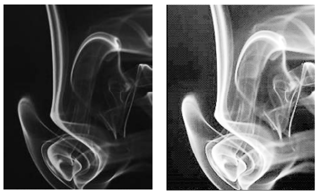
Three Classes of Image Processing Operations by Direct Manipulation of Gray Levels

- Any image-processing operation transforms the gray values of the pixels.
- Image-processing operations may be divided into three classes based on the information required to perform the transformation.
- From the simplest to the most complex, they are as follows:
  - **Point operations : chap.4**
  - **Neighborhood processing (spatial filter) : chap.5**
  - **Geometrical transforms: chap.6**



Chapter 4: Point Processing

Arithmetic Operations

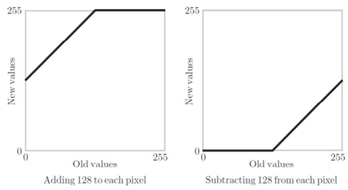
- These operations act by applying a simple function

$$y = f(x)$$

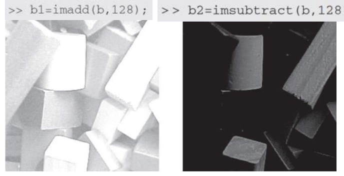
- In each case we may have to adjust the output slightly in order to ensure that the results are integers in the 0 . . . 255 range **(type uint8)**

$$y \leftarrow \begin{cases} 255 & \text{if } y > 255, \\ 0 & \text{if } y < 0. \end{cases}$$

Arithmetic Operations

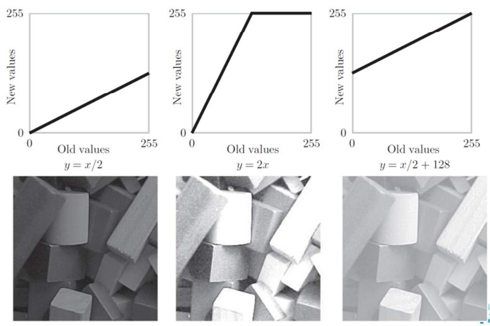



```
>> b=imread('blocks.tif');
>> b1=imadd(b,128);   >> b2=imsubtract(b,128);
```



Arithmatic Op: Multiplication and Division

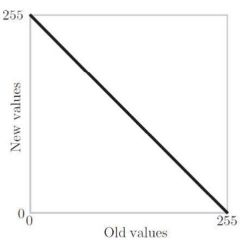
$y = x/2$	`b3=immultiply(b,0.5);` or `b3=imdivide(b,2)`
$y = 2x$	`b4=immultiply(b,2);`
$y = x/2 + 128$	`b5=imadd(immultiply(b,0.5),128);` or `b5=imadd(imdivide(b,2),128);`

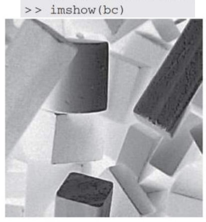


4.2 Arithmetic Op: Complements

- The **complement** of a grayscale image is its photographic negative (= **image negative**)
- **type double** (0.0~1.0) : `1-m`
- **type uint8** (0~255) : `255-m`

```
>> bc=imcomplement(b);
>> imshow(bc)
```





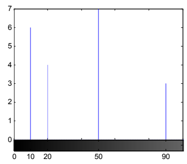
4.3 Histogram

- A graph representing pixel value vs the number of its occurrance in the image
  (= 확률 밀도 함수, Probability Density Function (PDF))

$$p_r(r_k) = \frac{n_k}{n} \qquad k = 0,1,2,\ldots,L-1$$

  - n : total number of pixels in an image
  - n_k : number of pixel value r_k
  - L : number of intensity levels (pixel values)
- Graph of r_k vs. $p_r(r_k)$

EX: a = [10 10 10 10 10;
20 20 20 20 10;
50 50 50 50 50;
90 90 90 50 50]



Histogram Example

```
>> p=imread('pout.tif');
>> imshow(p),figure,imhist(p),axis tight
```

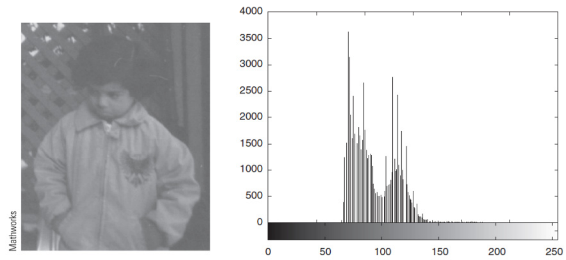
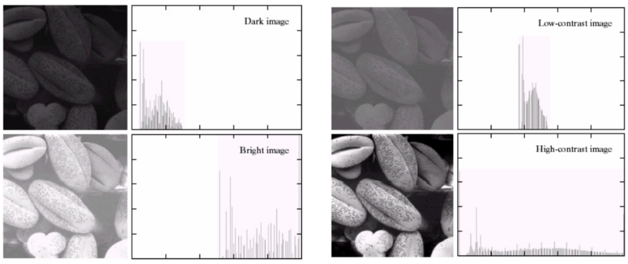


FIGURE 4.8 *The image* `pout.tif` *and its histogram.*

Histogram and Image Properties



Histogram(Contrast) Stretching

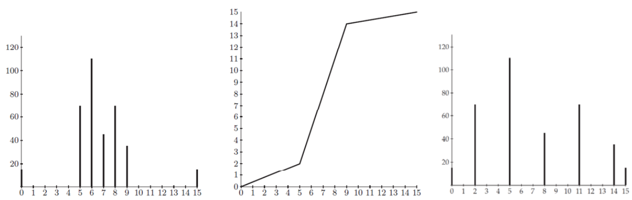
- A table of the numbers n_i of gray values

Gray level i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
n_i	15	0	0	0	0	70	110	45	70	35	0	0	0	0	0	15

(with n = 360, as before)

- We can stretch out the gray levels in the center of the range by applying the piecewise linear function

Contrast Stretching



Transformation function

$$j = \frac{14-2}{9-5}(i-5)+2$$

i : original gray level
j : its result after the transformation

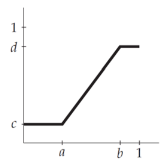
Transformaiton Lookup table

i	5	6	7	8	9
j	2	5	8	11	14

This function has the effect of stretching the gray levels 5–9 to gray levels 2–14

Constrast Stretching in Matlab

```
imadjust(im,[a,b],[c,d])
```



- **imadjust** is designed to work equally well on images of type `double`, `uint8`, or `uint16`
- the values of *a*, b, c, and d must be between 0 and 1
- **imadjust** automatically converts the image `im` (if needed) to be of type `double`

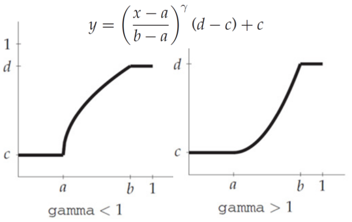
- Note that `imadjust` does not work quite in the same way as shown in the figure.
- The `imadjust` function has one other optional parameter: the `gamma` value.

$$y = \left(\frac{x-a}{b-a}\right)^{\gamma}(d-c)+c$$

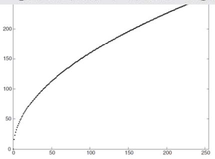
Contrast Stretching with γ

```
>> t=imread('tire.tif');
>> th=imadjust(t,[],[],0.5);
>> imshow(t),figure,imshow(th)
```

$$y = \left(\frac{x-a}{b-a}\right)^{\gamma}(d-c)+c$$



```
>> plot(t,th,'.'),axis tight
```



CS: Piecewise Linear Transformation Function

- How to implement in Matlab ?

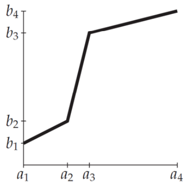
```
pix=find(im >= a(i) & im < a(i+1));
out(pix)=(im(pix)-a(i))*(b(i+1)-b(i))/(a(i+1)-a(i))+b(i)
```

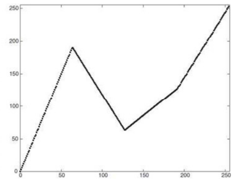
im : input image, **out** : output image

$$\boxed{y = \frac{b_{i+1}-b_i}{a_{i+1}-a_i}(x-a_i)+b_i}$$

defined in p.77

```
>> th=histpwl(t,[0 .25 .5 .75 1],[0 .75 .25 .5 1]);
>> imshow(th)
>> figure,plot(t,th,'.'),axis tight
```





4.3.2 Histogram Equalization

- An entirely automatic procedure
- Suppose our image has L different gray levels, 0, 1, 2, . . . , $L-1$, and gray level i occurs n_i times in the image

$$\left(\frac{n_0+n_1+\cdots+n_i}{n}\right)(L-1)$$

Where $n = n_0 + n_1 + n_2 + \cdots + n_{L-1}$

4.3.2 Histogram Equalization

- **WHY IT WORKS** If we were to treat the image as a continuous function $f(x, y)$ and the histogram as the area between different contours, then we can treat the histogram as a probability density function.

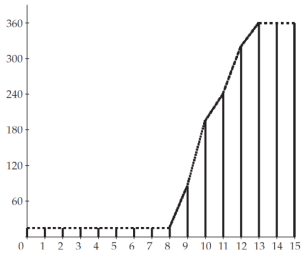


FIGURE 4.22 *The cumulative histogram.*

Histogram Equalization : How to stretch?

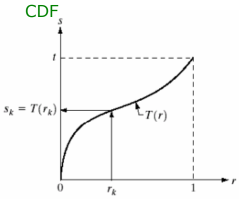
- Pixels r(n1,n2) are transformed to s(n1,n2)

$$s(n_1,n_2) = T[r(n_1,n_2)]$$

- Trans. func.(T): CDF(cumulative distribution function) of random variable ***r***

$$S = T(r) = \int_0^r p_r(w)dw$$

CDF



WHY?
CDF makes pdf (probability density function, pdf) $p_s(s)$ after HE uniform distribution
=> purpose of HE

HE Example

- Suppose a 4-bit grayscale image has the histogram, associated with a table of the numbers n_i of gray values

Gray level i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
n_i	15	0	0	0	0	0	0	0	0	70	110	45	80	40	0	0

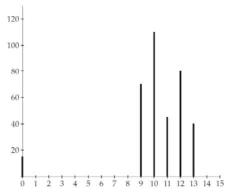


FIGURE 4.17 *Another histogram indicating poor contrast.*

Gray level i	pdf n_i	CDF Σn_i	$(1/24)\Sigma n_i$	Rounded value
0	15	15	0.63	1
1	0	15	0.63	1
2	0	15	0.63	1
3	0	15	0.63	1
4	0	15	0.63	1
5	0	15	0.63	1
6	0	15	0.63	1
7	0	15	0.63	1
8	0	15	0.63	1
9	70	85	3.65	4
10	110	195	8.13	8
11	45	240	10	10
12	80	320	13.33	13
13	40	360	15	15
14	0	360	15	15
15	0	360	15	15

$n_i/360$

HE Example

```
>> p=imread('pout.tif');
>> ph=histeq(p);
>> imshow(ph),figure,imhist(ph),axis tight
```

For the exam, you have to be able to do it by hand as well !

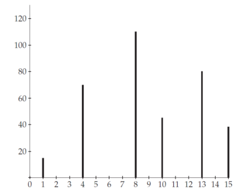
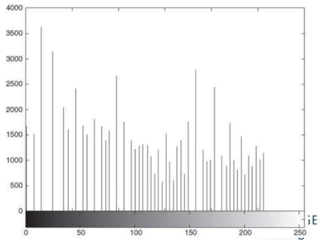
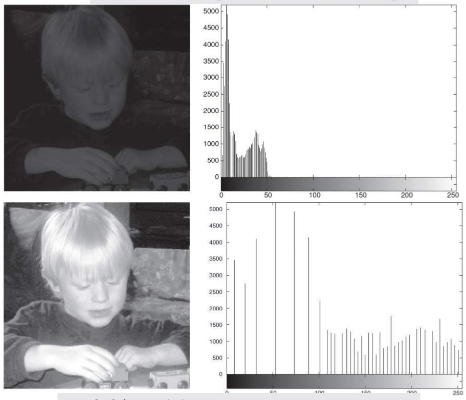


FIGURE 4.18 *The histogram of Figure 4.17 after equalization.*



```
>> en=imread('engineer.tif');
>> e=imdivide(en,4);
>> imshow(e),figure,imhist(e),axis tight
```



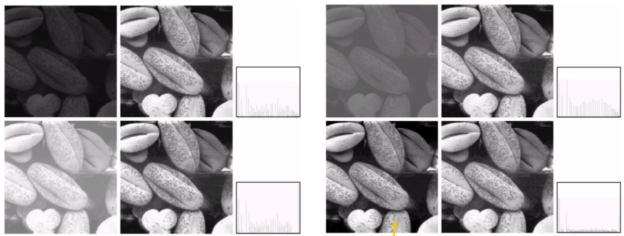
```
>> eh=histeq(e);
>> imshow(eh),figure,imhist(eh),axis tight
```

Histogram Equalization: Step by Step

1. Get histogram(pdf) of input image.
2. Get transformation function T() by calculating cdf and compose a lookup table.
3. Using T(), change input pixel value r -> output pixel value s.
4. Put the pixel value s at the corresponding pixel location of output image.

단골 시험문제 - HE를 손으로 할 수 있을 정도가 되야함

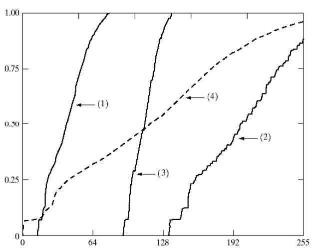
Histogram Equalization: Results



Drawback of HE
Transformation function is fixed.
does not work for high-contrast image

Histogram Equalization : CDFs of the previous slide

FIGURE 3.18 Transformation functions (1) through (4) were obtained from the histograms of the images in Fig.3.17(a), using Eq. (3.3-8).



Histogram Matching(Specification)

- Histogram equalization generates a uniform histogram
- For interactive image enhancement, the user may like to result in a customized histogram

---> Use Histogram Specification

Histogram Specification

- r : pixel values of original image
- u : pixel values of desired image : PDF should be pre-determined.
- s : pixel values of histogram-equalized image
- For continuous data, HE of r and u results in s. (uniform histogram)

$$s = T(r) = \int_0^r P_r(\omega)d\omega$$ CDF(누적분포함수)

$$v = G(u) = \int_0^u P_u(\omega)d\omega$$

$$\boxed{u = G^{-1}(s) = G^{-1}(T(r))}$$ r → u conversion

- Specify a particular probability density function G(u) then calculate $G^{-1}(T(r))$ for histogram specification.

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Graphical Interpretation of Histogram Specification

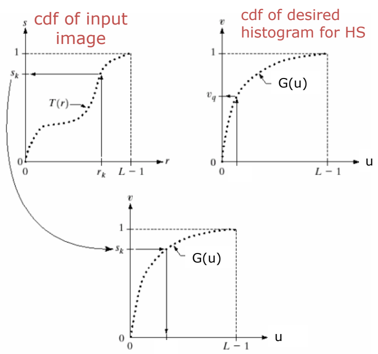
step1 : get histogram-equalized pixel values

$$s = T(r) = \int_0^r P_r(\omega)d\omega$$

$$v = G(u) = \int_0^u P_u(\omega)d\omega$$

$$u = G^{-1}(s) = G^{-1}(T(r))$$

step2 : using histogram equalized pixel values as input, get histogram-specified pixel values



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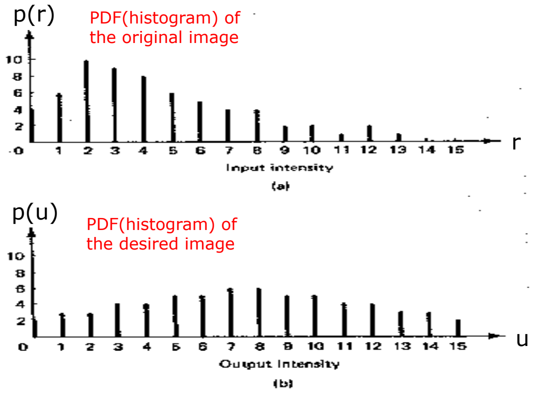
An Example of Histogram Specification with 15 intensity levels and 64 pixels



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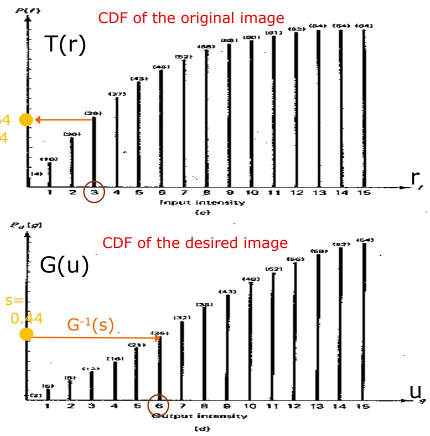
step1 : get histogram-equalized pixel values

$$s = T(r) = \int_0^r P_r(\omega)d\omega$$

$$v = G(u) = \int_0^u P_u(\omega)d\omega$$

$$u = G^{-1}(s) = G^{-1}(T(r))$$

step2 : using histogram equalized pixel values as input, get histogram-specified pixel values



- histogram-equalized pixel value of pixel value 3 : floor(15*28/64) = 7
- histogram-specified pixel value of pixel value 3 : $G^{-1}(28/64)$ = 6

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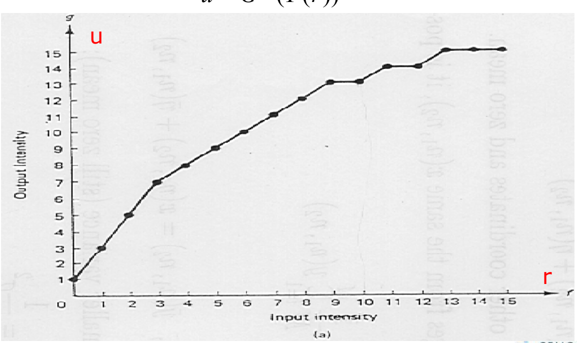
An Example of Histogram Specification : Look-up Table

$$u = G^{-1}(T(r))$$



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Histogram Specification: Step by Step

1. Get histogram P(r) of input image.
2. Get s=T(r) for histogram equalization.
3. Design the desired histogram P(u) and make transformation function G(u) for HS. (You might want to do it manually.)
4. Generate a look-up table(LUT) to convert input pixel values (r) to output values (u) by T(r) and $G^{-1}(s)$.
5. Put the converted pixel values in the output image.

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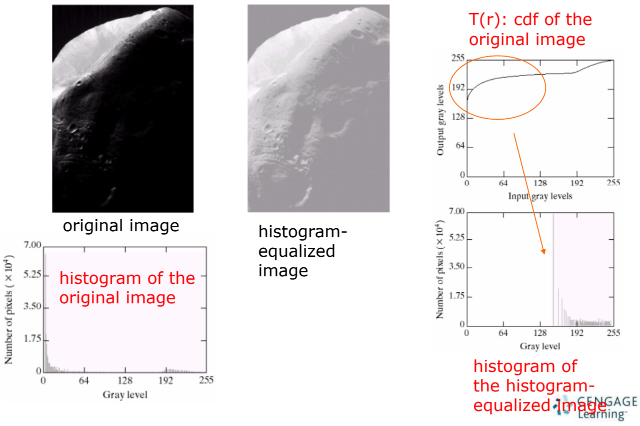
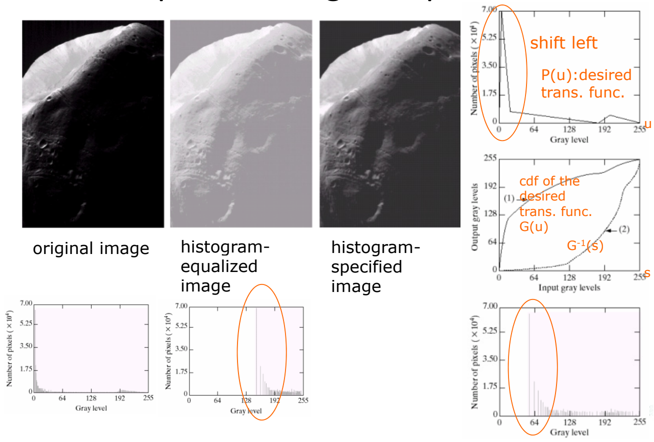
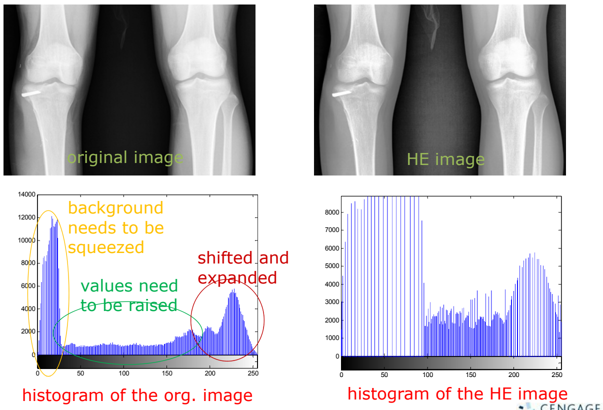
Example of Histogram Specification



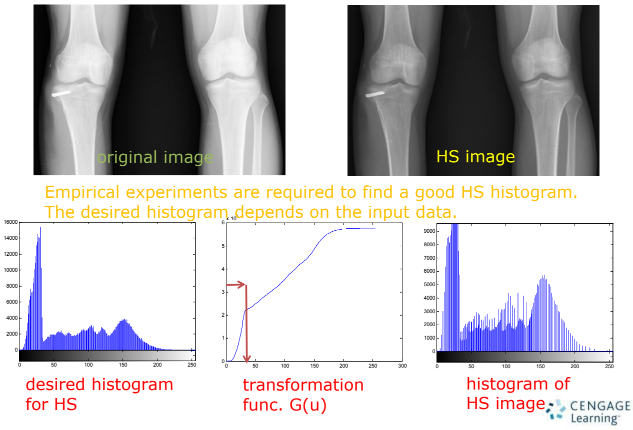
Example of Histogram Specification



A Practical Example of HS (1)



A Practical Example of HS (2)



4.4 Lookup Tables

- Point operations can be performed very effectively by using a **lookup table**, known more simply as an **LUT**
- e.g., the LUT corresponding to division by 2 looks like

Index:	0	1	2	3	4	5	. . .	250	251	252	253	254	255
LUT:	0	0	1	1	2	2	. . .	125	125	126	126	127	127

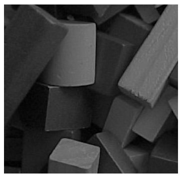
4.4 Lookup Tables

- If `T` is a lookup table in MATLAB and `im` is our image, the lookup table can be applied by the simple command

  `T(im+1)`

- e.g.,

```
>> T=uint8(floor([0:255]/2));
>> b = imread('image.tif');
>> b2 = T(b);
```



4.4 Lookup Tables

- As another example, suppose we wish to apply an LUT to implement the contrast-stretching function

$$y = \frac{64}{96}x,$$
$$y = \frac{192-64}{160-96}(x-96)+64,$$
$$y = \frac{255-192}{255-160}(x-160)+192$$

$$y = 0.6667x,$$
$$y = 2x - 128,$$
$$y = 0.6632x + 85.8947$$

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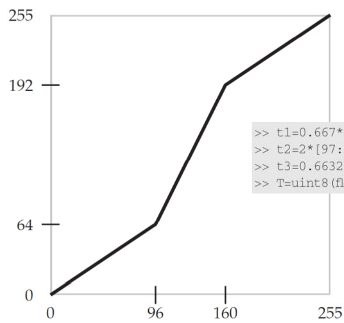
LUT Generation in Matlab




```
>> t1=0.667*[0:96];
>> t2=2*[97:160]-128;
>> t3=0.6632*[161:255]+85.8947;
>> T=uint8(floor([t1 t2 t3]));
```

FIGURE 4.23 *A piecewise linear contrast-stretching function.*

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