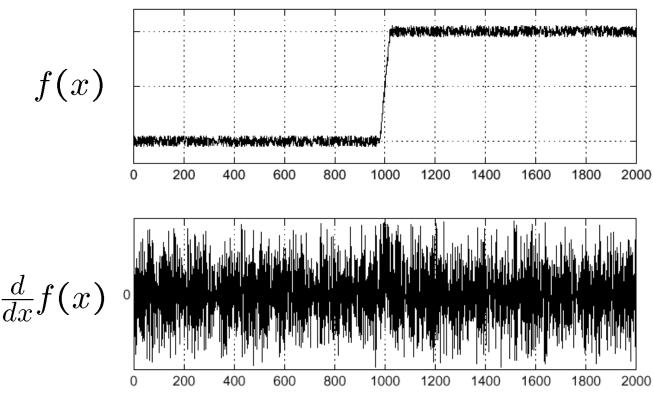
# Three Classes of Image Processing Operations by Direct Manipulation of Gray Levels

- Any image-processing operation transforms the gray values of the pixels.
- Image-processing operations may be divided into three classes based on the information required to perform the transformation.
- From the simplest to the most complex, they are as follows:
  - Point operations : chap.4
  - Neighborhood processing (spatial filter): chap.5
  - Geometrical transforms: chap.6



# Combined Filtering: Effects of Noise

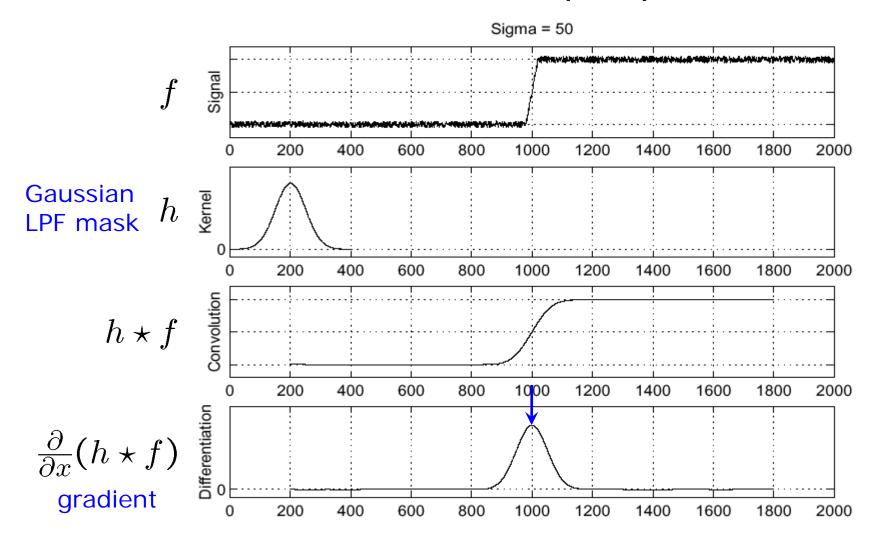
- Consider a single row or column of the image
  - Plotting intensity as a function of position gives a signal



Where is the edge??

Laplace operator may detect edges as well as noise.

# Solution: Smooth(LPF) First



Where is the edge?

Look for peaks in  $\frac{\partial}{\partial x}(h \star f)$ 

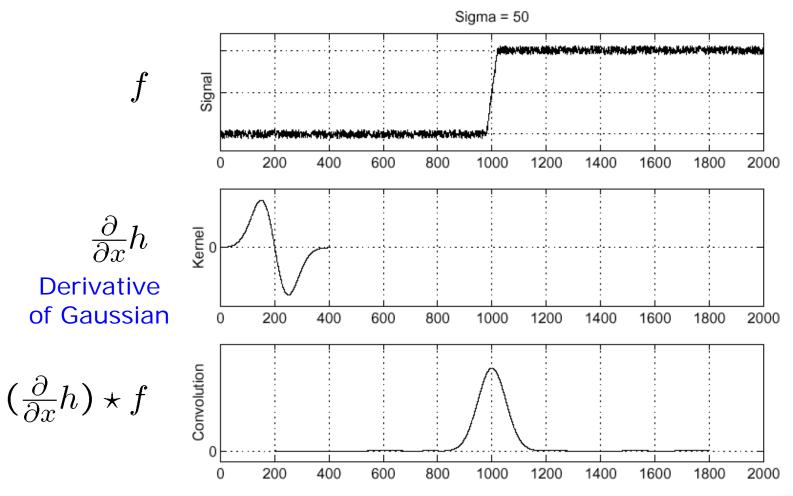
$$\frac{\partial}{\partial x}(h\star f)$$

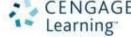


# Derivative of Gaussian (DoG)

$$\frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x}h) \star f$$

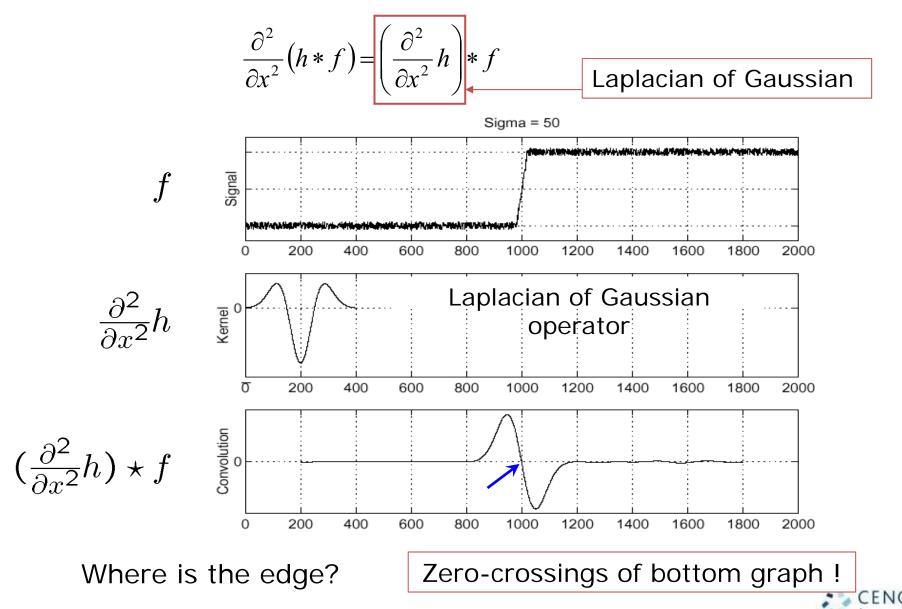
...saves us one operation.



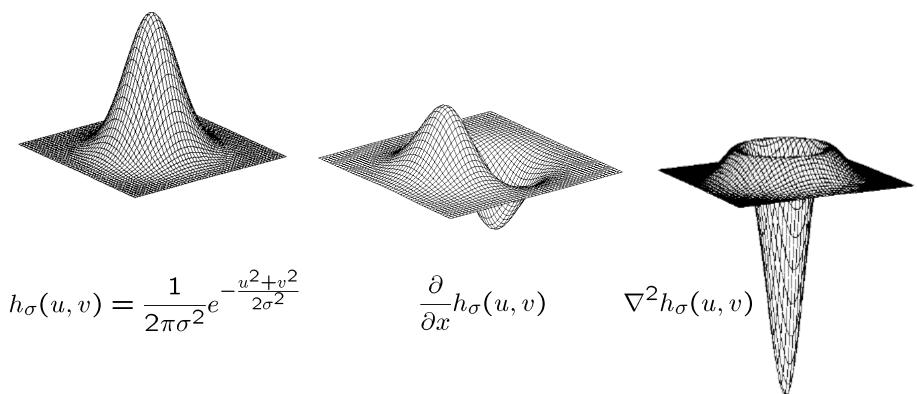


courtesy of S. Narasimhan at CMU

# Laplacian of Gaussian (LoG)



# 2D Gaussian Edge Operators



Gaussian

Derivative of Gaussian (DoG)

Laplacian of Gaussian Mexican Hat (Sombrero)

•  $\nabla^2$  is the **Laplacian** operator:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$



# High-Pass Filters in Matlab

```
>> f=fspecial('laplacian')
f =
                   0.1667
   0.1667
          0.6667
   0.6667 -3.3333
                   0.6667
   0.1667 0.6667
                   0.1667
>> cf=filter2(f,c);
>> imshow(cf/100)
>> f1=fspecial('log')
f1 =
   0.0448
           0.0468 0.0564
                                      0.0448
                              0.0468
                                      0.0468
   0.0468
          0.3167 0.7146
                              0.3167
   0.0564 0.7146 -4.9048
                              0.7146
                                      0.0564
   0.0468 0.3167 0.7146
                              0.3167
                                      0.0468
   0.0448 0.0468
                   0.0564
                              0.0468
                                      0.0448
>> cf1=filter2(f1,c);
>> figure, imshow(cf1/100)
```



# FIGURE 5.5

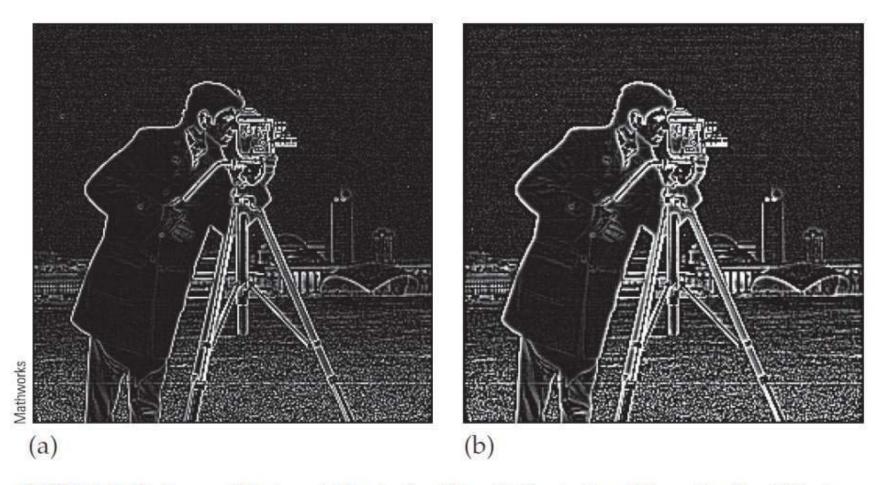


FIGURE 5.5 High-pass filtering. (a) Laplacian filter. (b) Laplacian of Gaussian (log) filtering.



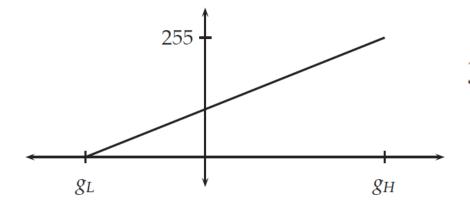
#### VALUES OUTSIDE THE RANGE 0–255

✓ Make negative values positive

method 1. Clip values

$$y = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } 0 \le x \le 255 \\ 255 & \text{if } x > 255 \end{cases}$$

method 2. 0-255 Scaling transformation (uint8)



$$y = 255 \frac{x - g_L}{g_H - g_L}$$



#### VALUES OUTSIDE THE RANGE 0–255

```
>> f2=[1 -2 1;-2 4 -2;1 -2 1];
>> cf2=filter2(f2,c);

>> figure, imshow(mat2gray(cf2));

O-1 Scaling transformation (double)
>> maxcf2=max(cf2(:));
>> mincf2=min(cf2(:));
>> cf2g=(cf2-mincf2)/(maxcf2-mncf2);

>> figure, imshow(cf2/60)
```



## FIGURE 5.6

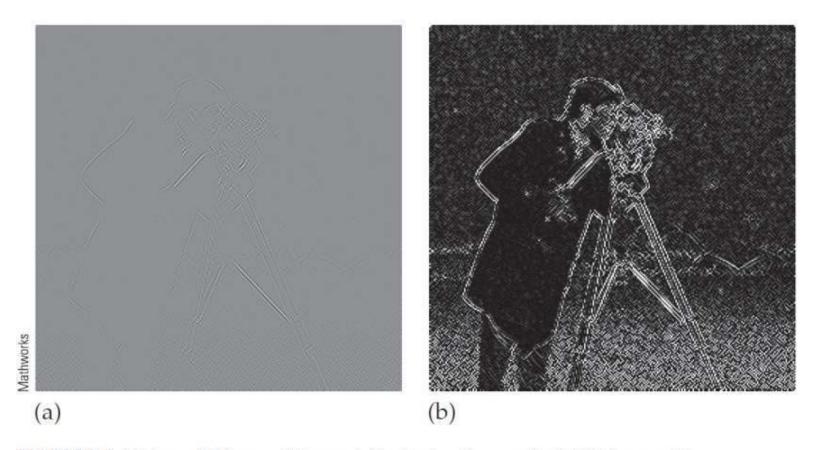


FIGURE 5.6 Using a high-pass filter and displaying the result. (a) Using mat2gray. (b) Dividing by a constant.



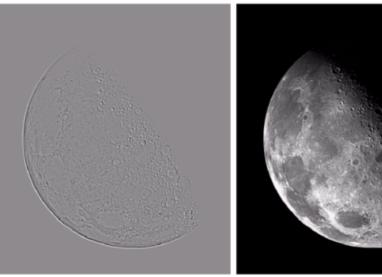
# HPF for Image Enhancement

(a) Moon



(b)
After
Laplacian
HPF
(2<sup>nd</sup> derivative)
: Neg. value
was set to 0.

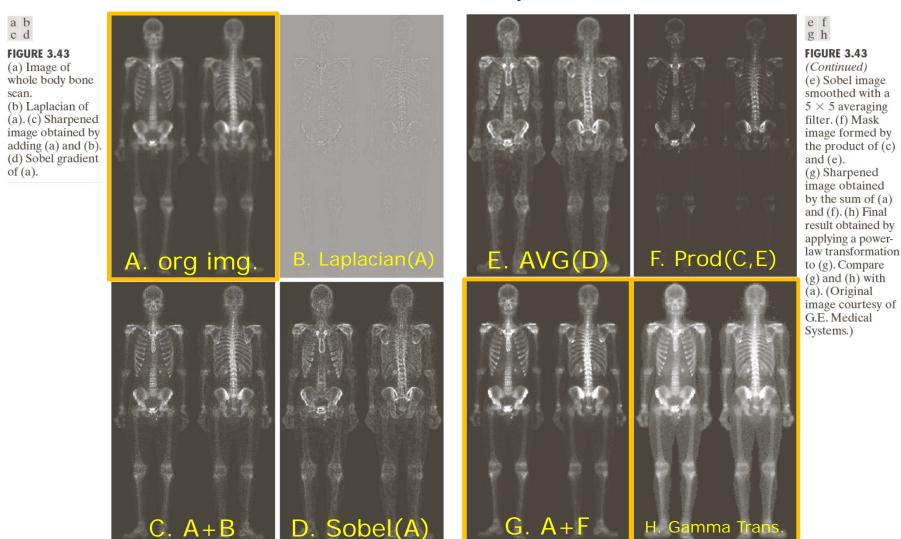
(C) Intensity scaling of (b)



(d) = (a)-(b) with negative center in the mask



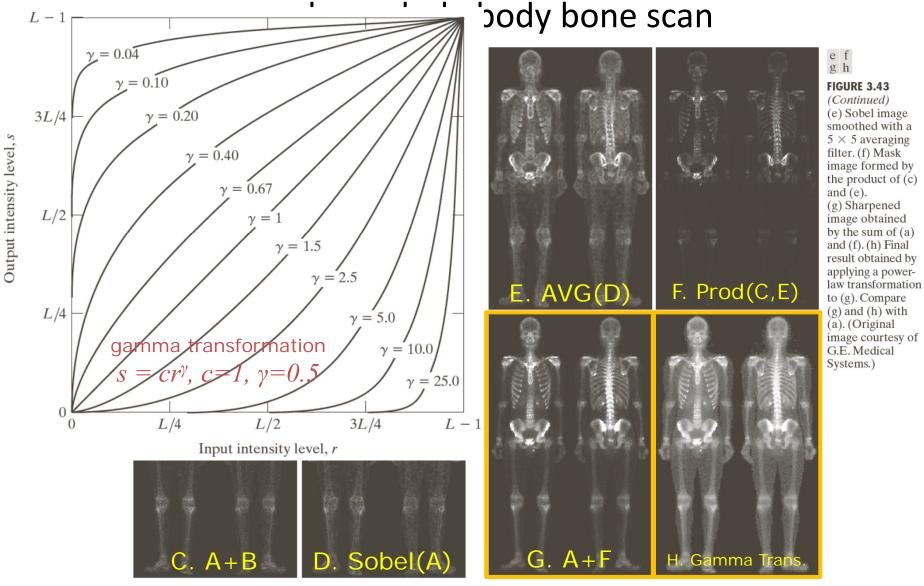
# Combined Spatial Enhancement : nuclear whole body bone scan



Sharpen the image and bring out more of the skeletal detail.

1) Laplacian to highlight fine detail 2) gradient to enhance the prominent edges 3) increase the dynamic range of intensity (3.2.3 gamma trans.)

# Combined Spatial Enhancement



Sharpen the image and bring out more of the skeletal detail.

1) Laplacian to highlight fine detail 2) gradient to enhance the prominent edges 3) increase the dynamic range of intensity (3.2.3 gamma trans.)

# 5.6 Edge Sharpening

## Unsharp Masking

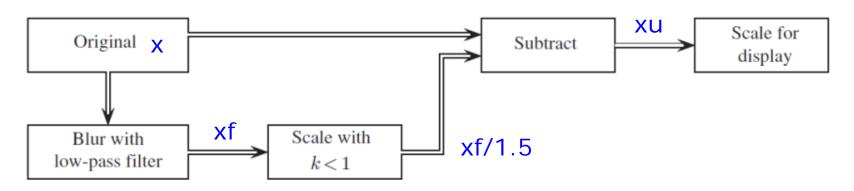
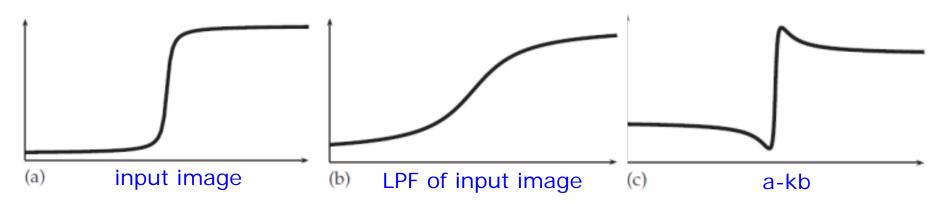


FIGURE 5.10 Schema for unsharp masking.

```
>> f=fspecial('average');
>> xf=filter2(f,x);
>> xu=double(x)-xf/1.5
>> imshow(xu/70)
```



# **Unsharp Masking**



**FIGURE 5.12** Unsharp masking. (a) Pixel values over an edge. (b) The edge blurred. (c) (a) - k(b).





Learning

# **Unsharp Masking**

• The unsharp option of fspecial produces such filters

$$\frac{1}{\alpha+1} \begin{bmatrix} -\alpha & \alpha-1 & -\alpha \\ \alpha-1 & \alpha+5 & \alpha-1 \\ -\alpha & \alpha-1 & -\alpha \end{bmatrix}$$

```
>> p=imread('pelicans.tif');
\frac{1}{\alpha+1} \begin{vmatrix} -\alpha & \alpha-1 & -\alpha \\ \alpha-1 & \alpha+5 & \alpha-1 \\ -\alpha & \alpha-1 & -\alpha \end{vmatrix} >> u=fspecial('unsharp', 0.5); \alpha = 0.5 \\ >> pu=filter2(u,p); \\ >> imshow(p), figure, imshow(pu/255)
```





CENGAGE Learning"

(a)

# High-Boost Filtering

 Allied to unsharp masking filters are the highboost filters

high boost = 
$$A(original) - (low pass)$$

- $\checkmark$  where A is an amplification factor.
- ✓ If A = 1, then the high-boost filter becomes an ordinary high-pass filter.



# **High-Boost Filtering**

```
>> id=[0 0 0;0 1 0;0 0 0];
>> f=fspecial('average');
>> hb1=3*id-2*f
hb1 =
  -0.2222
          -0.2222 -0.2222
  -0.2222 2.7778 -0.2222
  -0.2222 -0.2222 -0.2222
>> hb2=1.25*id-0.25*f
hb2 =
          -0.0278 -0.0278
  -0.0278
  -0.0278 1.2222 -0.0278
  -0.0278
          -0.0278
                   -0.0278
```

- >> x1=filter2(hb1, x); >> imshow(x1/255)
- - >> x2=filter2(hb2, x); >> imshow(x2/255)



Learning"

#### Nonlinear Filters

#### Maximum and minimum filter

```
>> cmax=nlfilter(c,[3,3],'max(x(:))');
>> cmin=nlfilter(c,[3,3],'min(x(:))');
```





FIGURE 5.15 Using nonlinear filters. (a) Using a maximum filter. (b) Using a minimum filter.

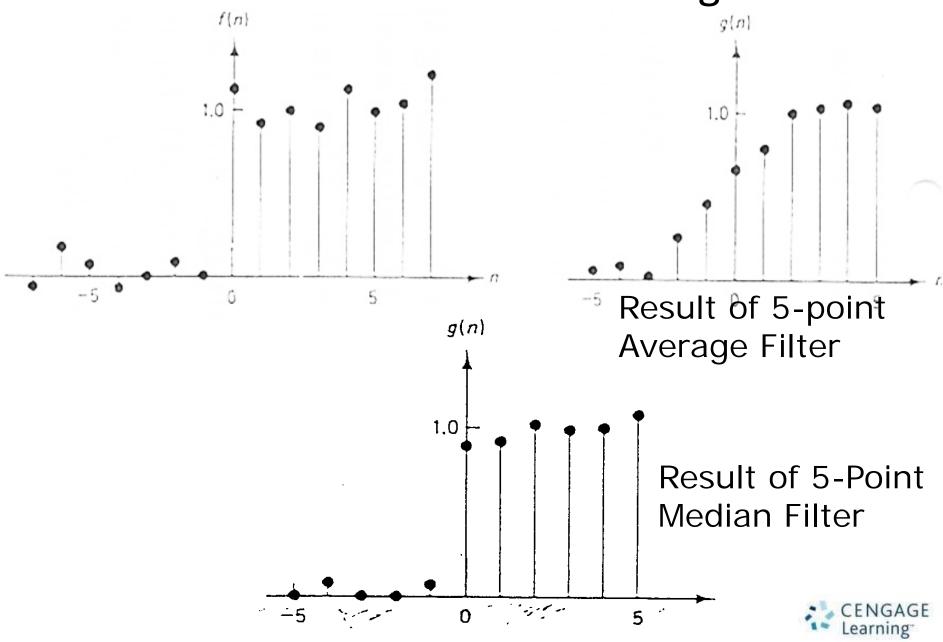


#### Non-linear LPF: Median Filter

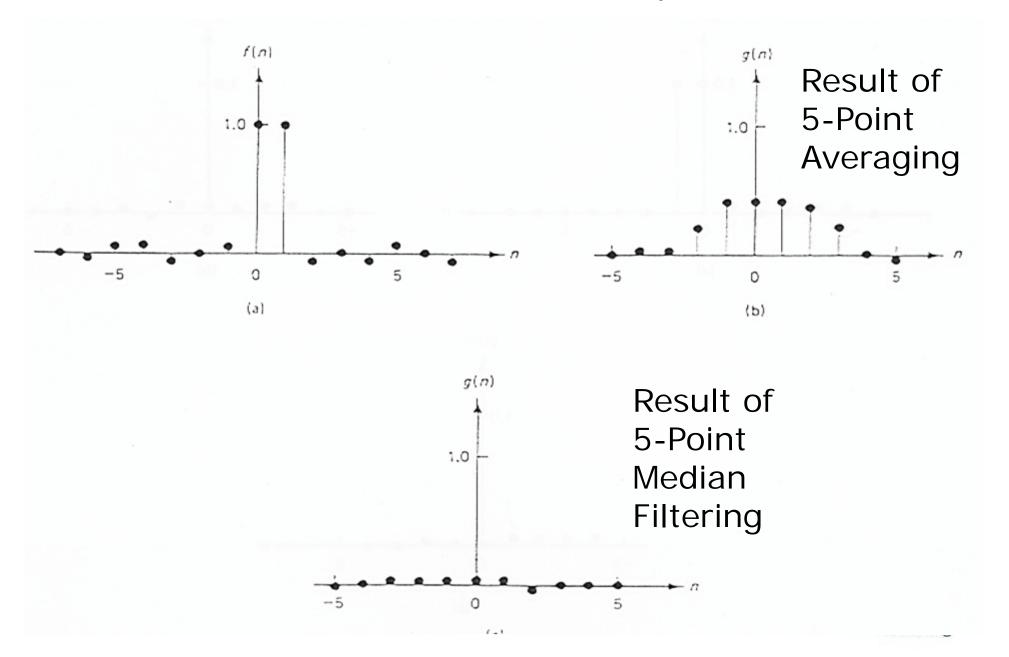
- LPF: blurred while noise is removed to some extent
  - -> solution : median filtering
  - -> avoids blurring in noise removing
- ex1: Average and median value of [ 1, 2, 3, 4, 100] ?
- ex2: median value of [10,20,20,20,15,20,20,25,100]?
- nonlinear process
- especially useful for reducing impulse or salt-and-pepper noise.
- Median filtering: with a sliding window passing through the image, choose the median (middle) intensity value -> nonlinear.
- It preserves **edges**.



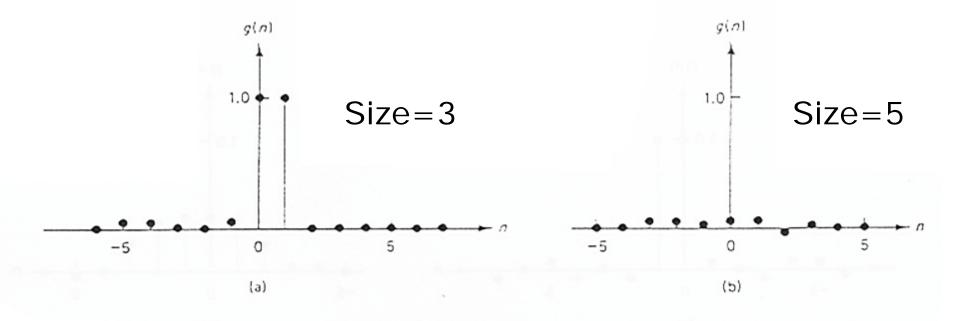
# Median Filters Preserve Edges

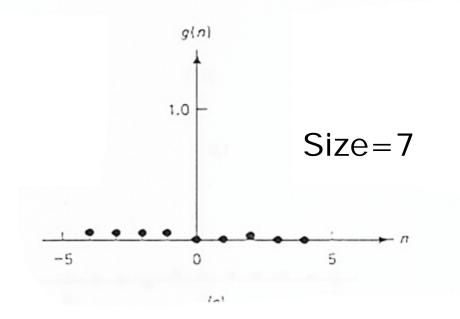


# Median Filters Remove Impulse Noise



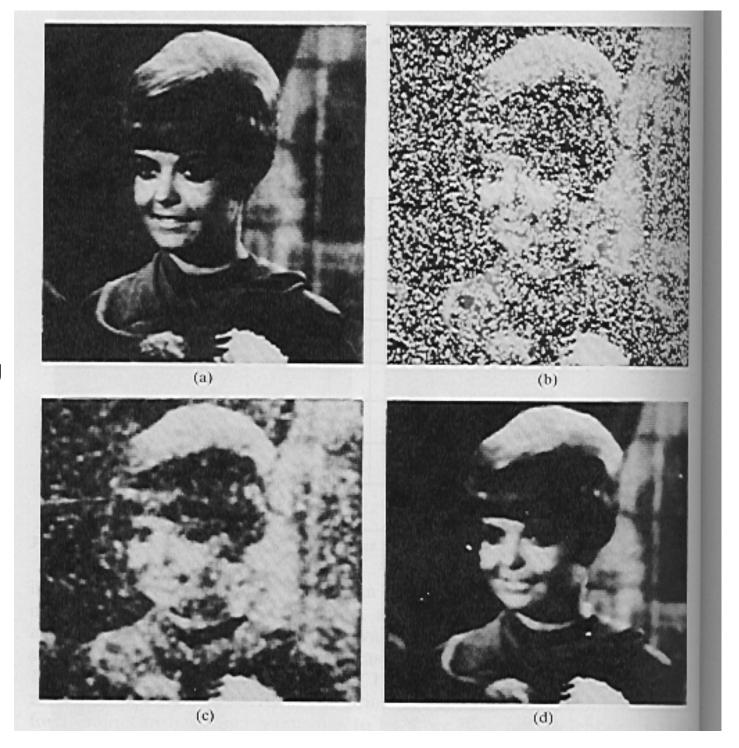
#### Effect of Median Filter Window Sizes





# Removing Impulse Noise

- (a) original
- (b) impulse noise
- (c) 5x5 averaging
- (d) 5x5 median

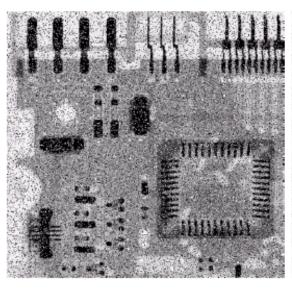


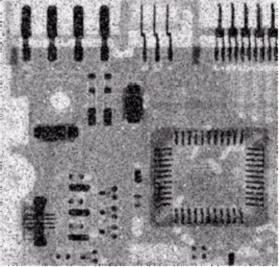
# Removing Impulse Noise

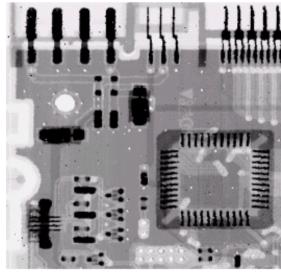
x-ray image salt-and-pepper noise

3x3 averaging

3x3 median







a b c

**FIGURE 3.37** (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3 × 3 averaging mask. (c) Noise reduction with a 3 × 3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)



# Region of Interest Processing

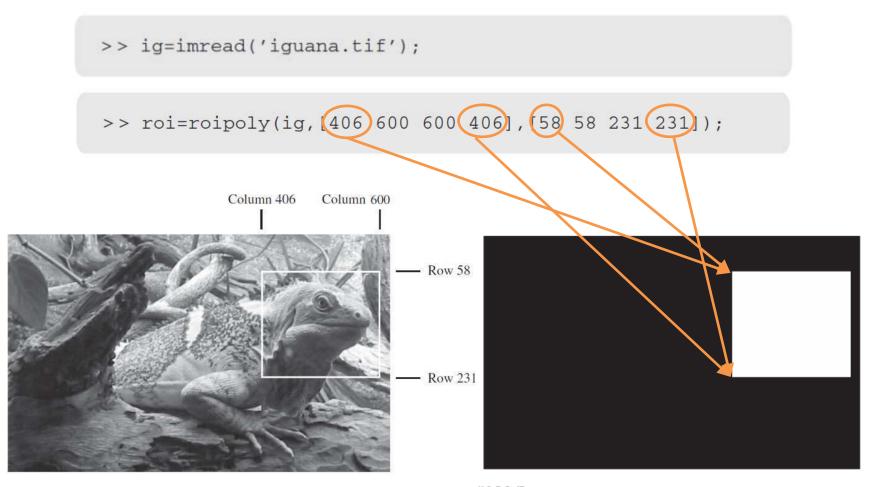


FIGURE 5.16 An image with a ROI.

URE 5.17 The mask corresponding to the ROI defined in Figure 5.16.



### Regions of Interest in MATLAB

```
>> roi=roipoly(ig);
```

- This will bring up the iguana image if it isn't shown already.
- Vertices of the ROI can be selected with the mouse.

```
>> a=fspecial('average',[15,15]);
>> iga=roifilt2(a,ig,roi);
>> imshow(iga)
>> u=fspecial('unsharp');
>> igu=roifilt2(u,ig,roi);
>> figure,imshow(igu)
>> l=fspecial('log');
>> igl=roifilt2(l,ig,roi);
>> figure,imshow(igl)
```



# **ROI** Processing: Results



iga: Average filtering



igu: Unsharp masking

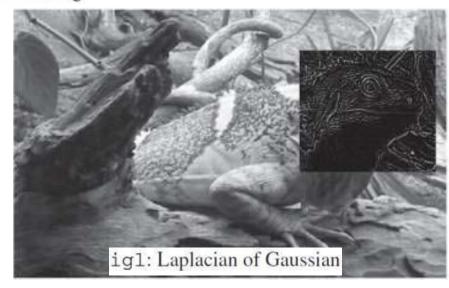


FIGURE 5.18 Examples of the use of roifilt2.



# Summary

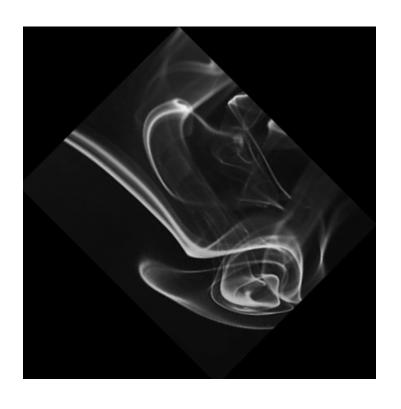
#### Chap 4. Point Processing

- √ Histogram(Contrast) Stretching
- ✓ Piecewise Contrast Stretching
- ✓ Histogram Equalization & Specification

#### Chap 5. Neighborhood Processing

- ✓ Convolution for Spatial Filtering
- ✓ Lowpass Filtering: Gaussian filter
- √ Highpass Filtering: Laplacian, LoG filter
- ✓ Edge Sharpening by Unsharp Masking
- ✓ High Boost Filtering
- ✓ Nonlinear Filtering: median, min, max





# Chapter 6: Image Geometry



## 6.1 Interpolation of Data

 Suppose we have a collection of four values that we wish to enlarge to eight

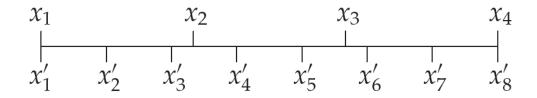


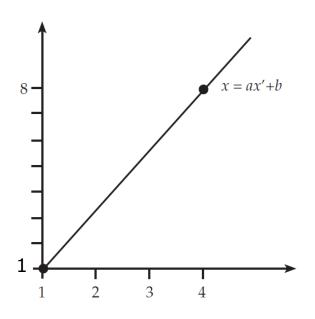
FIGURE 6.1 Replacing four points with eight.



FIGURE 6.2 Figure 6.1 slightly redrawn.



# 6.1 Interpolation of Data



**FIGURE 6.3** Filled circles are the coincide points of x and x'.

 The a and b of the linear function can be solved by

$$1 = a + b$$
$$4 = 8a + b$$

 Then we can obtain the linear function -> continuous

$$x = \frac{3}{7}x' + \frac{4}{7} \qquad x' = \frac{1}{3}(7x - 4),$$
$$x = \frac{1}{7}(3x' + 4)$$



## 6.1 Interpolation of Data

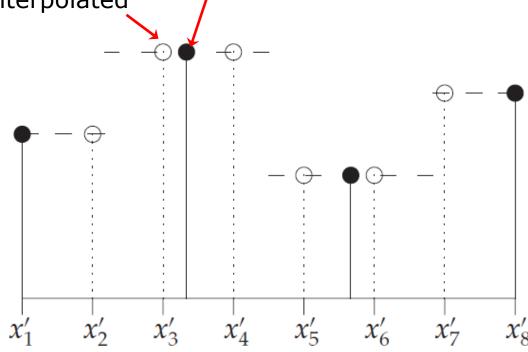
- In digital (discrete) signals, none of the  $x_i$  points coincide exactly with an original  $x_j$ , except for the first and last.
- We have to estimate function values  $f(x_i)$  based on the known values of nearby  $f(x_i)$
- Such estimation of function values based on surrounding values is called interpolation.



# **Nearest-neighbor interpolation**

$$f(x_i') = f(x_j)$$

 $x'_i$ : point to be  $x_j$ : original point closest to  $x'_i$  interpolated



**FIGURE 6.4** Nearest-neighbor interpolation.



# **Linear interpolation**

using distance( $\lambda$ ) as a weight

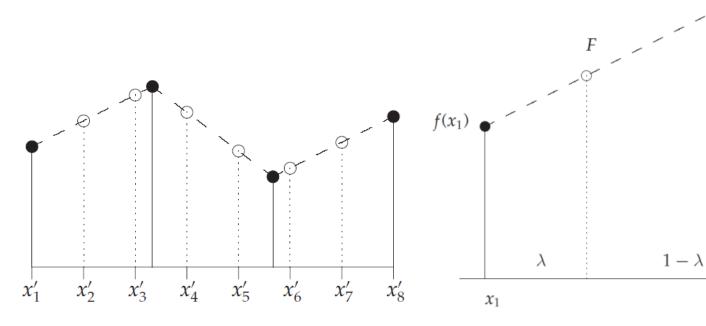


FIGURE 6.5 Linear interpolation.

FIGURE 6.6 Calculating linearly interpolated values.

$$\frac{F - f(x_1)}{\lambda} = \frac{f(x_2) - f(x_1)}{1}$$

$$F = \lambda f(x_2) + (1 - \lambda)f(x_1).$$



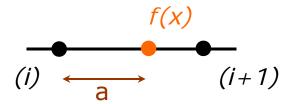
 $\bullet$   $f(x_2)$ 

 $x_2$ 

#### Interpolation

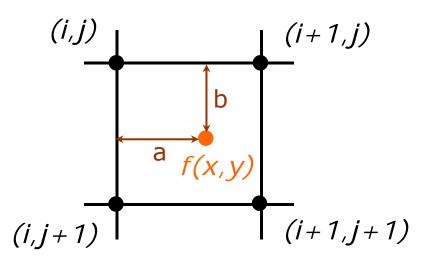
• linear interpolation: 1D

$$f(x) = (1-a) * f(i) + a * f(i+1)$$



bilinear interpolation : 2D uses 4 neighbors

$$f(x,y) = (1-a)(1-b)* f(i,j)$$
  
+  $a(1-b)$  \*  $f(i+1,j)$   
+  $(1-a)b$  \*  $f(i,j+1)$   
+  $ab$  \*  $f(i+1,j+1)$ 





#### Interpolation

[Q]  $2x2 \text{ image } -> 6x6, p_5$ ?

[A]

- i) nearest neighbor :  $p_5 = 8$
- ii) bilinear:

a = 2/3, b = 1/3  

$$p_5 = 1/3*2/3*2 + 2/3*2/3*8$$
  
 $+ 1/3*1/3*7 + 2/3*1/3*1$   
 $= 45/9 = 5$ 



#### 6.2 Image Interpolation

• Function imresize

```
imresize(A,k,'method')
```

 Where A is an image of any type, k is a scaling factor, and 'method' is either 'nearest' or 'bilinear', etc.

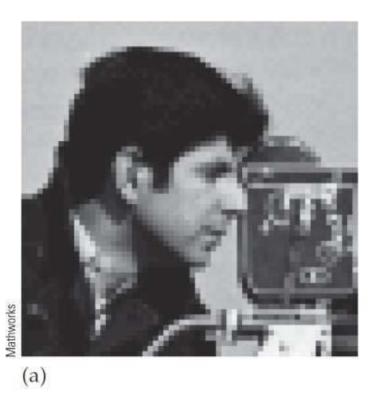
```
>> c=imread('cameraman.tif');
>> head=c(33:96,90:153);
>> imshow(head)
>> head4n=imresize(head,4,'nearest');imshow(head4n)
>> head4b=imresize(head,4,'bilinear');imshow(head4b)
```



#### FIGURE 6.9 & 6.10



FIGURE 6.9 The head.



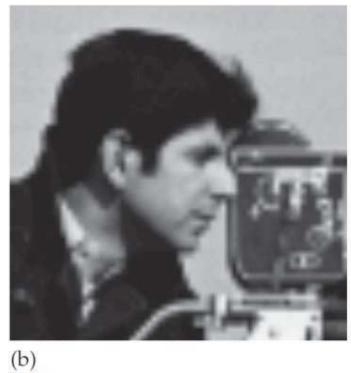


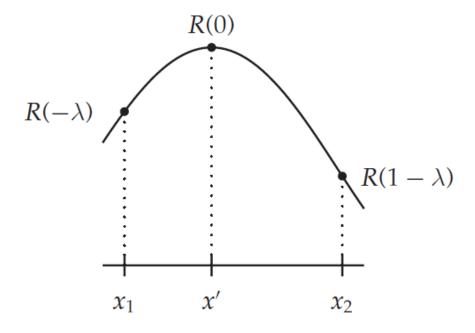
FIGURE 6.10 Scaling by interpolation. (a) Nearest neighbor scaling. (b) Bilinear interpolation.



# Generalized Weight Function, R(u)

$$x_1 \le x' \le x_2$$
$$x' - x_1 = \lambda$$

$$f(x') = R(-\lambda)f(x_1) + R(1-\lambda)f(x_2).$$



**FIGURE 6.11** *Using a general interpolation function.* 



# Weight function R(u)

#### nearest neighbor

$$R_0(u) = \begin{cases} 0 & \text{if } u \le -0.5, \\ 1 & \text{if } -0.5 < u \le 0.5, \\ 0 & \text{if } u > 0.5, \end{cases}$$

#### bilinear

$$R_1(u) = \begin{cases} 1 + u & \text{if } u \le 0, \\ 1 - u & \text{if } u \ge 0. \end{cases}$$

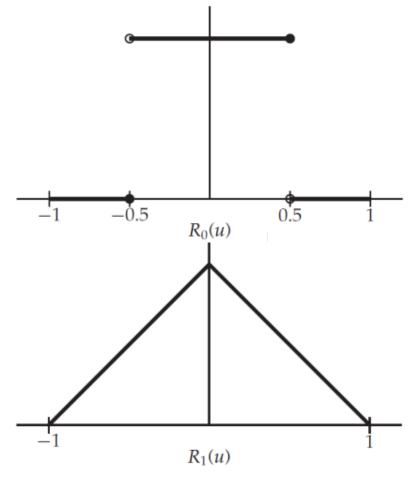


FIGURE 6.12 Two interpolation functions.



#### **Cubic Interpolation**

• defined over the interval  $-2 \le u \le 2$ 

$$R_3(u) = \begin{cases} 1.5|u|^3 - 2.5|u|^2 + 1 & \text{if } |u| \le 1, \\ -0.5|u|^3 + 2.5|u|^2 - 4|u| + 2 & \text{if } 1 < |u| \le 2. \end{cases}$$

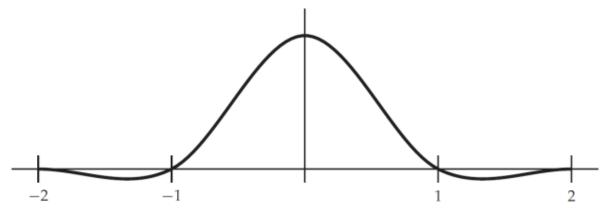
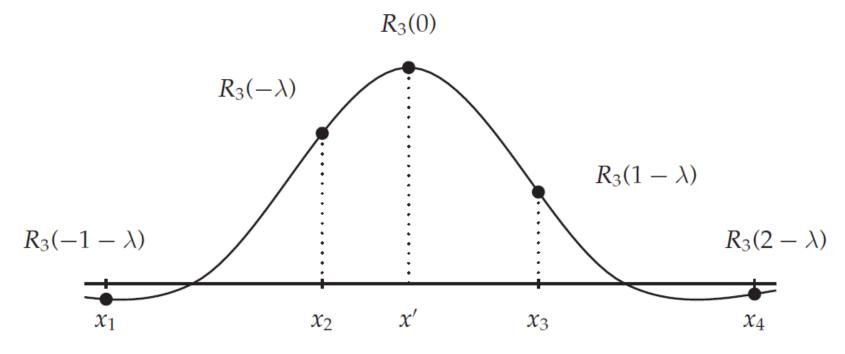


FIGURE 6.13 The cubic interpolation function  $R_3(u)$ .



# Deciding Weights by R(u) with 4 Neighboring Pixels

$$f(x') = R_3(-1 - \lambda)f(x_1) + R_3(-\lambda)f(x_2) + R_3(1 - \lambda)f(x_3) + R_4(2 - \lambda)f(x_4),$$



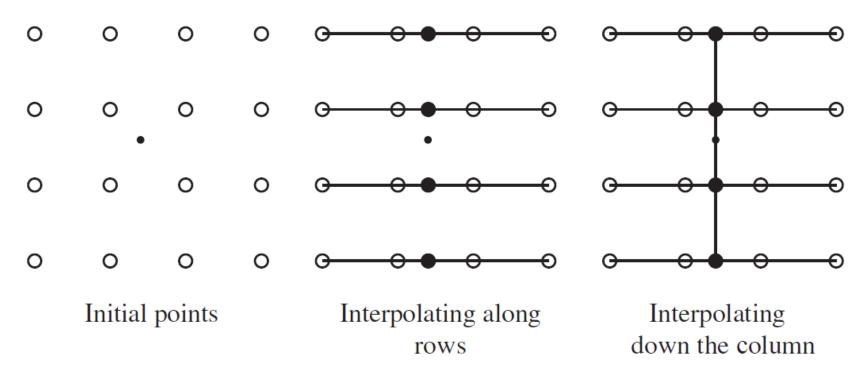
**FIGURE 6.14** Using  $R_3(u)$  for interpolation.

$$x' - x_2 = \lambda$$



# Bicubic interpolation for Image

uses 16 known values around the point (x', y')



**FIGURE 6.15** How to apply bicubic interpolation.



#### **FIGURE 6.16**

>> head4c=imresize(head,4,'bicubic');imshow(head4c)



FIGURE 6.16 Enlargement using bicubic interpolation.



## 6.4 Enlargement by Spatial Filtering

 If we merely wish to enlarge an image by a power of two, there is a quick and dirty method that uses linear filtering

✓ zero-interleaved (inserting a zero between elements)

$$m_2(i,j) = \begin{cases} m((i+1)/2, (j+1)/2) & \text{if } i \text{ and } j \text{ are both odd,} \\ 0 & \text{otherwise.} \end{cases}$$



## Zero-Interleaved Array in Matlab

#### This can be implemented with a simple function

```
function out=zeroint(a)
%
% ZEROINT(A) produces a zero-interleaved version of the matrix A.
% For example:
%
% a=[1 2 3; 4 5 6];
% zeroint(a)
%
% 1 0 2 0 3
% 0 0 0 0 0
% 4 0 5 0 6
%
[m,n]=size(a); a2=reshape([a;zeros(m,n)],m,2*n);
out=reshape([a2';zeros(2*n,m)],2*n,2*m)';
```

FIGURE 6.17 A simple function for implementing zero interleaving.



# 6.4 Enlargement by Spatial Filtering

We can now replace the zeros by applying a spatial filter to this matrix

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \frac{1}{4} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \qquad \frac{1}{64} \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}$$
 nearest-neighbor bilinear bicubic



# 6.4 Enlargement by Spatial Filtering

```
>> filter2([1 1 0;1 1 0;0 0 0],m2)
ans =
    16
          16
                                          13
                                                 13
    16
          16
                        2
                              3
                                    3
                                          13
                                                 13
     5
           5
                 11
                       11
                             10
                                    10
                                           8
                                                 8
     5
           5
                 11
                       11
                             10
                                    10
                                           8
                                                 8
     9
           9
                              6
                                          12
                                                 12
     9
           9
                                     6
                              6
                                          12
                                                 12
                             15
                 14
                       14
                 14
                       14
                             15
                                    15
>> filter2([1 2 1;2 4 2;1 2 1]/4,m2)
ans =
                                                                               6.5000
   16.0000
               9.0000
                         2.0000
                                    2.5000
                                               3.0000
                                                         8.0000
                                                                   13.0000
   10.5000
               8.5000
                         6.5000
                                    6.5000
                                               6.5000
                                                         8.5000
                                                                   10.5000
                                                                               5.2500
    5.0000
                                   10.5000
                                                                               4.0000
               8.0000
                        11.0000
                                              10.0000
                                                         9.0000
                                                                    8.0000
    7.0000
                         9.0000
                                                                   10.0000
               8.0000
                                    8.5000
                                               8.0000
                                                         9.0000
                                                                               5.0000
    9.0000
                         7.0000
                                    6.5000
                                               6.0000
                                                         9.0000
                                                                               6.0000
               8.0000
                                                                   12.0000
    6.5000
               8.5000
                        10.5000
                                   10.5000
                                              10.5000
                                                         8.5000
                                                                    6.5000
                                                                               3.2500
    4.0000
               9.0000
                        14.0000
                                   14.5000
                                              15.0000
                                                         8.0000
                                                                    1.0000
                                                                               0.5000
    2.0000
                         7.0000
                                    7.2500
                                               7.5000
                                                                    0.5000
                                                                               0.2500
               4.5000
                                                         4.0000
```



#### Image Enlargement in 4 Ways

```
>> imshow(hz)
>> imshow(filter2([1 1 0;1 1 0;0 0 0],hz)/255)
>> imshow(filter2([1 2 1;2 4 2;1 2 1]/4,hz)/255)
>> bfilt=[1 4 6 4 1;4 16 24 16 4;6 24 36 24 6;4 16 24 16 4;1 4 6 4 1]/64;
>> imshow(filter2(bfilt,hz)/255)
```



Zero interleaving Nearest neighbor





Bilinear



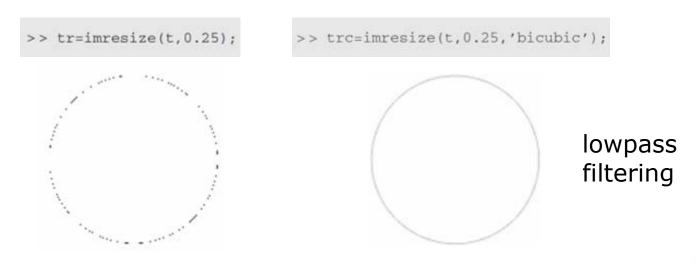
Bicubic

FIGURE 6.18 Enlargement by spatial filtering.



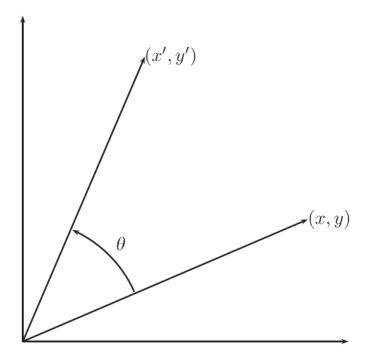
### 6.5 Scaling Smaller

- Making an image smaller is also called image minimization
- Subsampling
- problems in high freq. image





#### 6.6 Rotation



**FIGURE 6.20** Rotating a point through angle  $\theta$ .

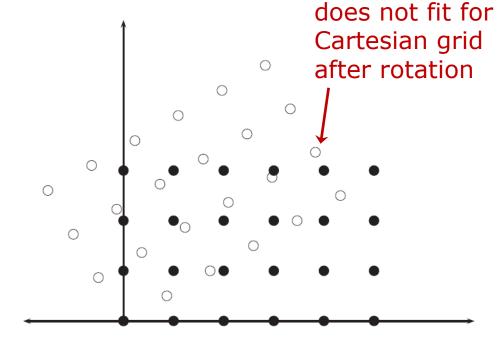


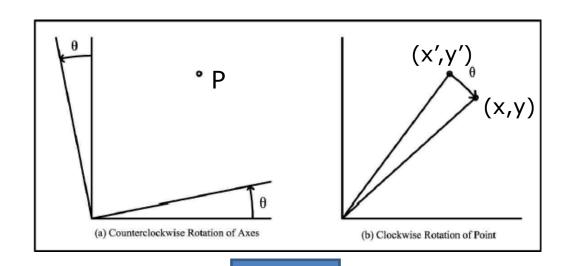
FIGURE 6.21 Rotating a rectangle.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$

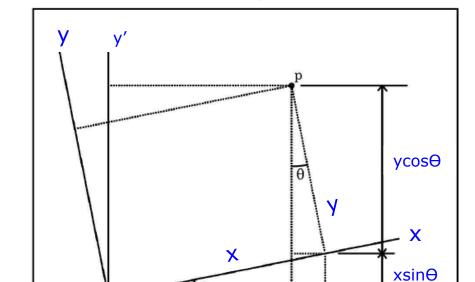
The rotation matrix is orthogonal so its transpose equals inverse matrix. Multiplying on the left by the transpose of the matrix.

counter clockwise rotation





# Derivation of Rotation Matrix



xcosθ

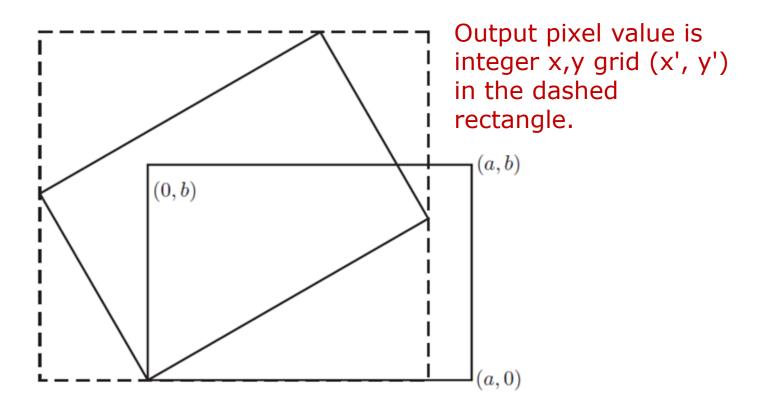
ysinθ

$$x' = x \cos \theta - y \sin \theta$$
$$y' = x \sin \theta + y \cos \theta$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



#### **Image Rotation**



 When the output is rotated back, all the values should lie within the original image limits

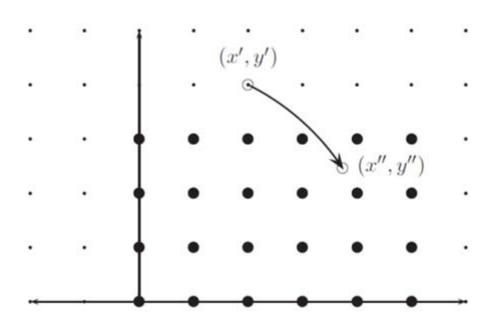
$$0 \le x' \cos \theta + y' \sin \theta \le a,$$

$$0 \le -x'\sin\theta + y'\cos\theta \le b$$



## Rotation: How to Get the Value at (x', y')

- 1. Get the location (x'', y'') by rotating back the (x',y') position.
- 2. Calculate the gray value at (x'', y'') by interpolation, using surrounding gray values.
- 3. The interpolated value (x'', y'') becomes the value for pixel at (x', y') in the rotated image





#### Rotation in Malab

```
>> cr=imrotate(c,60); % nearest neighbor
>> imshow(cr)
>> crc=imrotate(c,60,'bicubic');
>> imshow(crc)
```

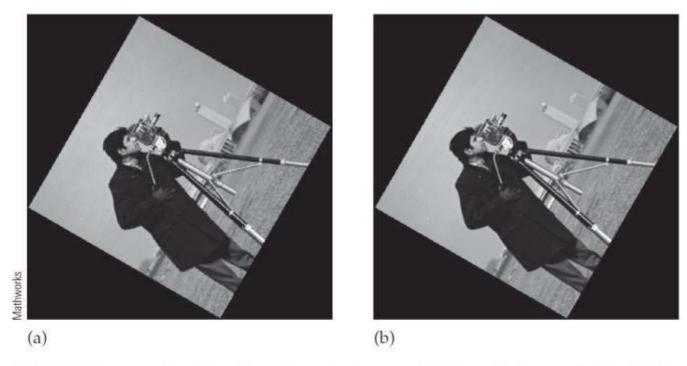


FIGURE 6.25 Rotation with interpolation. (a) Nearest neighbor. (b) Bicubic interpolation.



## Anamorphosis

• Deliberately distorted image of an object shape for artistic or dramatic effect.



FIGURE 6.26 The Ambassadors (1533) by Hans Holbein.



# Extraction of Anamorphosis Image

```
>> a=imread('AMBASSADORS.JPG');
>> a=rgb2gray(a);
>> skull=a(566:743,157:586);
```

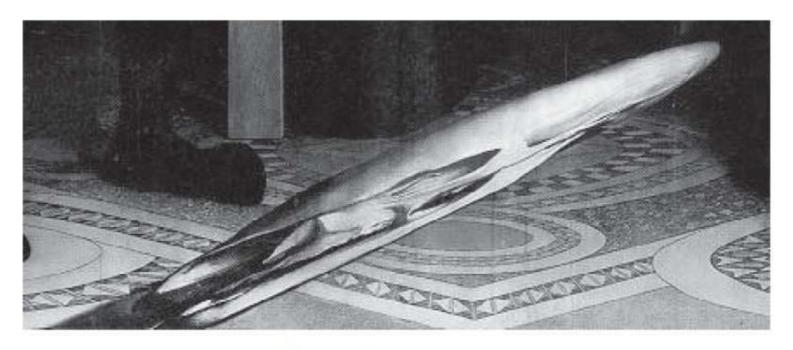


FIGURE 6.21 The skull alone.



## **Undoing Anamorphosis**

```
>> skull2=imresize(imrotate(skull,-22,'bicubic'),[500,150],'bicubic');
>> imshow(skull2(200:350,:)) % extraction of skull area
```

rotation followed by stretching -> needs trial and error



FIGURE 6.28 The corrected skull.



# Summary

#### Chap 6. Image Geometry

- ✓ Interpolation: nearest neighbor, bilinear, bicubic
- ✓ enlargement, minimization
- ✓ rotation
- ✓ anamorphosis

