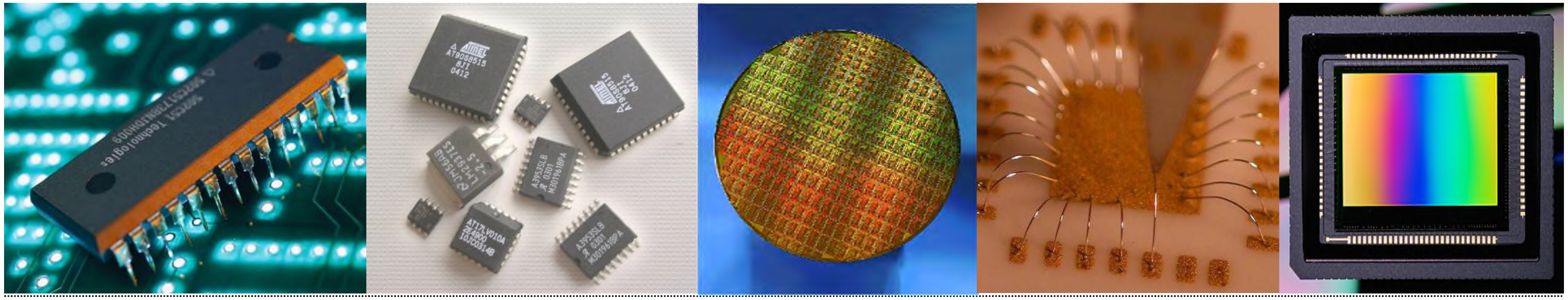


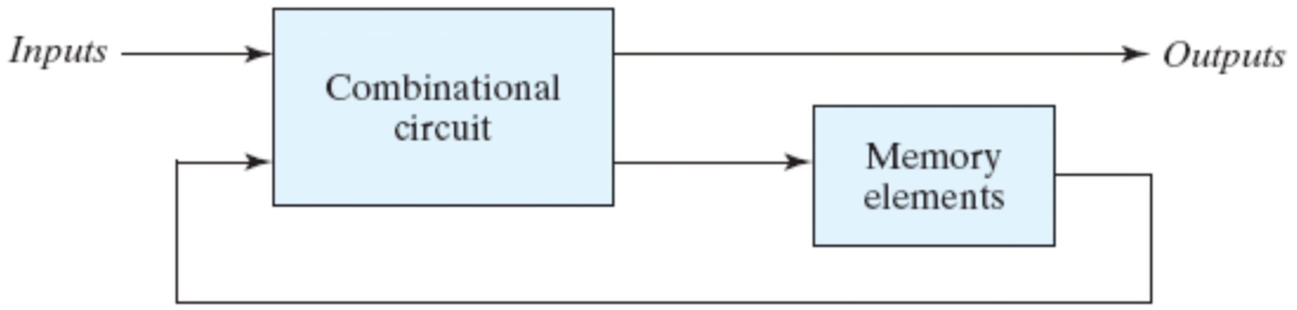
Ch 5. Synchronous sequential logic

5.1 Introduction

- In order to perform useful or flexible sequences of operations, we need to be able to construct circuits that can store information between the operations.
- latches and Flip-Flops
- Sequential circuits consisting of both flip-flop and combinational logic

5.2 Sequential circuits

- Outputs are function of inputs and present states
- Present states are supplied by memory elements



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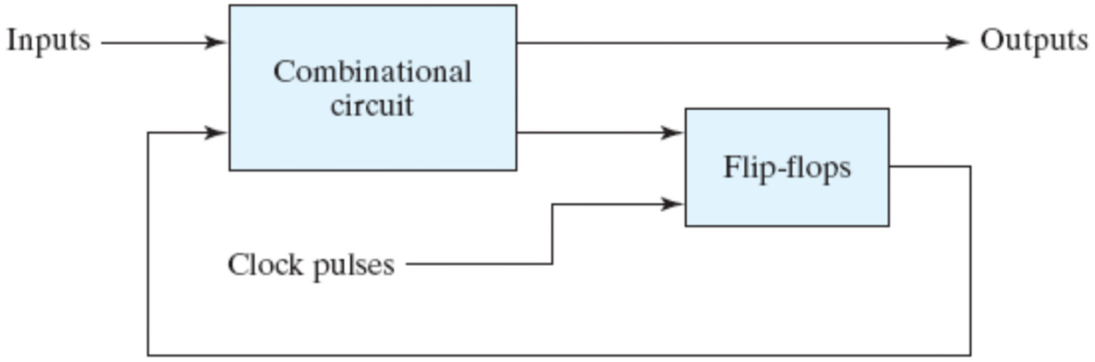
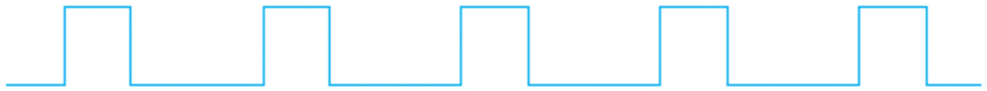
5.2 Sequential circuits

- Two types of sequential circuit
- Synchronous : behavior depends on the signals affecting storage elements at discrete time
- Asynchronous : behavior depends on inputs at any instance of time 시간에 관계 없이 인풋이 들어오는 대로 출력이 변함



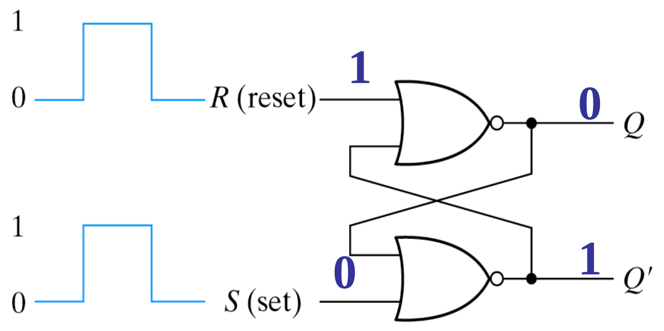
(a) Block diagram



(b) Timing diagram of clock pulses

5.3 Latches

- SR latch : consist of two cross-coupled NOR gates
- S=1,R=0 then Q=1(set)
- S=0,R=1 then Q=0(reset)
- S=0,R=0 then no change(keep condition)
- S=1,R=1 Q=Q′=0 (undefined)



(a) Logic diagram

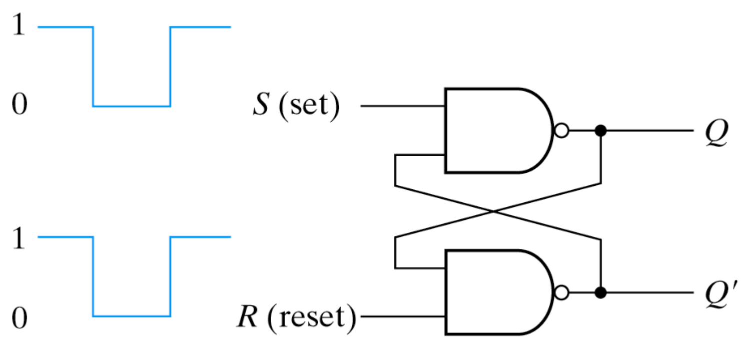
S	R	Q	Q'	
1	0	1	0	
0	0	1	0	(after $S = 1, R = 0$)
0	1	0	1	
0	0	0	1	(after $S = 0, R = 1$)
1	1	0	0	

(b) Function table

Fig. 5-3 *SR* Latch with NOR Gates

5.3 Latches - SR latch

- S′R′ latch with NAND gates
  - Require the complement value of NOR latch



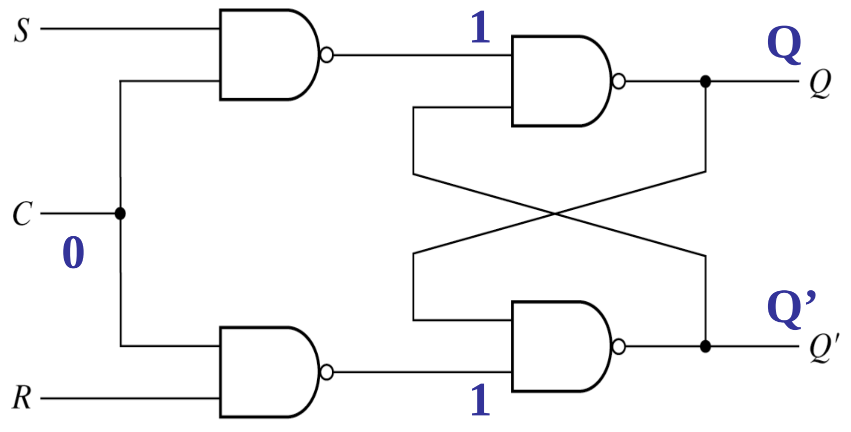
S	R	Q	Q'	
1	0	0	1	
1	1	0	1	(after $S = 1, R = 0$)
0	1	1	0	
1	1	1	0	(after $S = 0, R = 1$)
0	0	1	1	

(a) Logic diagram (b) Function table

Fig. 5-4 *SR* Latch with NAND Gates

5.3 Latches - SR latch

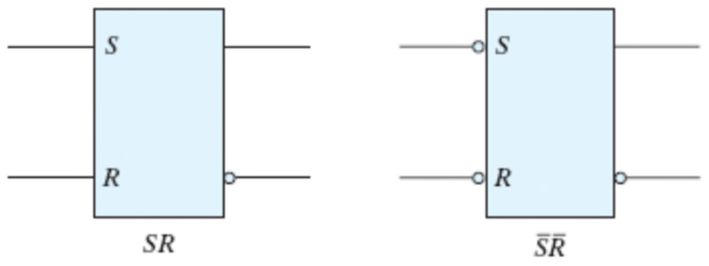
- SR latch with control input
  - Add two NAND gate and control signal
  - C=0(no action), C=1(act as SR latch)



(a) Logic diagram

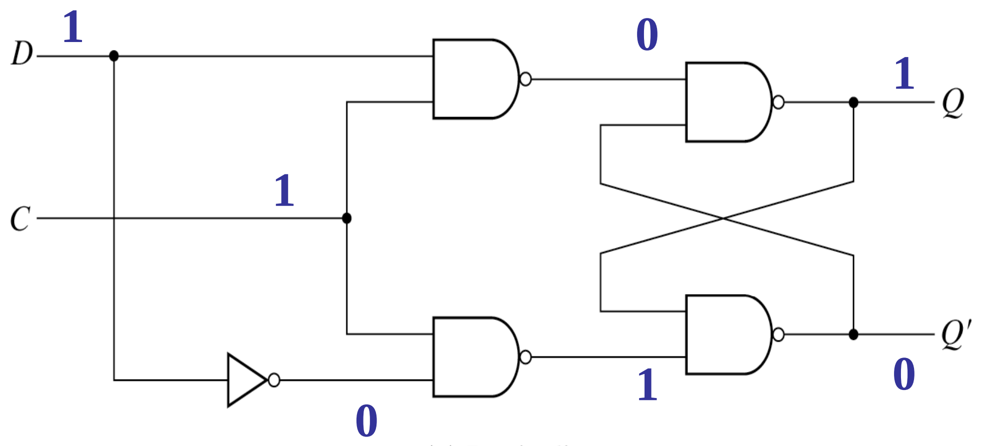
C	S	R	Next state of Q
0	X	X	No change
1	0	0	No change
1	0	1	$Q = 0$; Reset state
1	1	0	$Q = 1$; set state
1	1	1	Indeterminate

(b) Function table



5.3 Latches - D latch

- Eliminate indeterminate state in SR latch
  - C=1, output value is equal to D

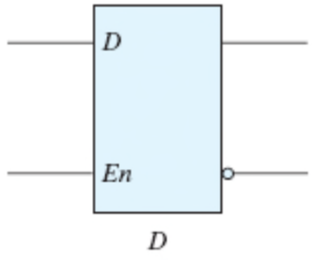


(a) Logic diagram

C	D	Next state of Q
0	X	No change
1	0	$Q = 0$; Reset state
1	1	$Q = 1$; Set state

(b) Function table

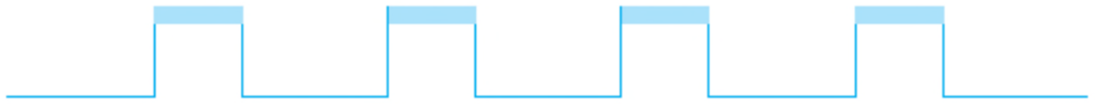
Fig. 5-6 D Latch

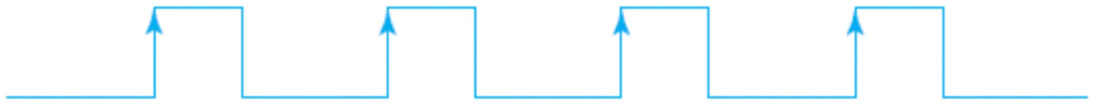


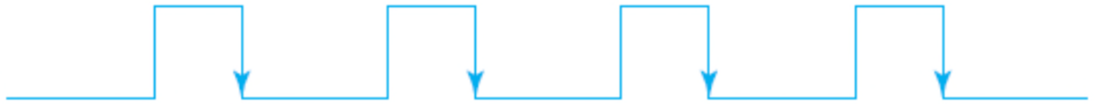
5.4 Flip-Flops

- Latch : case (a), output changes as input changes
- Flip-flop : output only changes at clock edge

Latch 는 값이 1인 동안에는 계속 출력이 변화한다 - asynchronous 에는 적합하지 않음


(a) Response to positive level


(b) Positive-edge response


(c) Negative-edge response

5.4 Flip-flops – Master-Slave Flip-Flop

- Negative edge triggered D flip-flop
- C=0 : master disable, slave enable
- Output has no relation with input
- C=1 : master enable, slave disable

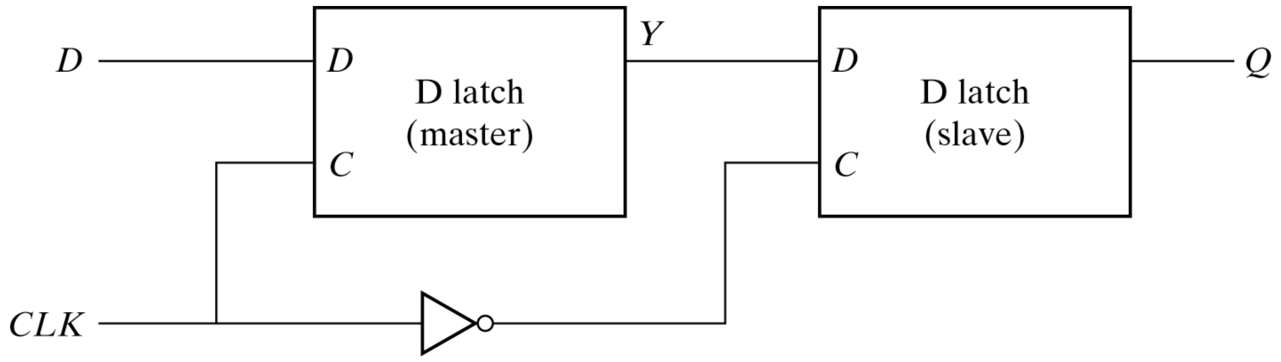
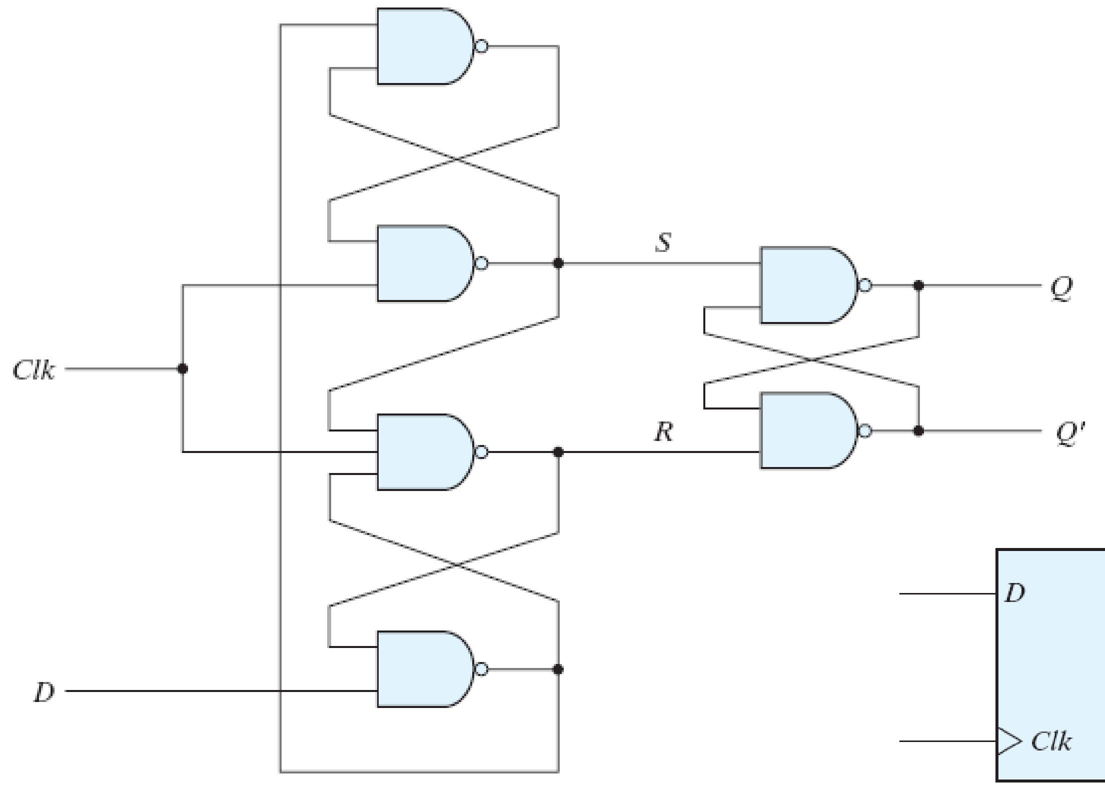
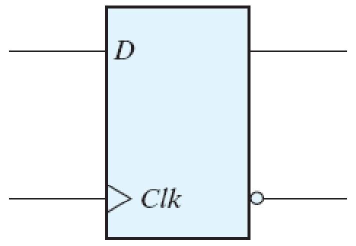


Fig. 5-9 Master-Slave *D* Flip-Flop

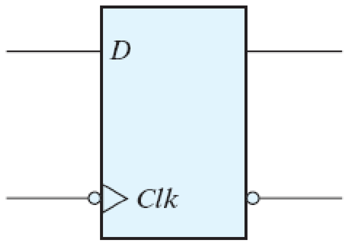
5.4 Flip-flops - Edge-Triggered Flip-Flop

- D-type positive edge triggered flip flop
  - Consist of 3 SR-latches
  - Q changes only when C becomes 0 to 1





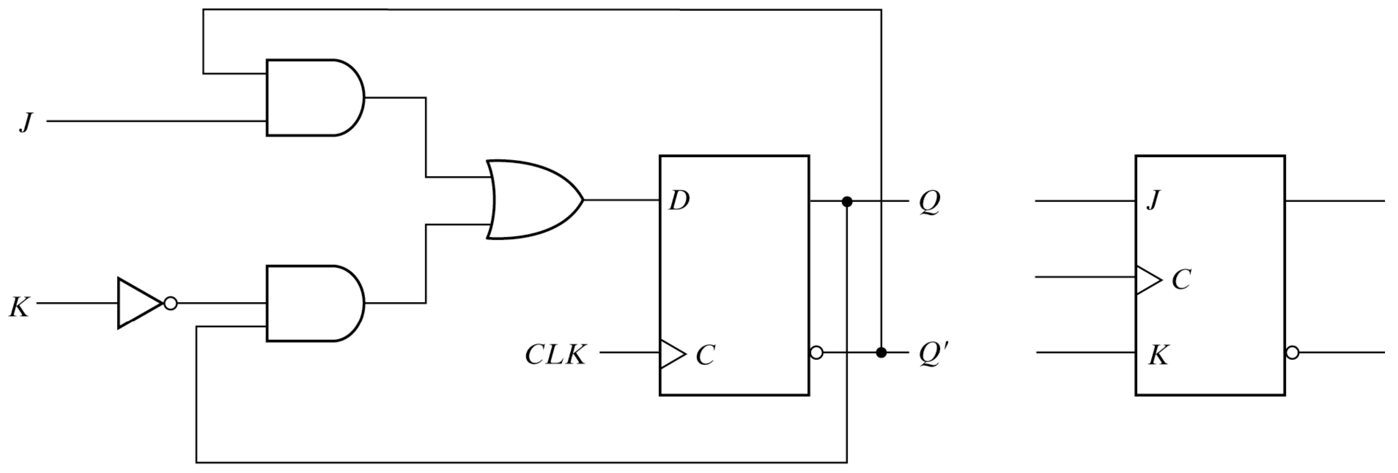
(a) Positive-edge



(a) Negative-edge

5.4 Flip-flops - Other Flip-Flop

- JK flip-flop
  - Performs three operations
  - Set(J), Reset(K), Complement(J=K=1)
  - D=JQ′+K′Q



(a) Circuit diagram (b) Graphic symbol

Fig. 5-12 *JK* Flip-Flop

5.4 Flip-flops - Other Flip-Flop

- T flip-flop
  - Complementing flip-flop
  - D=TQ′+T′Q

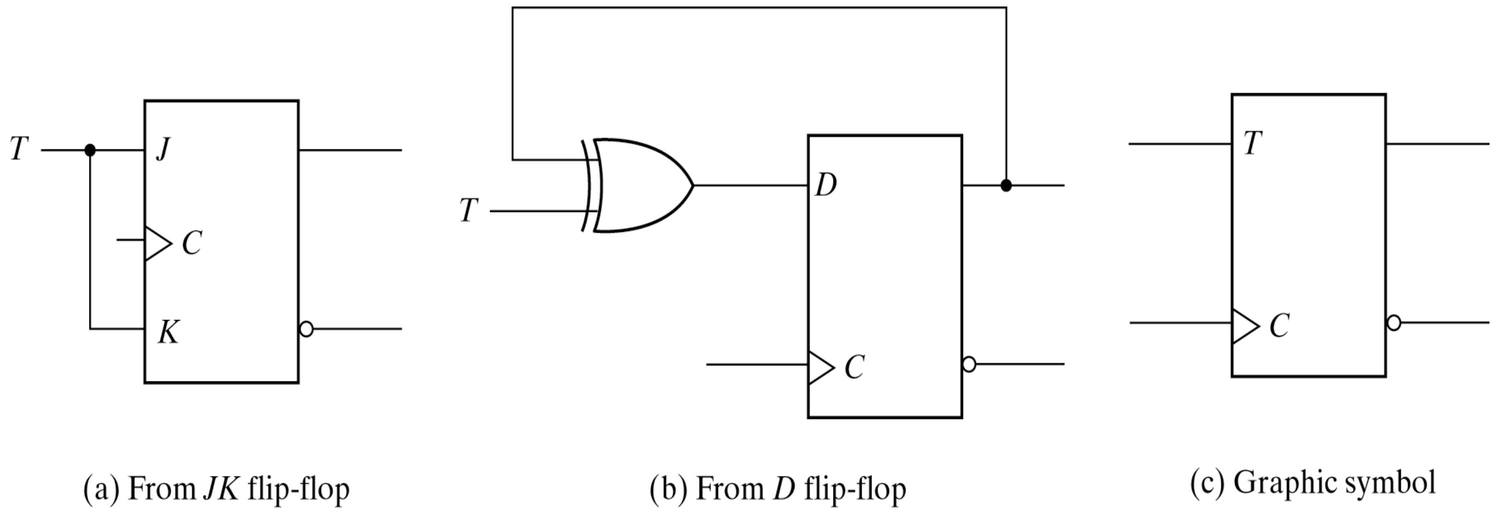
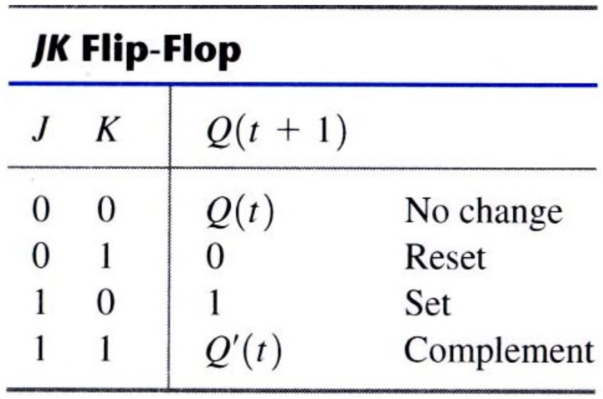


Fig. 5-13 T Flip-Flop

5.4 Flip-flops - Characteristic tables

- Flip-flop characteristic tables

$$Q(t+1) = JQ' + K'Q$$

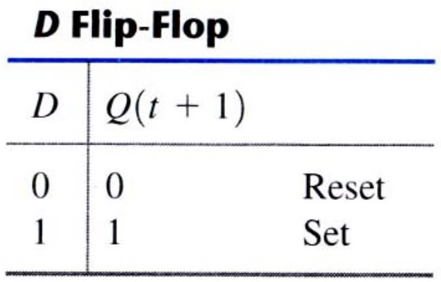


***JK* Flip-Flop**

J	K	$Q(t+1)$	
0	0	$Q(t)$	No change
0	1	0	Reset
1	0	1	Set
1	1	$Q'(t)$	Complement

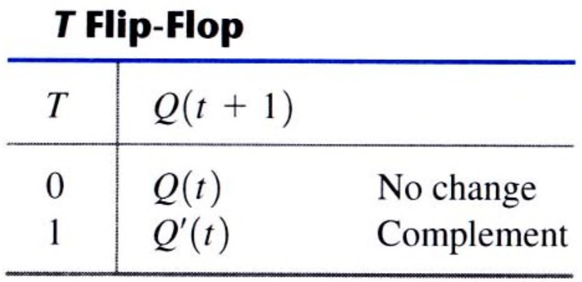
***SR* Flip-Flip**

S	R	Q(t+1)	Operation
0	0	Q(t)	No change
0	1	0	Reset
1	0	1	Set
1	1	?	Undefined



***D* Flip-Flop**

D	$Q(t+1)$	
0	0	Reset
1	1	Set



***T* Flip-Flop**

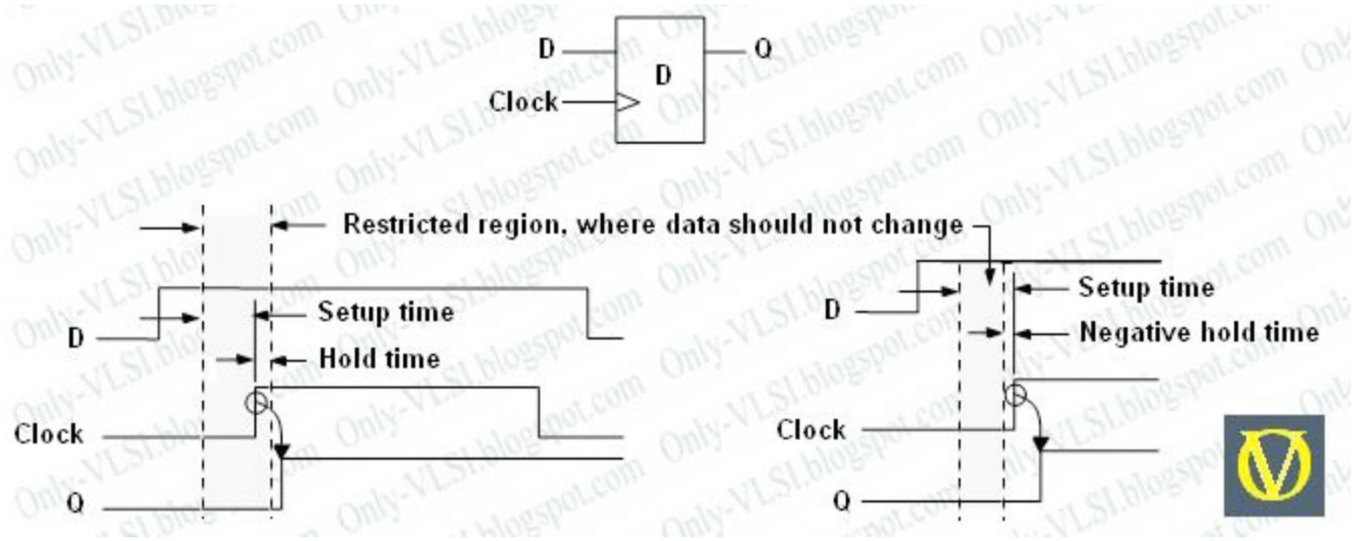
T	$Q(t+1)$	
0	$Q(t)$	No change
1	$Q'(t)$	Complement

$$Q(t+1) = D$$

$$Q(t+1) = TQ' + T'Q$$

5.4 Flip-flops - Terminologies

- *Setup time*: Time in which D input must be maintained at a constant value prior to applying the clock.
- *Hold time*: Time when D input must not change after the application of the positive transition of the pulse.
- *Propagation delay time*: Time interval between the trigger edge and the stabilization of the output to the new state.



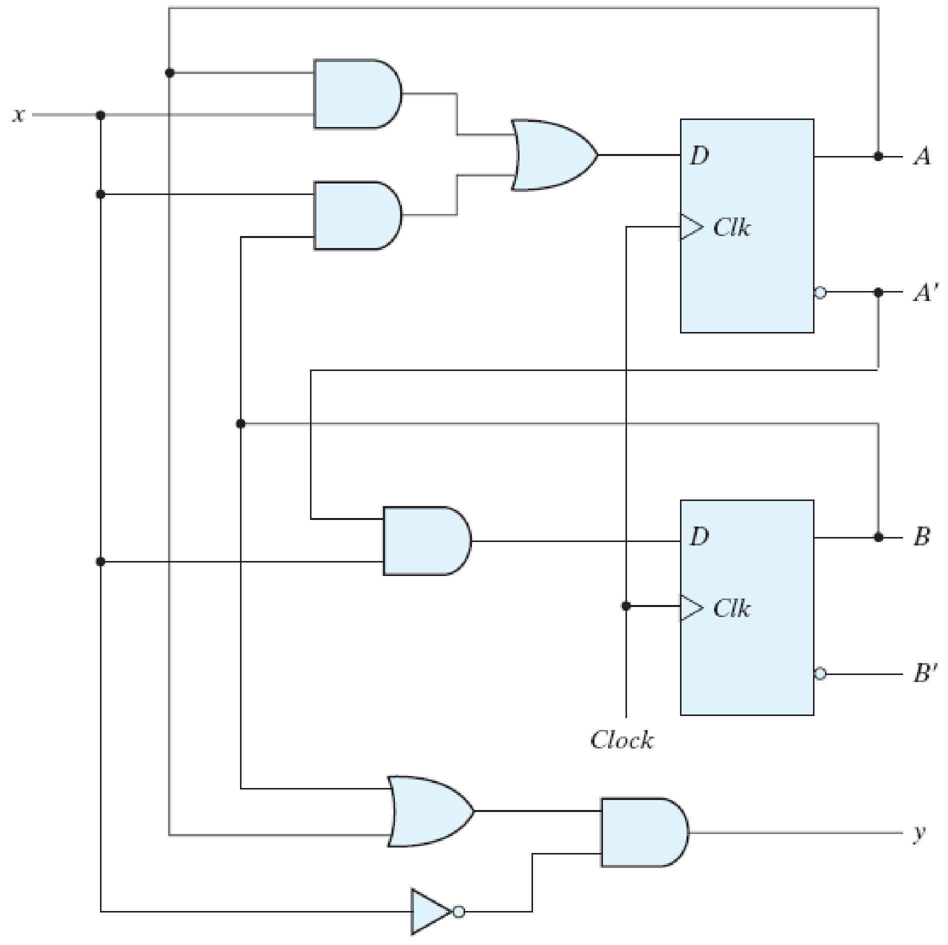
(Source: only-vlsi.blogspot.com)

5.5 Analysis of clocked sequential circuits

- Behavior of clocked sequential circuit is determined from input, output and present state
- Output, next state are a function of input and present state

5.5 Analysis of clocked sequential circuits - State equations

- Specifies the next state and output as a function of the present state and inputs
- A(t+1)=A*x* + B*x*
- B(t+1)=A′*x*
- *Y*=(A+B)*x*′



5.5 Analysis of clocked sequential circuits - State table

- Time sequence table of inputs, outputs and flip-flop states
- two types of state table exist

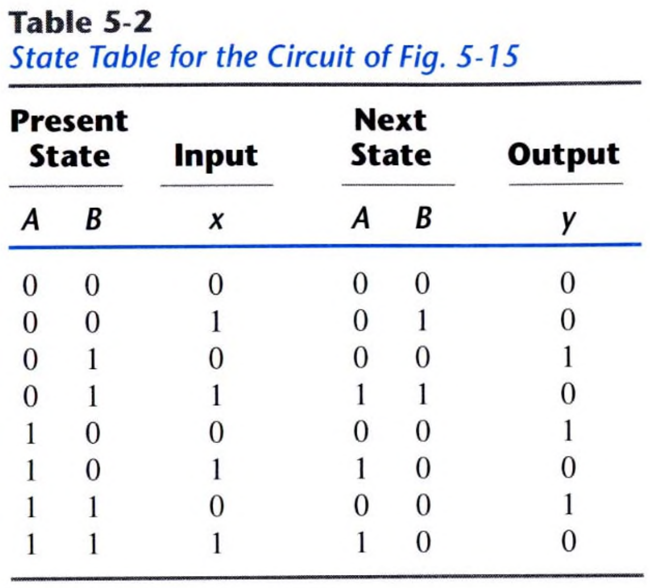


Table 5-2
State Table for the Circuit of Fig. 5-15

Present State		Input	Next State		Output
A	*B*	*x*	*A*	*B*	*y*
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

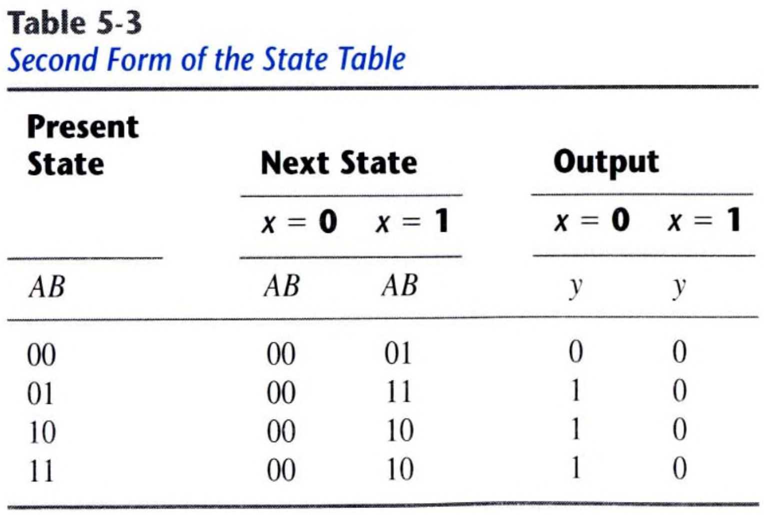


Table 5-3
Second Form of the State Table

Present State	Next State		Output	
	$x = 0$	$x = 1$	$x = 0$	$x = 1$
AB	*AB*	*AB*	*y*	*y*
00	00	01	0	0
01	00	11	1	0
10	00	10	1	0
11	00	10	1	0

5.5 Analysis of clocked sequential circuits - State diagram

- A kind of flow diagram
- Can be derived from state table
- State-circle, transition-line, I/O

Table 5-2
State Table for the Circuit of Fig. 5-15

Present State		Input	Next State		Output
A	*B*	*x*	*A*	*B*	*y*
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

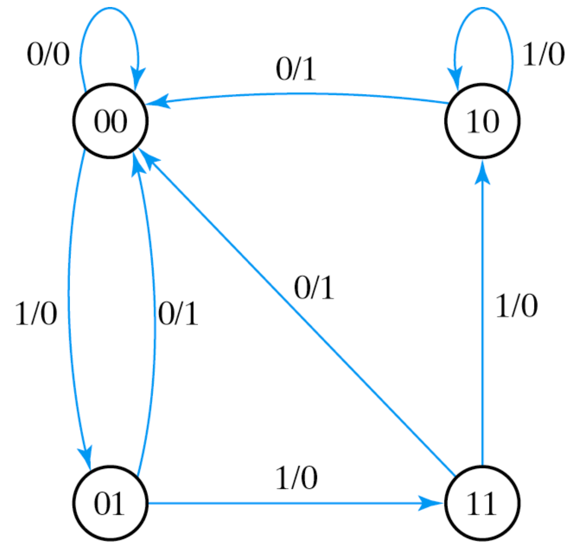
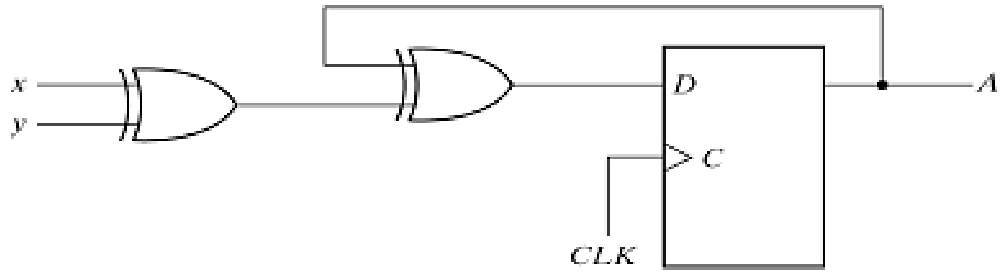


Fig. 5-16 State Diagram of the Circuit of Fig. 5-15

5.5 Analysis of clocked sequential circuits - Analysis with D flip-flops

- Input equation : $D_A = A \oplus x \oplus y$
- State equation is equal to input equation



(a) Circuit diagram

Present state	Inputs		Next state
A	x	y	A
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

(b) State table

$$\begin{tikzcd}
0 \arrow[loop left, "{00, 11}"] \arrow[r, bend left, "{01, 10}"] & 1 \arrow[loop right, "{00, 11}"] \arrow[l, bend left, "{01, 10}"]
\end{tikzcd}$$

(c) State diagram

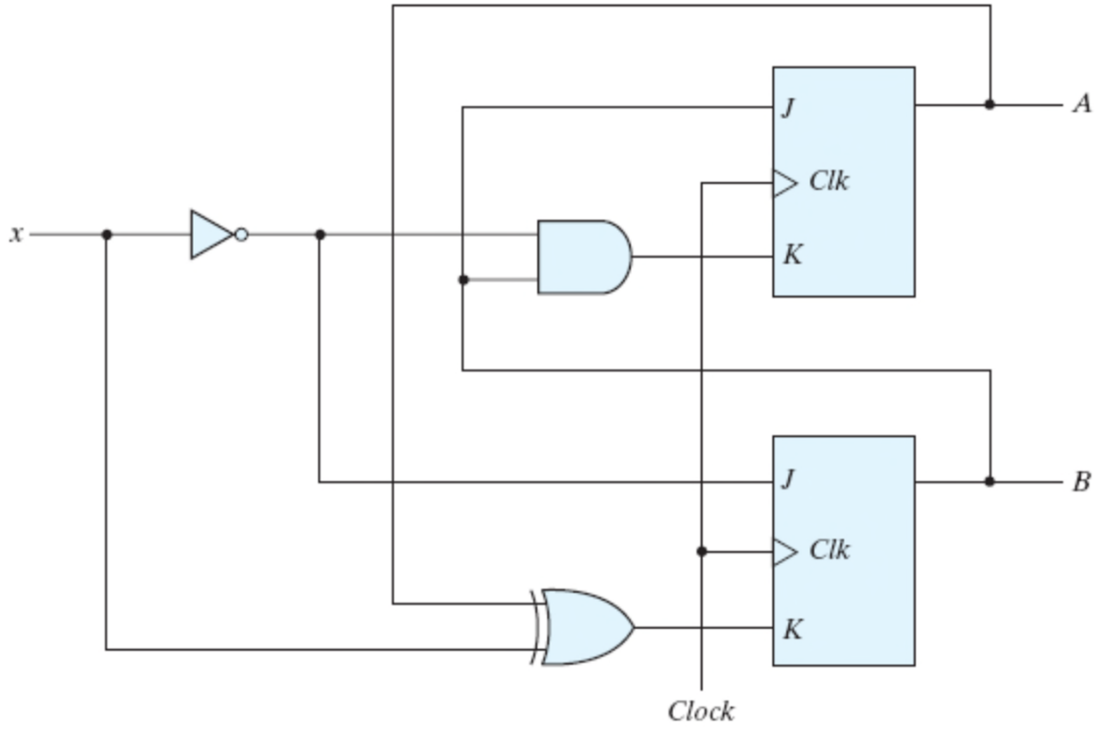
Fig. 5-17 Sequential Circuit with D Flip-Flop

5.5 Analysis of clocked sequential circuits - Analysis with JK flip-flops

- State equation is not the same as the input equation
- Have to refer characteristic table or characteristic equation
- Input equations

  $J_A = B \quad K_A = Bx'$

  $J_B = x' \quad K_B = A'x + Ax'$



5.5 Analysis of clocked sequential circuits - Analysis with JK flip-flops

- State table and state diagram

Table 5-4
State Table for Sequential Circuit with JK Flip-Flops

Present State		Input	Next State		Flip-Flop Inputs			
A	*B*	*x*	*A*	*B*	J_A	K_A	J_B	K_B
0	0	0	0	1	0	0	1	0
0	0	1	0	0	0	0	0	1
0	1	0	1	1	1	1	1	0
0	1	1	1	0	1	0	0	1
1	0	0	1	1	0	0	1	1
1	0	1	1	0	0	0	0	0
1	1	0	0	0	1	1	1	1
1	1	1	1	1	1	0	0	0

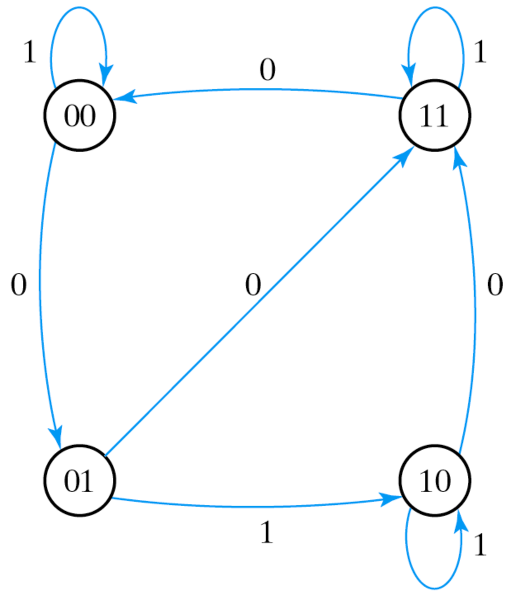


Fig. 5-19 State Diagram of the Circuit of Fig. 5-18

5.5 Analysis of clocked sequential circuits - Analysis with T flip-flops

- Input equations and output equation

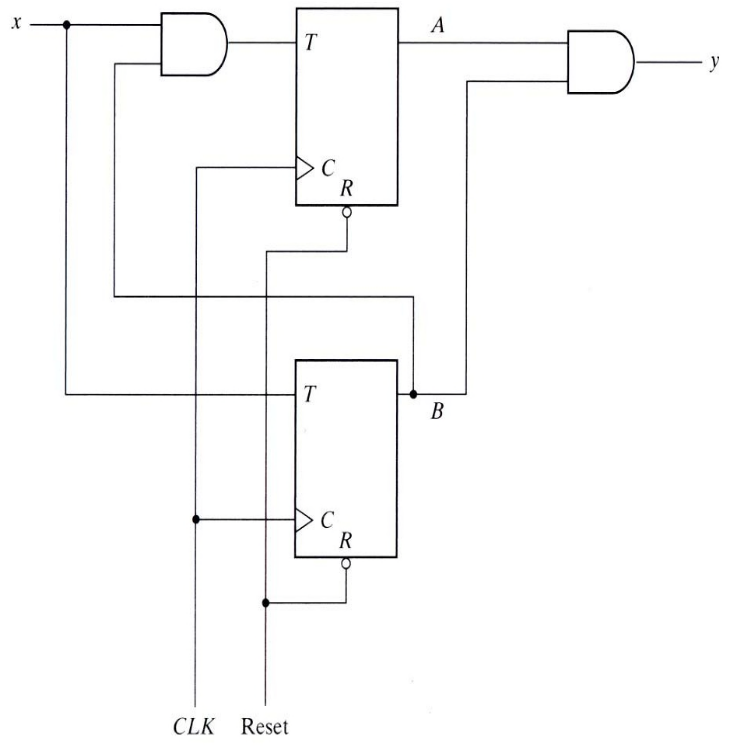
  $T_A = Bx$, $T_B = x$

  $y = AB$

- State equations are derived from characteristic equation

  $A(t+1) = T_A A' + T_A' A$

  $B(t+1) = T_B B' + T_B' B$

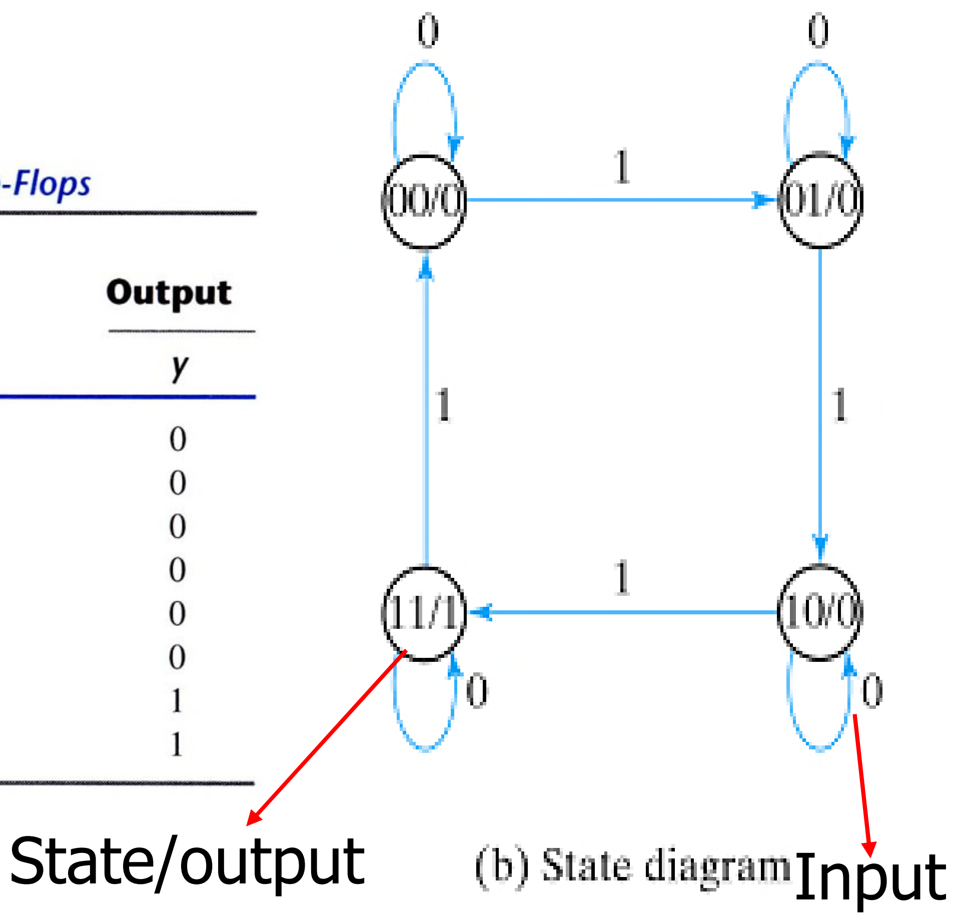


(a) Circuit diagram

5.5 Analysis of clocked sequential circuits - Analysis with T flip-flops

Table 5-5
State Table for Sequential Circuit with T Flip-Flops

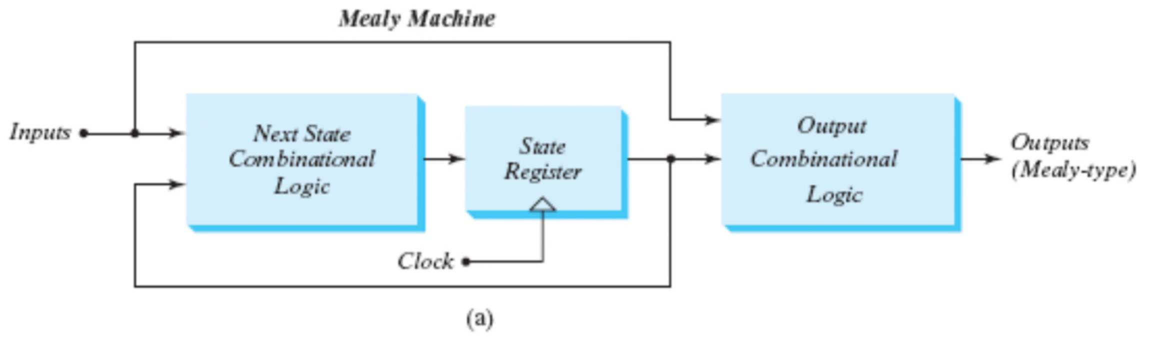
Present State		Input	Next State		Output
A	*B*	*x*	*A*	*B*	*y*
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	1	0
0	1	1	1	0	0
1	0	0	1	0	0
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1	0	0	1

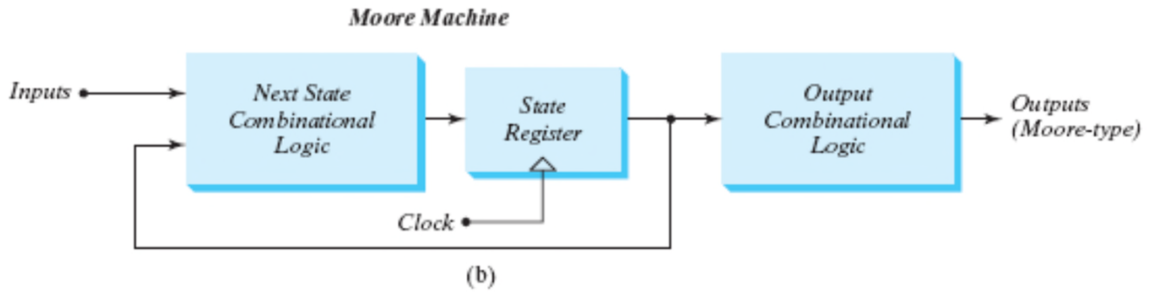


(b) State diagram

5.5 Analysis of clocked sequential circuits - Mealy and Moore models

- Mealy model : output is a function of the present state and input
- Inputs must be synchronized with the clock
- Outputs must be sampled at the clock edge
- Moore model : output is a function of the present state only
- Outputs are synchronized with the clock



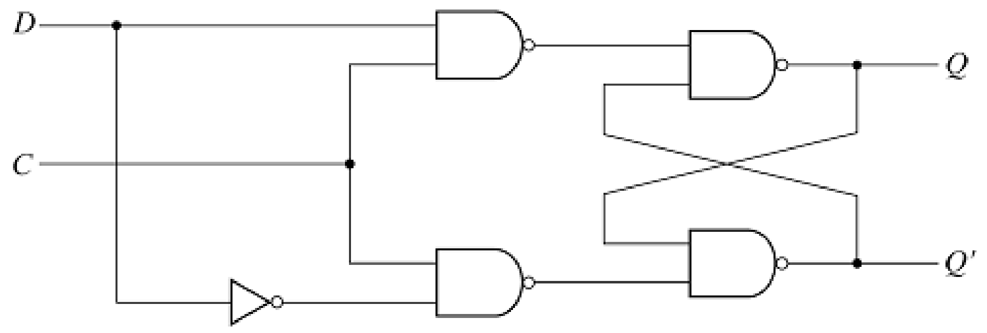


5.6 HDL for sequential circuits

- Two kinds of behavioral statements
- Initial : executes only once
- Always : executes repeatedly until the simulation terminates

5.6 HDL for sequential circuits - Flip-flops and latches

- D-latch



HDL Example 5-1

```
//Description of D latch (See Fig. 5-6)
module D_latch (Q,D,control);
   output Q;
   input D,control;
   reg Q;
   always @ (control or D)
   if (control) Q = D;      //Same as: if (control == 1)
endmodule
```

5.6 HDL for sequential circuits - Flip-flops and latches

- D flip-flop
  - D flip-flop with asynchronous reset

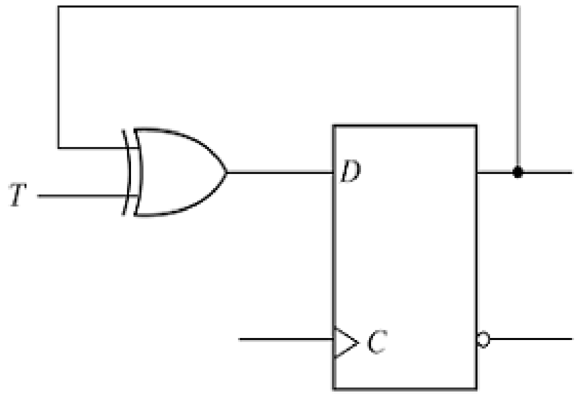
```
//D flip-flop
module D_FF (Q,D,CLK);
   output Q;
   input D,CLK;
   reg Q;
   always @ (posedge CLK)
     Q = D;
endmodule

//D flip-flop with asynchronous reset.
module DFF (Q,D,CLK,RST);
   output Q;
   input D,CLK,RST;
   reg Q;
   always @(posedge CLK or negedge RST)
     if (~RST) Q = 1'b0;     // Same as: if (RST == 0)
     else Q = D;
endmodule
```

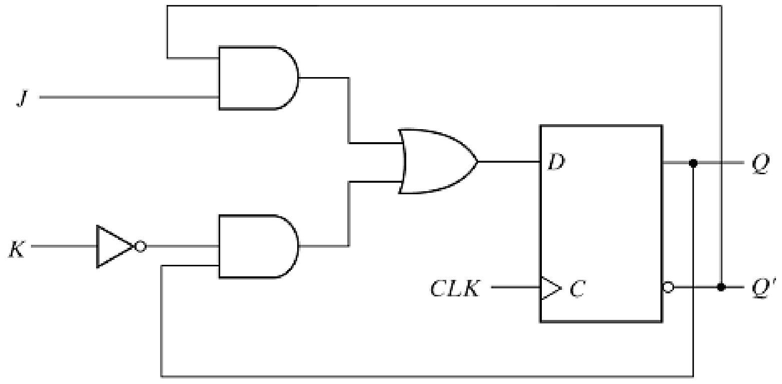
5.6 HDL for sequential circuits - Flip-flops and latches

T flip-flop from D flip-flop

```
//T flip-flop from D flip-flop and gates
module TFF (Q,T,CLK,RST);
   output Q;
   input T,CLK,RST;
   wire DT;
   assign DT = Q ^ T ;
//Instantiate the D flip-flop
   DFF TF1 (Q,DT,CLK,RST);
endmodule
```

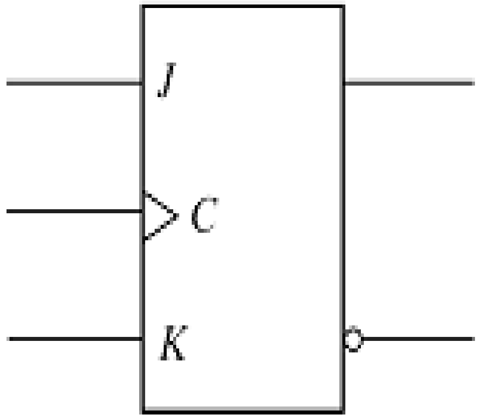


```
//JK flip-flop from D flip-flop and gates
module JKFF (Q,J,K,CLK,RST);
   output Q;
   input J,K,CLK,RST;
   wire JK;
   assign JK = (J & ~Q) | (~K & Q);
//Instantiate D flipflop
   DFF JK1 (Q,JK,CLK,RST);
endmodule
```



5.6 HDL for sequential circuits - Flip-flops and latches

- JK flip-flop



HDL Example 5-4

```
// Functional description of JK flip-flop
module  JK_FF (J,K,CLK,Q,Qnot);
   output Q,Qnot;
   input  J,K,CLK;
   reg  Q;
   assign Qnot = ~ Q ;
   always @ (posedge CLK)
           case ({J,K})
             2'b00: Q = Q;
             2'b01: Q = 1'b0;
             2'b10: Q = 1'b1;
             2'b11: Q = ~ Q;
           endcase
endmodule
```

5.6 HDL for sequential circuits - State diagram

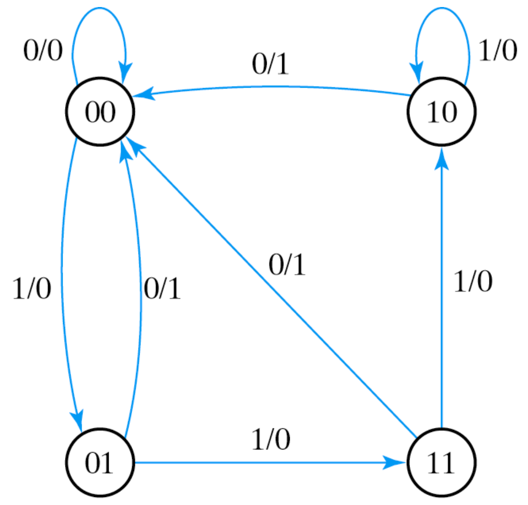


Fig. 5-16 State Diagram of the Circuit of Fig. 5-15

(Mealy state diagram)

```
module Mealy_mdl (x,y,CLK,RST);
  input x,CLK,RST;
  output y;
  reg y;
  reg [1:0] Prstate, Nxtstate;
  parameter S0 = 2'b00, S1 = 2'b01, S2 = 2'b10, S3 = 2'b11;
   always @ (posedge CLK or negedge RST)
      if (~RST) Prstate = S0;  //Initialize to state S0
      else Prstate = Nxtstate; //Clock operations
   always @ (Prstate or x)      //Determine next state
         case (Prstate)
            S0: if (x) Nxtstate = S1;
                  else Nxtstate = S0;
            S1: if (x) Nxtstate = S3;
                  else Nxtstate = S0;
            S2: if (~x)Nxtstate = S0;
                  else Nxtstate = S2;
            S3: if (x) Nxtstate = S2;
                  else Nxtstate = S0;
         endcase
   always @ (Prstate or x)      //Evaluate output
         case (Prstate)
            S0: y = 0;
            S1: if (x) y = 1'b0; else y = 1'b1;
            S2: if (x) y = 1'b0; else y = 1'b1;
            S3: if (x) y = 1'b0; else y = 1'b1;
         endcase
  endmodule
```

5.6 HDL for sequential circuits - State diagram

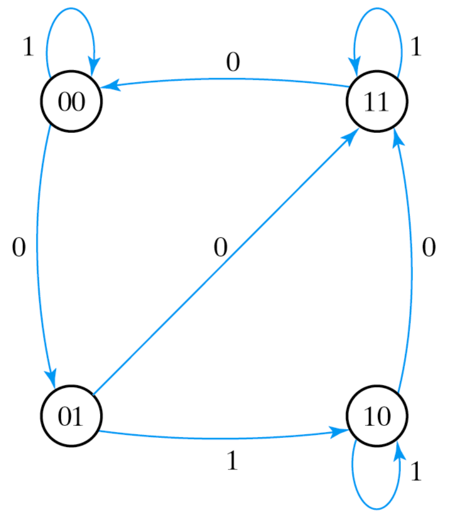
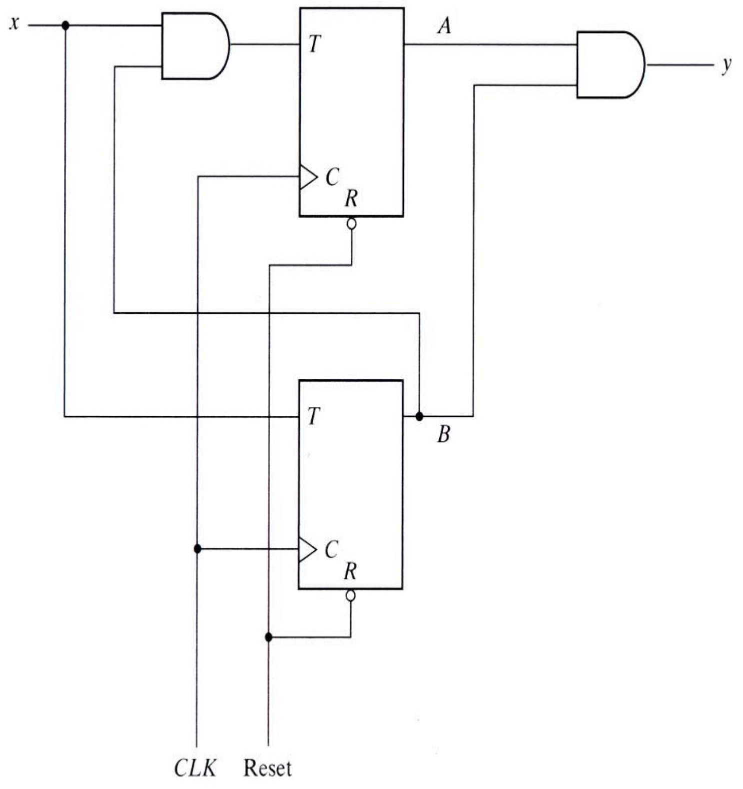


Fig. 5-19 State Diagram of the Circuit of Fig. 5-18

(Moore state diagram)

```
//Moore state diagram (Fig. 5-19)
module Moore_mdl (x,AB,CLK,RST);
   input x,CLK,RST;
   output [1:0]AB;
   reg [1:0] state;
   parameter S0 = 2'b00, S1 = 2'b01, S2 = 2'b10, S3 = 2'b11;
      always @ (posedge CLK or negedge RST)
         if (~RST) state = S0;  //Initialize to state S0
         else
         case (state)
           S0: if (~x) state = S1; else state = S0;
           S1: if (x)  state = S2; else state = S3;
           S2: if (~x) state = S3; else state = S2;
           S3: if (~x) state = S0; else state = S3;
         endcase
   assign AB = state;          //Output of flip-flops
endmodule
```

5.6 HDL for sequential circuits - Structural description



(a) Circuit diagram

```
module Tcircuit (x,y,A,B,CLK,RST);
   input x,CLK,RST;
   output y,A,B;
   wire TA,TB;
//Flip-flip input equations
   assign TB = x,
          TA = x & B;
//Output equation
   assign y = A & B;
//Instantiate T flip-flops
   T_FF BF (B,TB,CLK,RST);
   T_FF AF (A,TA,CLK,RST);
endmodule
```

```
module T_FF (Q,T,CLK,RST);
   output Q;
   input T,CLK,RST;
   reg Q;
     always @ (posedge CLK or negedge RST)
       if (~RST) Q = 1'b0;
       else Q = Q ^ T;
endmodule
```

5.6 HDL for sequential circuits - Structural description

```
module testTcircuit;
  reg x,CLK,RST;  //inputs for circuit
  wire y,A,B;    //output from circuit
  Tcircuit TC (x,y,A,B,CLK,RST);  // instantiate circuit
  initial
     begin
         RST = 0;
         CLK = 0;
      #5 RST = 1;
         repeat (16)
      #5 CLK = ~CLK;
     end
  initial
     begin
          x = 0;
      #15 x = 1;
          repeat (8)
      #10 x = ~ x;
     end
endmodule
```

(Stimulus and output)

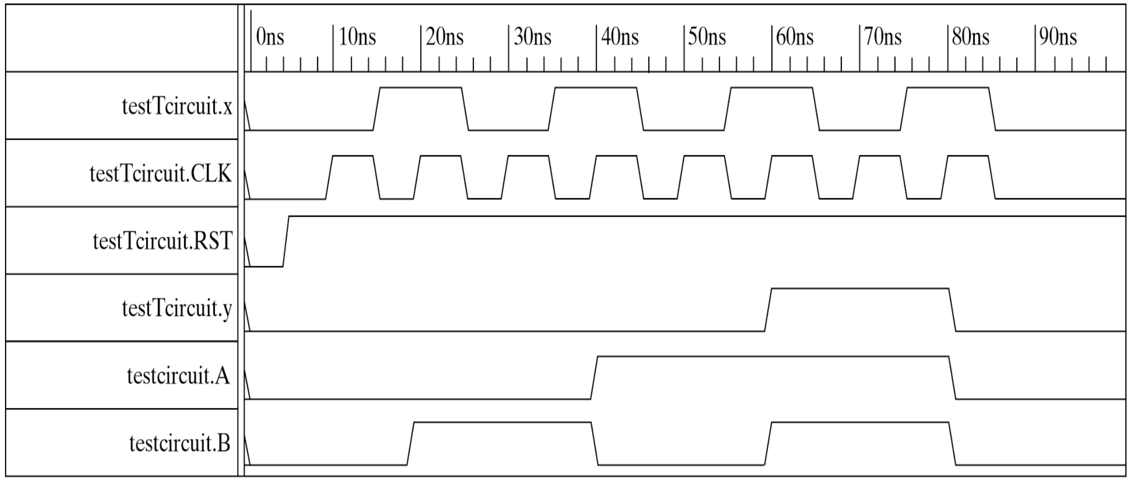


Fig. 5-21 Simulation Output of HDL Example 5-7

5.6 HDL for sequential circuits

```
module Mealy_Zero_Detector (
  output reg y_out,
  input            x_in, clock, reset
);
  reg [1: 0]       state, next_state;
  parameter        S0 = 2'b00, S1 = 2'b01, S2 = 2'b10, S3 = 2'b11;

  always @ (posedge clock, negedge reset)        // state transition
    if (reset == 0) state <= S0;
    else state <= next_state;

  always @ (state, x_in) // Form the next state
    case (state)
      S0:          if (x_in)   next_state = S1; else next_state = S0;
      S1: if (x_in)    next_state = S3; else next_state = S0;
      S2:          if (~x_in) next_state = S0; else next_state = S2;
      S3: if (x_in)   next_state = S2; else next_state = S0;
    endcase

  always @ (state, x_in) // Form the output
    case (state)
      S0:          y_out = 0;
      S1, S2, S3: y_out = ~x_in;
    endcase
endmodule

module t_Mealy_Zero_Detector;
  wire  t_y_out;
  reg   t_x_in, t_clock, t_reset;

Mealy_Zero_Detector M0 (t_y_out, t_x_in, t_clock, t_reset);

initial #200 $finish;
initial begin t_clock = 0; forever #5 t_clock = ~t_clock; end
```

```
initial fork
      t_reset = 0;
  #2 t_reset = 1;
  #87 t_reset = 0;
  #89 t_reset = 1;
  #10 t_x_in = 1;
  #30 t_x_in = 0;
  #40 t_x_in = 1;
  #50 t_x_in = 0;
  #52 t_x_in = 1;
  #54 t_x_in = 0;
  #70 t_x_in = 1;
  #80 t_x_in = 1;
  #70 t_x_in = 0;
  #90 t_x_in = 1;
  #100 t_x_in = 0;
  #120 t_x_in = 1;
  #160 t_x_in = 0;
  #170 t_x_in = 1;

  join
endmodule
```

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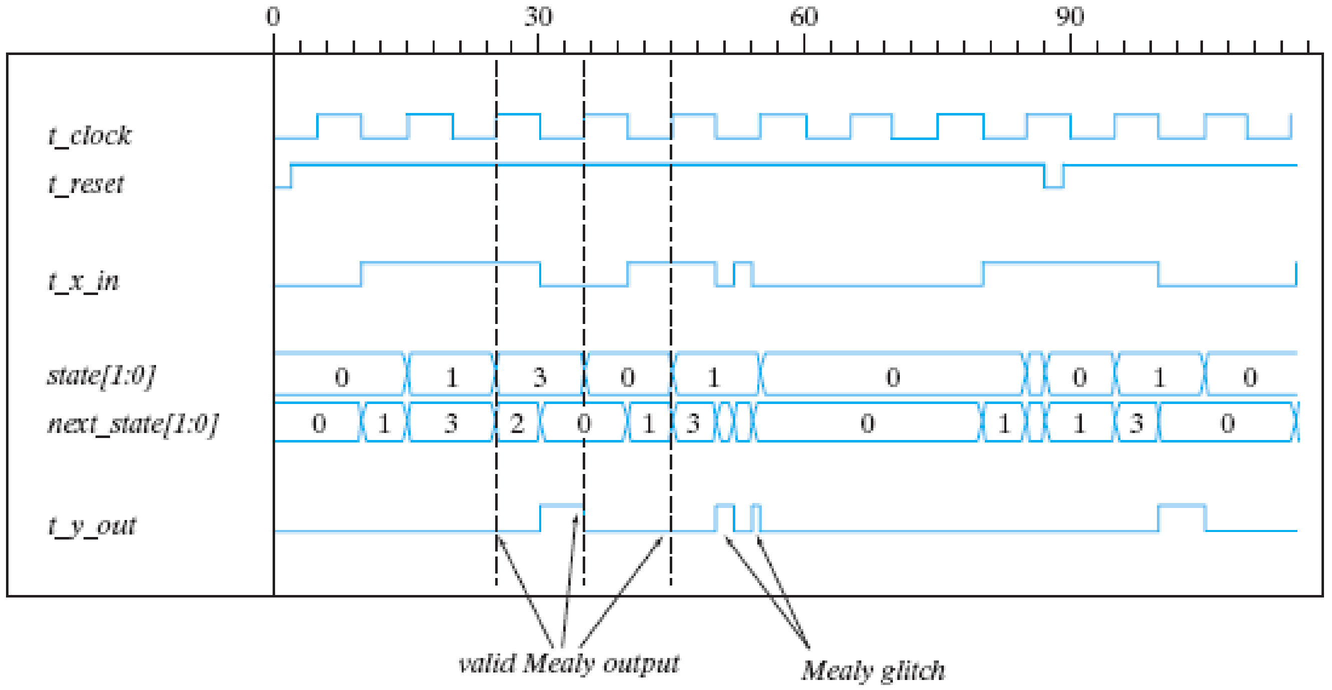
5.6 HDL for sequential circuits



5.7 State reduction and assignment

- State reduction is used to reduce the number of flip-flop
- Only input/output sequences are important
- Interested in present states that go to the same next state and have the same output

5.7 State reduction and assignment - State reduction

Table 5-6
State Table

	Next State		Output	
Present State	$x = 0$	$x = 1$	$x = 0$	$x = 1$
a	a	b	0	0
b	c	d	0	0
c	a	d	0	0
d	e	f	0	1
e	a	f	0	1
f	g	f	0	1
g	a	f	0	1

Reducing the State Table

	Next State		Output	
Present State	$x = 0$	$x = 1$	$x = 0$	$x = 1$
a	a	b	0	0
b	c	d	0	0
c	a	d	0	0
d	e	f	0	1
e	a	f	0	1
f	e	f	0	1

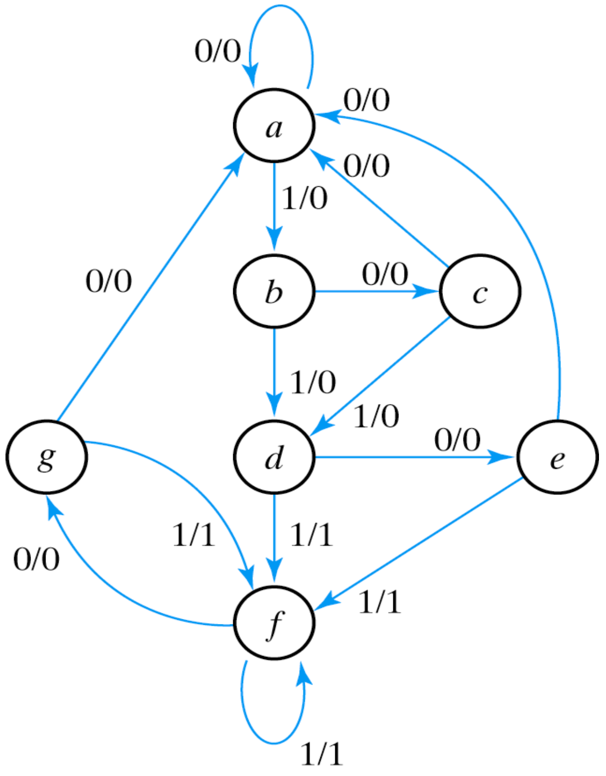


Fig. 5-22 State Diagram

5.7 State reduction and assignment - State reduction

Table 5-8
Reduced State Table

Present State	Next State $x = 0$	Next State $x = 1$	Output $x = 0$	Output $x = 1$
a	a	b	0	0
b	c	d	0	0
c	a	d	0	0
d	e	d	0	1
e	a	d	0	1

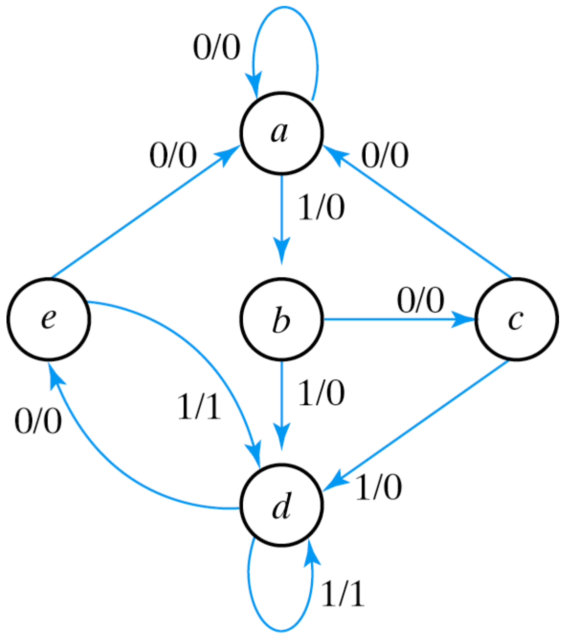


Fig. 5-23 Reduced State Diagram

5.7 State reduction and assignment - State assignment

- *m* states circuit, codes must contain *n* bits where $2^n \geq m$
- Three possible binary state assignments

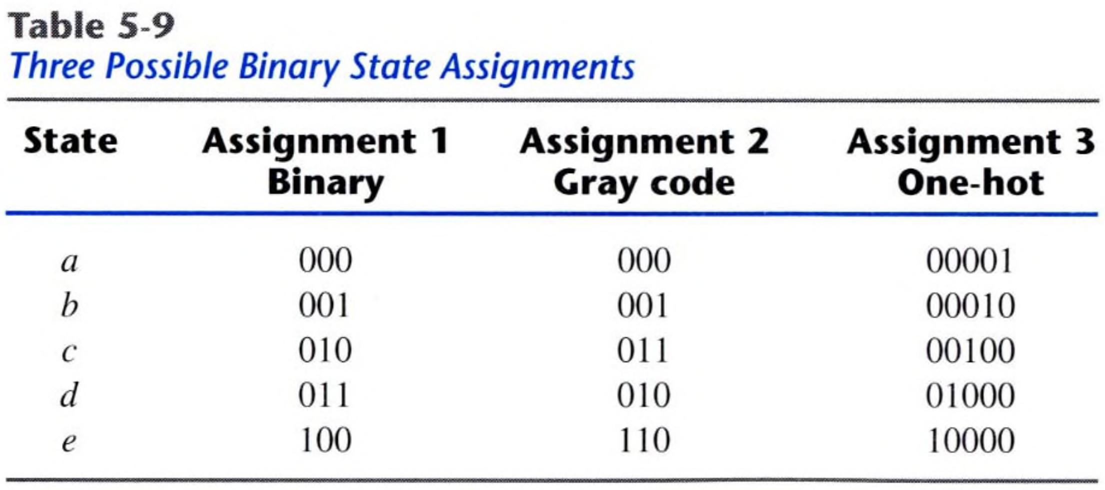


Table 5-9
Three Possible Binary State Assignments

State	Assignment 1 Binary	Assignment 2 Gray code	Assignment 3 One-hot
a	000	000	00001
b	001	001	00010
c	010	011	00100
d	011	010	01000
e	100	110	10000

5.8 Design procedure

- Sequential circuit design : requires state table

  ⇔ Combinational circuit : truth table
- The number of flip-flop is determined from the number of states in circuit
- If 2^n states exist, there are *n* flip-flops

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5.8 Design procedure

- Design steps
  1)Derive a state diagram or state table
  2)Reduce the number of states if necessary
  3)Assign binary code to the state
  4)Choose the type of flip-flops to be used
  5)Derive the flip-flop input equations and output equations
  6)Draw the logic diagram

5.8 Design procedure

- Derive a state diagram
  - Sequential detector
  - Three or more consecutive 1’s in a string of bits coming through an input line

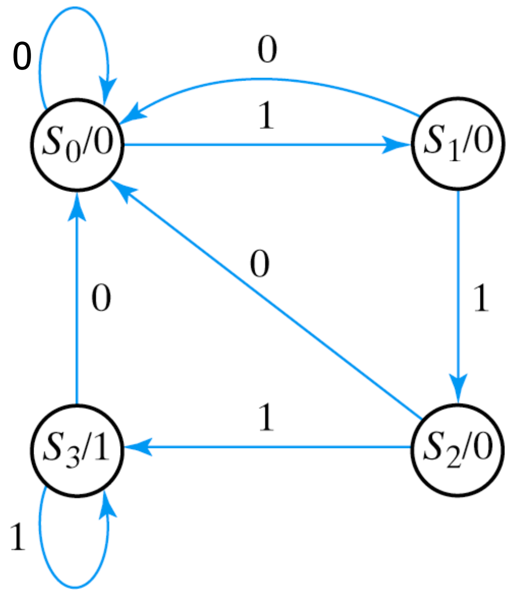


Fig. 5-24 State Diagram for Sequence Detector

5.8 Design procedure - Synthesis using D flip-flops

- Input equations are obtained directly from the next states

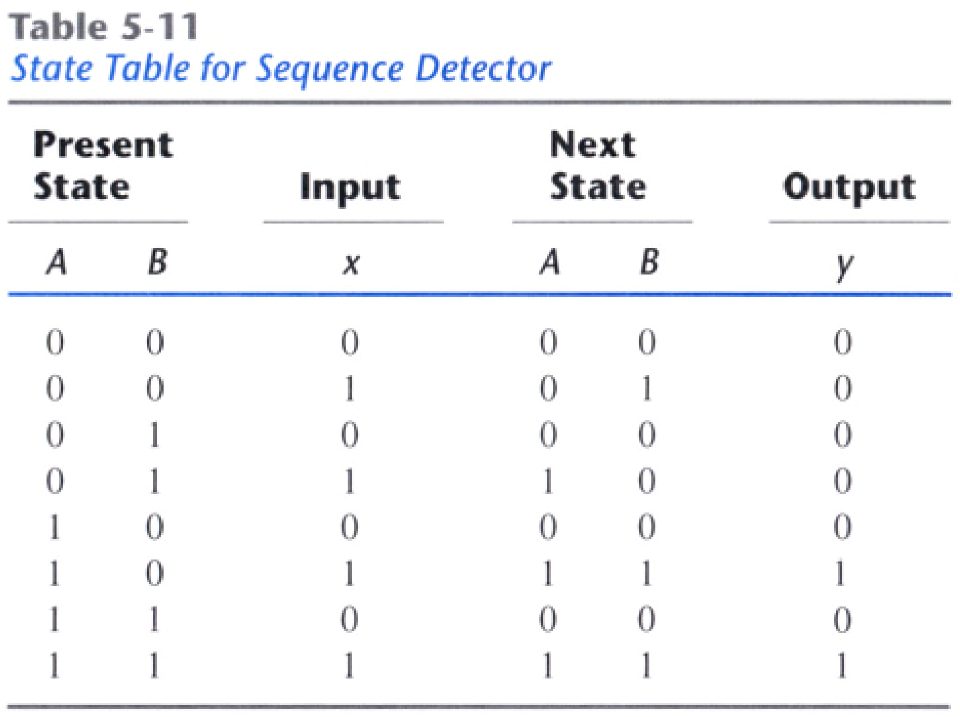
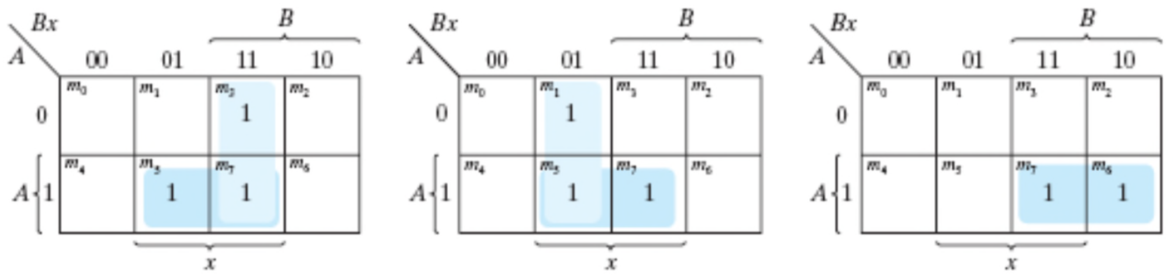


Table 5-11
State Table for Sequence Detector

Present State		Input	Next State		Output
A	*B*	*x*	*A*	*B*	*y*
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	0
0	1	1	1	0	0
1	0	0	0	0	0
1	0	1	1	1	1
1	1	0	0	0	0
1	1	1	1	1	1

5.8 Design procedure - Synthesis using D flip-flops

- K-maps and logic diagram



$D_A = Ax + Bx$ $D_B = Ax + B'x$ $y = AB$

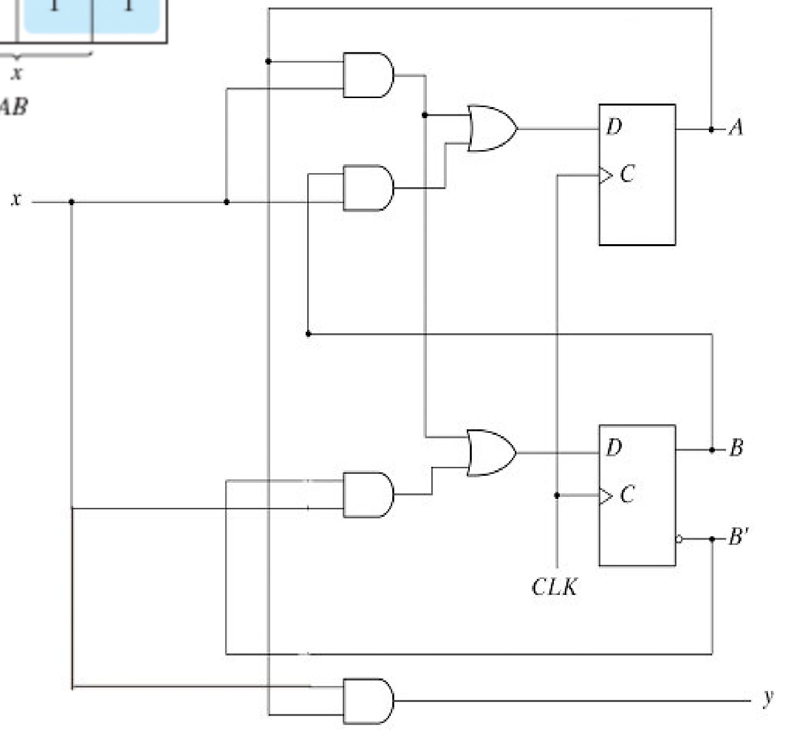


Fig. 5-26 Logic Diagram of Sequence Detector

5.8 Design procedure - Excitation table

- Excitation Table

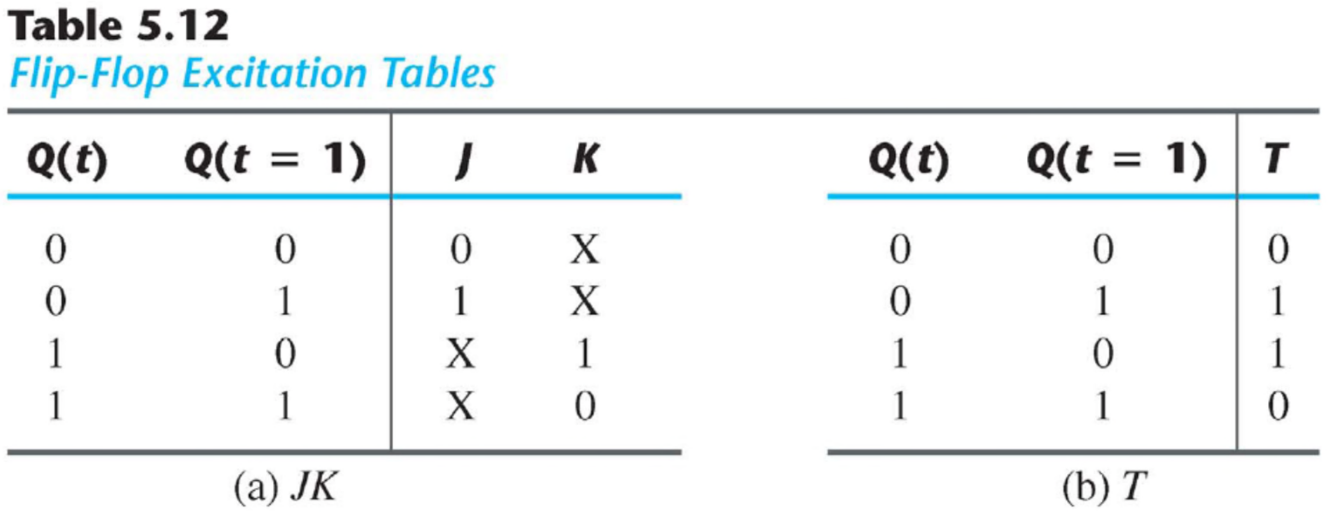


Table 5.12
Flip-Flop Excitation Tables

$Q(t)$	$Q(t = 1)$	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

(a) *JK*

$Q(t)$	$Q(t = 1)$	T
0	0	0
0	1	1
1	0	1
1	1	0

(b) *T*

5.8 Design procedure - Synthesis with JK flip-flops

- Input equations evaluated from the present state to next state transition

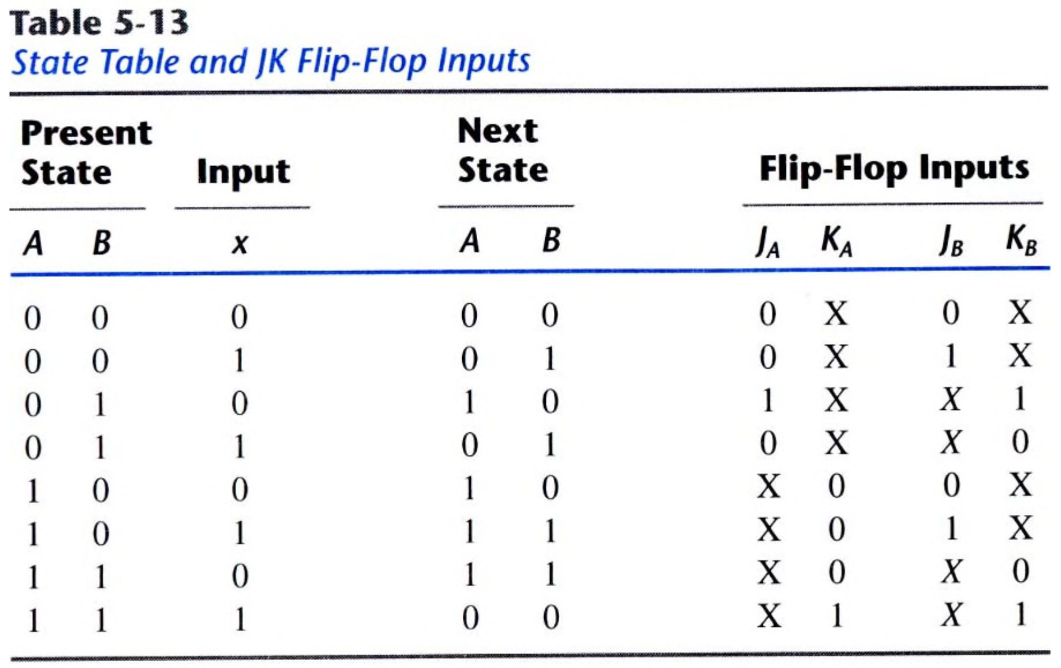


Table 5-13
State Table and JK Flip-Flop Inputs

Present State		Input	Next State		Flip-Flop Inputs			
A	B	x	A	B	J_A	K_A	J_B	K_B
0	0	0	0	0	0	X	0	X
0	0	1	0	1	0	X	1	X
0	1	0	1	0	1	X	X	1
0	1	1	0	1	0	X	X	0
1	0	0	1	0	X	0	0	X
1	0	1	1	1	X	0	1	X
1	1	0	1	1	X	0	X	0
1	1	1	0	0	X	1	X	1

5.8 Design procedure - Synthesis with JK flip-flops

- K-maps and logic diagram

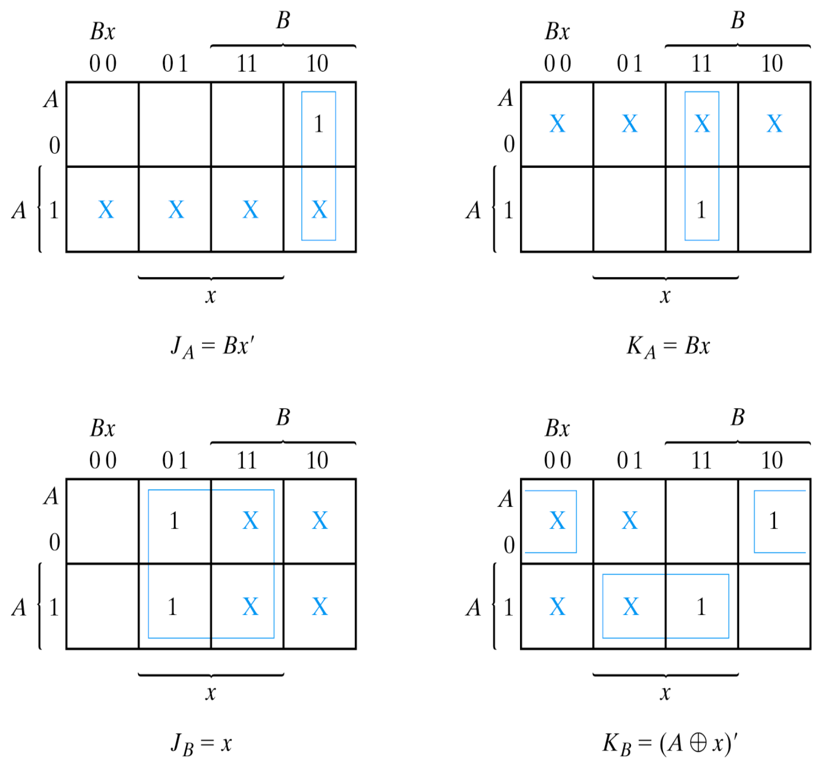
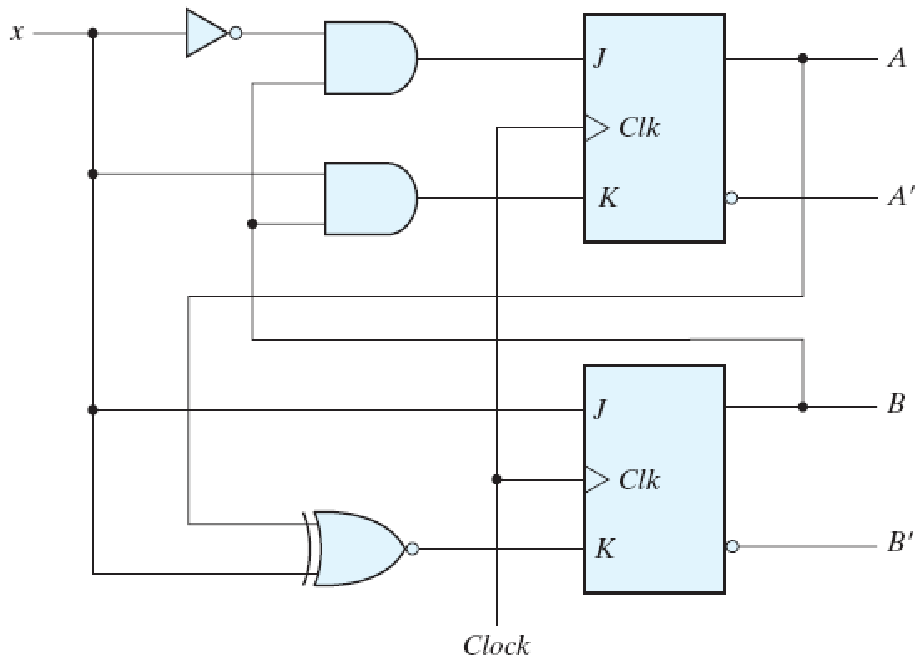


Fig. 5-27 Maps for *J* and *K* Input Equations



5.8 Design procedure - Synthesis using T flip-flops

- 3-bit binary counter
- 3-bit counter has 3 flip-flops and can count from 0 to 2^n-1(n=3)

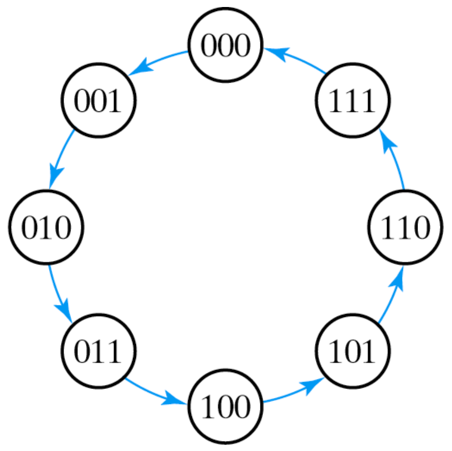


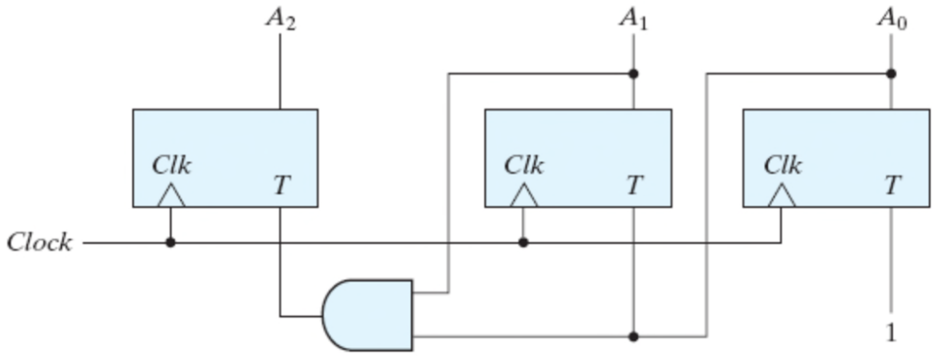
Fig. 5-29 State Diagram of 3-Bit Binary Counter

5.8 Design procedure - Synthesis using T flip-flops

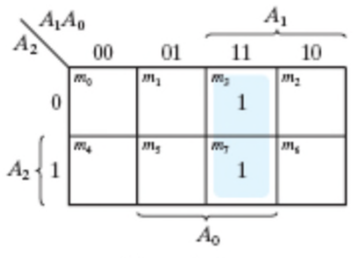
- State table and logic diagram

Table 5-14
State Table for 3-Bit Counter

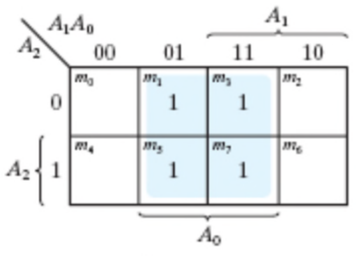
Present State			Next State			Flip-Flop Inputs		
A_2	A_1	A_0	A_2	A	A_0	T_{A2}	T_{A1}	T_{A0}
0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1	1
0	1	0	0	1	1	0	0	1
0	1	1	1	0	0	1	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	0	0	1	1
1	1	0	1	1	1	0	1	1
1	1	1	0	0	0	1	1	1



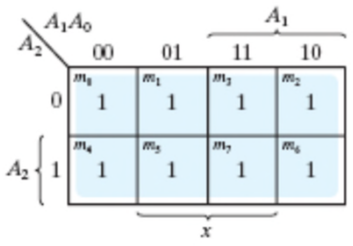
$T_{A2}=A_1A_0$, $T_{A1}=A_0$, $T_{A0}=1$



$T_{A2} = A_1A_0$



$T_{A1} = A_0$



$T_{A0} = 1$