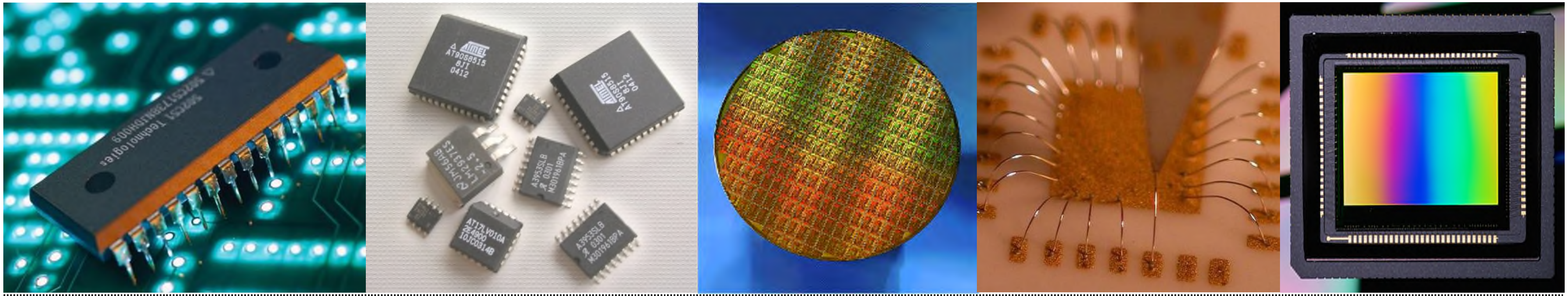

1. Digital System &Binary Numbers

1.1 Digital System

- Digital System
  - Communication, Business transaction, Traffic control, other commercial, industrial, scientific enterprises etc.
  - Discrete elements of information
- Signal
  - Discrete elements of information are represented in a digital system by physical quantities
- Binary Code
  - Discrete elements of information are represented with groups of bits
  - bit : binary + digit

1.2 Binary Number

- $a_5a_4a_3a_2a_1a_0.a_{-1}a_{-2}a_{-3}$
  $= a_nr^n+a_{n-1}r^{n-1}+...+a_2r^2+a_1r+a_0+a_{-1}r^{-1}+a_{-2}r^{-2}+...+a_{-m}r^{-m}$

$7392 = 7 \times 10^3 + 3 \times 10^2 + 9 \times 10^1 +2 \times 10^0$

$(11010.11)_2 =1\times2^4+ 1\times2^3+ 0\times2^2+ 1\times2^1+ 0\times2_0+ 1\times2_{-1}+ 1\times2_{-2}$

$= (26.75)_{10}$

$2^{10} = 1$Kilo

$2^{20} = 1$Mega

$2^{30} = 1$Giga

Table 1-1
Powers of Two

n	2^n	n	2^n	n	2^n
0	1	8	256	16	65,536
1	2	9	512	17	131,072
2	4	10	1,024	18	262,144
3	8	11	2,048	19	524,288
4	16	12	4,096	20	1,048,576
5	32	13	8,192	21	2,097,152
6	64	14	16,384	22	4,194,304
7	128	15	32,768	23	8,388,608

1.2 Binary Number

					오타, 1011
Augend	101101	Minuend:	101101	Multiplicand:	~~101~~
Addend	+100111	Subtrahend:	-100111	Multiplier:	*101
sum	1010100	Difference:	000110		1011
					0000
					1011
				Product:	10111

1.3 Number Base Conversions

- Ex 1-1) Convert decimal 41 to binary.

	Integer Quotient		Remainder	Coefficient
41/2 =	20	+	½	$a_0 = 1$
20/2 =	10	+	0	$a_1 = 0$
10/2 =	5	+	0	$a_2 = 0$
5/2 =	2	+	½	$a_3 = 1$
2/2 =	1	+	0	$a_4 = 0$
1/2 =	0	+	½	$a_5 = 1$

Integer	Remainder
41	
20	1
10	0
5	0
2	1
1	0
0	1

Answer

=101001

answer : $(41)_{10} = (a_5a_4a_3a_2a_1a_0)_2 = (101001)_2$

1.3 Number Base Conversions

- Ex 1-2) Convert decimal 153 to octal.

153	
19	1
2	3
0	2

$= (231)_8$

- Ex 1-3) Convert $(0.6875)_{10}$ to binary. **소수부부이 0이 될 때 까지 계속 2를 곱해주면 됨**

	Integer		Fraction	Coefficient
0.6875*2 =	1	+	0.3750	$a_{-1} = 1$
0.3750*2 =	0	+	0.7500	$a_{-2} = 0$
0.7500*2 =	1	+	0.5000	$a_{-3} = 1$
0.5000*2 =	1	+	0.0000	$a_{-4} = 1$

Answer: $(0.6875)_{10} = (0.a_{-1}a_{-2}a_{-3}a_{-4})_2 = (0.1011)_2$

1.4 Octal and Hexadecimal Numbers

Table 1-2
Numbers with Different Bases

Decimal (base 10)	Binary (base 2)	Octal (base 8)	Hexadecimal (base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

$$(\underset{2}{\underline{10}}\ \underset{6}{\underline{110}}\ \underset{1}{\underline{001}}\ \underset{5}{\underline{101}}\ \underset{3}{\underline{011}}\ .\ \underset{7}{\underline{111}}\ \underset{4}{\underline{100}}\ \underset{0}{\underline{000}}\ \underset{6}{\underline{110}})_2 = (26153.7460)_8$$

$$(\underset{2}{\underline{10}}\ \underset{C}{\underline{1100}}\ \underset{6}{\underline{0110}}\ \underset{B}{\underline{1011}}\ .\ \underset{F}{\underline{1111}}\ \underset{2}{\underline{0010}})_2 = (2C6B.F2)_{16}$$

1.5 Complements – Diminished Radix Complement

- (r-1)'s complements of N is $(r^n-1)-N$

- r=10, r-1=9, 9'complements of N is $(10^n-1)-N$

  Ex) the 9's complements of 546700 is 999999-546700 = 453299

  the 9's complements of 012398 is 999999-012398 = 987601

- For binary number, r=2, r-1=1

  1'complements of N is $(2^n-1)-N$

  Ex) the 1's complements of 1011000 is 0100111

  the 1's complements of 0101101 is 1010010

1.5 Complements – Radix Complement

- The r's complements of an n-digit number N is r^n-N, N≠0

  0, N=0

- $r^n - N = [(r^n - 1) - N] + 1$

  → The r's complements is obtained by adding 1 to the (r-1)'s complements

- Ex) The 10's complements of 012398 is 987602

  The 10's complements of 246700 is 753300

  The 2's complements of 1101100 is 0010100

  The 2's complements of 0110111 is 1001001

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- Ex1-5) using 10's complement, subtract 72532-3250.

M =	72532
10's complement of N =	+ 96750
Sum =	169282
Discard end carry 10^5 =	-100000
Answer =	69282

- Ex1-6) Using 10's complement, subtract 3250-72532.

M =	03250
10's complement of N =	+27468
Sum =	30718

There is no end carry

Therefore, the answer is –(10's complement of 30718)=-69282

- Ex1-7) X=1010100, Y=1000011, (a) X-Y, (b) Y-X

(a) X−Y

X =	1010100
2's complement of Y =	+0111101
Sum =	10010001
Discard end carry 2^7 =	-10000000
Answer: X-Y =	0010001

(b) Y−X

Y =	1000011
2's complement of X =	+0101100
Sum =	1101111

There is no carry.

The answer is Y−X = −(2's complement of 1101111)=−0010001

- Ex1-8) Repeat Example 1-7 using 1'complement.

(a) X−Y = 1010100−10000011

X =	1010100
1's complement of Y =	+0111100
Sum =	10010000
End-around carry =	+ 1
Answer: X-Y =	0010001

(b) Y−X = 10000011−1010100

Y =	1000011
1's complement of X =	+0101011
Sum =	1101110

There is no carry.

The answer is Y−X = −(1's complement of 1101110)=−0010001

1.6 Signed Binary Numbers

- Ex) The number 9 represented in binary with eight bit

  +9 : 00001001

  -9 : 10001001 (signed-magnitude representation)

  11110110 (signed-1's-complement representation)

  11110111 (signed-2's-complement representation)

Table 1-3
Signed Binary Numbers

Decimal	Signed-2's complement	Signed-1's complement	Signed magnitude
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
−0	—	1111	1000
−1	1111	1110	1001
−2	1110	1101	1010
−3	1101	1100	1011
−4	1100	1011	1100
−5	1011	1010	1101
−6	1010	1001	1110
−7	1001	1000	1111
−8	1000	—	—

1.6 Signed Binary Numbers

- Arithmetic Addition

  - signed-magnitude system follows the rules of ordinary arithmetic.
  - signed-complement system requires only addition.

+6	00000110	-6	11111010
+13	00001101	+13	00001101
+19	00010011	+7	00000111

+6	00000110	-6	11111010
-13	11110011	-13	11110011
-7	11111001	-19	11101101

- Arithmetic Subtraction

$$(\pm A)-(+B) = (\pm A)+(-B)$$
$$(\pm A)-(-B) = (\pm A)+(+B)$$

1.7 Binary Code-BCD code

2진수를 10진수처럼 사용하기 위해서

- the 4-bit code for one decimal

$$(185)_{10} = (0001\ 1000\ 0101)_{BCD} = (10111001)_2$$

Table 1-4
Binary Coded Decimal (BCD)

Decimal symbol	BCD digit
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

- BCD Addition

4	0100	4	0100	8	1000
+5	+0101	+8	+1000	+9	+1001
9	1001	12	1100	17	10001
			+0110		+0110
			10010		10111

- if the binary sum is greater or equal to 1010, we add 0110 to obtain the correct BCD

1.7 Binary Code-Other Decimal Codes

Table 1-5
Four Different Binary Codes for the Decimal Digits

Decimal digit	BCD 8421	2421	Excess-3	8 4-2-1
0	0000	0000	0011	0 0 0 0
1	0001	0001	0100	0 1 1 1
2	0010	0010	0101	0 1 1 0
3	0011	0011	0110	0 1 0 1
4	0100	0100	0111	0 1 0 0
5	0101	1011	1000	1 0 1 1
6	0110	1100	1001	1 0 1 0
7	0111	1101	1010	1 0 0 1
8	1000	1110	1011	1 0 0 0
9	1001	1111	1100	1 1 1 1
	1010	0101	0000	0 0 0 1
Unused	1011	0110	0001	0 0 1 0
bit	1100	0111	0010	0 0 1 1
combi-	1101	1000	1101	1 1 0 0
nations	1110	1001	1110	1 1 0 1
	1111	1010	1111	1 1 1 0

1.7 Binary Code-Gray Code

Table 1-6
Gray Code

Gray code	Decimal equivalent
0000	0
0001	1
0011	2
0010	3
0110	4
0111	5
0101	6
0100	7
1100	8
1101	9
1111	10
1110	11
1010	12
1011	13
1001	14
1000	15

그레이코드는 연속한 숫자 사이에 비트가 단 하나만 다르다.
-> 오류를 더 쉽게 발견하거나 최적화를 위해서 사용

1.7 Binary Code- ASCII Character Code

Table 1-7
American Standard Code for Information Interchange (ASCII)

	$b_7b_6b_5$							
$b_4b_3b_2b_1$	**000**	**001**	**010**	**011**	**100**	**101**	**110**	**111**
0000	NUL	DLE	SP	0	@	P	`	p
0001	SOH	DC1	!	1	A	Q	a	q
0010	STX	DC2	“	2	B	R	b	r
0011	ETX	DC3	#	3	C	S	c	s
0100	EOT	DC4	$	4	D	T	d	t
0101	ENQ	NAK	%	5	E	U	e	u
0110	ACK	SYN	&	6	F	V	f	v
0111	BEL	ETB	‘	7	G	W	g	w
1000	BS	CAN	(	8	H	X	h	x
1001	HT	EM	)	9	I	Y	i	y
1010	LF	SUB	*	:	J	Z	j	z
1011	VT	ESC	+	;	K	[	k	{
1100	FF	FS	,	<	L	\	l	|
1101	CR	GS	−	=	M	]	m	}
1110	SO	RS	.	>	N	∧	n	~
1111	SI	US	/	?	O	—	o	DEL

- Error-Detecting Code

데이터 전송에 있어서 비트를 하나를 더 붙여서 1의 갯수를 체크하여 데이터 전송에 오류가 있었는지 확인하는(Error-Detecting) 방식

	With even parity **1의 갯수가 짝수(even)**	With odd parity **1의 갯수가 홀수(odd)**
ASCII A = 1000001	01000001	11000001
ASCII T = 1010100	11010100	01010100

1.8 Binary Storage and Registers

- Registers – A register with n cells can store any discrete quantity of information that contains n bits.
- Register Transfer

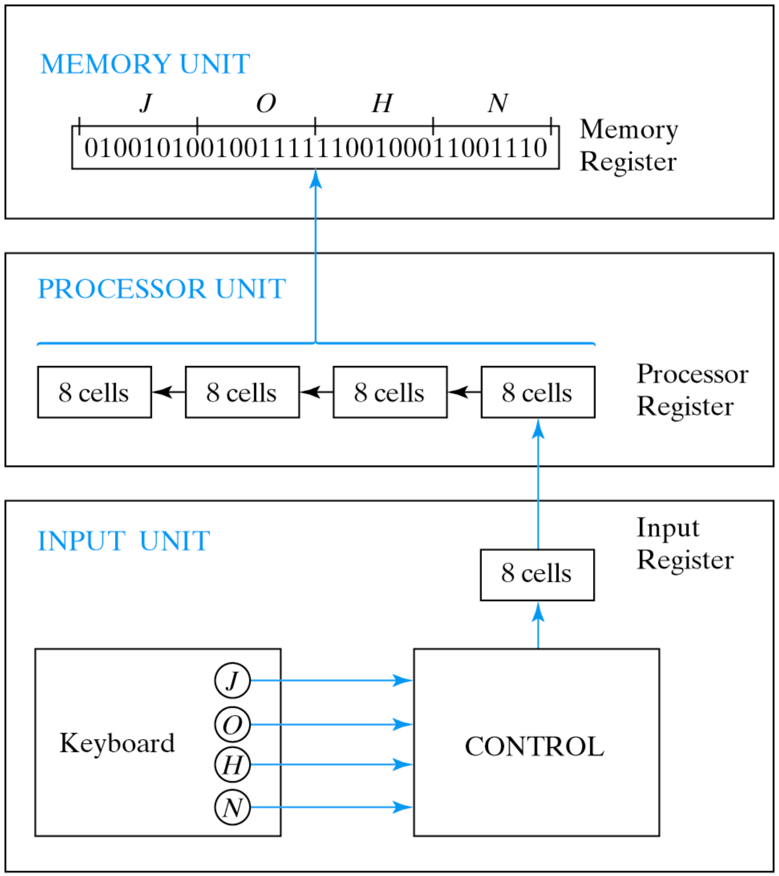


Fig. 1-1 Transfer of information with registers

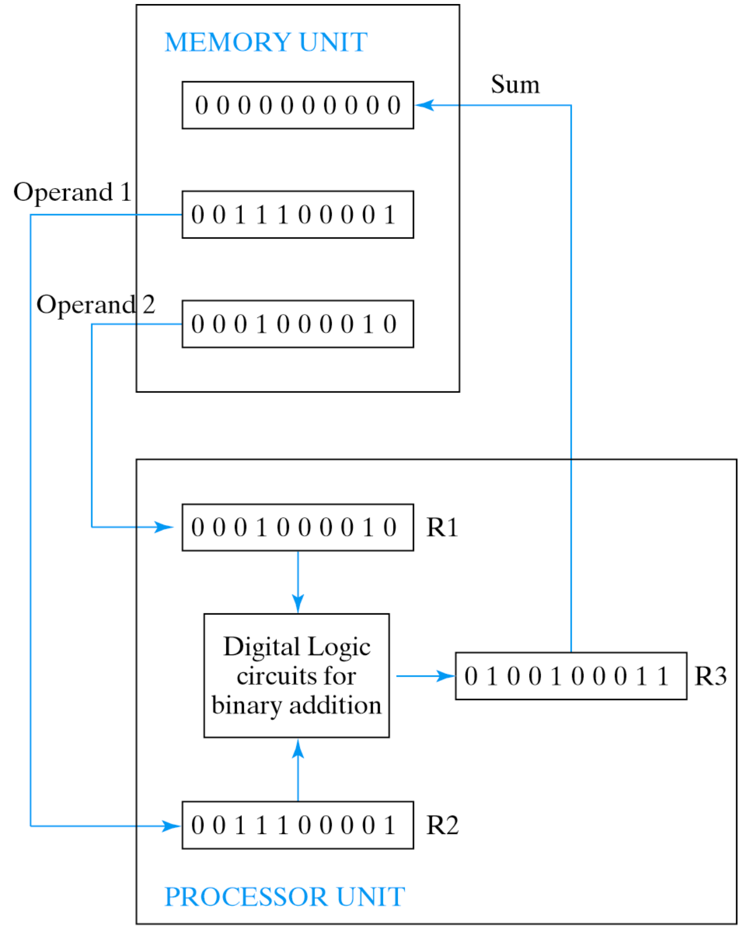


Fig. 1-2 Example of binary information processing

1.9 Binary Logic

- Definition of Binary Logic

Table 1-8
Truth Tables of Logical Operations

AND			OR			NOT	
x	y	$x \cdot y$	x	y	$x + y$	x	x'
0	0	0	0	0	0	0	1
0	1	0	0	1	1	1	0
1	0	0	1	0	1		
1	1	1	1	1	1		

- Logic Gates

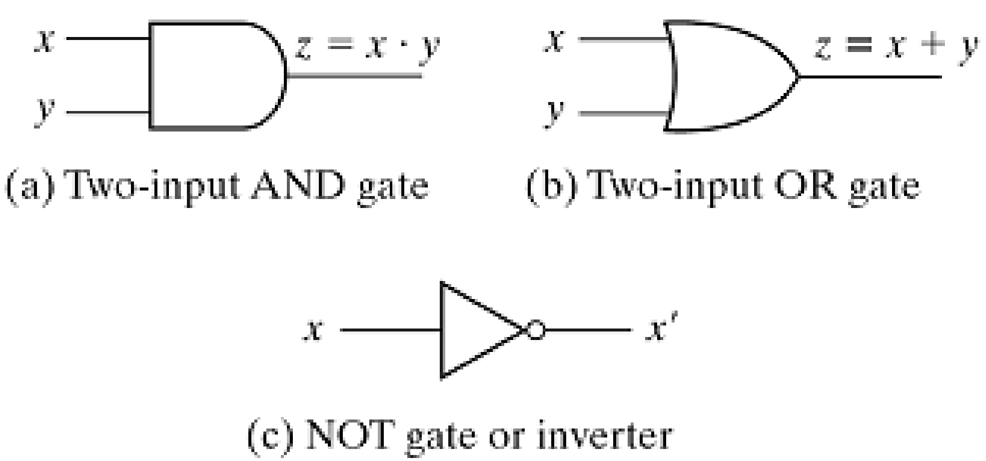


Fig. 1-4 Symbols for digital logic circuits

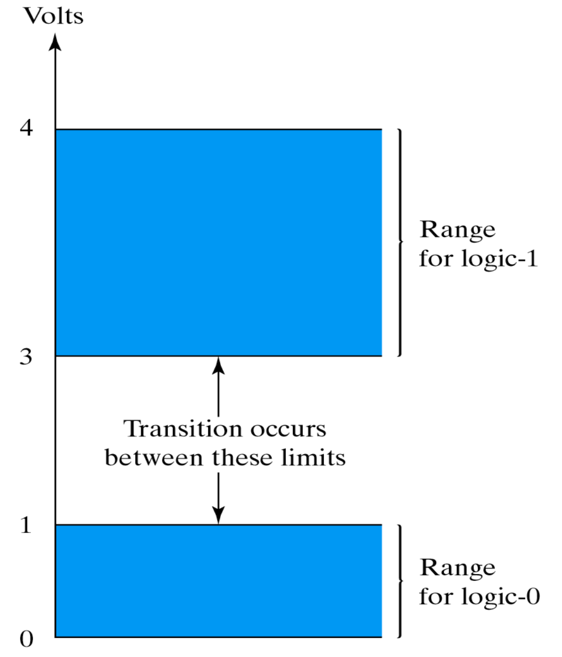


Fig. 1-3 Example of binary signals

1.9 Binary Logic

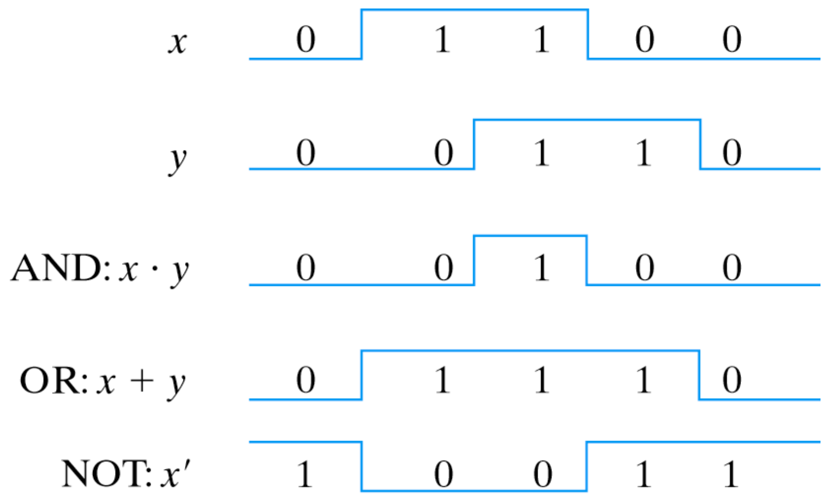
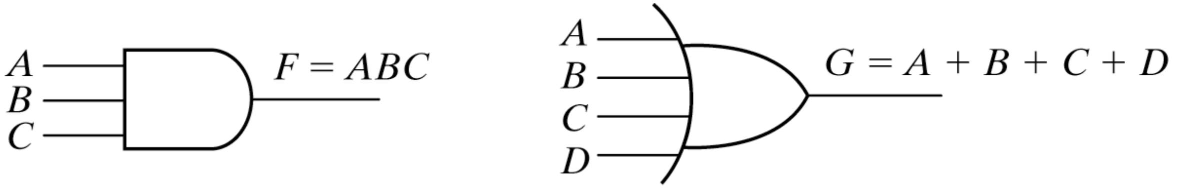


Fig. 1-5 Input-output signals for gates



(a) Three-input AND gate

(b) Four-input OR gate

Fig. 1-6 Gates with multiple inputs