Convex Optimization

Matlab Assignment

Feb. 28, 2018

Note: Use SeDuMi (solver) and Yalmip (solver interface) to solve the following optimization problems. Please read the usage of following commands in Yalmip: 'sdpvar', 'sdpsettings', 'optimize' etc.
(https://yalmip.github.io/tutorial/basics/).

Q1: Solve the following linear feasibility problem:

$$p^* \;=\; f_0(x_1^*,\; x_2^*) \;=\; \min_{x_1,\; x_2} \quad 0$$

$$\text{s.t.} \quad x_1 \;\geq\; 0, \quad x_2 \;\geq\; 0, \quad x_1 + x_2 \;\geq\; 1, \quad x_1 + x_2 \;\leq\; 2, \quad x_1 + 2x_2 \;\geq\; 2$$

(a) locate the feasible point $(x_1^*,\; x_2^*)$ in the polyhedron defined by the constraints.

Q2: Solve the following linear optimization problem:

$$p^* \;=\; f_0(x_1^*,\; x_2^*) \;=\; \min_{x_1,\; x_2} \quad 2x_1 + 3x_2$$

$$\text{s.t.} \quad x_1 \;\geq\; 0, \quad x_2 \;\geq\; 0, \quad x_1 + x_2 \;\geq\; 1, \quad x_1 + x_2 \;\leq\; 2, \quad x_1 + 2x_2 \;\geq\; 2$$

(a) locate the optimal point $(x_1^*,\; x_2^*)$ in the polyhedron defined by the constraints.

Q3: Solve the following linear optimization problem:

$$p^* \;=\; f_0(x_1^*,\; x_2^*) \;=\; \min_{x_1,\; x_2} \quad 2x_1 + 3x_2$$

$$\text{s.t.} \quad x_1 \;\geq\; 0, \quad x_2 \;\geq\; 0, \quad x_1 + x_2 \;\geq\; 1, \quad x_1 + x_2 \;\leq\; 2, \quad x_1 + 2x_2 \;\geq\; 2, \quad -x_1 + x_2 \;\leq\; 1$$

(a) locate the optimal point $(x_1^*,\ x_2^*)$ in the polyhedron defined by the constraints and,
(b) compare the optimal value $p^* = f_0(x_1^*,\ x_2^*)$ to the optimal value obtained in **Q2**.

Q4: Consider the following semidefinite program:

$$p^* \ = \ f_0(Q^*) \ = \ \min_{Q} \quad Trace(Q)$$

$$\text{s.t.} \quad Q \ \succeq \ 0, \quad x_1^T Q x_1 \ \geq \ \gamma_1, \quad x_2^T Q x_2 \ \geq \ \gamma_2, \quad x_3^T Q x_3 \ \leq \ \gamma_3,$$

where $x_1,\ x_2,\ x_3 \in \mathbb{R}^{n\times 1}$ are known vectors which can be generated randomly (e.g. using 'randn' function in Matlab); $\gamma_1 = 3,\ \gamma_2 = 2,\ \gamma_3 = 1$; $Q \in \mathbb{S}_+^n$ is unknown. Find:
(a) the optimal point Q^* and optimal value $p^* = f_0(Q^*)$ of the above semidefinite program and,
(b) using eigen vector decomposition (EVD), verify if the optimal point Q^* is a positive semidefinite matrix.