# Statistics in Healthcare

## Unit 7 (optional):

Math details of ANOVA, Wilcoxon rank-sum test, and Log-rank test

# Optional material: math details of the ANOVA test

# How to calculate ANOVA's by hand…

| Treatment 1 | Treatment 2 | Treatment 3 | Treatment 4 |
|---|---|---|---|
| $y_{11}$ | $y_{21}$ | $y_{31}$ | $y_{41}$ |
| $y_{12}$ | $y_{22}$ | $y_{32}$ | $y_{42}$ |
| $y_{13}$ | $y_{23}$ | $y_{33}$ | $y_{43}$ |
| $y_{14}$ | $y_{24}$ | $y_{34}$ | $y_{44}$ |
| $y_{15}$ | $y_{25}$ | $y_{35}$ | $y_{45}$ |
| $y_{16}$ | $y_{26}$ | $y_{36}$ | $y_{46}$ |
| $y_{17}$ | $y_{27}$ | $y_{37}$ | $y_{47}$ |
| $y_{18}$ | $y_{28}$ | $y_{38}$ | $y_{48}$ |
| $y_{19}$ | $y_{29}$ | $y_{39}$ | $y_{49}$ |
| $y_{110}$ | $y_{210}$ | $y_{310}$ | $y_{410}$ |

$n$=10 obs./group

$k$=4 groups

$$\bar{y}_{1\bullet} = \frac{\sum_{j=1}^{10} y_{1j}}{10} \qquad \bar{y}_{2\bullet} = \frac{\sum_{j=1}^{10} y_{2j}}{10} \qquad \bar{y}_{3\bullet} = \frac{\sum_{j=1}^{10} y_{3j}}{10} \qquad \bar{y}_{4\bullet} = \frac{\sum_{j=1}^{10} y_{4j}}{10}$$

The group means

$$\frac{\sum_{j=1}^{10} (y_{1j} - \bar{y}_{1\bullet})^2}{10-1} \qquad \frac{\sum_{j=1}^{10} (y_{2j} - \bar{y}_{2\bullet})^2}{10-1} \qquad \frac{\sum_{j=1}^{10} (y_{3j} - \bar{y}_{3\bullet})^2}{10-1} \qquad \frac{\sum_{j=1}^{10} (y_{4j} - \bar{y}_{4\bullet})^2}{10-1}$$

The (within) group variances

# Sum of Squares Within (SSW), or Sum of Squares Error (SSE)

$$\frac{\sum_{j=1}^{10}(y_{1j}-\bar{y}_{1\bullet})^2}{10-1} \quad \frac{\sum_{j=1}^{10}(y_{2j}-\bar{y}_{2\bullet})^2}{10-1} \quad \frac{\sum_{j=1}^{10}(y_{3j}-\bar{y}_{3\bullet})^2}{10-1} \quad \frac{\sum_{j=1}^{10}(y_{4j}-\bar{y}_{4\bullet})^2}{10-1}$$

The (within) group variances

$$\sum_{j=1}^{10}(y_{1j}-\bar{y}_{1\bullet})^2 + \sum_{j=1}^{10}(y_{2j}-\bar{y}_{2\bullet})^2 + \sum_{j=3}^{10}(y_{3j}-\bar{y}_{3\bullet})^2 + \sum_{j=1}^{10}(y_{4j}-\bar{y}_{4\bullet})^2$$

$$= \sum_{i=1}^{4}\sum_{j=1}^{10}(y_{ij}-\bar{y}_{i\bullet})^2$$

**Sum of Squares Within (SSW)**
**(or SS<u>E</u>, for chance <u>e</u>rror)**

# Sum of Squares Between (SSB), or Sum of Squares Regression (SSR)

Overall mean of all 40 observations ("grand mean")

$$\bar{\bar{y}}_{\bullet\bullet} = \frac{\sum_{i=1}^{4}\sum_{j=1}^{10} y_{ij}}{40}$$

$$10x\sum_{i=1}^{4}(\bar{y}_{i\bullet} - \bar{\bar{y}}_{\bullet\bullet})^2$$

Sum of Squares Between (SSB). Variability of the group means compared to the grand mean (the variability due to the treatment).

# Total Sum of Squares (SST)

$$\sum_{i=1}^{4}\sum_{j=1}^{10}(y_{ij} - \bar{\bar{y}}_{\bullet\bullet})^2$$

Total sum of squares(TSS). Squared difference of every observation from the overall mean. (numerator of variance of Y!)

# Partitioning of Variance

$$\sum_{i=1}^{4}\sum_{j=1}^{10}(y_{ij}-\bar{y}_{i\bullet})^2 + 10x\sum_{i=1}^{4}(\bar{y}_{i\bullet}-\bar{\bar{y}}_{\bullet\bullet})^2 = \sum_{i=1}^{4}\sum_{j=1}^{10}(y_{ij}-\bar{\bar{y}}_{\bullet\bullet})^2$$

**SSW + SSB = SST**

# ANOVA Table

| Source of variation | d.f. | Sum of squares | Mean Sum of Squares | F-statistic | p-value |
|---|---|---|---|---|---|
| Between (k groups) | k-1 | SSB (sum of squared deviations of group means from grand mean) | SSB/k-1 | $\boxed{\dfrac{SSB/k-1}{SSW/nk-k}}$ | Go to $F_{k-1,nk-k}$ chart |
| Within (n individuals per group) | nk-k | SSW (sum of squared deviations of observations from their group mean) | $s^2$=SSW/nk-k | | |
| Total variation | nk-1 | TSS (sum of squared deviations of observations from grand mean) | | | |

$\boxed{TSS=SSB + SSW}$

# ANOVA=t-test

| Source of variation | d.f. | Sum of squares | Mean Sum of Squares |
|---|---|---|---|
| Between (2 groups) | 1 | SSB (squared difference in means multiplied by n) | Squared difference in means times n |
| Within | 2n-2 | SSW<br><br>equivalent to numerator of pooled variance | Pooled variance |
| Total variation | 2n-1 | TSS | |

$$SSB = n\sum_{i=1}^{n}(\bar{X}_n - (\frac{\bar{X}_n + \bar{Y}_n}{2}))^2 + n\sum_{i=1}^{n}(\bar{Y}_n - (\frac{\bar{X}_n + \bar{Y}_n}{2}))^2 =$$

$$n\sum_{i=1}^{n}(\frac{\bar{X}_n}{2} - \frac{\bar{Y}_n}{2})^2 + n\sum_{i=1}^{n}(\frac{\bar{Y}_n}{2} - \frac{\bar{X}_n}{2})^2$$

$$n((\frac{\bar{X}_n}{2})^2 + (\frac{\bar{Y}_n}{2})^2 - 2\frac{\bar{X}_n * \bar{Y}_n}{2} + (\frac{\bar{Y}_n}{2})^2 + (\frac{\bar{X}_n}{2})^2 - 2\frac{\bar{X}_n * \bar{Y}_n}{2}) =$$

$$n(\bar{X}_n{}^2 - 2\bar{X}_n * \bar{Y}_n + \bar{Y}_n{}^2) = n(\bar{X}_n - \bar{Y}_n)^2$$

$$\frac{n(\bar{X} - \bar{Y})^2}{s_p{}^2} = (\frac{(\bar{X} - \bar{Y})}{\sqrt{\frac{s_p{}^2}{n} + \frac{s_p{}^2}{n}}})^2 = (t_{2n-2})^2$$

Go to **$F_{1, 2n-2}$** Chart→ notice values are just $(\mathbf{t}_{2n-2})^2$

# Example

| Treatment 1 | Treatment 2 | Treatment 3 | Treatment 4 |
|---|---|---|---|
| 60 inches | 50 | 48 | 47 |
| 67 | 52 | 49 | 67 |
| 42 | 43 | 50 | 54 |
| 67 | 67 | 55 | 67 |
| 56 | 67 | 56 | 68 |
| 62 | 59 | 61 | 65 |
| 64 | 67 | 61 | 65 |
| 59 | 64 | 60 | 56 |
| 72 | 63 | 59 | 60 |
| 71 | 65 | 64 | 65 |

# Example

**Step 1)** calculate the sum of squares between groups:

Mean for group 1 = 62.0

Mean for group 2 = 59.7

Mean for group 3 = 56.3

Mean for group 4 = 61.4

Grand mean= 59.85

| Treatment 1 | Treatment 2 | Treatment 3 | Treatment 4 |
|---|---|---|---|
| 60 inches | 50 | 48 | 47 |
| 67 | 52 | 49 | 67 |
| 42 | 43 | 50 | 54 |
| 67 | 67 | 55 | 67 |
| 56 | 67 | 56 | 68 |
| 62 | 59 | 61 | 65 |
| 64 | 67 | 61 | 65 |
| 59 | 64 | 60 | 56 |
| 72 | 63 | 59 | 60 |
| 71 | 65 | 64 | 65 |

SSB = $[(62-59.85)^2 + (59.7-59.85)^2 + (56.3-59.85)^2 + (61.4-59.85)^2]$ *xn per group*= 19.65*x10* = **196.5**

# Example

**Step 2)** calculate the sum of squares within groups:

$(60\text{-}62)^2 + (67\text{-}62)^2 + (42\text{-}62)^2 + (67\text{-}62)^2 + (56\text{-}62)^2 + (62\text{-}62)^2 + (64\text{-}62)^2 + (59\text{-}62)^2 + (72\text{-}62)^2 + (71\text{-}62)^2 + (50\text{-}59.7)^2 + (52\text{-}59.7)^2 + (43\text{-}59.7)^2 + 67\text{-}59.7)^2 + (67\text{-}59.7)^2 + (69\text{-}59.7)^2 \ldots + \ldots$ (sum of 40 squared deviations) = **2060.6**

| Treatment 1 | Treatment 2 | Treatment 3 | Treatment 4 |
|---|---|---|---|
| 60 inches | 50 | 48 | 47 |
| 67 | 52 | 49 | 67 |
| 42 | 43 | 50 | 54 |
| 67 | 67 | 55 | 67 |
| 56 | 67 | 56 | 68 |
| 62 | 59 | 61 | 65 |
| 64 | 67 | 61 | 65 |
| 59 | 64 | 60 | 56 |
| 72 | 63 | 59 | 60 |
| 71 | 65 | 64 | 65 |

# Step 3) Fill in the ANOVA table

| Source of variation | d.f. | Sum of squares | Mean Sum of Squares | F-statistic | p-value |
|---|---|---|---|---|---|
| Between | 3 | **196.5** | 65.5 | 1.14 | **.344** |
| Within | 36 | **2060.6** | 57.2 | - | - |
| Total | 39 | 2257.1 | - | - | - |

# Step 3) Fill in the ANOVA table

| Source of variation | d.f. | Sum of squares | Mean Sum of Squares | F-statistic | p-value |
|---|---|---|---|---|---|
| Between | 3 | **196.5** | 65.5 | 1.14 | **.344** |
| Within | 36 | **2060.6** | 57.2 | - | - |
| Total | 39 | 2257.1 | - | - | - |

**INTERPRETATION of ANOVA:**

**How much of the variance in height is explained by treatment group?**

**$R^2$="Coefficient of Determination" = SSB/TSS = 196.5/2275.1=9%**

# Coefficient of Determination

$$R^2 = \frac{SSB}{SSB + SSE} = \frac{SSB}{SST}$$

**SSE: Sum of Squares Within (SSW) (or SSE, for chance error)**

**The amount of variation in the outcome variable (dependent variable) that is explained by the predictor (independent variable).**

# Beyond one-way ANOVA

Often, you may want to test more than 1 treatment. ANOVA can accommodate more than 1 treatment or factor, so long as they are independent. Again, the variation partitions beautifully!

SST = SSB1 + SSB2 + SSW

# End optional material

# Optional material: math details for the Wilcoxon rank-sum test

# Wilcoxon rank-sum test

Rank all of the observations in order from 1 to n.

$T_1$ is the sum of the ranks from smaller population ($n_1$)

$T_2$ is the sum of the ranks from the larger population ($n_2$)

$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - T_1$$

$$U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - T_2$$

$$U_0 = \min(U_1, U_2)$$

Find $P(U \le U_0)$ in Mann-Whitney U tables
With $n_2$ = the bigger of the 2 populations (or website)

for $n_1 > 10, n_2 > 10,$

$$Z = \frac{U_0 - \frac{n_1 n_2}{2}}{\sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}}$$

## Table A5.07: Critical Values for the Wilcoxon/Mann-Whitney Test (U)

**Nondirectional α=.05 (Directional α=.025)**

| $n_1$ \ $n_2$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| **2** | - | - | - | - | - | - | - | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| **3** | - | - | - | - | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 | 6 | 7 | 7 | 8 |
| **4** | - | - | - | 0 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 11 | 12 | 13 | 13 |
| **5** | - | - | 0 | 1 | 2 | 3 | 5 | 6 | 7 | 8 | 9 | 11 | 12 | 13 | 14 | 15 | 17 | 18 | 19 | 20 |
| **6** | - | - | 1 | 2 | 3 | 5 | 6 | 8 | 10 | 11 | 13 | 14 | 16 | 17 | 19 | 21 | 22 | 24 | 25 | 27 |
| **7** | - | - | 1 | 3 | 5 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 | 32 | 34 |
| **8** | - | 0 | 2 | 4 | 6 | 8 | 10 | 13 | 15 | 17 | 19 | 22 | 24 | 26 | 29 | 31 | 34 | 36 | 38 | 41 |
| **9** | - | 0 | 2 | 4 | 7 | 10 | 12 | 15 | 17 | 21 | 23 | 26 | 28 | 31 | 34 | 37 | 39 | 42 | 45 | 48 |
| **10** | - | 0 | 3 | 5 | 8 | 11 | 14 | 17 | 20 | $\boxed{23}$ | 26 | 29 | 33 | 36 | 39 | 42 | 45 | 48 | 52 | 55 |
| **11** | - | 0 | 3 | 6 | 9 | 13 | 16 | 19 | 23 | 26 | 30 | 33 | 37 | 40 | 44 | 47 | 51 | 55 | 58 | 62 |
| **12** | - | 1 | 4 | 7 | 11 | 14 | 18 | 22 | 26 | 29 | 33 | 37 | 41 | 45 | 49 | 53 | 57 | 61 | 65 | 69 |
| **13** | - | 1 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 33 | 37 | 41 | 45 | 50 | 54 | 59 | 63 | 67 | 72 | 76 |
| **14** | - | 1 | 5 | 9 | 13 | 17 | 22 | 26 | 31 | 36 | 40 | 45 | 50 | 55 | 59 | 64 | 67 | 74 | 78 | 83 |
| **15** | - | 1 | 5 | 10 | 14 | 19 | 24 | 29 | 34 | 39 | 44 | 49 | 54 | 59 | 64 | 70 | 75 | 80 | 85 | 90 |
| **16** | - | 1 | 6 | 11 | 15 | 21 | 26 | 31 | 37 | 42 | 47 | 53 | 59 | 64 | 70 | 75 | 81 | 86 | 92 | 98 |
| **17** | - | 2 | 6 | 11 | 17 | 22 | 28 | 34 | 39 | 45 | 51 | 57 | 63 | 67 | 75 | 81 | 87 | 93 | 99 | 105 |
| **18** | - | 2 | 7 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 55 | 61 | 67 | 74 | 80 | 86 | 93 | 99 | 106 | 112 |
| **19** | - | 2 | 7 | 13 | 19 | 25 | 32 | 38 | 45 | 52 | 58 | 65 | 72 | 78 | 85 | 92 | 99 | 106 | 113 | 119 |
| **20** | - | 2 | 8 | 14 | 20 | 27 | 34 | 41 | 48 | 55 | 62 | 69 | 76 | 83 | 90 | 98 | 105 | 112 | 119 | 127 |

$U_{obt}$ is the lesser of the two calculated test statistics ($U_1$ & $U_2$). If $U_{obt} \le U_{crit}$, reject $H_0$.
Dashes (-) indicate that the sample size is too small to reject the Null Hypothesis at the chosen α level.

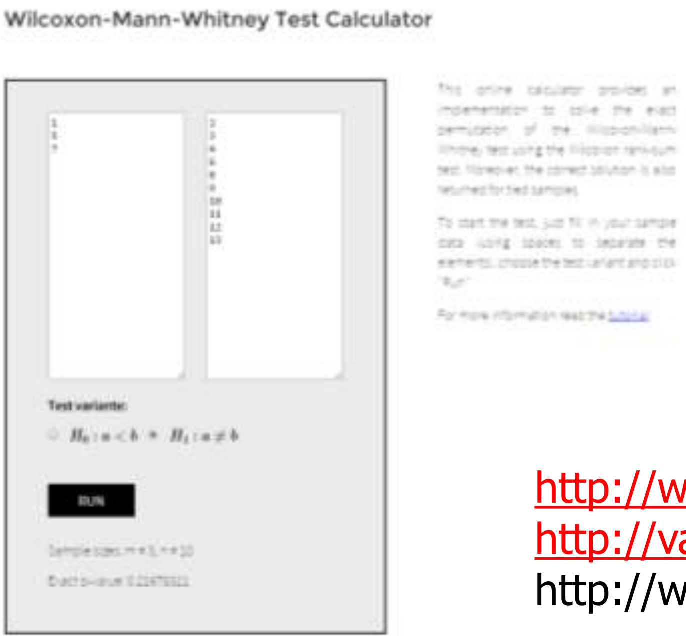
$U_o = 18 < U_{crit} = 23$

➔ reject $H_o$

# Example…

- Example: if team 1 and team 2 (two gymnastic teams) are competing, and the judges rank all the individuals in the competition, how can you tell if team 1 has done significantly better than team 2 or vice versa (unequal team size)?

# U calculator online…




http://www.ccb.uni-saarland.de/?page_id=812
http://vassarstats.net/utest.html
http://www.socscistatistics.com/tests/signedranks/Default2.aspx

# Applying the test to real data:

Example: If the girls on the two gymnastics teams were ranked as follows:

Team 1: 1, 5, 7 Observed $T_1 = 13$

Team 2: 2,3,4,6,8,9,10,11,12,13 Observed $T_2 = 78$

Are the teams significantly different?

Total sum of ranks = 13+78 = 91 $n_1n_2 = 3*10 = 30$

$U_1 = 30 + 6 - 13 = 23$

$U_2 = 30 + 55 - 78 = 7$

$\therefore U_0 = 7$

Not quite statistically significant in U table…p=.1084 *x2* for two-tailed test

## Table A5.07: Critical Values for the Wilcoxon/Mann-Whitney Test (U)

**Nondirectional α=.05 (Directional α=.025)**

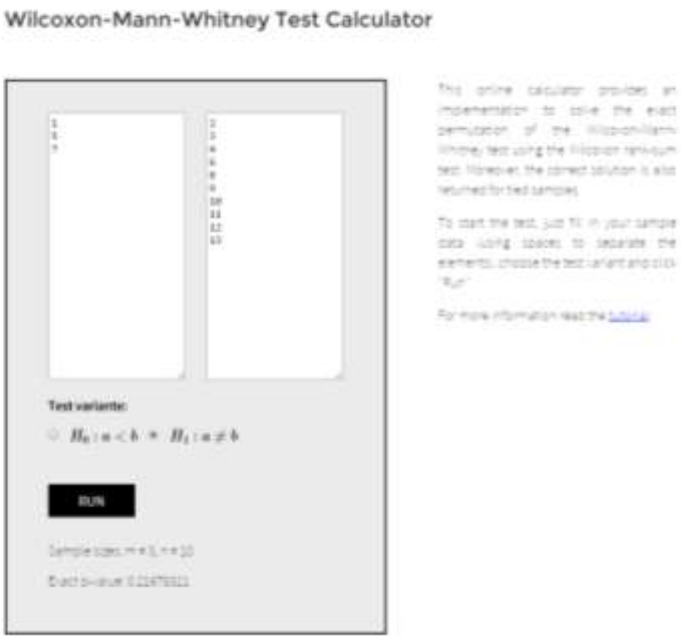
| $n_1$ \ $n_2$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| **2** | - | - | - | - | - | - | - | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| **3** | - | - | - | - | 0 | 1 | 1 | 2 | 2 | $\boxed{3}$ | 3 | 4 | 4 | 5 | 5 | 6 | 6 | 7 | 7 | 8 |
| **4** | - | - | - | 0 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 11 | 12 | 13 | 13 |
| **5** | - | - | 0 | 1 | 2 | 3 | 5 | 6 | 7 | 8 | 9 | 11 | 12 | 13 | 14 | 15 | 17 | 18 | 19 | 20 |
| **6** | - | - | 1 | 2 | 3 | 5 | 6 | 8 | 10 | 11 | 13 | 14 | 16 | 17 | 19 | 21 | 22 | 24 | 25 | 27 |
| **7** | - | - | 1 | 3 | 5 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 | 32 | 34 |
| **8** | - | 0 | 2 | 4 | 6 | 8 | 10 | 13 | 15 | 17 | 19 | 22 | 24 | 26 | 29 | 31 | 34 | 36 | 38 | 41 |
| **9** | - | 0 | 2 | 4 | 7 | 10 | 12 | 15 | 17 | 21 | 23 | 26 | 28 | 31 | 34 | 37 | 39 | 42 | 45 | 48 |
| **10** | - | 0 | 3 | 5 | 8 | 11 | 14 | 17 | 20 | 23 | 26 | 29 | 33 | 36 | 39 | 42 | 45 | 48 | 52 | 55 |
| **11** | - | 0 | 3 | 6 | 9 | 13 | 16 | 19 | 23 | 26 | 30 | 33 | 37 | 40 | 44 | 47 | 51 | 55 | 58 | 62 |
| **12** | - | 1 | 4 | 7 | 11 | 14 | 18 | 22 | 26 | 29 | 33 | 37 | 41 | 45 | 49 | 53 | 57 | 61 | 65 | 69 |
| **13** | - | 1 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 33 | 37 | 41 | 45 | 50 | 54 | 59 | 63 | 67 | 72 | 76 |
| **14** | - | 1 | 5 | 9 | 13 | 17 | 22 | 26 | 31 | 36 | 40 | 45 | 50 | 55 | 59 | 64 | 67 | 74 | 78 | 83 |
| **15** | - | 1 | 5 | 10 | 14 | 19 | 24 | 29 | 34 | 39 | 44 | 49 | 54 | 59 | 64 | 70 | 75 | 80 | 85 | 90 |
| **16** | - | 1 | 6 | 11 | 15 | 21 | 26 | 31 | 37 | 42 | 47 | 53 | 59 | 64 | 70 | 75 | 81 | 86 | 92 | 98 |
| **17** | - | 2 | 6 | 11 | 17 | 22 | 28 | 34 | 39 | 45 | 51 | 57 | 63 | 67 | 75 | 81 | 87 | 93 | 99 | 105 |
| **18** | - | 2 | 7 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 55 | 61 | 67 | 74 | 80 | 86 | 93 | 99 | 106 | 112 |
| **19** | - | 2 | 7 | 13 | 19 | 25 | 32 | 38 | 45 | 52 | 58 | 65 | 72 | 78 | 85 | 92 | 99 | 106 | 113 | 119 |
| **20** | - | 2 | 8 | 14 | 20 | 27 | 34 | 41 | 48 | 55 | 62 | 69 | 76 | 83 | 90 | 98 | 105 | 112 | 119 | 127 |

$U_o = 7 > U_{crit} = 3$

➔ fail to reject $H_o$

$U_{obt}$ is the lesser of the two calculated test statistics ($U_1$ & $U_2$). If $U_{obt} \leq U_{crit}$, reject $H_0$.
Dashes (-) indicate that the sample size is too small to reject the Null Hypothesis at the chosen α level.

# U calculator online…




http://vassarstats.net/utest.html

# End optional material

# Optional: Math detail of log-rank test

# Log-rank test example

Test of Equality over Strata

| Test | Chi-Square | DF | Pr > Chi-Square |
|---|---|---|---|
| **Log-Rank** | **4.6599** | **1** | **0.0309** |

Chi-square test (with 1 degree of freedom) of the (overall) difference between the two groups.

Groups are significantly different.

# The log-rank test

K Strata = unique event times

| | Event | No Event |
|---|---|---|
| Group 1 | **a** | b |
| Group 2 | c | d |

$N_k$

$$\frac{[\sum_{i=1}^{k}(a_k - E(a_k))]^2}{\sum_{i=1}^{k} Var(a_k)} \sim \chi_1^2$$

$$E(a_k) = \frac{row1_k * col1_k}{N_k}$$

$$Var(a_k) = \frac{row1_k * row2_k * col1_k * col2_k}{N_k^2(N_k - 1)}$$

# Log-rank test

| | Event | No Event |
|---|---|---|
| Group 1 | **a** | b |
| Group 2 | c | d |

How do you know that this is a chi-square with 1 df?

$$\frac{[\sum_{i=1}^{k}(a_k - E(a_k))]^2}{\sum_{i=1}^{k} Var(a_k)} \sim \chi_1^2$$

$$E(a_k) = \frac{row1_k * col1_k}{N_k}$$

$$Var(a_k) = \frac{row1_k * row2_k * col1_k * col2_k}{N_k^2(N_k - 1)}$$

Variance is the variance of a hypergeometric distribution

# Event time 1 (2 months), control group:

At risk=22

1st event at month 2.

| time | Survival | Failure | Survival Standard Error | Number Failed | Number Left |
|---|---|---|---|---|---|
| 0.000 | 1.0000 | 0 | 0 | 0 | **22** |
| **2.000** | **0.9545** | **0.0455** | **0.0444** | **1** | **21** |
| 3.000 | 0.9091 | 0.0909 | 0.0613 | 2 | 20 |
| 4.000 | 0.8636 | 0.1364 | 0.0732 | 3 | 19 |
| 7.000 | 0.8182 | 0.1818 | 0.0822 | 4 | 18 |
| 10.000 | 0.7727 | 0.2273 | 0.0893 | 5 | 17 |
| 22.000 | 0.7273 | 0.2727 | 0.0950 | 6 | 16 |
| 28.000 | 0.6818 | 0.3182 | 0.0993 | 7 | 15 |
| 29.000 | 0.6364 | 0.3636 | 0.1026 | 8 | 14 |
| 32.000 | 0.5909 | 0.4091 | 0.1048 | 9 | 13 |
| 37.000 | 0.5455 | 0.4545 | 0.1062 | 10 | 12 |
| 40.000 | 0.5000 | 0.5000 | 0.1066 | 11 | 11 |
| 41.000 | 0.4545 | 0.5455 | 0.1062 | 12 | 10 |
| 54.000 | 0.4091 | 0.5909 | 0.1048 | 13 | 9 |
| 61.000 | 0.3636 | 0.6364 | 0.1026 | 14 | 8 |
| 63.000 | 0.3182 | 0.6818 | 0.0993 | 15 | 7 |
| 71.000 | 0.2727 | 0.7273 | 0.0950 | 16 | 6 |
| 127.000* | . | . | . | 16 | 5 |

# Event time 1 (2 months), treated group:

1st event at month 2.

At risk=22

| time | Survival | Failure | Survival Standard Error | Number Failed | Number Left |
|---|---|---|---|---|---|
| 0.000 | 1.0000 | 0 | 0 | 0 | **22** |
| **2.000** | **0.9545** | **0.0455** | **0.0444** | **1** | **21** |
| 6.000 | 0.9091 | 0.0909 | 0.0613 | 2 | 20 |
| 12.000 | 0.8636 | 0.1364 | 0.0732 | 3 | 19 |
| 54.000 | 0.8182 | 0.1818 | 0.0822 | 4 | 18 |
| 56.000* | . | . | . | 4 | 17 |
| 68.000 | 0.7701 | 0.2299 | 0.0904 | 5 | 16 |
| 89.000 | 0.7219 | 0.2781 | 0.0967 | 6 | 15 |
| 96.000 | . | . | . | 7 | 14 |
| 96.000 | 0.6257 | 0.3743 | 0.1051 | 8 | 13 |
| 125.000* | . | . | . | 8 | 12 |
| 128.000* | . | . | . | 8 | 11 |
| 131.000* | . | . | . | 8 | 10 |
| 140.000* | . | . | . | 8 | 9 |
| 141.000* | . | . | . | 8 | 8 |
| 143.000 | 0.5475 | 0.4525 | 0.1175 | 9 | 7 |
| 145.000* | . | . | . | 9 | 6 |

# Stratum 1= event time 1

Event time 1:

1 died from each group. (22 at risk in each group)

| | Event | No Event |
|---|---|---|
| treated | **1** | 21 |
| control | 1 | 21 |

44

$$a_1 = 1$$

$$E(a_1) = \frac{(22)*(2)}{44} = 1$$

$$Var(a_1) = \frac{(22)*(22)*(2)*(42)}{44^2(43)} = .244$$

# Event time 2 (3 months), control group:

At risk=21

Next event at month 3.

| time | Survival | Failure | Survival Standard Error | Number Failed | Number Left |
|---|---|---|---|---|---|
| 0.000 | 1.0000 | 0 | 0 | 0 | 22 |
| 2.000 | 0.9545 | 0.0455 | 0.0444 | 1 | **21** |
| **3.000** | **0.9091** | **0.0909** | **0.0613** | **2** | **20** |
| 4.000 | 0.8636 | 0.1364 | 0.0732 | 3 | 19 |
| 7.000 | 0.8182 | 0.1818 | 0.0822 | 4 | 18 |
| 10.000 | 0.7727 | 0.2273 | 0.0893 | 5 | 17 |
| 22.000 | 0.7273 | 0.2727 | 0.0950 | 6 | 16 |
| 28.000 | 0.6818 | 0.3182 | 0.0993 | 7 | 15 |
| 29.000 | 0.6364 | 0.3636 | 0.1026 | 8 | 14 |
| 32.000 | 0.5909 | 0.4091 | 0.1048 | 9 | 13 |
| 37.000 | 0.5455 | 0.4545 | 0.1062 | 10 | 12 |
| 40.000 | 0.5000 | 0.5000 | 0.1066 | 11 | 11 |
| 41.000 | 0.4545 | 0.5455 | 0.1062 | 12 | 10 |
| 54.000 | 0.4091 | 0.5909 | 0.1048 | 13 | 9 |
| 61.000 | 0.3636 | 0.6364 | 0.1026 | 14 | 8 |
| 63.000 | 0.3182 | 0.6818 | 0.0993 | 15 | 7 |
| 71.000 | 0.2727 | 0.7273 | 0.0950 | 16 | 6 |
| 127.000* | . | . | . | 16 | 5 |

# Event time 2 (3 months), treated group:

No events at 3 months

At risk=21

| time | Survival | Failure | Survival Standard Error | Number Failed | Num Le |
|---|---|---|---|---|---|
| 0.000 | 1.0000 | 0 | 0 | 0 | 22 |
| 2.000 | 0.9545 | 0.0455 | 0.0444 | 1 | **21** |
| 6.000 | 0.9091 | 0.0909 | 0.0613 | 2 | 20 |
| 12.000 | 0.8636 | 0.1364 | 0.0732 | 3 | 19 |
| 54.000 | 0.8182 | 0.1818 | 0.0822 | 4 | 18 |
| 56.000* | . | . | . | 4 | 17 |
| 68.000 | 0.7701 | 0.2299 | 0.0904 | 5 | 16 |
| 89.000 | 0.7219 | 0.2781 | 0.0967 | 6 | 15 |
| 96.000 | . | . | . | 7 | 14 |
| 96.000 | 0.6257 | 0.3743 | 0.1051 | 8 | 13 |
| 125.000* | . | . | . | 8 | 12 |
| 128.000* | . | . | . | 8 | 11 |
| 131.000* | . | . | . | 8 | 10 |
| 140.000* | . | . | . | 8 | 9 |
| 141.000* | . | . | . | 8 | 8 |
| 143.000 | 0.5475 | 0.4525 | 0.1175 | 9 | 7 |
| 145.000* | . | . | . | 9 | 6 |

# Stratum 2= event time 2

Event time 2:

At 3 months, 1 died in the control group.

At that time 21 from each group were at risk

| | Event | No Event |
|---|---|---|
| treated | **0** | 21 |
| control | 1 | 20 |

42

$$a_1 = 0$$

$$E(a_1) = \frac{(1)*(21)}{42} = .5$$

$$Var(a_1) = \frac{(21)*(21)*(1)*(41)}{42^2(41)} = .25$$

# Event time 3 (4 months), control group:

At risk=20

| time | Survival | Failure | Survival Standard Error | Number Failed | Num Le |
|---|---|---|---|---|---|
| 0.000 | 1.0000 | 0 | 0 | 0 | 22 |
| 2.000 | 0.9545 | 0.0455 | 0.0444 | 1 | 21 |
| 3.000 | 0.9091 | 0.0909 | 0.0613 | 2 | **20** |
| **4.000** | **0.8636** | **0.1364** | **0.0732** | **3** | **19** |
| 7.000 | 0.8182 | 0.1818 | 0.0822 | 4 | 18 |
| 10.000 | 0.7727 | 0.2273 | 0.0893 | 5 | 17 |
| 22.000 | 0.7273 | 0.2727 | 0.0950 | 6 | 16 |
| 28.000 | 0.6818 | 0.3182 | 0.0993 | 7 | 15 |
| 29.000 | 0.6364 | 0.3636 | 0.1026 | 8 | 14 |
| 32.000 | 0.5909 | 0.4091 | 0.1048 | 9 | 13 |
| 37.000 | 0.5455 | 0.4545 | 0.1062 | 10 | 12 |
| 40.000 | 0.5000 | 0.5000 | 0.1066 | 11 | 11 |
| 41.000 | 0.4545 | 0.5455 | 0.1062 | 12 | 10 |
| 54.000 | 0.4091 | 0.5909 | 0.1048 | 13 | 9 |
| 61.000 | 0.3636 | 0.6364 | 0.1026 | 14 | 8 |
| 63.000 | 0.3182 | 0.6818 | 0.0993 | 15 | 7 |
| 71.000 | 0.2727 | 0.7273 | 0.0950 | 16 | 6 |
| 127.000* | . | . | . | 16 | 5 |

1 event at month 4.

# Event time 3 (4 months), treated group:

At risk=21

| time | Survival | Failure | Survival Standard Error | Number Failed | Number Left |
|---|---|---|---|---|---|
| 0.000 | 1.0000 | 0 | 0 | 0 | 22 |
| 2.000 | 0.9545 | 0.0455 | 0.0444 | 1 | **21** |
| 6.000 | 0.9091 | 0.0909 | 0.0613 | 2 | 20 |
| 12.000 | 0.8636 | 0.1364 | 0.0732 | 3 | 19 |
| 54.000 | 0.8182 | 0.1818 | 0.0822 | 4 | 18 |
| 56.000* | . | . | . | 4 | 17 |
| 68.000 | 0.7701 | 0.2299 | 0.0904 | 5 | 16 |
| 89.000 | 0.7219 | 0.2781 | 0.0967 | 6 | 15 |
| 96.000 | . | . | . | 7 | 14 |
| 96.000 | 0.6257 | 0.3743 | 0.1051 | 8 | 13 |
| 125.000* | . | . | . | 8 | 12 |
| 128.000* | . | . | . | 8 | 11 |
| 131.000* | . | . | . | 8 | 10 |
| 140.000* | . | . | . | 8 | 9 |
| 141.000* | . | . | . | 8 | 8 |
| 143.000 | 0.5475 | 0.4525 | 0.1175 | 9 | 7 |
| 145.000* | . | . | . | 9 | 6 |

# Stratum 3= event time 3 (4 months)

Event time 3:

At 4 months, 1 died in the control group.

At that time 21 from the treated group and 20 from the control group were at-risk.

| | Event | No Event |
|---|---|---|
| treated | **0** | 21 |
| control | 1 | 19 |

41

$$a_1 = 0$$

$$E(a_1) = \frac{(1)*(21)}{41} = .51$$

$$Var(a_1) = \frac{(21)*(20)*(1)*(40)}{41^2(40)} = .25$$

# Etc., 1 stratum per event time

$$\frac{[\sum_{i=1}^{22}(a_k - E(a_k))]^2}{\sum_{i=1}^{22} Var(a_k)} = \frac{[(1-1)+(0-.5)+(0-.51)+..............]^2}{.244+.25+.25+.....} = 4.66$$

Then find P value → P < 0.05 ➔ Significant