BMED318 Biomedical Image Processing

2D Image Reconstruction for Tomographic Imaging: Filtered Back-projection

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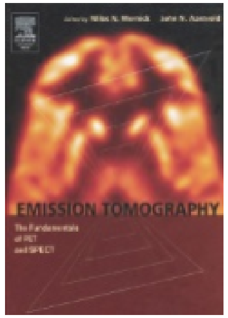
Emission Tomography : The Fundamentals of PET and SPECT



저자	Aarsvold, John N. Wernick, Miles N.
주제어	Tomography, Emission
초록	PET and SPECT are two of today's most important medical-imaging methods, providing images that reveal subtle information about physiological processes in humans and animals. Emission Tomography: The Fundamentals of PET and SPECT explains the physics and engineering principles of these important functional-imaging methods. The technology of emission tomography is covered in detail, including historical origins, scientific and mathematical foundations, imaging systems and their components, image reconstruction and analysis, simulation techniques, and clinical and lal describes the state of the art of emission t of conventional SPECT and PET, as well iterative image reconstruction, small-anim systems. This book is intended as a textb graduate students, researchers, medical p engineers, and professional engineers an imaging industry. Thorough tutorials of fur are presented by dozens of the leading re SPECT has long been a mainstay of clinic one of the world's fastest growing medica its dramatic contributions to cancer imagir Emission Tomography: The Fundamental essential resource for understanding the t the most widely used forms of molecular i tutorial treatments, coupled with coverage the four holders of the prestigious Institute Engineers Medical Imaging Scientist Awa contributors•Include color artwork
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CHAPTER

20

Analytic Image Reconstruction Methods

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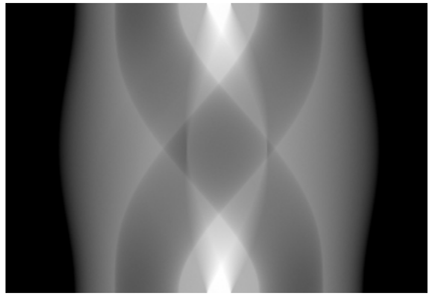
Two Steps in Tomography Imaging

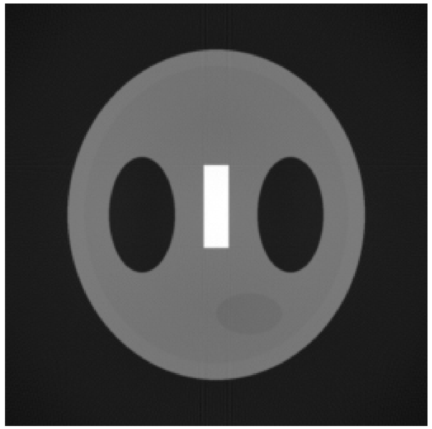
1. Data acquisition

- Record of projections
- Result : a set of angular projections.
- The set of projections of a single slice is called <u>sinogram</u>.

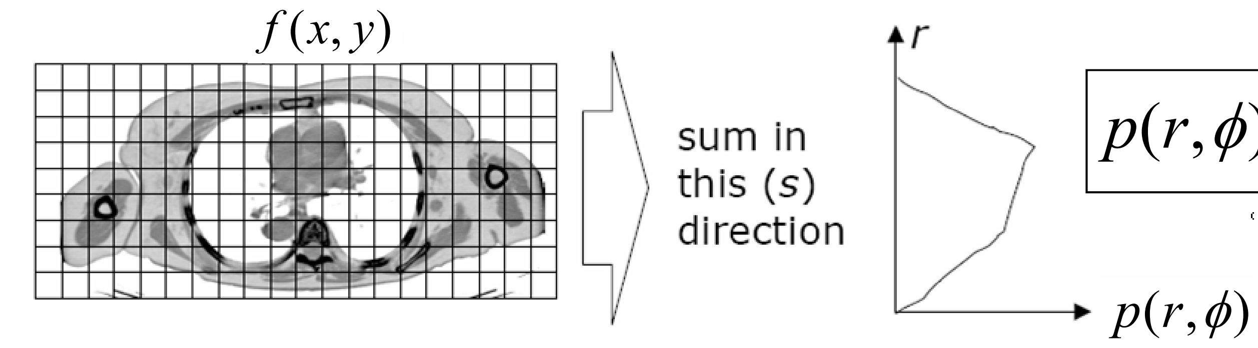


2. Image reconstruction from projections

- Analytic method (e.g. FBP: filtered backprojection)
- Iterative method (e.g. OSEM: ordered subset expectation maximization algorithm).

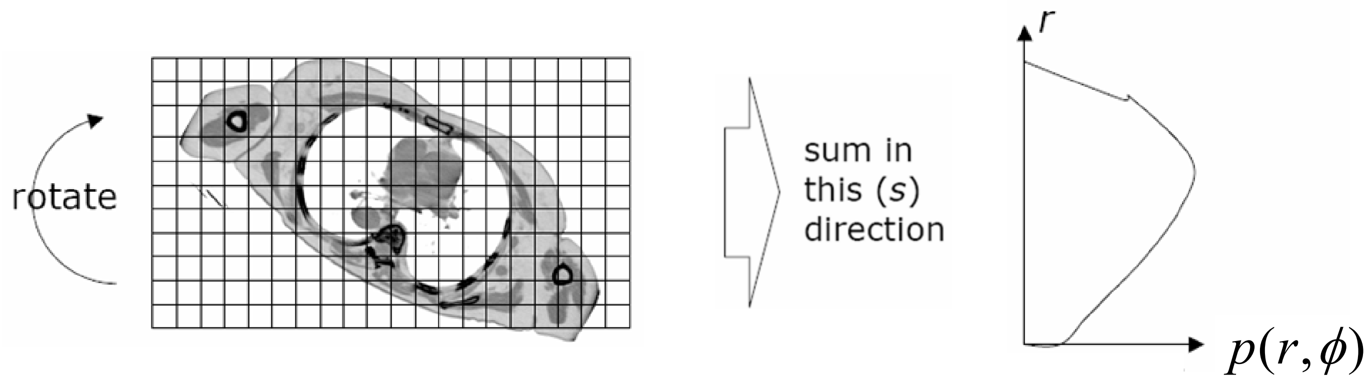


Data Acquisition: Projection



$$p(r,\phi) = \int f(x,y)ds$$

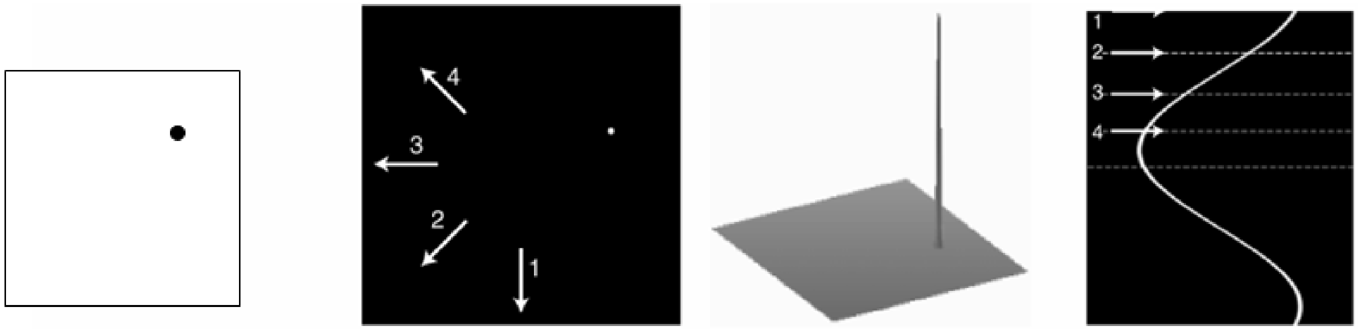
- $p(r,\Phi)$: A projection of $f(x,y)$ along the angle Φ (equivalently saying, along the direction s). → Radon transform
- Row or column sum of the matrix below
- Easiest way to calculate a projection numerically is to rotate f(x,y) and sum in columns or rows.



Data Acquisition: Sinogram

- 2D data grouping all p(r,Φ) at all Φ angles (or in all s directions)
- Group of projections acquired at all Φ angles.
- Needs to determine the original object f(x,y).

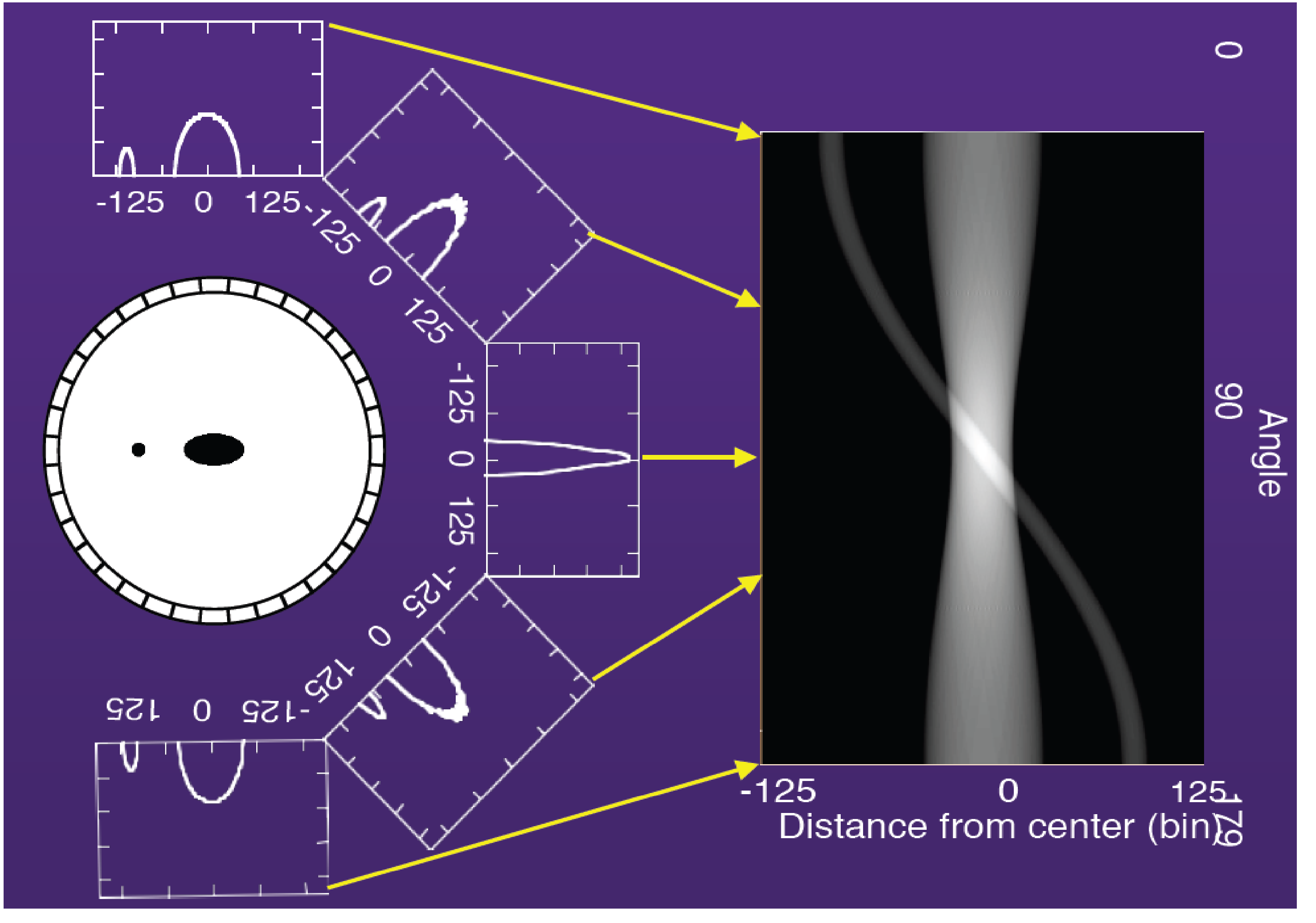
  => input data of image reconstruction



(a) (b) (c)

$$\boxed{p(r,\phi) = \int f(x,y)ds}$$

Sinogram : How It Works



Analytic reconstruction methods

(projection-backprojection algorithms)

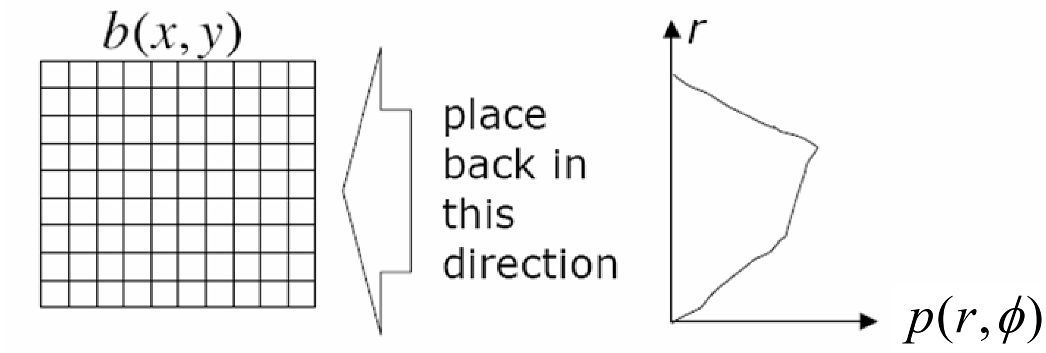
- ☐ Backprojection Filtering (BPF)
- ☐ Filtered Backprojection (FBP)

Analytic Image Reconstruction

- Solving inverse problem to estimate $f(x,y)$ from $p(r,\Phi)$, knowing the relation,

$$\boxed{p(r,\phi) = \int f(x,y)ds}$$

- In other words, given $p(r,\Phi)$, how do we get back to $f(x,y)$?
- The core concept of the solution is backprojection, which is
  - placing back the values of $p(r,\Phi)$ into the image matrix along the angle Φ

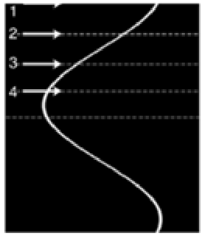


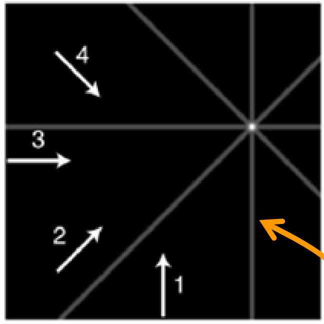
  - Then, repeating for all angles (all s directions)

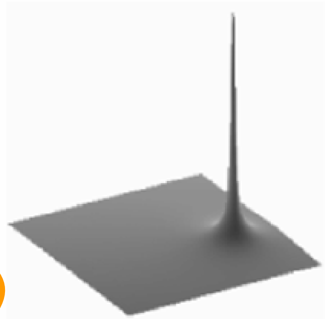
Backprojection

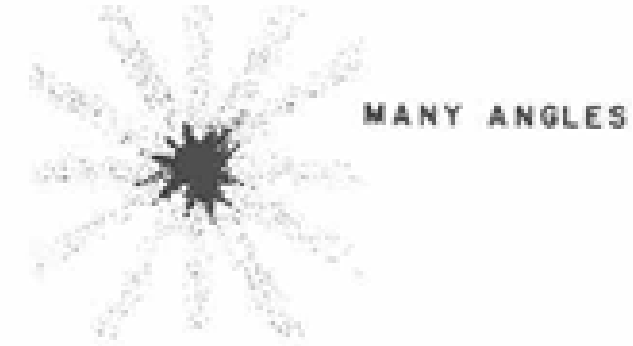
원본에 비해 다소 blurring이 생김 - 한계

- Since the knowledge where the values(f(x,y)) came from was lost, best we can do is to place a constant value p(r,Φ) into elements along the angle Φ.



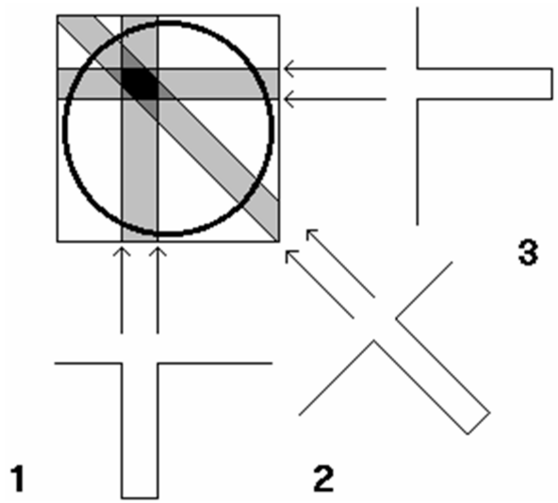






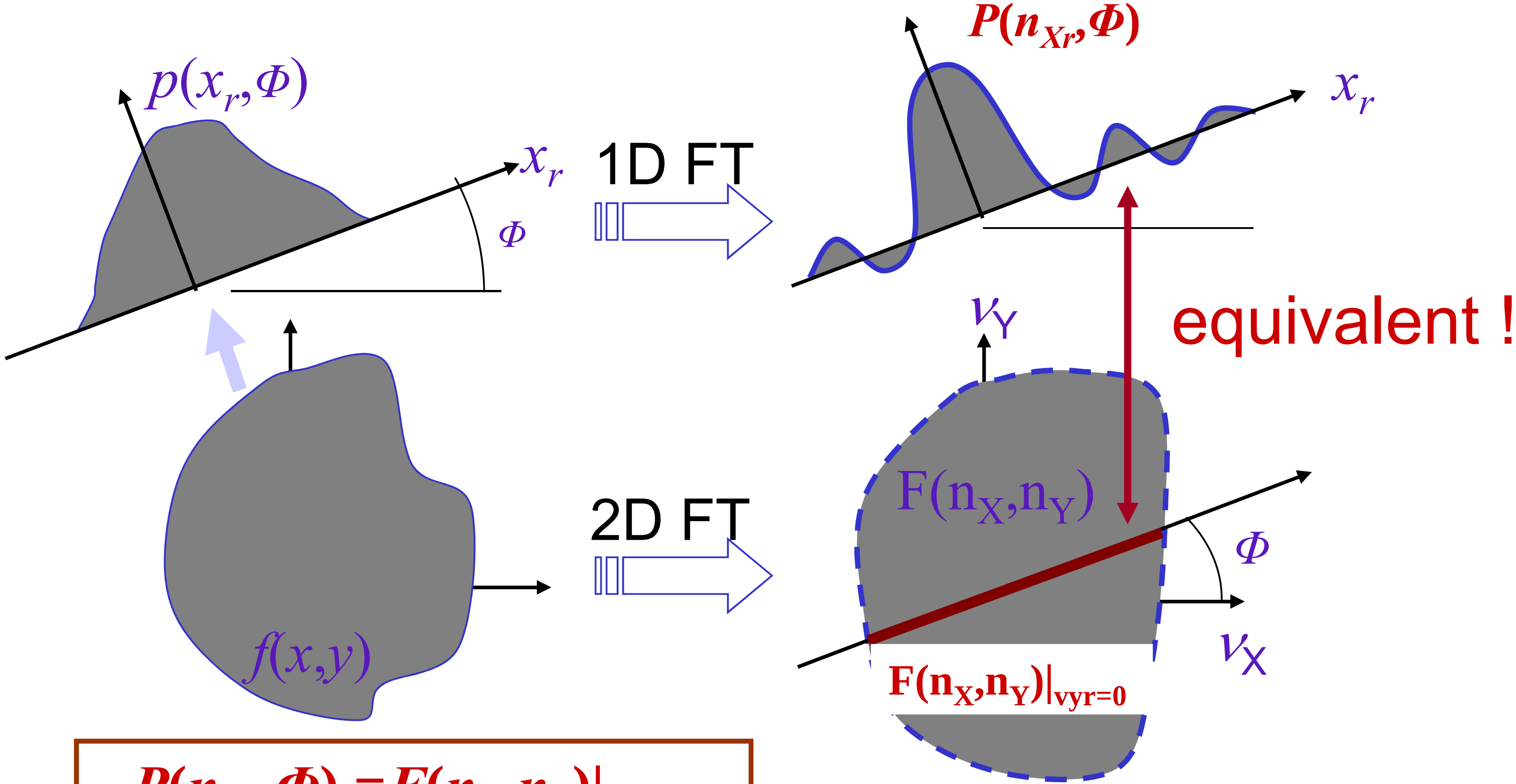
$$b(x,y,\phi) = p(x_r,\phi)$$

$$b(x,y) = \int_0^{\pi} b(x,y,\phi)\,d\phi$$



- b(x,y) = f(x,y) ?
- Something must have happened during projection and backprojection.

Central Section Theorem (= Projection Slice Theorem)



$$P(n_{Xr},\Phi) = F(n_X,n_Y)|_{vyr=0}$$

- Key to solve the relationship
- 1D Fourier transform of a projection at angle Φ **=** the central-section at the same angle Φ through the 2D Fourier transform of the object.

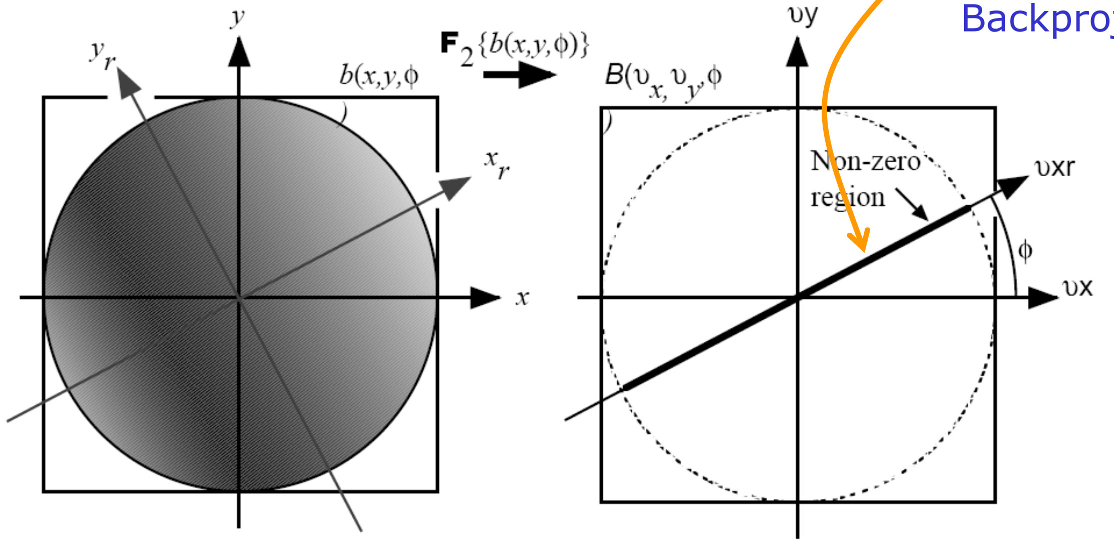
Relationship between b(x,y) and f(x,y)

- 2D Fourier transform of b(x,y,Φ), a backprojection

$$\boxed{B(\upsilon_x,\upsilon_y,\phi)} = \mathbf{F}_2\{b(x,y,\phi)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} b(x,y,\phi)e^{-2\pi i(x\upsilon_x + y\upsilon_y)}dx\,dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x_r,\phi)e^{-2\pi i(x_r\upsilon_{xr} + y_r\upsilon_{yr})}dx_r\,dy_r = \boxed{P(\upsilon_{xr},\phi)\delta(\upsilon_{yr})} \quad \upsilon_{yr} = -\upsilon_x \sin\phi + \upsilon_y \cos\phi$$

Backprojection theorem



- By central section theorem $\boxed{P(\upsilon_{xr},\phi) = F(\upsilon_x,\upsilon_y)\big|_{\upsilon_{yr}=0}}$

$$B(\upsilon_x,\upsilon_y,\phi) = F(\upsilon_x,\upsilon_y)\delta(\upsilon_{yr})$$

Relationship between b(x,y) and f(x,y):
Backprojection Theorem

$$B(\upsilon_x,\upsilon_y,\phi) = \mathbf{F}_2\{b(x,y,\phi)\} = P(\upsilon_{xr},\phi)\delta(\upsilon_{yr})$$

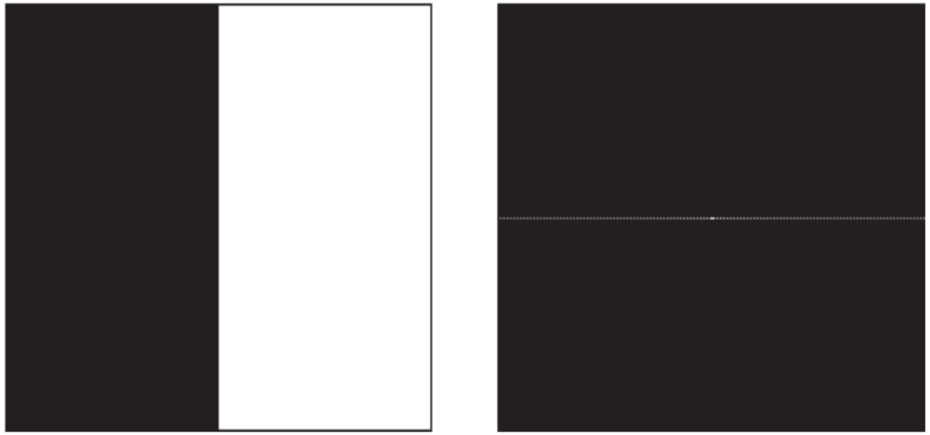


FIGURE 7.10 *A single edge and its DFT.*

- We assume $b(x, y, \Phi)$ is constant in the y_r direction.
- In other words, the Fourier transform of a single back-projection is non-zero along a line through the origin.
- Along the line(v_{xr}), the values are equal to the Fourier transform of the projection multiplied by the delta function in the perpendicular direction (v_{yr}) .

Relationship between b(x,y) and f(x,y)

- $B(v_x,v_y)$ is the sum of $B(v_x,v_y,\Phi)$ in all Φ.

$$B(\upsilon_x,\upsilon_y) = \int_0^\pi B(\upsilon_x,\upsilon_y,\phi)\,d\phi = \int_0^\pi F(\upsilon_x,\upsilon_y)\delta(\upsilon_{yr})\,d\phi \quad \Leftarrow \quad B(\upsilon_x,\upsilon_y,\phi) = F(\upsilon_x,\upsilon_y)\delta(\upsilon_{yr})$$

$$\boxed{B(\upsilon_x,\upsilon_y) = \frac{F(\upsilon_x,\upsilon_y)}{\upsilon} \qquad \upsilon = \sqrt{\upsilon_x^2 + \upsilon_y^2}}$$

FT of backprojection $b(x,y)$
= *FT* of $f(x,y)$ x (1/distance from the origin in freq. domain)

- Inverse FT of $B(v_x,v_y)$

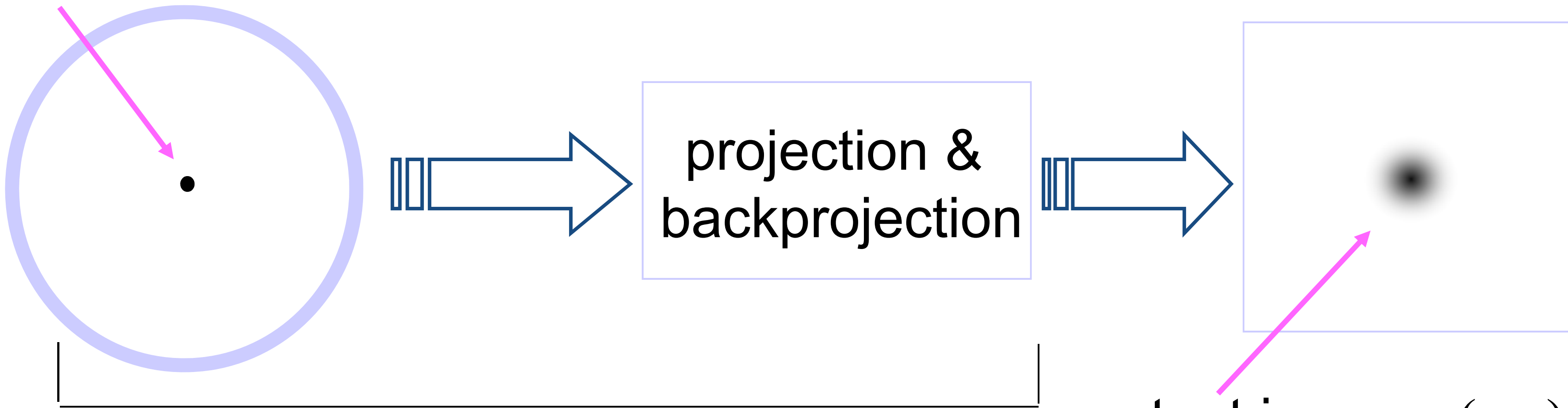
$$b(x,y) = F_2^{-1}\{B(v_x,v_y)\} = F_2^{-1}\{\frac{F(v_x,v_y)}{v}\}$$
$$= f(x,y) * h(x,y)$$
$$h(x,y) = \frac{1}{r}, \quad r = \sqrt{x^2 + y^2}$$

2D Hankel transform

$$\mathbf{F}_2^{-1}\left\{\frac{1}{\upsilon}\right\} = \frac{1}{r} \qquad r = \sqrt{x^2 + y^2}$$

Relationship between b(x,y) and f(x,y) : Linear System Approach (Linear Shift Invariant System)

$f(x,y) = \delta(x,y)$ (spatial impulse input)

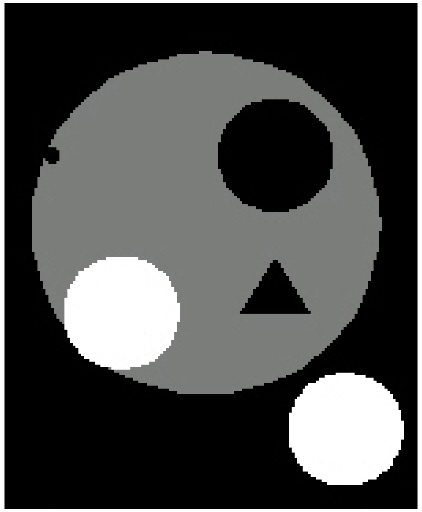


transfer function: $h(x,y) = 1/r$

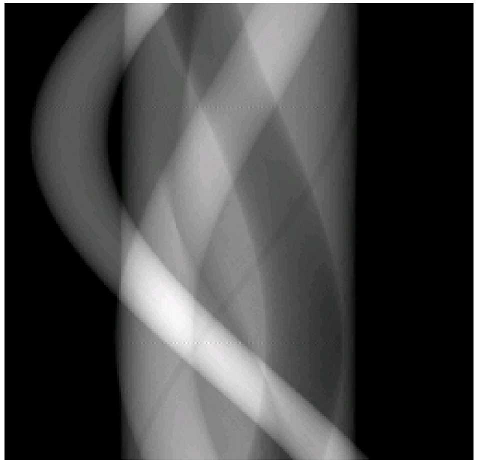
output image $g(x,y)$ is point-spread-function $h(x,y)$

The projection+backprojection operations causes 'smoothing' (or blurring) wrt the original image by impulse response $1/r$.

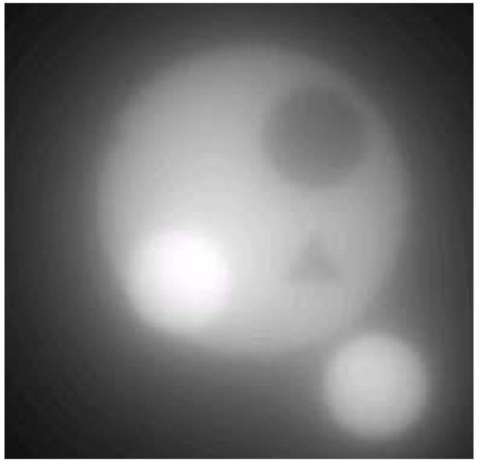
Relationship between b(x,y) and f(x,y): Example



f(x,y)



Sinogram



b(x,y)

- backprojection image, b(x,y)

$$\boxed{b(x,y) = f(x,y) * h(x,y) = f(x,y) * \frac{1}{r}, \quad r = \sqrt{x^2 + y^2}}$$

How to Approximate f(x,y) from b(x,y) in LSI System

- Solution 1: Backprojection Filtering(BPF)
  - Deblur the b(x,y) by filtering
    - F(true image) = F(BP image) / F(1/r) = F(BP image) / (1/ν)
    - F(true image) = F(BP image) $\times$ v
    - true image = F^{-1}{F(BP image) $\times$ v}

$$f(x,y) = \mathbf{F}_2^{-1}\left\{ \upsilon \mathbf{F}_2 \left\{ b(x,y) = \int_0^{\pi} p(x_r,\phi)\, d\phi \right\} \right\}$$

  - Drawback: b(x,y) has larger support than f(x,y) due to the convolution with 1/r. → <u>significantly larger matrix size</u> to compute and store b(x,y) than the size needed for the final result
  - Solution: interchanging the backprojection and filtering steps

How to Approximate f(x,y) from b(x,y) in LSI System

- Solution 2: Filtered Backprojection(FBP)
  - Filtering p(x,y) before backprojection
  - F(true image) = F(BP image) / F(1/r)
  - F(true image) = F(BP image) $\times$ v
  - true image = BP[F^{-1} {F(projection) $\times$ v}]

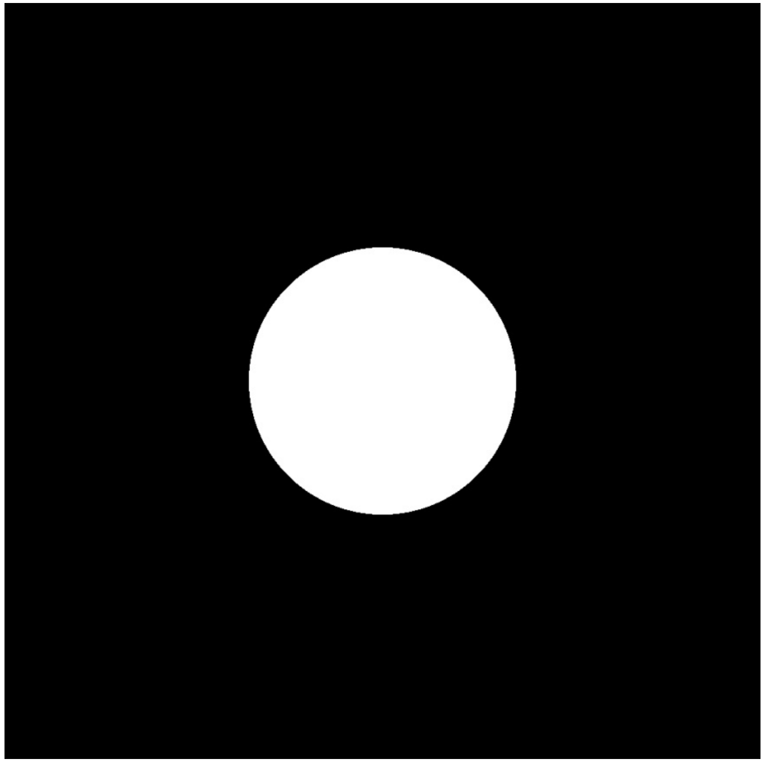
$$f(x,y) = \int_0^{\pi} p^F(x_r,\phi)\,d\phi$$

$$p^F(x_r,\phi) = \mathbf{F}_1^{-1}\left\{\left|\upsilon_{xr}\right|\mathbf{F}_1\left\{p(x_r,\phi)\right\}\right\}$$

  - More efficient than BPF
  - Most common in analytic image reconstruction

Analytic Reconstruction Demo

- Disk object



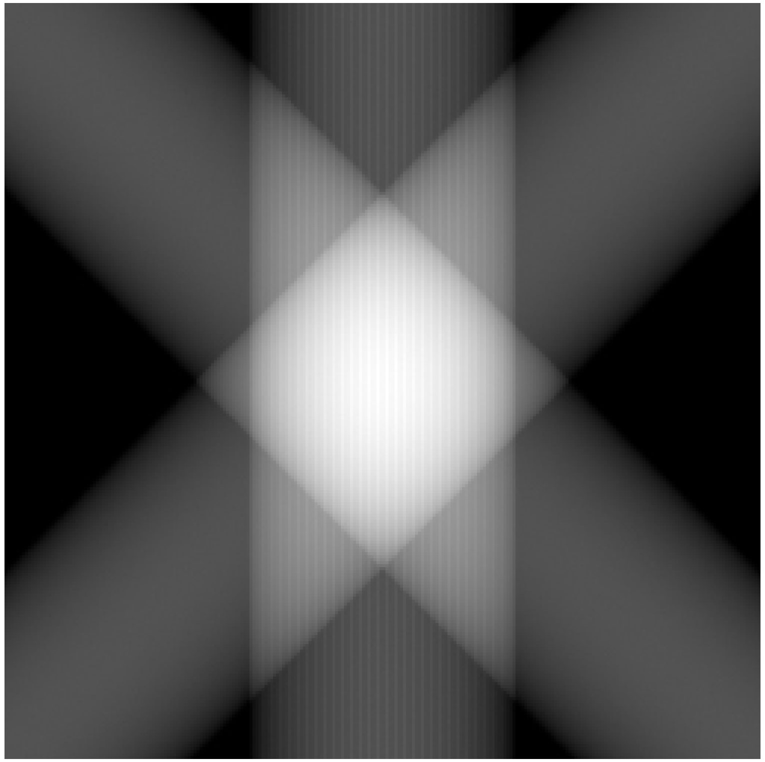
<True Image>

<Sinogram>

Courtesy of Dr. Hyung-Gu Lee

Analytic Reconstruction Demo

- Linear Superposition of Back Projection (LSBP)



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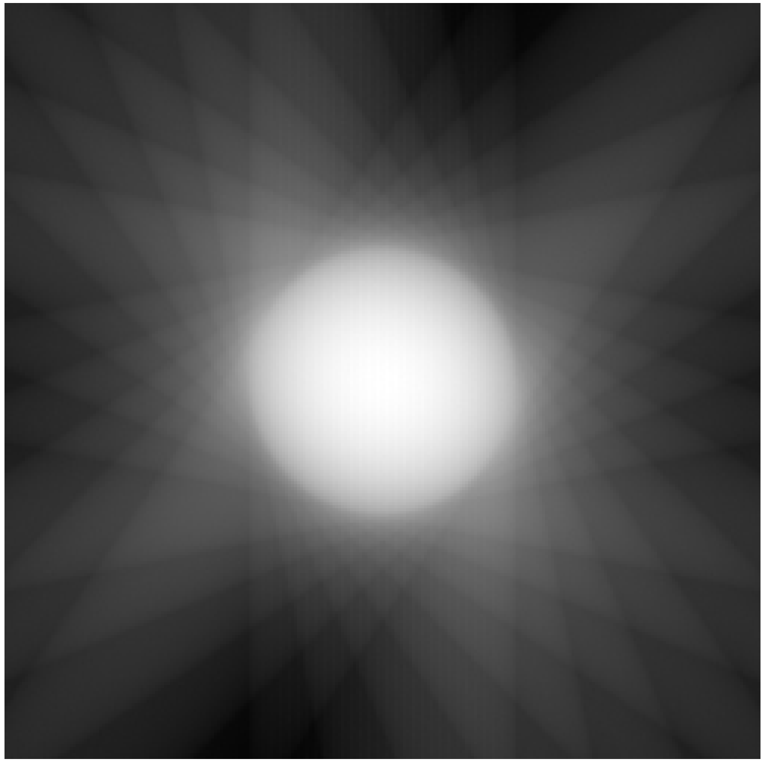
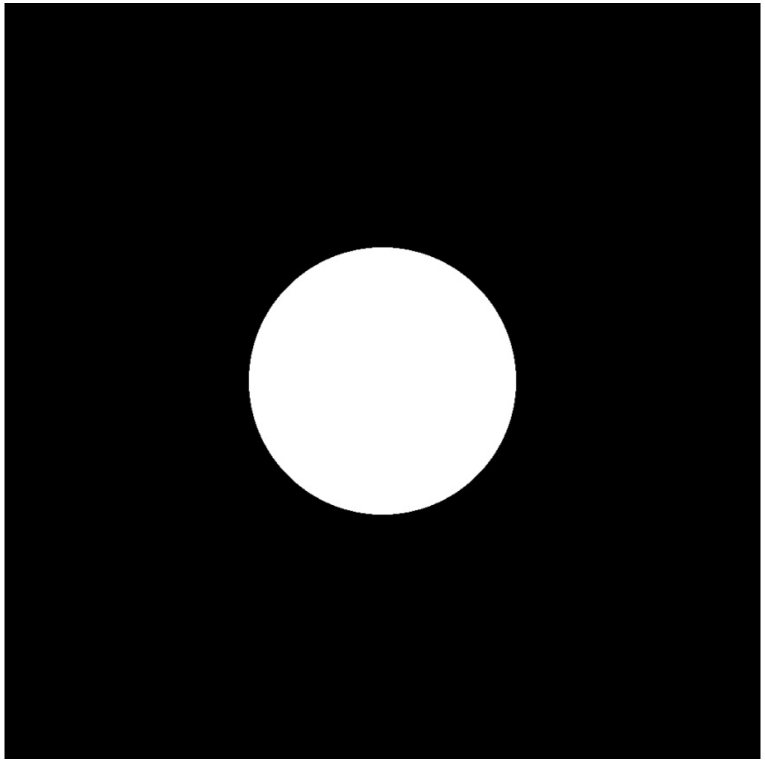


<No. of projection : 16>

Courtesy of Dr. Hyung-Gu Lee

Analytic Reconstruction Demo



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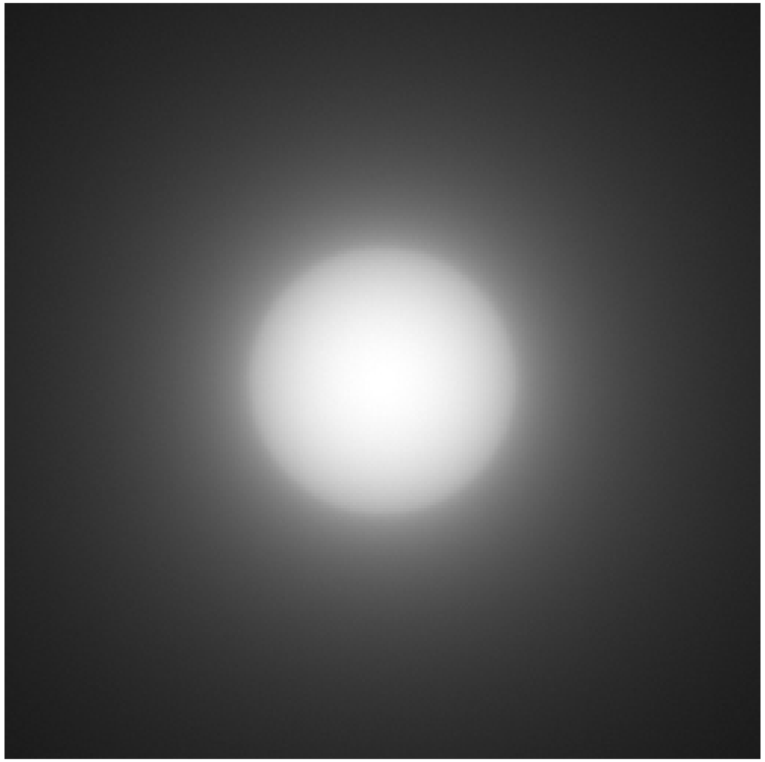
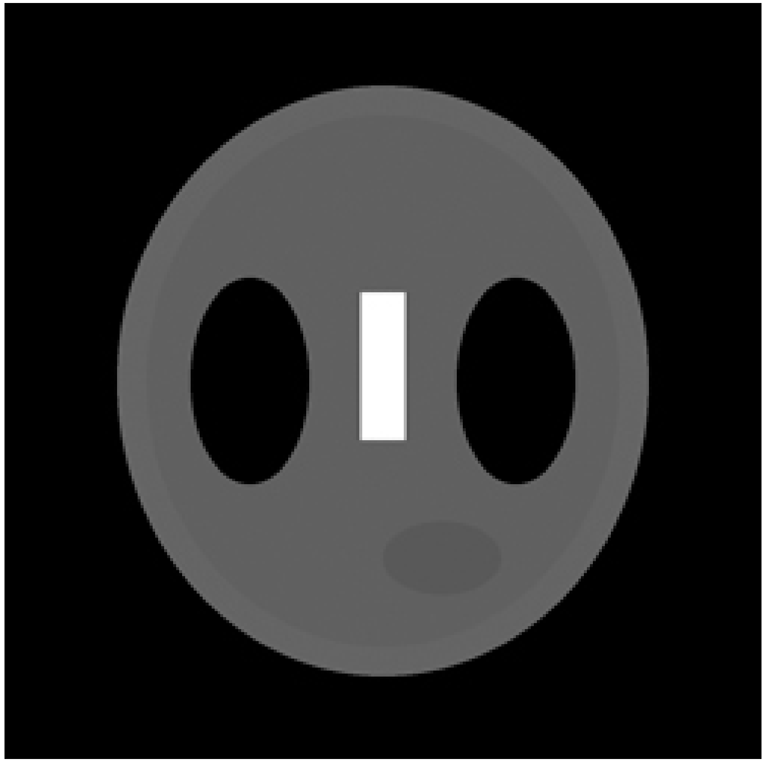


<Back Projected Image>

Courtesy of Dr. Hyung-Gu Lee

Analytic Reconstruction Demo

- General CT Phantom



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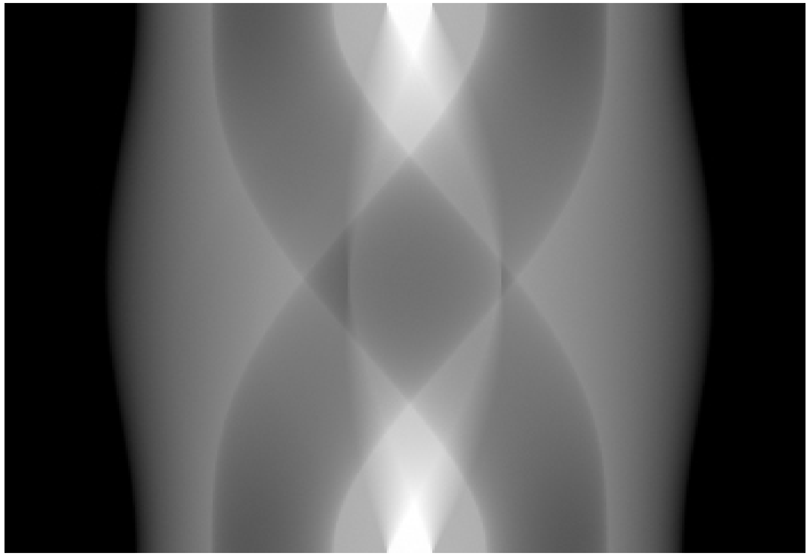
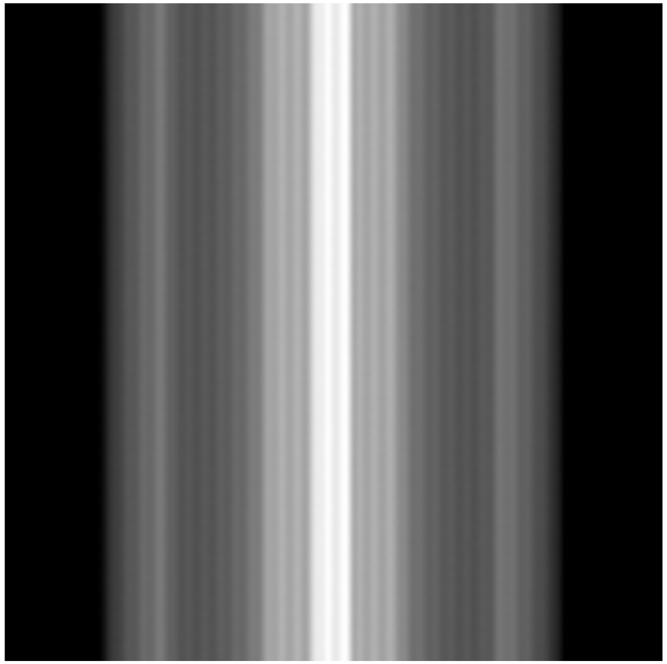


<Sinogram>

Courtesy of Dr. Hyung-Gu Lee

Analytic Reconstruction Demo

- Linear Superposition of Back Projection (LSBP)



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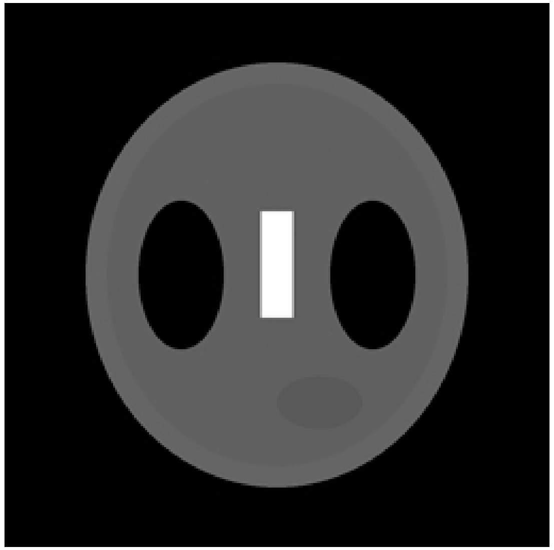
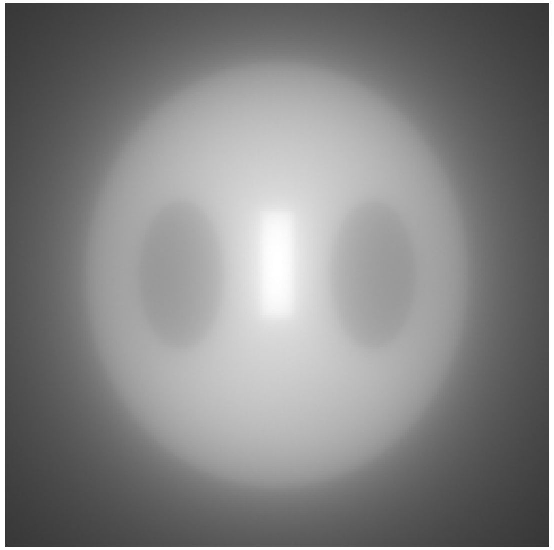
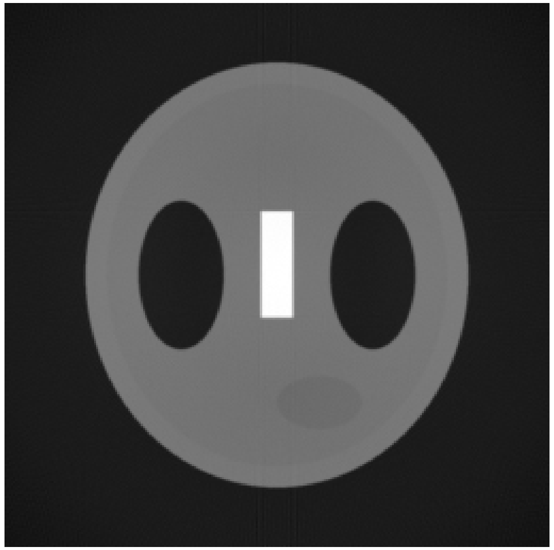
Analytic Reconstruction Demo





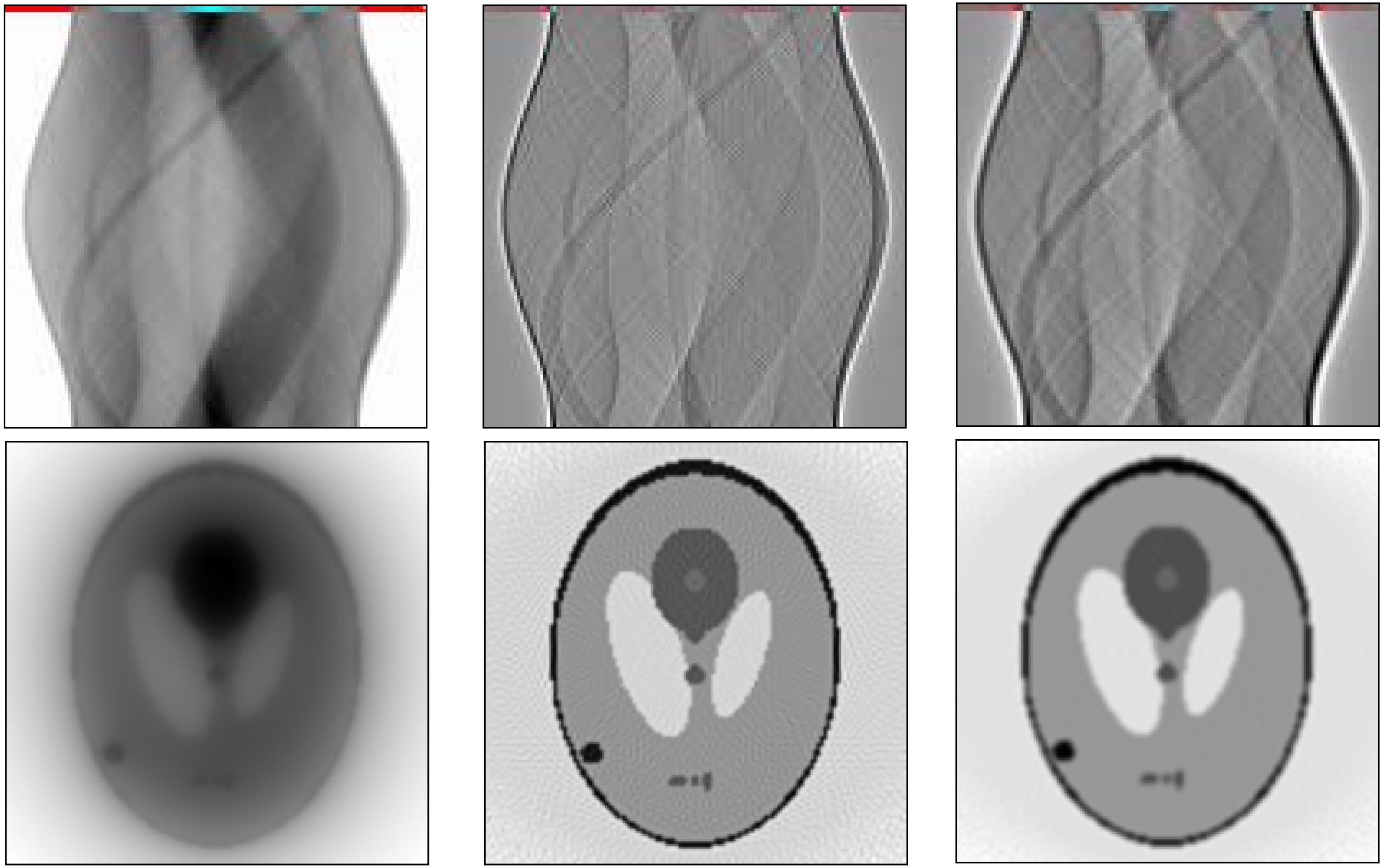


<True Image> (LSBP Image) (Shepp-Logan filtered)

Courtesy of Dr. Hyung-Gu Lee

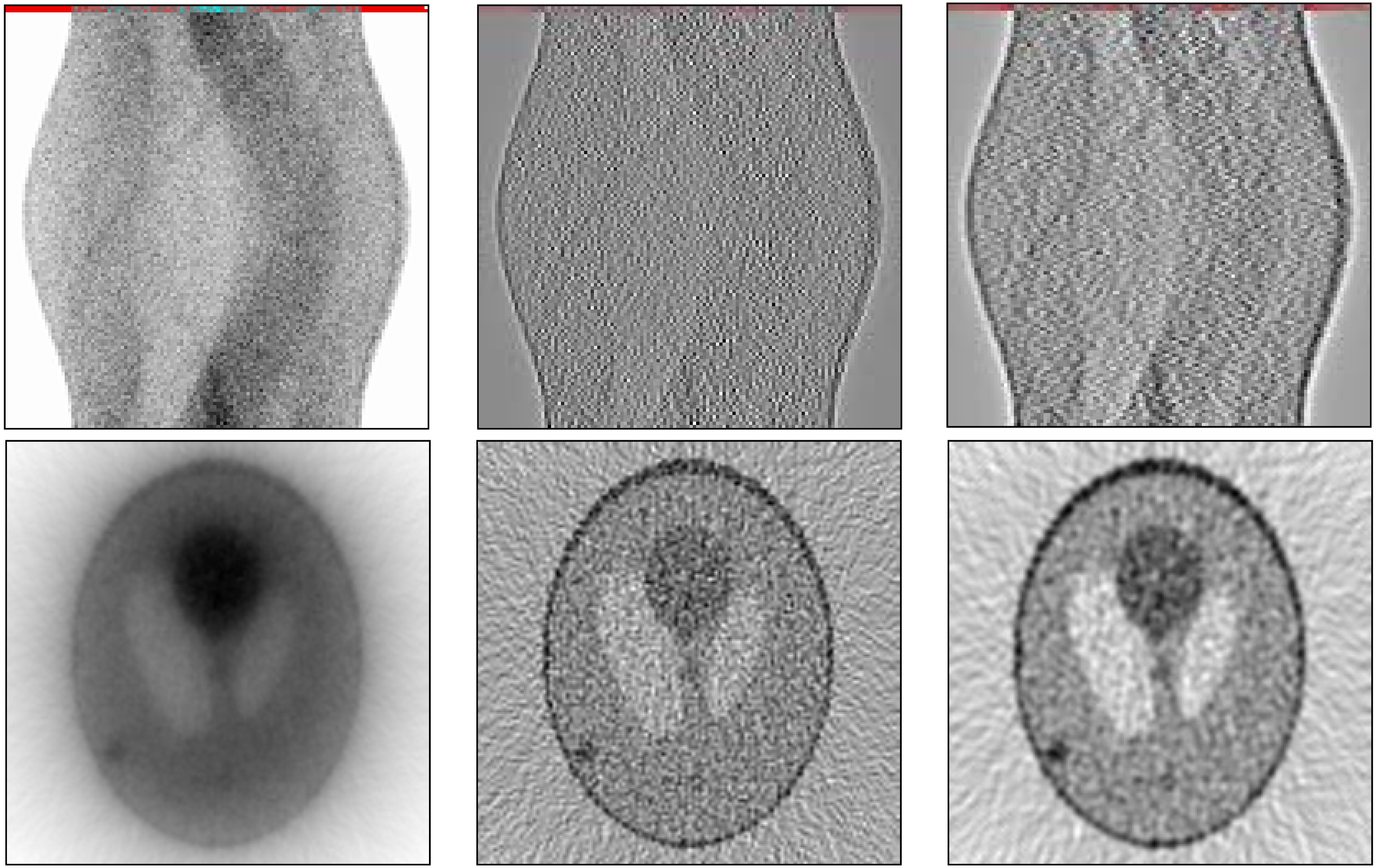
Filtered Backprojection Demo: Noiseless Data

No filter | Ramp filter | Hanning filter

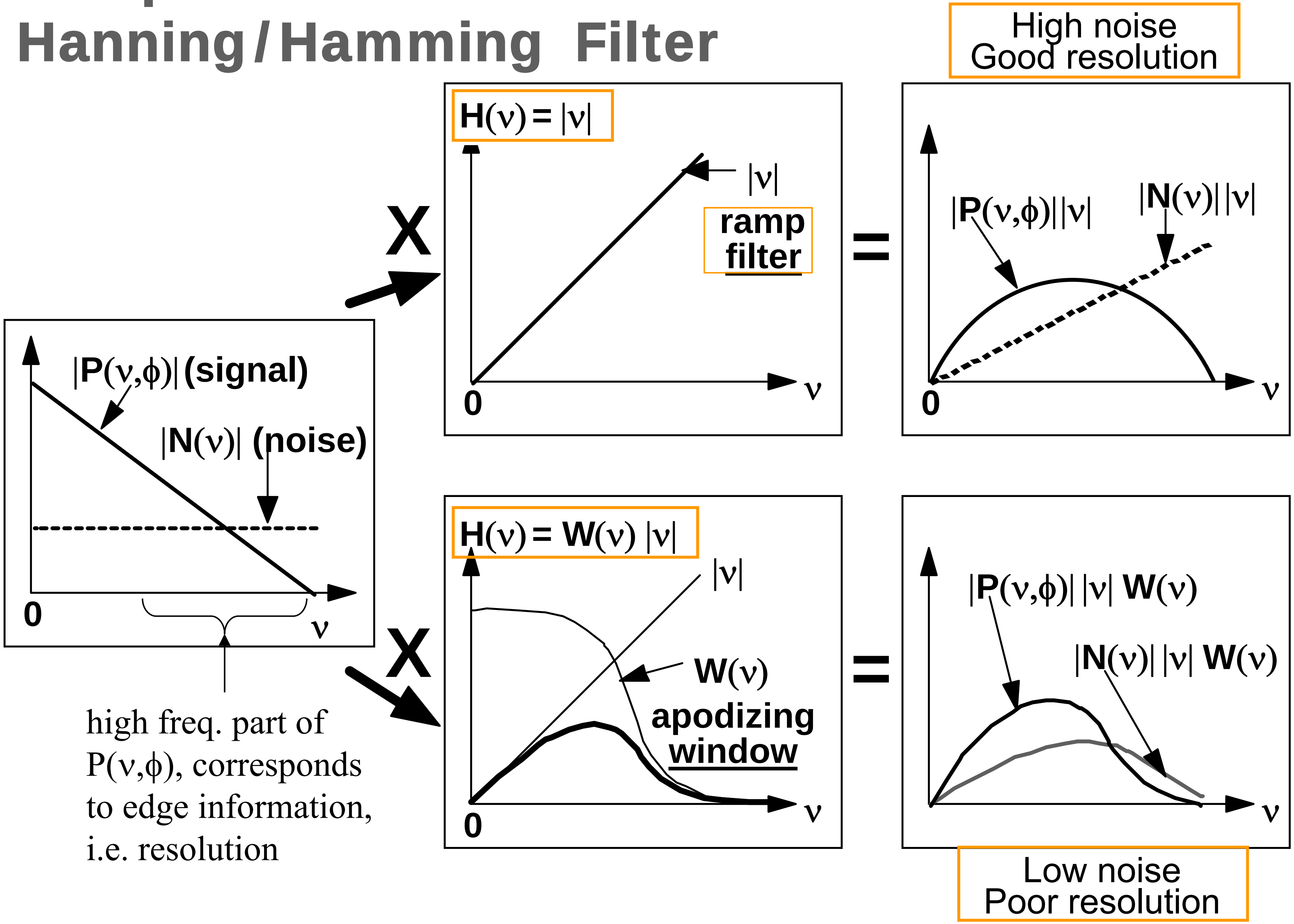


Filtered Backprojection Demo

No filter | Ramp filter | Hanning filter

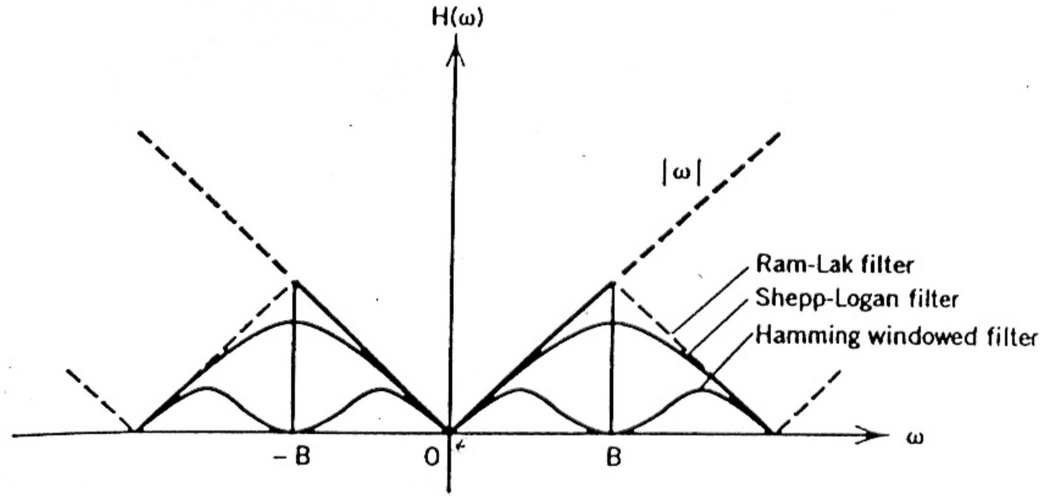


Ramp Filter vs. Hanning / Hamming Filter



What Type of Windowing to Use?

- A typical ‘window’ (sometimes called a smoothing filter) is the Hamming window. It is not the name, or the equation, but the *shape* that determines the trade-off between noise and resolution.
- How do you pick the right shape? A very difficult question, that is not considered polite to ask in some circles.



Different Filters in FBP

http://www.impactscan.org/slides/xrayct/sld056.htm

FBP : Pros and Cons

- Pros
  - straightforward, one free parameter (filter)
  - the noise propagation properties are well understood (linear)
  - a lot of experience with it.
  - very time and memory efficient
- Cons
  - a very simple model of the imaging system
  - does not account for the statistical noise

    : Linear approximation of non-linear behavior of noise causes streak artifact.
  - can't include other factors as constraints
- Solution to overcome the drawbacks of FBP

  => iterative reconstruction

FBP : Pros and Cons