**M1**	For substituting y = 0 and rearranging to $2x=\ln 9$ or $x=\frac{1}{2}\ln 9$
	A1*cso	For a fully correct solution with no errors – must see $x=\frac{1}{2}\ln 9$ *or* $2x=2\ln 3$
ALT	**M1**	For substituting y = 0 and manipulating to give $e^x=3$
	A1*cso	For a fully correct solution with no errors
(b)	**M1**	For $\pi\int_0^{\ln 3}\left(e^{2x}-9\right)^2\,dx$
	A1	For $\pi\int_0^{\ln 3}\left(e^{4x}-18e^{2x}+81\right)dx$ oe. If π seen for first M1, can be omitted.
	M1	For minimally acceptable attempt to integrate two terms of their expression (at least one term integrated correctly). π and limits can be omitted.
	A1ft	For $\frac{1}{4}e^{4x}-9e^{2x}+81x$ Ft their expression of the form $e^{4x}+ce^{2x}+d$ π and limits can be omitted
	M1	For correct substitution of 0 and their *b* into a changed expression. The substitution must be fully shown for this mark. π can be omitted. This mark can be implied by a correct final answer.
	A1	For obtaining the correct expression

π must only be present for the final A1 mark and the first M1 mark (can be implied by first or final A1)

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