| <b>Question</b> number | Scheme  | Marks       |  |
|------------------------|---|-------------|--|
| 8 a                    | Height of the waste paper basket = $\sqrt{(5x)^2 - (3x)^2} = 4x$ (cm)                                   | M1          |  |
|                        | $V = \frac{1}{2}(2x+8x) \times 4x \times h = 20x^2h = 2250 \Rightarrow h = \frac{2250}{20x^2}$ oe       | M1          |  |
|                        | $S = 2 \times 20x^2 + 2xh + 2(5xh)$   | M1          |  |
|                        | $S = 40x^2 + 12x \left(\frac{2250}{20x^2}\right) \text{ oe}$  | M1          |  |
|                        | $S = 40x^{2} + \frac{1350}{x} *$ $\frac{dS}{dx} = 80x - \frac{1350}{x^{2}} \text{ oe}$                  | A1 cso (5)  |  |
| b                      | $\frac{\mathrm{d}S}{\mathrm{d}x} = 80x - \frac{1350}{x^2} \text{ oe}$                                   | M1          |  |
|                        | $\frac{dS}{dx} = 80x - \frac{1350}{x^2} = 0$ so $x^3 = \frac{135}{8} \Rightarrow x =$                   | M1          |  |
|                        | x = 2.56 awrt   | A1          |  |
|                        | $\frac{d^2S}{dx^2} = 80 + \frac{2700}{x^3} > 0 \text{ for all positive values of } x.: \text{ minimum}$ | M1 A1ft (5) |  |
| c                      | When $x = 2.56$ $S = 40(2.56)^2 + \frac{1350}{2.56} = 789$ awrt   | M1 A1 (2)   |  |
|                        | Total 12 marks  |             |  |

| Part | Mark      | Notes   |
|------|-----------|---|
| (a)  |           | For finding the height of the waste-paper basket using Pythagoras theorem.  |
|      | M1        | $\sqrt{(5x)^2 - (3x)^2} = 4x \text{ (cm)}$  |
|      | M1        | For finding the volume which must come from;  |
|      |           | W   |
|      |           | $V = $ correct area of trapezium (using their height) $\times h$  |
|      |           | $V = \frac{1}{2}(2x + 8x) \times '4x' \times h = 2250 \Longrightarrow ('20' x^2 h = 2250)$  |
|      | 1.22      | and for obtaining an expression for the height h: $h = \frac{2250}{20'x^2}$ or $xh = \frac{2250}{20'x}$   |
|      |           | Please check their algebra carefully, as some may even substitute $hx^2 =$<br><b>NB:</b> This is an A mark in Epen.   |
|      | M1        | For writing the surface area in terms of 2 unknowns $[x \text{ and } h]$ which need not be simplified. This must be <b>correct</b> using their expression in terms of $x$ for the height of |
|      |           | the prism. $(8x + 2x) \times (4x)$  |
|      |           | e.g. $S = 2 \times \frac{(8x+2x) \times '4x'}{2} + 2xh + 2(5xh) = [40x^2 + 12xh]$   |
|      | M1        | For eliminating $h$ from their expression for $S$   |
|      |           | S must be in the form $Ax^2 + Bxh$ and h must be of the form $\frac{C}{r^2}$ where A, B and C are   |
|      |           | constants.  |
|      | A1<br>cso | For the given result exactly as written. $S = 40x^2 + \frac{1350}{x}$   |
| (b)  | M1        | For an acceptable attempt to differentiate the <b>given</b> expression for <i>S</i> . See General Guidance.   |
|      | M1        | For setting their $\frac{dS}{dx} = 0$ and attempting to find a value for x  |
|      |           | The minimum acceptable expression is $\frac{dS}{dx} = Px \pm \frac{Q}{x^2}$ where P and Q are constants.  |
|      | A1        | For awrt $x = 2.56$   |
|      | M1        | For differentiating their $\frac{dS}{dx}$ , which must be as a minimum $\frac{dS}{dx} = Px \pm \frac{Q}{x^2}$ to find the   |
|      |           | second derivative to achieve as a minimum $\frac{d^2S}{dx^2} = \pm M \pm \frac{N}{x^3}$   |
|      |           | Concludes that as $\frac{d^2S}{dx^2}$ will always be positive, [either by substitution, or by inference]  |
|      |           | so the value of x obtained will be a minimum.   |
|      | A1ft      | $\left[ \frac{d^2S}{dx^2} = 80 + \frac{2700}{2.56^3} = 240.9 \right]$ with a conclusion.  |
|      |           | <b>NOTE:</b> The ft only applies to their value of x. Do not ft an incorrect $\frac{d^2S}{dx^2}$  |
| (c)  | M1        | Substitutes their value of x, obtained using a correct method) into the <b>given</b> expression for S [provided it is a positive value of NOT allow positive values of x]                   |
|      | A1        | for S [provided it is a positive value, do <b>NOT</b> allow negative values of x]  For awrt 789   |
| L    | 1 4 4     | 2 0 2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0   |