Sbaig1_Assignment3

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Problem Statement:

Heart Start produces automated external defibrillators (AEDs) in each of two different plants (A and B). The unit production costs and monthly production capacity of the two plants are indicated in the table below. The AEDs are sold through three wholesalers. The shipping cost from each plant to the warehouse of each wholesaler along with the monthly demand from each wholesaler are also indicated in the table. How many AEDs should be produced in each plant, and how should they be distributed to each of the three wholesaler warehouses so as to minimize the combined cost of production and shipping?

- Formulate a transportation problem.
- Solve a transportation problem
- Formulate the dual of the transportation problem
- Interpret the dual of transportation problem

```r
HeartStart <- matrix(c(22,14,30,600,100 ,
16,20,24,625,120,
80,60,70,"-","-" ), ncol=5, byrow=TRUE)
colnames(HeartStart) <- c("Warehouse 1", "Warehouse 2", "Warehouse
3","Production cost","Production Capacity")
rownames(HeartStart) <- c("Plant A", "Plant B","Monthly Demand")
HeartStart <- as.table(HeartStart)
HeartStart
```

```
##                Warehouse 1 Warehouse 2 Warehouse 3 Production cost
## Plant A        22          14          30          600
## Plant B        16          20          24          625
## Monthly Demand 80          60          70          -
##                Production Capacity
## Plant A        100
## Plant B        120
## Monthly Demand -
```

Formulating the LP below:

$$Min\ TC = 22x_{11} + 14x_{12} + 30x_{13}$$

$$+16x_{21} + 20x_{22} + 24x_{23}$$

$$+80x_{31} + 60x_{32} + 70x_{33}$$

Supply constraints

$$x_{11} + x_{12} + x_{13} \leq 100$$

$$x_{21} + x_{22} + x_{23} \leq 120$$

Demand Constraints

$$x_{11} + x_{21} \geq 80$$

$$x_{12} + x_{22} \geq 60$$

$$x_{13} + x_{23} \geq 70$$

Non-Negativity of the variables

$$x_{ij} \geq 0$$

$$i = 1,2,3$$

$$j = 1,2,3$$

```
library(lpSolve)

## Warning: package 'lpSolve' was built under R version 4.1.3

#Cost matrix

costs <- matrix(c(622,614,630,0,
641,645,649,0), ncol = 4,byrow = TRUE)

#Plant names

colnames(costs) <- c("Warehouse 1", "Warehouse 2","Warehouse 3","Dummy")
rownames(costs) <- c("Plant A", "Plant B")
costs

##         Warehouse 1 Warehouse 2 Warehouse 3 Dummy
## Plant A         622         614         630     0
## Plant B         641         645         649     0
```

#Supply side constraints

```
row.signs <- rep("<=", 2)
row.rhs <- c(100,120)
```

#Demand (sinks) side constraints

```
col.signs <- rep(">=", 4)
col.rhs <- c(80,60,70,10)
```

Solving the Transportation Problem

```
lptrans <- lp.transport(costs, "min", row.signs, row.rhs, col.signs, col.rhs)
```

#Variable values

```
lptrans$solution

##      [,1] [,2] [,3] [,4]
## [1,]    0   60   40    0
## [2,]   80    0   30   10
```

objective function value

```
lptrans$objval

## [1] 132790
```

#Constraints value

```
lptrans$solution

##      [,1] [,2] [,3] [,4]
## [1,]    0   60   40    0
## [2,]   80    0   30   10
```

Summary of the optimal solution to minimize the cost of production and shipping.

1. **Plant 2: Ware House 1 should produce 80 AEDs**
2. **Plant 1: Ware House 2 should produce 60 AEDs**
3. **Plant 1: Ware House 3 should produce 40 AEDs**
4. **Plant 2: Ware House 3 should produce 30 AEDs**
5. **Plant 2: Ware House 4 should produce 10 AEDs**

Formulating the dual of Transportation Problem

$$\text{Maximize VA } = 100P_1 + 120P_2 - 80W_1 - 60W_2 - 70W_3$$

Subject to the following constraints

Total Profit Constraints

$$MR_1 - MC_1 \geq= 622$$

$$MR_2 - MC_1 \geq= 614$$

$$MR_3 - MC_1 \geq= 630$$

$$MR_1 - MC_2 \geq= 641$$

$$MR_2 - MC_2 \geq= 645$$

$$MR_3 - MC2 \geq= 649$$

Where MR1 = Marginal Revenue from Warehouse1

MR2 = Marginal Revenue from Warehouse2

MR3 = Marginal Revenue from Warehouse3

MC1 = Marginal Cost from Plant1

MC2 = Marginal Cost from Plant2

Economic Interpretation of Dual

$$MR_1 >= MC_1 + 622$$

$$MR_2 >= MC_1 + 614$$

$$MR_3 >= MC_1 + 630$$

$$MR_1 >= MC_2 + 641$$

$$MR_2 >= MC_2 + 645$$

$$MR_3 \geq MC_2 + 649$$

The universal rule of Profit maximization states that MR >= MC where “MR” is the Marginal Revenue and “MC” is the Marginal Cost.

$$MR_1 >= MC_1 + 621 \quad i.e. \quad MR_1 >= MC_1$$