

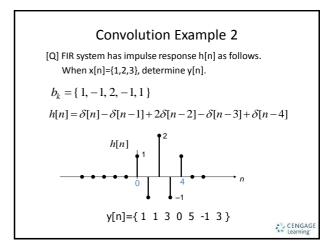
Convolution Example 1

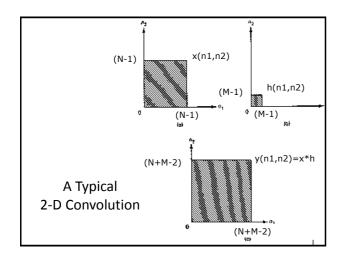
• one of the most common methods for *filtering* a function

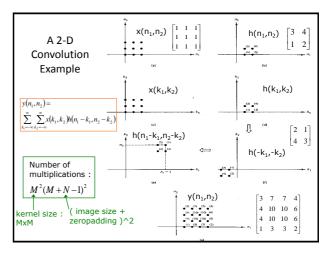
$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$
 x(n) $\xrightarrow{h(n)}$ x(n)*h(n) if n is contant, the equation is the function of k

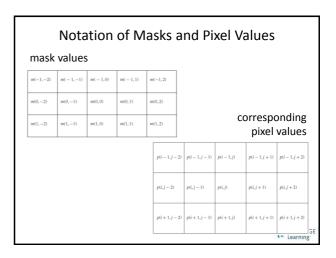
- x(n) = 1 0 0 0 0 1 0 0 0 0 1, h(n) = 1 2 3 1 x*h = 1 2 3 1 0 1 2 3 1 0 1 2 3 1
- $x(n) = 1 \ 1 \ 1$, $h(n) = 1 \ 1$ $x*h = 1 \ 2 \ 2 \ 1$
- Number of multiplications : M(M+N-1)

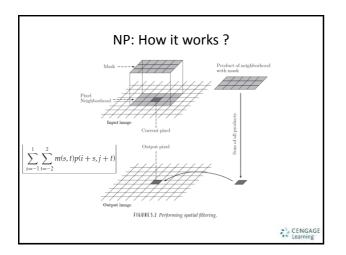
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Spatial Filtering: Convolution

- Allied to spatial filtering is spatial convolution
 - ✓ The filter must be rotated by 180° (flips in both horizontal and vertical directions) before multiplying and adding

$$\begin{split} & \sum_{s=-1}^{1} \sum_{t=-2}^{2} m(-s,-t) p(i+s,j+t) \\ & \sum_{s=-1}^{1} \sum_{t=-2}^{2} m(s,t) p(i-s+j-t) \end{split}$$

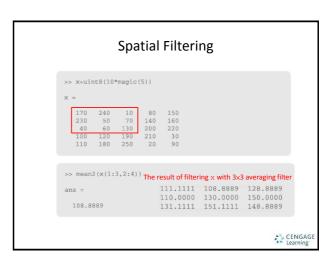
$$\sum_{s=-1}^{1} \sum_{t=-2}^{2} m(s,t) p(i-s+j-t)$$



Spatial Filtering

• EXAMPLE One important linear filter is to use a 3x3 mask and take the average of all nine values within the mask



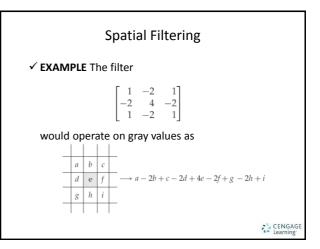


Spatial Filtering

- It is convenient to describe a linear filter simply in terms of the coefficients of all the gray values of pixels within the mask
 - √ The averaging filter

$$\begin{split} \frac{1}{9}a + \frac{1}{9}b + \frac{1}{9}c + \frac{1}{9}d + \frac{1}{9}e + \frac{1}{9}f + \frac{1}{9}g + \frac{1}{9}h + \frac{1}{9}i \\ \begin{bmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix} &= \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{split}$$

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Edges of the Image

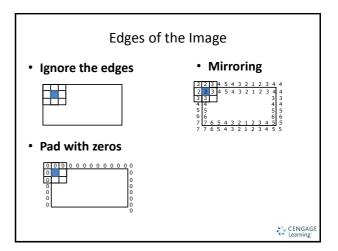
 What happens at the edge of the image, where the mask partly falls outside the image?



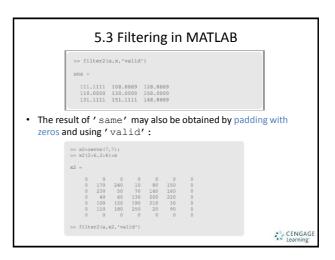
FIGURE 5.3 A mask at the edge of an image.

 There are a number of different approaches to dealing with this problem





5.3 Filtering in MATLAB • filter2 (filter, image, shape) • shape is optional and describes the method for dealing with edges. ✓ `same' -pad with zeros ✓ `valid' -ignore the edges >> a=ones(3,3)/9 a = 0.1111 0.1111 0.1111 0.1111 0.1111 0.1111 0.1111 0.1111 0.1111 >> filter2(a,x, 'same') ans = 76.6667 85.5556 65.5556 67.7778 58.8889 87.7778 111.1111 108.8889 128.8889 105.5556 66.6667 110.0000 130.0000 150.0000 160.6667 67.7778 131.1111 151.1111 148.8889 18.5556 56.6667 105.5556 107.7778 87.7778 38.8889



5.3 Filtering in MATLAB

- filter2 (filter, image, 'full') returns a result larger than the original.
- It does this by padding with zero and applying the filter at all places on and around the image where the mask intersects the image matrix.

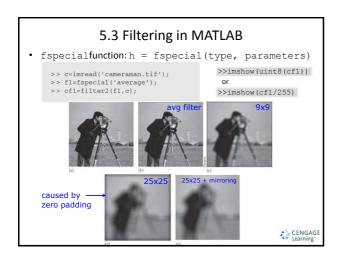
```
>> filter2(a,x,'full')
              76.6667
87.7778
   44.4444
                         85.5556
                                     65.5556
                                                            58.8889
                                                                       34.4444
   41.1111
              66.6667
                                               150.0000
                                                          106.6667
                                                                       45.5556
   27.7778
              67.7778
56.6667
                                    151.1111
                                                            85.5556
38.8889
                                                                       37.7778
                         131.1111
                                               148.8889
                        105.5556
                         60.0000
                                     50.0000
                                                                       Learnin
```

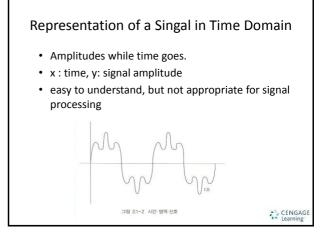
5.3 Filtering in MATLAB

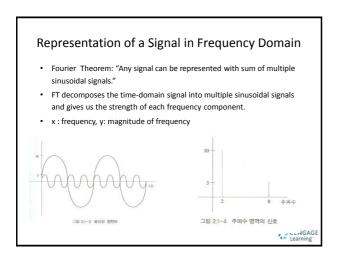
- filter2 provides no mirroring option
- The mirroring approach can be realized by placing the following codes before filter2 (filter, image, 'valid')

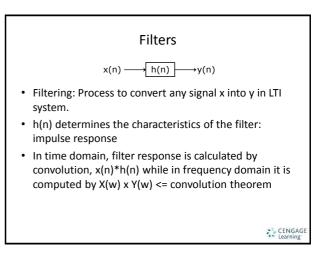
 Where matrix x is extended to m_x, wr/wc is defined as one half total column/row number of the mask (chopping the decimal)

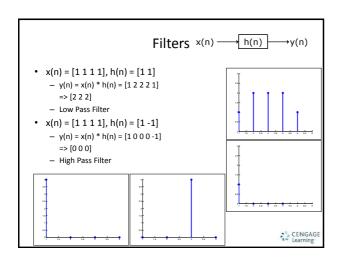
```
>> filter2(a,m_x,'valid')
ans=
185.5556 132.2222 102.2222 94.4444 135.5556
136.6667 111.1111 108.8889 128.8889 164.4444
107.7778 110.0000 130.0000 150.0000 152.2222
95.5556 131.1111 151.1111 148.8889 123.3333
124.4444 165.5556 157.7778 127.7778 74.4444
```

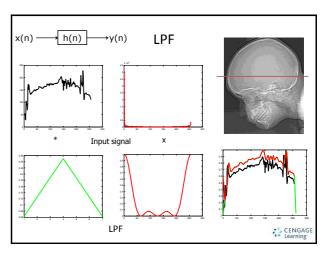


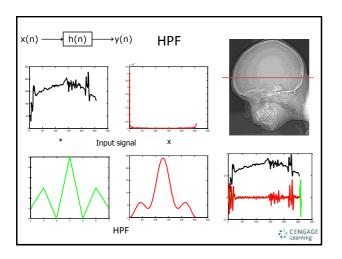


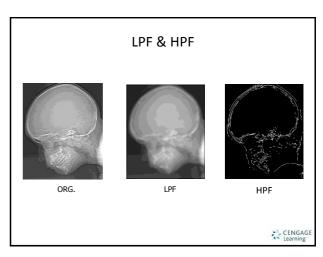












5.4 Frequencies: Low- and High-Pass Filters

- Frequencies of an image are a measure of the amount by which gray values change with distance
 - ✓ high-pass filter $\begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}$
 - ✓ low-pass filter $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

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Lowpass Filtering for Noise Smoothing

- Most image energy: e,g, bit plane slicing
 - -> is in its **low frequency components** (high spatial correlation among neighboring pixels),
- Most image noise
 - -> wideband (e.g., white Gaussian noise)
 - -> lowpass filtering (LPF) required

(it cannot remove the noise in the low frequency range though !).

- LPFs cause blurring
 - critical for some images with sharp edges
 - not critical for images of smooth contrast.
- Another use: remove blocky effect (e.g., 8 by 8 DCT based coding) created in lossy image coding.

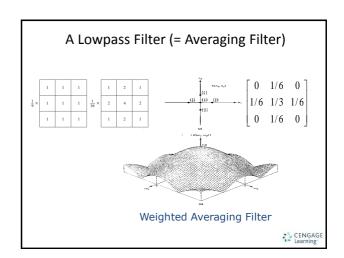


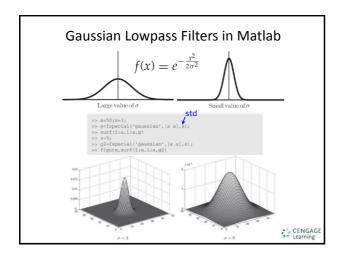
Some Typical Filter Masks of LPFs:

Averaging LPFs:

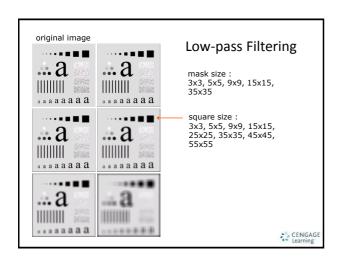
• Some Other (Gaussian) LPFs:

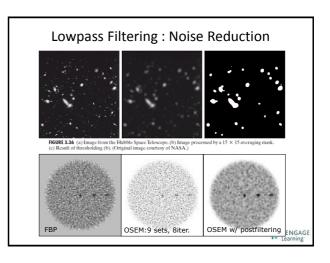
$$\frac{1}{10} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad \qquad \frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

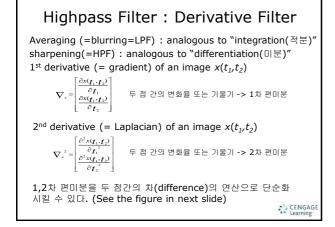


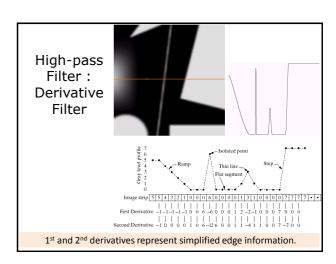












Edge Detection by Gradient (1st derivative)

• Edges: the image portions which have large gradients

$$\begin{split} &x(\boldsymbol{n}_1,\boldsymbol{n}_2) \underset{\text{gradient}}{\text{magnitude of}} \\ &\nabla_x(\boldsymbol{n}_1,\boldsymbol{n}_2) = mag(\nabla_x(\boldsymbol{n}_1,\boldsymbol{n}_2)) = \sqrt{\left[\frac{\partial x(\boldsymbol{n}_1,\boldsymbol{n}_2)}{\partial \boldsymbol{n}_1}\right]^2 + \left[\frac{\partial x(\boldsymbol{n}_1,\boldsymbol{n}_2)}{\partial \boldsymbol{n}_2}\right]^2} \\ &\nabla_x(\boldsymbol{n}_1,\boldsymbol{n}_2) \approx \left|\frac{\partial x(\boldsymbol{n}_1,\boldsymbol{n}_2)}{\partial \boldsymbol{n}_1}\right| + \left|\frac{\partial x(\boldsymbol{n}_1,\boldsymbol{n}_2)}{\partial \boldsymbol{n}_2}\right| \end{split}$$

You can apply the directional derivative edge detector only

apply two directional detectors together and sum their square magnitudes to get the omni-directional (isotropic) edges.



1st Derivative (gradient) Filters: Prewitt

Typically, we want to average over several neighboring rows, i.e.,

$$\begin{split} \frac{\partial x(\underline{t_1},\underline{t_2})}{\partial t_1} &\approx & \left[x(\underline{n_1}+1,\underline{n_2}-1) - x(\underline{n_1}-1,\underline{n_2}-1) \right] \\ &+ \left[x(\underline{n_1}+1,\underline{n_2}) - x(\underline{n_1}-1,\underline{n_2}) \right] \\ &+ \left[x(\underline{n_1}+1,\underline{n_2}+1) - x(\underline{n_1}-1,\underline{n_2}+1) \right] \end{split}$$

• These result in Prewitt operators (directional filters):

$$\begin{array}{c} \text{gradient for} \\ \text{y direction} \end{array} \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ \end{bmatrix} \qquad \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \\ \end{bmatrix} \\ \text{gradient for} \\ \text{x direction} \\ \text{cENGAGE} \\ \text{tearning}$$

1st Derivative (gradient) Filters: Sobel

• How about emphasize the center row more:

$$\frac{\partial x(\mathbf{f}_{1}, \mathbf{f}_{2})}{\partial \mathbf{f}_{1}} \approx \left[x(\mathbf{n}_{1}+1, \mathbf{n}_{2}-1) - x(\mathbf{n}_{1}-1, \mathbf{n}_{2}-1) \right] \\
+ 2 \left[x(\mathbf{n}_{1}+1, \mathbf{n}_{2}) - x(\mathbf{n}_{1}-1, \mathbf{n}_{2}) \right] \\
+ \left[x(\mathbf{n}_{1}+1, \mathbf{n}_{2}+1) - x(\mathbf{n}_{1}-1, \mathbf{n}_{2}+1) \right]$$

· These result in Sobel operators:

$$\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \qquad \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

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1st Derivative (gradient) Filters: Robert

• We can also deal with both derivatives simultaneously, i.e. emphasize diagonal edge information.:

$$\frac{\partial x(\boldsymbol{t}_1,\boldsymbol{t}_2)}{\partial \boldsymbol{t}_1} + \frac{\partial x(\boldsymbol{t}_1,\boldsymbol{t}_2)}{\partial \boldsymbol{t}_2} \approx \left[x(\boldsymbol{\eta}_1,\boldsymbol{\eta}_2) - x(\boldsymbol{\eta}_1 - 1,\boldsymbol{\eta}_2 - 1) \right]$$

• This results in **Robert** operators:

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

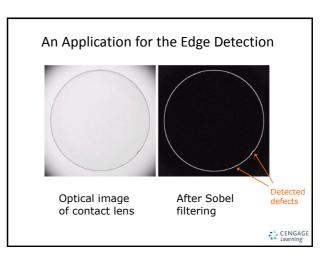
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Edge Extraction via Gradients (a) original (b) vertical Sobel filtering

(c) horizontal Sobel filter

(d) magnitude of gradients





Laplacian Edge Detectors: 2nd Derivative

$$\nabla_x^2(\boldsymbol{\eta}_1,\boldsymbol{\eta}_2) \cong \frac{\partial^2 x(\boldsymbol{\eta}_1,\boldsymbol{\eta}_2)}{\partial \boldsymbol{\eta}_1^2} + \frac{\partial^2 x(\boldsymbol{\eta}_1,\boldsymbol{\eta}_2)}{\partial \boldsymbol{\eta}_2^2}$$

Difference Eq. Approximation of Laplacian:

$$\begin{split} \frac{\partial x(n_1,n_2)}{\partial n_1} &\approx x(n_1+1,n_2) - x(n_1,n_2) \\ \frac{\partial^2 x(n_1,n_2)}{\partial n^2} &\approx \left[x(n_1+1,n_2) - x(n_1,n_2) \right] - \left[x(n_1,n_2) - x(n_1-1,n_2) \right] \\ &= x(n_1+1,n_2) - 2x(n_1,n_2) + x(n_1-1,n_2) \end{split}$$

$$\nabla_x^2(\boldsymbol{\eta}_1, \boldsymbol{\eta}_2) \cong x(\boldsymbol{\eta}_1 + 1, \boldsymbol{\eta}_2) + x(\boldsymbol{\eta}_1 - 1, \boldsymbol{\eta}_2) + x(\boldsymbol{\eta}_1, \boldsymbol{\eta}_2 + 1) + x(\boldsymbol{\eta}_1, \boldsymbol{\eta}_2 - 1) - 4x(\boldsymbol{\eta}_1, \boldsymbol{\eta}_2)$$

Isotropic (omni directional) edge detection



Some Laplacian Operators gain

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 & -1 \\ 2 & -4 & 2 \\ -1 & 2 & -1 \end{bmatrix}$$

- gain Note that: The edges are indicated by zero crossing, instead of large gradients (1st derivative).
- Detecting the zero crossing: low operated values with large variance in a local 5-by-5 window.

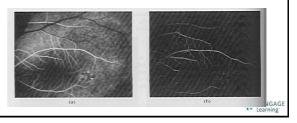
$$\sigma_{x}^{2} = \frac{1}{(2M+1)^{2}} \sum_{i=n_{1}-M}^{n_{1}+M} \sum_{j=n_{2}-M}^{n_{2}+M} \left[x(i,j) - \overline{x}(n_{1},n_{2}) \right]^{2}$$



A Laplacian Filter Mask: Graphical Representation $\underbrace{ \begin{array}{c} \overset{\sigma_1}{\longrightarrow} \overset{\sigma_1$

HPF: Laplacian Filter Mask

- Goal: highlight or enhance fine details in an image.
- A basic highpass spatial filtering: $\frac{1}{9}\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$
- Note that the sum of coefficients is 0! The DC term of H(w1,w2) is zero, i.e., the global contrast of the images is reduced.
- Results need scaling or clipping to remain in [0,L-1].



Summary

- · Chap 4. Point Processing
 - √ Histogram(Contrast) Stretching
 - ✓ Piecewise Contrast Stretching
 - ✓ Histogram Equalization & Specification
- · Chap 5. Neighborhood Processing
 - ✓ Convolution for Spatial Filtering
 - ✓ Lowpass Filtering: Gaussian filter
 - √ Highpass Filtering: Laplacian, LoG filter
 - ✓ Edge Sharpening by Unsharp Masking
 - ✓ High Boost Filtering
 - ✓ Nonlinear Filtering: median, min, max

