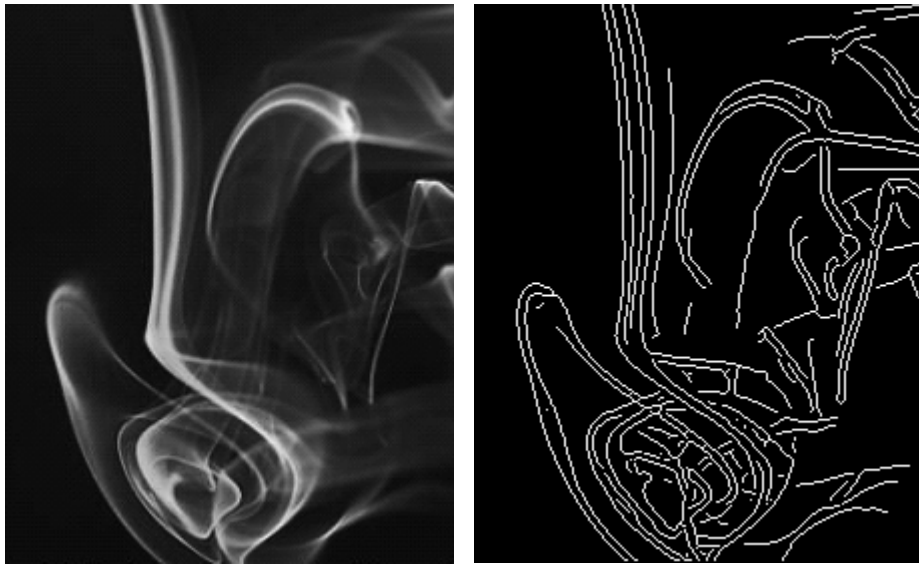


Chapter 9:

Image Segmentation

9.1 Introduction

- **Segmentation** is the operation of partitioning an image into component parts or into separate objects.

- In this chapter, we will investigate two very important topics: **thresholding** and **edge detection**
  - Single & double thresholding
  - How to determine the threshold value
  - Adaptive thresholding
  - Edge detection: 1st and 2nd derivatives
  - Canny edge detector
  - Hough Transform

Single Thresholding

- grayscale image -> binary image

$$\text{A pixel becomes } \begin{cases} \text{white if its gray level is} > T, \\ \text{black if its gray level is} \leq T. \end{cases}$$

```
>> r=imread('rice.tif');
>> imshow(r),figure,imshow(r>110)
```

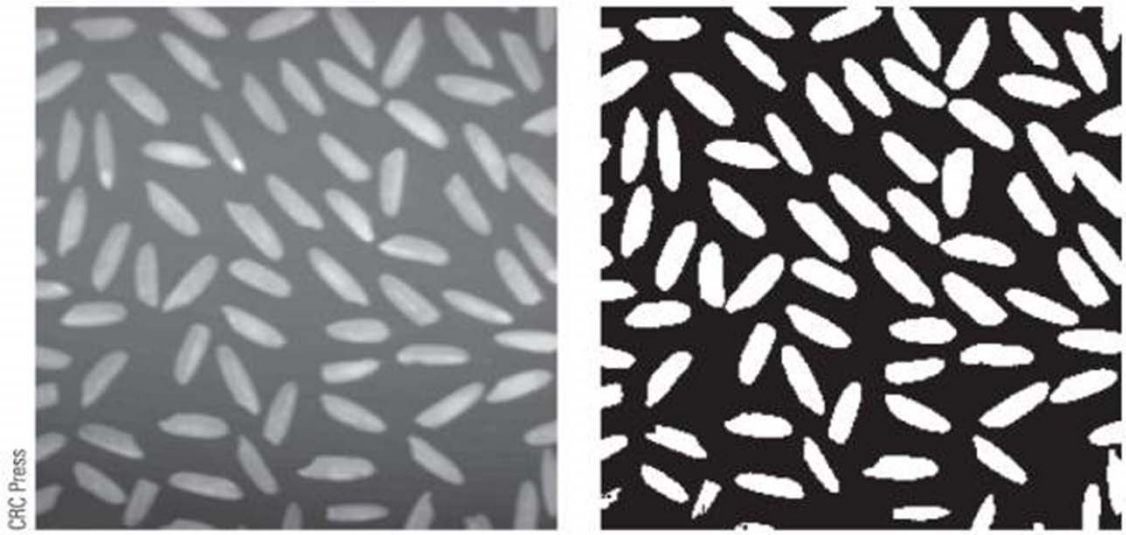
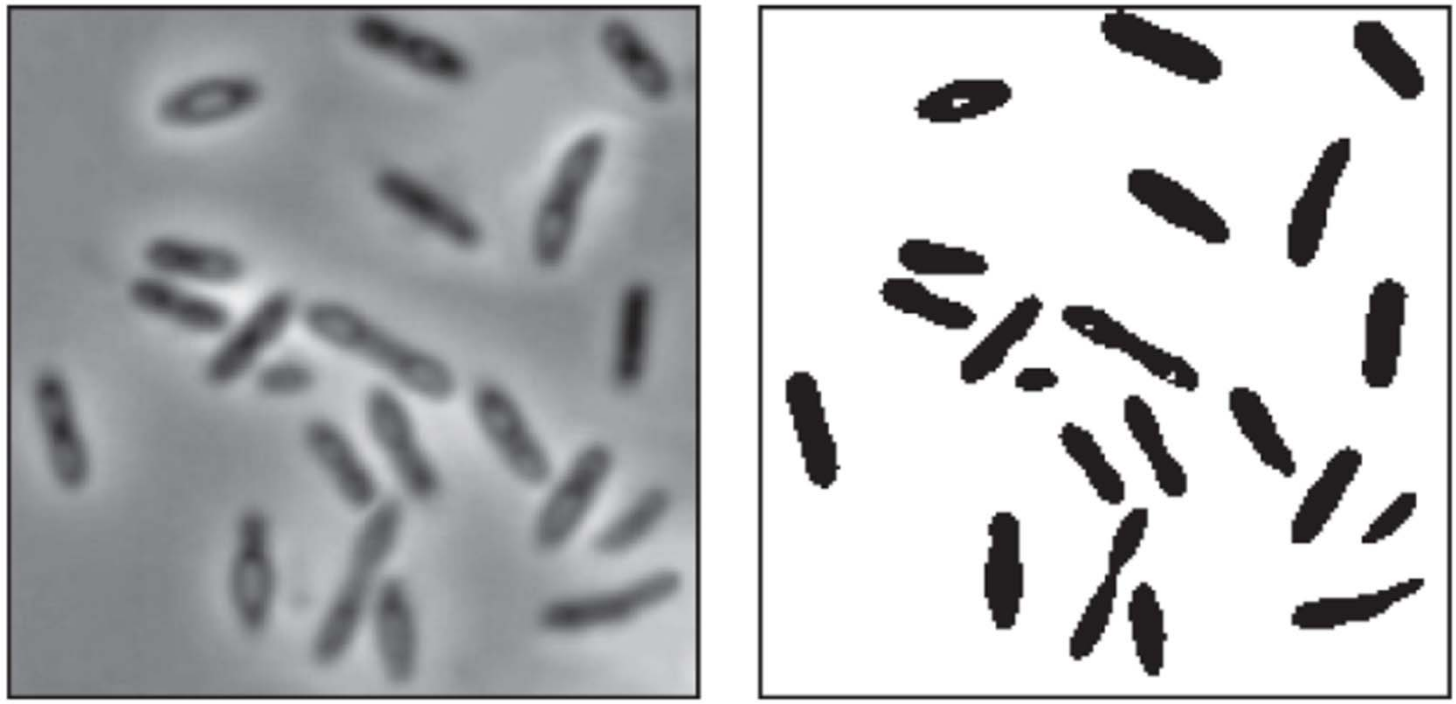


FIGURE 9.1 *Thresholded image of rice grains.*

Single Thresholding

```
>> b=imread('bacteria.tif');
>> imshow(b),figure,imshow(b>100)
```



CRC Press

FIGURE 9.2 *Thresholded image of bacteria.*

Single Thresholding in Matlab: im2bw()

- MATLAB has the **im2bw** function, which thresholds an image, using the general syntax.
- It works on grayscale, colored, and indexed images of data type uint8, uint16, or double.

```
im2bw(image,level)
```

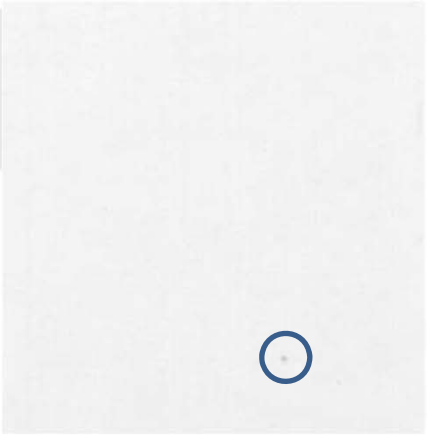
e.g.

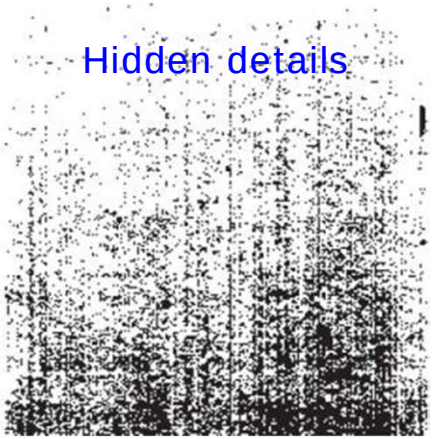
```
>> im2bw(r,0.43);
>> im2bw(b,0.39);
```

- choosing an appropriate value of T is important.

```
>> p=imread('paper1.tif');
>> imshow(p),figure,imshow(p>241)
```

Nearly all gray values are very high.



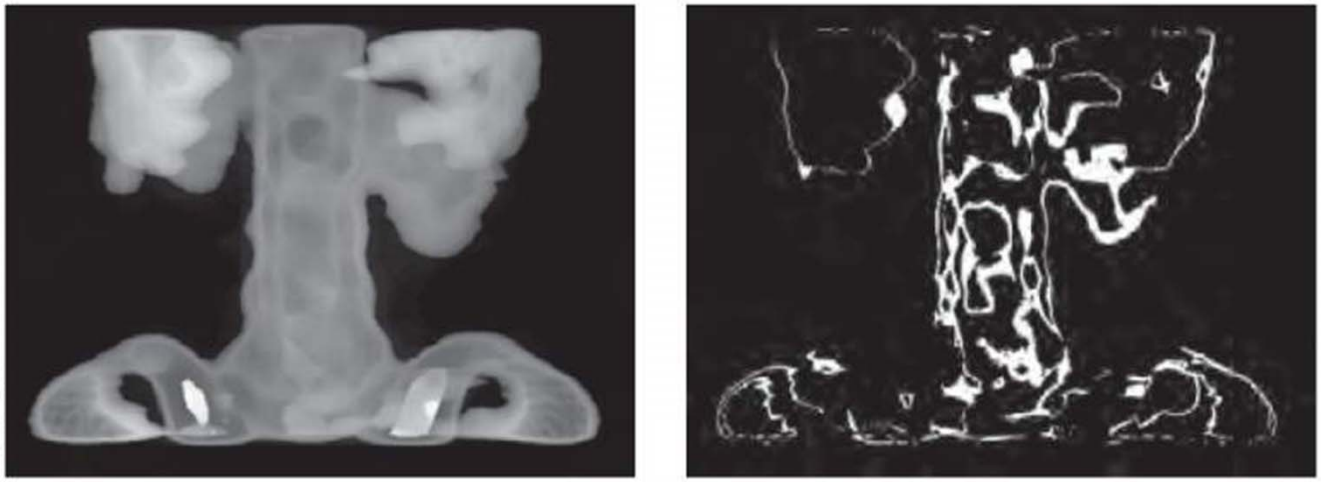


Double Thresholding

- grayscale image with two thresholds -> binary image

$$\text{a pixel becomes } \begin{cases} \text{white if its gray level is between } T_1 \text{ and } T_2, \\ \text{black if its gray level is otherwise.} \end{cases}$$

```
>> [x,map]=imread('spine.tif');
>> s=ind2gray(x,map);
>> imshow(s),figure,imshow(s>115 & s<125)
```



Mathworks

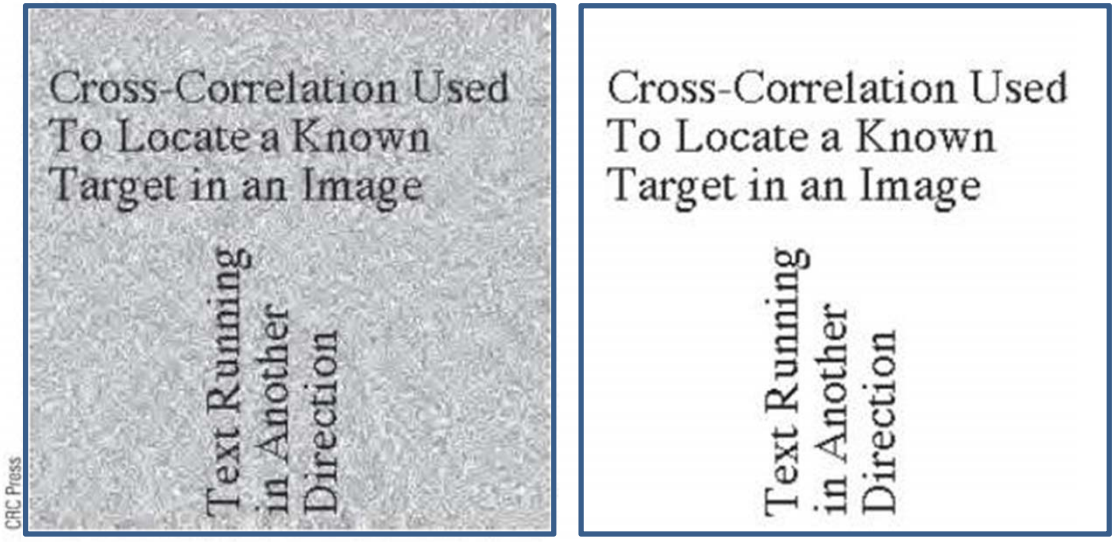
FIGURE 9.4 *The image* `spine.tif` *as the result after double thresholding.*

Applications of Thresholdng

- Remove unnecessary detail: e.g. rice and bacteria images
- bring out hidden detail : e.g. paper and spine images
- remove a varying background from text or a drawing

```
>> r=rand(256)*128+127;
>> t=imread('text.tif');
>> tr=uint8(r.*double(not(t));
>> imshow(tr)
```

```
>> imshow(tr>100)
```



text in noisy background

segmentation by single thresholding

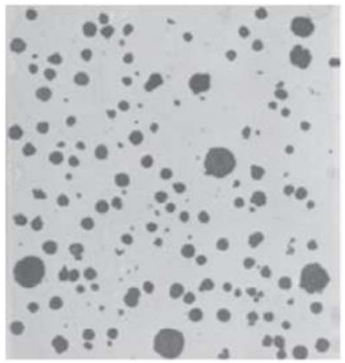
FIGURE 9.5 *Text on a varying background and thresholding.*

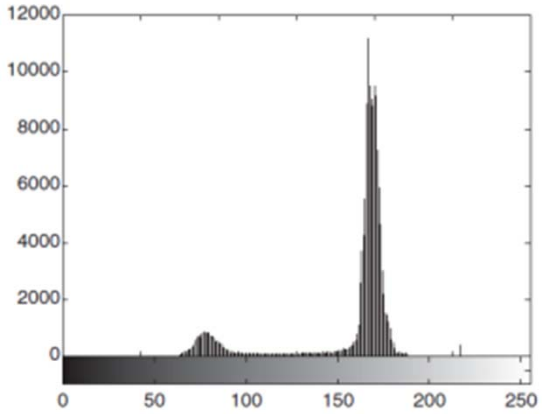
Choosing an Appropriate Thresholding Value

```
>> n=imread('nodules1.tif');
>> imshow(n);
>> n1=im2bw(n,0.35);
>> n2=im2bw(n,0.75);
>> figure,imshow(n1),figure,imshow(n2)
```

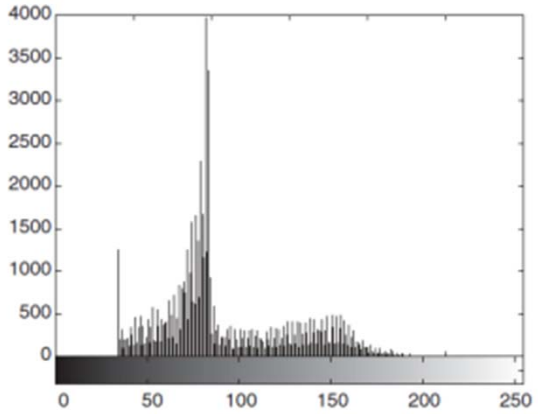
FIGURE 9.6 *Attempts at thresholding.*

Histogram Analysis

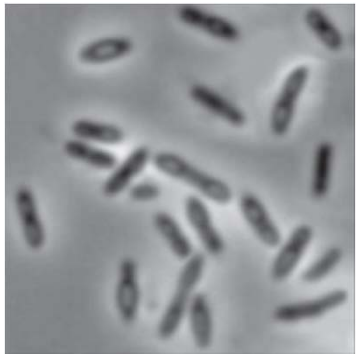


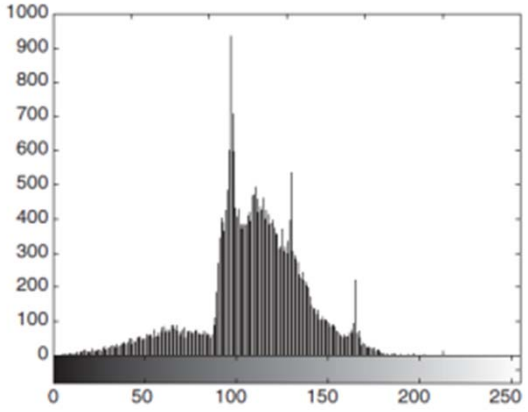


Nodules image : `nodules.tif`

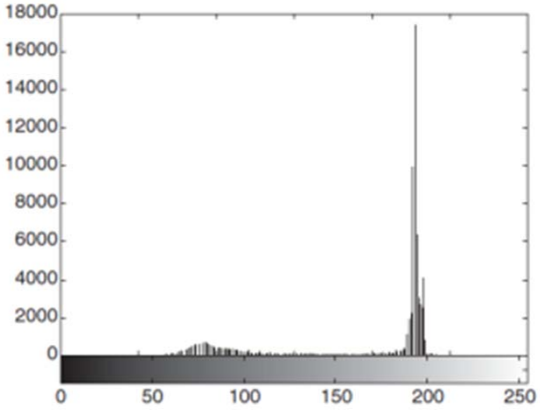


Rice image : `rice.tif`





Bacteria image : `bacteria.tif`



Coins image : `eight.tif`

FIGURE 9.7 *Histograms.*

Histogram Analysis

- Choosing a threshold value is easy only if we have a prior knowledge of individual histograms of objects and background.
- However, in general, histogram of object and background is overlaped so that we cannot easily determine the appropriate threshold values.

-> needs more intelligent way to choose an optimal threshold.

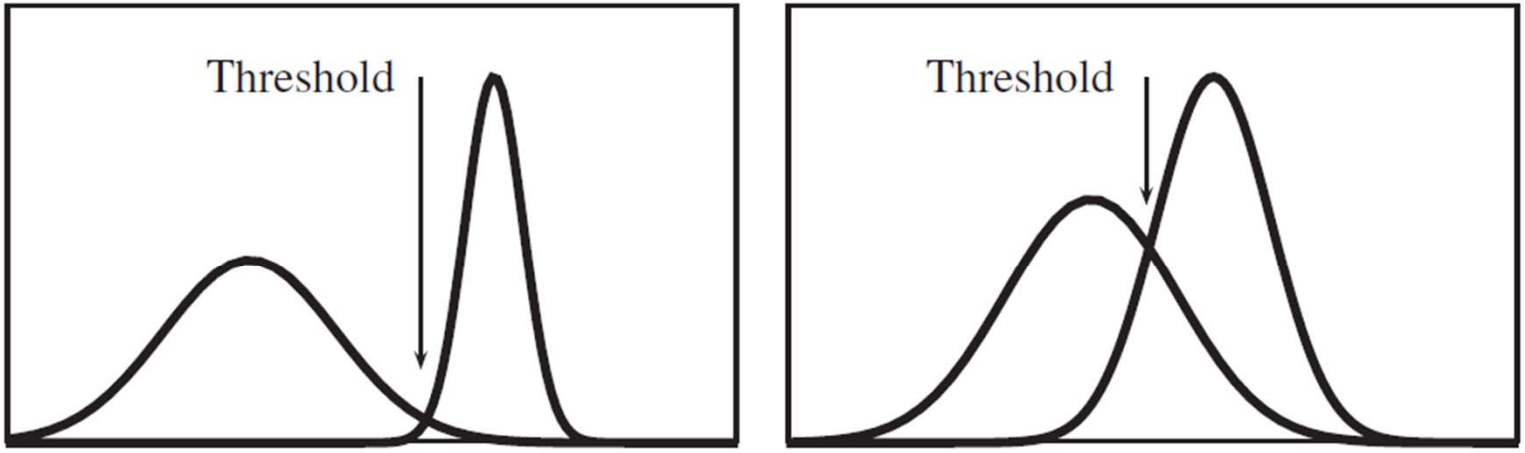


FIGURE 9.8 *Splitting up a histogram for thresholding.*

Otsu's Method:
Choosing an Appropriate Thresholding Value

- Good classifier should be able to divide two classes so that each class has a small variance.
- method 1: Find threshold value t that makes sum of variances of two classes becomes minimum.

$$t = \arg\min_{k}\left[\omega_1(k)\,\mathrm{var}_1(k) + \omega_2(k)\,\mathrm{var}_2(k)\right]$$

$$\omega_1(k) = \sum_{i=0}^{k} p_i, \quad \omega_2(k) = \sum_{i=k+1}^{255} p_i$$

- method 2(faster): Find threshold value t that makes difference of weighted average of two classes becomes maximum.

$$t = \arg\max_{k}\left[\omega_1(k)\omega_2(k)(\mu_1(k) - \mu_2(k))^2\right]$$

- Matlab command: `graythresh()`
- http://en.wikipedia.org/wiki/Otsu's_method

Example of Otsu's Method

```
>> tn=graythresh(n)

tn =

    0.5804

>> r=imread('rice.tif');
>> tr=graythresh(r)

tr =

    0.4902

>> b=imread('bacteria.tif');
>> tb=graythresh(b)

tb =

    0.3765

>> e=imread('eight.tif');
>> te=graythresh(e)

te =

    0.6490
```

```
>> imshow(im2bw(n,tn))
>> figure,imshow(im2bw(r,tr))
>> figure,imshow(im2bw(b,tb))
>> figure,imshow(im2bw(e,te))
```

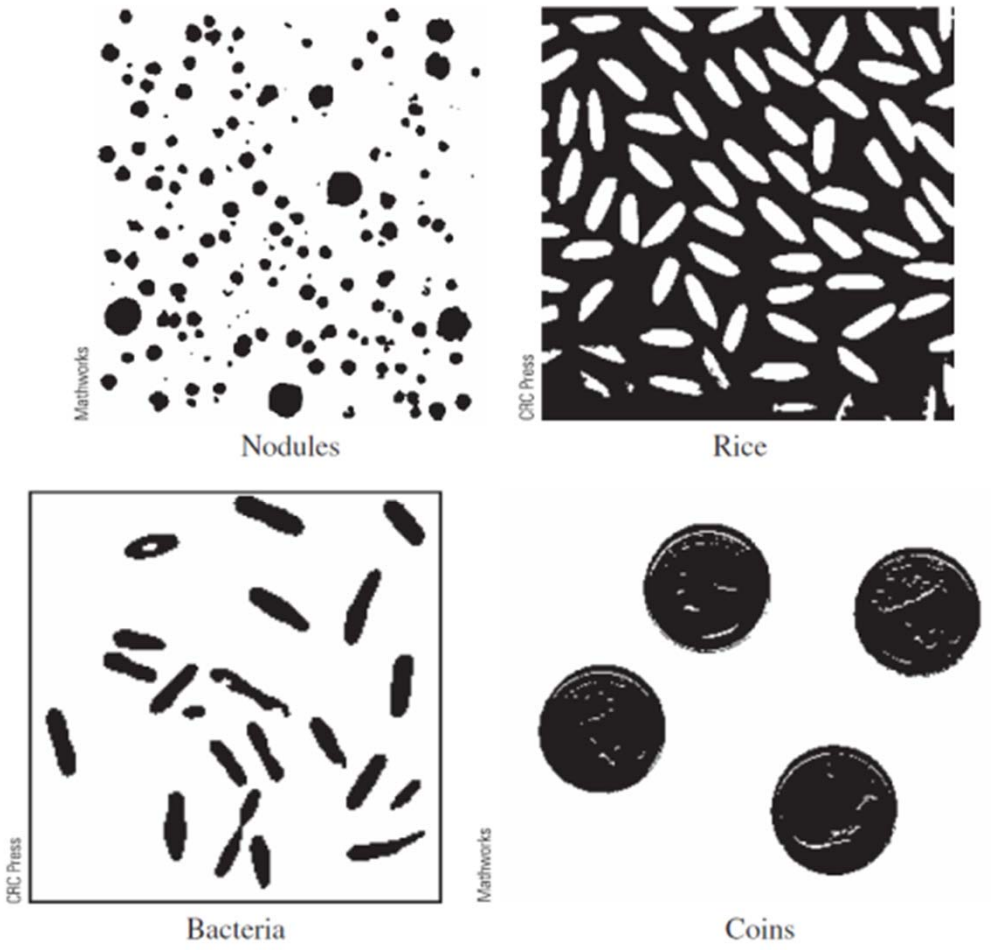


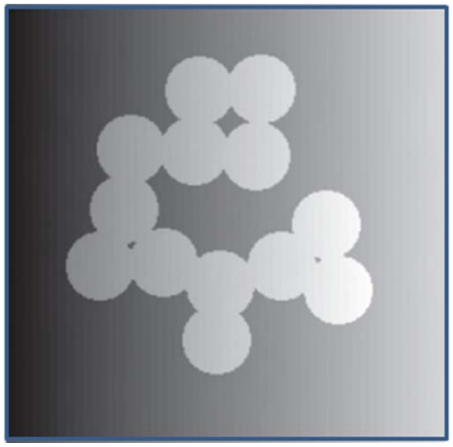
FIGURE 9.9 *Thresholding with values from* `graythresh`.

Adaptive Thresholding

- Sometimes a single global threshold values (even if it was extracted by Otsu's method) does not work.
- Needs to determine the threshold values in each image block depending on the characteristic of each block.

  -> adaptively thresholding

```
>> c=imread('circles.tif');
>> x=ones(256,1)*[1:256];
>> c2=double(c).*(x/2+50)+(1-double(c)).*x/2;
>> c3=uint8(255*mat2gray(c2));
```



c3 from circles.tif

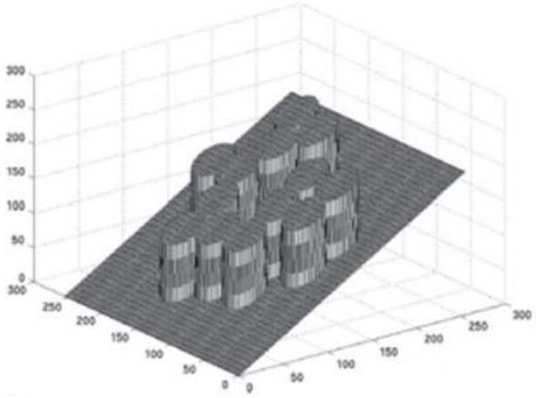
global thresholding

```
>> t=graythresh(c3)

t =

    0.4196

>> ct=im2bw(c3,t);
```

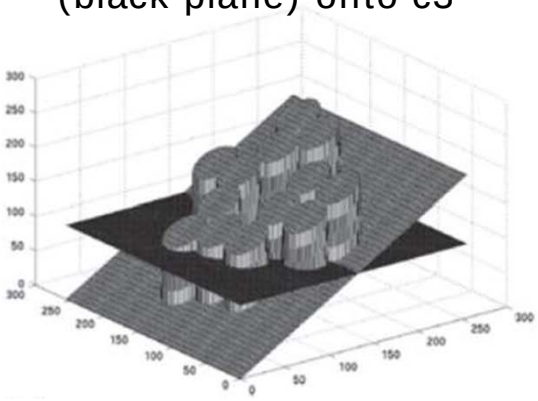
Adaptive Thresholding

Problem determination by 3D surface plots

surface plot of c3



global threshold (black plane) onto c3



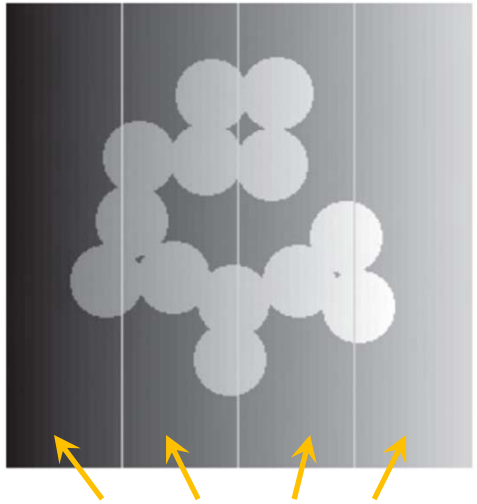
solution: apply blockwise thresholding adaptively

blkproc and im2bw: pp233

```
>> p1=c3(:,1:64);
>> p2=c3(:,65:128);
>> p3=c3(:,129:192);
>> p4=c3(:,193:256);
```

```
>> g1=im2bw(p1,graythresh(p1));
>> g2=im2bw(p2,graythresh(p2));
>> g3=im2bw(p3,graythresh(p3));
>> g4=im2bw(p4,graythresh(p4));
```

```
>> imshow([g1 g2 g3 g4])
```



4 sub-blocks

Edge Detection in Matlab

- The general MATLAB command for finding edges is

```
edge(image,'method',parameters...)
```

51	52	53	59
54	52	53	62
50	52	53	68
55	52	53	55

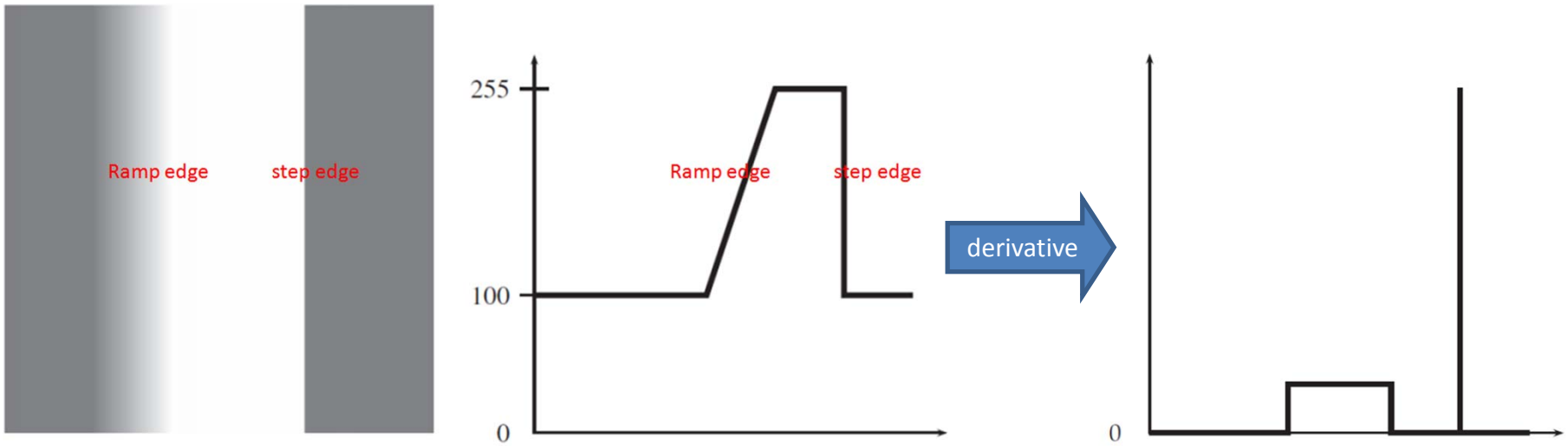
(a)

50	53	155	160
51	53	160	170
52	53	167	190
51	53	162	155

(b)

FIGURE 9.13 *Blocks of pixels.*

Derivatives and Edges



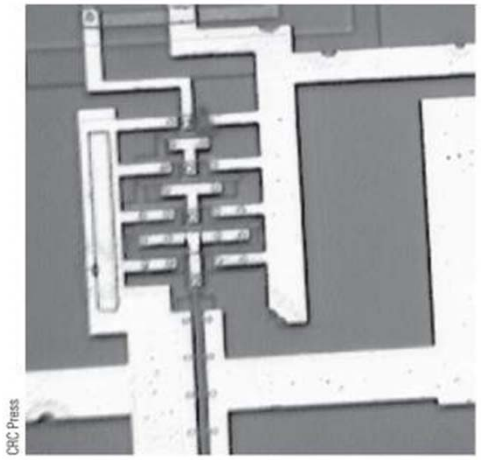
- derivative f'(x)=df/dx
- discrete version: f(x+1)-f(x), f(x)-f(x-1), (f(x+1)-f(x-1))/2
- Expansion into 2D image -> gradient

$$\left[\frac{\partial f}{\partial x} \quad \frac{\partial f}{\partial y}\right] \xrightarrow{\text{magnitude}} \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

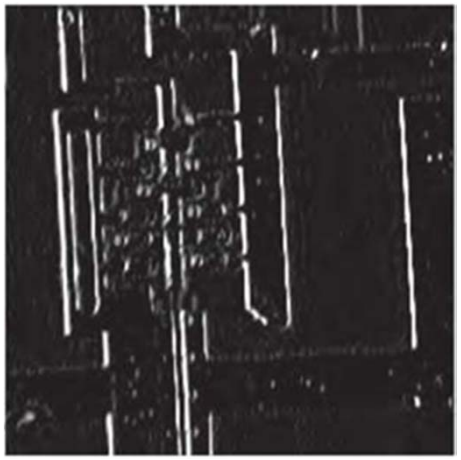
Edge Detection Filters: Prewitt

$$P_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad P_y = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

```
>> ic=imread('ic.tif');
```

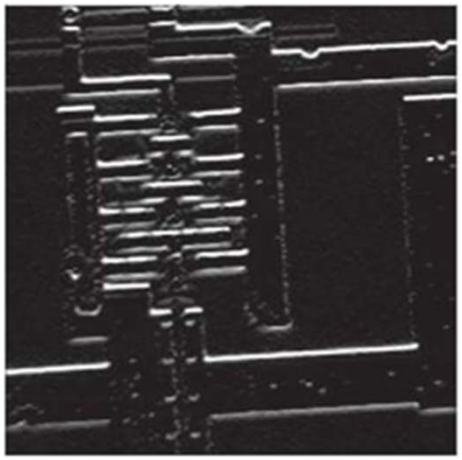



```
>> px=[-1 0 1;-1 0 1;-1 0 1];
>> icx=filter2(px,ic);
>> figure,imshow(icx/255)
```



Prewitt filter: Px

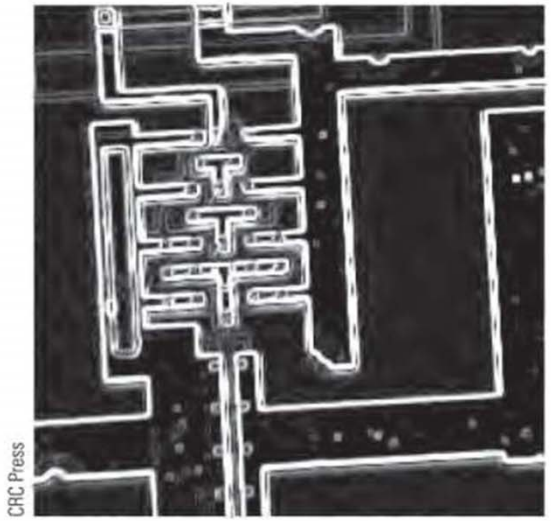
```
>> py=px';
>> icy=filter2(py,ic);
>> figure,imshow(icy/255)
```



Prewitt filter: Py

Edge Detection by Prewitt Filter

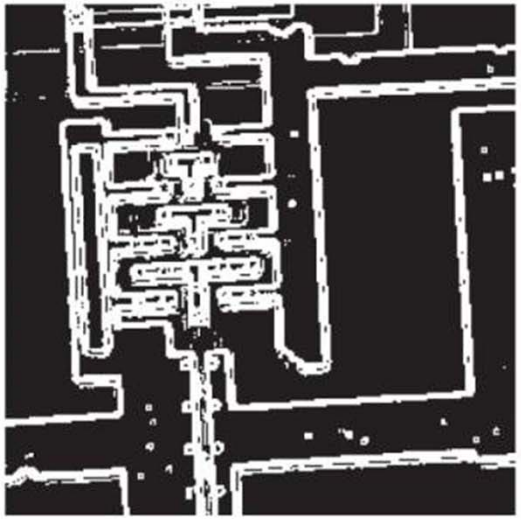
magnitude of gradient images (icx and icy) -> gray scale



(a)

```
>> edge_p=sqrt(icx.^2+icy.^2);
>> figure,imshow(edge_p/255)
```

Thresholding magnitude of gradient images -> binary



(b)

```
>> edge_t=im2bw(edge_p/255,0.3);
```

edge() in matlab: Prewitt magnitude + thresholding + some extra processing

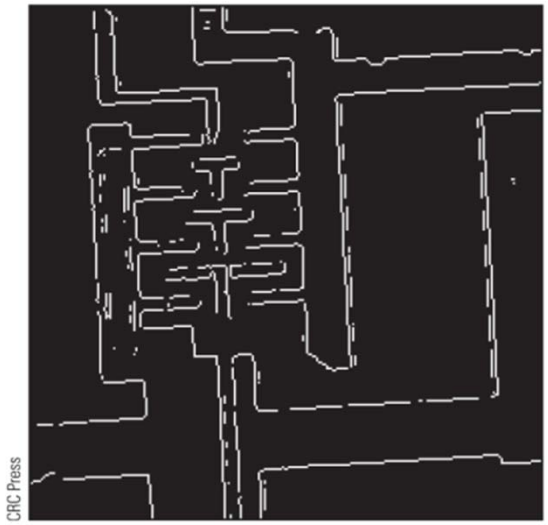


FIGURE 9.19 *The* `prewitt` *option of* `edge`.

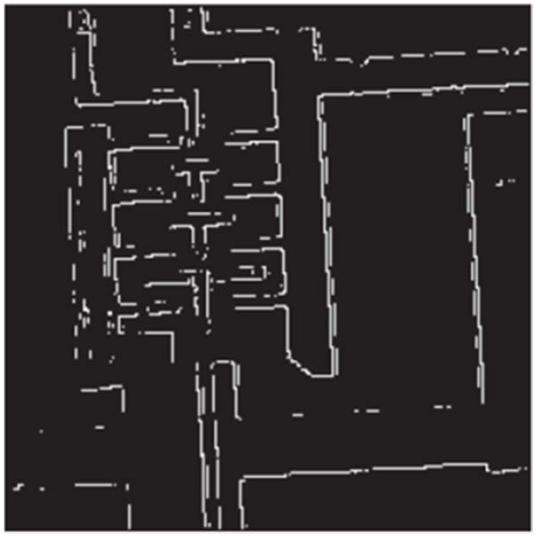
```
>> edge_p=edge(ic,'prewitt');
```

More Edge Detection Filters in Matlab

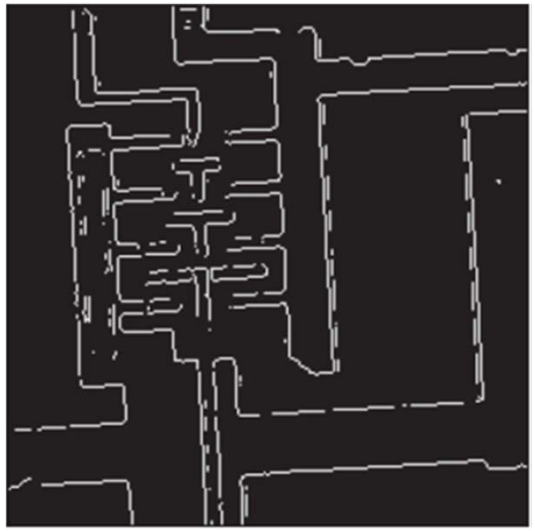
- Roberts cross-gradient filters: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

- Sobel filters: $\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} -1 & -2 & 1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

```
>> edge_r=edge(ic,'roberts');
>> figure,imshow(edge_r)
```



```
>> edge_s=edge(ic,'sobel');
>> figure,imshow(edge_s)
```

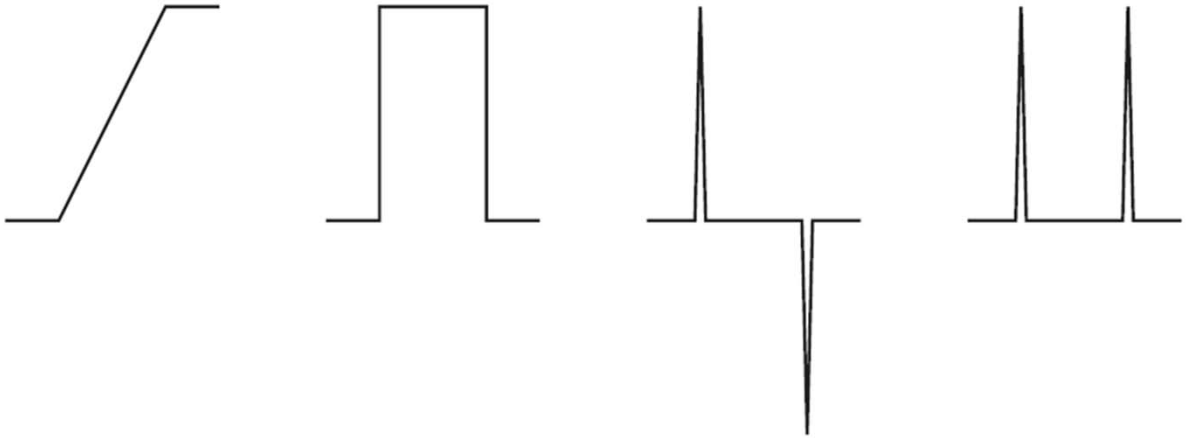


Second Derivatives for Edge Detection

- The Laplacian: $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ → discrete version → $\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

  ✓ Pros: isotropic(= rotation invariant) filter

  ✓ Cons: very sensitive to noise



```
>> l=fspecial('laplacian',0);
>> ic_l=filter2(l,ic);
>> figure,imshow(mat2gray(ic_l))
```

Edge Detection by Zero Crossing

- The position where the result of the filter **changes sign**.
- E.g., the simple image given below and the result after filtering with a Laplacian mask
- What if we simply take all zero crossing points as edges ?

50	50	50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50	50	50
50	50	200	200	200	200	200	200	50	50
50	50	200	200	200	200	200	200	50	50
50	50	200	200	200	200	200	200	50	50
50	50	200	200	200	200	200	200	50	50
50	50	50	50	200	200	200	200	50	50
50	50	50	50	200	200	200	200	50	50
50	50	50	50	50	50	50	50	50	50
50	50	50	50	50	50	50	50	50	50

(a)

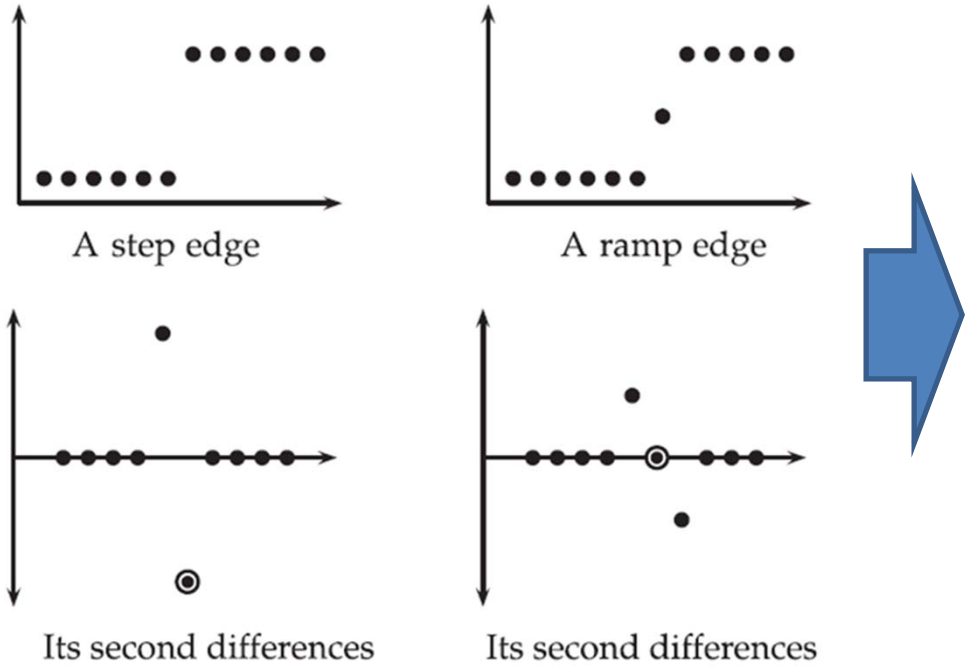
−100	−50	−50	−50	−50	−50	−50	−50	−50	−100
−50	0	150	150	150	150	150	150	0	−50
−50	150	−300	−150	−150	−150	−150	−300	150	−50
−50	150	−150	0	0	0	0	−150	150	−50
−50	150	−150	0	0	0	0	−150	150	−50
−50	150	−300	−150	0	0	0	−150	150	−50
−50	0	150	300	−150	0	0	−150	150	−50
−50	0	0	150	−300	−150	−150	−300	150	−50
−50	0	0	0	150	150	150	150	0	−50
−100	−50	−50	−50	−50	−50	−50	−50	−50	−100

(b)

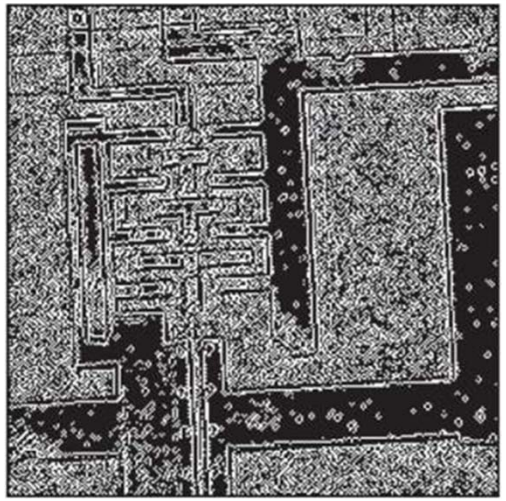
FIGURE 9.23 *Locating zero crossings in an image. (a) A simple image. (b) After laplace filtering.*

Edge Detection by Zero Crossing

- Two cases of zero crossing
  - ✓ They have a negative gray value and are orthogonally adjacent to a pixel whose gray value is positive.
  - ✓ They have a value of zero and are between negative- and positive-valued pixels




```
>> l=fspecial('laplace',0);
>> icz=edge(ic,'zerocross',l);
>> imshow(icz)
```



Too many edges are detected.
How can we improve the results?
-> Marr-Hildreth Method

Edge Detection by Zero Crossing

- **Marr-Hildreth** method
  - ✓ Smooth the image with a Gaussian filter.
  - ✓ Convolve the result with a Laplacian filter.

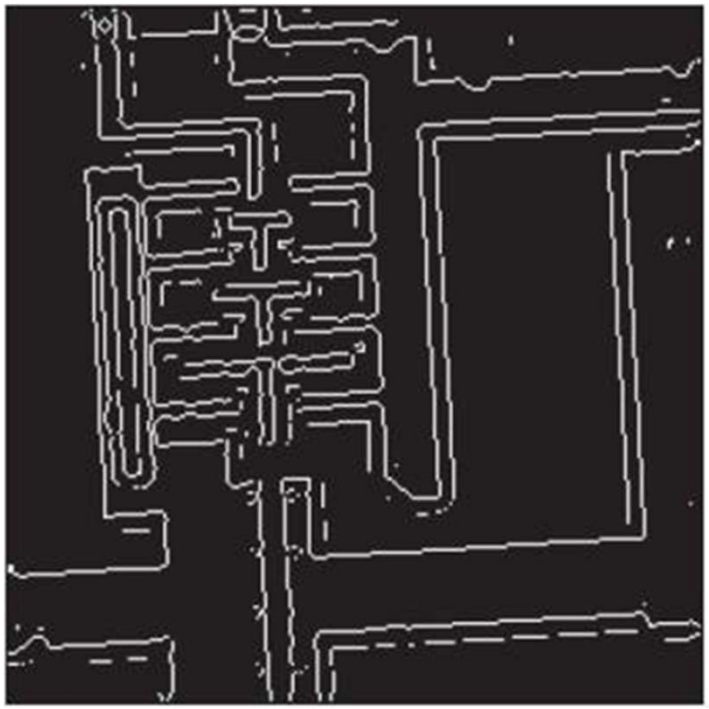
  } Laplacian of Gaussian (LoG)

  - ✓ Find the zero crossings.

```
>> fspecial('log', 13, 2);
>> edge(ic, 'log');
```

or

```
>> log=fspecial('log',13,2);
>> edge(ic,'zerocross',log);
```



Canny Edge Detector

- Proposed by John Canny in 1986.
- The most popular edge filter ever !
- Stage 1: 1D DoG filters -> result: xe(x,y)
  - ✓ Take our image x
  - ✓ Create a one-dimensional Gaussian filter g
  - ✓ Create a 1D filter dg corresponding to the expression given in

$$\frac{d}{dx}e^{-\frac{x^2}{2\sigma^2}} = \left(-\frac{x^2}{\sigma^2}\right)e^{-\frac{x^2}{2\sigma^2}}$$

- Gaussian, then derivative of Gaussian
- to smooth the noise and find possible candidate pixels for edges
- separable (1D filtering twice)

  - ✓ Convolve g with dg to obtain gdg
  - ✓ Apply gdg to x producing $x1$
  - ✓ Apply gdg' to x producing $x2$
  - ✓ Form an magnitude of edge xe with the equation

$$\mathrm{xe} = \sqrt{\mathrm{x1}^2 + \mathrm{x2}^2}$$

Canny Edge Detector

- Stage 2: Non-maxima suppression -> result: $g_N(x,y)$
  - Rejects pixels detected as edges in the Stage 1 by detecting the maximum derivative among adjacent pixels.
  - Thresholding alone is not a good way to detection edges in the result *xe*.

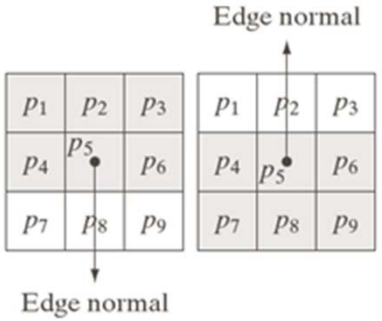
✓Calculate edge normal(gradient vector) *xg* from the 1D gradient images *x1* and *x2*.

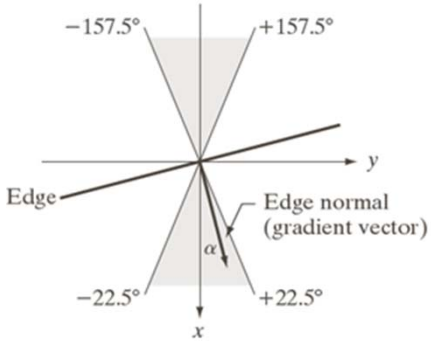
$$xg = \tan^{-1}\left(\frac{x2}{x1}\right)$$

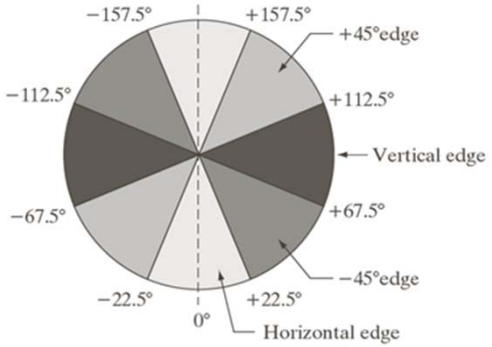
✓Because it is generated using the gradient, xg typically contains wide ridges around local maxima. Thus, we needs to thin those ridges.

Based on xg, quantize edge orientations into the predetermined steps. (e.g. four orientations in 3x3 region: horizontal, vertical, +45°, -45°)

✓If a value of xe(x,y) is less than at least one of its two neighbors along xg_i, let $g_N(x,y) = 0$ (suppression); otherwise, let $g_N(x,y) = xe(x,y)$.







Canny Edge Detector

- Stage 3: Hysteresis thresholding for binary edge image
  - uses two threshold values: T_L and T_H for lower and higher bounds respectively.

  ✓ $g_{NH}(x,y) = g_N(x,y) >= T_H$: strong edge pixels

  ✓ $g_{NL}(x,y) = g_N(x,y) >= T_L$

  ✓ $g_{NL}(x,y) = g_{NL}(x,y) - g_{NH}(x,y)$: weak edge pixels

  ✓ Connectivity analysis in $g_{NH}(x,y)$

  1. Locate the next unvisited edge pixel, p, in $g_{NH}(x,y)$.
  2. Mark as valid edge pixels all the weak pixels in $g_{NL}(x,y)$ that are connected to p using 8 connectivity.
  3. If all nonzero pixels in $g_{NH}(x,y)$ have been processed, go to the next step. Otherwise, return to the first step.
  4. Set to 0 all pixels in $g_{NL}(x,y)$ that were not marked as valid edge pixels.

  ✓ Append all the nonzero pixels from $g_{NL}(x,y)$ to $g_{NH}(x,y)$.

Canny Edge Detector in Matlab

```
>> [icc,t]=edge(ic,'canny');
>> t

t =

    0.0500    0.1250

>> imshow(icc)
```

- BW = EDGE(I,'canny',[low, high],SIGMA)
  - [low, high]: threshold limits
  - SIGMA: standard deviation
- The higher we make the upper threshold, the fewer edges will be shown.

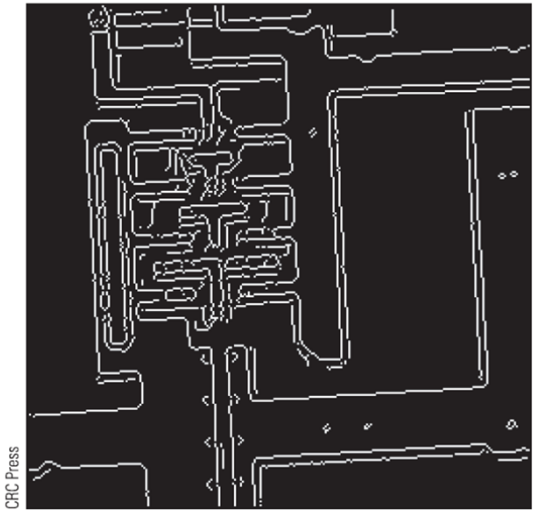
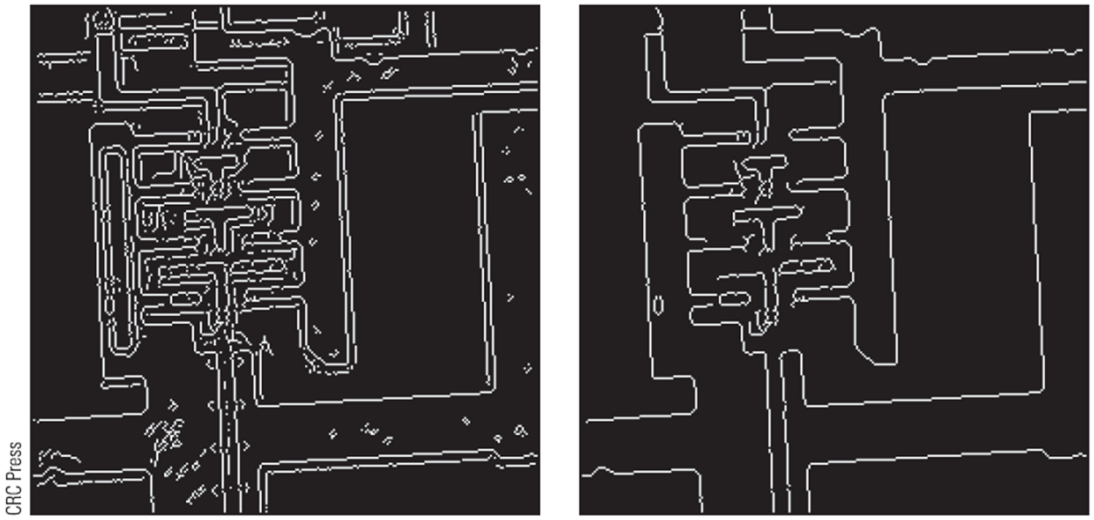


FIGURE 9.29 *Canny edge detection.*

default threshold and standard deviation



edge(ic,'canny',[0,0.05]) edge(ic,'canny',[0.01,0.5])

FIGURE 9.30 *Canny edge detection with different thresholds.*

Hough Transform

- If the detected edge points are sparse, we might need to fit a line to those points. However it is a time-consuming and computationally inefficient process.
- Is there any way to find boundary lines to compensate the problem described above ? => Hough Transform
- Basic idea of Hough Transform
  - A coordinate (x,y) in an image can be transformed into a line y = ax + b with all possible pairs of (a,b).
  - Every pixel at (x,y) coordinate in the image can be mapped into a line in the transform array.

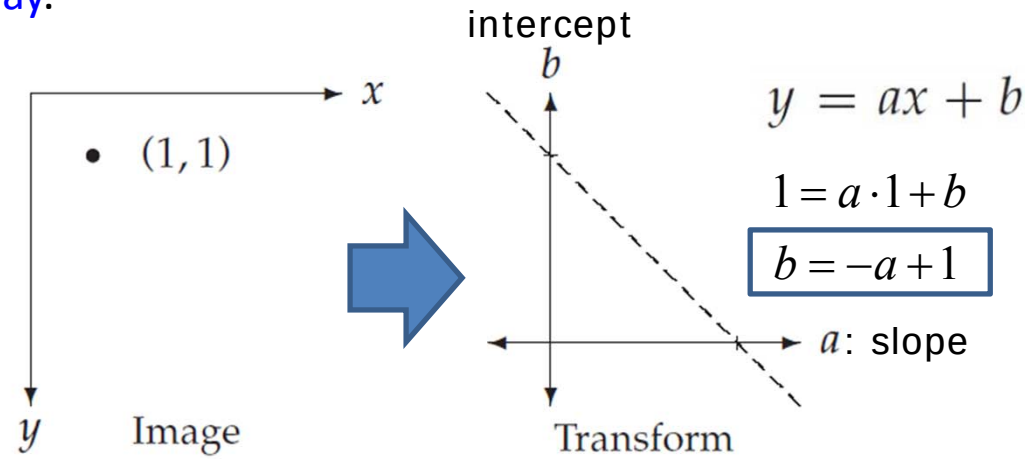


FIGURE 9.31 *A point in an image and its corresponding line in the transform.*

Basic Idea of Hough Transform

5 pixels are transformed into 5 lines

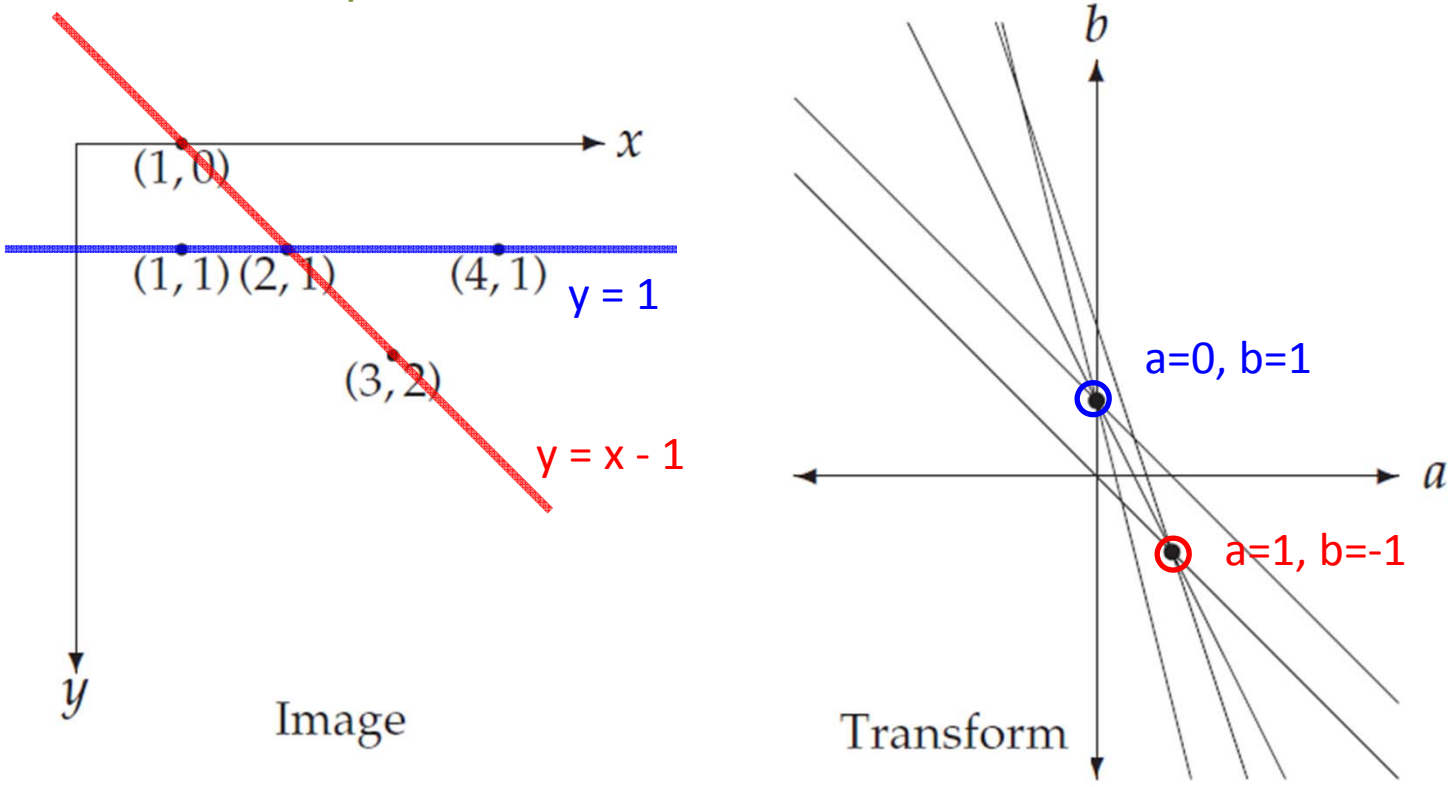
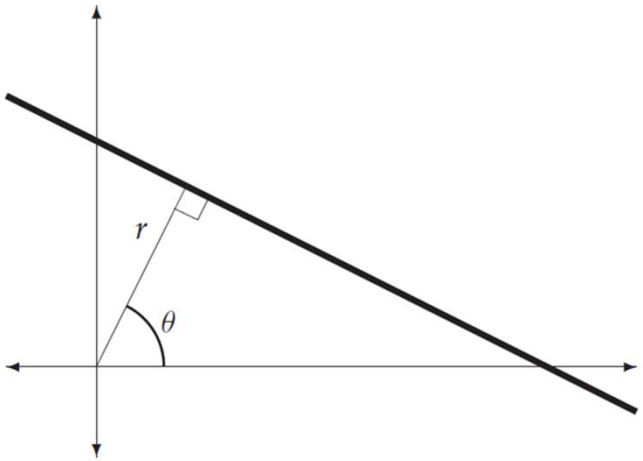


FIGURE 9.32 *An image and its corresponding lines in the transform.*

- Higher value of cross point (a,b) in the transformed array indicates more distinguished lines (more pixels through the line) in the image !

Hough Transform for Generalization

- We cannot express a vertical line in the form $y = mx+c$, because m represents the gradient and a vertical line has infinite gradient.
- Any line can be described in terms of the two parameters r and θ.
  - ✓ r is the perpendicular distance from the line to the origin
  - ✓ θ is the angle of the line's perpendicular to the x axis



$$-90 < \theta \leq 90$$

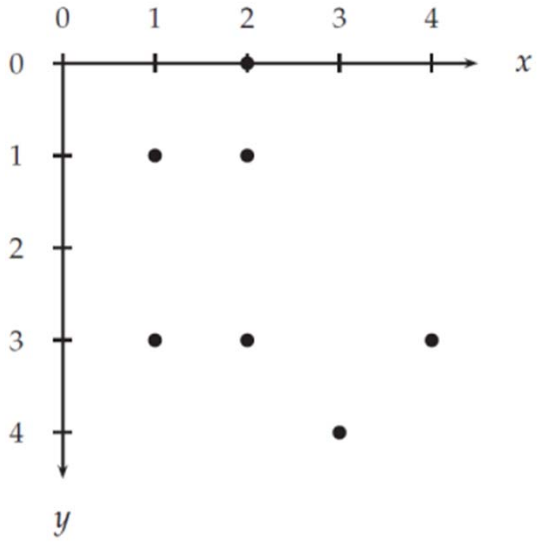
$$x\cos\theta + y\sin\theta = r$$

We can calculate θ and r based on the image coordinate (x,y).
See pp.253 for the derivation.

- Higher value of (r,θ) in the transformed array indicates stronger lines (more pixels through the line) in the image.

Hough Transform Example

- If we discretize θ to use only four values: -45°, 0°, 45°, 90°



(x, y)	$-45°$	$0°$	$45°$	$90°$
(2, 0)	1.4	2	1.4	0
(1, 1)	0	1	1.4	1
(2, 1)	0.7	2	2.1	1
(1, 3)	−1.4	1	2.8	3
(2, 3)	−0.7	2	3.5	3
(4, 3)	0.7	4	4.9	3
(3, 4)	−0.7	3	4.9	4

θ

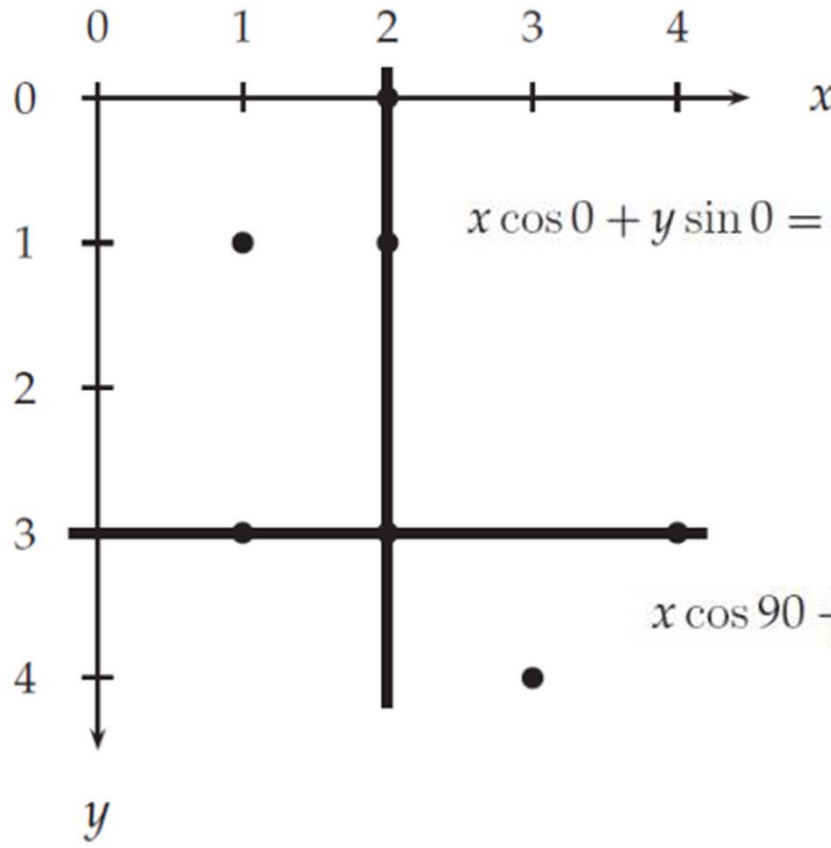
r

$x\cos\theta + y\sin\theta = r$

- The accumulator array contains the number of times each value of (r, θ) appears in the above table.

	−1.4	−0.7	0	0.7	1	1.4	2	2.1	2.8	3	3.5	4	4.9
−45°	1		2	1	2		1						
0°					2		(3)			1		1	
45°						2		1	1		1		2
90°			1		2					(3)		2	

Hough Transform Example



Detected lines

Implementing Hough Transform in MATLAB

1. Decide on a discrete set of values of θ and r to use.
2. At each foreground pixel (x, y) in the image, calculate, the values $r = x\cos\theta + y\sin\theta$ for all θ.
3. Create an accumulator array whose sizes are the numbers of angles θ and values r in the chosen discretizations from Step 1, and
4. Step through all of our r values, updating the accumulator array as we go.

Hough Transform in MATLAB

- Discretizing r and θ
  - Let θ with the range of 0 <= θ < 2π.
  - Restrict r to non-negative value.

```
>> angles = [-90:180]*pi/180;
```

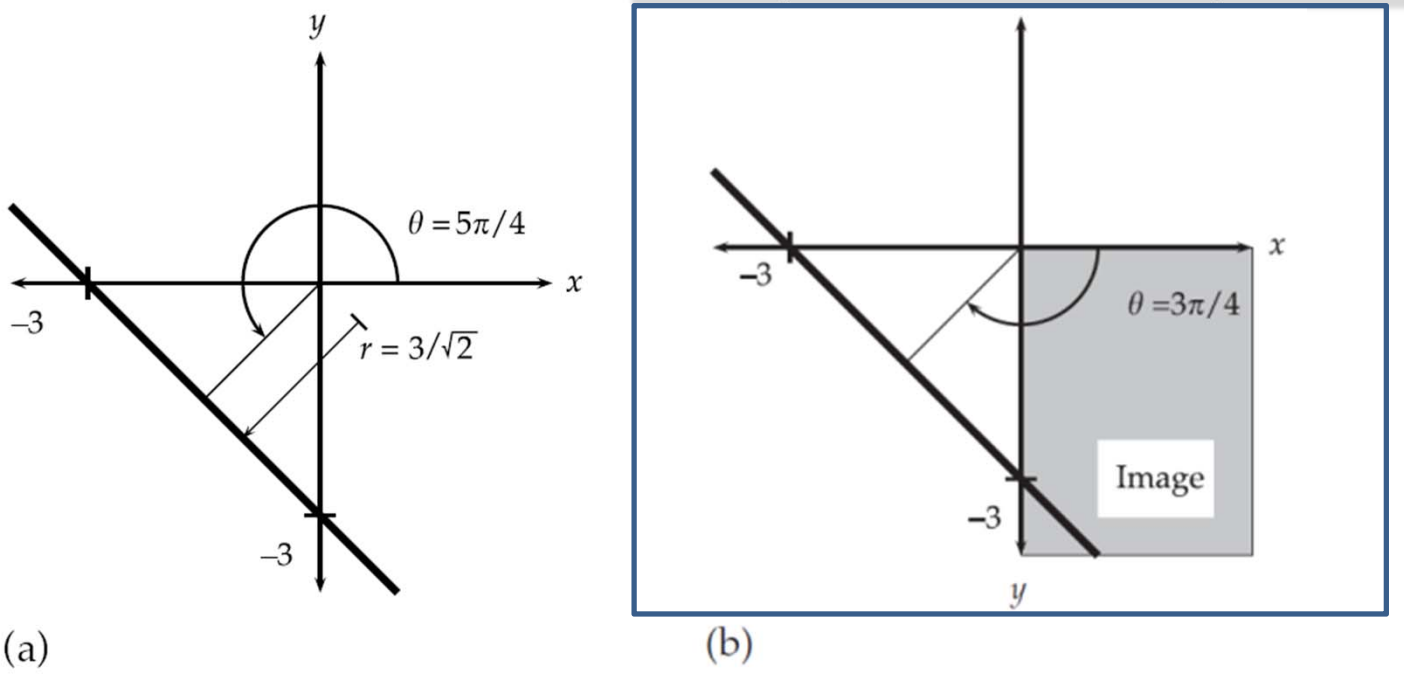


FIGURE 9.37 *A line parameterized with r and θ. (a) Using ordinary Cartesian axes. (b) Using matrix axes.*

Example of Hough Transform in MATLAB

- Calculating the r values
  - ✓ If im is a **binary image**

```
>> [x,y]=find(im);
```

We can create a binary edge image by use of the edge function beforehand.

```
>> r=floor(x*cos(angles)+y*sin(angles));
```

- Forming the Accumulator Array

```
>> rmax=max(r(find(r>0)));
>> acc=zeros(rmax+1,270);
```

- Updating the Accumulator Array

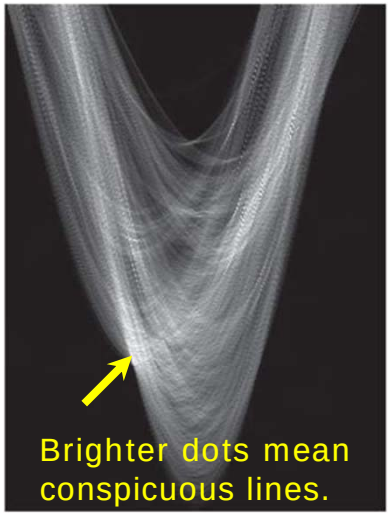
```
function res=hough2(image)

%
% HOUGH2(IMAGE) creates the Hough transform corresponding to the image IMAGE
%

if ~isbw(image)
  edges=edge(image,'canny');
else
  edges=image;
end;
[x,y]=find(edges);
angles=[-90:180]*pi/180;
r=floor(x*cos(angles)+y*sin(angles));
rmax=max(r(find(r>0)));
acc=zeros(rmax+1,270);
for i=1:length(x),
  for j=1:270,
    if r(i,j)>=0
      acc(r(i,j)+1,j)=acc(r(i,j)+1,j)+1;
    end;
  end;
end;
res=acc;
```

FIGURE 9.38 *A simple MATLAB function for implementing the Hough transform.*

Example of Hough Transform in MATLAB

```
>> c=imread('cameraman.tif');
>> hc=hough2(c);
```

```
>> imshow(mat2gray(hc)*1.5)
```




```
>> max(hc(:))

ans =

    91
```

```
>> [r,theta]=find(hc==91)

r =

   138


theta =

   181
```

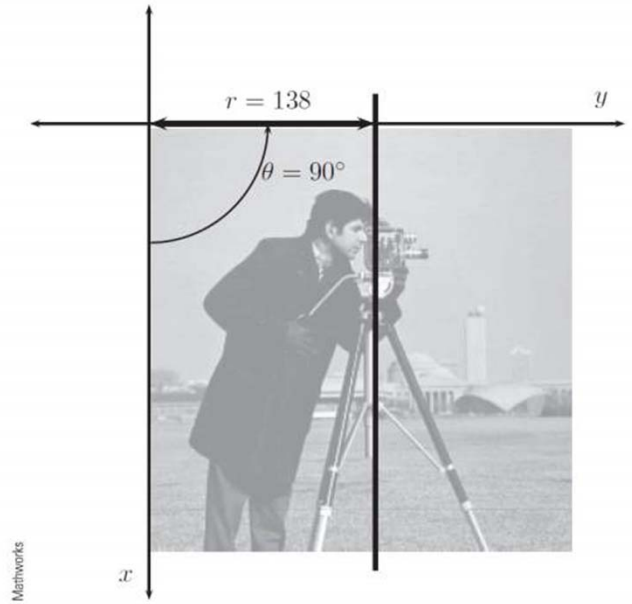


FIGURE 9.40 *A line from the Hough Transform.*

How to Display the Detected Line in Matlab

```
function houghline(image,r,theta)
%
% Draws a line at perpendicular distance R from the upper left corner of the
% current figure, with perpendicular angle THETA to the left vertical axis.
% THETA is assumed to be in degrees.
%
[x,y]=size(image);
angle=pi*(181-theta)/180;
X=[1:x];
if sin(angle)==0
  line([r r],[0,y],'Color','black')
else
  line([0,y],[r/sin(angle),(r-y*cos(angle))/sin(angle)],'Color','black')
end;
```

(a) (b)

See pp. 261 for more detailed information to use houghline().

Summary

- **Chap 9. Image Segmentation**
  - Single & double thresholding
  - How to determine the threshold value
  - Adaptive thresholding
  - Edge detection: 1st and 2nd derivatives
  - Canny edge detector
  - Hough Transform
- **Chap 10. Mathematical Morphology**
  - Erosion & dilation -> boundary detection
  - Opening & closing -> noise removal
  - Hit-or-miss transform -> shape detection
  - Binary applications: region filling, connected components, skeletons
  - Grayscale morphology -> edge detection, noise removal

Chapter 10:

Mathematical Morphology

10.1 Introduction

- *Morphology* is a branch of image processing that is particularly useful for analyzing **shapes** in images.
- We will develop basic morphological tools for investigation of **binary images** and then show how to extend these tools to **grayscale images.**
- In this chapter, we will learn,
  - What is morphology?
  - Simple morphological operations: erosion, dilation
  - Compound operations: opening, closing
  - Morphological algorithms: boundary detection, region filling, connected components, skeleton

Translation & Reflection

- Translation

$$A_w = \{(a,b) + (x,y) : (a,b) \in A\}$$

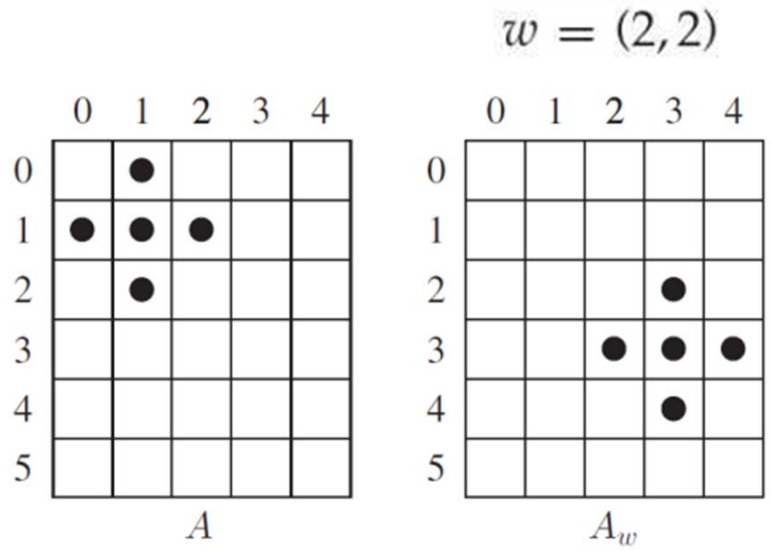


FIGURE 10.1 *Translation.*

- Reflection

$$\hat{A} = \{(-x,-y) : (x,y) \in A\}$$

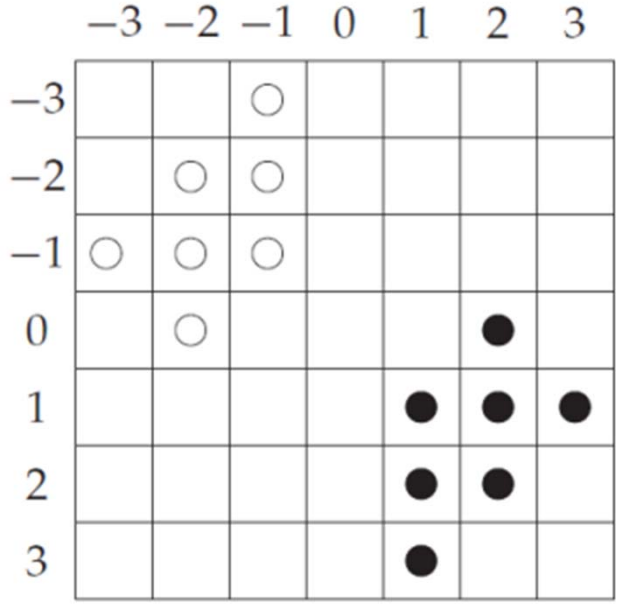


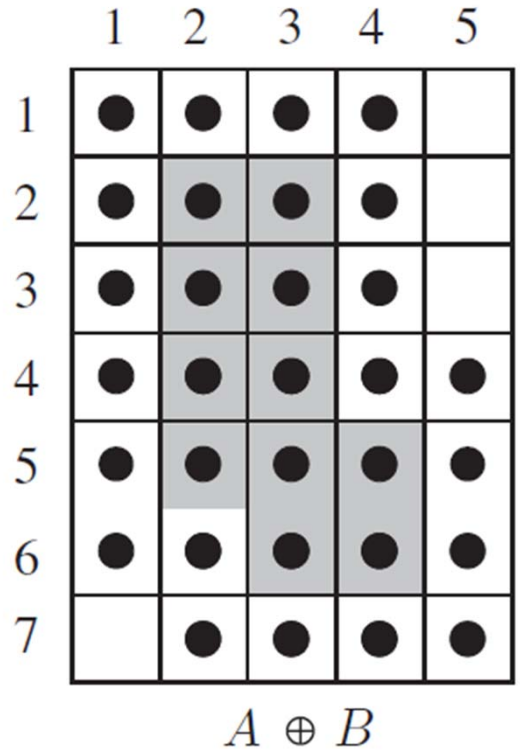
FIGURE 10.2 *Reflection.*

Dilation

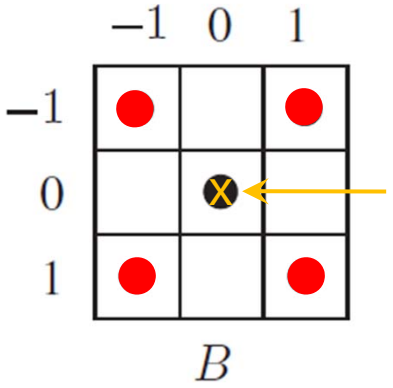
- A and B are sets of pixels
- Also known as **Minkowski addition**

$$A \oplus B = \{(x,y) + (u,v) : (x,y) \in A, (u,v) \in B\}$$

$$A \oplus B = \bigcup_{x \in B} A_x$$



structuring element (SE)



Center of SE should be specified.

FIGURE 10.3 *Dilation.*

Dilation in Matlab

```
>> t=imread('text.tif');
>> sq=ones(3,3);
>> td=imdilate(t,sq);
>> subplot(1,2,1),imshow(t)
>> subplot(1,2,2),imshow(td)
```

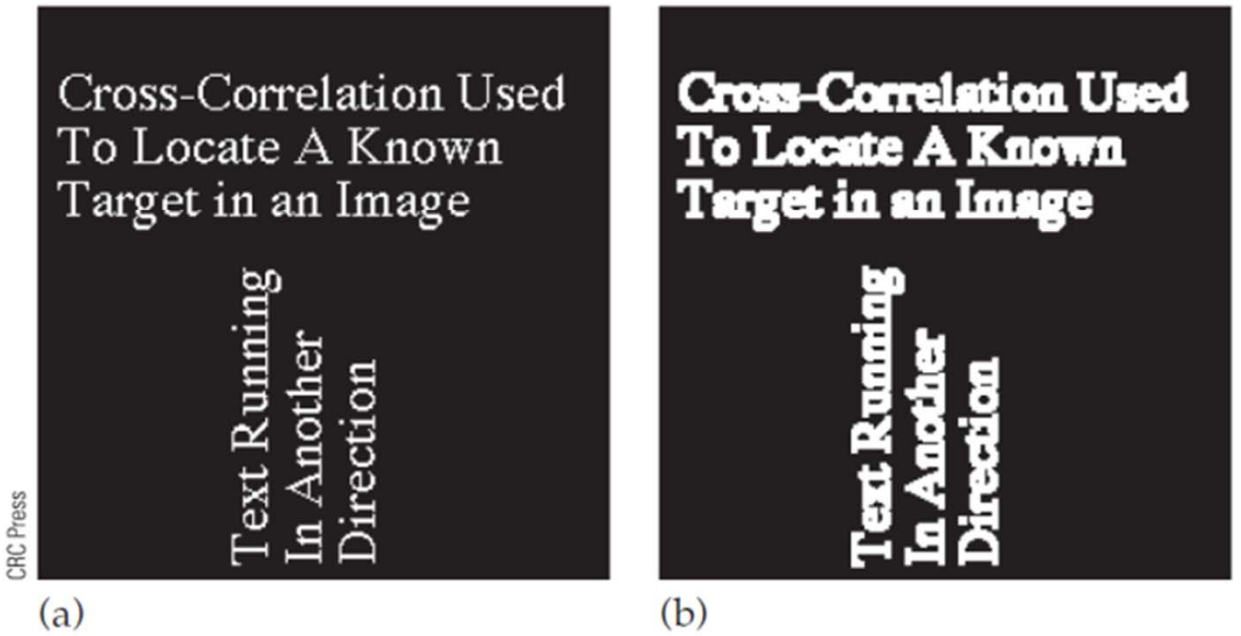


FIGURE 10.5 *Dilation of a binary image. (a) Text image. (b) Result of dilation.*

Erosion

- Also known as Minkowski substraction $A \ominus B = \{w : B_w \subseteq A\}$

$$A - B = \bigcap_{b \in B} A_b$$

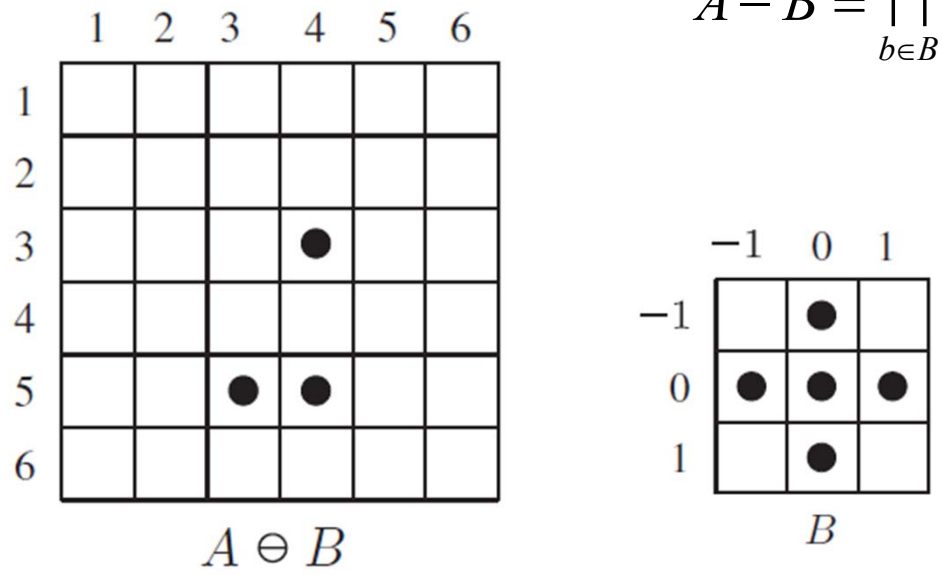


FIGURE 10.6 *Erosion with a cross-shaped structuring element.*

Erosion in Matlab

```
>> c=imread('circbw.tif');
>> ce=imerode(c,sq);
>> subplot(1,2,1),imshow(c)
>> subplot(1,2,2),imshow(ce)
```

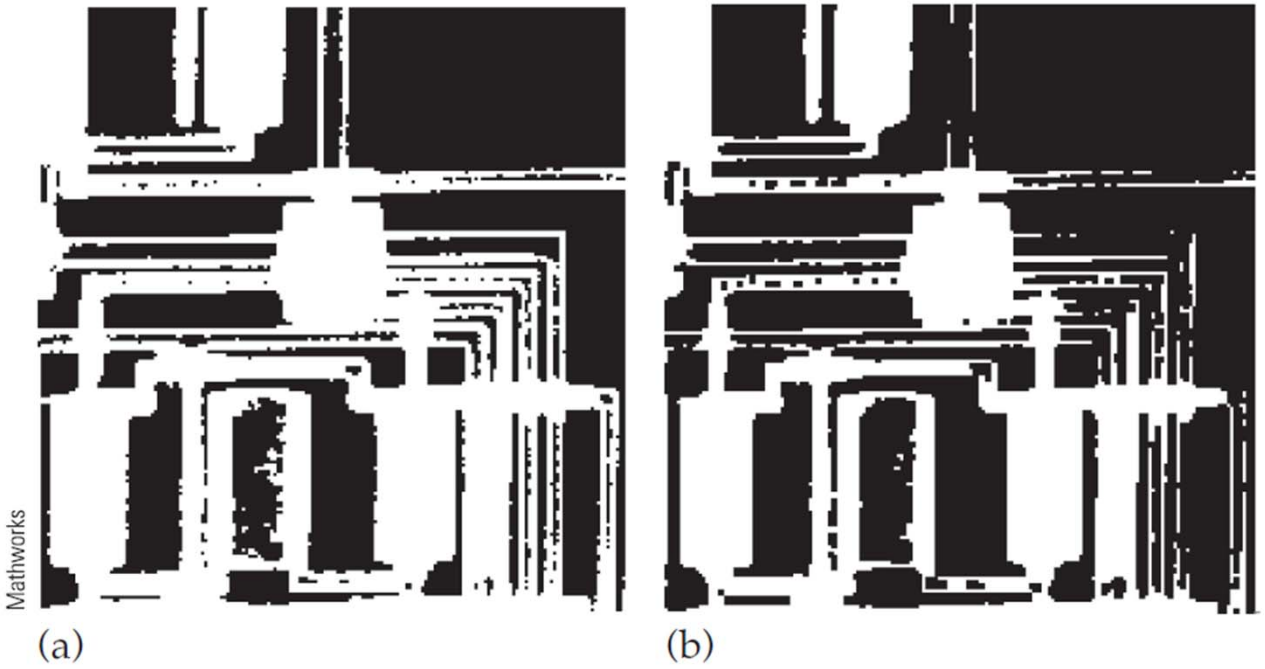
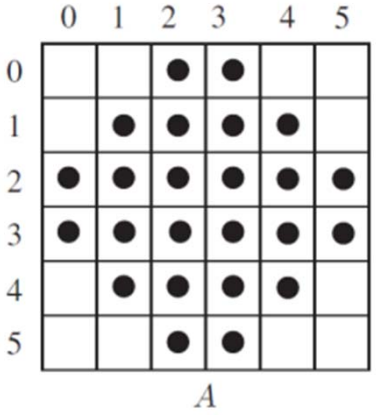


FIGURE 10.8 *Erosion of a binary image. (a) Original image. (b) Result of erosion.*

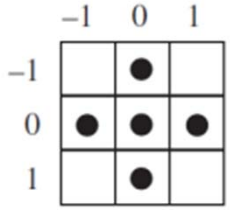
Application of Dilation and Erosion: Boundary Detection

- If A is an image and B a small structuring element,

(i)	$A - (A \ominus B)$	internal boundary
(ii)	$(A \oplus B) - A$	external boundary
(iii)	$(A \oplus B) - (A \ominus B)$	morphological gradient

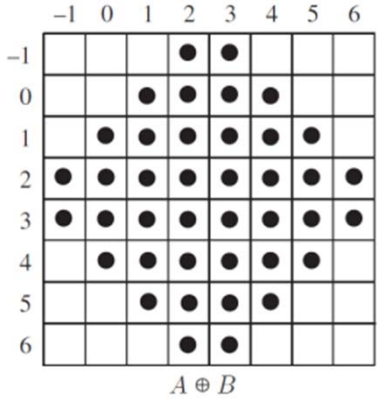


A

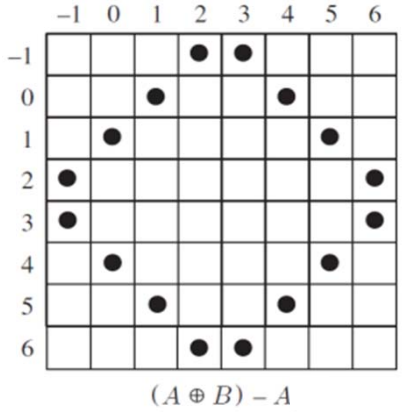


B

external boundary detection: outside of the object

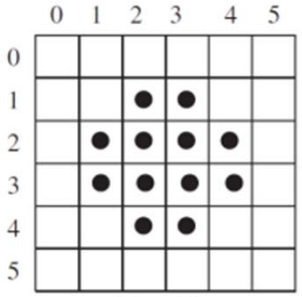


$A \oplus B$

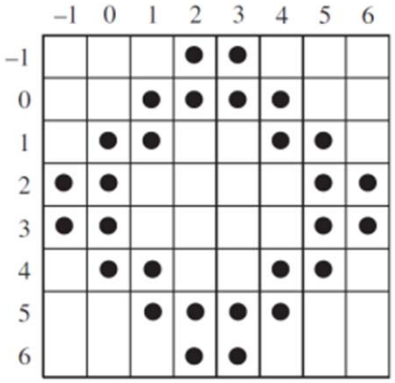


$(A \oplus B) - A$

morphological gradient: both int. and ext. boundaries



$(A \ominus B)$



$(A \oplus B) - (A \ominus B)$

Internal Boundary Detection in Matlab

```
>> re=imerode(r,sq);
>> r_int=r&~re;
>> subplot(1,2,1),imshow(r)
>> subplot(1,2,2),imshow(r_int)
```

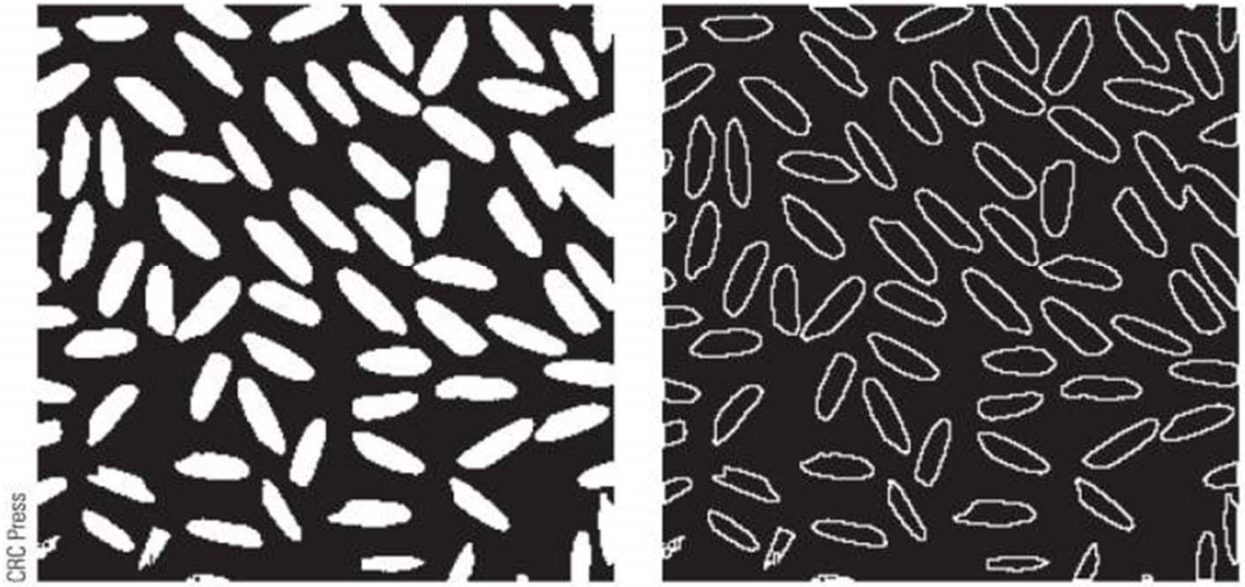


FIGURE 10.10 *Morphological edge detection. (a) The rice grains image. (b) The internal boundary.*

External and Gradient Boundary Detection

```
>> rd=imdilate(r,sq);
>> r_ext=rd&~r;
>> r_grad=rd&~re;
>> subplot(1,2,1),imshow(r_ext)
>> subplot(1,2,2),imshow(r_grad)
```

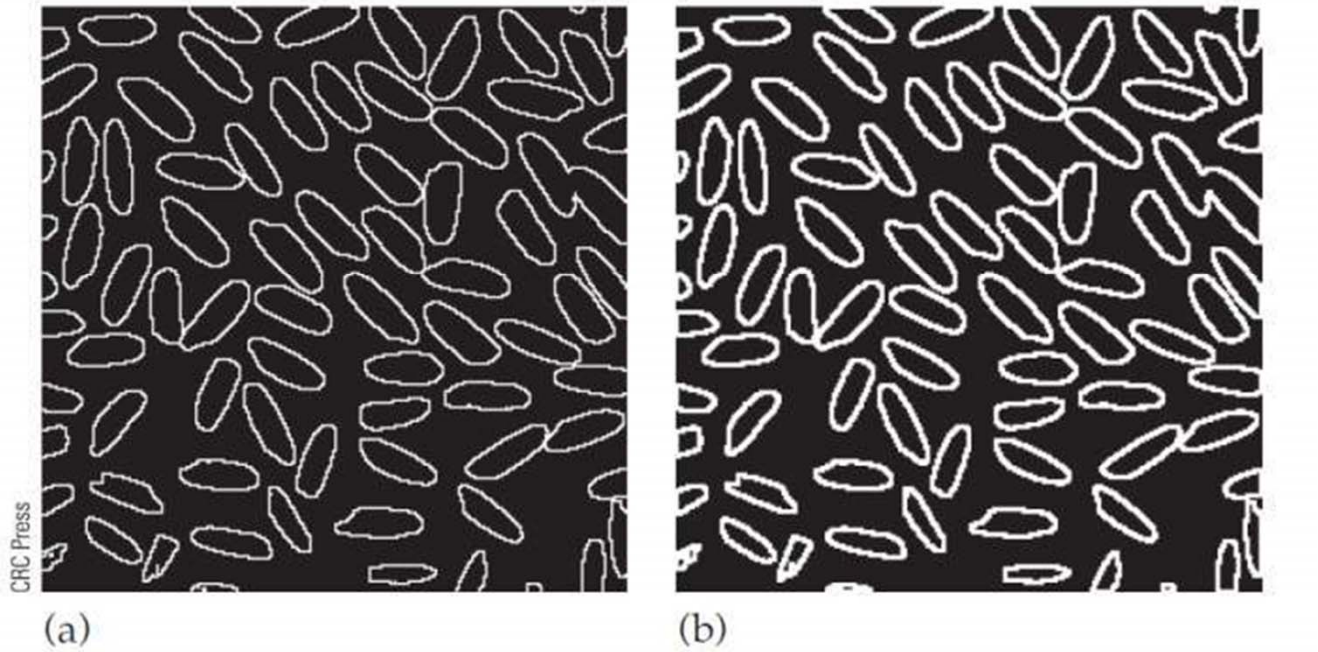


FIGURE 10.11 *More morphological edge detection. (a) External boundary. (b) Morphological gradient.*

Opening

- **Opening** (function: `imopen( )`)

$$A \circ B = (A \ominus B) \oplus B$$

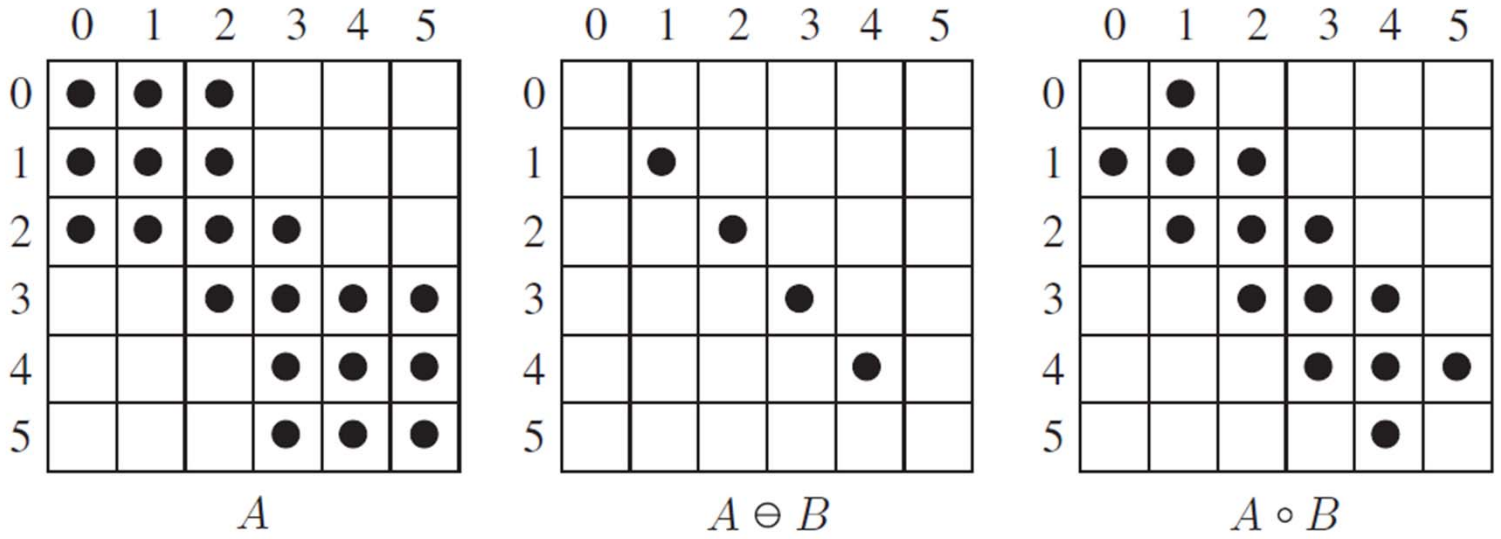
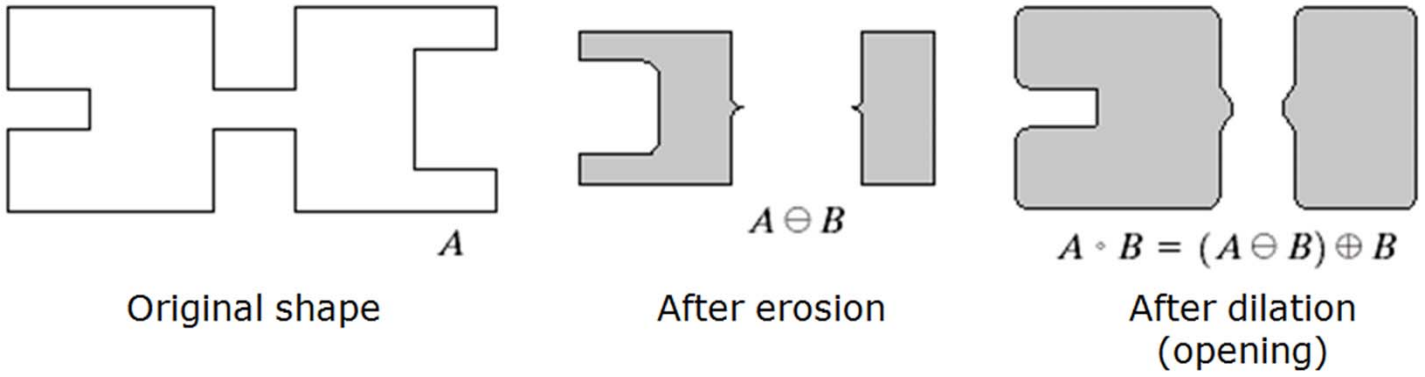


FIGURE 10.12 *Opening.*

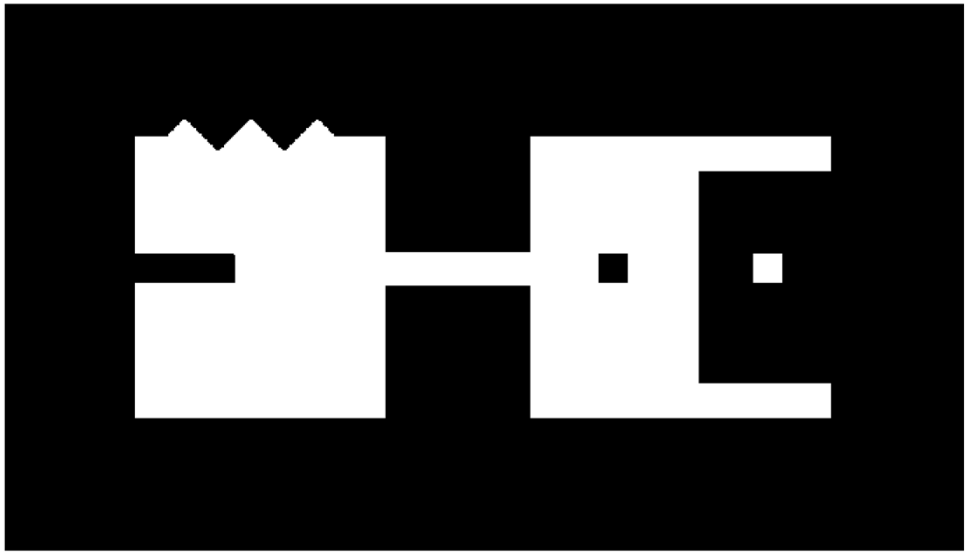
Properties of Opening

- ✓ $(A \circ B) \subseteq A$
  - ✓ Note that this is not the case with erosion. As we have seen, an erosion may not necessarily be a subset.
- ✓ $(A \circ B) \circ B = A \circ B$
  - ✓ That is, an opening can never be done more than once (**idempotent**).
- ✓ If $A \subseteq C$, then $(A \circ B) \subseteq (C \circ B)$
- ✓ Opening tends to smooth an image, to break narrow joins, and to remove thin protrusions.



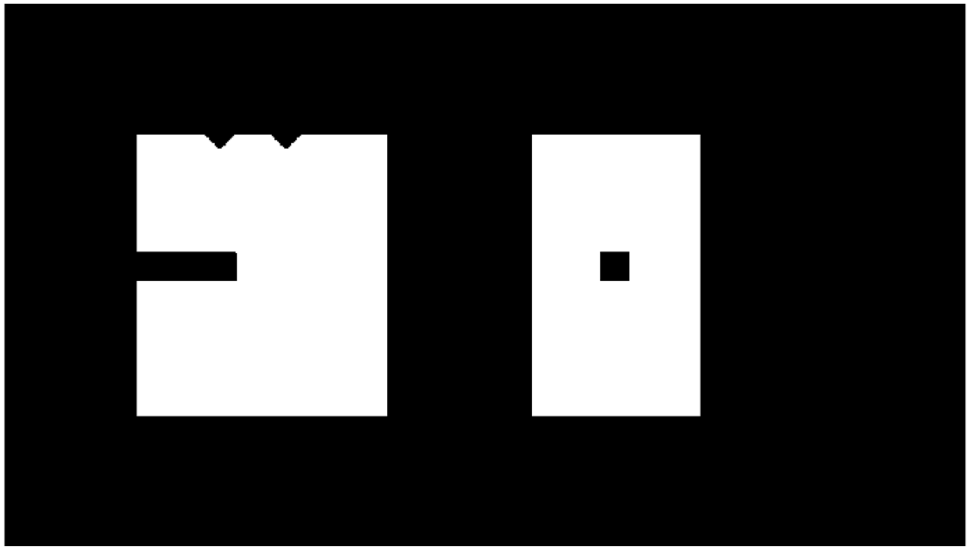
Opening Example

Original Image



se

Image After Opening



Closing

- **Closing** (function: `imclose()`)

$$A \bullet B = (A \oplus B) \ominus B$$

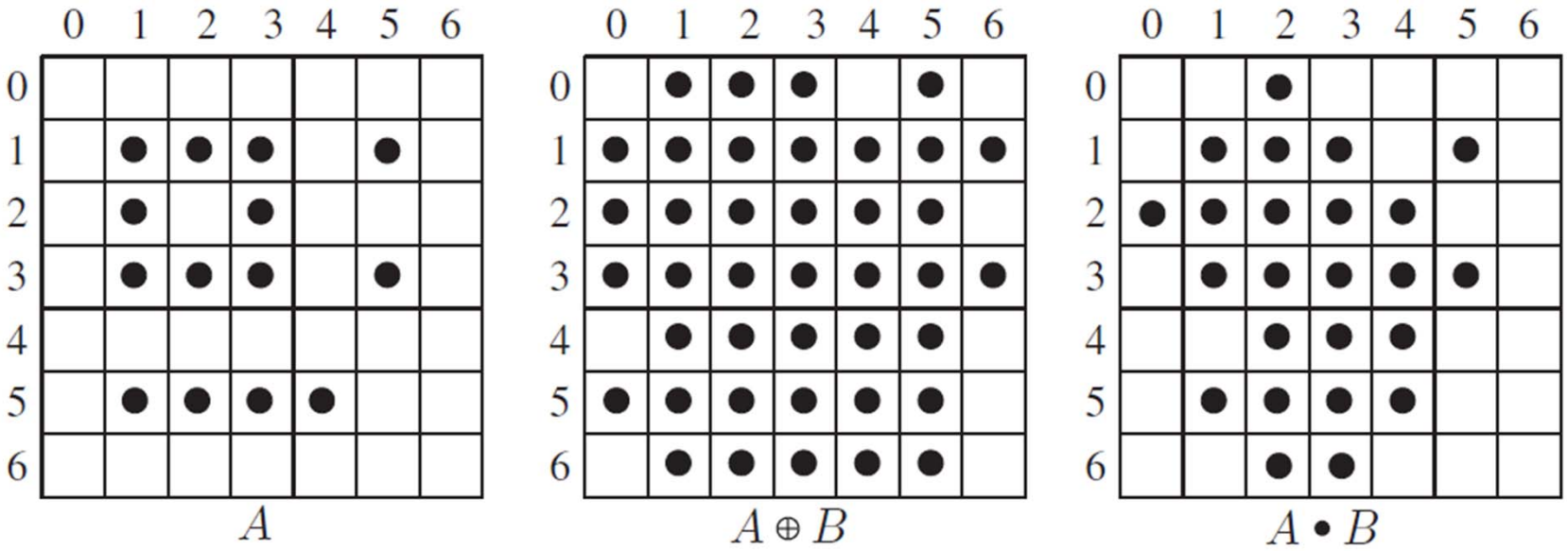
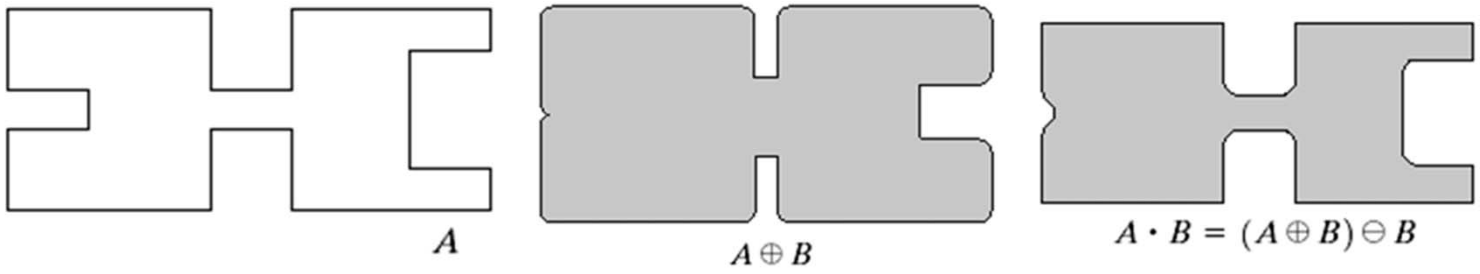


FIGURE 10.13 *Closing.*

Properties of Closing

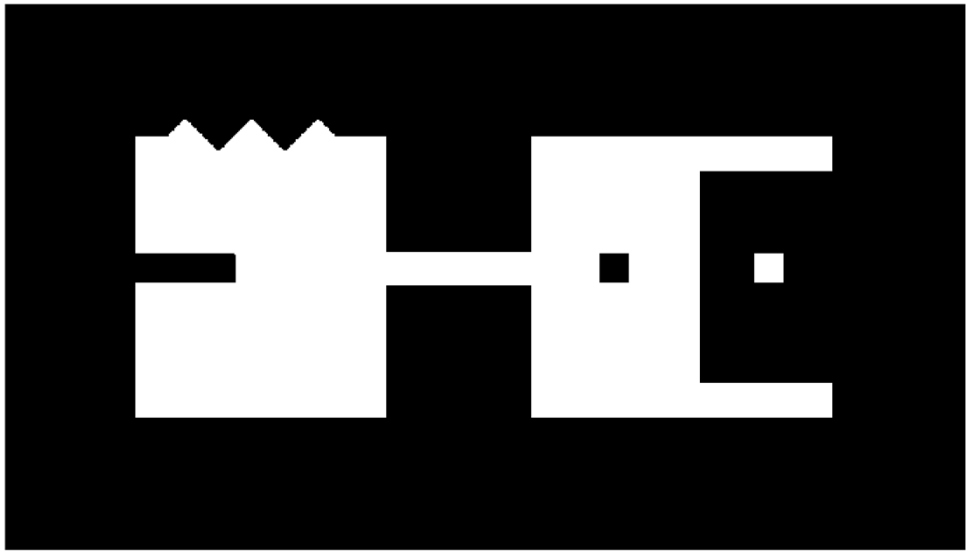
- ✓ $A \subseteq (A \bullet B)$
- ✓ $(A \bullet B) \bullet B = A \bullet B$
  - ✓ That is, closing, like opening, is **idempotent.**
- ✓ If $A \subseteq C$, then $(A \bullet B) \subseteq (C \bullet B)$
- ✓ Closing also tends to smooth an image, but it fuses narrow breaks and thin gulfs and eliminates small holes.



Original shape | After dilation | After erosion (closing)

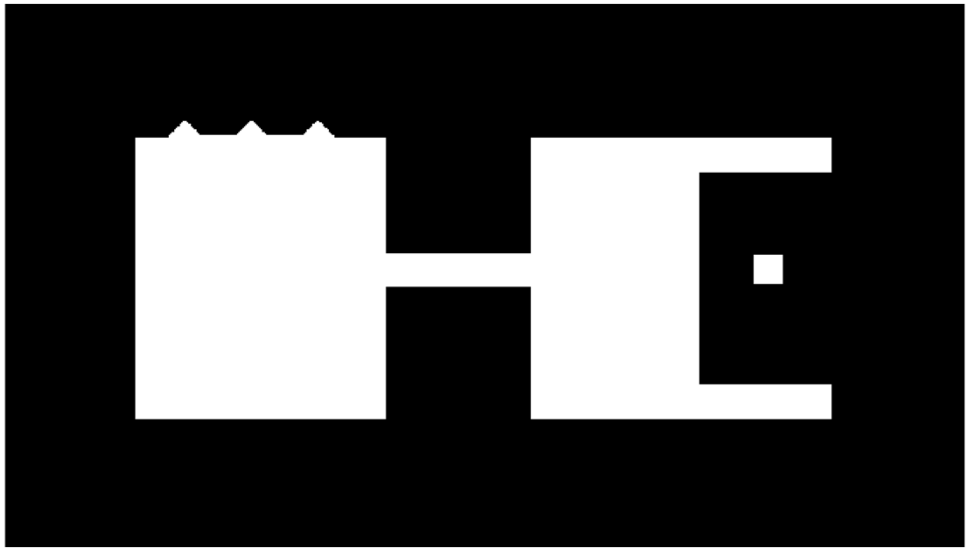
Closing Example

Original Image



se

Image After Closing

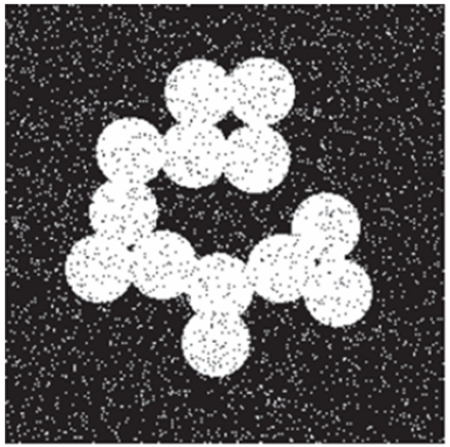


Application of Opening and Closing: Noise Removal

- Morphological filtering

```
>> c=imread('circles.tif');
>> x=rand(size(c));
>> d1=find(x<=0.05);
>> d2=find(x>=0.95);
>> c(d1)=0;
>> c(d2)=1;
>> imshow(c)
```

```
>> cf1=imclose(imopen(c,sq),sq);
>> figure,imshow(cf1)
>> cf2=imclose(imopen(c,cr),cr);
>> figure,imshow(cf2)
```



square SE

cross SE

Hit-or-Miss Transform

- Very useful for finding same shapes in the image
- defined with dilation and erosion
- If *B* is the 3x3 square structuring element,

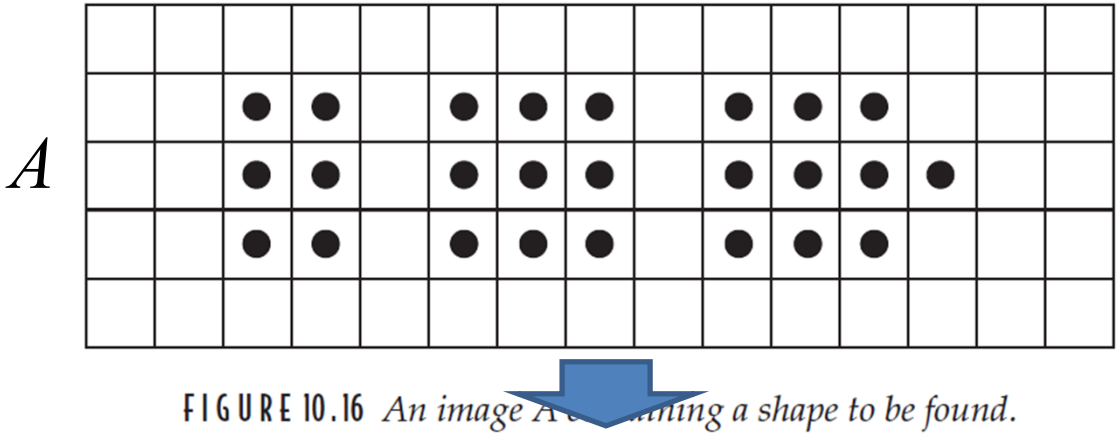
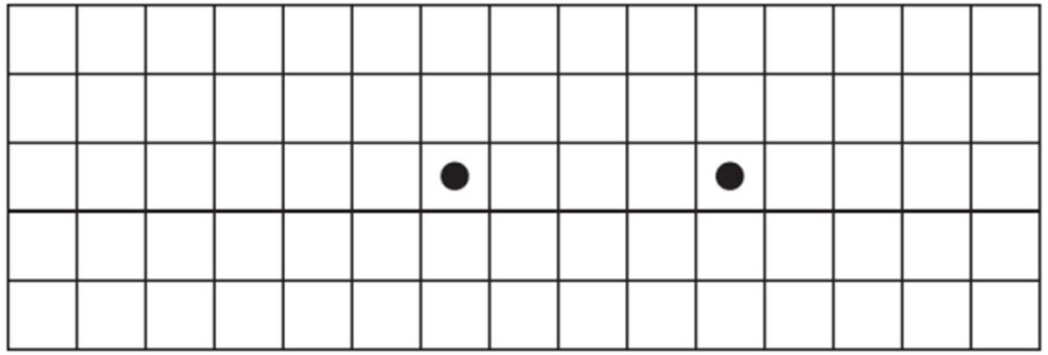
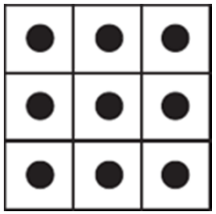


FIGURE 10.16 *An image A containing a shape to be found.*





B
square SE

FIGURE 10.17 *The erosion* $A \ominus B$.

Hit-or-Miss Transform

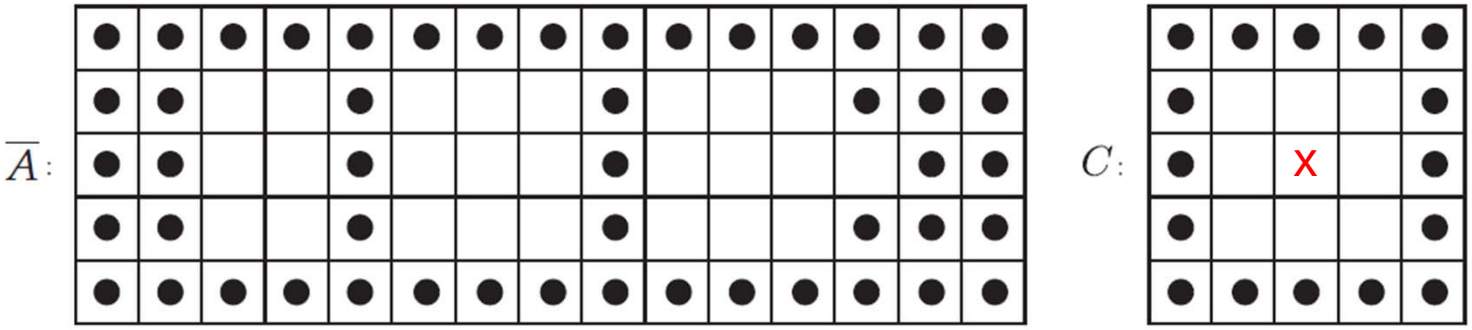
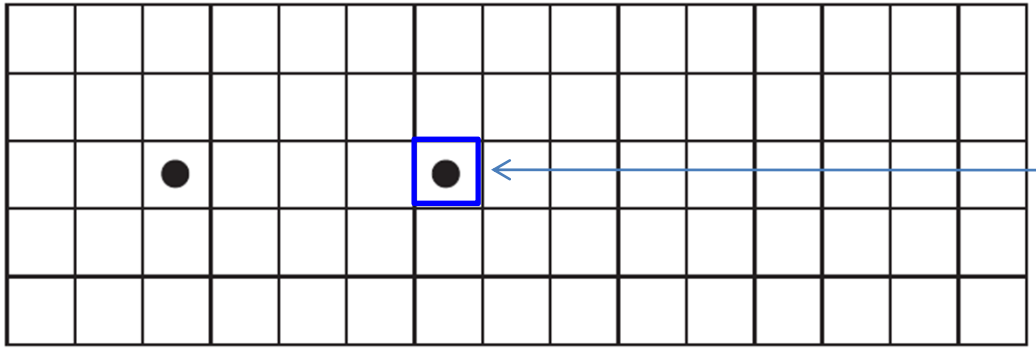


FIGURE 10.18 *The complement and the second structuring element.*



a point of detected object which has the same shape as B

FIGURE 10.19 *The erosion* $\overline{A} \ominus C$.

$$A \circledast B = (A \ominus B_1) \cap (\overline{A} \ominus B_2)$$

In this example, $B_1 = B$, $B_2 = C$.

Hit-or-Miss Transform

- In general, if we are looking for a particular shape in an image, we can design two structuring elements, B1 and B2.
- B1 should be the same shape that we want to find.
- B2 should fit around the shape.

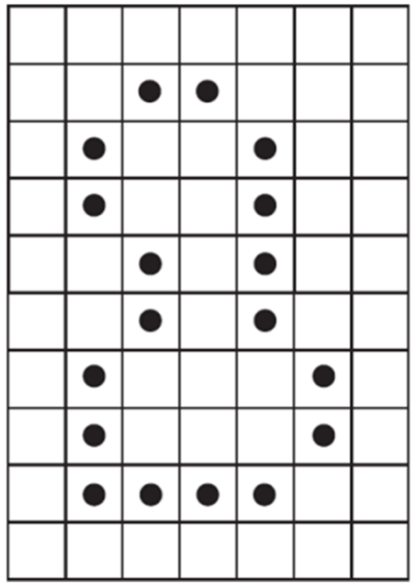
$$A \circledast B = (A \ominus B_1) \cap (\overline{A} \ominus B_2)$$

```
>> b1=ones(1,6);
>> b2=[1 1 1 1 1 1 1 1;1 0 0 0 0 0 0 1; 1 1 1 1 1 1 1 1];
>> tb1=imerode(t,b1);
>> tb2=imerode(~t,b2);
>> hit_or_miss=tb1&tb2;
>> [x,y]=find(hit_or_miss==1)
```

```
>> tb1=imerode(t,b1);
```

Region Filling

- How to fill inside of the boundary ?



- Given a pixel p within the region, we wish to fill up the entire region.

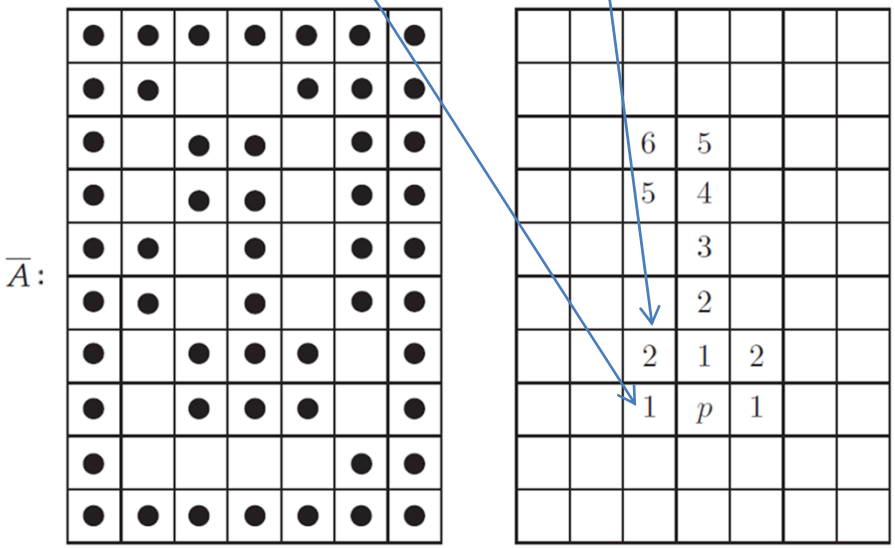
1. Dilate p with the cross-shaped structuring element B.
2. Take an intersection with complement of A.

$$\{p\} = X_0, X_1, X_2, \ldots, X_k = X_{k+1},$$

$$X_n = (X_{n-1} \oplus B) \cap \overline{A}$$

3. Repeat 1 and 2 until the result image is converged. $X_n = X_{n-1}$

$X_0 = \{p\}$, $X_1 = \{p,1\}$, $X_2 = \{p,1,2\}$, ...



Region Filling in Matlab: **`regfill()`**

```
function out=regfill(im,pos,kernel)
% REGFILL(IM,POS,KERNEL) performs region filling of binary
% image IMAGE,with kernel KERNEL, starting at point with
% coordinates given by POS.
% Example:
%           n=imread('nicework.tif');
%           nb=n&~imerode(n,ones(3,3));
%           nr=regfill(nb,[74,52],ones(3,3));
%
current=zeros(size(im));
last=zeros(size(im));
last(pos(1),pos(2))=1;
current=imdilate(last,kernel)&~im;
while any(current(:)~=last(:)),
  last=current;
  current=imdilate(last,kernel)&~im;
end;
out=current;
```

FIGURE 10.24 *A simple program for filling regions.*

Region Filling in Matlab

```
>> n=imread('nicework.tif');
>> imshow(n),pixval on
>> nb=n&~imerode(n,sq);
>> figure,imshow(nb)
>> nf=regfill(nb,[74,52],sq);
>> figure,imshow(nf)
```

FIGURE 10.25 *Region filling.*

Connected Components

- Cross shape SE for 4-connected components
- Square shape SE for 8-connected components

$$X_0 = p$$

$$X_n = (X_{n-1} \oplus B) \cap A, \quad n = 1,2,3,\ldots \quad \text{Until} \quad X_n = X_{n-1}$$

It is not A^c !

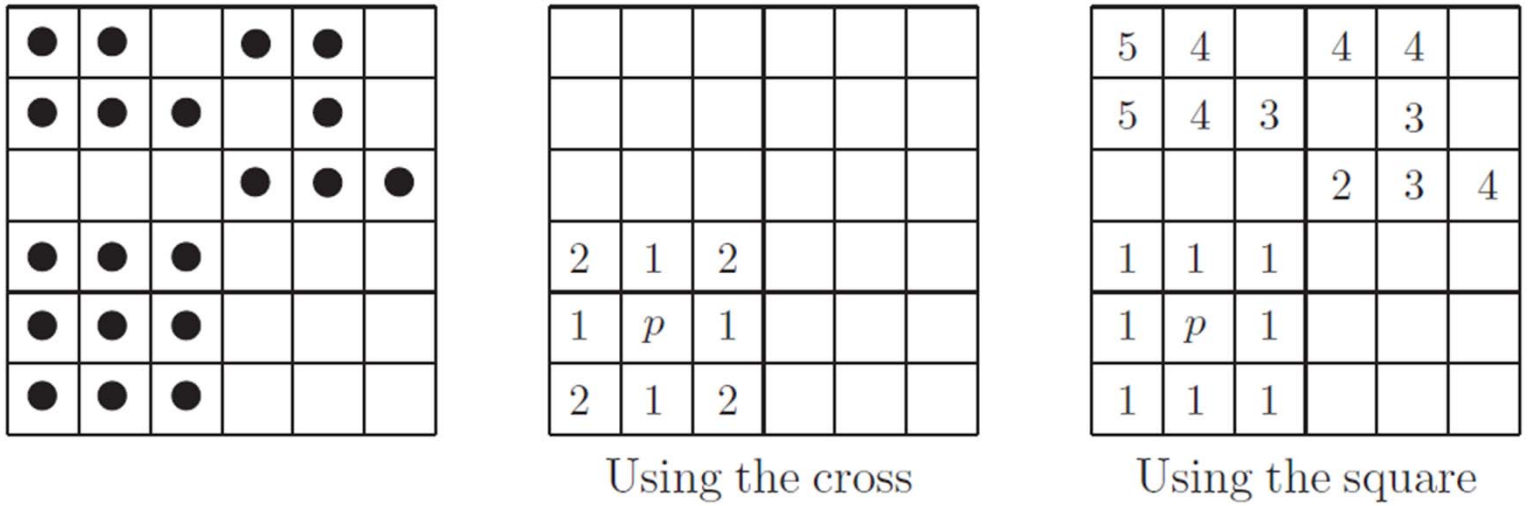


FIGURE 10.23 *Filling connected components.*

Connected Components in Matlab: **components()**

```
function out=components(im,pos,kernel)
% COMPONENTS(IM,POS,KERNEL) produces the connected component
% of binary image IMAGE which nicludes the point with coordinates given
% by POS, using kernel KERNEL.
%
% Example:
%           n=imread('nicework.tif');
%           nc=components(nb,[74,52],ones(3,3));
%
current=zeros(size(im));
last=zeros(size(im));
last(pos(1),pos(2))=1;
current=imdilate(last,kernel)&im;
while any(current(:)~=last(:)),
  last=current;
  current=imdilate(last,kernel)&im;
end;
out=current;
```

FIGURE 10.26 *A simple program for connected components.*

Connected Components in Matlab

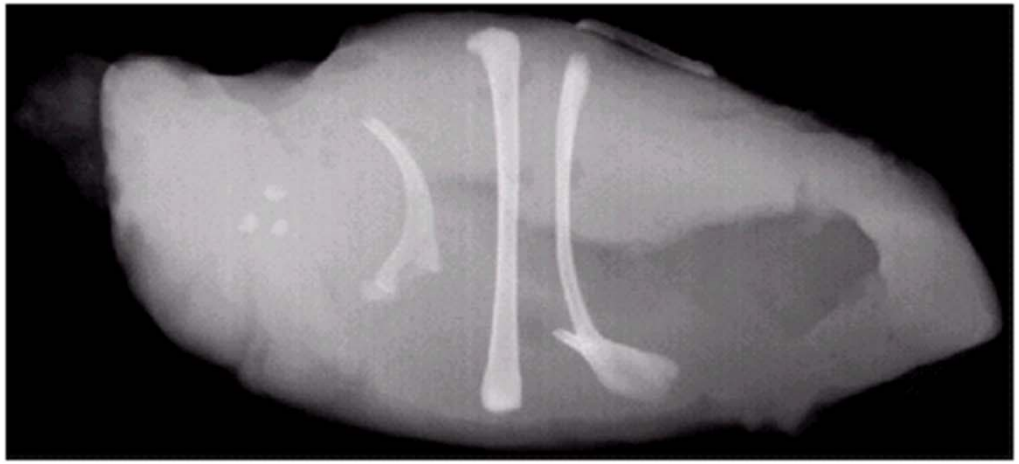
```
>> sq2=ones(11,11);
>> nc=components(n,[57,97],sq);
>> imshow(nc)
>> nc2=components(n,[57,97],sq2);
>> figure,imshow(nc2)
```

(a) 3x3 square SE (b) 11x11 square SE

FIGURE 10.27 *Connected components.*

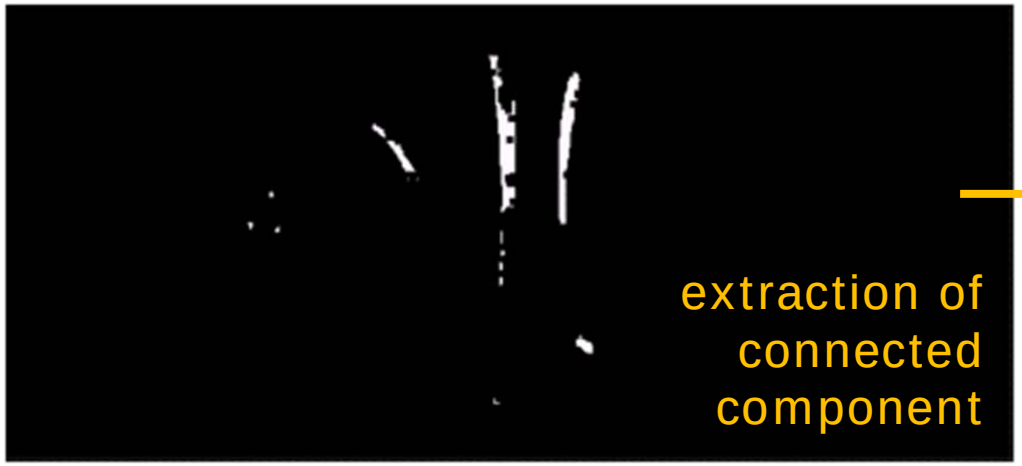
Connected Components Example

original image



thresholding

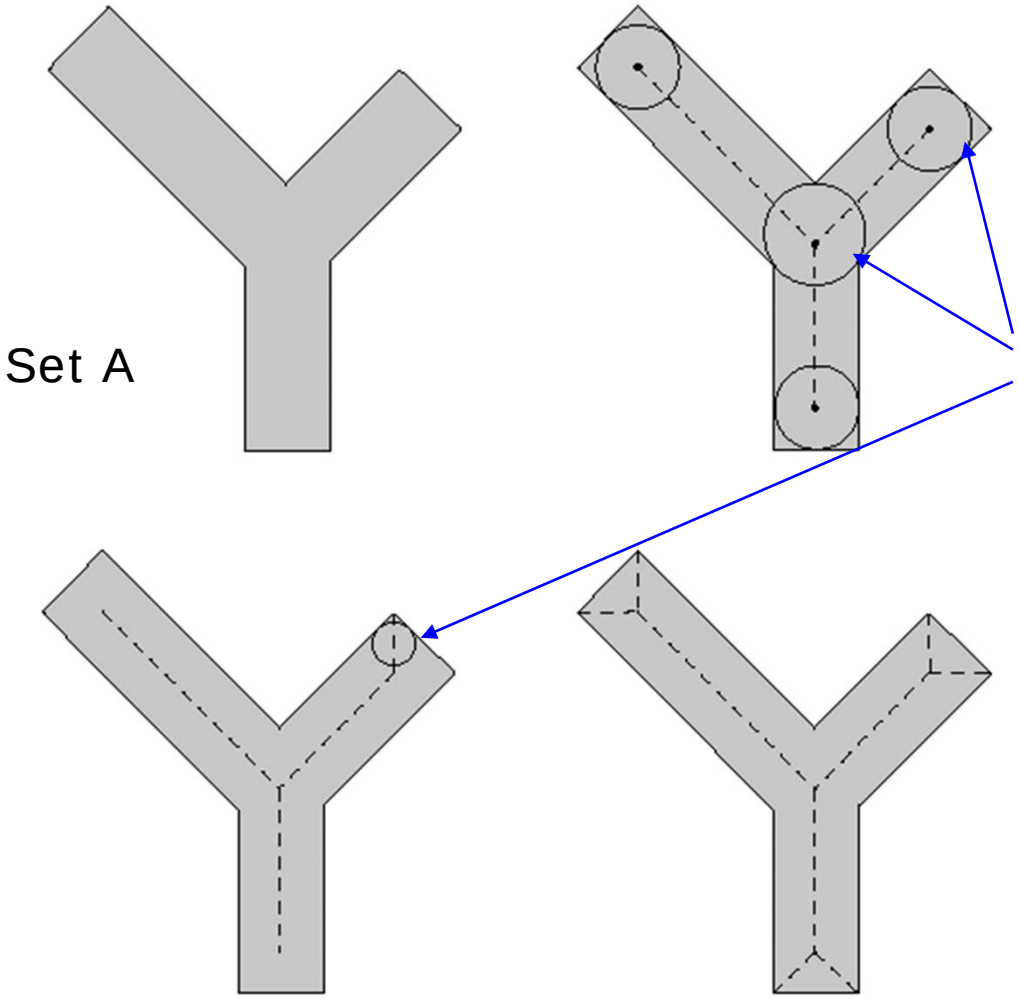
erosion with a 5x5 SE



extraction of connected component

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

Skeletons



How to define a Skeletons?

Maximum disk

1. The largest disk centered at a pixel

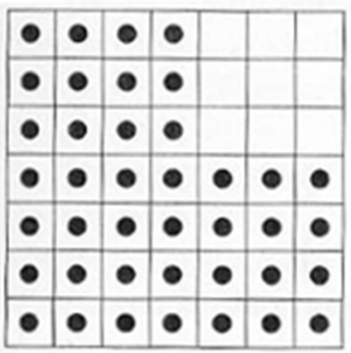
2. Touch the boundary of A at two or more places

Skeleton

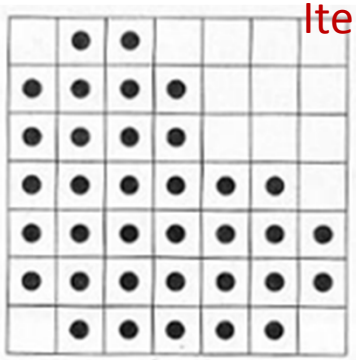
1. iterative erosions followed by an opening
2. eroded image - opened image

Erosions	Openings	Set differences
A	$A \circ B$	$A - (A \circ B)$
$A \ominus B$	$(A \ominus B) \circ B$	$(A \ominus B) - ((A \ominus B) \circ B)$
$A \ominus 2B$	$(A \ominus 2B) \circ B$	$(A \ominus 2B) - ((A \ominus 2B) \circ B)$
$A \ominus 3B$	$(A \ominus 3B) \circ B$	$(A \ominus 3B) - ((A \ominus 3B) \circ B)$
$\vdots$	$\vdots$	$\vdots$
$A \ominus kB$	$(A \ominus kB) \circ B$	$(A \ominus kB) - ((A \ominus kB) \circ B)$

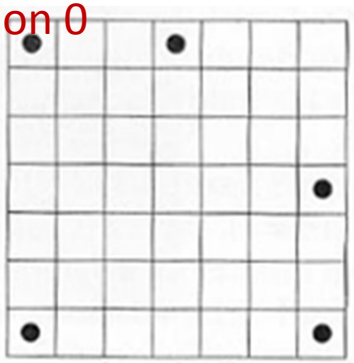
Iteration 0



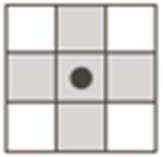
A



$A \circ B$



$A-(A \circ B)$

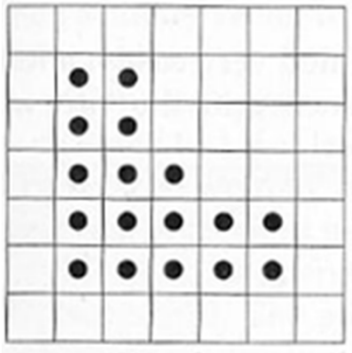


B

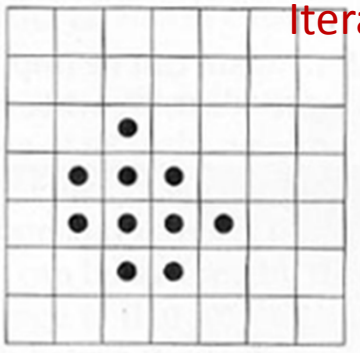
$$S_k(A) = (A \Theta kB) - (A \Theta kB) \circ B$$

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

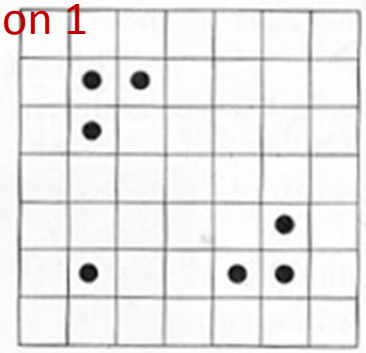
Iteration 1



$A \ominus B$

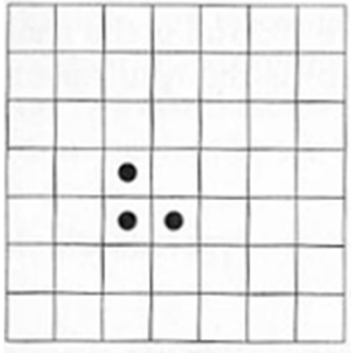


$(A \ominus B) \circ B$

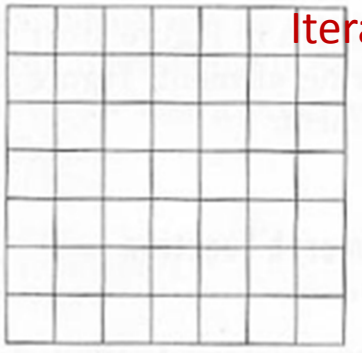


$(A \ominus B) - ((A \ominus B) \circ B)$

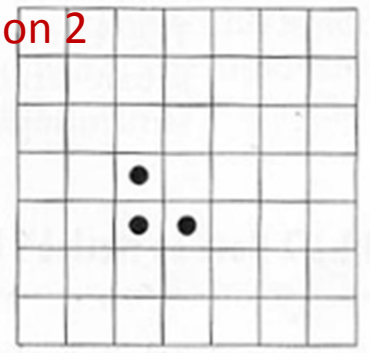
Iteration 2



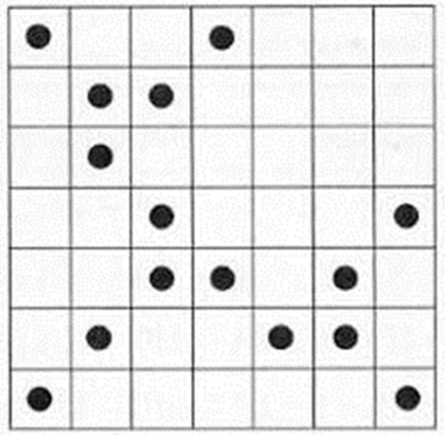
$A \ominus 2B$



$(A \ominus 2B) \circ B$



$(A \ominus 2B) - ((A \ominus 2B) \circ B)$



Final skeleton, S(A)

Skeleton in Matlab

```
function skel = imskel(image,str)
% IMSKEL(IMAGE,STR) - Calculates the skeleton of binary image IMAGE using
% structuring element STR.  This function uses Lantejoul's algorithm.
%
skel=zeros(size(image));
e=image;
while (any(e(:))),
    o=imopen(e,str);
    skel=skel | (e&~o);
    e=imerode(e,str);
end
```

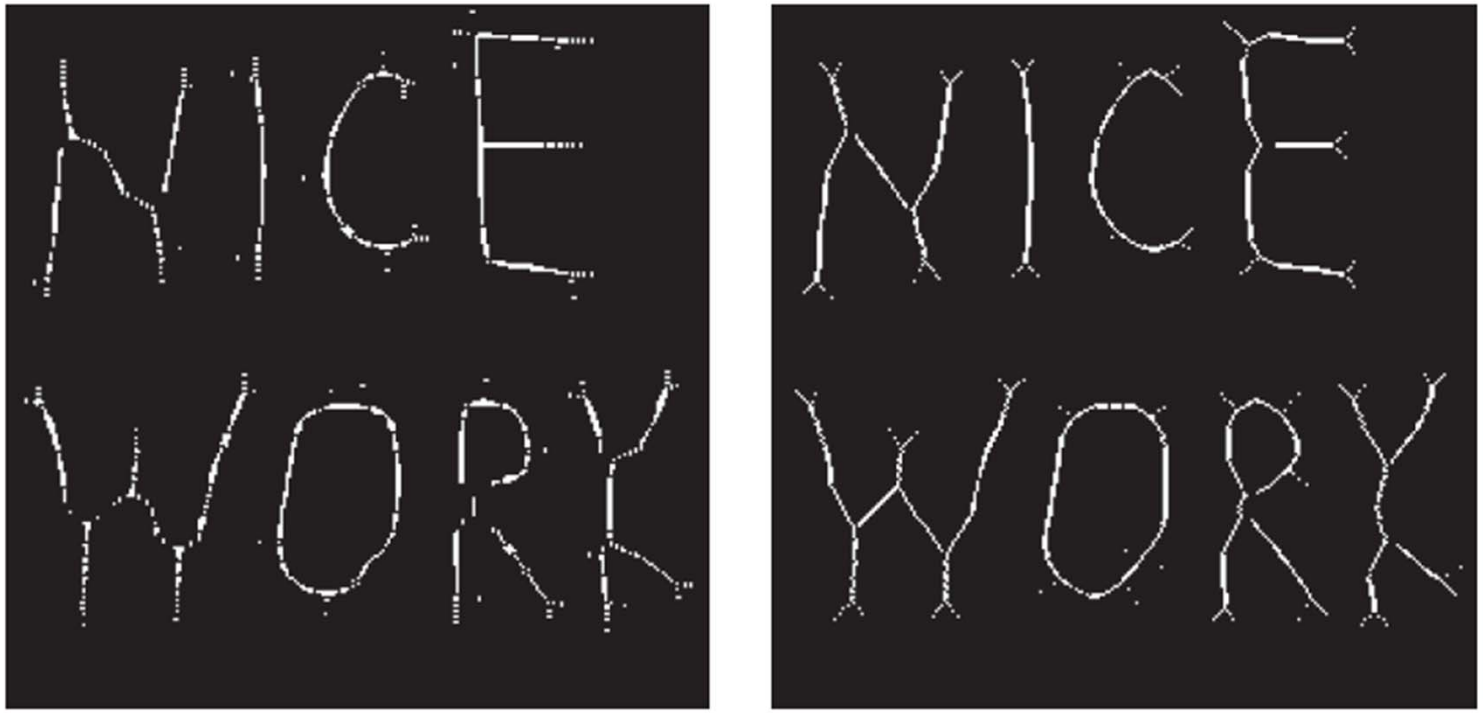
FIGURE 10.30 *A simple program for computing skeletons.*

```
>> nk=imskel(n,sq);
>> imshow(nk)
>> nk2=imskel(n,cr);
>> figure,imshow(nk2)
```

FIGURE 10.31

Square-structuring element

Cross-structuring element



(a) (b)

FIGURE 10.31 *Skeletonization of a binary image.*

A Note on MATLAB's bwmorph Function

- Based on **lookup tables** (ch11)
  - ✓ Consider the 3×3 neighborhood of a pixel.
  - ✓ Since each pixel in the neighborhood can have only two values, there are $2^9 = 512$ different possible neighborhoods.
  - ✓ Define a morphological operation to be a function that maps these neighborhoods to the values 0 and 1.
  - ✓ Each possible neighborhood state can be associated with a numeric value from 0 (all pixels have value 0) to 511 (all pixels have value 1).
  - ✓ The lookup table is then a binary vector of length 512. Its kth element is the value of the function for state k.

- Many other operations can be defined by this method (see the help file for `bwmorph`)

Grayscale Morphology

- **Another way for Erosion**
  - ✓ 1. Find the 3×3 neighborhood N_p of p
  - ✓ 2. Compute the matrix $\{N_p - B\}$
  - ✓ 3. Find the minimum value in the matrix $\{N_p - B\}$

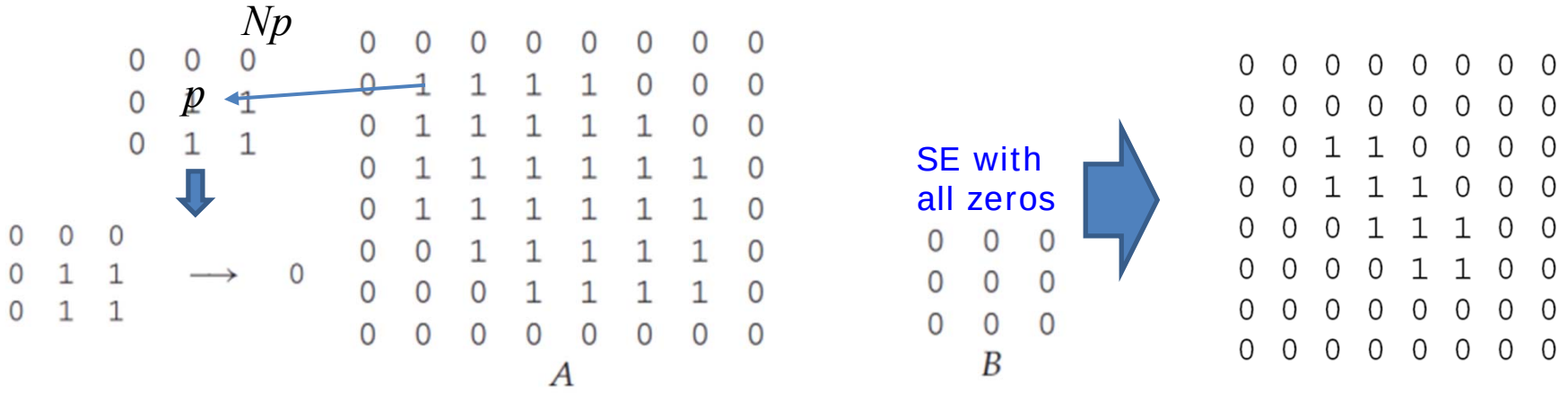


FIGURE 10.32 *An example for erosion.*

Grayscale Morphology

- **Another way for dilation**
  - ✓ 1. Find the 3×3 neighborhood N_p of p
  - ✓ 2. Compute the matrix $N_p + B$
  - ✓ 3. Find the maximum of that result

- **Summary**

$$(A \ominus B)(x,y) = \min\{A(x+s, y+t) - B(s,t), (s,t) \in D_B\},$$
$$(A \oplus B)(x,y) = \max\{A(x+s, y+t) + B(s,t), (s,t) \in D_B\}.$$

Grayscale Morphology Example

x \ y	1	2	3	4	5
1	10	20	20	20	30
2	20	30	30	40	50
3	20	30	30	50	60
4	20	40	50	50	60
5	30	50	60	60	70

A

s \ t	−1	0	1
−1	1	2	3
0	4	5	6
1	7	8	9

B

FIGURE 10.33 *An example for grayscale erosion and dilation.*

$$A \ominus B = \begin{matrix} 5 & 6 & 14 & 15 & 16 \\ 8 & 9 & 17 & 18 & 19 \\ 12 & 13 & 25 & 26 & 39 \\ 15 & 16 & 28 & 29 & 46 \\ 18 & 19 & 39 & 48 & 49 \end{matrix}$$

$$A \oplus B = \begin{matrix} 39 & 39 & 49 & 59 & 58 \\ 39 & 39 & 59 & 69 & 68 \\ 49 & 59 & 59 & 69 & 68 \\ 59 & 69 & 69 & 79 & 78 \\ 56 & 66 & 66 & 76 & 75 \end{matrix}$$

Grayscale Morphology in Matlab

- The `arbitrary` parameter of `strel` allows us to create a structuring element containing any values we like.
- `ones(3,3)` provides the neighborhood.
- Matrix as third parameter provides the values of 3x3 SE.

```
>> str=strel('arbitrary',ones(3,3),[1 2 3;4 5 6;7 8 9])

str =

Nonflat STREL object containing 9 neighbors.

Neighborhood:
     1     1     1
     1     1     1
     1     1     1


Height:
     1     2     3
     4     5     6
     7     8     9
```

```
>> A=[10 20 20 20 30;20 30 30 40 50;20 30 30 50 60;20 40 50 50 60;30 50 60 60 70];
>> imerode(A,str)

ans =

     5     6    14    15    16
     8     9    17    18    19
    12    13    25    26    39
    15    16    28    29    46
    18    19    39    48    49
```

Grayscale Morphology in Matlab

```
>> str2=strel('arbitrary',ones(3,3),[9 8 7;6 5 4;3 2 1])
str2 =
Nonflat STREL object containing 9 neighbors.

Neighborhood:
     1     1     1
     1     1     1
     1     1     1

Height:
     9     8     7
     6     5     4
     3     2     1

>> imdilate(A,str2)

ans =

    39    39    49    59    58
    39    39    59    69    68
    49    59    59    69    68
    59    69    69    79    78
    56    66    66    76    75
```

Grayscale Morphology in Matlab: Erosion and Dilation

```
>> c=imread('caribou.tif');
>> str=strel('square',5)
```

```
>> cd=imdilate(c,str);
>> ce=imerode(c,str);
>> imshow(cd),figure,imshow(ce)
```

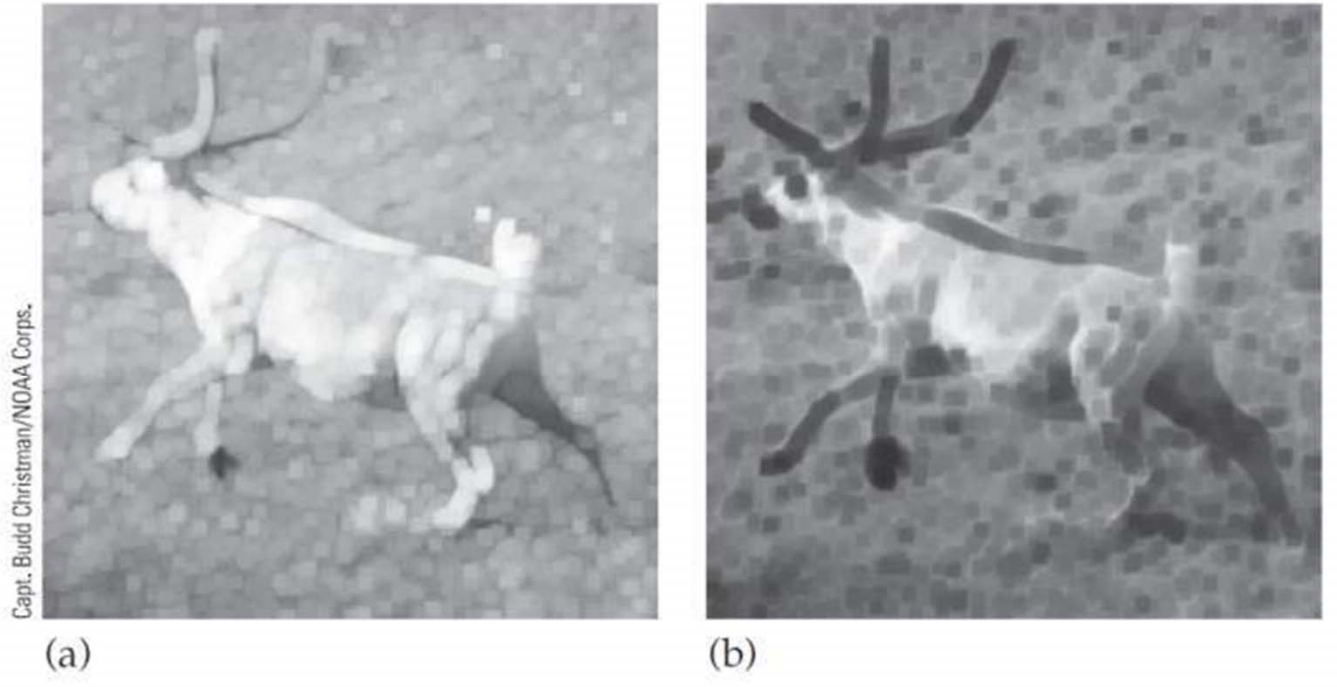


FIGURE 10.34 *Morphology. (a) Dilation. (b) Erosion.*

Grayscale Morphology: Opening and Closing

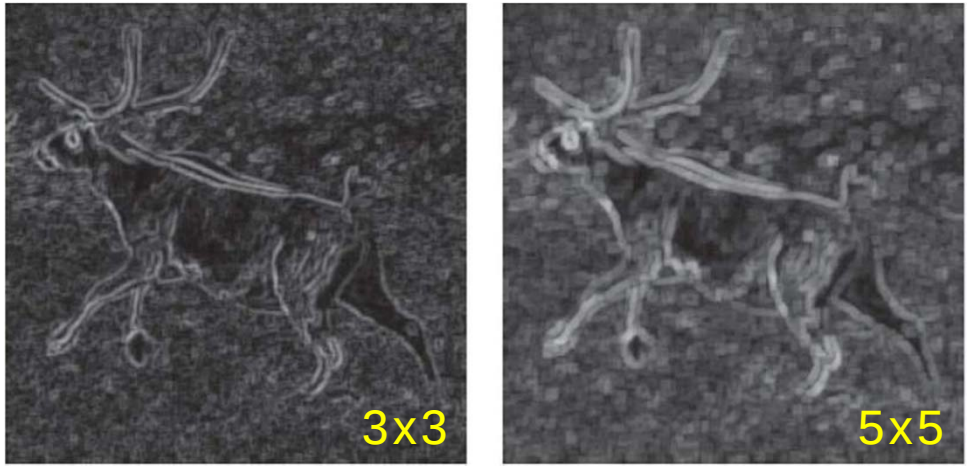
```
>> co=imopen(c,str);
>> cc=imclose(c,str);
>> imshow(co),figure,imshow(cc)
```

FIGURE 10.35 *Grayscale opening and closing. (a) Opening. (b) Closing.*

Applications of Grayscale Morphology: Edge Detection

- morphological gradient in grayscale $(A \oplus B) - (A \ominus B)$
- 3x3 and 5x5 square SE

```
>> str1=strel('square',3);
>> str2=strel('square',5);
>> ce1=imerode(c,str1);
>> ce2=imerode(c,str2);
>> cd1=imdilate(c,str1);
>> cd2=imdilate(c,str2);
>> cg1=imsubtract(cd1,ce1);
>> cg2=imsubtract(cd2,ce2);
>> imshow(cg1),figure,imshow(cg2)
```



Applications of Grayscale Morphology: Noise Removal

- opening followed by a closing in grayscale
- In general, it does not performed well on Gaussian noise

```
>> cn=imnoise(c,'salt & pepper');
>> cf=imclose(imopen(cn,str),str);
>> imshow(cn),figure,imshow(cf)
```

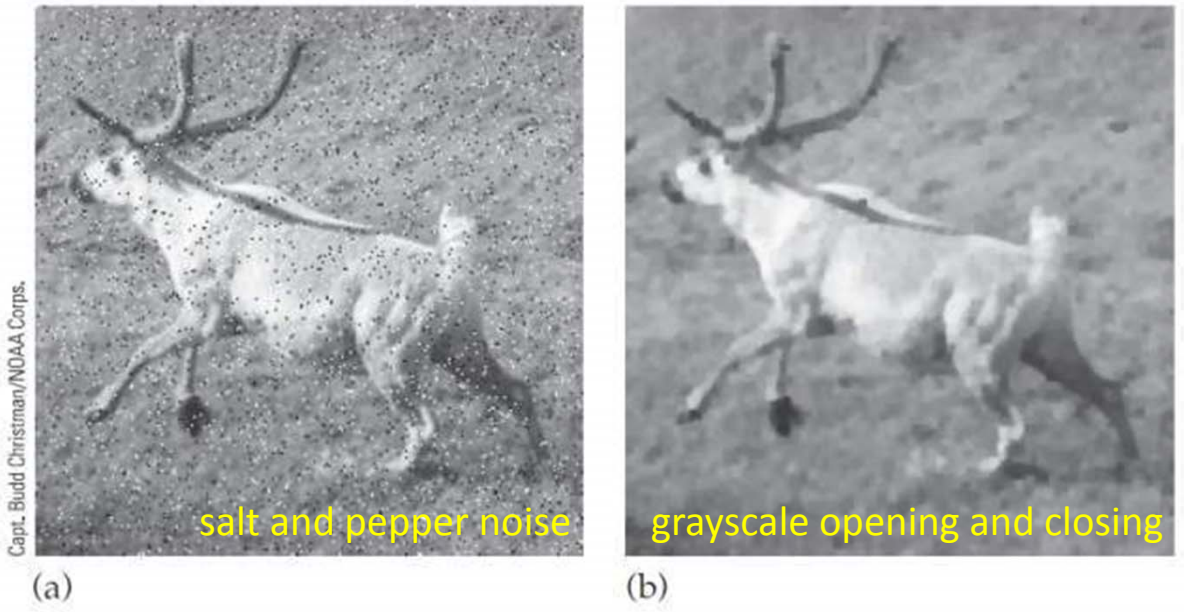


FIGURE 10.37 *Use of morphological filtering to remove noise.*

Summary

- **Chap 9. Image Segmentation**
  - Single & double thresholding
  - How to determine the threshold value
  - Adaptive thresholding
  - Edge detection: 1st and 2nd derivatives
  - Canny edge detector
  - Hough Transform
- **Chap 10. Mathematical Morphology**
  - Erosion & dilation -> boundary detection
  - Opening & closing -> noise removal
  - Hit-or-miss transform -> shape detection
  - Binary applications: region filling, connected components, skeletons
  - Grayscale morphology -> edge detection, noise removal