

# Graph Algorithms (Flow)

Tuesday, April 16, 2024      6:36 in the evening

Maximum Flow

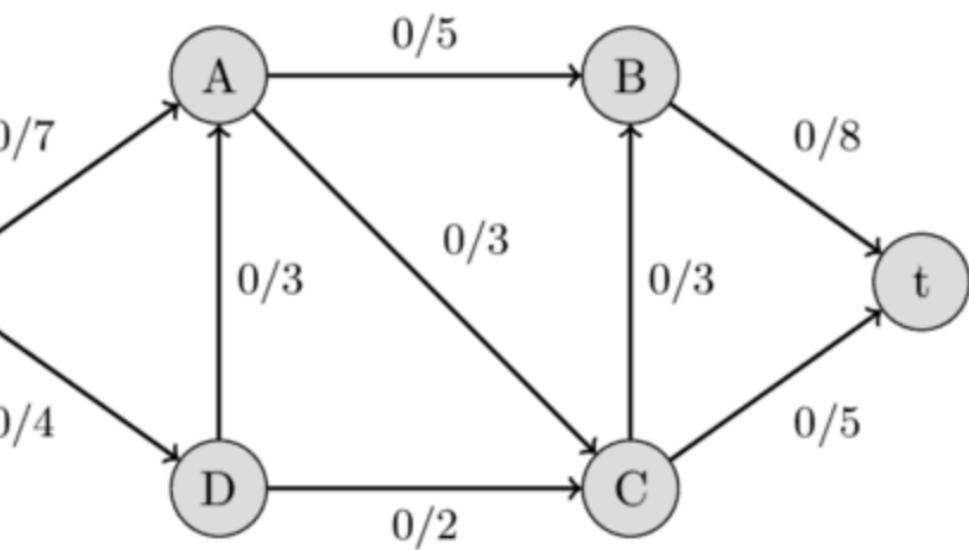
$$G = (V, E)$$

$$(s, t)$$

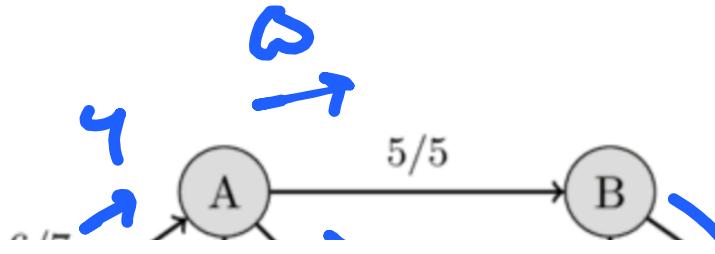
$$w : E \rightarrow \mathbb{N}$$

$$\forall e \in E : c(e) \leq w(e)$$

$$\forall v \in V \setminus \{s, t\} \quad \sum_{e \in \text{out}(v)} c(e) =$$



$\sum' c(e)$   
 $c \in \text{inv}(V)$



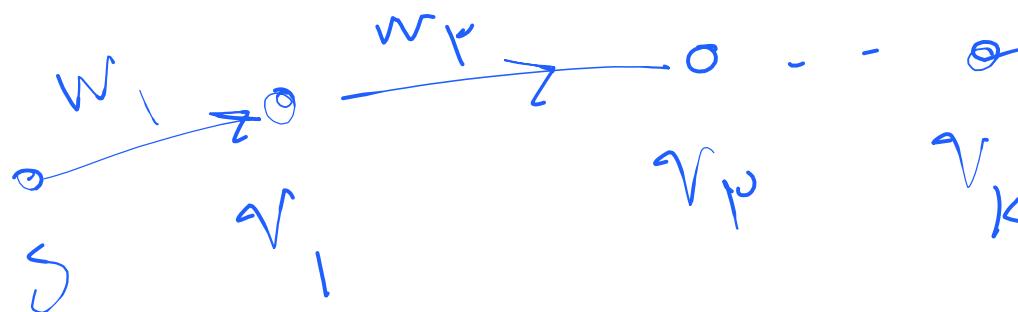


$$\text{maximize} \sum_{e \in \text{out}(s)} c(e) = \sum_{e \in}$$

Augmenting Path

دیگر قابل بهبود نیست و تا  $s$

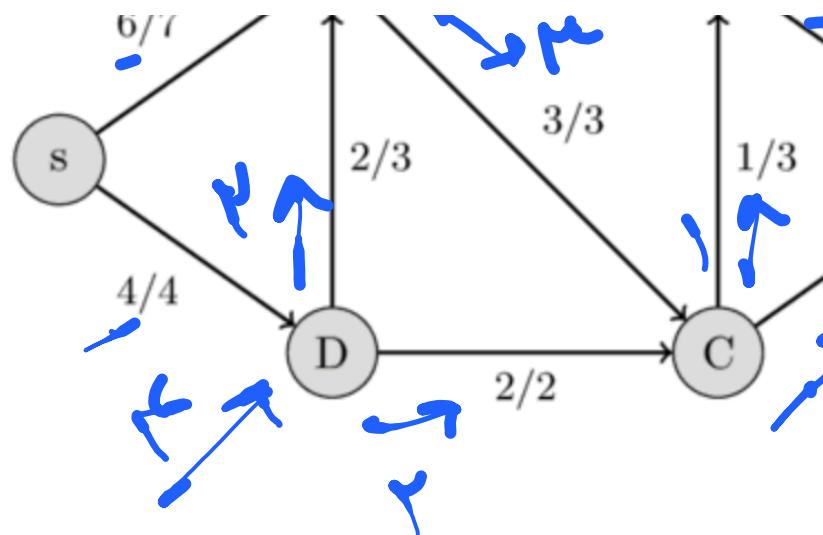
$\forall e \in E \Rightarrow \text{add } e'$   
 $(v_i, u)$



$\leftarrow \curvearrowright \curvearrowleft \curvearrowright \curvearrowleft \curvearrowright$

$c(e)$

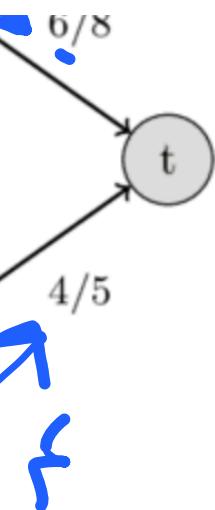
$\text{in}(t)$



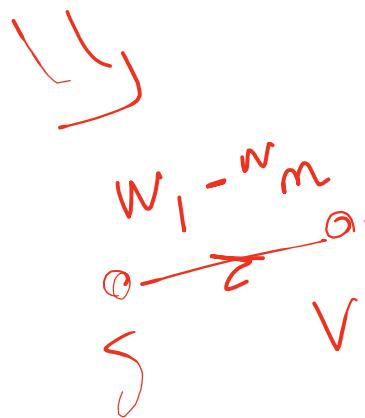
$\exists t \text{ such that}$

$$= (u, v) \quad w(e') = 0$$

$w_{k+1} \rightarrow 0$   
+



$$w_m = \min_i w_i$$



Flow = 0

While (there exists augmenting path from S to T) {

    Let S, v1, v2, ..., vk, T be an augmenting path

    wm = min(w1, w2, ..., wk+1)

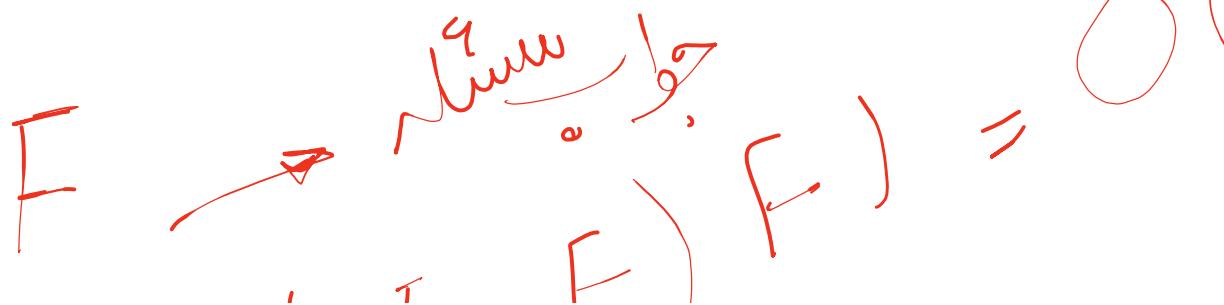
    flow += wm

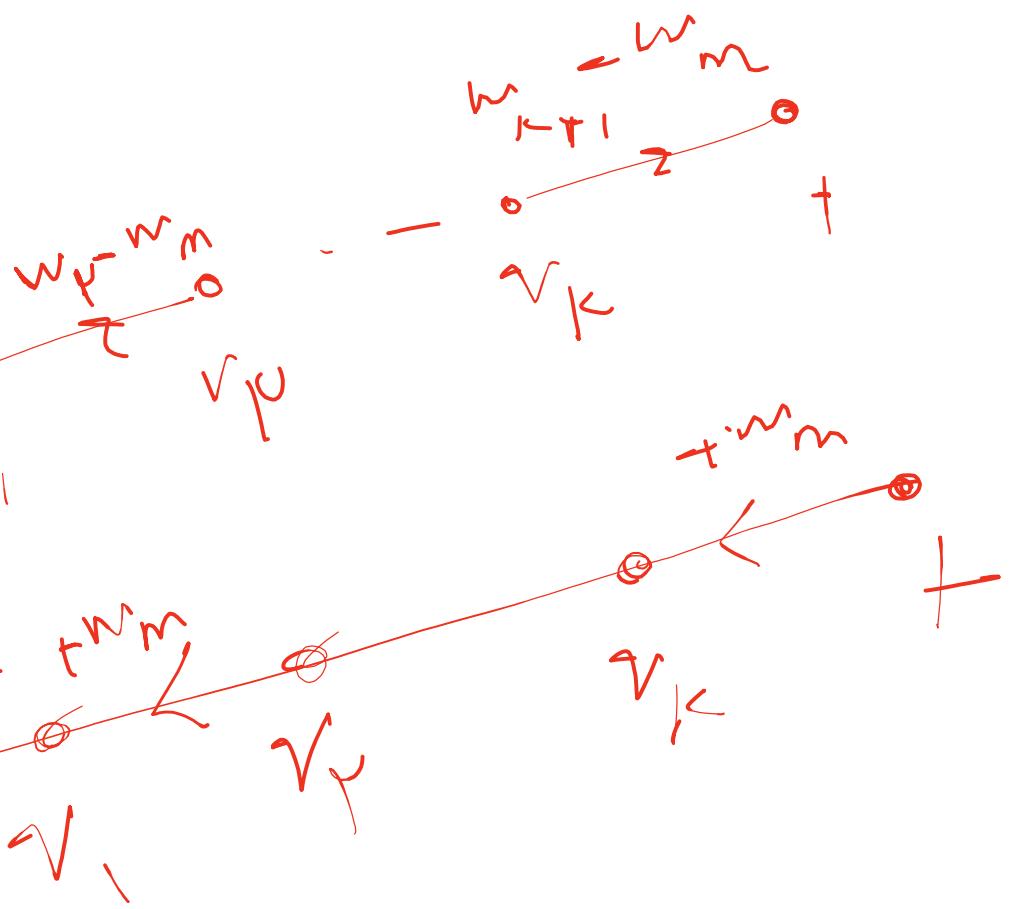
    w1 -= wm, w2 -= wm, ..., wk+1 -= wm

    w'1 += wm, w'2 += wm, ..., w'k+1 += wm

}

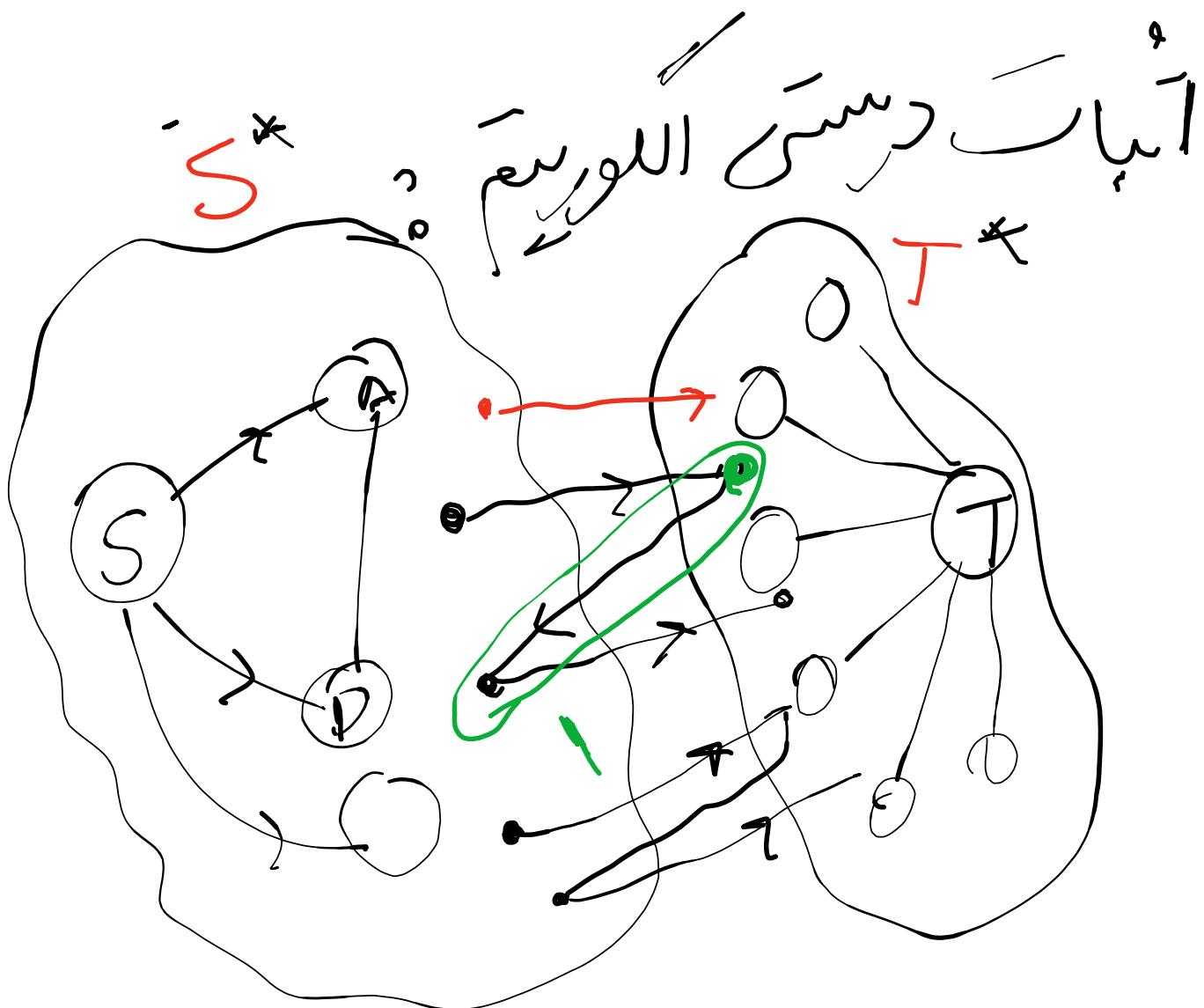
Return flow



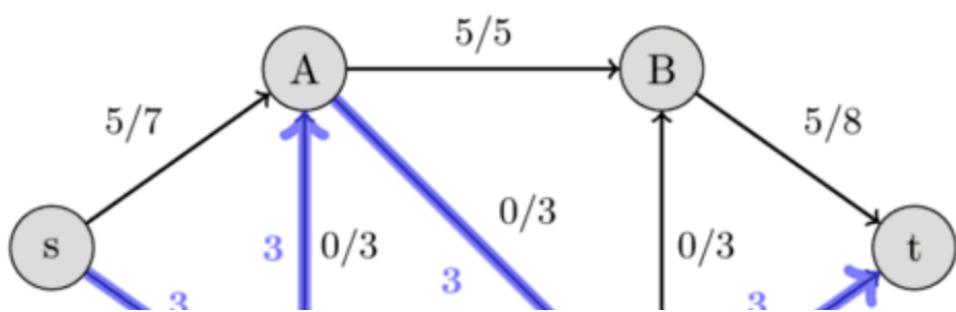
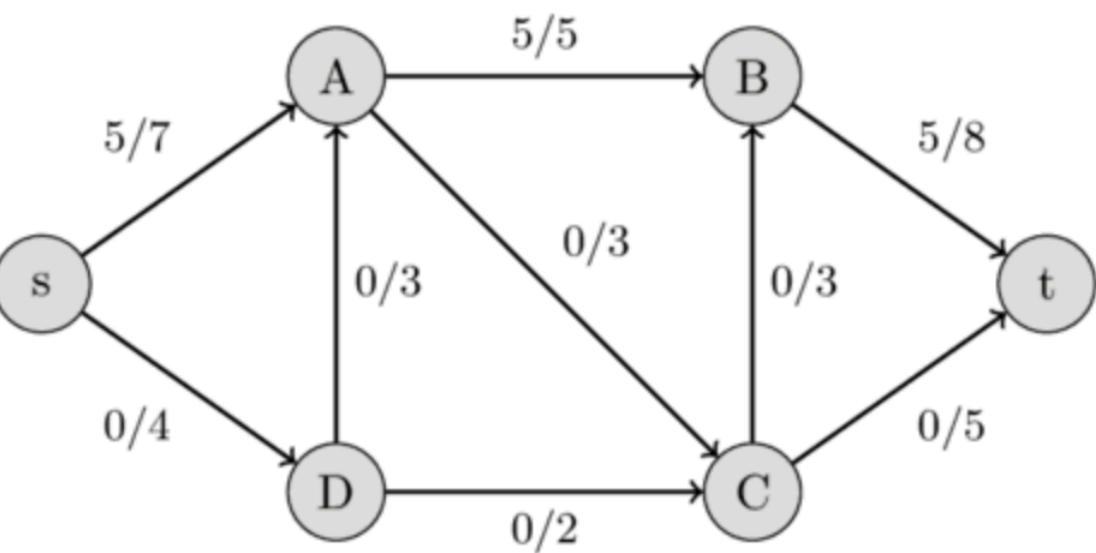
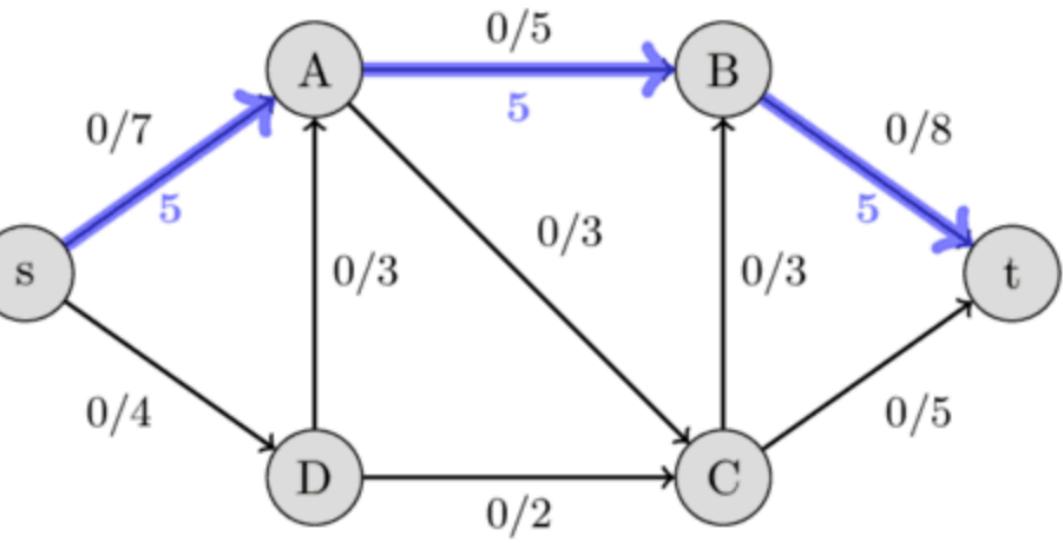


(E F)

O((V + L))



$$cut = \sum_{e(v,u) : v \in S, u \in T} w_e = Flow$$

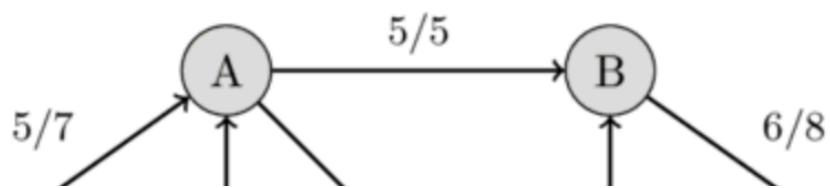
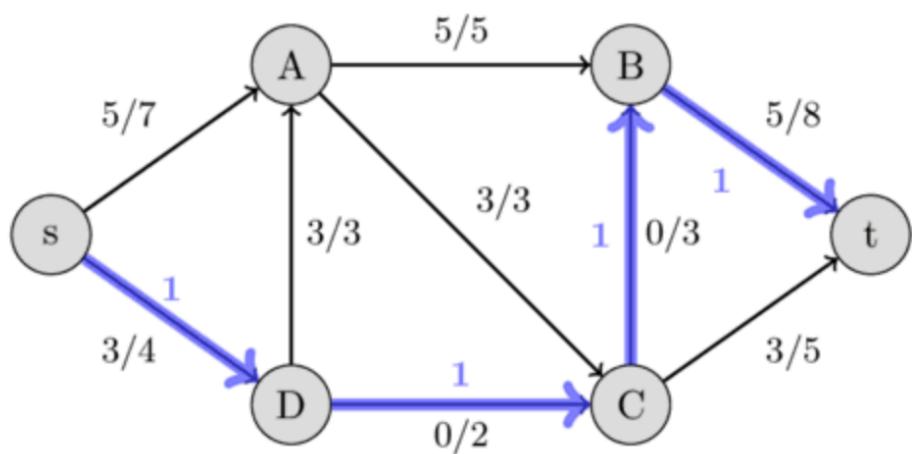
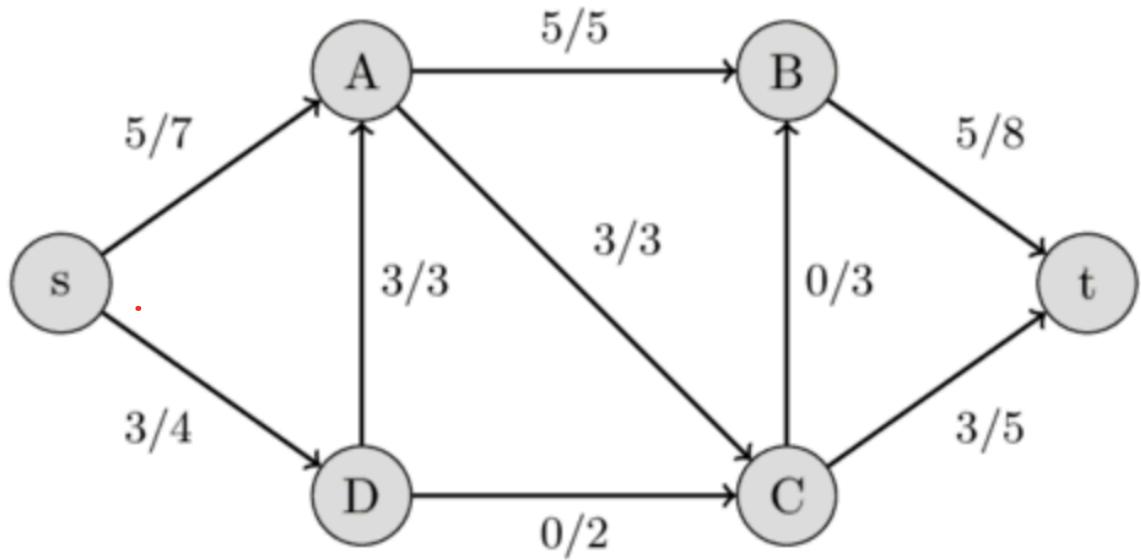
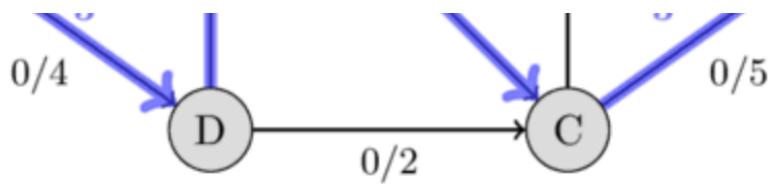


$\forall$  flow  $\leq \{$  we  
s.t.

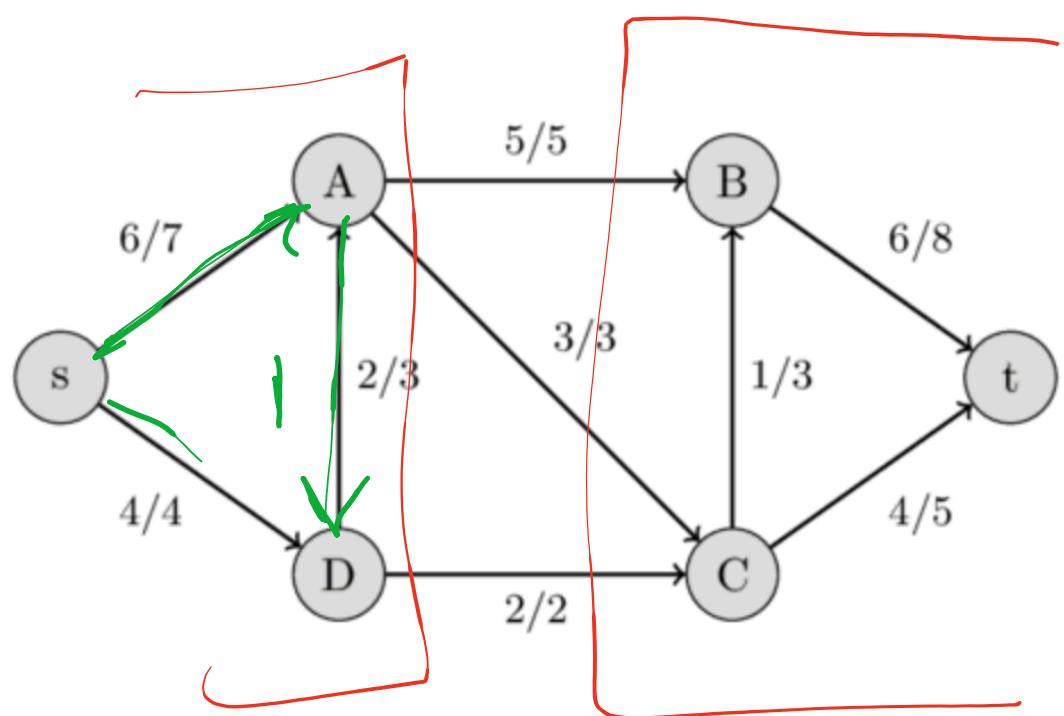
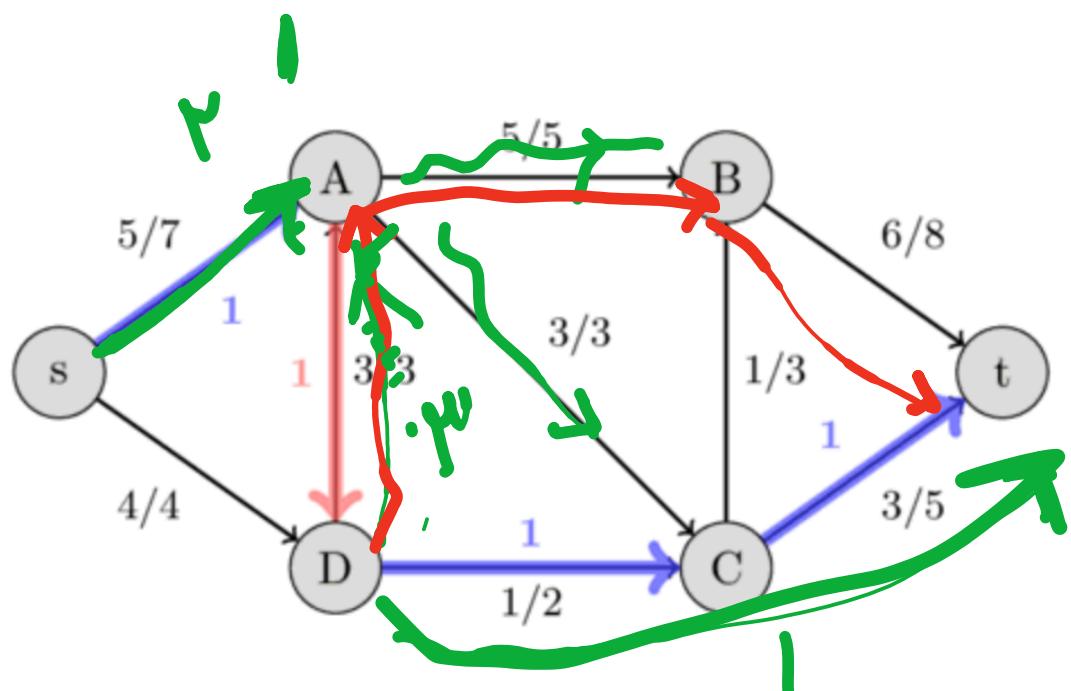
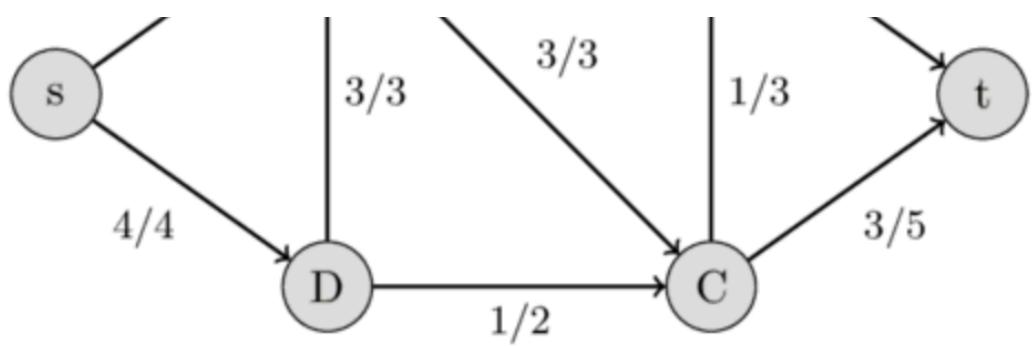
Flow = min cut

= max flow

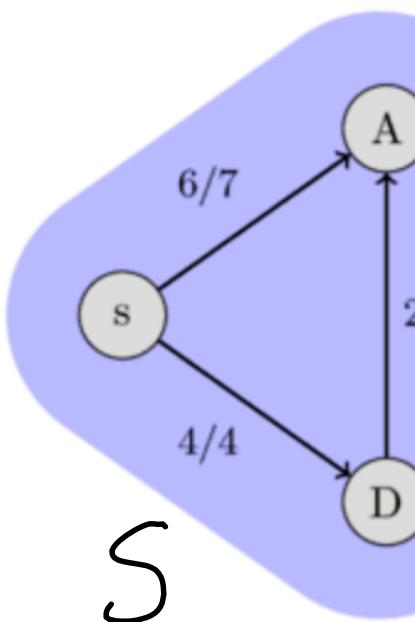
min cut  $\leq$  cut =  $F$   $\leq$  max flow  
min cut = Flow





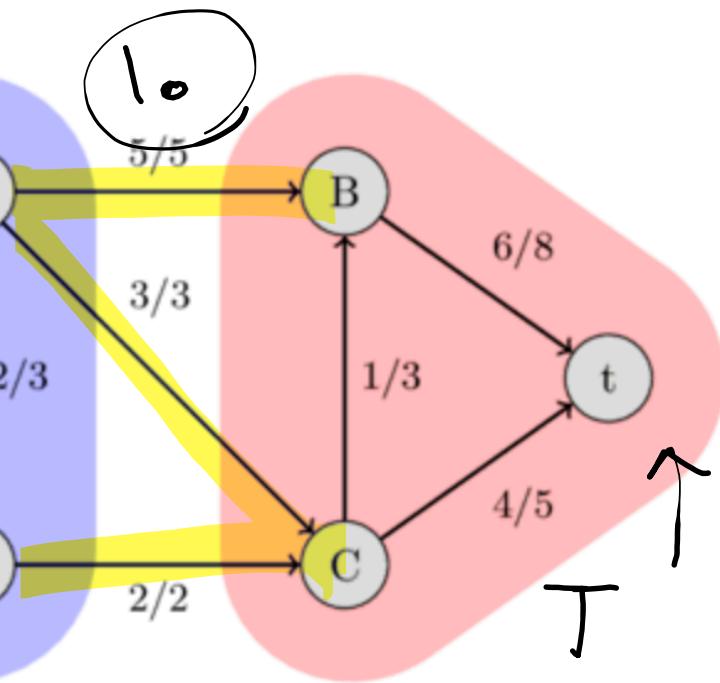


Maximum Flow = Minimum C.



$$\min_{S, T : s \in S, t \in T, S \cup T = V} \sum_{e \in E : (v, u) : v \in S, u \in T} w(e)$$

نیز ایجاد کردن



min s-t cut

$$S \cap T = \emptyset$$

: یکی

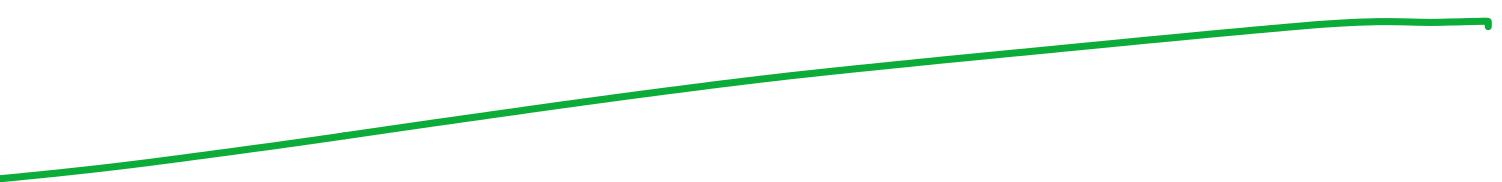
x

Edmonds Karp

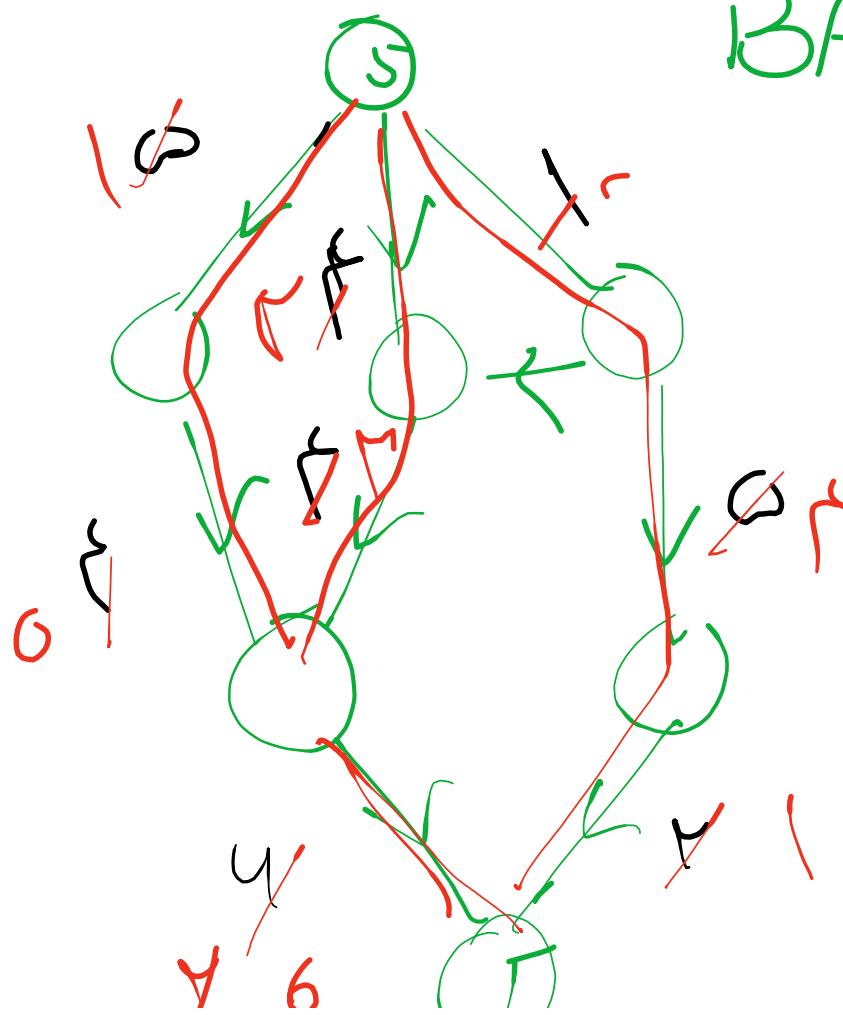
Augmenting path

$O(EV)$

$\downarrow \text{Aug}^{ij}$

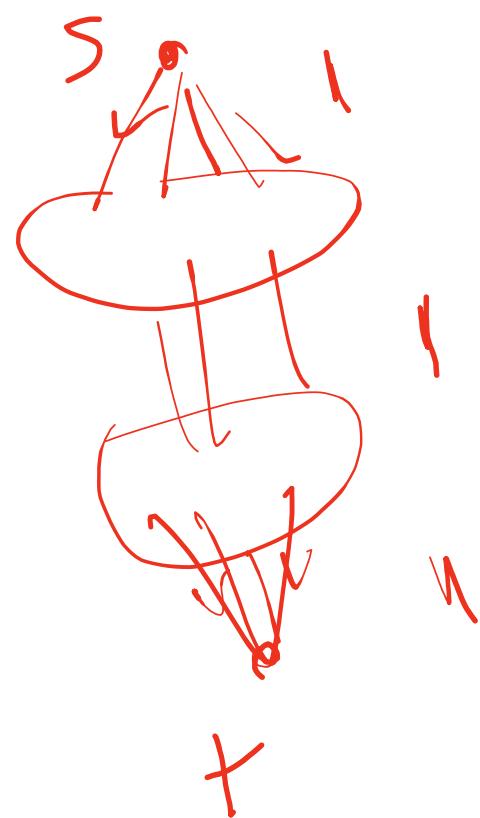


BFS



# Dinic's Algorithm

---



Λ ~ ☺



maximum matching