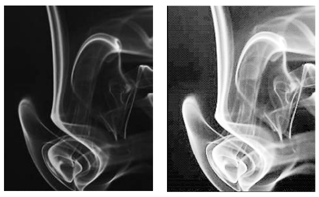
Three Classes of Image Processing Operations by Direct Manipulation of Gray Levels

- Any image-processing operation transforms the gray values of the pixels.
- Image-processing operations may be divided into three classes based on the information required to perform the transformation.
- From the simplest to the most complex, they are as follows:
  - **Point operations : chap.4**
  - **Neighborhood processing (spatial filter) : chap.5**
  - **Geometrical transforms: chap.6**



Chapter 4: Point Processing

Arithmetic Operations

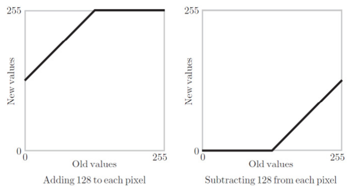
- These operations act by applying a simple function

$$y = f(x)$$

- In each case we may have to adjust the output slightly in order to ensure that the results are integers in the 0 . . . 255 range **(type uint8)**

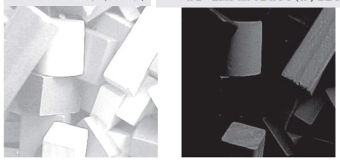
$$y \leftarrow \begin{cases} 255 & \text{if } y > 255, \\ 0 & \text{if } y < 0. \end{cases}$$

Arithmetic Operations



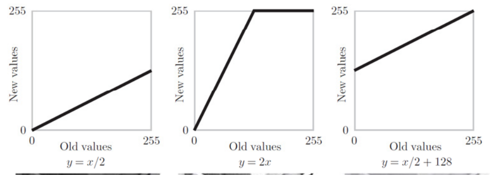
Adding 128 to each pixel   Subtracting 128 from each pixel

```
>> b=imread('blocks.tif');
>> b1=imadd(b,128);
>> b2=imsubtract(b,128);
```

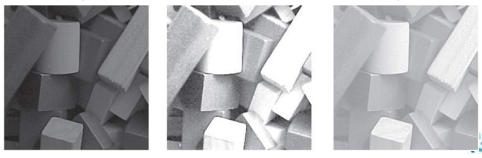


Arithmatic Op: Multiplication and Division

$y = x/2$	`b3=immultiply(b,0.5);` or `b3=imdivide(b,2)`
$y = 2x$	`b4=immultiply(b,2);`
$y = x/2 + 128$	`b5=imadd(immultiply(b,0.5),128);` or `b5=imadd(imdivide(b,2),128);`

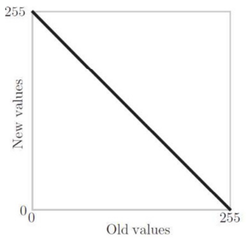


$y = x/2$ $y = 2x$ $y = x/2 + 128$



4.2 Arithmetic Op: Complements

- The **complement** of a grayscale image is its photographic negative (= **image negative**)
- **type double** (0.0~1.0) : `1-m`
- **type uint8** (0~255) : `255-m`



```
>> bc=imcomplement(b);
>> imshow(bc)
```

4.3 Histogram

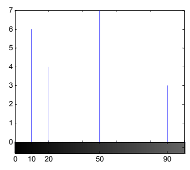
- A graph representing pixel value vs the number of its occurrance in the image
  (= 확률 밀도 함수, Probability Density Function (PDF))

$$p_r(r_k) = \frac{n_k}{n} \qquad k = 0,1,2,\ldots,L-1$$

- n : total number of pixels in an image
- n_k : number of pixel value r_k
- L : number of intensity levels (pixel values)

- Graph of r_k vs. $p_r(r_k)$

```
EX: a = [10 10 10 10 10;
         20 20 20 20 10;
         50 50 50 50 50;
         90 90 90 50 50]
```



Histogram Example

```
>> p=imread('pout.tif');
>> imshow(p),figure,imhist(p),axis tight
```

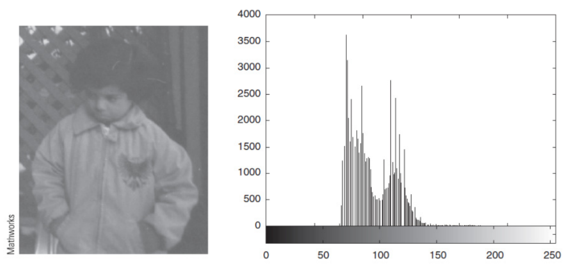
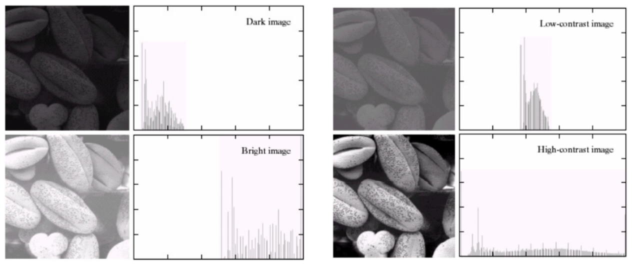


FIGURE 4.8 *The image* `pout.tif` *and its histogram.*

Histogram and Image Properties



Histogram(Contrast) Stretching

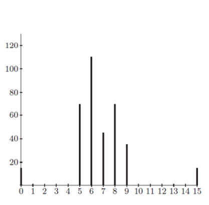
- A table of the numbers n_i of gray values

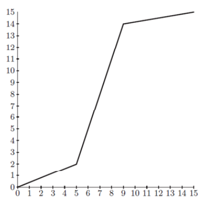
Gray level i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
n_i	15	0	0	0	0	70	110	45	70	35	0	0	0	0	0	15

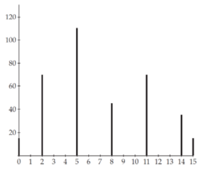
(with n = 360, as before)

- We can stretch out the gray levels in the center of the range by applying the piecewise linear function

Contrast Stretching







Transformation function

$$j = \frac{14-2}{9-5}(i-5)+2$$

i : original gray level
j : its result after the transformation

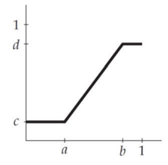
Transformaiton Lookup table

i	5	6	7	8	9
j	2	5	8	11	14

This function has the effect of stretching the gray levels 5–9 to gray levels 2–14

Constrast Stretching in Matlab

```
imadjust(im,[a,b],[c,d])
```



- **imadjust** is designed to work equally well on images of type `double`, `uint8`, or `uint16`
- the values of *a*, b, c, and d must be between 0 and 1
- **imadjust** automatically converts the image `im` (if needed) to be of type `double`

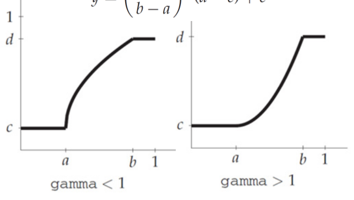
- Note that `imadjust` does not work quite in the same way as shown in the figure.
- The `imadjust` function has one other optional parameter: the `gamma` value.

$$y = \left(\frac{x-a}{b-a}\right)^{\gamma}(d-c)+c$$

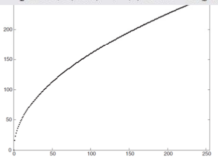
Contrast Stretching with γ

```
>> t=imread('tire.tif');
>> th=imadjust(t,[],[],0.5);
>> imshow(t),figure,imshow(th)
```

$$y = \left(\frac{x-a}{b-a}\right)^{\gamma}(d-c)+c$$



```
>> plot(t,th,'.'),axis tight
```



CS: Piecewise Linear Transformation Function

- How to implement in Matlab ?

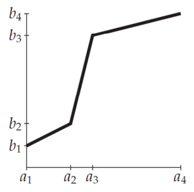
```
pix=find(im >= a(i) & im < a(i+1));
out(pix)=(im(pix)-a(i))*(b(i+1)-b(i))/(a(i+1)-a(i))+b(i)
```

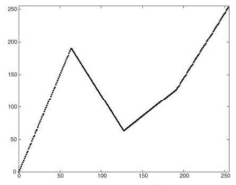
`im` : input image, **`out`** : output image

$$\boxed{y = \frac{b_{i+1}-b_i}{a_{i+1}-a_i}(x-a_i)+b_i}$$

defined in p.77

```
>> th=histpwl(t,[0 .25 .5 .75 1],[0 .75 .25 .5 1]);
>> imshow(th)
>> figure,plot(t,th,'.'),axis tight
```





4.3.2 Histogram Equalization

- An entirely automatic procedure
- Suppose our image has L different gray levels, 0, 1, 2, . . . , $L-1$, and gray level i occurs n_i times in the image

$$\left(\frac{n_0+n_1+\cdots+n_i}{n}\right)(L-1)$$

Where $n=n_0+n_1+n_2+\cdots+n_{L-1}$

4.3.2 Histogram Equalization

- **WHY IT WORKS** If we were to treat the image as a continuous function $f(x, y)$ and the histogram as the area between different contours, then we can treat the histogram as a probability density function.

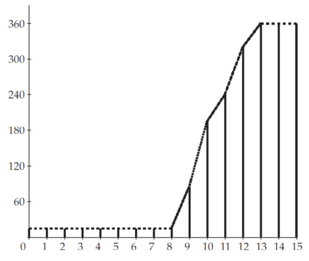


FIGURE 4.22 *The cumulative histogram.*

Histogram Equalization : How to stretch?

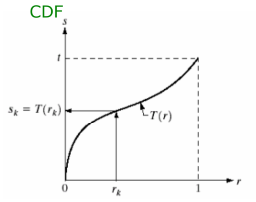
- Pixels r(n1,n2) are transformed to s(n1,n2)

$$s(n_1,n_2)=T[r(n_1,n_2)]$$

- Trans. func.(T): CDF(cumulative distribution function) of random variable ***r***

$$S=T(r)=\int_0^r p_r(w)dw$$

CDF



WHY?
CDF makes pdf (probability density function, pdf) $p_s(s)$ after HE uniform distribution
=> purpose of HE

HE Example

- Suppose a 4-bit grayscale image has the histogram, associated with a table of the numbers n_i of gray values

Gray level i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
n_i	15	0	0	0	0	0	0	0	0	70	110	45	80	40	0	0

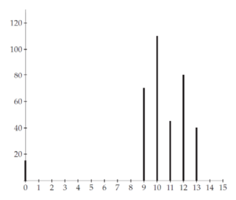


FIGURE 4.17 *Another histogram indicating poor contrast.*

	pdf	CDF		
Gray level i	n_i	Σn_i	$(1/24)\Sigma n_i$	Rounded value
0	15	15	0.63	1
1	0	15	0.63	1
2	0	15	0.63	1
3	0	15	0.63	1
4	0	15	0.63	1
5	0	15	0.63	1
6	0	15	0.63	1
7	0	15	0.63	1
8	0	15	0.63	1
9	70	85	3.65	4
10	110	195	8.13	8
11	45	240	10	10
12	80	320	13.33	13
13	40	360	15	15
14	0	360	15	15
15	0	360	15	15

$n_i/360$

HE Example

```
>> p=imread('pout.tif');
>> ph=histeq(p);
>> imshow(ph),figure,imhist(ph),axis tight
```

For the exam, you have to be able to do it by hand as well !

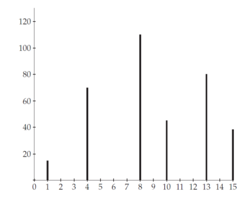
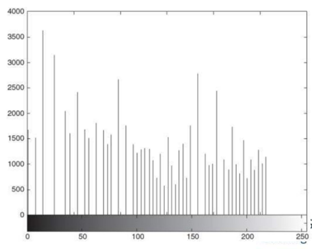
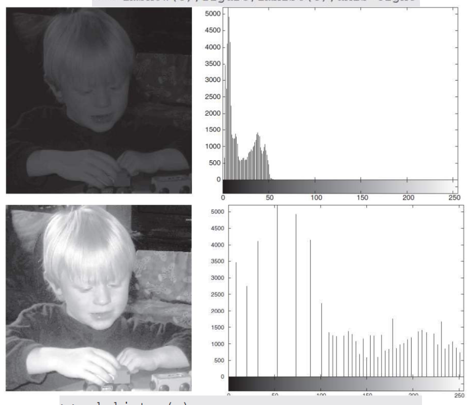


FIGURE 4.18 *The histogram of Figure 4.17 after equalization.*



```
>> en=imread('engineer.tif');
>> e=imdivide(en,4);
>> imshow(e),figure,imhist(e),axis tight
```

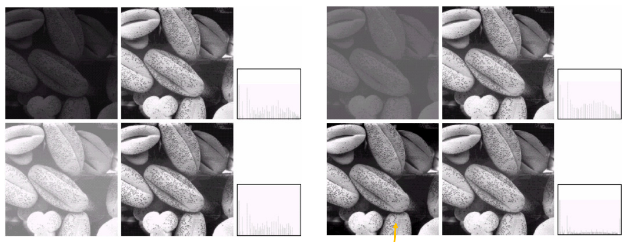


```
>> eh=histeq(e);
>> imshow(eh),figure,imhist(eh),axis tight
```

Histogram Equalization: Step by Step

1. Get histogram(pdf) of input image.
2. Get transformation function T() by calculating cdf and compose a lookup table.
3. Using T(), change input pixel value r -> output pixel value s.
4. Put the pixel value s at the corresponding pixel location of output image.

Histogram Equalization: Results



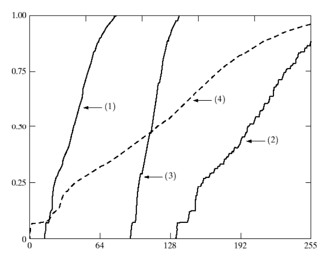
Drawback of HE

Transformation function is fixed.

does not work for high-contrast image

Histogram Equalization : CDFs of the previous slide

FIGURE 3.18 Transformation functions (1) through (4) were obtained from the histograms of the images in Fig.3.17(a), using Eq. (3.3-8).



Histogram Matching(Specification)

- Histogram equalization generates a uniform histogram
- For interactive image enhancement, the user may like to result in a customized histogram

---> Use Histogram Specification

Histogram Specification

- r : pixel values of original image
- u : pixel values of desired image : PDF should be pre-determined.
- s : pixel values of histogram-equalized image
- For continuous data, HE of r and u results in s. (uniform histogram)

$$s = T(r) = \int_0^r P_r(\omega)d\omega \longleftarrow \text{CDF(누적분포함수)}$$

$$v = G(u) = \int_0^u P_u(\omega)d\omega$$

$$\boxed{u = G^{-1}(s) = G^{-1}(T(r))} \quad r \rightarrow u \text{ conversion}$$

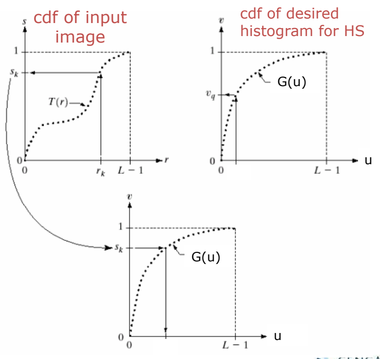
- Specify a particular probability density function G(u) then calculate $G^{-1}(T(r))$ for histogram specification.

Graphical Interpretation of Histogram Specification

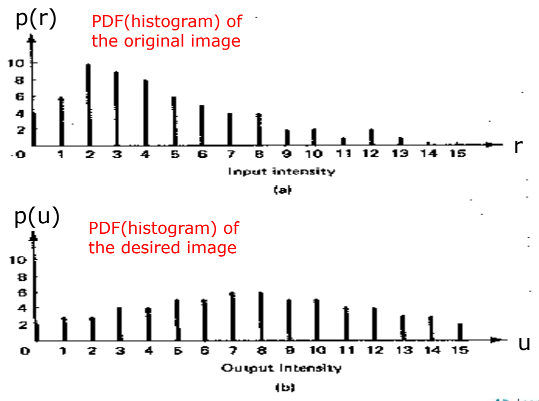
step1 : get histogram-equalized pixel values

$$s = T(r) = \int_0^r P_r(\omega)d\omega$$
$$v = G(u) = \int_0^u P_u(\omega)d\omega$$
$$u = G^{-1}(s) = G^{-1}(T(r))$$

step2 : using histogram equalized pixel values as input, get histogram-specified pixel values



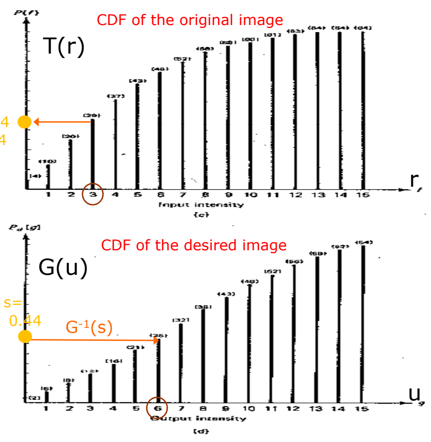
An Example of Histogram Specification with 15 intensity levels and 64 pixels



step1 : get histogram-equalized pixel values

$$s = T(r) = \int_0^r P_r(\omega)d\omega$$
$$v = G(u) = \int_0^u P_u(\omega)d\omega$$
$$u = G^{-1}(s) = G^{-1}(T(r))$$

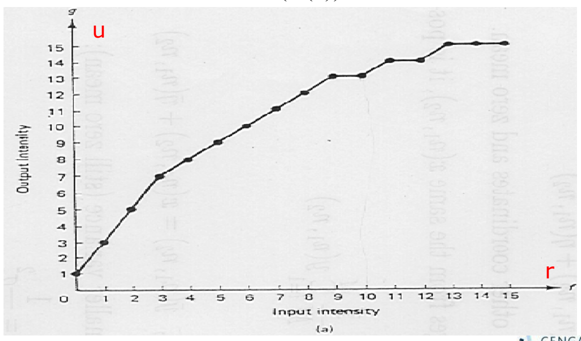
step2 : using histogram equalized pixel values as input, get histogram-specified pixel values



- histogram-equalized pixel value of pixel value 3 : floor(15*28/64) = 7
- histogram-specified pixel value of pixel value 3 : $G^{-1}(28/64) = 6$

An Example of Histogram Specification : Look-up Table

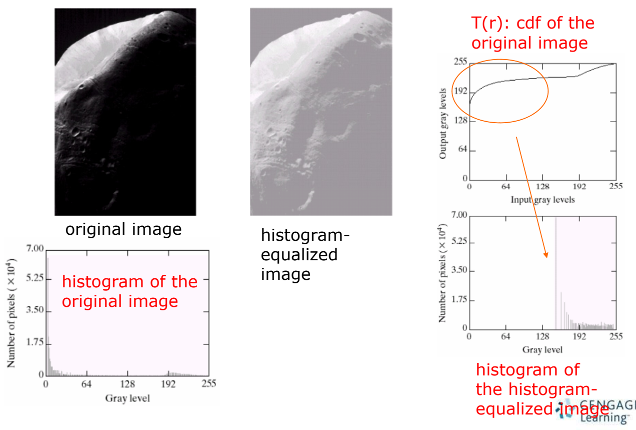
$$u = G^{-1}(T(r))$$



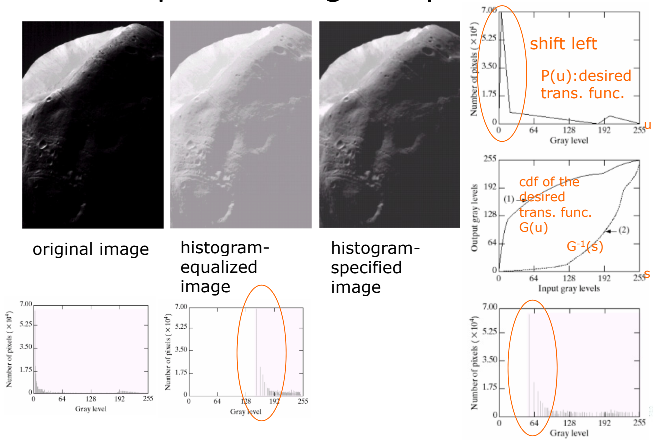
Histogram Specification: Step by Step

1. Get histogram P(r) of input image.
2. Get s=T(r) for histogram equalization.
3. Design the desired histogram P(u) and make transformation function G(u) for HS. (You might want to do it manually.)
4. Generate a look-up table(LUT) to convert input pixel values (r) to output values (u) by T(r) and $G^{-1}(s)$.
5. Put the converted pixel values in the output image.

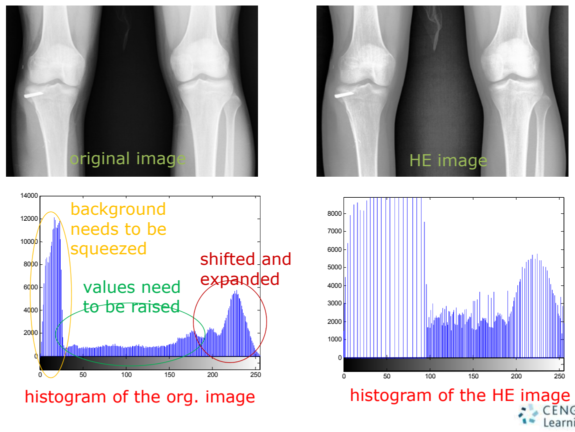
Example of Histogram Specification



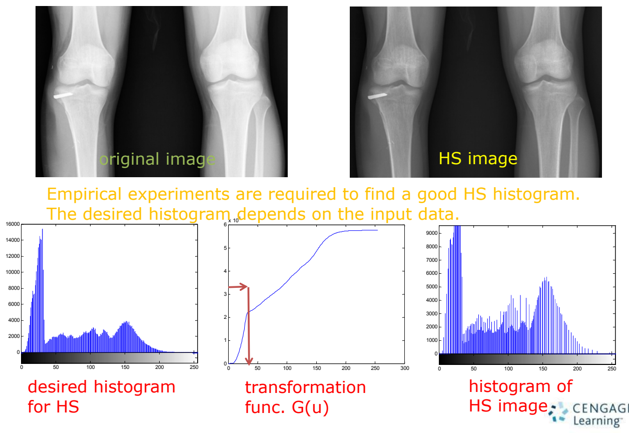
Example of Histogram Specification



A Practical Example of HS (1)


original image
HE image
background needs to be squeezed
shifted and expanded
values need to be raised
histogram of the org. image
histogram of the HE image

A Practical Example of HS (2)


original image
HS image
Empirical experiments are required to find a good HS histogram.
The desired histogram depends on the input data.
desired histogram for HS
transformation func. G(u)
histogram of HS image

4.4 Lookup Tables

- Point operations can be performed very effectively by using a **lookup table**, known more simply as an **LUT**
- e.g., the LUT corresponding to division by 2 looks like

Index:	0	1	2	3	4	5	. . .	250	251	252	253	254	255
LUT:	0	0	1	1	2	2	. . .	125	125	126	126	127	127

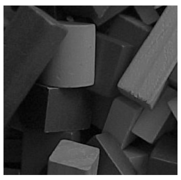
4.4 Lookup Tables

- If `T` is a lookup table in MATLAB and `im` is our image, the lookup table can be applied by the simple command

`T(im+1)`

- e.g.,

```
>> T=uint8(floor([0:255]/2));
>> b = imread('image.tif');
>> b2 = T(b);
```



4.4 Lookup Tables

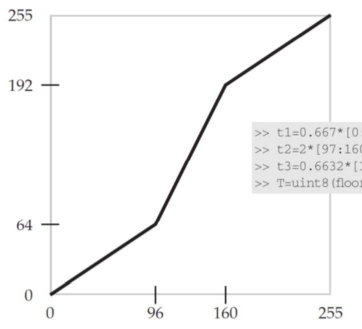
- As another example, suppose we wish to apply an LUT to implement the contrast-stretching function

$$y = \frac{64}{96}x,$$
$$y = \frac{192-64}{160-96}(x-96)+64,$$
$$y = \frac{255-192}{255-160}(x-160)+192$$

$$\Rightarrow$$

$$y = 0.6667x,$$
$$y = 2x-128,$$
$$y = 0.6632x+85.8947$$

LUT Generation in Matlab



```
>> t1=0.667*[0:96];
>> t2=2*[97:160]-128;
>> t3=0.6632*[161:255]+85.8947;
>> T=uint8(floor([t1 t2 t3]));
```

FIGURE 4.23 *A piecewise linear contrast-stretching function.*