

## PROBLEMS

**1.1** Find the independent nonlinearity, as defined in Figure 1.4(b), for the following set of inputs and outputs from a nearly linear system. Instrument was designed for an output equal to twice the input. Full scale is 20.

Inputs	0.50	1.50	2.00	5.00	10.00
Outputs	0.90	3.05	4.00	9.90	20.50

**1.2** Find the correlation coefficient  $r$  for the set of five inputs and outputs given in Problem 1.1.

**1.3** Derive the operational transfer function and the sinusoidal transfer function for an  $RC$  high-pass filter. Plot the step response and the complete frequency response (magnitude and phase).



**1.4** A first-order low-pass instrument must measure hummingbird wing displacements (assume sinusoidal) with frequency content up to 100 Hz with an amplitude inaccuracy of less than 5%. What is the maximal allowable time constant for the instrument? What is the phase angle at 50 Hz and at 100 Hz?

**1.5** A mercury thermometer has a cylindrical capillary tube with an internal diameter of 0.2 mm. If the volume of the thermometer and that of the bulb are not affected by temperature, what volume must the bulb have if a sensitivity of  $2 \text{ mm}/^\circ\text{C}$  is to be obtained? Assume operation near  $24^\circ\text{C}$ . Assume that the stem volume is negligible compared with the bulb internal volume. Differential expansion coefficient of  $\text{Hg} = 1.82 \times 10^{-4} \text{ ml}/(\text{ml} \cdot ^\circ\text{C})$ .

**1.6** For the spring scale shown in Figure 1.7(a), find the transfer function when the mass is negligible.

**1.7** Find the time constant from the following differential equation, given that  $x$  is the input,  $y$  is the output, and  $a$  through  $h$  are constants.

$$a \frac{dy}{dt} + bx + c + hy = e \frac{dy}{dt} + fx + g$$



**1.8** A low-pass first-order instrument has a time constant of 20 ms. Find the frequency, in hertz, of the input at which the output will be 93% of the dc output. Find the phase angle at this frequency.

**1.9** A second-order instrument has a damping ratio of 0.4 and an undamped natural frequency of 85 Hz. Sketch the step response, and give numerical values for the amplitude and time of the first two positive maxima. Assume that the input goes from 0 to 1 and that the static sensitivity is 10.

**1.10** Consider an underdamped second-order system with step response as shown in Figure 1.7(c). A different way to define logarithmic decrement is

$$\Gamma = \ln \frac{y_n}{y_{n+1}}$$

Figure 2.21(d) shows several results of this type of calculation. One of the examples shown is an efficient system capable of making measurements in the dark without stimulating the eye. Such a device can be used for tracking eye movements. It can be formed from a tungsten source, a Kodak 87 Wratten filter, and a silicon sensor. If GaAs provides enough output, it can replace both the tungsten source and the Kodak 87 Wratten filter (Borah, 2006).

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**2.1** For Figure P2.1, plot the ratio of the output voltage to the input voltage  $v_o/v_i$  as a function of the displacement  $x_i$  of a potentiometer with a total displacement  $x_t$  for ranges of  $R_m$ , the input resistance of the meter. Show that the maximal error occurs in the neighborhood of  $x_i/x_t = 0.67$ . What is the value of this maximal error?

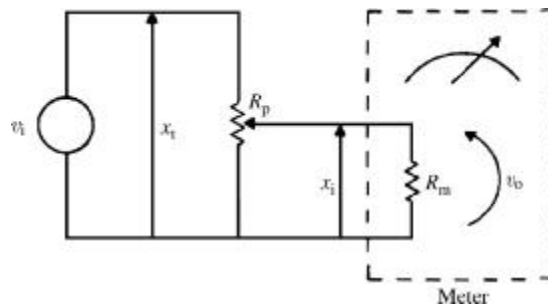


Figure P2.1

**2.2** The practical limitation for wire spacing in potentiometer construction is between 20 and 40 turns/mm. Find the resolution limitation for a translational and a rotational potentiometer. Propose a way to increase the resolution of a rotational potentiometer.

**2.3** Discuss some of the possible problems involved in elastic-resistance strain-gage sensors and their solutions.



**2.4** The electromotive force  $E$  for a thermocouple is given by (2.21). Calculate and plot  $E$  for conditions in which the reference source is at  $0^\circ\text{C}$  and temperature varies from  $0^\circ\text{C}$  to  $50^\circ\text{C}$ . The thermocouple material is copper constantan with  $a = 38.7 \mu\text{V}/^\circ\text{C}$  and  $b = 0.082 \mu\text{V}/^\circ\text{C}^2$ . How significant is the second term in your calculations? Note that these calculated curves are not exactly satisfied in the practical situation. Thus an experimental calibration must be measured over the range of interest.

**2.5** Using the results of Problem 2.4, calculate the sensitivity  $\alpha$  at  $37^\circ\text{C}$  for the copper-constantan thermocouple.

2.21  
가

0

50

$a = 38.7, b = 0.082$

가?

E

E

0

**2.6** Calculate the value of the thermistor temperature coefficient  $\alpha$  for  $T = 300\text{ K}$  and  $b = 4000\text{ K}$ .



**2.7** For the LVDT shown in Figure 2.6(c), sketch the voltages c-e, d-e, and c-d as the core is displaced through its normal range.

**2.8** For Example 2.1, what size shunting capacitor should be added to extend the low-corner frequency to 0.05 Hz, as is required to detect pulse waveforms? How is the sensitivity changed?

2.1

**2.9** Design a charge amplifier for a piezoelectric sensor that has 500 pF capacitance. It should pass frequencies from 0.05 to 100 Hz so that it can detect carotid pulses, and it should not drift into saturation.

0.05Hz

**2.10** Sketch typical thermistor  $v$ - $i$  characteristics with and without a heat sink. Explain why there is a difference.

**2.11** Calculate and sketch a curve for the radiant output of the skin at 300 K at 2000, 5000, 10,000, and 20,000 nm.

**2.12** Sketch an optical system using curved mirrors instead of lenses that could replace the system shown in Figure 2.20(b).

**2.13** Sketch the circuit for a photo Darlington transistor, which is two cascaded emitter-follower transistors. Estimate its linearity and response time.

**2.14** For the solar cell shown in Figure 2.25, what value load resistor would receive the maximal power?

**2.15** For the photomultiplier shown in Figure 2.24, when  $R_L$  is high enough for adequate sensitivity, the stray capacity- $R_L$  product produces a time constant that is too long. Design a circuit that is 10 times faster and has no loss in sensitivity.

**2.16** For Figure 2.21(d), plot the relative combination product for GaP, HbO, CdS.

## REFERENCES

- Alihanka, J., K. Vaahtoranta, and S.-E. Björkqvist, Apparatus in medicine for the monitoring and/or recording of the body movements of a person on a bed, for instance of a patient. United States Patent 4,320,766, 1982.
- Anonymous, *Manual on the Use of Thermocouples in Temperature Measurement*. Publication 470A. Philadelphia: American Society for Testing and Materials, 1974.
- Borah, J., "Eye movement, measurement techniques for." In J. G. Webster (ed.), *Encyclopedia of Medical Devices and Instrumentation*, 2nd ed. New York: Wiley, 2006, Vol. 3, pp. 263-286.
- Bowman, L., and J. D. Meindl, "Capacitive sensors." In J. G. Webster (ed.), *Encyclopedia of Medical Devices and Instrumentation*. New York: Wiley, 1988, pp. 551-556.
- Cobbold, R. S. C., *Transducers for Biomedical Measurements*. New York: John Wiley & Sons, Inc., 1974.
- Conway, J. M., K. H. Norris, and C. E. Bodwell, "A new approach for the estimation of body composition: Infrared interactance." *Am. J. Clin. Nutr.*, 1984, 40, 1123-1130.
- Dechow, P. C., and Q. Wang, "Strain gages." In J. G. Webster (ed.), *Encyclopedia of Medical Devices and Instrumentation*. 2nd ed. New York: Wiley, 2006, Vol. 6, pp. 282-290.
- Doebelin, E. O., *Measurement Systems: Application and Design*, 4th ed. New York: McGraw-Hill, 1990.
- Fraden, J., "Noncontact temperature measurements in medicine." In D. L. Wise (ed.), *Bio-instrumentation and Biosensors*. New York: Marcel Dekker, 1991, pp. 511-550.