KUET_Effervescent Team Notebook

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1 Data Structure

1.1 Dsu With Rollback [89 lines] - bc2588

```
struct dsu_save {
  int v, rnkv, u, rnku;
  dsu_save() {}
  dsu_save(int _v, int _rnkv, int _u, int _rnku)
    : v(_v), rnkv(_rnkv), u(_u), rnku(_rnku) { }
};
struct dsu_with_rollbacks {
  vector<int> p, rnk;
  int comps;
  stack<dsu_save> op;
  dsu_with_rollbacks() {}
  dsu_with_rollbacks(int n) {
    p.resize(n);
    rnk.resize(n);
    for (int i = 0; i < n; i++) {
      p[i] = i;
      rnk[i] = 0;
    }
    comps = n;
  }
  int find_set(int v) { return (v == p[v]) ? v :
      find_set(p[v]); }
  bool unite(int v, int u) {
    v = find_set(v);
    u = find_set(u);
    if (v == u) return false;
    comps--;
    if (rnk[v] > rnk[u]) swap(v, u);
    op.push(dsu_save(v, rnk[v], u, rnk[u]));
    p[v] = u;
    if (rnk[u] == rnk[v]) rnk[u]++;
    return true;
  }
  void rollback() {
    if (op.empty()) return;
    dsu_save x = op.top();
    op.pop();
    comps++;
    p[x.v] = x.v;
    rnk[x.v] = x.rnkv;
    p[x.u] = x.u;
    rnk[x.u] = x.rnku;
  }
};
struct query {
  int v, u;
  bool united;
  query(int _v, int _u) : v(_v), u(_u) {}
};
struct QueryTree {
  vector<vector<query>> t;
  dsu_with_rollbacks dsu;
  int T;
  QueryTree() {}
  QueryTree(int _T, int n) : T(_T) {
    dsu = dsu_with_rollbacks(n);
    t.resize(4 * T + 4);
  }
  void add_to_tree(int v, int l, int r, int ul, int ur,
      query& q) {
    if (ul > ur) return;
    if (l == ul && r == ur) {
      t[v].push_back(q);
      return;
    }
    int mid = (l + r) / 2;
    add_to_tree(2 * v, l, mid, ul, min(ur, mid), q);
    add_to_tree(2 * v + 1, mid + 1, r, max(ul, mid +
        1), ur, q);
  }
  void add_query(query q, int l, int r) {
      add_to_tree(1, 0, T - 1, l, r, q); }
  void dfs(int v, int l, int r, vector<int>& ans) {
    for (query& q : t[v]) {
      q.united = dsu.unite(q.v, q.u);
    }
    if (l == r)
      ans[l] = dsu.comps;
    else {
      int mid = (l + r) / 2;
      dfs(2 * v, l, mid, ans);
      dfs(2 * v + 1, mid + 1, r, ans);
    }
    for (query q : t[v]) {
      if (q.united) dsu.rollback();
    }
  }
  vector<int> solve() {
    vector<int> ans(T);
    dfs(1, 0, T - 1, ans);
    return ans;
```

```
  }
};
```

1.2 MO with Update [43 lines] - 2fbf87

```
//1 indexed
//Complexity:O(S x Q + Q x N^2/S^2)
//S = (2*n^2)^(1/3)
const int block_size = 2720;  // 4310 for 2e5
const int mx = 1e5 + 5;
struct Query {
  int L, R, T, id;
  Query() {}
  Query(int _L, int _R, int _T, int _id) : L(_L),
      R(_R), T(_T), id(_id) {}
  bool operator<(const Query &x) const {
    if (L / block_size == x.L / block_size) {
      if (R / block_size == x.R / block_size) return T <
          x.T;
      return R / block_size < x.R / block_size;
    }
    return L / block_size < x.L / block_size;
  }
} Q[mx];
struct Update {
  int pos;
  int old, cur;
  Update(){};
  Update(int _p, int _o, int _c) : pos(_p), old(_o),
      cur(_c){};
} U[mx];
int ans[mx];
inline void add(int id) {}
inline void remove(int id) {}
inline void update(int id, int L, int R) {}
inline void undo(int id, int L, int R) {}
inline int get() {}
void MO(int nq, int nu) {
  sort(Q + 1, Q + nq + 1);
  int L = 1, R = 0, T = nu;
  for (int i = 1; i <= nq; i++) {
    Query q = Q[i];
    while (T < q.T) update(++T, L, R);
    while (T > q.T) undo(T--, L, R);
    while (L > q.L) add(--L);
    while (R < q.R) add(++R);
    while (L < q.L) remove(L++);
    while (R > q.R) remove(R--);
    ans[q.id] = get();
  }
}
```

1.3 MO [28 lines] - bed3e5

```
const int N = 2e5 + 5;
const int Q = 2e5 + 5;
const int SZ = sqrt(N) + 1;
struct qry {
  int l, r, id, blk;
  bool operator<(const qry& p) const {
    return blk == p.blk ? r < p.r : blk < p.blk;
  }
};
qry query[Q];
ll ans[Q];
void add(int id) {}
void remove(int id) {}
ll get() {}
int n, q;
void MO() {
  sort(query, query + q);
  int cur_l = 0, cur_r = -1;
  for (int i = 0; i < q; i++) {
    qry q = query[i];
    while (cur_l > q.l) add(--cur_l);
    while (cur_r < q.r) add(++cur_r);
    while (cur_l < q.l) remove(cur_l++);
    while (cur_r > q.r) remove(cur_r--);
    ans[q.id] = get();
  }
}
/* 0 indexed. */
```

1.4 Persistent Segment Tree [64 lines] - f58bc9

```
const int mxn = 4e5+5;
int root[mxn], leftchild[25*mxn], rightchild[25*mxn],
    value[25*mxn], a[mxn];
int now = 0, n, sz = 1;
int l, r;

int build(int L, int R){
  int node = ++now;
  if(L == R){
    //initialize
    //value[node] = a[L];
    return node;
  }
  int mid = (L+R)>>1;
  leftchild[node] = build(L, mid);
  rightchild[node] = build(mid+1, R);
  //combine
  //value[node] = value[leftchild[node]] +
      value[rightchild[node]];
  return node;
}

int update(int nownode, int L, int R, int ind, int val){
  int node = ++now;
  if(L == R){
    //value[node] = value[nownode]+val;
    //update value[node]
    return node;
  }
  int mid = (L+R)>>1;
  leftchild[node] = leftchild[nownode];
  rightchild[node] = rightchild[nownode];
  if(mid >= ind){//change condition as required
    leftchild[node] = update(leftchild[nownode], L,
        mid, ind, val);
  }
  else{
    rightchild[node] = update(rightchild[nownode],
        mid+1, R, ind, val);
  }
  //value[node] = value[leftchild[node]] +
      value[rightchild[node]];
  //combine value[node]
  return node;
}


int query(int nownode, int L, int R){
  if(l > R || r < L) return 0;

  if(L>=l && r >= R){
    return value[nownode];
  }
  int mid = (L+R)>>1;
  //change as required
  return query(leftchild[nownode], L, mid) +
      query(rightchild[nownode], mid+1, R);
}

void persistant(){
  root[0] = build(1, n);
  while(m--){
    if(ck == 2){
      cout << query(root[idx], 1, n) << "\n";
    }
    else{
        root[sz++] = update(root[idx], 1, n, ind,
            val);
    }
  }
}
```

1.5 SQRT Decomposition [96 lines] - a772d3

```
struct sqrtDecomposition {
  static const int sz = 320;//sz = sqrt(N);
  int numberofblocks;

  struct node {
    int L, R;
    bool islazy = false;
    ll lazyval=0;
    //extra data needed for different problems
    void ini(int l, int r) {
      for(int i=l; i<=r; i++) {
        //...initialize as need
      }
      L=l, R=r;
    }
    void semiupdate(int l, int r, ll val) {
      if(l>r) return;
      if(islazy){
        for(int i=L; i<=R; i++){
          //...distribute lazy to everyone
        }
        islazy = 0;
        lazyval = 0;
      }
      for(int i=l; i<=r; i++){
        //...do it manually
      }
    }
    void fullupdate(ll val){
      if(islazy){
        //...only update lazyval
      }
      else{
        for(int i=L; i<=R; i++){
          //...everyone are not equal, make them equal
        }
        islazy = 1;
        //update lazyval
```

```
    }
  }
  void update(int l, int r, ll val){
    if(l<=L && r>=R) fullupdate(val);
    else semiupdate(max(l, L), min(r, R), val);
  }
  ll semiquery(int l, int r){
    if(l>r) return 0;
    if(islazy){
      for(int i=L; i<=R; i++){
        //...distribute lazy to everyone
      }
      islazy = 0;
      lazyval = 0;
    }
    ll ret = 0;
    for(int i=l; i<=r; i++){
      //...take one by one
    }
    return ret;
  }
  ll fullquery(){
    //return stored value;
  }
  ll query(int l, int r){
    if(l<=L && r>=R) return fullquery();
    else return semiquery(max(l, L), min(r, R));
  }
};
vector<node> blocks;
void init(int n){
  numberofblocks = (n+sz-1)/sz;
  int curL = 1, curR = sz;
  blocks.resize(numberofblocks+5);
  for(int i=1; i<=numberofblocks; i++){
    curR = min(n, curR);
    blocks[i].ini(curL, curR);
    curL += sz;
    curR += sz;
  }
}
void update(int l, int r, ll val){
  int left = (l-1)/sz+1;
  int right = (r-1)/sz+1;
  for(int i=left; i<=right; i++){
    blocks[i].update(l, r, val);
  }
}
ll query(int l, int r){
  int left = (l-1)/sz+1;
  int right = (r-1)/sz+1;
  ll ret = 0;
  for(int i=left; i<=right; i++){
    ret += blocks[i].query(l, r);
  }
  return ret;
}
};
```

1.6 Segment Tree [73 lines] - c1fe4f

```
/*edit:data,combine,build check datatype*/
template<typename T>
struct SegmentTree {
#define lc (C << 1)
#define rc (C << 1 | 1)
#define M ((L+R)>>1)
  struct data {
    T sum;
    data() :sum(0) {};
  };
  vector<data>st;
  vector<bool>isLazy;
  vector<T>lazy;
  int N;
  SegmentTree(int _N) :N(_N) {
    st.resize(4 * N);
    isLazy.resize(4 * N);
    lazy.resize(4 * N);
  }
  void combine(data& cur, data& l, data& r) {
    cur.sum = l.sum + r.sum;
  }
  void push(int C, int L, int R) {
    if (!isLazy[C]) return;
    if (L != R) {
      isLazy[lc] = 1;
      isLazy[rc] = 1;
      lazy[lc] += lazy[C];
      lazy[rc] += lazy[C];
    }
    st[C].sum = (R - L + 1) * lazy[C];
    lazy[C] = 0;
    isLazy[C] = false;
  }
  void build(int C, int L, int R) {
    if (L == R) {
      st[C].sum = 0;
      return;
    }
    build(lc, L, M);
    build(rc, M + 1, R);
    combine(st[C], st[lc], st[rc]);
  }
  data Query(int i, int j, int C, int L, int R) {
    push(C, L, R);
    if (j < L || i > R || L > R) return data();  //
      default val 0/INF
    if (i <= L && R <= j) return st[C];
    data ret;
    data d1 = Query(i, j, lc, L, M);
    data d2 = Query(i, j, rc, M + 1, R);
    combine(ret, d1, d2);
    return ret;
  }
  void Update(int i, int j, T val, int C, int L, int R)
     {
    push(C, L, R);
    if (j < L || i > R || L > R) return;
    if (i <= L && R <= j) {
      isLazy[C] = 1;
      lazy[C] = val;
      push(C, L, R);
      return;
    }
    Update(i, j, val, lc, L, M);
    Update(i, j, val, rc, M + 1, R);
    combine(st[C], st[lc], st[rc]);
  }
  void Update(int i, int j, T val) {
    Update(i, j, val, 1, 1, N);
  }
  T Query(int i, int j) {
    return Query(i, j, 1, 1, N).sum;
  }
};
```

1.7 Sqrt Tricks [8 lines] - addf19

```
1. Size of the block is not always Sqrt, adjust it as
    necessary. if o(n/b+b) then take n/b = b and
    calculate b.
2. MO's Algorithm
   *it is possible to solve a Mo problem without any
       remove operation. For L in one block R only
       increases, for every range we can start L from
       the last of that block
3. Sqrt Decomposition by time of queries.
   *keep overall solution and sqrt(n) updates in a
       vector and for a query iterate over all of them,
       when the vector size exceeds sqrt(n) you can add
       these updates with overall solution using o(n)
4. If sum of N positive numbers are S, there are at most
    sqrt(S) distinct values.
5. Randomization
6. Baby step, gaint step
```

1.8 Treap [166 lines] - 8eef59

```
struct Treap {
  struct Node {
    int val, priority, cnt;  // value, priority, subtree
        size
    Node* l, * r;             // left child,right child
        pointer
    Node() {} //rng from template
    Node(int key) : val(key), priority(rng()),
        l(nullptr), r(nullptr) {}
  };
  typedef Node* node;
  node root;
  Treap() : root(0) {}
  int cnt(node t) { return t ? t->cnt : 0; }  // return
      subtree size
  void updateCnt(node t) {
    if (t) t->cnt = 1 + cnt(t->l) + cnt(t->r);  //
        update subtree size
  }
  void push(node cur) {
    ;  // Lazy Propagation
  }

  void combine(node& cur, node l, node r) {
    if (!l) {
      cur = r;
      return;
    }
    if (!r) {
      cur = l;
      return;
    }
    // Merge Operations like in segment tree
  }

  void reset(node& cur) {
```

```
  if (!cur) return;  // To reset other fields of cur except value and cnt
}

void operation(node& cur) {
  if (!cur) return;
  reset(cur);
  combine(cur, cur->l, cur);
  combine(cur, cur, cur->r);
}
// Split(T,key): split the tree in two tree. Left pointer contains all value
// less than or equal to key.Right pointer contains the rest.
void split(node t, node& l, node& r, int key) {
  if (!t)
    return void(l = r = nullptr);
  push(t);
  if (t->val <= key) {
    split(t->r, t->r, r, key), l = t;

  }
  else {
    split(t->l, l, t->l, key), r = t;
  }
  updateCnt(t);
  operation(t);
}
void splitPos(node t, node& l, node& r, int k, int add = 0) {
  if (!t) return void(l = r = 0);
  push(t);
  int idx = add + cnt(t->l);
  if (idx <= k)
    splitPos(t->r, t->r, r, k, idx + 1), l = t;
  else
    splitPos(t->l, l, t->l, k, add), r = t;
  updateCnt(t);
  operation(t);
}
// Merge(T1,T2): merges 2 tree into one.The tree with root of higher
// priority becomes the new root.
void merge(node& t, node l, node r) {
  push(l);
  push(r);
  if (!l || !r)
    t = l ? l : r;
  else if (l->priority > r->priority)
    merge(l->r, l->r, r), t = l;
  else
    merge(r->l, l, r->l), t = r;
  updateCnt(t);
  operation(t);
}

node merge_treap(node l, node r) {
  if (!l) return r;
  if (!r) return l;
  if (l->priority < r->priority) swap(l, r);
  node L, R;
  split(r, L, R, l->val);
  l->r = merge_treap(l->r, R);
  l->l = merge_treap(L, l->l);
  updateCnt(l);
  operation(l);
  return l;
}
// insert creates a set.all unique value.
void insert(int val) {
  if (!root) {
    root = new Node(val);
    return;
  }
  node l, r, mid, mid2, rr;
  mid = new Node(val);
  split(root, l, r, val);
  merge(l, l, mid);  // these 3 lines will create multiset.
  merge(root, l, r);
  /*split(root, l, r, val - 1);  // l contains all small values.
  merge(l, l, mid);             // l contains new val too.
  split(r, mid2, rr, val);      // rr contains all greater values.
  merge(root, l, rr);*/
}
// removes all similar values.
void erase(int val) {
  node l, r, mid;
  /* Removes all similar element*/
  split(root, l, r, val - 1);
  split(r, mid, r, val);
  merge(root, l, r);
  /*Removes single instance*/
  /*split(root, l, r, val - 1);
  split(r, mid, r, val);
  merge(mid,mid->l,mid->r);
  merge(l, l, mid);
  merge(root, l, r);*/
}
void clear(node cur) {
  if (!cur) return;
  clear(cur->l), clear(cur->r);
  delete cur;
}

void clear() { clear(root); }
void inorder(node t) {
  if (!t) return;
  inorder(t->l);
  cout << t->val << ' ';
  inorder(t->r);
}
void inorder() {
  inorder(root);
  puts("");
}
//1 indexed - xth element after sorting.
int find_by_order(int x) {
  if (!x) return -1;
  x--;
  node l, r, mid;
  splitPos(root, l, r, x - 1);
  splitPos(r, mid, r, 0);
  int ans = -1;
  if (cnt(mid) == 1) ans = mid->val;
  merge(r, mid, r);
  merge(root, l, r);
}
// 1 indexed. index of val in sorted array. -1 if not found.
int order_of_key(int val) {
  node l, r, mid;
  split(root, l, r, val - 1);
  split(r, mid, r, val);
  int ans = -1;
  if (cnt(mid) == 1) ans = 1 + cnt(l);
  merge(r, mid, r);
  merge(root, l, r);
  return ans;
}
};
```

1.9 Trie Bit [61 lines] - 390174

```
struct Trie {
  struct node {
    int next[2];
    int cnt, fin;
    node() :cnt(0), fin(0) {
      for (int i = 0; i < 2; i++) next[i] = -1;
    }
  };
  vector<node>data;
  Trie() {
    data.push_back(node());
  }
  void key_add(int val) {
    int cur = 0;
    for (int i = 30; i >= 0; i--) {
      int id = (val >> i) & 1;
      if (data[cur].next[id] == -1) {
        data[cur].next[id] = data.size();
        data.push_back(node());
      }
      cur = data[cur].next[id];
      data[cur].cnt++;
    }
    data[cur].fin++;
  }
  int key_search(int val) {
    int cur = 0;
    for (int i = 30; ~i; i--) {
      int id = (val >> i) & 1;
      if (data[cur].next[id] == -1) return 0;
      cur = data[cur].next[id];
    }
    return data[cur].fin;
  }
  void key_delete(int val) {
    int cur = 0;
    for (int i = 30; ~i; i--) {
      int id = (val >> i) & 1;
      cur = data[cur].next[id];
      data[cur].cnt--;
    }
    data[cur].fin--;
  }
  bool key_remove(int val) {
    if (key_search(val)) return key_delete(val), 1;
```

```
    return 0;
  }
  int maxXor(int x) {
    int cur = 0;
    int ans = 0;
    for (int i = 30; ~i; i--) {
      int b = (x >> i) & 1;
      if (data[cur].next[!b] + 1 &&
          data[data[cur].next[!b]].cnt > 0) {
        ans += (1LL << i);
        cur = data[cur].next[!b];
      }
      else cur = data[cur].next[b];
    }
    return ans;
  }
};
```

2 Dynamic Programming

2.1 Divide and Conquer DP [26 lines] - 6d8559

```
ll G,L;///total group,cell size
ll dp[8001][801],cum[8001];
ll C[8001];///value of each cell
inline ll cost(ll l,ll r){
  return(cum[r]-cum[l-1])*(r-l+1);
}
void fn(ll g,ll st,ll ed,ll r1,ll r2){
  if(st>ed)return;
  ll mid=(st+ed)/2,pos=-1;
  dp[mid][g]=inf;
  for(int i=r1;i<=r2;i++){
    ll tcost=cost(i,mid)+dp[i-1][g-1];
    if(tcost<dp[mid][g]){
        dp[mid][g]=tcost,pos=i;
    }
  }
  fn(g,st,mid-1,r1,pos);
  fn(g,mid+1,ed,pos,r2);
}
int main(){
  for(int i=1;i<=L;i++)
    cum[i]=cum[i-1]+C[i];
  for(int i=1;i<=L;i++)
    dp[i][1]=cost(1,i);
  for(int i=2;i<=G;i++)fn(i,1,L,1,L);
}
```

2.2 Dynamic Convex Hull Trick [66 lines] - c283fc

```
const int N = 3e5 + 9;
const int mod = 1e9 + 7;

//add lines with -m and -b and return -ans to
//make this code work for minimums.(not -x)
const ll inf = -(1LL << 62);
struct line {
  ll m, b;
  mutable function<const line*() > succ;
  bool operator < (const line& rhs) const {
    if (rhs.b != inf) return m < rhs.m;
    const line* s = succ();
    if (!s) return 0;
    ll x = rhs.m;
    return b - s->b < (s->m - m) * x;
  }
};
struct CHT : public multiset<line> {
  bool bad(iterator y) {
    auto z = next(y);
    if (y == begin()) {
      if (z == end()) return 0;
      return y -> m == z -> m && y -> b <= z -> b;
    }
    auto x = prev(y);
    if (z == end()) return y -> m == x -> m && y -> b
        <= x -> b;
    return 1.0 * (x -> b - y -> b) * (z -> m - y -> m)
        >= 1.0 * (y -> b - z -> b) * (y -> m - x -> m);
  }
  void add(ll m, ll b) {
    auto y = insert({ m, b });
    y->succ = [ = ] { return next(y) == end() ? 0 :
        &*next(y); };
    if (bad(y)) {
      erase(y);
      return;
    }
    while (next(y) != end() && bad(next(y)))
        erase(next(y));
    while (y != begin() && bad(prev(y))) erase(prev(y));
  }
  ll query(ll x) {
    assert(!empty());
    auto l = *lower_bound((line) {
      x, inf
    });
    return l.m * x + l.b;
  }
};
CHT* cht;
ll a[N], b[N];
int32_t main() {
  ios_base::sync_with_stdio(0);
  cin.tie(0);

  int n;
  cin >> n;
  for(int i = 0; i < n; i++) cin >> a[i];
  for(int i = 0; i < n; i++) cin >> b[i];
  cht = new CHT();
  cht -> add(-b[0], 0);
  ll ans = 0;
  for(int i = 1; i < n; i++) {
    ans = -cht -> query(a[i]);
    cht -> add(-b[i], -ans);
  }
  cout << ans << nl;
  return 0;
}
```

2.3 Knuth Optimization [32 lines] - 911417

```
/*It is applicable where recurrence is in the form :
dp[i][j]=mini<k<j{dp[i][k]+dp[k][j]}+C[i][j]
condition for applicability is:
A[i,j-1]<=A[i,j]<=A[i+1,j]
Where,
A[i][j]-the smallest k that gives optimal answer,like-
dp[i][j]=dp[i-1][k]+C[k][j]
C[i][j]-given cost function
also applicable if: C[i][j]satisfies the following 2
    conditions:
C[a][c]+C[b][d]<=C[a][d]+C[b][c],a<=b<=c<=d
C[b][c]<=C[a][d],a<=b<=c<=d
reduces time complexity from O(n^3)to O(n^2)*/
for(int s=0;s<=k;s++)//s-length(size)of substring
  for(int l=0;l+s<=k;l++){//l-left point
    int r=l+s;//r-right point
    if(s<2){
      res[l][r]=0;//DP base-nothing to break
      mid[l][r]=l;/*mid is equal to left border*/
      continue;
    }
    int mleft=mid[l][r-1];/*Knuth's trick: getting
        bounds on m*/
    int mright=mid[l+1][r];
    res[l][r]=inf;
    for(int m=mleft;m<=mright;m++){/*iterating for m in
        the bounds only*/
      int64 tres=res[l][m]+res[m][r]+(x[r]-x[l]);
      if(res[l][r]>tres){//relax current solution
        res[l][r]=tres;
        mid[l][r]=m;
      }
    }
  }
int64 answer=res[0][k];
```

2.4 LIS O(nlogn) with full path [17 lines] - e7e81f

```
int num[MX],mem[MX],prev[MX],array[MX],res[MX],maxlen;
void LIS(int SZ,int num[]){
  CLR(mem),CLR(prev),CLR(array),CLR(res);
  int i,k;
  maxlen=1;
  array[0]=-inf;
  RFOR(i,1,SZ+1)  array[i]=inf;
  prev[0]=-1,mem[0]=num[0];
  FOR(i,SZ){
    k=lower_bound(array,array+maxlen+1,num[i])-array;
    if(k==1)  array[k]=num[i],mem[k]=i,prev[i]=-1;
    else array[k]=num[i],mem[k]=i,prev[i]=mem[k-1];
    if(k>maxlen)  maxlen=k;
  }
  k=0;
  for(i=mem[maxlen];i!=-1;i=prev[i])res[k++]=num[i];
}
```

2.5 SOS DP [18 lines] - 5063f0

```
//iterative version
for(int mask = 0; mask < (1<<N); ++mask){
  dp[mask][-1] = A[mask];  //handle base case separately
      (leaf states)
  for(int i = 0;i < N; ++i){
    if(mask & (1<<i))
      dp[mask][i] = dp[mask][i-1] +
          dp[mask^(1<<i)][i-1];
    else
      dp[mask][i] = dp[mask][i-1];
  }
  F[mask] = dp[mask][N-1];
}
//memory optimized, super easy to code.
for(int i = 0; i<(1<<N); ++i)
  F[i] = A[i];
```

```
for(int i = 0;i < N; ++i) for(int mask = 0; mask <
    (1<<N); ++mask){
  if(mask & (1<<i))
    F[mask] += F[mask^(1<<i)];
}
```

2.6 Sibling DP [26 lines] - cfc5ff

```
/*/dividing tree into min group such that each group
    cost not exceed k*/
ll n,k,dp[mx][mx];
vector<pair<ll,ll>>adj[mx];///must be rooted tree
ll sibling_dp(ll par,ll idx,ll remk){
  if(remk<0)return inf;
  if(adj[par].size()<idx+1)return 0;
  ll u=adj[par][idx].first;
  if(dp[u][remk]!=-1)
    return dp[u][remk];
  ll ret=inf,under=0,sibling=0;
  if(par!=0){//creating new group
    under=1+dfs(u,0,k);
    sibling=dfs(par,idx+1,remk);
    ret=min(ret,under+sibling);
  }
  //divide the current group
  ll temp=remk-adj[par][idx].second;
  for(ll chk=temp;chk>=0;chk--){
    ll siblingk=temp-chk;
    under=0,sibling=0;
    under=dfs(u,0,chk);
    sibling=dfs(par,idx+1,siblingk);
    ret=min(ret,under+sibling);
  }
  return dp[u][remk]=ret;
}
```

3 Flow

3.1 Blossom [58 lines] - 1b2a6f

```
// Finds Maximum matching in General Graph
// Complexity O(NM)
// mate[i] = j means i is paired with j
// source: https://codeforces.com/blog/entry
    /92339?#comment-810242
vector<int> Blossom(vector<vector<int>>& graph) {
  //mate contains matched edge.
  int n = graph.size(), timer = -1;
  vector<int> mate(n, -1), label(n), parent(n),
    orig(n), aux(n, -1), q;
  auto lca = [&](int x, int y) {
    for (timer++; ; swap(x, y)) {
      if (x == -1) continue;
      if (aux[x] == timer) return x;
      aux[x] = timer;
      x = (mate[x] == -1 ? -1 : orig[parent[mate[x]]]);
    }
  };
  auto blossom = [&](int v, int w, int a) {
    while (orig[v] != a) {
      parent[v] = w; w = mate[v];
      if (label[w] == 1) label[w] = 0, q.push_back(w);
      orig[v] = orig[w] = a; v = parent[w];
    }
  };
  auto augment = [&](int v) {
    while (v != -1) {
      int pv = parent[v], nv = mate[pv];
      mate[v] = pv; mate[pv] = v; v = nv;
    }
  };
  auto bfs = [&](int root) {
    fill(label.begin(), label.end(), -1);
    iota(orig.begin(), orig.end(), 0);
    q.clear();
    label[root] = 0; q.push_back(root);
    for (int i = 0; i < (int)q.size(); ++i) {
      int v = q[i];
      for (auto x : graph[v]) {
        if (label[x] == -1) {
          label[x] = 1; parent[x] = v;
          if (mate[x] == -1)
            return augment(x), 1;
          label[mate[x]] = 0; q.push_back(mate[x]);
        }
        else if (label[x] == 0 && orig[v] != orig[x]) {
          int a = lca(orig[v], orig[x]);
          blossom(x, v, a); blossom(v, x, a);
        }
      }
    }
    return 0;
  };
  // Time halves if you start with (any) maximal
      matching.
  for (int i = 0; i < n; i++)
    if (mate[i] == -1)
      bfs(i);
  return mate;
}
```

3.2 Dinic [72 lines] - a786f1

```
/*.Complexity: O(V^2 E)
  .Call Dinic with total number of nodes.
  .Nodes start from 0.
  .Capacity is long long data.
  .make graph with create edge(u,v,capacity).
  .Get max flow with maxFlow(src,des).*/
#define eb emplace_back
struct Dinic {
  struct Edge {
    int u, v;
    ll cap, flow = 0;
    Edge() {}
    Edge(int u, int v, ll cap) :u(u), v(v), cap(cap) {}
  };
  int N;
  vector<Edge>edge;
  vector<vector<int>>adj;
  vector<int>d, pt;
  Dinic(int N) :N(N), edge(0), adj(N), d(N), pt(N) {}
  void addEdge(int u, int v, ll cap) {
    if (u == v) return;
    edge.eb(u, v, cap);
    adj[u].eb(edge.size() - 1);
    edge.eb(v, u, 0);
    adj[v].eb(edge.size() - 1);
  }
  bool bfs(int s, int t) {
    queue<int>q({ s });
    fill(d.begin(), d.end(), N + 1);
    d[s] = 0;
    while (!q.empty()) {
      int u = q.front();q.pop();
      if (u == t) break;
      for (int k : adj[u]) {
        Edge& e = edge[k];
        if (e.flow<e.cap && d[e.v]>d[e.u] + 1) {
          d[e.v] = d[e.u] + 1;
          q.emplace(e.v);
        }
      }
    }
    return d[t] != N + 1;
  }
  ll dfs(int u, int T, ll flow = -1) {
    if (u == T || flow == 0) return flow;
    for (int& i = pt[u];i < adj[u].size();i++) {
      Edge& e = edge[adj[u][i]];
      Edge& oe = edge[adj[u][i] ^ 1];
      if (d[e.v] == d[e.u] + 1) {
        ll amt = e.cap - e.flow;
        if (flow != -1 && amt > flow) amt = flow;
        if (ll pushed = dfs(e.v, T, amt)) {
          e.flow += pushed;
          oe.flow -= pushed;
          return pushed;
        }
      }
    }
    return 0;
  }
  ll maxFlow(int s, int t) {
    ll total = 0;
    while (bfs(s, t)) {
      fill(pt.begin(), pt.end(), 0);
      while (ll flow = dfs(s, t)) {
        total += flow;
      }
    }
    return total;
  }

};
```

3.3 Flow [6 lines] - 6ebca7

```
Covering Problems:
> Maximum Independent Set(Bipartite): Largest set of
    nodes which do not have any edge between them. sol:
    V-(MaxMatching)
> Minimum Vertex Cover(Bipartite): -Smallest set of
    nodes to cover all the edges -sol: MaxMatching
> Minimum Edge Cover(General graph): -Smallest set of
    edges to cover all the nodes -sol: V-(MaxMatching)
    (if edge cover exists, does not exit for isolated
    nodes)
> Minimum Path Cover(Vertex disjoint) DAG: -Minimum
    number of vertex disjoint paths that visit all the
    nodes -sol: make a bipartite graph using same nodes
    in two sides, one side is "from" other is "to", add
    edges from "from" to "to", then ans is
    V-(MaxMatching)
> Minimum Path Cover(Vertex Not Disjoint) General graph:
    -Minimum number of paths that visit all the nodes
    -sol: consider cycles as nodes then it will become a
    path cover problem with vertex disjoint on DAG
```

3.4 HopCroftKarp [67 lines] - fac9fc

```cpp
/*.Finds Maximum Matching In a bipartite graph
  .Complexity O(E√V)
  .1-indexed
  .No default constructor
  .add single edge for (u, v)*/
struct HK {
  static const int inf = 1e9;
  int n;
  vector<int>matchL, matchR, dist;
  //matchL contains value of matched node for L part.
  vector<vector<int>>adj;
  HK(int n) :n(n), matchL(n + 1),
  matchR(n + 1), dist(n + 1), adj(n + 1) {
  }

  void addEdge(int u, int v) {
    adj[u].push_back(v);
  }
  bool bfs() {
    queue<int>q;
    for (int u = 1;u <= n;u++) {
      if (!matchL[u]) {
        dist[u] = 0;
        q.push(u);
      }
      else dist[u] = inf;
    }
    dist[0] = inf;///unmatched node matches with 0.
    while (!q.empty()) {
      int u = q.front();
      q.pop();
      for (auto v : adj[u]) {
        if (dist[matchR[v]] == inf) {
          dist[matchR[v]] = dist[u] + 1;
          q.push(matchR[v]);
        }
      }
    }
    return dist[0] != inf;
  }

  bool dfs(int u) {
    if (!u) return true;
    for (auto v : adj[u]) {
      if (dist[matchR[v]] == dist[u] + 1
          && dfs(matchR[v])) {
        matchL[u] = v;
        matchR[v] = u;
        return true;
      }
    }
    dist[u] = inf;
    return false;
  }
  int max_match() {
    int matching = 0;
    while (bfs()) {
      for (int u = 1;u <= n;u++) {
        if (!matchL[u])
          if (dfs(u))
            matching++;
      }
    }
    return matching;
  }
};
```

3.5 Hungarian [116 lines] - 64902f

```cpp
/* Complexity: O(n^3) but optimized
   It finds minimum cost maximum matching.
   For finding maximum cost maximum matching
   add -cost and return -matching()
   1-indexed */
struct Hungarian {
  long long c[N][N], fx[N], fy[N], d[N];
  int l[N], r[N], arg[N], trace[N];
  queue<int> q;
  int start, finish, n;
  const long long inf = 1e18;
  Hungarian() {}
  Hungarian(int n1, int n2) : n(max(n1, n2)) {
    for (int i = 1; i <= n; ++i) {
      fy[i] = l[i] = r[i] = 0;
      for (int j = 1; j <= n; ++j) c[i][j] = inf;
    }
  }
  void add_edge(int u, int v, long long cost) {
    c[u][v] = min(c[u][v], cost);
  }
  inline long long getC(int u, int v) {
    return c[u][v] - fx[u] - fy[v];
  }
  void initBFS() {
    while (!q.empty()) q.pop();
    q.push(start);
    for (int i = 0; i <= n; ++i) trace[i] = 0;
    for (int v = 1; v <= n; ++v) {
      d[v] = getC(start, v);
      arg[v] = start;
    }
    finish = 0;
  }
  void findAugPath() {
    while (!q.empty()) {
      int u = q.front();
      q.pop();
      for (int v = 1; v <= n; ++v) if (!trace[v]) {
        long long w = getC(u, v);
        if (!w) {
          trace[v] = u;
          if (!r[v]) {
            finish = v;
            return;
          }
          q.push(r[v]);
        }
        if (d[v] > w) {
          d[v] = w;
          arg[v] = u;
        }
      }
    }
  }
  void subX_addY() {
    long long delta = inf;
    for (int v = 1; v <= n; ++v) if (trace[v] == 0 &&
        d[v] < delta) {
      delta = d[v];
    }
    // Rotate
    fx[start] += delta;
    for (int v = 1; v <= n; ++v) if (trace[v]) {
      int u = r[v];
      fy[v] -= delta;
      fx[u] += delta;
    }
    else d[v] -= delta;
    for (int v = 1; v <= n; ++v) if (!trace[v] && !d[v])
      {
      trace[v] = arg[v];
      if (!r[v]) {
        finish = v;
        return;
      }
      q.push(r[v]);
    }
  }
  void Enlarge() {
    do {
      int u = trace[finish];
      int nxt = l[u];
      l[u] = finish;
      r[finish] = u;
      finish = nxt;
    } while (finish);
  }
  long long maximum_matching() {
    for (int u = 1; u <= n; ++u) {
      fx[u] = c[u][1];
      for (int v = 1; v <= n; ++v) {
        fx[u] = min(fx[u], c[u][v]);
      }
    }
    for (int v = 1; v <= n; ++v) {
      fy[v] = c[1][v] - fx[1];
      for (int u = 1; u <= n; ++u) {
        fy[v] = min(fy[v], c[u][v] - fx[u]);
      }
    }
    for (int u = 1; u <= n; ++u) {
      start = u;
      initBFS();
      while (!finish) {
        findAugPath();
        if (!finish) subX_addY();
      }
      Enlarge();
    }
    long long ans = 0;
    for (int i = 1; i <= n; ++i) {
      if (c[i][l[i]] != inf) ans += c[i][l[i]];
      else l[i] = 0;
    }
    return ans;
  }
};
```

3.6 MCMF [116 lines] - 466389

```cpp
/*Credit: ShahjalalShohag
  .Works for both directed, undirected and with negative
      cost too
```

```
   .doesn't work for negative cycles
   .for undirected edges just make the directed flag
     false
   .Complexity: O(min(E^2 *V log V, E logV * flow))*/
using T = long long;
const T inf = 1LL << 61;
struct MCMF {
  struct edge {
    int u, v;
    T cap, cost;
    int id;
    edge(int _u, int _v, T _cap, T _cost, int _id) {
      u = _u;
      v = _v;
      cap = _cap;
      cost = _cost;
      id = _id;
    }
  };
  int n, s, t, mxid;
  T flow, cost;
  vector<vector<int>> g;
  vector<edge> e;
  vector<T> d, potential, flow_through;
  vector<int> par;
  bool neg;
  MCMF() {}
  MCMF(int _n) { // 0-based indexing
    n = _n + 10;
    g.assign(n, vector<int>());
    neg = false;
    mxid = 0;
  }
  void add_edge(int u, int v, T cap, T cost, int id =
      -1, bool directed = true) {
    if (cost < 0) neg = true;
    g[u].push_back(e.size());
    e.push_back(edge(u, v, cap, cost, id));
    g[v].push_back(e.size());
    e.push_back(edge(v, u, 0, -cost, -1));
    mxid = max(mxid, id);
    if (!directed) add_edge(v, u, cap, cost, -1, true);
  }
  bool dijkstra() {
    par.assign(n, -1);
    d.assign(n, inf);
    priority_queue<pair<T, T>, vector<pair<T, T>>,
        greater<pair<T, T>> > q;
    d[s] = 0;
    q.push(pair<T, T>(0, s));
    while (!q.empty()) {
      int u = q.top().second;
      T nw = q.top().first;
      q.pop();
      if (nw != d[u]) continue;
      for (int i = 0; i < (int)g[u].size(); i++) {
        int id = g[u][i];
        int v = e[id].v;
        T cap = e[id].cap;
        T w = e[id].cost + potential[u] - potential[v];
        if (d[u] + w < d[v] && cap > 0) {
          d[v] = d[u] + w;
          par[v] = id;
          q.push(pair<T, T>(d[v], v));
        }
      }
    }
    for (int i = 0; i < n; i++) { // update potential
      if (d[i] < inf) potential[i] += d[i];
    }
    return d[t] != inf;
  }
  T send_flow(int v, T cur) {
    if (par[v] == -1) return cur;
    int id = par[v];
    int u = e[id].u;
    T w = e[id].cost;
    T f = send_flow(u, min(cur, e[id].cap));
    cost += f * w;
    e[id].cap -= f;
    e[id ^ 1].cap += f;
    return f;
  }
  //returns {maxflow, mincost}
  pair<T, T> solve(int _s, int _t, T goal = inf) {
    s = _s;
    t = _t;
    flow = 0, cost = 0;
    potential.assign(n, 0);
    if (neg) {
      // run Bellman-Ford to find starting potential
      d.assign(n, inf);
      for (int i = 0, relax = true; i < n && relax; i++)
          {
        for (int u = 0; u < n; u++) {
          for (int k = 0; k < (int)g[u].size(); k++) {
            int id = g[u][k];
            int v = e[id].v;
            T cap = e[id].cap, w = e[id].cost;
            if (d[v] > d[u] + w && cap > 0) {
              d[v] = d[u] + w;
              relax = true;
            }
          }
        }
      }
      for (int i = 0; i < n; i++) if (d[i] < inf)
          potential[i] = d[i];
    }
    while (flow < goal && dijkstra()) flow +=
        send_flow(t, goal - flow);
    flow_through.assign(mxid + 10, 0);
    for (int u = 0; u < n; u++) {
      for (auto v : g[u]) {
        if (e[v].id >= 0) flow_through[e[v].id] = e[v ^
            1].cap;
      }
    }
    return make_pair(flow, cost);
  }
};
```

4 Game Theory

4.1 Points to be noted [14 lines] - 6fe124

```
>[First Write a Brute Force solution]
>Nim = all xor
>Misere Nim = Nim + corner case: if all piles are 1,
   reverse(nim)
>Bogus Nim = Nim
>Staircase Nim = Odd indexed pile Nim (Even indexed pile
    doesnt matter, as one player can give bogus moves to
    drop all even piles to ground)
>Sprague Grundy: [Every impartial game under the normal
    play convention is equivalent to a one-heap game of
    nim]
Every tree = one nim pile = tree root value; tree leaf
    value = 0; tree node value = mex of all child nodes.
[Careful: one tree node can become multiple new tree
    roots(multiple elements in one node), then the value
    of that node = xor of all those root values]
>Hackenbush(Given a rooted tree; cut an edge in one
    move; subtree under that edge gets removed; last
    player to cut wins):
 Colon:  //G(u) = (G(v1) + 1) ⊕ (G(v2) + 1) ⊕ ··· [v1, v2, ···
    are childs of u]
For multiple trees ans is their xor
>Hackenbush on graph (instead of tree given an rooted
    graph):
fusion: All edges in a cycle can be fused to get a tree
    structure; build a super node, connect some single
    nodes with that super node, number of single nodes
    is the number of edges in the cycle.
Sol: [Bridge component tree] mark all bridges, a group
    of edges that are not bridges, becomes one component
    and contributes number of edges to the hackenbush.
    (even number of edges contributes 0, odd number of
    edges contributes 1)
```

5 Geometry

5.1 Geometry [384 lines] - 6bfd7b

```
namespace Geometry
{
  #define M_PI(acos(-1.0))
  double eps=1e-8;
  typedef double T;//coordinate point type
  struct pt //Point
  {
    T x,y;
    pt(){}
    pt(T _x,T _y):x(_x),y(_y){}
    pt operator+(pt p){
      return{x+p.x,y+p.y};
    }
    pt operator-(pt p){
      return{x-p.x,y-p.y};
    }
    pt operator*(T d){
      return{x*d,y*d};
    }
    pt operator*(pt d){/*I added for General linear
        transformation,not sure about that function*/
      return{x*d.x,y*d.y};
    }
    pt operator/(T d){
      return{x/d,y/d};/*only for floating point*/
    }
    pt operator/(pt d){/*I added for General linear
        transformation,not sure about that function*/
      return{x/d.x,y/d.y};
    }
    bool operator<(const pt& p)const {
```

```
    if(x!=p.x)
      return x<p.x;
    return y<p.y;
  }
  bool operator==(pt b){
    return x==b.x && y==b.y;
  }
  bool operator!=(pt b){
    return!(*(this)==b);
  }
  friend ostream& operator<<(ostream& os,const pt p){
    return os<<"("<<p.x<<","<<p.y<<")";
  }
  friend istream& operator>>(istream& is,pt &p){
    is>>p.x>>p.y;
    return is;
  }
};
 T sq(pt p){
   return p.x*p.x+p.y*p.y;
 }
 double Abs(pt p){
   return sqrtl(sq(p));
 }
 pt translate(pt v,pt p){ /*To translate an object by a
     vector v*/
   return p+v;
 }
 pt scale(pt c,double factor,pt p){/*To scale an object
     by a certain ratio factor around a center*/
   return c+(p-c)*factor;
 }
 pt rot(pt p,double a){/*To rotate a point by angle
     a*/
   return{p.x*cos(a)-p.y*sin(a),p.x*sin(a)+p.y*
       cos(a)};
 }
 pt perp(pt p){/*To rotate a point 90 degree*/
   return{-p.y,p.x};
 }
 pt linearTransfo(pt p,pt q,pt r,pt fp,pt fq){/*so far
     don't know about that function*/
   return fp+(r-p)*(fq-fp)/(q-p);
 }
 T dot(pt v,pt w){
   return v.x*w.x+v.y*w.y;
 }
 bool isPerp(pt v,pt w){
   return dot(v,w)==0;
 }
 double angle(pt v,pt w){/*Find the smallest angle of
     two vector*/
   double cosTheta=dot(v,w)/Abs(v)/Abs(w);
   return acos(max(-1.0,min(1.0,cosTheta)));
 }
 T cross(pt v,pt w){
   return v.x*w.y-v.y*w.x;
 }
 T orient(pt a,pt b,pt c){
   return cross(b-a,c-a); /*if c is left side+ve,c is
       right side-ve,on line 0*/
 }
 bool inAngle(pt a,pt b,pt c,pt p){/*if p is in the
     angle*/
   assert(orient(a,b,c)!=0);
   if(orient(a,b,c)<0)
     swap(b,c);
   return orient(a,b,p)>=0 && orient(a,c,p)<=0;
 }
 double orientedAngle(pt a,pt b,pt c){/*the actual
     angle from ab to ac*/
   if(orient(a,b,c)>=0)
       return angle(b-a,c-a);
   else
       return 2*M_PI-angle(b-a,c-a);
 }
///line
 struct line{
   pt v;
   T c;
   line(){}
   line(pt p,pt q){/*From points P and Q*/
     v=(q-p),this->c=cross(v,p);
   }
   line(T a,T b,T c){/*From equation ax+by=c*/
     v=pt(b,-a),this->c=c;
   }
   line(pt v,T c){/*From direction vector v and offset
       c*/
     this->v=v,this->c=c;
   }
   double getY(double x){/*self made,not sure if it is
       okay*/
     assert(v.x!=0);
     double ret=(double)(c+v.y*x)/v.x;
     return ret;
   }
   double getX(double y){/*self made,not sure if it is
       okay*/
     assert(v.y!=0);
     double ret=(double)(c-v.x*y)/-v.y;
     return ret;
   }
   T side(pt p){/*which side a point is*/
     return cross(v,p)-c;
   }
   double dist(pt p){/*point to line dist*/
     return abs(side(p))/Abs(v);
   }
   double sqDist(pt p){/*square dist*/
     return side(p)*side(p)/(double)sq(v);
   }
   line perpThrough(pt p){/*perpendicular line with
       point p*/
       return line(p,p+perp(v));
   }
   bool cmpProj(pt p,pt q){/*compare function to sort
       points on a line*/
       return dot(v,p)<dot(v,q);
   }
   line translate(pt t){/*translate with vector t*/
       return line(v,c+cross(v,t));
   }
   line shiftLeft(double dist){/*translate with
       distance dist*/
       return line(v,c+dist*Abs(v));
   }
   pt proj(pt p){
       return p-perp(v)*side(p)/sq(v);
   }
   pt refl(pt p){
       return p-perp(v)*2*side(p)/sq(v);
   }
};
 bool areParallel(line l1,line l2){
   return(l1.v.x*l2.v.y==l1.v.y*l2.v.x);
 }
 bool areSame(line l1,line l2){
   return areParallel(l1,l2)and(l1.v.x*l2.c==l2.v.x*
       l1.c)and(l1.v.y*l2.c==l2.v.y*l1.c);
 }
 bool inter(line l1,line l2,pt& out){
   T d=cross(l1.v,l2.v);
   if(d==0)return false;
   out=(l2.v*l1.c-l1.v*l2.c)/d;
   return true;
 }
 line intBisector(line l1,line l2,bool interior){/*if
     change sign then returns the other one*/
   assert(cross(l1.v,l2.v)!=0);
   double sign=interior?1:-1;
   return line(l2.v/Abs(l2.v)+l1.v*sign/Abs(l1.v),
           l2.c/Abs(l2.v)+l1.c*sign/Abs(l1.v));
 }
 //segment
 bool inDisk(pt a,pt b,pt p){/*check weather point p is
     in diameter AB*/
   return dot(a-p,b-p)<=0;
 }
 bool onSegment(pt a,pt b,pt p){/*check weather point p
     is in segment AB*/
   return orient(a,b,p)==0 and inDisk(a,b,p);
 }
 bool properInter(pt a,pt b,pt c,pt d,pt& i){
   double oa=orient(c,d,a),
          ob=orient(c,d,b),
          oc=orient(a,b,c),
          od=orient(a,b,d);
 //Proper intersection exists iff opposite signs
   if(oa*ob<0 and oc*od<0){
     i=(a*ob-b*oa)/(ob-oa);
     return 1;
   }
   return 0;
 }
/*To create sets of points we need a comparison
    function*/
 struct cmpX{
   bool operator()(pt a,pt b){
       return make_pair(a.x,a.y)<make_pair(b.x,b.y);
   }
};
 set<pt,cmpX>inters(pt a,pt b,pt c,pt d){
   pt out;
   if(properInter(a,b,c,d,out))
     return{out};
   set<pt,cmpX>s;
   if(onSegment(c,d,a))s.insert(a);
   if(onSegment(c,d,b))s.insert(b);
   if(onSegment(a,b,c))s.insert(c);
   if(onSegment(a,b,d))s.insert(d);
   return s;
```

```cpp
}
bool LineSegInter(line l,pt a,pt b,pt& out){
  if(l.side(a)*l.side(b)>eps)return 0;
  return inter(l,line(a,b),out);
}
double segPoint(pt a,pt b,pt p){/*returns distance from a point p to segment AB*/
  if(a!=b){
      line l(a,b);
      if(l.cmpProj(a,p)and l.cmpProj(p,b))
        return l.dist(p);
  }
  return min(Abs(p-a),Abs(p-b));
}
double segSeg(pt a,pt b,pt c,pt d){/*returns distance from a segment AB to segment CD*/
  pt dummy;
  if(properInter(a,b,c,d,dummy))return 0;
  return min(min(min(segPoint(a,b,c),segPoint(a,b,d)),segPoint(c,d,a)),segPoint(c,d,b));
}
/*int latticePoints(pt a,pt b){
  // requires int representation
   return __gcd(abs(a.x-b.x),abs(a.y-b.y))+1;
 } // A = i+(b/2)-1; here
     A=area,i=pointsinside,b=pointsonline
 Polygon*/
bool isConvex(vector<pt>&p){
  bool hasPos=0,hasNeg=0;
  for(int i=0,n=p.size();i<n;i++){
    int o=orient(p[i],p[(i+1)%n],p[(i+2)%n]);
    if(o>0)hasPos=1;
    if(o<0)hasNeg=true;
  }
  return!(hasPos and hasNeg);
}
double areaTriangle(pt a,pt b,pt c){
  return abs(cross(b-a,c-a))/2.0;
}
double areaPolygon(const vector<pt>&p){
  double area=0.0;
  for(int i=0,n=p.size();i<n;i++){
    area+=cross(p[i],p[(i+1)%n]);
  }
  return fabs(area)/2.0;
}
bool pointInPolygon(const vector<pt>&p,pt q){/*returns true if pt q is in polygon p*/
  bool c=false;
  for(int i=0,n=p.size();i<n;i++){
    int j=(i+1)%p.size();
    if((p[i].y<=q.y and q.y<p[j].y or p[j].y<=q.y and
        q.y<p[i].y)and
      q.x<p[i].x+(p[j].x-p[i].x)*(q.y-p[i].y)/
          (p[j].y-p[i].y))
        c=!c;
  }
  return c;
}
ll is_point_in_convex(vector<pt>& p, pt &x) { // O(log n)
    ll n = p.size(); /*this function from YouKnowWho*/
    if (n < 3) return 1;
    ll a =orient(p[0], p[1], x), b = orient(p[0], p[n - 1], x);
    if (a < 0 || b > 0) return 1;
    ll l = 1, r = n - 1;
    while (l + 1 < r) {
        int mid = l + r >> 1;
        if (orient(p[0], p[mid], x) >= 0) l = mid;
        else r = mid;
    }
    ll k = orient(p[l], p[r], x);
    if (k <= 0) return -k;
    if (l == 1 && a == 0) return 0;
    if (r == n - 1 && b == 0) return 0;
    return -1;
}
 pt centroidPolygon(vector<pt>&p){/*from rezaul,i don't know about that*/
   pt c(0,0);
   double scale=6.0*areaPolygon(p);
//  if(scale<eps)return c;
   for(int i=0,n=p.size();i<n;i++){
     int j=(i+1)%n;
     c=c+(p[i]+p[j])*cross(p[i],p[j]);
   }
   return c/scale;
 }
///Circle
 pt circumCenter(pt a,pt b,pt c){/*return the center of the circle go through point a,b,c*/
   b=b-a,c=c-a;
   assert(cross(b,c)!=0);
   return a+perp(b*sq(c)-c*sq(b))/cross(b,c)/2;
 }
 bool circle2PtsRad(pt p1,pt p2,double r,pt& c){
   double d2=sq(p1-p2);
   double det=r*r/d2-0.25;
   if(det<0.0)return false;
   double h=sqrt(det);
   c.x=(p1.x+p2.x)*0.5+(p1.y-p2.y)*h;
   c.y=(p1.y+p2.y)*0.5+(p2.x-p1.x)*h;
   return true;
 }
 int circleLine(pt c,double r,line l,pair<pt,pt>& out){/*circle line intersection*/
   double h2=r*r-l.sqDist(c);
   if(h2<0)return 0;/*the line doesn't touch the circle;*/
   pt p=l.proj(c);
   pt h=l.v*sqrt(h2)/Abs(l.v);
   out=make_pair(p-h,p+h);
   return 1+(h2>0);
 }
 int circleCircle(pt c1,double r1,pt c2,double r2,pair<pt,pt>& out){/*circle circle intersection*/
   pt d=c2-c1;
   double d2=sq(d);
   if(d2==0){//concentric circles
     assert(r1!=r2);
     return 0;
   }
   double pd=(d2+r1*r1-r2*r2)/2;
   double h2=r1*r1-pd*pd/d2;//h ^ 2
   if(h2<0)return 0;
   pt p=c1+d*pd/d2,h=perp(d)*sqrt(h2/d2);
   out=make_pair(p-h,p+h);
   return 1+h2>0;
 }
 int tangents(pt c1,double r1,pt c2,double r2,bool inner,vector<pair<pt,pt>>&out){
   if(inner)r2=-r2;/*returns tangent(the line which touch a circle in one point)of two circle*/
   pt d=c2-c1;/*the same code can be used to find the tangent to a circle passing through a point by setting r2 to 0*/
   double dr=r1-r2,d2=sq(d),h2=d2-dr*dr;
   if(d2==0 or h2<0){
     assert(h2!=0);
     return 0;
   }
   for(int sign :{-1,1}){
       pt v=pt(d*dr+perp(d)*sqrt(h2)*sign)/d2;
       out.push_back(make_pair(c1+v*r1,c2+v*r2));
   }
   return 1+(h2>0);
 }
//Convex Hull-Monotone Chain
 pt H[100000+5];
 vector<pt>monotoneChain(vector<pt>&points){
   sort(points.begin(),points.end());
   vector<pt>ret;
   ret.clear();
   int st=0;
   for(int i=0,sz=points.size();i<sz;i++){
     while(st>=2 and
         orient(H[st-2],H[st-1],points[i])<0)st--;
     H[st++]=points[i];
   }
   int taken=st-1;
   for(int i=points.size()-2;i>=0;i--){
     while(st>=taken+2 and
         orient(H[st-2],H[st-1],points[i])<0)st--;
     H[st++]=points[i];
   }
   for(int i=0;i<st;i++)ret.push_back(H[i]);
   return ret;
 }
 //Convex Hull-Monotone Chain from you_know_who
 int sz;
 vector<pt> monotoneChain(vector<pt> &v) {
     if(v.size()==1) return v;
     sort(v.begin(), v.end());
     vector<pt> up(2*v.size()+2), down(2*v.size()+2);
     int szup=0, szdw=0;
     for(int i=0;i<v.size();i++) {
         while(szup>1 && orient(up[szup-2],
             up[szup-1], v[i])>=0)
             szup--;
         while(szdw>1 && orient(down[szdw-2],
             down[szdw-1], v[i])<=0)
             szdw--;
         up[szup++]=v[i];
         down[szdw++]=v[i];
     }
     if(szdw>1) szdw--;
     reverse(up.begin(), up.begin()+szup);
     for(int i=0;i<szup-1;i++) down[szdw++]= up[i];
```

```
    if(szdw==2 && down[0].x==down[1].x &&
        down[0].y==down[1].y)
        szdw--;
    sz = szdw;
    return down;
  }
  double cosA(double a,double b,double c){
      double val=b*b+c*c-a*a;
      val/=(2*b*c);
      return acos(val);
  }
  double triangle(double a,double b,double c){
      double s=(a+b+c)/2;
      return sqrtl(s*(s-a)*(s-b)*(s-c));
  }
}
using namespace Geometry;
```

5.2 Rotation Matrix [39 lines] - f97f03

```
struct { double x; double y; double z; } Point;
double rMat[4][4];
double inMat[4][1] = {0.0, 0.0, 0.0, 0.0};
double outMat[4][1] = {0.0, 0.0, 0.0, 0.0};
void mulMat() {
  for(int i = 0; i < 4; i++ ){
    for(int j = 0; j < 1; j++){
      outMat[i][j] = 0;
      for(int k = 0; k < 4; k++)
        outMat[i][j] += rMat[i][k] * inMat[k][j];
    }
  }
}
void setMat(double ang, double u, double v, double w){
  double L = (u * u + v * v + w * w);
  ang = ang * PI / 180.0; /*converting to radian
      value*/
  double u2 = u*u; double v2 = v*v; double w2 = w*w;
  rMat[0][0]=(u2+(v2+w2)*cos(ang))/L;
  rMat[0][1]=(u*v*(1-cos(ang))-w*sqrt(L)*sin(ang))/L;
  rMat[0][2]=(u*w*(1-cos(ang))+v*sqrt(L)*sin(ang))/L;
  rMat[0][3]=0.0;
  rMat[1][0]=(u*v*(1-cos(ang))+w*sqrt(L)*sin(ang))/L;
  rMat[1][1]=(v2+(u2+w2)*cos(ang))/L;
  rMat[1][2]=(v*w*(1-cos(ang))-u*sqrt(L)*sin(ang))/L;
  rMat[1][3]=0.0;
  rMat[2][0]=(u*w*(1-cos(ang))-v*sqrt(L)*sin(ang))/L;
  rMat[2][1]=(v*w*(1-cos(ang))+u*sqrt(L)*sin(ang))/L;
  rMat[2][2]=(w2 + (u2 + v2) * cos(ang)) / L;
  rMat[2][3]=0.0; rMat[3][0]=0.0; rMat[3][1]=0.0;
  rMat[3][2]=0.0; rMat[3][3]=1.0;
}
/*double ang;
  double u, v, w; //points = the point to be rotated
  Point point, rotated; //u,v,w=unit vector of line
  inMat[0][0] = points.x; inMat[1][0] = points.y;
  inMat[2][0] = points.z; inMat[3][0] = 1.0;
  setMat(ang, u, v, w); mulMat();
  rotated.x = outMat[0][0]; rotated.y = outMat[1][0];
  rotated.z = outMat[2][0];*/
```

6 Graph

6.1 2SAT [92 lines] - 5289ec

```
struct TwoSat {
  vector<bool>vis;
  vector<vector<int>>adj, radj;
  vector<int>dfs_t, ord, par;
  int n, intime;//For n node there will be 2*n node in
      SAT.
  void init(int N) {
    n = N;
    intime = 0;
    vis.assign(N * 2 + 1, false);
    adj.assign(N * 2 + 1, vector<int>());
    radj.assign(N * 2 + 1, vector<int>());
    dfs_t.resize(N * 2 + 1);
    ord.resize(N * 2 + 1);
    par.resize(N * 2 + 1);
  }
  inline int neg(int x) {
    return x <= n ? x + n : x - n;
  }
  inline void add_implication(int a, int b) {
    if (a < 0) a = n - a;
    if (b < 0) b = n - b;
    adj[a].push_back(b);
    radj[b].push_back(a);
  }
  inline void add_or(int a, int b) {
    add_implication(-a, b);
    add_implication(-b, a);
  }
  inline void add_xor(int a, int b) {
    add_or(a, b);
    add_or(-a, -b);
  }
  inline void add_and(int a, int b) {
    add_or(a, b);
    add_or(a, -b);
    add_or(-a, b);
  }
  inline void force_true(int x) {
    if (x < 0) x = n - x;
    add_implication(neg(x), x);
  }
  inline void add_xnor(int a, int b) {
    add_or(a, -b);
    add_or(-a, b);
  }
  inline void add_nand(int a, int b) {
    add_or(-a, -b);
  }
  inline void add_nor(int a, int b) {
    add_and(-a, -b);
  }
  inline void force_false(int x) {
    if (x < 0) x = n - x;
    add_implication(x, neg(x));
  }
  inline void topsort(int u) {
    vis[u] = 1;
    for (int v : radj[u]) if (!vis[v]) topsort(v);
    dfs_t[u] = ++intime;
  }
  inline void dfs(int u, int p) {
    par[u] = p, vis[u] = 1;
    for (int v : adj[u]) if (!vis[v]) dfs(v, p);
  }
  void build() {
    int i, x;
    for (i = n * 2, intime = 0;i >= 1;i--) {
      if (!vis[i]) topsort(i);
      ord[dfs_t[i]] = i;
    }
    vis.assign(n * 2 + 1, 0);
    for (i = n * 2;i > 0;i--) {
      x = ord[i];
      if (!vis[x]) dfs(x, x);
    }
  }
  bool satisfy(vector<int>& ret)//ret contains the value
      that are true if the graph is satisfiable.
  {
    build();
    vis.assign(n * 2 + 1, 0);
    for (int i = 1; i <= n * 2; i++) {
      int x = ord[i];
      if (par[x] == par[neg(x)]) return 0;
      if (!vis[par[x]]) {
        vis[par[x]] = 1;
        vis[par[neg(x)]] = 0;
      }
    }
    for (int i = 1;i <= n;i++) if (vis[par[i]])
        ret.push_back(i);
    return 1;
  }
};
```

6.2 BridgeTree [66 lines] - f8e197

```
int N, M, timer, compid;
vector<pair<int, int>> g[mx];
bool used[mx], isBridge[mx];
int comp[mx], tin[mx], minAncestor[mx];
vector<int> Tree[mx]; // Store 2-edge-connected
    component tree.(Bridge tree).
void markBridge(int v, int p) {
  tin[v] = minAncestor[v] = ++timer;
  used[v] = 1;
  for (auto& e : g[v]) {
    int to, id;
    tie(to, id) = e;
    if (to == p) continue;
    if (used[to]) minAncestor[v] = min(minAncestor[v],
        tin[to]);
    else {
      markBridge(to, v);
      minAncestor[v] = min(minAncestor[v],
          minAncestor[to]);
      if (minAncestor[to] > tin[v]) isBridge[id] = true;
      // if (tin[u] <= minAncestor[v]) ap[u] = 1;
    }
  }
}
void markComp(int v, int p) {
  used[v] = 1;
  comp[v] = compid;
  for (auto& e : g[v]) {
    int to, id;
    tie(to, id) = e;
    if (isBridge[id]) continue;
    if (used[to]) continue;
    markComp(to, v);
```

```
  }
}
vector<pair<int, int>> edges;
void addEdge(int from, int to, int id) {
  g[from].push_back({ to, id });
  g[to].push_back({ from, id });
  edges[id] = { from, to };
}
void initB() {
  for (int i = 0; i <= compid; ++i) Tree[i].clear();
  for (int i = 1; i <= N; ++i) used[i] = false;
  for (int i = 1; i <= M; ++i) isBridge[i] = false;
  timer = compid = 0;
}
void bridge_tree() {
  initB();
  markBridge(1, -1); //Assuming graph is connected.
  for (int i = 1; i <= N; ++i) used[i] = 0;
  for (int i = 1; i <= N; ++i) {
    if (!used[i]) {
      markComp(i, -1);
      ++compid;
    }
  }
  for (int i = 1; i <= M; ++i) {
    if (isBridge[i]) {
      int u, v;
      tie(u, v) = edges[i];
      // connect two componets using edge.
      Tree[comp[u]].push_back(comp[v]);
      Tree[comp[v]].push_back(comp[u]);
      int x = comp[u];
      int y = comp[v];
    }
  }
}
```

6.3 Centroid Decomposition [49 lines] - 3fb5b1

```
ll n,subsize[mx];
vector<ll>adj[mx];
char ans[mx];
bool brk[mx];
void calculatesize(ll u,ll par){
  subsize[u]=1;
  for(ll i=0;i<(ll)adj[u].size();i++){
    ll v=adj[u][i];
    if(v==par or brk[v]==true)continue;
    calculatesize(v,u);
    subsize[u]+=subsize[v];
  }
}
ll getcentroid(ll u,ll par,ll n){
  ll ret=u;
  for(ll i=0;i<(ll)adj[u].size();i++){
    ll v=adj[u][i];
    if(v==par or brk[v]==true)continue;
    if(subsize[v]>(n/2)){
      ret=getcentroid(v,u,n);
      break;
    }
  }
  return ret;
}
void decompose(ll u,char rank){
  calculatesize(u,-1);
  ll c=getcentroid(u,-1,subsize[u]);
  brk[c]=true;
  ans[c]=rank;
  for(ll i=0;i<(ll)adj[c].size();i++){
    ll v=adj[c][i];
    if(brk[v]==true)continue;
    decompose(v,rank+1);
  }
}
int main(){
  scanf("%lld",&n);
  for(ll i=0;i<n-1;i++){
    ll a,b;
    scanf("%lld %lld",&a,&b);
    adj[a].push_back(b);
    adj[b].push_back(a);
  }
  decompose(1,'A');
  for(ll i=1;i<=n;i++){
    printf("%c",ans[i]);
  }
}
```

6.4 DSU on Tree [56 lines] - 391fb6

```
int n;
//extra data you need
vector<int> adj[mxn];
vector<int> *dsu[mxn];
void call(int u, int p=-1){
  sz[u] = 1;
  for(auto v: adj[u]){
    if(v != p){
      dep[v] = dep[u]+1;
      call(v, u);
      sz[u] += sz[v];
    }
  }
}
void dfs(int u, int p = -1, int isb = 1){
  int mx=-1, big=-1;
  for(auto v: adj[u]){
    if(v != p && sz[v]>mx){
      mx = sz[v];
      big = v;
    }
  }
  for(auto v: adj[u]){
    if(v != p && v != big){
      dfs(v, u, 0);
    }
  }
  if(big != -1){
    dfs(big, u, 1);
    dsu[u] = dsu[big];
  }
  else{
    dsu[u] = new vector<int>();
  }
  dsu[u]->push_back(u);
  //calculation
  for(auto v: adj[u]){
    if(v == p || v == big) continue;
    for(auto x: *dsu[v]){
      dsu[u]->push_back(x);
      //calculation
    }
  }
  //calculate ans for node u
  if(isb == 0){
    for(auto x: *dsu[u]){
      //reverse calculation
    }
  }
}
int main() {
  //input graph
  dep[1] = 1;
  call(1);
  dfs(1);
}
```

6.5 Heavy Light Decomposition [73 lines] - d0e24f

```
/*Heavy Light Decomposition
Build Complexity O(n)
Query Complexity O(lg^2 n)
Call init()with number of nodes
It's probably for the best to not do"using namespace
    hld"*/
namespace hld {
  //N is the maximum number of nodes
  /*par,lev,size corresponds to
      parent,depth,subtree-size*/
  //head[u]is the starting node of the chain u is in
  //in[u]to out[u]keeps the subtree indices
  const int N=100000+7;
  vector<int>g[N];
  int par[N],lev[N],head[N],size[N],in[N],out[N];
  int cur_pos,n;
  //returns the size of subtree rooted at u
  /*maintains the child with the largest subtree at the
      front of g[u]*/
  //WARNING: Don't change anything here specially with
      size[]if Jon Snow
  int dfs(int u,int p){
    size[u]=1,par[u]=p;
    lev[u]=lev[p]+1;
    for(auto &v : g[u]){
      if(v==p)continue;
      size[u]+=dfs(v,u);
      if(size[v]>size[g[u].front()]){
        swap(v,g[u].front());
      }
    }
    return size[u];
  }
  //decomposed the tree in an array
  //note that there is no physical array here
  void decompose(int u,int p){
    in[u]=++cur_pos;
    for(auto &v : g[u]){
      if(v==p)continue;
      head[v]=(v==g[u].front()? head[u]: v);
      decompose(v,u);
    }
    out[u]=cur_pos;
  }
  //initializes the structure with _n nodes
  void init(int _n,int root=1){
```

```
    n=_n;
    cur_pos=0;
    dfs(root,0);
    head[root]=root;
    decompose(root,0);
  }
  //checks whether p is an ancestor of u
  bool isances(int p,int u){
    return in[p]<=in[u]and out[u]<=out[p];
  }
  //Returns the maximum node value in the path u-v
  ll query(int u,int v){
    ll ret=-INF;
    while(!isances(head[u],v)){
      ret=max(ret,seg.query(1,1,n,in[head[u]],in[u]));
      u=par[head[u]];
    }
    swap(u,v);
    while(!isances(head[u],v)){
      ret=max(ret,seg.query(1,1,n,in[head[u]],in[u]));
      u=par[head[u]];
    }
    if(in[v]<in[u])swap(u,v);
    ret=max(ret,seg.query(1,1,n,in[u],in[v]));
    return ret;
  }
  //Adds val to subtree of u
  void update(int u,ll val){
    seg.update(1,1,n,in[u],out[u],val);
  }
};
```

6.6 K'th Shortest path [40 lines] - 9f3788

```
int m,n,deg[MM],source,sink,K,val[MM][12];
struct edge{
  int v,w;
}adj[MM][500];
struct info{
  int v,w,k;
  bool operator<(const info &b)const{
    return w>b.w;
  }
};
priority_queue<info,vector<info>>Q;
void kthBestShortestPath(){
  int i,j;
  info u,v;
  for(i=0;i<n;i++)
    for(j=0;j<K;j++)val[i][j]=inf;
  u.v=source,u.k=0,u.w=0;
  Q.push(u);
  while(!Q.empty()){
    u=Q.top();
    Q.pop();
    for(i=0;i<deg[u.v];i++){
      v.v=adj[u.v][i].v;
      int cost=adj[u.v][i].w+u.w;
      for(v.k=u.k;v.k<K;v.k++){
        if(cost==inf)break;
        if(val[v.v][v.k]>cost){
          swap(cost,val[v.v][v.k]);
          v.w=val[v.v][v.k];
          Q.push(v);
          break;
        }
      }
      for(v.k++;v.k<K;v.k++){
        if(cost==inf)break;
        if(val[v.v][v.k]>cost)swap(cost, val[v.v][v.k]);
      }
    }
  }
}
```

6.7 LCA [46 lines] - 9de12b

```
const int Lg = 22;
vector<int>adj[mx];
int level[mx];
int dp[Lg][mx];
void dfs(int u) {
  for (int i = 1;i < Lg;i++)
    dp[i][u] = dp[i - 1][dp[i - 1][u]];
  for (int v : adj[u]) {
    if (dp[0][u] == v)continue;
    level[v] = level[u] + 1;
    dp[0][v] = u;
    dfs(v);
  }
}
int lca(int u, int v) {
  if (level[v] < level[u])swap(u, v);
  int diff = level[v] - level[u];
  for (int i = 0;i < Lg;i++)
    if (diff & (1 << i))
      v = dp[i][v];
  for (int i = Lg - 1;i >= 0;i--)
    if (dp[i][u] != dp[i][v])
      u = dp[i][u], v = dp[i][v];
  return u == v ? u : dp[0][u];
}
int kth(int u, int k) {
  for (int i = Lg - 1;i >= 0;i--)
    if (k & (1 << i))
      u = dp[i][u];
  return u;
}
//kth node from u to v. 0th is u.
int go(int u, int v, int k) {
  int l = lca(u, v);
  int d = level[u] + level[v] - (level[l] << 1);
  assert(k <= d);
  if (level[l] + k <= level[u]) return kth(u, k);
  k -= level[u] - level[l];
  return kth(v, level[v] - level[l] - k);
}
/*
   LCA(u,v) with root r:
   lca(u,v)^lca(u,r)^lca(v,r)
   Distance between u,v:
   level(u) + level(v) - 2*level(lca(u,v))
   */
```

6.8 SCC [43 lines] - 4da431

```
/*components: number of SCC.
sz: size of each SCC.
comp: component number of each node.
Create reverse graph.
Run find_scc() to find SCC.
Might need to create condensation graph by
    create_condensed().
Think about indeg/outdeg
for multiple test cases- clear
    adj/radj/comp/vis/sz/topo/condensed.*/
vector<int>adj[mx], radj[mx];

int comp[mx], vis[mx], sz[mx], components;
vector<int>topo;
void dfs(int u) {
  vis[u] = 1;
  for (int v : adj[u])
    if (!vis[v]) dfs(v);
  topo.push_back(u);
}
void dfs2(int u, int val) {
  comp[u] = val;
  sz[val]++;
  for (int v : radj[u])
    if (comp[v] == -1)
      dfs2(v, val);
}
void find_scc(int n) {
  memset(vis, 0, sizeof vis);
  memset(comp, -1, sizeof comp);
  for (int i = 1;i <= n;i++)
    if (!vis[i])
      dfs(i);
  reverse(topo.begin(), topo.end());
  for (int u : topo)
    if (comp[u] == -1)
      dfs2(u, ++components);
}
vector<int>condensed[mx];
void create_condensed(int n) {
  for (int i = 1;i <= n;i++)
    for (int v : adj[i])
      if (comp[i] != comp[v])
        condensed[comp[i]].push_back(comp[v]);
}
```

7 Math

7.1 Big Sum [13 lines] - 8d9520

```
ll bigsum(ll a, ll b, ll m) {
  if (b == 0) return 0;
  ll sum; a %= m;
  if (b & 1) {
    sum = bigsum((a * a) % m, (b - 1) / 2, m);
    sum = (sum + (a * sum) % m) % m;
    sum = (1 + (a * sum) % m) % m;
  } else {
    sum = bigsum((a * a) % m, b / 2, m);
    sum = (sum + (a * sum) % m) % m;
  }
  return sum;
}
```

7.2 CRT [52 lines] - 59a568

```
ll ext_gcd(ll A, ll B, ll* X, ll* Y) {
  ll x2, y2, x1, y1, x, y, r2, r1, q, r;
  x2 = 1; y2 = 0;
  x1 = 0; y1 = 1;
  for (r2 = A, r1 = B; r1 != 0; r2 = r1, r1 = r, x2 =
      x1, y2 = y1, x1 = x, y1 = y) {
    q = r2 / r1;
```

```
    r = r2 % r1;
    x = x2 - (q * x1);
    y = y2 - (q * y1);
  }
  *X = x2; *Y = y2;
  return r2;
}
/*------------BlackBox-------------*/
class ChineseRemainderTheorem {
  typedef long long vlong;
  typedef pair<vlong, vlong> pll;
  /** CRT Equations stored as pairs of vector. See
      addEquation()*/
  vector<pll> equations;
  public:
  void clear() {
    equations.clear();
  }
  /** Add equation of the form x = r (mod m)*/
  void addEquation(vlong r, vlong m) {
    equations.push_back({ r, m });
  }
  pll solve() {
    if (equations.size() == 0) return { -1,-1 }; /// No
        equations to solve
    vlong a1 = equations[0].first;
    vlong m1 = equations[0].second;
    a1 %= m1;
    /** Initially x = a_0 (mod m_0)*/
    /** Merge the solution with remaining equations */
    for (int i = 1; i < equations.size(); i++) {
      vlong a2 = equations[i].first;
      vlong m2 = equations[i].second;
      vlong g = __gcd(m1, m2);
      if (a1 % g != a2 % g) return { -1,-1 }; ///
          Conflict in equations
      /** Merge the two equations*/
      vlong p, q;
      ext_gcd(m1 / g, m2 / g, &p, &q);
      vlong mod = m1 / g * m2;
      vlong x = ((__int128)a1 * (m2 / g) % mod * q % mod
          + (__int128)a2 * (m1 / g) % mod * p % mod) %
          mod;
      /** Merged equation*/
      a1 = x;
      if (a1 < 0) a1 += mod;
      m1 = mod;
    }
    return { a1, m1 };
  }
};
```

7.3 FFT [85 lines] - 4ca8f0

```
template<typename float_t>
struct mycomplex {
  float_t x, y;
  mycomplex<float_t>(float_t _x = 0, float_t _y = 0) :
      x(_x), y(_y) {}
  float_t real() const { return x; }
  float_t imag() const { return y; }
  void real(float_t _x) { x = _x; }
  void imag(float_t _y) { y = _y; }
  mycomplex<float_t>& operator+=(const
      mycomplex<float_t> &other) { x += other.x; y +=
      other.y; return *this; }
  mycomplex<float_t>& operator-=(const
      mycomplex<float_t> &other) { x -= other.x; y -=
      other.y; return *this; }
  mycomplex<float_t> operator+(const mycomplex<float_t>
      &other) const { return mycomplex<float_t>(*this)
      += other; }
  mycomplex<float_t> operator-(const mycomplex<float_t>
      &other) const { return mycomplex<float_t>(*this)
      -= other; }
  mycomplex<float_t> operator*(const mycomplex<float_t>
      &other) const {
    return {x * other.x - y * other.y, x * other.y +
        other.x * y};
  }
  mycomplex<float_t> operator*(float_t mult) const {
    return {x * mult, y * mult};
  }
  friend mycomplex<float_t> conj(const
      mycomplex<float_t> &c) {
    return {c.x, -c.y};
  }
  friend ostream& operator<<(ostream &stream, const
      mycomplex<float_t> &c) {
    return stream << '(' << c.x << ", " << c.y << ')';
  }
};
using cd = mycomplex<double>;
void fft(vector<cd> & a, bool invert) {
  int n = a.size();
  for (int i = 1, j = 0; i < n; i++) {
    int bit = n >> 1;
    for (; j & bit; bit >>= 1)
      j ^= bit;
    j ^= bit;
    if (i < j)
      swap(a[i], a[j]);
  }
  for (int len = 2; len <= n; len <<= 1) {
    double ang = 2 * PI / len * (invert ? -1 : 1);
    cd wlen(cos(ang), sin(ang));
    for (int i = 0; i < n; i += len) {
      cd w(1);
      for (int j = 0; j < len / 2; j++) {
        cd u = a[i+j], v = a[i+j+len/2] * w;
        a[i+j] = u + v;
        a[i+j+len/2] = u - v;
        w = w*wlen;
      }
    }
  }
  if (invert) {
    for (cd & x : a){
      double z = n;
      z=1/z;
      x = x*z;
    }
    // x /= n;
  }
}
void multiply (const vector<bool> & a, const
    vector<bool> & b, vector<bool> & res) {//change all
    the bool to your type needed
  vector<cd> fa (a.begin(), a.end()),  fb (b.begin(),
      b.end());
  size_t n = 1;
  while (n < max (a.size(), b.size()))  n <<= 1;
  n <<= 1;
  fa.resize (n),  fb.resize (n);
  fft (fa, false),  fft (fb, false);
  for (size_t i=0; i<n; ++i)
    fa[i] =fa[i] * fb[i];
  fft (fa, true);
  res.resize (n);
  for (size_t i=0; i<n; ++i)
    res[i] = round(fa[i].real());
  while(res.back()==0) res.pop_back();
}
void pow(const vector<bool> &a, vector<bool> &res, long
    long int k){
  vector<bool> po=a;
  res.resize(1);
  res[0] = 1;
  while(k){
    if(k&1){
      multiply(po, res, res);
    }
    multiply(po, po, po);
    k/=2;
  }
}
```

7.4 GaussElimination [39 lines] - aa53e0

```
template<typename ld>
int gauss(vector<vector<ld>>& a, vector<ld>& ans) {
  const ld EPS = 1e-9;
  int n = a.size();///number of equations
  int m = a[0].size() - 1;///number of variables
  vector<int>where(m, -1);///indicates which row
      contains the solution
  int row, col;
  for (col = 0, row = 0;col < m && row < n;++col) {
    int sel = row;///which row contains the maximum
        value/
    for (int i = row + 1;i < n;i++)
      if (abs(a[i][col]) > abs(a[sel][col]))
        sel = i;
    if (abs(a[sel][col]) < EPS) continue;///it's
        basically 0.
    a[sel].swap(a[row]);///taking the max row up
    where[col] = row;
    ld t = a[row][col];
    for (int i = col;i <= m;i++) a[row][i] /= t;
    for (int i = 0;i < n;i++) {
      if (i != row) {
        ld c = a[i][col];
        for (int j = col;j <= m;j++)
          a[i][j] -= a[row][j] * c;
      }
    }
    row++;
  }
  ans.assign(m, 0);
  for (int i = 0;i < m;i++)
    if (where[i] != -1)
      ans[i] = a[where[i]][m] / a[where[i]][i];
  for (int i = 0;i < n;i++) {
    ld sum = 0;
```

```
    for (int j = 0;j < m;j++)
      sum += ans[j] * a[i][j];
    if (abs(sum - a[i][m]) > EPS) ///L.H.S!=R.H.S
      ans.clear();//No solution
  }
  return row;
}
```

7.5 GaussMod2 [44 lines] - e8fae4

```
template<typename T>
struct Gauss {
  int bits = 60;
  vector<T>table;
  Gauss() {
    table = vector<T>(bits, 0);
  }
  //call with constructor to define bit size.
  Gauss(int _bits) {
    bits = _bits;
    table = vector<T>(bits, 0);
  }
  int basis()//return rank/size of basis
  {
    int ans = 0;
    for (int i = 0;i < bits;i++)
      if (table[i])
        ans++;
    return ans;
  }
  bool can(T x)//can x be obtained from the basis
  {
    for (int i = bits - 1;i >= 0;i--) x = min(x, x ^
        table[i]);
    return x == 0;
  }
  void add(T x) {
    for (int i = bits - 1;i >= 0 && x;i--) {
      if (table[i] == 0) {
        table[i] = x;
        x = 0;
      }
      else x = min(x, x ^ table[i]);
    }
  }
  T getBest() {
    T x = 0;
    for (int i = bits - 1;i >= 0;i--)
      x = max(x, x ^ table[i]);
    return x;
  }
  void Merge(Gauss& other) {
    for (int i = bits - 1;i >= 0;i--)
      add(other.table[i]);
  }
};
```

7.6 Karatsuba Idea [5 lines] - 6944e1

```
Three subproblems:
a = xH yH
d = xL yL
e = (xH + xL)(yH + yL) - a - d
Then xy = a rn + e rn/2 + d
```

7.7 Linear Diophatine [19 lines] - 7c6f05

```
int extended_gcd(ll a, ll b, ll& x, ll& y) {
  if (b == 0){x = 1;y = 0;return a;}
  ll x1, y1;
  ll d = extended_gcd(b, a % b, x1, y1);
  x = y1;y = x1 - y1 * (a / b);
  return d;
}
/*x'=x+(k*B/g),y'=y-(k*A/g);infinite soln
if A=B=0,C must equal 0 and any x,y is solution;
if A|B=0,(x,y)=(C/A,k)|(k,C/B)*/
bool LDE(ll A,ll B,ll C,ll &x,ll &y){
  int g=gcd(A,B);
  if(C%g!=0)return false;
  int a=A/g,b=B/g,c=C/g;
  extended_gcd(a,b,x,y);//ax+by=1
  if(g<0){a*=-1;b*=-1;c*=-1;}//Ensure gcd(a,b)=1
  x*=c;y*=c;//ax+by=c
  return true;//Solution Exists
}
```

7.8 Matrix [100 lines] - a33f18

```
template<typename T>
struct Matrix {
  T MOD = 1e9 + 7;///change if necessary
  T add(T a, T b) const {
    T res = a + b;
    if (res >= MOD) return res - MOD;
    return res;
  }
  T sub(T a, T b) const {
    T res = a - b;
    if (res < 0) return res + MOD;
    return res;
  }
  T mul(T a, T b) const {
    T res = a * b;
    if (res >= MOD) return res % MOD;
    return res;
  }
  int R, C;
  vector<vector<T>>mat;
  Matrix(int _R = 0, int _C = 0) {
    R = _R, C = _C;
    mat.resize(R);
    for (auto& v : mat) v.assign(C, 0);
  }
  void print() {
    for (int i = 0;i < R;i++)
      for (int j = 0;j < C;j++)
        cout << mat[i][j] << " \n"[j == C - 1];
  }
  void createIdentity() {
    for (int i = 0; i < R; i++)
      for (int j = 0; j < C; j++)
        mat[i][j] = (i == j);
  }
  Matrix operator+(const Matrix& o) const {
    Matrix res(R, C);
    for (int i = 0; i < R; i++)
      for (int j = 0; j < C; j++)
        res[i][j] = add(mat[i][j] + o.mat[i][j]);
  }
  Matrix operator-(const Matrix& o) const {
    Matrix res(R, C);
    for (int i = 0; i < R; i++)
      for (int j = 0; j < C; j++)
        res[i][j] = sub(mat[i][j] + o.mat[i][j]);
  }
  Matrix operator*(const Matrix& o) const {
    Matrix res(R, o.C);
    for (int i = 0; i < R; i++)
      for (int j = 0; j < o.C; j++)
        for (int k = 0;k < C;k++)
          res.mat[i][j] = add(res.mat[i][j],
              mul(mat[i][k], o.mat[k][j]));
    return res;
  }
  Matrix pow(long long x) {
    Matrix res(R, C);
    res.createIdentity();
    Matrix<T> o = *this;
    while (x) {
      if (x & 1) res = res * o;
      o = o * o;
      x >>= 1;
    }
    return res;
  }
  Matrix inverse()///Only square matrix && non-zero
      determinant
  {
    Matrix res(R, R + R);
    for (int i = 0;i < R;i++) {
      for (int j = 0;j < R;j++)
        res.mat[i][j] = mat[i][j];
      res.mat[i][R + i] = 1;
    }
    for (int i = 0;i < R;i++) {
      ///find row 'r' with highest value at [r][i]
      int tr = i;
      for (int j = i + 1;j < R;j++)
        if (abs(res.mat[j][i]) > abs(res.mat[tr][i]))
          tr = j;
      ///swap the row
      res.mat[tr].swap(res.mat[i]);
      ///make 1 at [i][i]
      T val = res.mat[i][i];
      for (int j = 0;j < R + R;j++) res.mat[i][j] /=
          val;
      ///eliminate [r][i] from every row except i.
      for (int j = 0;j < R;j++) {
        if (j == i) continue;
        for (int k = R + R - 1;k >= i;k--) {
          res.mat[j][k] -= res.mat[i][k] * res.mat[j][i]
              / res.mat[i][i];
        }
      }
    }
    Matrix ans(R, R);
    for (int i = 0;i < R;i++)
      for (int j = 0;j < R;j++)
        ans.mat[i][j] = res.mat[i][R + j];
    return ans;
  }
};
```

7.9 Miller-Rabin-Pollard-Rho [68 lines] - 3e3e5f

```
ll powmod(ll a, ll p, ll m) {///(a^p % m)
  ll result = 1;
  a %= m;
  while (p) {
    if (p & 1)
      result = (vll)result * a % m;
    a = (vll)a * a % m;
    p >>= 1;
  }
  return result;
}
bool check_composite(ll n, ll a, ll d, int s) {
  ll x = powmod(a, d, n);
  if (x == 1 || x == n - 1)
    return false;
  for (int r = 1; r < s; r++) {
    x = (vll)x * x % n;
    if (x == n - 1)
      return false;
  }
  return true;
}
bool MillerRabin(ll n) {
  if (n < 2) return false;
  int r = 0;
  ll d = n - 1;
  while ((d & 1) == 0) {
    d >>= 1;
    r++;
  }
  for (int a : {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31,
      37}) {
    if (n == a) return true;
    if (check_composite(n, a, d, r))
      return false;
  }
  return true;
}
ll mult(ll a, ll b, ll mod) {
  return (vll)a * b % mod;
}
ll f(ll x, ll c, ll mod) {
  return (mult(x, x, mod) + c) % mod;
}
ll rho(ll n) {
  if (n % 2 == 0) return 2;
  ll x = myrand() % n + 1, y = x, c = myrand() % n + 1,
      g = 1;
  while (g == 1) {
    x = f(x, c, n);
    y = f(y, c, n);
    y = f(y, c, n);
    g = __gcd(abs(x - y), n);
  }
  return g;
}
set<ll>prime;
void prime_factorization(ll n) {
  if (n == 1) return;
  if (MillerRabin(n)) {
    prime.insert(n);
    return;
  }
  ll x = n;
  while (x == n) x = rho(n);
  prime_factorization(x);
  prime_factorization(n / x);
}
//call prime_factorization(n) for prime factors.
//call MillerRabin(n) to check if prime.
```

7.10 Mod Inverse [5 lines] - 772679

```
int modInv(int a, int m) {
    int x, y; //if g==1 Inverse doesn't exist
    int g = gcdExt(a, m, x, y);
    return (x % m + m) % m;
}
```

7.11 NTT [96 lines] - 6faca3

```
ll power(ll a, ll p, ll mod) {
  if (p==0)   return 1;
  ll ans = power(a, p/2, mod);
  ans = (ans * ans)%mod;
  if(p%2)    ans = (ans * a)%mod;
  return ans;
}
int primitive_root(int p) {
  vector<int> factor;
  int phi = p-1, n = phi;
  for (int i=2; i*i<=n; i++) {
    if (n%i)    continue;
    factor.push_back(i);
    while (n%i==0)  n/=i;
  }
  if (n>1)    factor.push_back(n);
  for (int res =2; res<=p; res++) {
    bool ok = true;
    for (int i=0; i<factor.size() && ok; i++)
      ok &= power(res, phi/factor[i], p) != 1;
    if (ok) return res;
  }
  return -1;
}
int nttdata(int mod, int &root, int &inv, int &pw) {
  int c = 0, n = mod-1;
  while (n%2==0)  c++, n/=2;
  pw = (mod-1)/n;
  int g = primitive_root(mod);
  root = power(g, n, mod);
  inv = power(root, mod-2, mod);
  return c;
}
const int M = 786433;
struct NTT {
  int N;
  vector<int> perm;
  int mod, root, inv, pw;
  NTT(){}
  NTT(int mod, int root, int inv, int pw) : mod(mod),
      root(root), inv(inv), pw(pw) {}
  void precalculate() {
    perm.resize(N);
    perm[0] = 0;
    for (int k=1; k<N; k<<=1) {
      for (int i=0; i<k; i++) {
        perm[i] <<= 1;
        perm[i+k] = 1 + perm[i];
      }
    }
  }
  void fft(vector<ll> &v, bool invert = false) {
    if (v.size() != perm.size()) {
      N = v.size();
      assert(N && (N&(N-1)) == 0);
      precalculate();
    }
    for (int i=0; i<N; i++)
      if (i < perm[i])
        swap(v[i], v[perm[i]]);
    for (int len = 2; len <= N; len <<=1) {
      ll factor = invert ? inv: root;
      for (int i=len; i<pw; i<<=1)
        factor = (factor * factor) % mod;
      for (int i=0; i<N; i+=len) {
        ll w = 1;
        for (int j=0; j<len/2; j++) {
          ll x = v[i+j], y = (w*v[i+j+len/2])%mod;
          v[i+j] = (x+y)%mod;
          v[i+j+len/2] = (x-y+mod)%mod;
          w = (w*factor)%mod;
        }
      }
    }
    if (invert) {
      ll n1 = power(N, mod-2, mod);
      for (ll &x: v)  x = (x*n1)%mod;
    }
  }
  vector<ll> multiply(vector<ll> a, vector<ll> &b) {
    while (a.size() && a.back() == 0)   a.pop_back();
    while (b.size() && b.back() == 0)   b.pop_back();
    int n = 1;
    while (n < a.size() + b.size()) n<<=1;
    a.resize(n);
    b.resize(n);
    fft(a);
    fft(b);
    for (int i=0; i<n; i++) a[i] = (a[i] * b[i])%M;
    fft(a, true);
    while (a.size() && a.back() == 0)   a.pop_back();
    return a;
  }
  //     int mod=786433, root, inv, pw;
  //     nttdata(mod, root, inv, pw);
  //     NTT nn = NTT(mod, root, inv, pw);
};
```

7.12 No of Digits in n! in base B [7 lines] - 86bfaf

```
ll NoOfDigitInNFactInBaseB(ll N,ll B){
  ll i;
  double ans=0;
  for(i=1;i<=N;i++)ans+=log(i);
  ans=ans/log(B),ans=ans+1;
  return(ll)ans;
}
```

7.13 SOD Upto N [16 lines] - d8aa2c

```
ll SOD_UpTo_N(ll N){
  ll i,j,ans=0;///upto N in Sqrt(N)
  for(i=1;i*i<=N;i++){
    j=N/i;
    ans+=((j*(j+1))/2)-(((i-1)*i)/2);
```

```
    ans+=((j-i)*i);
  }
  return ans;
}
ll SODUptoN(ll N){
  ll res=0,u=sqrt(N);
  for(ll i=1;i<=u;i++)
    res+=(N/i)-i;
  res*=2,res+=u;
  return res;
}
```

7.14 Sieve Phi Mobius [26 lines] - 353c39

```
const int N = 1e7;
vector<int>pr;
int mu[N + 1], phi[N + 1], lp[N + 1];
void sieve() {
  phi[1] = 1, mu[1] = 1;
  for (int i = 2; i <= N; i++) {
    if (lp[i] == 0) {
      lp[i] = i;
      phi[i] = i - 1;
      pr.push_back(i);
    }
    for (int j = 0; j < pr.size() && i * pr[j] <= N;
        j++) {
      lp[i * pr[j]] = pr[j];
      if (i % pr[j] == 0) {
        phi[i * pr[j]] = phi[i] * pr[j];
        break;
      }
      else
        phi[i * pr[j]] = phi[i] * phi[pr[j]];
    }
  }
  for (int i = 2;i <= N;i++) {
    if (lp[i / lp[i]] == lp[i]) mu[i] = 0;
    else mu[i] = -1 * mu[i / lp[i]];
  }
}
```

8 Misc

8.1 Bit hacks [12 lines] - dd22ef

```
# x & -x is the least bit in x.
# iterate over all the subsets of the mask
for (int s=m; ; s=(s-1)&m) {
 ... you can use s ...
 if (s==0)  break;
}
# c = x&-x, r = x+c; (((r^x) >> 2)/c) | r is the
next number after x with the same number of bits set.
# __builtin_popcount(x) //number of ones in binary
  __builtin_popcountll(x) // for long long
# __builtin_clz(x) // number of leading zeros
  __builtin_ctz(x) // number of trailing zeros, they
     also have long long version
```

8.2 Bitset C++ [13 lines] - a6a7a4

```
bitset<17>BS;
BS[1] = BS[7] = 1;
cout<<BS._Find_first()<<endl; // prints 1
bs._Find_next(idx). This function returns first set bit
    after index idx.for example:

bitset<17>BS;
BS[1] = BS[7] = 1;
cout<<BS._Find_next(1)<<','<<BS._Find_next(3)<<endl; //
    prints 7,7
So this code will print all of the set bits of BS:

for(int i=BS._Find_first();i< BS.size();i =
    BS._Find_next(i))
    cout<<i<<endl;
//Note that there isn't any set bit after idx,
    BS._Find_next(idx) will return BS.size(); same as
    calling BS._Find_first() when bitset is clear;
```

8.3 Template [33 lines] - 7aea62

```
// #pragma GCC optimize("O3,unroll-loops")
// #pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;

template <typename A, typename B> ostream&
    operator<<(ostream& os, const pair<A, B>& p) {
    return os << '(' << p.first << ", " << p.second <<
    ')'; }
template <typename T_container, typename T = typename
    enable_if<!is_same<T_container, string>::value,
    typename T_container::value_type>::type> ostream&
    operator<<(ostream& os, const T_container& v) { os
    << '{'; string sep; for (const T& x : v) os << sep
    << x, sep = ", "; return os << '}'; }
void dbg_out() { cerr << endl; }
template <typename Head, typename... Tail> void
    dbg_out(Head H, Tail... T) { cerr << " " << H;
    dbg_out(T...); }

#ifdef SMIE
#define debug(args...) cerr << "(" << #args << "):",
    dbg_out(args)
#else
#define debug(args...)
#endif

template <typename T> inline T gcd(T a, T b) { T c;while
    (b) { c = b;b = a % b;a = c; }return a; } // better
    than __gcd
ll powmod(ll a, ll b, ll MOD) { ll res = 1;a %=
    MOD;assert(b >= 0);for (; b; b >>= 1) { if (b &
    1)res = res * a % MOD;a = a * a % MOD; }return res;
    }
template <typename T>using orderedSet = tree<T,
    null_type, less_equal<T>, rb_tree_tag,
    tree_order_statistics_node_update>;
//order_of_key(k) - number of element strictly less than
    k
//find_by_order(k) - k'th element in set.(0
    indexed)(iterator)

mt19937
rng(chrono::steady_clock::now().time_since_epoch()
    .count());
//uniform_int_distribution<int>(0, i)(rng)
int main(int argc, char* argv[]) {
  ios_base::sync_with_stdio(false);//DON'T CC++
  cin.tie(NULL);//DON'T use for interactive
  int seed = atoi(argv[1]);
}
```

8.4 build [2 lines] - 801989

```
#!/bin/bash
>&2 echo -e "Making [$2]\t: $1.cpp" && g++ -std=gnu++17
    -Wshadow -Wall -Wextra -Wno-unused-result -O2 -g
    -fsanitize=undefined -fsanitize=address $2 "$1.cpp"
    -o "$1"
```

8.5 check [15 lines] - 478053

```
#!/bin/bash
build $1
TESTNO=0
for INP in $1.in*; do
  printf "\n=========\n"
  printf "INPUT %d" $TESTNO
  printf "\n=========\n"
    cat $INP
  printf "\n=========\n"
  printf "OUTPUT %d" $TESTNO
  printf "\n=========\n"
    ./$1 < $INP
    mv $INP $1.in$TESTNO 2>/dev/null
    TESTNO=$((TESTNO+1))
done
```

8.6 debug [3 lines] - 859f78

```
#!/bin/bash
build "$1" -DSMIE && >&2 echo -e "Running\t\t:
    $1\n------------------------------" && "./$1"
```

8.7 stress [15 lines] - 62e61a

```
#!/bin/bash
build $1 $2 && build $1_gen $2 && build $1_brute $2 &&
for((i = 1; ; ++i)); do
    echo -e "\nTest Case "$i
    ./$1_gen $i > inp
    ./$1 < inp > out1
    ./$1_brute < inp > out2
    diff -w out1 out2 || break
done
echo -e "===========\nINPUT\n-----"
cat inp
echo -e "\nOUTPUT\n------"
cat out1
echo -e "\nEXPECTED\n--------"
cat out2
```

8.8 vimrc [14 lines] - ffdf4e

```
filetype plugin indent on
set rnu wfw hls is ar aw wrap mouse=a

let mapleader=' '
im jk <esc>
tno jk <c-w>N
no <leader>d "_d
im {<cr> {<cr>}<esc>O
nn ff :let @+ = expand("%:p")<cr>
nn cd :cd %:h<cr>

au BufNewFile *.cpp -r ./template.cpp | 14

ca hash w !cpp -dD -P -fpreprocessed \| tr -d
    '[:space:]' \| md5sum \| cut -c-6
```

9 String

9.1 Aho-Corasick [124 lines] - 2d8d6c

```
const int NODE=3000500;///Maximum Nodes
const int LGN=30;     ///Maximum Number of Tries
const int MXCHR=53;   ///Maximum Characters
const int MXP=5005;   ///
struct node {
  int val;
  int child[MXCHR];
  vector<int>graph;
  void clear(){
    CLR(child,0);
    val=0;
    graph.clear();
  }
}Trie[NODE+10];
int maxNodeId,fail[NODE+10],par[NODE+10];
int nodeSt[NODE+10],nodeEd[NODE+10];
vlong csum[NODE+10],pLoc[MXP];
void resetTrie(){
  maxNodeId=0;
}
int getNode(){
  int curNodeId=++maxNodeId;
  Trie[curNodeId].clear();
  return curNodeId;
}
inline void upd(vlong pos){
  csum[pos]++;
}
inline vlong qry(vlong pos){
  vlong res=csum[pos];
  return res;
}
struct AhoCorasick {
  int root,size,euler;
  void clear(){
    root=getNode();
    size=euler=0;
  }
  inline int getname(char ch){
    if(ch=='-')return 52;
    else if(ch>='A' && ch<='Z')return 26+(ch-'A');
    else return(ch-'a');
  }
  void addToTrie(string &s,int id){
  //Add string s to the Trie in general way
    int len=SZ(s),cur=root;
    FOR(i,0,len-1){
      int c=getname(s[i]);
      if(Trie[cur].child[c]==0){
        int curNodeId=getNode();
        Trie[curNodeId].val=c;
        Trie[cur].child[c]=curNodeId;
      }
      cur=Trie[cur].child[c];
    }
    pLoc[id]=cur;
    size++;
  }
  void calcFailFunction(){
    queue<int>Q;
    Q.push(root);
    while(!Q.empty()){
      int s=Q.front();
      Q.pop();
    //Add all the children to the queue:
      FOR(i,0,MXCHR-1){
        int t=Trie[s].child[i];
        if(t!=0){
          Q.push(t);
          par[t]=s;
        }
      }
      if(s==root){/*Handle special case when s is
          root*/
        fail[s]=par[s]=root;
        continue;
      }
  //Find fall back of s:
      int p=par[s],f=fail[p];;
      int val=Trie[s].val;
  /*Fall back till you found a node who has got val as a
      child*/
      while(f!=root && Trie[f].child[val]==0){
        f=fail[f];
      }
      fail[s]=(Trie[f].child[val]==0)? root :
          Trie[f].child[val];
  //Self fall back not allowed
      if(s==fail[s]){
        fail[s]=root;
      }
      Trie[fail[s]].graph.push_back(s);
    }
  }
  void dfs(int pos){
    ++euler;
    nodeSt[pos]=euler;
    for(auto x: Trie[pos].graph){
      dfs(x);
    }
    nodeEd[pos]=euler;
  }
  //Returns the next state
  int goTo(int state,int c){
    if(Trie[state].child[c]!=0){/*No need to fall
        back*/
      return Trie[state].child[c];
    }
  //Fall back now:
    int f=fail[state];
    while(f!=root && Trie[f].child[c]==0){
      f=fail[f];
    }
    int res=(Trie[f].child[c]==0)?
        root:Trie[f].child[c];
    return res;
  }
  /*Iterate through the whole text and find all the
      matchings*/
  void findmatching(string &s){
    int cur=root,idx=0;
    int len=SZ(s);
    while(idx<len){
      int c=getname(s[idx]);
      cur=goTo(cur,c);
      upd(nodeSt[cur]);
      idx++;
    }
  }
}acorasick;
```

9.2 Double Hasing [50 lines] - 1a70c1

```
struct SimpleHash {
    int len;
    long long base, mod;
    vector<int> P, H, R;
    SimpleHash() {}
    SimpleHash(string str, long long b, long long m) {
        base = b, mod = m, len = str.size();
        P.resize(len + 4, 1), H.resize(len + 3, 0),
            R.resize(len + 3, 0);
        for (int i = 1; i <= len + 3; i++)
            P[i] = (P[i - 1] * base) % mod;
        for (int i = 1; i <= len; i++)
            H[i] = (H[i - 1] * base + str[i - 1] + 1007)
                % mod;
        for (int i = len; i >= 1; i--)
            R[i] = (R[i + 1] * base + str[i - 1] + 1007)
                % mod;
    }
    inline int range_hash(int l, int r) {
        int hashval = H[r + 1] - ((long long)P[r - l +
            1] * H[l] % mod);
        return (hashval < 0 ? hashval + mod : hashval);
    }
    inline int reverse_hash(int l, int r) {
        int hashval = R[l + 1] - ((long long)P[r - l +
            1] * R[r + 2] % mod);
        return (hashval < 0 ? hashval + mod : hashval);
    }
};
struct DoubleHash {
    SimpleHash sh1, sh2;
    DoubleHash() {}
    DoubleHash(string str) {
        sh1 = SimpleHash(str, 1949313259, 2091573227);
        sh2 = SimpleHash(str, 1997293877, 2117566807);
    }
    long long concate(DoubleHash& B , int l1 , int r1 ,
        int l2 , int r2) {
        int len1 = r1 - l1+1 , len2 = r2 - l2+1;
        long long x1 = sh1.range_hash(l1, r1) ,
        x2 = B.sh1.range_hash(l2, r2);
        x1 = (x1 * B.sh1.P[len2]) % 2091573227;
        long long newx1 = (x1 + x2) % 2091573227;
        x1 = sh2.range_hash(l1, r1);
        x2 = B.sh2.range_hash(l2, r2);
        x1 = (x1 * B.sh2.P[len2]) % 2117566807;
        long long newx2 = (x1 + x2) % 2117566807;
        return (newx1 << 32) ^ newx2;
    }
    inline long long range_hash(int l, int r) {
        return ((long long)sh1.range_hash(l, r) << 32) ^
            sh2.range_hash(l, r);
    }
    inline long long reverse_hash(int l, int r) {
        return ((long long)sh1.reverse_hash(l, r) << 32)
            ^ sh2.reverse_hash(l, r);
    }
};
```

9.3 KMP [23 lines] - 99c570

```
char P[maxn],T[maxn];
int b[maxn],n,m;
void kmpPreprocess(){
  int i=0,j=-1;
  b[0]=-1;
  while(i<m){
    while(j>=0 and P[i]!=P[j])
      j=b[j];
      i++;j++;
      b[i]=j;
  }
}
void kmpSearch(){
  int i=0,j=0;
  while(i<n){
    while(j>=0 and T[i]!=P[j])
      j=b[j];
      i++;j++;
    if(j==m){
      //pattern found at index i-j
    }
  }
}
```

9.4 Manacher [16 lines] - 2b3cab

```
vector<int> manacher_odd(string s) {
  int n = s.size();
  s = "$" + s + "^";
  vector<int> p(n + 2);
  int l = 1, r = 1;
  for(int i = 1; i <= n; i++) {
    p[i] = max(0, min(r - i, p[l + (r - i)]));
    while(s[i - p[i]] == s[i + p[i]]) {
      p[i]++;
    }
    if(i + p[i] > r) {
      l = i - p[i], r = i + p[i];
    }
  }
  return vector<int>(begin(p) + 1, end(p) - 1);
}
```

9.5 Palindromic Tree [30 lines] - 9ebc05

```
struct PalindromicTree{
  int n,idx,t;
  vector<vector<int>> tree;
  vector<int> len,link;
  string s; // 1-indexed
  PalindromicTree(string str){
    s="$"+str;
    n=s.size();
    len.assign(n+5,0);
    link.assign(n+5,0);
    tree.assign(n+5,vector<int>(26,0));
  }
  void extend(int p){
    while(s[p-len[t]-1]!=s[p]) t=link[t];
    int x=link[t],c=s[p]-'a';
    while(s[p-len[x]-1]!=s[p]) x=link[x];
    if(!tree[t][c]){
      tree[t][c]=++idx;
      len[idx]=len[t]+2;
      link[idx]=len[idx]==1?2:tree[x][c];
    }
    t=tree[t][c];
  }
  void build(){
    len[1]=-1,link[1]=1;
    len[2]=0,link[2]=1;
    idx=t=2;
    for(int i=1;i<n;i++) extend(i);
  }
};
```

9.6 Prefix Function Automaton [21 lines] - b65c0e

```
/* create prefix function array in 26n.*/

int aut[mxn][26];
int lps[mxn];

void automaton(string &s){
  int n = s.size();
  aut[0][s[0] - 'a'] = 1;
  for(int i = 1; i < n; i++){
    for(int j = 0; j < 26; j++){
      if(j == s[i] - 'a'){
        aut[i][j] = i + 1;
        lps[i + 1] = aut[lps[i]][j];
      }
      else {
        aut[i][j] = aut[lps[i]][j];
      }
    }
    cout << lps[i + 1] << endl;
  }
}
```

9.7 Suffix Array [78 lines] - f2f7a0

```
struct SuffixArray {
  vector<int> p, c, rank, lcp;
  vector<vector<int>> st;
  SuffixArray(string const& s) {
    build_suffix(s + char(1));
    build_rank(p.size());
    build_lcp(s + char(1));
    build_sparse_table(lcp.size());
  }
  void build_suffix(string const& s) {
    int n = s.size();
    const int MX_ASCII = 256;
    vector<int> cnt(max(MX_ASCII, n), 0);
    p.resize(n); c.resize(n);
    for (int i = 0; i < n; i++) cnt[s[i]]++;
    for (int i=1; i<MX_ASCII; i++) cnt[i]+=cnt[i-1];
    for (int i = 0; i < n; i++) p[--cnt[s[i]]] = i;
    c[p[0]] = 0;
    int classes = 1;
    for (int i = 1; i < n; i++) {
      if (s[p[i]] != s[p[i-1]]) classes++;
      c[p[i]] = classes - 1;
    }
    vector<int> pn(n), cn(n);
    for (int h = 0; (1 << h) < n; ++h) {
      for (int i = 0; i < n; i++) {
        pn[i] = p[i] - (1 << h);
        if (pn[i] < 0) pn[i] += n;
      }
      fill(cnt.begin(), cnt.begin() + classes, 0);
      for (int i = 0; i < n; i++) cnt[c[pn[i]]]++;
      for (int i=1; i<classes; i++) cnt[i]+=cnt[i-1];
      for (int i=n-1;i>=0;i--) p[--cnt[c[pn[i]]]]=pn[i];
      cn[p[0]] = 0; classes = 1;
      for (int i = 1; i < n; i++) {
        pair<int, int> cur = {c[p[i]], c[(p[i] + (1 <<
            h)) % n]};
        pair<int, int> prev = {c[p[i-1]], c[(p[i-1] + (1
            << h)) % n]};
        if (cur != prev) ++classes;
        cn[p[i]] = classes - 1;
      }
      c.swap(cn);
    }
  }
  void build_rank(int n) {
    rank.resize(n, 0);
    for (int i = 0; i < n; i++) rank[p[i]] = i;
  }
  void build_lcp(string const& s) {
    int n = s.size(), k = 0;
    lcp.resize(n - 1, 0);
    for (int i = 0; i < n; i++) {
      if (rank[i] == n - 1) {
        k = 0;
        continue;
      }
      int j = p[rank[i] + 1];
      while (i + k < n && j + k < n && s[i+k] == s[j+k])
        k++;
      lcp[rank[i]] = k;
      if (k) k--;
    }
  }
  void build_sparse_table(int n) {
    int lim = __lg(n);
    st.resize(lim + 1, vector<int>(n)); st[0] = lcp;
    for (int k = 1; k <= lim; k++)
      for (int i = 0; i + (1 << k) <= n; i++)
        st[k][i] = min(st[k - 1][i], st[k - 1][i + (1 <<
            (k - 1))]);
  }
  int get_lcp(int i) { return lcp[i]; }
  int get_lcp(int i, int j) {
    if (j < i) swap(i, j);
    j--; /*for lcp from i to j we don't need last lcp*/
    int K = __lg(j - i + 1);
    return min(st[K][i], st[K][j - (1 << K) + 1]);
  }
};
```

9.8 Suffix Automata [109 lines] - 600ddc

```
const int mxc = 26;
/*
  + link     - longest suffix belonging to another
      endpos-equivalent class.
  + len      - largest string length ending in current
      state.
  + firstPos - first occurance of substring ending at
      current state.
  + adj      - suffix link tree.
  + sz       - number of states.
  + occ      - number of times state occured in string.
```

```
  + dist     - number of distinct substring.
  + cnt & SA - for count sorting the nodes.
*/
struct SuffixAutomata{
  struct state{
    int link, len, firstPos;
    int next[mxc];
    bool is_clone;
    state(){}
    state(int l){
      len = l, link = -1;
      is_clone = false;
      for(int i=0;i<mxc;i++)next[i] = -1;
    }
  };
  vector<state>t;
  int sz, last;
  vector<ll>cnt,dist, occ,SA;
  vector<vector<int>> adj;
  SuffixAutomata(){
    t.pb(state(0));
    occ.pb(0);
    last = sz = 0;
  }
  int getID(char c){ return c - 'a';}
  void extend(char c){
    int idx = ++sz, p = last, id = getID(c);
    t.pb(state(t[last].len + 1));
    t[idx].firstPos = t[idx].len - 1;
    occ.pb(1);
    while(p!= -1 and t[p].next[id] == - 1){
      t[p].next[id] = idx;
      p = t[p].link;
    }
    if(p==-1) t[idx].link = 0;
    else{
      int q = t[p].next[id];
      if(t[p].len+1 == t[q].len) t[idx].link = q;
      else{
        int clone = ++sz;
        state x = t[q];
        x.len = t[p].len+1;
        t.pb(x);
        t[clone].firstPos = t[q].firstPos;
        t[clone].is_clone = true;
        occ.pb(0);
        while(p!=-1 and t[p].next[id]==q){
          t[p].next[id] = clone;
          p = t[p].link;
        }
        t[idx].link = t[q].link = clone;
      }
    }
    last = idx;
  }
  void build(string &s){
    for(char c:s) extend(c);
    cnt = dist = SA = vector<ll>(sz+1);
    adj.resize(sz+1);
    for(int i=0;i<=sz;i++)cnt[t[i].len]++;
    for(int i=1;i<=sz;i++)cnt[i]+=cnt[i-1];
    for(int i=0;i<=sz;i++) SA[--cnt[t[i].len]] = i;

    for(int i=sz;i>0;i--){
      int idx = SA[i];
      occ[t[idx].link]+=occ[idx];
      adj[t[idx].link].pb(idx);
      dist[idx] = 1;
      for(int j=0;j<mxc;j++){
        if(t[idx].next[j]+1){
          dist[idx]+=dist[t[idx].next[j]];
        }
      }
    }
    for(int i=0;i<mxc;i++){
      if(t[0].next[i]+1) dist[0]+=dist[t[0].next[i]];
    }
  }
  pair<int,int> LCS( string& s){
    int mxlen = 0, bestpos = -1, pos = 0,len = 0;
    int u = 0;
    for(char c:s){
      int v = getID(c);
      while( u and t[u].next[v]!=-1){
        u = t[u].link;
        len = t[u].len;
      }
      if(t[u].next[v]+1){
        len++;
        u = t[u].next[v];
      }
      if(len>mxlen){
        mxlen = len;
        bestpos = pos;
      }
      pos++;
    }
    return {bestpos - mxlen + 1, mxlen};
  }
  state &operator[](int index) { return t[index];}
};
```

9.9 Trie [28 lines] - 408ef5

```
const int maxn=100005;
struct Trie{
  int next[27][maxn];
  int endmark[maxn],sz;
  bool created[maxn];
  void insertTrie(string& s){
    int v=0;
    for(int i=0;i<(int)s.size();i++){
      int c=s[i]-'a';
      if(!created[next[c][v]]){
        next[c][v]=++sz;
        created[sz]=true;
      }
      v=next[c][v];
    }
    endmark[v]++;
  }
  bool searchTrie(string& s){
    int v=0;
    for(int i=0;i<(int)s.size();i++){
      int c=s[i]-'a';
      if(!created[next[c][v]])
        return false;
      v=next[c][v];
    }
    return(endmark[v]>0);
  }
};
```

9.10 Z-Algorithm [19 lines] - e04285

```
void compute_z_function(const char*S,int N){
  int L=0,R=0;
  for(int i=1;i<N;++i){
    if(i>R){
      L=R=i;
      while(R<N && S[R-L]==S[R])++R;
      Z[i]=R-L,--R;
    }
    else{
      int k=i-L;
      if(Z[k]<R-i+1)Z[i]=Z[k];
      else{
        L=i;
        while(R<N && S[R-k]==S[R])++R;
        Z[i]=R-L,--R;
      }
    }
  }
}
```

Theoretical Computer Science Cheat Sheet

Definitions

$f(n) = O(g(n))$	iff $\exists$ positive c, n_0 such that $0 \le f(n) \le cg(n)\ \forall n \ge n_0$.
$f(n) = \Omega(g(n))$	iff $\exists$ positive c, n_0 such that $f(n) \ge cg(n) \ge 0\ \forall n \ge n_0$.
$f(n) = \Theta(g(n))$	iff $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$.
$f(n) = o(g(n))$	iff $\lim_{n\to\infty} f(n)/g(n) = 0$.
$\lim\limits_{n\to\infty} a_n = a$	iff $\forall \epsilon > 0$, $\exists n_0$ such that $\lvert a_n - a\rvert < \epsilon$, $\forall n \ge n_0$.
$\sup S$	least $b \in \mathbb{R}$ such that $b \ge s$, $\forall s \in S$.
$\inf S$	greatest $b \in \mathbb{R}$ such that $b \le s$, $\forall s \in S$.
$\liminf\limits_{n\to\infty} a_n$	$\lim\limits_{n\to\infty} \inf\{a_i \mid i \ge n, i \in \mathbb{N}\}$.
$\limsup\limits_{n\to\infty} a_n$	$\lim\limits_{n\to\infty} \sup\{a_i \mid i \ge n, i \in \mathbb{N}\}$.
$\binom{n}{k}$	Combinations: Size k subsets of a size n set.
$\left[{n \atop k}\right]$	Stirling numbers (1st kind): Arrangements of an n element set into k cycles.
$\left\{{n \atop k}\right\}$	Stirling numbers (2nd kind): Partitions of an n element set into k non-empty sets.
$\left\langle{n \atop k}\right\rangle$	1st order Eulerian numbers: Permutations $\pi_1\pi_2\ldots\pi_n$ on $\{1,2,\ldots,n\}$ with k ascents.
$\left\langle\!\left\langle{n \atop k}\right\rangle\!\right\rangle$	2nd order Eulerian numbers.
C_n	Catalan Numbers: Binary trees with $n+1$ vertices.

Series

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}.$$

In general:

$$\sum_{i=1}^{n} i^m = \frac{1}{m+1}\left[(n+1)^{m+1} - 1 - \sum_{i=1}^{n}\left((i+1)^{m+1} - i^{m+1} - (m+1)i^m\right)\right]$$

$$\sum_{i=1}^{n-1} i^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k} B_k n^{m+1-k}.$$

Geometric series:

$$\sum_{i=0}^{n} c^i = \frac{c^{n+1}-1}{c-1}, \quad c \ne 1, \quad \sum_{i=0}^{\infty} c^i = \frac{1}{1-c}, \quad \sum_{i=1}^{\infty} c^i = \frac{c}{1-c}, \quad |c| < 1,$$

$$\sum_{i=0}^{n} ic^i = \frac{nc^{n+2} - (n+1)c^{n+1} + c}{(c-1)^2}, \quad c \ne 1, \quad \sum_{i=0}^{\infty} ic^i = \frac{c}{(1-c)^2}, \quad |c| < 1.$$

Harmonic series:

$$H_n = \sum_{i=1}^{n} \frac{1}{i}, \qquad \sum_{i=1}^{n} iH_i = \frac{n(n+1)}{2}H_n - \frac{n(n-1)}{4}.$$

$$\sum_{i=1}^{n} H_i = (n+1)H_n - n, \quad \sum_{i=1}^{n}\binom{i}{m}H_i = \binom{n+1}{m+1}\left(H_{n+1} - \frac{1}{m+1}\right).$$

1. $\binom{n}{k} = \frac{n!}{(n-k)!k!}$, **2.** $\sum_{k=0}^{n}\binom{n}{k} = 2^n$, **3.** $\binom{n}{k} = \binom{n}{n-k}$,

4. $\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}$, **5.** $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$,

6. $\binom{n}{m}\binom{m}{k} = \binom{n}{k}\binom{n-k}{m-k}$, **7.** $\sum_{k=0}^{n}\binom{r+k}{k} = \binom{r+n+1}{n}$,

8. $\sum_{k=0}^{n}\binom{k}{m} = \binom{n+1}{m+1}$, **9.** $\sum_{k=0}^{n}\binom{r}{k}\binom{s}{n-k} = \binom{r+s}{n}$,

10. $\binom{n}{k} = (-1)^k\binom{k-n-1}{k}$, **11.** $\left\{{n \atop 1}\right\} = \left\{{n \atop n}\right\} = 1$,

12. $\left\{{n \atop 2}\right\} = 2^{n-1} - 1$, **13.** $\left\{{n \atop k}\right\} = k\left\{{n-1 \atop k}\right\} + \left\{{n-1 \atop k-1}\right\}$,

14. $\left[{n \atop 1}\right] = (n-1)!$, **15.** $\left[{n \atop 2}\right] = (n-1)!H_{n-1}$, **16.** $\left[{n \atop n}\right] = 1$, **17.** $\left[{n \atop k}\right] \ge \left\{{n \atop k}\right\}$,

18. $\left[{n \atop k}\right] = (n-1)\left[{n-1 \atop k}\right] + \left[{n-1 \atop k-1}\right]$, **19.** $\left\{{n \atop n-1}\right\} = \left[{n \atop n-1}\right] = \binom{n}{2}$, **20.** $\sum_{k=0}^{n}\left[{n \atop k}\right] = n!$, **21.** $C_n = \frac{1}{n+1}\binom{2n}{n}$,

22. $\left\langle{n \atop 0}\right\rangle = \left\langle{n \atop n-1}\right\rangle = 1$, **23.** $\left\langle{n \atop k}\right\rangle = \left\langle{n \atop n-1-k}\right\rangle$, **24.** $\left\langle{n \atop k}\right\rangle = (k+1)\left\langle{n-1 \atop k}\right\rangle + (n-k)\left\langle{n-1 \atop k-1}\right\rangle$,

25. $\left\langle{0 \atop k}\right\rangle = \begin{cases} 1 & \text{if } k = 0, \\ 0 & \text{otherwise} \end{cases}$ **26.** $\left\langle{n \atop 1}\right\rangle = 2^n - n - 1$, **27.** $\left\langle{n \atop 2}\right\rangle = 3^n - (n+1)2^n + \binom{n+1}{2}$,

28. $x^n = \sum_{k=0}^{n}\left\langle{n \atop k}\right\rangle\binom{x+k}{n}$, **29.** $\left\langle{n \atop m}\right\rangle = \sum_{k=0}^{m}\binom{n+1}{k}(m+1-k)^n(-1)^k$, **30.** $m!\left\{{n \atop m}\right\} = \sum_{k=0}^{n}\left\langle{n \atop k}\right\rangle\binom{k}{n-m}$,

31. $\left\langle{n \atop m}\right\rangle = \sum_{k=0}^{n}\left\{{n \atop k}\right\}\binom{n-k}{m}(-1)^{n-k-m}k!$, **32.** $\left\langle\!\left\langle{n \atop 0}\right\rangle\!\right\rangle = 1$, **33.** $\left\langle\!\left\langle{n \atop n}\right\rangle\!\right\rangle = 0$ for $n \ne 0$,

34. $\left\langle\!\left\langle{n \atop k}\right\rangle\!\right\rangle = (k+1)\left\langle\!\left\langle{n-1 \atop k}\right\rangle\!\right\rangle + (2n-1-k)\left\langle\!\left\langle{n-1 \atop k-1}\right\rangle\!\right\rangle$, **35.** $\sum_{k=0}^{n}\left\langle\!\left\langle{n \atop k}\right\rangle\!\right\rangle = \frac{(2n)^{\underline{n}}}{2^n}$,

36. $\left\{{x \atop x-n}\right\} = \sum_{k=0}^{n}\left\langle\!\left\langle{n \atop k}\right\rangle\!\right\rangle\binom{x+n-1-k}{2n}$, **37.** $\left\{{n+1 \atop m+1}\right\} = \sum_{k}\binom{n}{k}\left\{{k \atop m}\right\} = \sum_{k=0}^{n}\left\{{k \atop m}\right\}(m+1)^{n-k}$, coprime ($k$ + 1)-tuple together with n. It is a generalization of Euler's totient, $\phi(n) = J_1(n)$.

$$J_k(n) = n^k \prod_{p|n}\left(1-\frac{1}{p^k}\right)$$

- $$\sum_{d|n} J_k(d) = n^k$$
- $$\sum_{d|n}\varphi(d)=n$$
- $$\varphi(n)=\sum_{d|n}\mu(d)\cdot\frac{n}{d} = n\sum_{d|n}\frac{\mu(d)}{d}$$

- $a \mid b \implies \varphi(a) \mid \varphi(b)$
- $n \mid \varphi(a^n-1)$ for $a, n > 1$
- $\varphi(mn) = \varphi(m)\varphi(n)\cdot\frac{d}{\varphi(d)}$ where $d = \gcd(m,n)$

Note the special cases

- $\varphi(2m) = \begin{cases} 2\varphi(m) & \text{if } m \text{ is even} \\ \varphi(m) & \text{if } m \text{ is odd} \end{cases}$
- $\varphi(n^m) = n^{m-1}\varphi(n)$

- $\varphi(\operatorname{lcm}(m,n))\cdot\varphi(\gcd(m,n)) = \varphi(m)\cdot\varphi(n)$

Compare this to the formula

- $\operatorname{lcm}(m,n)\cdot\gcd(m,n) = m\cdot n$

(See least common multiple.)

- $\varphi(n)$ is even for $n \ge 3$. Moreover, if n has r distinct odd prime factors, $2^r \mid \varphi(n)$

- $\varphi(n) = \sum_{d|n} d.\mu(\frac{n}{d})$
- $\sum_{d|n}\frac{\mu^2(d)}{\varphi(d)} = \frac{n}{\varphi(n)}$
- $\sum_{\substack{1\le k\le n \\ (k,n)=1}} k = \frac{1}{2}n\varphi(n) \quad \text{for } n>1$
- $$\frac{\varphi(n)}{n} = \frac{\varphi(\operatorname{rad}(n))}{\operatorname{rad}(n)}$$ where, $$\operatorname{rad}(n) = \prod_{\substack{p|n \\ p \text{ prime}}} p$$
- $\left\lfloor\frac{n}{\varphi(n)}\right\rfloor$ is periodic. 1,2,1,2,1,3,1,2,1,2,1,3...
- $$\sum_{\substack{1\le k\le n \\ \gcd(k,n)=1}} \gcd(k-1,n) = \varphi(n)d(n)$$ where d(n) is number of divisors.
  same equation for gcd(ak-1,n) where a and n are coprime.
- for every n there is at least one other integer m ≠ n such that ϕ(m) = ϕ(n).

Divisor function

- $$\sigma_x(n) = \sum_{d|n} d^x,$$
- It is **multiplicative** i.e. if $\gcd(a,b)=1 \Longrightarrow \sigma_x(ab)=\sigma_x(a)\sigma_x(b)$.

- Any three consecutive Fibonacci numbers are pairwise coprime, which means that, for every n, $\gcd(F_n, F_{n+1}) = \gcd(F_n, F_{n+2}) = \gcd(F_{n+1}, F_{n+2}) = 1$.
- If p is a prime,
  $$\begin{cases} p = 5 & \Rightarrow p \mid F_p, \\ p \equiv \pm 1 \pmod 5 & \Rightarrow p \mid F_{p-1} \\ p \equiv \pm 2 \pmod 5 & \Rightarrow p \mid F_{p+1} \end{cases}$$
- The only nontrivial square Fibonacci number is 144. Attila Pethő proved in 2001 that there is only a finite number of perfect power Fibonacci numbers.In 2006, Y. Bugeaud, M. Mignotte, and S. Siksek proved that 8 and 144 are the only such non-trivial perfect powers.
- If the members of the Fibonacci sequence are taken mod n, the resulting sequence is periodic with period at most 6n.

Sum of floors

$\sum_{i=1}^{n}\left\lfloor\frac{n}{i}\right\rfloor$=?

```
int32_t main()
{
  BeatMeScanf;
  int i,j,k,n,m;
  cin>>n;
  ///complexity O(sqrt(n))
  for (int i = 1, last; i <= n; i = last + 1) {
    last = n / (n / i);
    debug(i,last,n/i);
    ///n / x yields the same value for i <= x <= la.
  }
  return 0;
}
```

Mobius Function and Inversion

Notes

- For any positive integer n, define μ(n) as the sum of the primitive nth roots of unity. It has values in {−1, 0, 1} depending on the factorization of n into prime factors:
  - ✓ μ(n) = 1 if n is a square-free positive integer with an even number of prime factors.
  - ✓ μ(n) = −1 if n is a square-free positive integer with an odd number of prime factors.
  - ✓ μ(n) = 0 if n has a squared prime factor.

  Here, a root of unity, occasionally called a de Moivre number, is any complex number that gives 1 when raised to some positive integer power n.
  An nth root of unity, where n is a positive integer (i.e. n = 1,2,3,...), is a number z(maybe complex) satisfying the equation $z^n = 1$.
  An nth root of unity is said to be primitive if it is not a kth root of unity for some smaller k, that is if
  $$z^n = 1 \quad \text{and} \quad z^k \ne 1 \text{ for } k = 1,2,3,\ldots,n-1.$$
- It is a multiplicatuve function.
- $$\sum_{d|n}\mu(d) = \begin{cases} 1 & \text{if } n = 1, \\ 0 & \text{if } n > 1. \end{cases}$$

- $\sum_{i=1}^{n}[\gcd(i,n) = k] = \varphi(\frac{n}{k})$
- $\sum_{k=1}^{n}\gcd(k,n) = \sum_{d|n} d.\ \varphi(\frac{n}{d})$
- $\sum_{k=1}^{n}\frac{1}{\gcd(k,n)} = \sum_{d|n}\frac{1}{d}.\ \varphi\left(\frac{n}{d}\right) = \frac{1}{n}\sum_{d|n} d.\varphi(d)$
- $\sum_{k=1}^{n}\frac{k}{\gcd(k,n)} = \frac{n}{2}.\sum_{d|n}\frac{1}{d}.\ \varphi\left(\frac{n}{d}\right) = \frac{n}{2}.\frac{1}{n}.\sum_{d|n} d.\varphi(d)$
- $\sum_{k=1}^{n}\frac{n}{\gcd(k,n)} = 2 * \sum_{k=1}^{n}\frac{k}{\gcd(k,n)} - 1, for\ n > 1$
-
- Given several integers, with integer x appears c_x times, and some fixed integer m. It is asked that how many integers that are co-prime to m.so,
  $$\sum_{i=1}^{n} c_i[\gcd(i,m)=1] = \sum_{d|m}\mu(d)\sum_{i=1}^{\lfloor n/d \rfloor} c_{id}$$
  The classic version states that if g and f are arithmetic functions satisfying
  $$g(n) = \sum_{d|n} f(d) \quad \text{for every integer } n \ge 1$$
  then
  $$f(n) = \sum_{d|n}\mu(d)g\left(\frac{n}{d}\right) \quad \text{for every integer } n \ge 1$$
- $\sum_{d|n}\mu(d) = [n=1]$
- $\sum_{i=1}^{n}\sum_{j=1}^{n}[\gcd(i,j)=1] = \sum_{d=1}^{n}\mu(d)\left\lfloor\frac{n}{d}\right\rfloor^2$
- $\sum_{i=1}^{n}\sum_{j=1}^{n}\gcd(i,j) = \sum_{d=1}^{n}\varphi(d)\left\lfloor\frac{n}{d}\right\rfloor^2$
- $$\text{if } F(n) = \prod_{d|n} f(d), \text{ then } f(n) = \prod_{d|n} F\left(\frac{n}{d}\right)^{\mu(d)}$$

mobius function

```
int mob[N];
void mobius()
{
  for(int i=1;i<N;i++) mob[i]=3;
  mob[1]=1;
  for(int i=2;i<N;i++){
    if(mob[i]==3){
      mob[i]=-1;
      for(int j=2*i;j<N;j+=i) mob[j]=(mob[j]==3?-1:mob[j]*(-1));
      if(i<=(N-1)/i) for(int j=i*i;j<N;j+=i*i) mob[j]=0;
    }
  }
}
```

GCD and LCM

- $\gcd(a,0) = a$
  $\gcd(a,b) = \gcd(b, a \bmod b)$
- Every common divisor of a and b is a divisor of gcd(a, b).
- If a divides the product b·c, and gcd(a, b) = d, then a/d divides c.
- If m is any integer, then gcd(a + m·b, b) = gcd(a, b)
- The gcd is a multiplicative function in the following sense: if a1 and a2 are relatively prime, then gcd(a1·a2, b) = gcd(a1, b)·gcd(a2, b).
- gcd(a, b)·lcm(a, b) = |a·b|
- gcd(a, lcm(b, c)) = lcm(gcd(a, b), gcd(a, c))
- lcm(a, gcd(b, c)) = gcd(lcm(a, b), lcm(a, c)).
- For non-negative integers a and b, where a and b are not both zero,
  $\gcd(n^a - 1, n^b - 1) = n^{\gcd(a,b)} - 1$.

$$r_8(n) = 16\sum_{d|n}(-1)^{n+d}d^3$$

Gauss Circle Theorem

- The Gauss circle problem is the problem of determining how many integer lattice points there are in a circle centered at the origin and with radius r.
- Since the equation of this circle is given in Cartesian coordinates by $x^2+y^2=r^2$, the question is equivalently asking how many pairs of integers m and n there are such that $m^2+n^2<=r^2$
- If the answer for a given r is denoted by $N(r)$ then
$$N(r) = 1 + 4\sum_{i=0}^{\infty}\left(\left\lfloor\frac{r^2}{4i+1}\right\rfloor - \left\lfloor\frac{r^2}{4i+3}\right\rfloor\right)$$
- A much simpler sum appears if the sum of squares function r2(n) is defined as the number of ways of writing the number n as the sum of two squares. Then
$$N(r) = \sum_{n=0}^{r^2} r_2(n).$$

3. Combinatorics

Notes

- $\sum_{0\le k\le n}\binom{n-k}{k}$ =Fib_{n+1}
- $\binom{n}{k} = \binom{n}{n-k}$
- $\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$
- $k\binom{n}{k} = n\binom{n-1}{k-1}$
- $\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}$
- $\sum_{i=0}^{n}\binom{n}{i} = 2^n$
- $\sum_{i\ge 0}\binom{n}{2i} = 2^{n-1}$
- $\sum_{i\ge 0}\binom{n}{2i+1} = 2^{n-1}$
- $\sum_{i=0}^{k}(-1)^i\binom{n}{i} = (-1)^k\binom{n-1}{k}$
- $\sum_{i=0}^{k}\binom{n+i}{i} = \binom{n+k+1}{k}$
- $\sum_{i=0}^{k}\binom{n+i}{n} = \binom{n+k+1}{k}$
- $\sum_{i=0}^{k}\binom{i}{n} = \binom{k+1}{n+1}$
- $1.\binom{n}{1} + 2.\binom{n}{2} + 3.\binom{n}{3} + \dots + n.\binom{n}{n} = n.2^{n-1}$
- $1^2.\binom{n}{1} + 2^2.\binom{n}{2} + 3^2.\binom{n}{3} + \dots + n^2.\binom{n}{n} = (n+n^2).2^{n-2}$
- Vandermonde's Identity: $\sum_{k=0}^{r}\binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r}$
- Hockey-Stick Identity: $n,r\in\mathbb{N}, n>r, \sum_{i=r}^{n}\binom{i}{r} = \binom{n+1}{r+1}$
- $\sum_{i=0}^{k}\binom{k}{i}^2 = \binom{2k}{k}$
- $\sum_{k=0}^{n}\binom{n}{k}\binom{n}{n-k} = \binom{2n}{n}$
- $\sum_{k=q}^{n}\binom{n}{k}\binom{k}{q} = 2^{n-q}\binom{n}{q}$
- $\sum_{i=0}^{n}3^i\binom{n}{i} = 4^i$
- $\sum_{i=0}^{n}\binom{2n}{i} = 2^{2n-1} + \frac{1}{2}\binom{2n}{n}$
- $\sum_{i=1}^{n}\binom{n}{i}\binom{n-1}{i-1} = \binom{2n-1}{n-1}$
- $\sum_{i=0}^{n}\binom{2n}{i}^2 = \frac{1}{2}\left\{\binom{4n}{2n} + \binom{2n}{n}^2\right\}$
- An integer n ≥ 2 is prime if and only if all the intermediate binomial coefficients $\binom{n}{1}, \binom{n}{2}, \dots, \binom{n}{n-1}$ are divisible by n.
- $\binom{n+k}{k}$divides $\frac{lcm(n,n+1,\dots,n+k)}{n}$
- Kummer's theorem states that for given integers n ≥ m ≥ 0 and a prime number p, the largest power of p dividing $\binom{n}{m}$ is equal to the number of carries when m is added to n − m in base p.
- Number of different binary sequences of length n such that no two 0's are adjacent=Fib_{n+1}
- Combination with repetition: Let's say we choose k elements from an n-element set, the order doesn't matter and each element can be chosen more than once. In that case, the number of different combinations is: $\binom{n+k-1}{k}$
- Number of ways to divide n different persons in n/k equal groups i.e. each having size k is $\binom{n-1}{k-1}$
- The number non-negative solution of the equation $x_1+x_2+x_3+\dots+x_k=n$ is $\binom{n+k-1}{n}$
- Number of binary sequence of length n and with k '1' is $\binom{n}{k}$
- The number of ordered pairs (a, b) of binary sequences of length n, such that the distance between them is k, can be calculated as follows: $\binom{n}{k}\cdot 2^n$.
The distance between a and b is the number of components that differs in a and b — for example, the distance between (0, 0, 1, 0) and (1, 0, 1, 1) is 2).

Catalan numbers

- ✓ $C_n = \frac{1}{n+1}\binom{2n}{n}$
- ✓ C_0 = 1,C_1=1 and $C_n = \sum_{k=0}^{n-1}C_kC_{n-1-k}$
- ✓ 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786
- ✓ Number of correct bracket sequence consisting of n opening and n closing brackets.
- ✓ The number of ways to completely parenthesize n+1 factors.
- ✓ The number of triangulations of a convex polygon with +2 sides (i.e. the number of partitions of polygon into disjoint triangles by using the diagonals).
- ✓ The number of ways to connect the 2n points on a circle to form n disjoint i.e. non-intersecting chords.
- ✓ The number of monotonic lattice paths from point (0,0) to point (n,n) in a square lattice of size n×n, which do not pass above the main diagonal (i.e. connecting (0,0) to (n,n)).
- ✓ Number of permutations of length n that can be stack sorted (i.e. it can be shown that the rearrangement is stack sorted if and only if there is no such index i<j<k, such that $a_k<a_i<a_j$).
- ✓ The number of non-crossing partitions of a set of n elements.
- ✓ The number of rooted full binary trees with n+1 leaves (vertices are not numbered). A rooted binary tree is full if every vertex has either two children or no children.
- ✓ The number of Dyck words of length 2n. A Dyck word is a string consisting of n X's and n Y's such that no initial segment of the string has more Y's than X's For example, the following are the Dyck words of length 6: XXXYYY XYXXYY XYXYXY XXYYXY XXYXYY.
- ✓ The number of different ways a convex polygon with n + 2 sides can be cut into triangles by connecting vertices with straight lines (a form of Polygon triangulation)
- ✓ Number of permutations of {1, …, n} that avoid the pattern 123 (or any of the other patterns of length 3); that is, the number of permutations with no three-term increasing subsequence. For n = 3, these permutations are 132, 213, 231, 312 and 321. For n = 4, they are 1432, 2143, 2413, 2431, 3142, 3214, 3241, 3412, 3421, 4132, 4213, 4231, 4312 and 4321
- ✓ Number of ways to tile a stairstep shape of height n with n rectangles.
- ✓ N(n,k) = $\frac{1}{n}\binom{n}{k}\binom{n}{k-1}$
- ✓ The number of expressions containing n pairs of parentheses, which are correctly matched and which contain k distinct nestings. For instance, $N(4, 2) = 6$ as with four pairs of parentheses six sequences can be created which each contain two times the sub-pattern '()':

```
()((())) (())(()) (()(())) ((()())) ((())()) ((()))()
```

- ✓ The number of paths from (0, 0) to (2n, 0), with steps only northeast and southeast, not straying below the x-axis, with k peaks. And sum of all number of peaks is Catalan number.

Stirling numbers of the first kind

- ✓ The Stirling numbers of the first kind count permutations according to their number of cycles (counting fixed points as cycles of length one).
- ✓ S(n,k) counts the number of permutations of n elements with k disjoint cycles.
- ✓ $S(n,k) = (n-1)*S(n-1,k) + S(n-1,k-1),$ $where\ S(0,0) = 1, S(n,0) = S(0,n) = 0$
- ✓ $\sum_{k=0}^{n}S(n,k) = n!$

Identities Cont.

38. $\left[{n+1 \atop m+1}\right] = \sum_k \left[{n \atop k}\right]\binom{k}{m} = \sum_{k=0}^{n}\left[{k \atop m}\right] n^{\underline{n-k}} = n!\sum_{k=0}^{n}\frac{1}{k!}\left[{k \atop m}\right]$,

39. $\left[{x \atop x-n}\right] = \sum_{k=0}^{n}\left\langle\!\!\left\langle {n \atop k} \right\rangle\!\!\right\rangle\binom{x+k}{2n}$,

40. $\left\{{n \atop m}\right\} = \sum_k \binom{n}{k}\left\{{k+1 \atop m+1}\right\}(-1)^{n-k}$,

41. $\left[{n \atop m}\right] = \sum_k \left[{n+1 \atop k+1}\right]\binom{k}{m}(-1)^{m-k}$,

42. $\left\{{m+n+1 \atop m}\right\} = \sum_{k=0}^{m} k\left\{{n+k \atop k}\right\}$,

43. $\left[{m+n+1 \atop m}\right] = \sum_{k=0}^{m} k(n+k)\left[{n+k \atop k}\right]$,

44. $\binom{n}{m} = \sum_k \left\{{n+1 \atop k+1}\right\}\left[{k \atop m}\right](-1)^{m-k}$,

45. $(n-m)!\binom{n}{m} = \sum_k \left[{n+1 \atop k+1}\right]\left\{{k \atop m}\right\}(-1)^{m-k}$, for $n \geq m$,

46. $\left\{{n \atop n-m}\right\} = \sum_k \binom{m-n}{m+k}\binom{m+n}{n+k}\left[{m+k \atop k}\right]$,

47. $\left[{n \atop n-m}\right] = \sum_k \binom{m-n}{m+k}\binom{m+n}{n+k}\left\{{m+k \atop k}\right\}$,

48. $\left\{{n \atop \ell+m}\right\}\binom{\ell+m}{\ell} = \sum_k \left\{{k \atop \ell}\right\}\left\{{n-k \atop m}\right\}\binom{n}{k}$,

49. $\left[{n \atop \ell+m}\right]\binom{\ell+m}{\ell} = \sum_k \left[{k \atop \ell}\right]\left[{n-k \atop m}\right]\binom{n}{k}$.

Trees

Every tree with n vertices has $n-1$ edges.

Kraft inequality: If the depths of the leaves of a binary tree are $d_1, \ldots, d_n$:

$$\sum_{i=1}^{n} 2^{-d_i} \leq 1,$$

and equality holds only if every internal node has 2 sons.

Geometry

Projective coordinates: triples (x, y, z), not all x, y and z zero. $(x, y, z) = (cx, cy, cz)$ $\forall c \neq 0$.

Cartesian	Projective
(x, y)	$(x, y, 1)$
$y = mx + b$	$(m, -1, b)$
$x = c$	$(1, 0, -c)$

Distance formula, L_p and L_∞ metric:

$$\sqrt{(x_1-x_0)^2 + (y_1-y_0)^2},$$

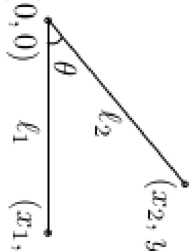
$$\left[|x_1-x_0|^p + |y_1-y_0|^p\right]^{1/p},$$

$$\lim_{p\to\infty}\left[|x_1-x_0|^p + |y_1-y_0|^p\right]^{1/p}.$$

Area of triangle (x_0, y_0), (x_1, y_1) and (x_2, y_2):

$$\frac{1}{2}\,\text{abs}\begin{vmatrix} x_1-x_0 & y_1-y_0 \\ x_2-x_0 & y_2-y_0 \end{vmatrix}.$$

Angle formed by three points:



$$\cos\theta = \frac{(x_1, y_1)\cdot(x_2, y_2)}{\ell_1\ell_2}.$$

Line through two points (x_0, y_0) and (x_1, y_1):

$$\begin{vmatrix} x & y & 1 \\ x_0 & y_0 & 1 \\ x_1 & y_1 & 1 \end{vmatrix} = 0.$$

Area of circle, volume of sphere:

$$A = \pi r^2, \quad V = \frac{4}{3}\pi r^3.$$

Stirling numbers of the second kind

- ✓ Stirling number of the second kind is the number of ways to partition a set of n objects into k non-empty subsets.
- ✓ $S(n,k) = k * S(n-1,k) + S(n-1,k-1)$, where $S(0,0) = 1, S(n,0) = S(0,n) = 0$
- ✓ $S(n,2)=2^{n-1}-1$
- ✓ $S(n,k)*k!$ = number of ways to color n nodes using colors from 1 to k such that each color is used at least once.

Bell number

- ✓ Counts the number of partitions of a set.
- ✓ $B_{n+1} = \sum_{k=0}^{n}\binom{n}{k} * B_k$
- ✓ $B_n = \sum_{k=0}^{n} S(n,k)$,where S(n,k) is stirling number of second kind.
- ✓ The number of multiplicative partitions of a squarefree number with i prime factors is the i-th Bell number, B_i.
- ✓ If a deck of n cards is shuffled by repeatedly removing the top card and reinserting it anywhere in the deck (including its original position at the top of the deck), with exactly n repetitions of this operation, then there are n^n different shuffles that can be performed. Of these, the number that return the deck to its original sorted order is exactly B_n. Thus, the probability that the deck is in its original order after shuffling it in this way is B_n/n^n.

Lucas Theorem

- ✓ If p is prime the $\binom{p^a}{k} \equiv 0 \ (mod\ p)$
- ✓ For non-negative integers m and n and a prime p, the following congruence relation holds:

$$\binom{m}{n} \equiv \prod_{i=0}^{k}\binom{m_i}{n_i} \pmod p,$$

where,
$m = m_k p^k + m_{k-1}p^{k-1} + \ldots + m_1 p + m_0,$
and
$n = n_k p^k + n_{k-1}p^{k-1} + \ldots + n_1 p + n_0$
are the base p expansions of m and n respectively. This uses the convention that $\binom{m}{n} = 0$, when $m < n$.

Derangement

- ✓ A derangement is a permutation of the elements of a set, such that no element appears in its original position.
- ✓ $d(n) = (n-1) * (d(n-1) + d(n-2))$, where $d(0) = 1, d(1) = 0$
- ✓ $d(n) = \left\lfloor \frac{n!}{e} \right\rceil, n \geq 1$

4. Burnside Lemma

The task is to count the number of different necklaces from n beads, each of which can be painted in one of the k colors. When comparing two necklaces, they can be rotated, but not reversed (i.e. a cyclic shift is permitted).

Solution:

$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

Combinatorial (5)

5.1 Permutations

5.1.1 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
n	11	12	13	14	15	16	17			
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
n	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

IntPerm.h

Description: Permutation -> integer conversion. (Not order preserving.)

Time: $\mathcal{O}(n)$

6 lines

```
int permToInt(vi& v) {
  int use = 0, i = 0, r = 0;
  trav(x,v) r=r * ++i + __builtin_popcount(use & -(1 << x)),
    use |= 1 << x;                    // (note: minus, not ~!)
  return r;
} // hash-cpp-all = e1b8eaea02324af14a3da94f409019b8
```

5.1.2 Cycles

Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp\left(\sum_{n \in S} \frac{x^n}{n}\right)$$

5.1.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1)+D(n-2)) = nD(n-1)+(-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

5.1.4 Burnside's lemma

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1} \sum_{k=0}^{m} \binom{m+1}{k} B_k (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty} \frac{B_k}{k!} f^{(k-1)}(m)$$

$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

5.3.2 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k), \quad c(0,0) = 1$$
$\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\dots(x+n-1)$

$c(8,k) =$
$8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) =$
$0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \dots$

5.3.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k} (-1)^j \binom{n+1}{j} (k+1-j)^n$$

5.3.4 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} j^n$$

5.3.5 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

5.3.6 Labeled unrooted trees

\# on n vertices: n^{n-2}
\# on k existing trees of size n_i: $n_1 n_2 \cdots n_k n^{k-2}$
\# with degrees d_i: $(n-2)!/((d_1-1)! \cdots (d_n-1)!)$

5.3.7 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1, \ C_{n+1} = \frac{2(2n+1)}{n+2} C_n, \ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.