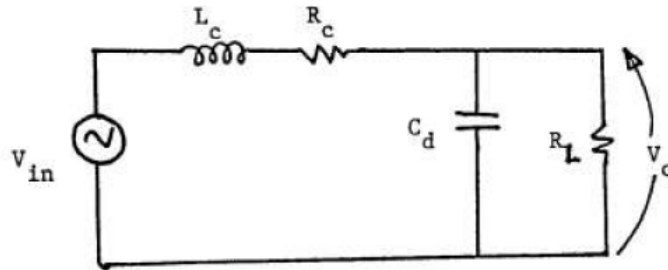


7.4 This system is the same as that of Example 7.1. Thus we have $f_n = 91$ Hz and $\zeta = 0.033$ for the system without a leak. The system with a leak may be modeled as shown below.



This circuit is similar to the circuit in Fig. 7.8 with R_L connected in parallel to C_d . R_L represents the pinhole leak at the junction of the catheter and sensor. We need to find new relationships for f_n and ζ for this circuit.

$$V_o(j\omega)/V_i(j\omega) = \frac{(1/C_d)L_c}{-\omega^2 + j\omega \left(\frac{R_L R_c C_d + L_c}{R_L C_d L_c} \right) + \frac{R_L + R_c}{R_L C_d L_c}}$$

The standard form for a second order system is

$$V_o(j\omega)/V_i(j\omega) = \frac{K \omega_n^2}{-\omega^2 + 2\zeta \omega_n(j\omega) + \omega_n^2}$$

Thus $\omega_n = \sqrt{(R_L + R_c)/R_L C_d L_c}$

and $\zeta = (R_L R_c C_d)/(2 R_L C_d L_c) \sqrt{(R_L C_d L_c)/(R_L + R_c)}$

The values for R_L , R_c , C_d and L_c must be determined

$$C_d = 1/E_d = 2.04 \times 10^{-15} \text{ m}^5/\text{N}$$

$$L_c = \rho L/\pi r^2 = (1 \times 10^3)(1)/\pi (0.046 \times 10^{-2})^2 = 1.5 \times 10^9 \text{ Pa}\cdot\text{s}^2/\text{m}^3$$

$$R_c = 8\eta L/\pi r^4 = 8(0.001(1)/\pi (0.046 \times 10^{-2})^4 = 4.69 \times 10^{10} \text{ Pa}\cdot\text{s}/\text{m}^3$$

$$R_L = \text{Pressure/Flow} = 13.3 \text{ kPa}/(0.4 \text{ ml/min})(1 \text{ min}/60 \text{ s}) \\ \times (1 \times 10^{-3} \text{ m}^3/1000 \text{ ml}) \\ = 2 \times 10^{12} \text{ Pa}\cdot\text{s}/\text{m}^3$$

Thus for the system with a leak

$$\omega_n = 580 \text{ r/s or } f_n = 92.4 \text{ Hz}$$

and

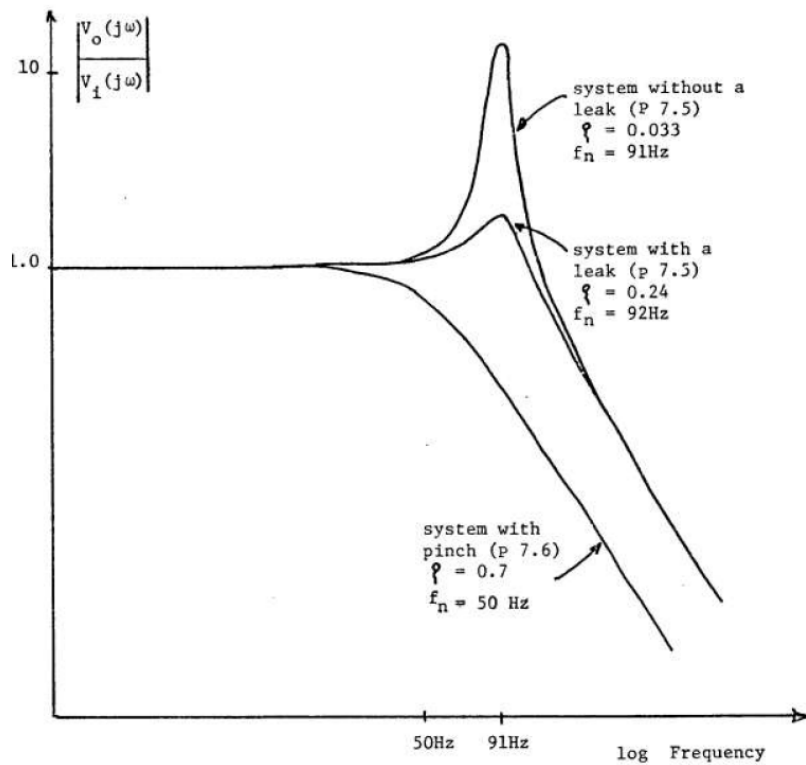
$$\zeta = 0.24$$

For reference the parameters for the system without a leak

$$f_n = 91 \text{ Hz}$$

$$\zeta = 0.033$$

The frequency response curves for the system with and without the leak are given below.



8.4

$$\begin{aligned}
 F &= \frac{Q}{\rho_b c_b \int_0^{t_1} \Delta T_b dt} = \frac{V_i T_i \rho_i c_i}{\rho_b c_b \int_0^{t_1} \Delta T_b dt} \\
 &= \frac{(10 \text{ ml})(-30 \text{ K})(1.005 \text{ g/ml})(4.172 \text{ J/(g}\cdot\text{K)}}{(1.060 \text{ g/ml})(3.640 \text{ J/(g}\cdot\text{K)})(-5.0 \text{ s}\cdot\text{K)}} \\
 &= 65.2 \text{ ml/s} \\
 (62.5 \text{ ml/s})(60 \text{ s/min}) &= 3912 \text{ ml/min} = 3.9 \text{ l/min}
 \end{aligned}$$

8.7

$$\begin{aligned}
 e &= Blu \\
 &= (0.03 \text{ T})(0.015 \text{ m})(1 \text{ m/s}) \\
 &= 0.00045 \text{ V} = 450 \mu\text{V}
 \end{aligned}$$