

Statistics in Healthcare

Unit 7 (optional):

Math details of ANOVA, Wilcoxon rank-sum test, and Log-rank test

Optional material: math details of the ANOVA test

How to calculate ANOVA's by hand...

Treatment 1	Treatment 2	Treatment 3	Treatment 4
y ₁₁	y ₂₁	y ₃₁	y ₄₁
y ₁₂	y ₂₂	y ₃₂	y ₄₂
y ₁₃	y ₂₃	y ₃₃	y ₄₃
y ₁₄	y ₂₄	y ₃₄	y ₄₄
y ₁₅	y ₂₅	y ₃₅	y ₄₅
y ₁₆	y ₂₆	y ₃₆	y ₄₆
y ₁₇	y ₂₇	y ₃₇	y ₄₇
y ₁₈	y ₂₈	y ₃₈	y ₄₈
y ₁₉	y ₂₉	y ₃₉	y ₄₉
y ₁₁₀	y ₂₁₀	y ₃₁₀	y ₄₁₀

$$n=10$$
 obs./group

k=4 groups

$$\overline{y}_{1\bullet} = \frac{\sum_{j=1}^{10} y_{1j}}{10}$$

$$\bar{y}_{2\bullet} = \frac{\sum_{j=1}^{10} y_{2j}}{10}$$

$$\overline{y}_{1\bullet} = \frac{\sum_{j=1}^{10} y_{1j}}{10} \qquad \overline{y}_{2\bullet} = \frac{\sum_{j=1}^{10} y_{2j}}{10} \qquad \overline{y}_{3\bullet} = \frac{\sum_{j=1}^{10} y_{3j}}{10} \qquad \overline{y}_{4\bullet} = \frac{\sum_{j=1}^{10} y_{4j}}{10}$$

$$\bar{y}_{4\bullet} = \frac{\sum_{j=1}^{10} y_{4j}}{10}$$

The group means

$$\sum_{j=1}^{10} (y_{1j} - \bar{y}_{1\bullet})^{2}$$

$$\frac{\sum_{j=1}^{10} (y_{1j} - \bar{y}_{1\bullet})^2}{10 - 1} \qquad \frac{\sum_{j=1}^{10} (y_{2j} - \bar{y}_{2\bullet})^2}{10 - 1} \qquad \frac{\sum_{j=1}^{10} (y_{3j} - \bar{y}_{3\bullet})^2}{10 - 1} \qquad \frac{\sum_{j=1}^{10} (y_{4j} - \bar{y}_{4\bullet})^2}{10 - 1}$$

$$-\frac{\sum_{j=1}^{10}(y)}{y}$$

$$\frac{\sum_{j=1}^{10} (y_{4j} - \overline{y}_{4\bullet})^2}{10 - 1}$$

The (within) group variances

Sum of Squares Within (SSW), or Sum of Squares Error (SSE)

$$\frac{\sum_{j=1}^{10} (y_{1j} - \bar{y}_{1\bullet})^2}{10 - 1} \qquad \frac{\sum_{j=1}^{10} (y_{2j} - \bar{y}_{2\bullet})^2}{10 - 1} \qquad \frac{\sum_{j=1}^{10} (y_{3j} - \bar{y}_{3\bullet})^2}{10 - 1} \qquad \frac{\sum_{j=1}^{10} (y_{4j} - \bar{y}_{4\bullet})^2}{10 - 1}$$

i=1 j=1

group variances

The (within)

$$\sum_{i=1}^{10} (y_{1j} - \overline{y}_{1\bullet})^2 + \sum_{j=1}^{10} (y_{2j} - \overline{y}_{2\bullet})^2 + \sum_{j=3}^{10} (y_{3j} - \overline{y}_{3\bullet})^2 + \sum_{j=1}^{10} (y_{4j} - \overline{y}_{4\bullet})^2$$

$$= \sum_{i=1}^{4} \sum_{j=1}^{10} (y_{ij} - \overline{y}_{i\bullet})^2$$

Sum of Squares Within (SSW) (or SSE, for chance error)

Sum of Squares Between (SSB), or Sum of Squares Regression (SSR)

Overall mean of all 40 observations ("grand mean")

$$\overline{\overline{y}}_{\bullet\bullet} = \frac{\sum_{i=1}^{4} \sum_{j=1}^{10} y_{ij}}{40}$$

$$10x\sum_{i\bullet}^{4}(\overline{y}_{i\bullet}-\overline{\overline{y}}_{\bullet\bullet})^{2} \leftarrow \boxed{}$$

Sum of Squares Between (SSB). Variability of the group means compared to the grand mean (the variability due to the treatment).

Total Sum of Squares (SST)

$$\sum_{i=1}^{4} \sum_{j=1}^{10} (y_{ij} - \overline{\overline{y}}_{\bullet \bullet})^2$$

Total sum of squares(TSS). Squared difference of every observation from the overall mean. (numerator of variance of Y!)

1

Partitioning of Variance

$$\sum_{i=1}^{4} \sum_{j=1}^{10} (y_{ij} - \overline{y}_{i\bullet})^2 + 10 \sum_{i=1}^{4} (\overline{y}_{i\bullet} - \overline{\overline{y}}_{\bullet\bullet})^2 = \sum_{i=1}^{4} \sum_{j=1}^{10} (y_{ij} - \overline{\overline{y}}_{\bullet\bullet})^2$$

SSW + SSB = SST

ANOVA Table

Source of variation	d.f.	Sum of squares	Mean Sum of Squares		
Between (k groups)	k-1	SSB (sum of squared deviations of group means from grand mean)	SSB/k-1	F-statistic $\frac{\frac{SSB}{k-1}}{\frac{SSW}{nk-k}}$	p-value Go to F _{k-1,nk-k} chart
Within (n individuals per group)	nk-k	SSW (sum of squared deviations of observations from their group mean)			
Total variation	nk-1	` -	ed deviations of rom grand mean)	TSS=SSB +	SSW

ANOVA=t-test

d.f.

Sum of squares

Sum of **Squares**

Mean

(squared difference in means multiplied by

SSB

n)

Squared difference in means

times n

Pooled variance $SSB = n\sum_{n=1}^{\infty} (\overline{X}_n - (\frac{\overline{X}_n + \overline{Y}_n}{2}))^2 + n\sum_{n=1}^{\infty} (\overline{Y}_n - (\frac{\overline{X}_n + \overline{Y}_n}{2}))^2 =$ $\left| n \sum_{i=1}^{n} \left(\frac{\overline{X}_{n}}{2} - \frac{\overline{Y}_{n}}{2} \right)^{2} + n \sum_{i=1}^{n} \left(\frac{\overline{Y}_{n}}{2} - \frac{\overline{X}_{n}}{2} \right)^{2} \right|$ $n((\frac{\overline{X}_n}{2})^2 + (\frac{\overline{Y}_n}{2})^2 - 2\frac{\overline{X}_n * \overline{Y}_n}{2} + (\frac{\overline{Y}_n}{2})^2 + (\frac{\overline{X}_n}{2})^2 - 2\frac{\overline{X}_n * \overline{Y}_n}{2}) =$ $n(\overline{X}_{n}^{2}-2\overline{X}_{n}*\overline{Y}_{n}+\overline{Y}_{n}^{2})=n(\overline{X}_{n}-\overline{Y}_{n})^{2}$

$$\frac{n(\overline{X} - \overline{Y})^{2}}{s_{p}^{2}} = (\frac{(\overline{X} - \overline{Y})}{\sqrt{\frac{s_{p}^{2} + s_{p}^{2}}{n}}})^{2} = (t_{2n-2})^{2}$$

$$\begin{array}{c} \mathbf{F}_{1, 2n-2} \\ \text{Chart} \rightarrow \\ \text{notice} \\ \text{values are} \\ \text{just } (\mathbf{t}_{2n-2})^{2} \end{array}$$

variance

Total variation

Within

2n-1 **TSS**

Example

Treatment 1	Treatment 2	Treatment 3	Treatment 4
60 inches	50	48	47
67	52	49	67
42	43	50	54
67	67	55	67
56	67	56	68
62	59	61	65
64	67	61	65
59	64	60	56
72	63	59	60
71	65	64	65

Example

Step 1) calculate the sum of squares between groups:

Mean for group 1 = 62.0

Mean for group 2 = 59.7

Mean for group 3 = 56.3

Mean for group 4 = 61.4

Grand mean = 59.85

Treatment 1	Treatment 2	Treatment 3	Treatment 4
60 inches	50	48	47
67	52	49	67
42	43	50	54
67	67	55	67
56	67	56	68
62	59	61	65
64	67	61	65
59	64	60	56
72	63	59	60
71	65	64	65

SSB =
$$[(62-59.85)^2 + (59.7-59.85)^2 + (56.3-59.85)^2 + (61.4-59.85)^2] xn per$$

 $group= 19.65x10 = 196.5$

Example

Step 2) calculate the sum of squares within groups:

deviations) = 2060.6

Treatment 1	Treatment 2	Treatment 3	Treatment 4
60 inches	50	48	47
67	52	49	67
42	43	50	54
67	67	55	67
56	67	56	68
62	59	61	65
64	67	61	65
59	64	60	56
72	63	59	60
71	65	64	65

Step 3) Fill in the ANOVA table

Source of variation	<u>d.f.</u>	Sum of squares	Mean Sum of Squares	F-statistic	p-value
Between	3	196.5	65.5	1.14	.344
Within	36	2060.6	57.2	-	-
Total	39	2257.1	_	_	-

Step 3) Fill in the ANOVA table

Source of variation	<u>d.f.</u>	Sum of squares	Mean Sum of Squares	F-statistic	p-value
Between	3	196.5	65.5	1.14	.344
Within	36	2060.6	57.2	-	-
Total	39	2257.1	_	_	_

INTERPRETATION of ANOVA:

How much of the variance in height is explained by treatment group?

 R^2 "Coefficient of Determination" = SSB/TSS = 196.5/2275.1=9%

-

Coefficient of Determination

$$R^2 = \frac{SSB}{SSB + SSE} = \frac{SSB}{SST}$$

SSE: Sum of Squares Within (SSW) (or SSE, for chance error)

The amount of variation in the outcome variable (dependent variable) that is explained by the predictor (independent variable).

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Beyond one-way ANOVA

Often, you may want to test more than 1 treatment. ANOVA can accommodate more than 1 treatment or factor, so long as they are independent. Again, the variation partitions beautifully!

SST = SSB1 + SSB2 + SSW





Optional material: math details for the Wilcoxon rank-sum test



Wilcoxon rank-sum test

Rank all of the observations in order from 1 to n.

 T_1 is the sum of the ranks from smaller population (n_1)

 T_2 is the sum of the ranks from the larger population (n_2)

$$\mathbf{U}_1 = n_1 n_2 + \frac{n_1 (n_1 + 1)}{2} - T_1$$

$$U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - T_2$$

Find
$$P(U \le U_0)$$
 in Mann-Whitney U tables
$$U_0 = \min(U_1, U_2)$$
With n_2 = the bigger of the 2 populations (or website)

$$Z = \frac{U_0 - \frac{n_1 n_2}{2}}{(n_1 + n_2 + 1)}$$

for $n_1 > 10$, $n_2 > 10$,

Table A5.07: Critical Values for the Wilcoxon/Mann-Whitney Test (U)

							None	direc	tion	ıl α=	.05 (0	Direc	tiona	l a=.	025)			163		36 38
8											r	12			220					
n ₁	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1			-	*	5.3	0.00			*	5.5				20	0.0		2.5	55		50
2		1	-				-	0	0	0	0	1	1	1	1	1	2	2	2	2
3	-		25		0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8
4	-	-	-	0	1	2	3	4	4	5	6	7	8	9	10	11	11	12	13	13
5			0	1	2	3	5	6	7	8	9	11	12	13	14	15	17	18	19	20
6			1	2	3 5	5	6	8	10	11	13	14	16	17	19	21	22	24	25	27
7	0.00	3.5	1	3	5	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34
8	-	0	2	4	6	8	10	13	15	17	19	22	24	26	29	31	34	36	38	41
9		0	2	4	7	10	12	15	17	21	23	26	28	31	34	37	39	42	45	48
10	-	0	3	5	8	11	14	17	20	23	26	29	33	36	39	42	45	48	52	55
11		0	3	6	9	13	16	19	23	26	30	33	37	40	44	47	51	55	58	62
12		1	4	7	11	14	18	22	26	29	33	37	41	45	49	53	57	61	65	69
13		1	4	8	12	16	20	24	28	33	37	41	45	50	54	59	63	67	72	76
14	-	1	5	9	13	17	22	26	31	36	40	45	50	55	59	64	67	74	78	83
15	-	1	5	10	14	19	24	29	34	39	44	49	54	59	64	70	75	80	85	90
16		1	6	11	15	21	26	31	37	42	47	53	59	64	70	75	81	86	92	98
17		2	6	11	17	22	28	34	39	45	51	57	63	67	75	81	87	93	99	105
18	-	2	7	12	18	24	30	36	42	48	55	61	67	74	80	86	93	99	106	112
19	_	2	7	13	19	25	32	38	45	52	58	65	72	78	85	92	99	106	113	119
20		2	8	14	20	27	34	41	48	55	62	69	76	83	90	98	105	112	119	127

$$U_o=18 < U_{crit} = 23$$
 \rightarrow reject H_o

 U_{obt} is the lesser of the two calculated test statistics ($U_1 \& U_2$). If $U_{obt} \le U_{orit}$, reject H_0 . Dashes (-) indicate that the sample size is too small to reject the Null Hypothesis at the chosen α level.

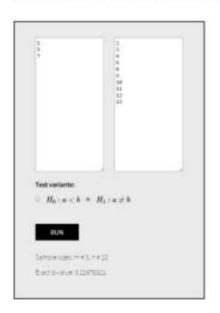
Example...

• Example: if team 1 and team 2 (two gymnastic teams) are competing, and the judges rank all the individuals in the competition, how can you tell if team 1 has done significantly better than team 2 or vice versa (unequal team size)?

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U calculator online...

Wilcoxon-Mann-Whitney Test Calculator



The online calculator provides an insperientation to solve the exact permission of the Misconnillativi Whome, text using the Misconnillativi text Miscoles the correct bouton is also recurred to text amount.

To man the text you fill in your range data lighty spaces to separate the sements choose the text larger and of the "Ryo".

For now information resorted to the

http://www.ccb.uni-saarland.de/?page_id=812

http://vassarstats.net/utest.html

http://www.socscistatistics.com/tests/signedranks/Default2.aspx

Applying the test to real data:

Example: If the girls on the two gymnastics teams were ranked as follows:

Team 1: 1, 5, 7

Observed $T_1 = 13$

Tedm 2: 2,3,4,6,8,9,10,11,12,13

Observed $T_2 = 78$

Are the teams significantly different?

Total sum of ranks = 13+78 = 91

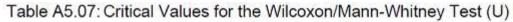
 $n_1 n_2 = 3*10 = 30$

$$U_1 = 30 + 6 - 13 = 23$$

$$U_2 = 30 + 55 - 78 = 7$$

$$\therefore U_0 = 7$$

Not quite statistically significant in U table...p=.1084 x2 for two-tailed test



Nondirectional a=.05 (Directional a=.025)

-											r	12			20					
n ₁	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1		. *	-	*	5.3	0.00			100	5.5	300				-		- 60	50		50
2			-					0	0	0	0	1	1	1	1	1	2	2	2	2
3	-	100	200	363	0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8
4	-	-	_	0	1	2	3	2	4	5	6	7	8	9	10	11	11	12	13	13
5		2.0	0	1	2	3	5	6	7	8	9	11	12	13	14	15	17	18	19	20
6			1	2	3	5	6	8	10	11	13	14	16	17	19	21	22	24	25	27
7		2.25	1	3	5	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34
8	-	0	2	4	6	8	10	13	15	17	19	22	24	26	29	31	34	36	38	41
9		0	2	4	7	10	12	15	17	21	23	26	28	31	34	37	39	42	45	48
10		0	3	5	8	11	14	17	20	23	26	29	33	36	39	42	45	48	52	55
11		0	3	6	9	13	16	19	23	26	30	33	37	40	44	47	51	55	58	62
12		1	4	7	11	14	18	22	26	29	33	37	41	45	49	53	57	61	65	69
13		1	4	8	12	16	20	24	28	33	37	41	45	50	54	59	63	67	72	76
14	-	1	5	9	13	17	22	26	31	36	40	45	50	55	59	64	67	74	78	83
15		1	5	10	14	19	24	29	34	39	44	49	54	59	64	70	75	80	85	90
16	-	1	6	11	15	21	26	31	37	42	47	53	59	64	70	75	81	86	92	98
17		2	6	11	17	22	28	34	39	45	51	57	63	67	75	81	87	93	99	105
18	-	2	7	12	18	24	30	36	42	48	55	61	67	74	80	86	93	99	106	112
19		2	7	13	19	25	32	38	45	52	58	65	72	78	85	92	99	106	113	119
20		2	8	14	20	27	34	41	48	55	62	69	76	83	90	98	105	112	119	127

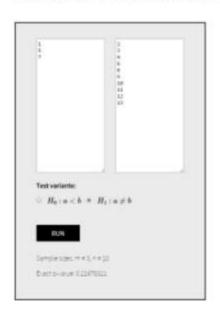
$$U_o = 7 > U_{crit} = 3$$

 \rightarrow fail to reject H_o

 U_{obt} is the lesser of the two calculated test statistics ($U_1 \& U_2$). If $U_{obt} \le U_{crit}$, reject H_0 . Dashes (-) indicate that the sample size is too small to reject the Null Hypothesis at the chosen α level.

U calculator online...

Wilcoxon-Mann-Whitney Test Calculator



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For more information read the bundle

http://vassarstats.net/utest.html





Optional: Math detail of logrank test

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Log-rank test example

Test of Equality over Strata

Log-Rank	4.6599	1	0.0309
Test	Chi-Square	DF	Pr > Chi-Square

Chi-square test (with 1 degree of freedom) of the (overall) difference between the two groups.

Groups are significantly different.

The log-rank test

K Strata =	
unique event	Group 1
times	Group 2

Event	No Event
→ a	b
С	d

$\left[\sum_{i=1}^{k} (a_k - E(a_k))\right]^2$
$\frac{1}{\sum_{k=1}^{k} Var(a_k)} \sim \chi_1$

$$E(a_k) = \frac{row1_k * col1_k}{N_k}$$

$$Var(a_k) = \frac{row1_k * row2_k * col1_k * col2_k}{N_k^2 (N_k - 1)}$$

Log-rank test

How do you know that this is a chi-square with 1 df?

	Event	No Event
Froup 1	a	b
Froup 2	c	d

$$\frac{\left[\sum_{i=1}^{k} (a_k - E(a_k))\right]^2}{\sum_{k=1}^{k} Var(a_k)} \sim \chi_1^2$$

$$E(a_k) = \frac{row1_k * col1_k}{N_k}$$

$$Var(a_k) = \frac{row1_k * row2_k * col1_k * col2_k}{N_k^2(N_k - 1)}$$

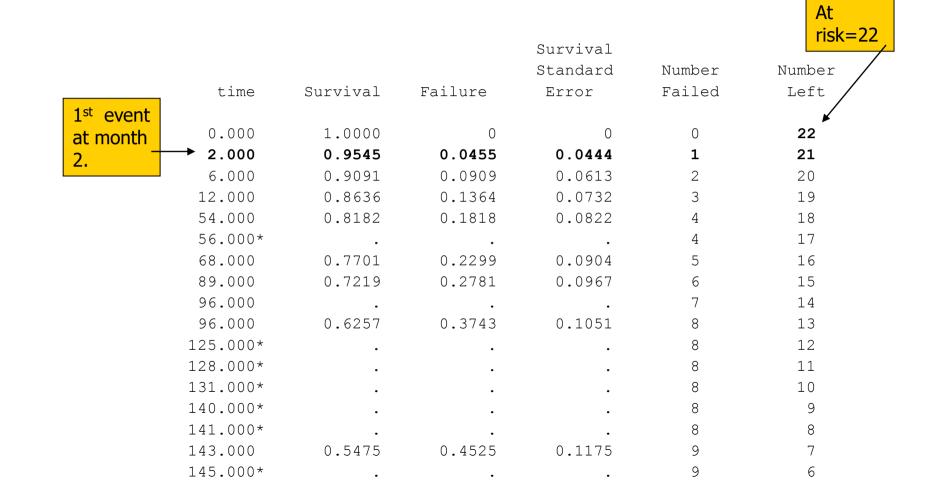
Variance is the variance of a hypergeometric distribution

Event time 1 (2 months), control group:

					/
			Standard	Number	Number
time	Survival	Failure	Error	Failed	Left /
0.000	1.0000	0	0	0	22
→ 2.000	0.9545	0.0455	0.0444	1	21
3.000	0.9091	0.0909	0.0613	2	20
4.000	0.8636	0.1364	0.0732	3	19
7.000	0.8182	0.1818	0.0822	4	18
10.000	0.7727	0.2273	0.0893	5	17
22.000	0.7273	0.2727	0.0950	6	16
28.000	0.6818	0.3182	0.0993	7	15
29.000	0.6364	0.3636	0.1026	8	14
32.000	0.5909	0.4091	0.1048	9	13
37.000	0.5455	0.4545	0.1062	10	12
40.000	0.5000	0.5000	0.1066	11	11
41.000	0.4545	0.5455	0.1062	12	10
54.000	0.4091	0.5909	0.1048	13	9
61.000	0.3636	0.6364	0.1026	14	8
63.000	0.3182	0.6818	0.0993	15	7
71.000	0.2727	0.7273	0.0950	16	6
127.000*				16	5
_	0.000 2.000 3.000 4.000 7.000 10.000 22.000 28.000 29.000 37.000 40.000 41.000 54.000 61.000 63.000 71.000	0.000 1.0000 → 2.000 0.9545 3.000 0.9091 4.000 0.8636 7.000 0.7727 22.000 0.7273 28.000 0.6818 29.000 0.6364 32.000 0.5455 40.000 0.5455 40.000 0.4545 54.000 0.4545 54.000 0.3636 63.000 0.3182 71.000 0.2727	0.000 1.0000 0 → 2.000 0.9545 0.0455 3.000 0.9091 0.0909 4.000 0.8636 0.1364 7.000 0.8182 0.1818 10.000 0.7727 0.2273 22.000 0.7273 0.2727 28.000 0.6818 0.3182 29.000 0.6364 0.3636 32.000 0.5909 0.4091 37.000 0.5455 0.4545 40.000 0.5000 0.5000 41.000 0.4545 0.5455 54.000 0.4091 0.5909 61.000 0.3636 0.6364 63.000 0.3182 0.6818 71.000 0.2727 0.7273	time Survival Failure Error 0.000 1.0000 0 0 → 2.000 0.9545 0.0455 0.0444 3.000 0.9091 0.0909 0.0613 4.000 0.8636 0.1364 0.0732 7.000 0.8182 0.1818 0.0822 10.000 0.7727 0.2273 0.0893 22.000 0.7273 0.2727 0.0950 28.000 0.6818 0.3182 0.0993 29.000 0.6364 0.3636 0.1026 32.000 0.5909 0.4091 0.1048 37.000 0.5455 0.4545 0.1062 40.000 0.5000 0.5000 0.1066 41.000 0.4545 0.5455 0.1062 54.000 0.4091 0.5909 0.1048 61.000 0.3636 0.6364 0.1026 63.000 0.3182 0.6818 0.0993 71.000 0.2727 0.7273 0.0950 </td <td>time Survival Failure Error Failed 0.000 1.0000 0 0 0 0 2.000 0.9545 0.0455 0.0444 1 3.000 0.9091 0.0909 0.0613 2 4.000 0.8636 0.1364 0.0732 3 7.000 0.8182 0.1818 0.0822 4 10.000 0.7727 0.2273 0.0893 5 22.000 0.7273 0.2727 0.0950 6 28.000 0.6818 0.3182 0.0993 7 29.000 0.6364 0.3636 0.1026 8 32.000 0.5909 0.4091 0.1048 9 37.000 0.5455 0.4545 0.1062 10 40.000 0.5000 0.5000 0.1066 11 41.000 0.4545 0.5455 0.1062 12 54.000 0.4091 0.5909 0.1048 13 61.000 0.3636 0.6364 0.1026 14 63.000 0.3182 0.6818 0.0993 15 71.000 0.2727 0.7273 0.0950 16</td>	time Survival Failure Error Failed 0.000 1.0000 0 0 0 0 2.000 0.9545 0.0455 0.0444 1 3.000 0.9091 0.0909 0.0613 2 4.000 0.8636 0.1364 0.0732 3 7.000 0.8182 0.1818 0.0822 4 10.000 0.7727 0.2273 0.0893 5 22.000 0.7273 0.2727 0.0950 6 28.000 0.6818 0.3182 0.0993 7 29.000 0.6364 0.3636 0.1026 8 32.000 0.5909 0.4091 0.1048 9 37.000 0.5455 0.4545 0.1062 10 40.000 0.5000 0.5000 0.1066 11 41.000 0.4545 0.5455 0.1062 12 54.000 0.4091 0.5909 0.1048 13 61.000 0.3636 0.6364 0.1026 14 63.000 0.3182 0.6818 0.0993 15 71.000 0.2727 0.7273 0.0950 16

Survival

Event time 1 (2 months), treated group:



4

Stratum 1= event time 1

Event time 1:

1 died from each group. (22 at risk in each group)

treated control

Event	No Event
1	21
1	21

$$a_{1} = 1$$

$$E(a_{1}) = \frac{(22) * (2)}{44} = 1$$

$$Var(a_{1}) = \frac{(22) * (22) * (2) * (42)}{44^{2} (43)} = .244$$

Event time 2 (3 months), control group:

				Survival Standard	Number	risk=21
	time	Survival	Failure	Error	Failed	Number Left
Next	0.000	1.0000	0	0	0	22
event at	2.000	0.9545	0.0455	0.0444	1	21 🗸
month	→ 3.000	0.9091	0.0909	0.0613	2	20
3.	4.000	0.8636	0.1364	0.0732	3	19
3.	7.000	0.8182	0.1818	0.0822	4	18
	10.000	0.7727	0.2273	0.0893	5	17
	22.000	0.7273	0.2727	0.0950	6	16
	28.000	0.6818	0.3182	0.0993	7	15
	29.000	0.6364	0.3636	0.1026	8	14
	32.000	0.5909	0.4091	0.1048	9	13
	37.000	0.5455	0.4545	0.1062	10	12
	40.000	0.5000	0.5000	0.1066	11	11
	41.000	0.4545	0.5455	0.1062	12	10
	54.000	0.4091	0.5909	0.1048	13	9
	61.000	0.3636	0.6364	0.1026	14	8
	63.000	0.3182	0.6818	0.0993	15	7
	71.000	0.2727	0.7273	0.0950	16	6
	127.000*	•	•	•	16	5

At

Event time 2 (3 months), treated group:

	time	Survival	Failure	Survival Standard Error	Number Failed	Nun risk=21
No events at 3 months	0.000 2.000 6.000 12.000 54.000 56.000* 68.000 89.000 96.000 125.000* 128.000* 131.000* 141.000* 143.000	1.0000 0.9545 0.9091 0.8636 0.8182 0.7701 0.7219 0.6257	0 0.0455 0.0909 0.1364 0.1818 0.2299 0.2781 0.3743	0 0.0444 0.0613 0.0732 0.0822 0.0904 0.0967 0.1051	0 1 2 3 4 4 5 6 7 8 8 8 8	22 21 20 19 18 17 16 15 14 13 12 11 10 9 8 7
	145.000*	•	•	•	9	6

Stratum 2= event time 2

Event time 2:

At 3 months, 1 died in the control group.

At that time 21 from each group were at risk

treated control

Event	No Event
0	21
1	20

 $a_{1} = 0$ $E(a_{1}) = \frac{(1) * (21)}{42} = .5$ $Var(a_{1}) = \frac{(21) * (21) * (1) * (41)}{42^{2} (41)} = .25$

Event time 3 (4 months), control group:

				Survival		A L
				Standard	Number	Num At
	time	Survival	Failure	Error	Failed	Le risk=20
	0.000	1.0000	0	0	0	22
	2.000	0.9545	0.0455	0.0444	1	21 /
1 event at	3.000	0.9091	0.0909	0.0613	2	20 🖟
month 4.	→ 4.000	0.8636	0.1364	0.0732	3	19
	7.000	0.8182	0.1818	0.0822	4	18
	10.000	0.7727	0.2273	0.0893	5	17
	22.000	0.7273	0.2727	0.0950	6	16
	28.000	0.6818	0.3182	0.0993	7	15
	29.000	0.6364	0.3636	0.1026	8	14
	32.000	0.5909	0.4091	0.1048	9	13
	37.000	0.5455	0.4545	0.1062	10	12
	40.000	0.5000	0.5000	0.1066	11	11
	41.000	0.4545	0.5455	0.1062	12	10
	54.000	0.4091	0.5909	0.1048	13	9
	61.000	0.3636	0.6364	0.1026	14	8
	63.000	0.3182	0.6818	0.0993	15	7
	71.000	0.2727	0.7273	0.0950	16	6
	127.000*	•	•		16	5

Event time 3 (4 months), treated group:

			Survival Standard	Number	At risk=21
time	Survival	Failure	Error	Failed	Left
0.000	1.0000	0	0	0	22
2.000	0.9545	0.0455	0.0444	1	21
6.000	0.9091	0.0909	0.0613	2	20
12.000	0.8636	0.1364	0.0732	3	19
54.000	0.8182	0.1818	0.0822	4	18
56.000*		•	•	4	17
68.000	0.7701	0.2299	0.0904	5	16
89.000	0.7219	0.2781	0.0967	6	15
96.000		•	•	7	14
96.000	0.6257	0.3743	0.1051	8	13
125.000*				8	12
128.000*		•	•	8	11
131.000*		•	•	8	10
140.000*		•	•	8	9
141.000*		•		8	8
143.000	0.5475	0.4525	0.1175	9	7
145.000*	•		•	9	6

Stratum 3= event time 3 (4 months)

Event time 3:

At 4 months, 1 died in the control group.

At that time 21 from the treated group and 20 from the control group were at-risk.

treated control

Event	No Event
0	21
1	19

$$a_{1} = 0$$

$$E(a_{1}) = \frac{(1) * (21)}{41} = .51$$

$$Var(a_{1}) = \frac{(21) * (20) * (1) * (40)}{41^{2} (40)} = .25$$

41

4

Etc., 1 stratum per event time

$$\frac{\left[\sum_{i=1}^{22} (a_k - E(a_k))\right]^2}{\sum_{i=1}^{22} Var(a_k)} = \frac{\left[(1-1) + (0-.5) + (0-.51) + \dots \right]^2}{.244 + .25 + .25 + \dots} = 4.66$$

Then find P value \rightarrow P < 0.05 \rightarrow Significant