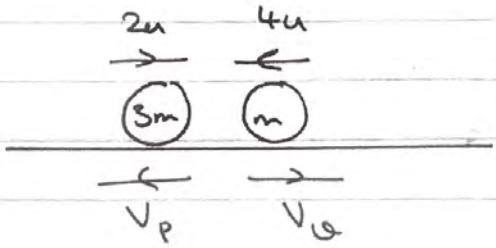
M1 June 2018 (MA)

Q1(a)



$\xrightarrow{}$ C.L.M : $3m(2u) - 4mu = mV_Q - 3mV_P$

$$2u = V_Q - 3V_P \quad \sim (1)$$

$I = m(v-u)$

for P $\rightarrow$ $\frac{21mu}{4} = 3m(V_P - -2u)$

$\Rightarrow \frac{21\cancel{m}u}{4} = 3\cancel{m}V_P + 6\cancel{m}u$

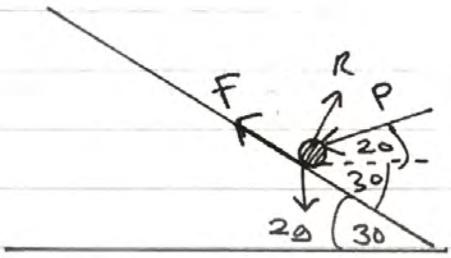
$\Rightarrow -\frac{3u}{4} = 3V_P \quad \therefore V_P = -\frac{u}{4}$

so speed $= \boxed{\frac{u}{4}}$

b) (1) : $V_Q = 2u + 3V_P = 2u + 3\left(-\frac{u}{4}\right)$

$\therefore \boxed{V_Q = \frac{5u}{4} = \text{speed}}$

Q2)



note that we want the least possible value of P. This least value occurs when friction acts up the plane.

R(Parallel to plane) : $P\cos 50 + F = 2g\sin 30$ — ①

R(Perp. to plane) : $R = 2g\cos 30 + P\sin 50$

$F = \frac{1}{4}R$

① : $P\cos 50 + \frac{1}{4}R = 2g\sin 30$

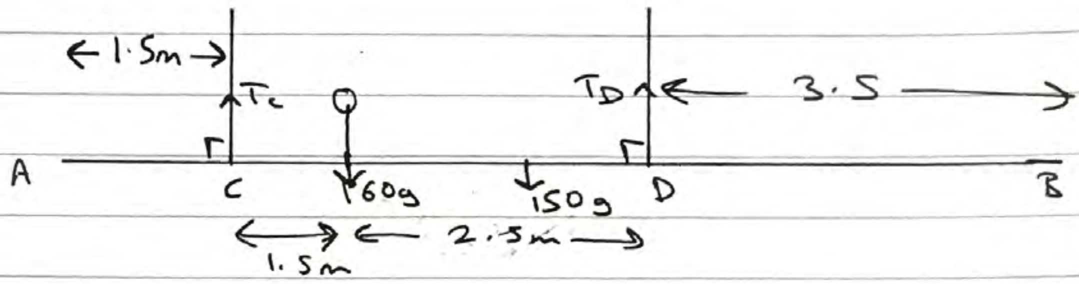
×4 : $4P\cos 50 + (R) = 4g$

sub ② : $4P\cos 50 + (2g\cos 30) + (P\sin 50) = 4g$

$P(4\cos 50 + \sin 50) = 4g - 2g\cos 30$

$P = \dfrac{4g - 2g\cos 30}{4\cos 50 + \sin 50} = \boxed{6.66\text{N}}$

Q3a)

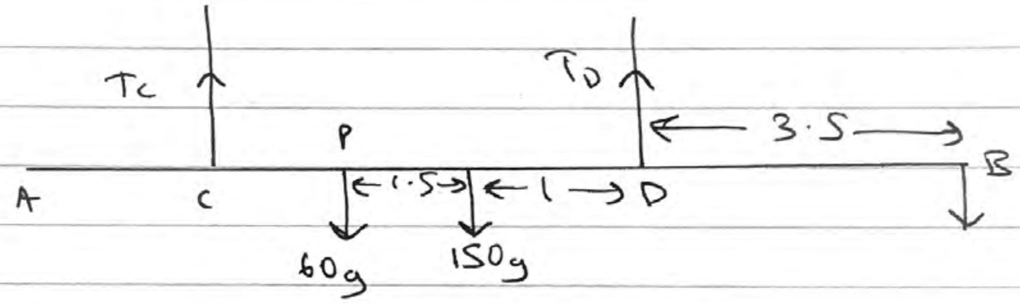


$\underline{M(D)}$: $T_C(4) = 150g(1) + 60g(2.5)$

$T_C = \frac{150g + 60g(2.5)}{4} = 75g$

$= \boxed{735N}$

b)



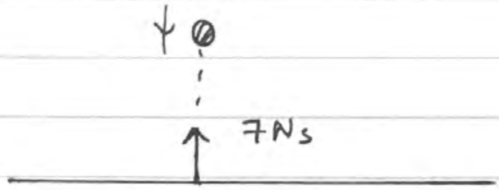
mass of gymnast at B is largest possible for beam to remain horizontal $\rightarrow T_C = 0$

$\underline{M(B)}$: $150g(4.5) + 60g(6) = T_D(3.5)$

$\therefore T_D = \frac{150g(4.5) + 60g(6)}{3.5}$

$= \boxed{2900N}$

(Q4a)



$\downarrow$
$s = 2.5$
$u = u$
$v = v_1$
$a = g$
$t =$

$v_1^2 = u^2 + 2as$

$v_1^2 = u^2 + 5g$

$v_1 = \sqrt{u^2 + 5g}$

$I = m(v - u)$

$7 = 0.2(v - -v_1)$

$7 = 0.2\left(10 + \sqrt{u^2 + 5g}\right)$

$35 = 10 + \sqrt{u^2 + 5g}$

$25 = \sqrt{u^2 + 5g}$

$u^2 + 5g = 625$

$u^2 = 576 \quad \therefore \boxed{u = 24}$

b) $\uparrow$
$s = 1$
$u = 10$
$v =$
$a = -g$
$t = t$

$s = ut + \frac{1}{2}at^2$

$1 = 10t - 4.9t^2$

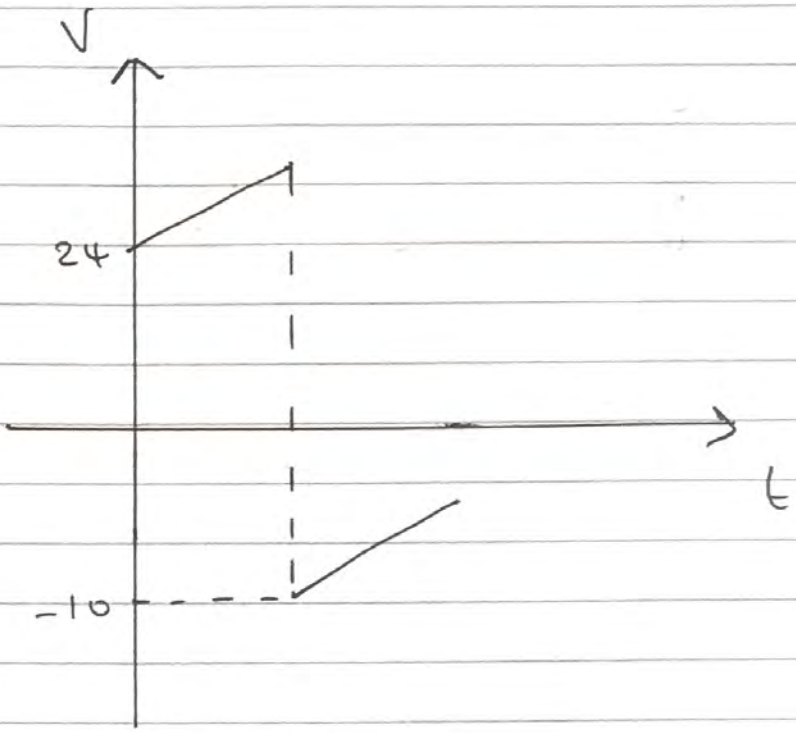
$4.9t^2 - 10t + 1 = 0$

By Quadratic Formula : $t = 1.94, \; 0.105$

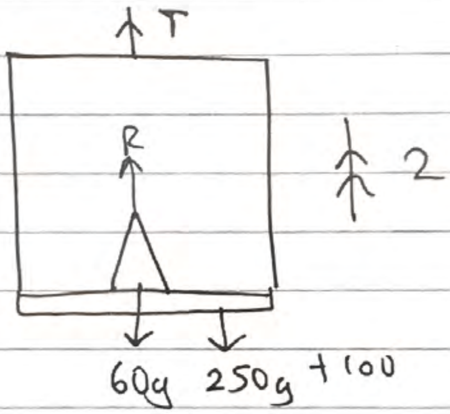
reject!

$t = 0.11s$

c)



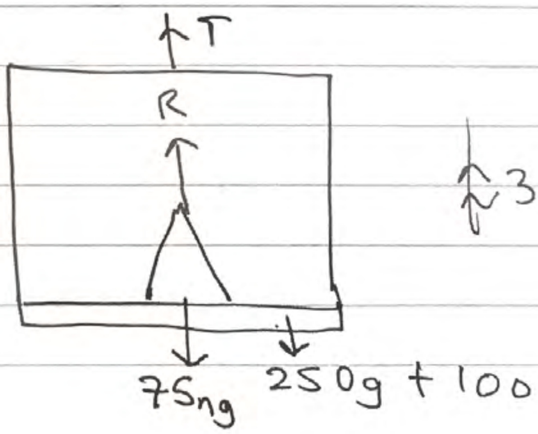
Q5a)



N2L $\uparrow +$ (woman) : $R - 60g = 60(2)$

$R = 60g + 120 = \boxed{708N}$

b)



$\uparrow^{+}$ N2L (system) : $T - 250g - 100 = 75ng = (250 + 75n)a$

$T = 10000$ for max no. of occupants to be carried...

$$10000 - 250g - 100 = 75ng + 250(3) + 75n(3)$$

$$6700 = n(75g + 3(75))$$

$$n = \frac{6700}{75g + 3(75)} = 6.979\ldots$$

this isn't quite 7!

so max $\boxed{n = 6}$

Q6a) $F_1 + F_2 = R = \begin{pmatrix} 4 \\ -6 \end{pmatrix} + \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} p+4 \\ q-6 \end{pmatrix}$

we are told R acts in the direction $-2\underline{i} - \underline{j}$

so... $2(q-6) = p + 4$

$\underline{i}$ component is double the $\underline{j}$.

$\Rightarrow 2q - 12 = p + 4$

$\Rightarrow p - 2q = -16$

b) $q = 3$: $p = 2q - 16 = 2(3) - 16 = -10$ //

so $R = \begin{pmatrix} -6 \\ -3 \end{pmatrix} = \Sigma F$ //

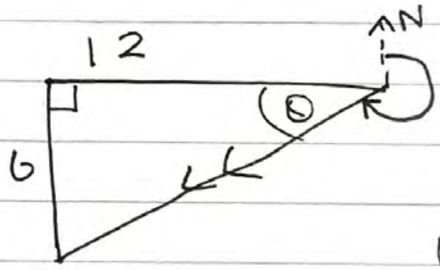
$\sum F = ma$

$|R| = \sqrt{6^2+3^2} = 3\sqrt{5}$

$R = ma$; $3\sqrt{5} = 0.5a$

$\therefore a = 6\sqrt{5} = \boxed{13.4\,ms^{-2}}$

c)



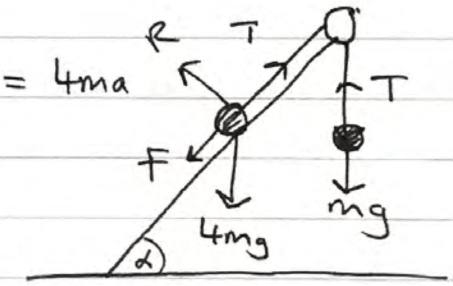
$\tan\theta = \frac{6}{12} = \frac{1}{2}$

$\theta = \tan^{-1}\left(\frac{1}{2}\right) = 26.6°$

Bearing: $270 - 26.6 = \boxed{243°}$

Q7a) Inextensible string

b) N2L(P) ↙+ : $4mg\sin\alpha - T - F = 4ma$ ~①



N2L(Q) ↑+ : $T - mg = ma$ ~②

c) ① + ② : $4mg\sin\alpha - T + T - F - mg = 5ma$

$\sin\alpha = \frac{3}{5}$ $\therefore \frac{12mg}{5} - F - mg = 5ma$

$F = \frac{1}{4}R$, $R = 4mg\cos\alpha$ //

$\therefore \frac{7}{5}\cancel{m}g - \frac{1}{4}(4\cancel{m}g)\left(\frac{4}{5}\right) = 5\cancel{m}a$

$\frac{3}{5}g = 5a \quad \therefore \quad a = \boxed{\frac{3g}{25}} \ ms^{-2}$

d) (for P)

$s = h$
$u = 0$
$v = v$
$a = \frac{3g}{25}$
$t =$

$v^2 = u^2 + 2as$

$v^2 = 0^2 + \frac{6gh}{25}$ //

(for Q)

$s = s$
$u = \sqrt{\frac{6gh}{25}}$
$v = 0$
$a = -g$
$t =$

$v^2 = u^2 + 2as$

$0^2 = \frac{6gh}{25} - 2gs$

$s = \frac{3h}{25}$ //

(Q is under influence of g only once P hits the ground.)

so after P hits the ground (and string is not taut), Q travels an extra distance of $\frac{3h}{25}$.

Q does not reach the pulley so d > total distance travelled

$\therefore d > \frac{3h}{25} + h$

$$\boxed{d > \frac{28h}{25}}$$