Chalmers University of Technology

SSY281 - Model predictive control

Assignment 1

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1 Discretization of a state-space model

1.1 a)

The discrete system model is given by the following equations:

$$A = \begin{pmatrix} 1.4421 & 0.1143 & 0 & -0.0045 \\ 9.4370 & 1.4421 & 0 & -0.0908 \\ 0.0237 & 0.0008 & 1.0000 & 0.0833 \\ 0.4814 & 0.0237 & 0 & 0.6845 \end{pmatrix} \tag{1}$$

$$B = \begin{pmatrix} 0.0677 \\ 1.3715 \\ 0.2530 \\ 4.7660 \end{pmatrix} \tag{2}$$

Where the discrete matrices A and B are calculated according to:

$$\begin{pmatrix} A & B \\ 0 & I \end{pmatrix} = e^{\begin{pmatrix} A & B \\ 0 & 0 \end{pmatrix} h} \tag{3}$$

Where $h = 0.1$ is the sampling interval

1.2 b)

$$A_a = \begin{pmatrix} 1.4421 & 0.1143 & 0 & -0.0045 & 0.0649 \\ 9.4370 & 1.4421 & 0 & -0.0908 & 1.0958 \\ 0.0237 & 0.0008 & 1.0000 & 0.0833 & 0.2419 \\ 0.4814 & 0.0237 & 0 & 0.6845 & 36.6657 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{4}$$

$$B_a = \begin{pmatrix} 0.0028 \\ 0.2757 \\ 0.0111 \\ 1.1002 \\ 1.0000 \end{pmatrix} \tag{5}$$

Where A_a and B_a are calculated using the same formula as in (3) with $h = h - \tau$ and $\tau = 0.8h$.
The eigenvalues of A and A_a are:

$$eig(A) = \begin{pmatrix} 1.0000 \\ 2.4781 \\ 0.4008 \\ 0.6899 \end{pmatrix} \tag{6}$$

$$eig(A_a) = \begin{pmatrix} 1.0000 \\ 2.4781 \\ 0.4008 \\ 0.6899 \\ 0 \end{pmatrix} \tag{7}$$

From the system with time delay, there is an extra eigenvalue at 0. this mean that the state associated with the eigenvalue zero never updates.

2 Dynamic Programming solution of the LQ problem

2.1 a)

The shortest N that makes the system asymptotically stable is

$$N = 33$$

with the optimal gain K :

$$K = \begin{pmatrix} -46.1566 & -5.2278 & 0.0155 & 0.4031 \end{pmatrix} \tag{8}$$

2.2 b)

Stationary Ricatti solution obtained by the function *idare()* is :

$$P_\infty = 10^4 \times \begin{pmatrix} 4.8730 & 0.5323 & -0.1038 & -0.1154 \\ 0.5323 & 0.0587 & -0.0114 & -0.0127 \\ -0.1038 & -0.0114 & 0.0125 & 0.0027 \\ -0.1154 & -0.0127 & 0.0027 & 0.0030 \end{pmatrix} \tag{9}$$

Stationary Ricatti solution obtained by dynamic programming with tolerance 10^{-1} is:

$$P_\infty = 10^4 \times \begin{pmatrix} 4.8726 & 0.5322 & -0.1037 & -0.1154 \\ 0.5322 & 0.0587 & -0.0114 & -0.0127 \\ -0.1037 & -0.0114 & 0.0125 & 0.0027 \\ -0.1154 & -0.0127 & 0.0027 & 0.0030 \end{pmatrix} \tag{10}$$

From (9) and (10), the values obtained by using the dynamic programming and Matlab function *idare()* are very close. The tolerance used in the DP approach is 10^{-1}, changing the tolerance to 10^{-2} will give the same stationary Ricatti solution with precision 5.

The number of iteration required to find the stationary solution P_∞ using DP with tolerance 10^{-1} is:

$$\mathrm{N} = 427$$

2.3 c)

Changing the terminal cost matrix P_f to the stationary solution P_∞ will reduce the number of iterations to

$$N = 1$$

. With optimal control gain vector :

$$K_\infty = \begin{pmatrix} -67.7456 & -7.5229 & 0.6939 & 0.9025 \end{pmatrix}$$

Since the Ricatti stationary solution is used as the terminal cost matrix P_f,the optimal control gain that makes the system stable will be found in one iteration.

3 Batch solution of the LQ problem

3.1 a)

The shortest N that makes the system asymptotically stable is:

$$N = 33$$

With vector gain K_0 as :

$$K_0 = \begin{pmatrix} -46.1566 & -5.2278 & 0.0155 & 0.4031 \end{pmatrix} \tag{11}$$

As expected the solution obtained form the batched solution (11) is the same as in the dynamic programming approach (8).

4 Receding horizon control

In this task, the dynamic programming approach was used to simulate the systems with 4 different sets of parameters. Namely the horizon length and the input control cost matrix.

$$R = 1 \ and \ N = 40,$$
$$R = 1 \ and \ N = 80,$$
$$R = 0.1 \ and \ N = 40,$$
$$R = 0.1 \ and \ N = 80.$$

After simulation for 2000-time steps, the states and input control trajectory are shown in the following figure:

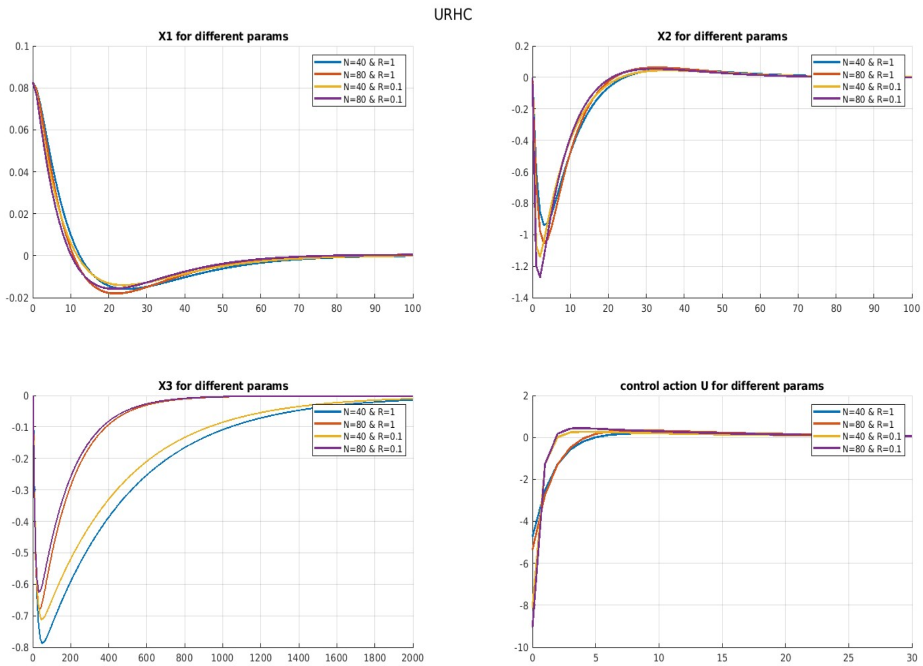

Figure 1: Unconstrained system simulation

From figure 1 the controller with the longest horizon and cheapest control penalty is the fastest one to drive the state X_3 to 0, at the cost of more control action than the other controllers. For the other states, the different controllers behaves more or less the same, still controllers with lower penalty R and higher horizon N are faster to regulate the system.
Which controller is the best depends on which state is more important. For states x_1 and x_2, all the controller behaves more or less the same but with different control action.

5 Constrained receding horizon control

In this question, the process was similar to the previous one with the addition of some constraints on the state X_2 and the control signal u The following parameters have been used for simulation.

$$
\begin{aligned}
R = 1 \ \ and \ \ N = 40, \\
R = 1 \ \ and \ \ N = 80, \\
R = 0.1 \ \ and \ \ N = 40, \\
R = 0.1 \ \ and \ \ N = 80.
\end{aligned}
$$

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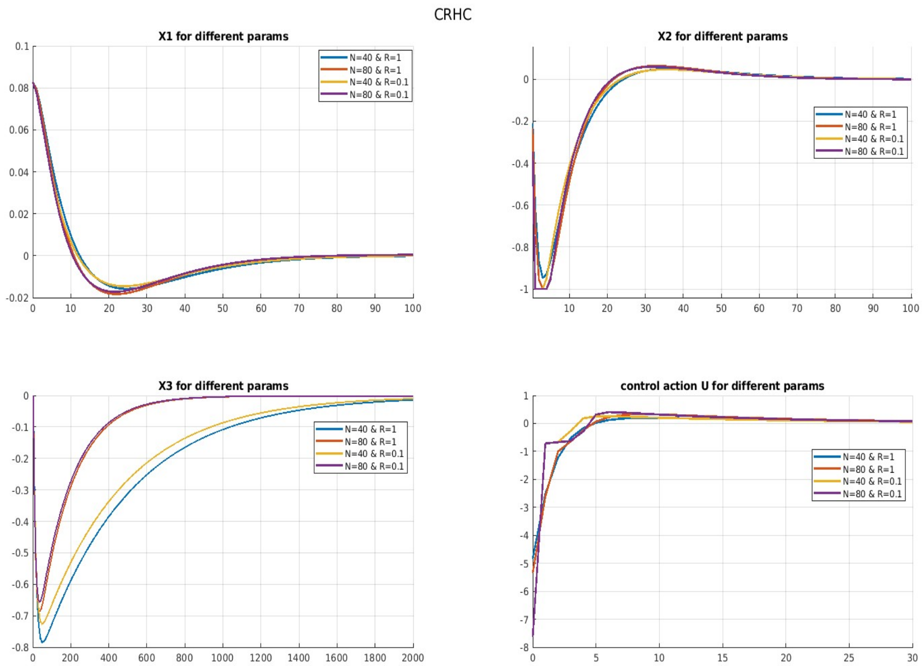

Figure 2: Constrained system simulation

The first major difference in comparison to the unconstrained case, now the control input is bounded as well as the state x_2. Where in the unconstrained case the state x_2 exceeds the absolute value of 1. In general, the constrained case is better since it allows to limit the control activity as well as keep the state trajectory bounded.