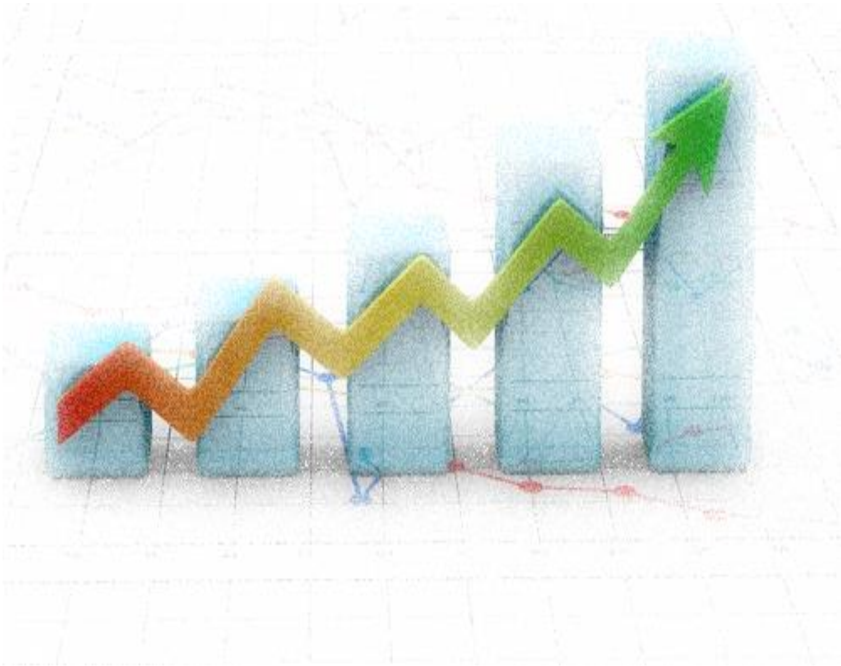
Data Science

Business Analytics: Time-Series Analysis

What is Forecasting?

- Forecasting is the process of making predictions or estimates about future events or conditions based on past and present data.
- It can be used in a variety of fields, including science, engineering, economics, and finance.
- For example, scientists use forecasting to predict the weather, the climate, and the occurrence of natural disasters. Engineers use forecasting to predict the performance of structures and systems. Economists use forecasting to predict economic growth, inflation, and unemployment. Financial analysts use forecasting to predict stock prices, interest rates, and currency exchange rates.



Businesses Forecasting

- Business Forecasting is the technique used to cast the foremost business scenarios to ease out the business decisions and management in future. It is used in decision-making processes across various industries and sectors.
- **The ongoing development in the field of predictive analytics using data analytics and machine learning techniques helps to shape and analyse the historical data to know the future business possibilities.**
- Business forecasting allows organizations to handle the uncertainty better and find new possibilities for sustainable growth of business.
- **Forecasting helps businesses and organizations plan for the future, optimize resource allocation, manage risks, and make informed decisions based on expected outcomes.**
- Forecasting methods can range from simple, intuitive approaches to complex statistical models and machine learning algorithms.
- These methods aim to capture patterns, relationships, and underlying factors that can influence the future.

Time-Series

1. Introduction to Time Series Forecasting
2. Time Series Analysis:
    - Components of a Time Series, Measures of Forecast
    - Accuracy, Stationary *Vs* Non-stationary Data
3. Moving Average Methods
4. Smoothing Methods: concepts and practices
5. What is seasonality & trends?
    - Trend Projections; Seasonal variations with Trend
6. Multiplicative Decomposition Method
7. Autocorrelation – AFC and PAFC
8. AR, MA and ARIMA Model

Case Examples using Excel Spreadsheet

Time Series Analysis and Forecasting

- Time series analysis is a statistical method used to analyze and interpret data points collected over time.
- Time series data consists of observations or measurements taken at regular intervals, such as daily, monthly, quarterly, or yearly.
- Examples of time series data include stock prices, weather conditions, economic indicators, sales figures, and population trends.
- Businesses are often very interested in forecasting time series variables.
- Time series focuses on studying the patterns, trends, and relationships within the data to make predictions or understand the underlying dynamics of the series
- The goal of time series analysis is to extract meaningful information from the data and use it to make forecasts, detect anomalies, identify patterns, and understand the underlying processes driving the series.

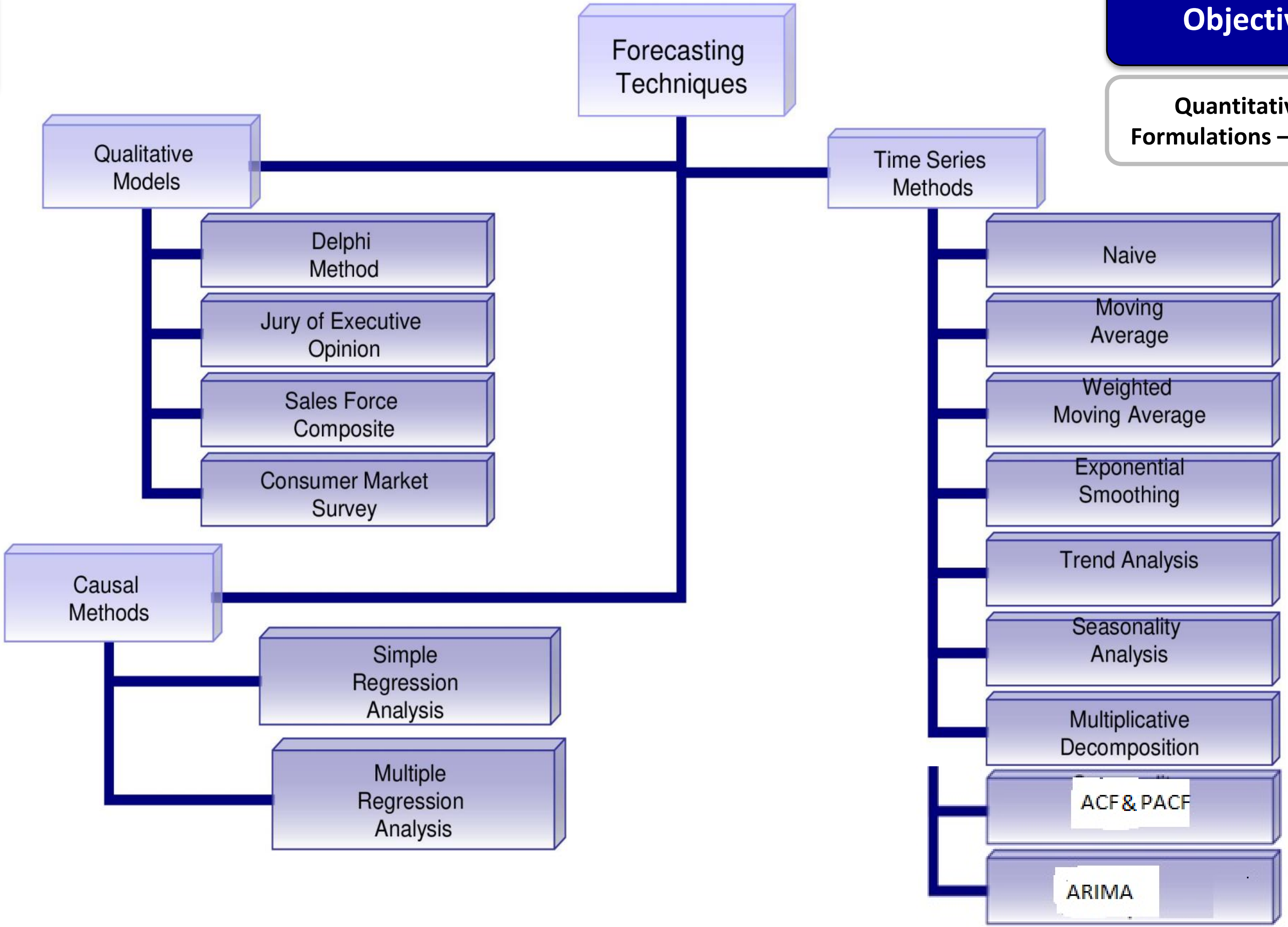
Forecasting Models

Subjective Approach:

Qualitative in nature and usually based on the opinions of people

Objective Approach:

Quantitative / Mathematical Formulations – Statistical forecasting



Qualitative Forecasting Methods

- **Customer Survey/Market Research**:

It utilizes the feedback and responses from customers to predict future trends. Businesses can use this technique to know the needs and expectations of customers. E.g. Survey to gather data on customer preferences, usage patterns, and satisfaction levels

- **Sales Force Composite:**

Input from the sales team to estimate future sales. Sales representatives provide their expert opinions and insights based on their knowledge of the market, customer behaviour, and their own sales experience.

- **Executive Committee Consensus**:

Inputs from key executives or decision-makers within an organization to make future sales predictions. These executives provide their insights and expertise to collectively reach a consensus on sales forecasts.

- **Delphi Method**:

Inputs from a panel of experts to reach a consensus on future outcomes. The experts participate in multiple rounds of anonymous surveys and feedback, allowing for a systematic exchange of opinions.

Examples of Qualitative Research Methods

Consumer Market Survey

An e-commerce business conducts a survey among its target customers to gather feedback on their online shopping experience, including factors like website usability, product selection, customer service, and delivery speed.

Sales Force Composite

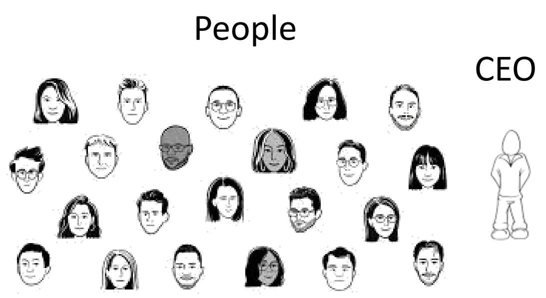
The sales team of an e-commerce company individually forecasts their sales figures for different product categories for the upcoming quarter. These individual forecasts are then aggregated to create a composite sales forecast for the entire company.

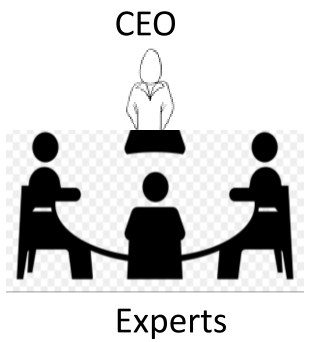
Jury of Expert Opinion

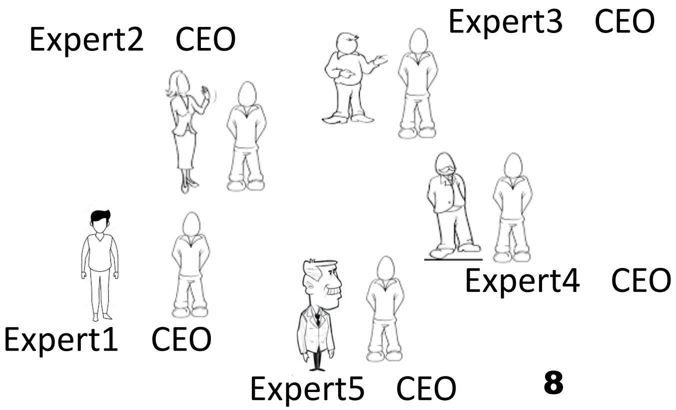
A group of industry experts, including e-commerce specialists, marketing professionals, and consumer behavior analysts, is brought together to evaluate the potential impact of implementing a new payment method for an e-commerce platform.

Delphi Method

An e-commerce association gathers a panel of industry experts, including e-commerce entrepreneurs, logistics professionals, and digital marketing specialists, to forecast the future trends and challenges in the e-commerce industry. The experts provide their predictions independently, and their responses are then anonymously compiled and shared for further discussion and refinement.

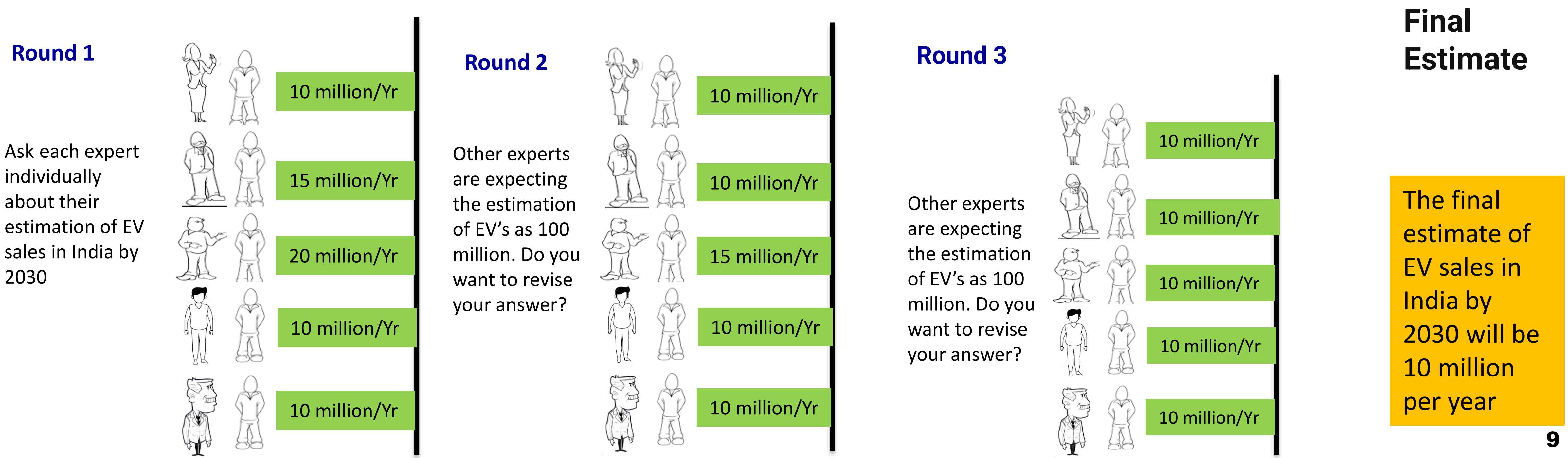






Delphi Method: EV Projection in India by 2030

- Inputs from a panel of experts to reach a consensus on future outcomes. The experts participate in multiple rounds of anonymous surveys and feedback, allowing for a systematic exchange of opinions.
- The Delphi method is a way of forecasting the future or reaching a consensus by asking a group of experts to answer several rounds of questions.
- The experts can revise their answers based on the feedback from other experts.
- The process stops when the experts agree or the results are stable.
- **Estimate the EV's sales in India by 2030.**



Quantitative Forecasting Models

- **Time Series**

Time series forecasting refers to the process of predicting future values or trends based on historical time-stamped data.

- **Causal Relationship**

Causal relationship forecasting analyzes the cause-and-effect relationship between variables to predict future outcomes.

- **Simulation**

Simulation forecasting involves creating a model that imitates real-world scenarios to predict outcomes.

Time Series & Cross-Sectional Data

- Time series data consist of observations of a single subject at multiple time intervals.
- Cross sectional data consist of observations of many subjects at the same point in time.
- Time series data focuses on the same variable over a period.
- On the other hand, cross sectional data focuses on several variables at the same point in time.

Cross-Sectional Data

City	Maximum Temperature	Humidity	Wind Speed
City X	30	62%	22 mph
City Y	25	58%	27 mph
City Z	32	61%	23 mph

Time Series Data

Year	Profit
2001	55000
2002	58000
2003	62000
2004	59000
2005	65000
2006	67000

Stationary Vs Non-Stationary Time Series Data

- **Stationary Data** - a time series variable exhibiting no significant upward or downward trend over time.

  A stationary time series is one whose statistical properties such as mean, variance, autocorrelation, etc. are all constant over time. Such statistics are useful as descriptors of future behavior only if the series is stationary.

- **Nonstationary Data** - a time series variable exhibiting a significant upward or downward trend over time.

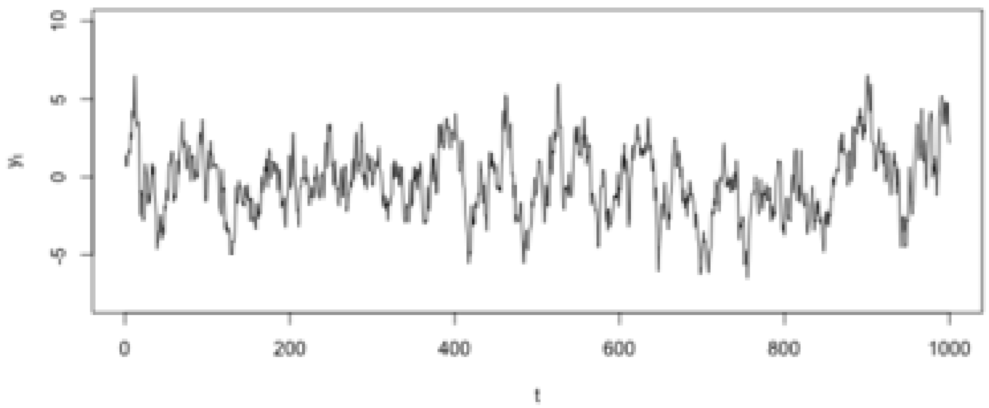
  Data points are often non-stationary or have means, variances, and covariances that change over time. Non-stationary behaviors can be trends, cycles, random walks, or combinations of the three. Thus, time series with trends, or with seasonality, are not stationary - the trend and seasonality will affect the value of the time series at different times.

Stationary Vs Non-Stationary Time Series Data

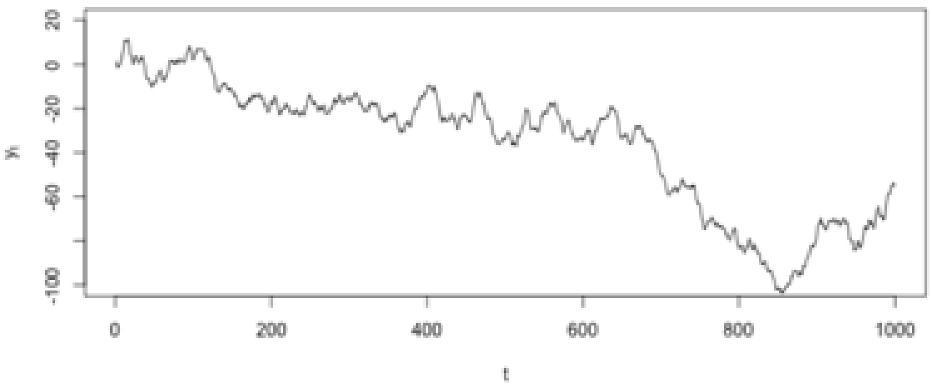
- How to check if the data is stationary or not?

- Plot the data and check whether the mean and variance remain constant (same distributions) for the whole series no matter where you chose a period.
- If it satisfies the above condition then the time series is said to be stationary.

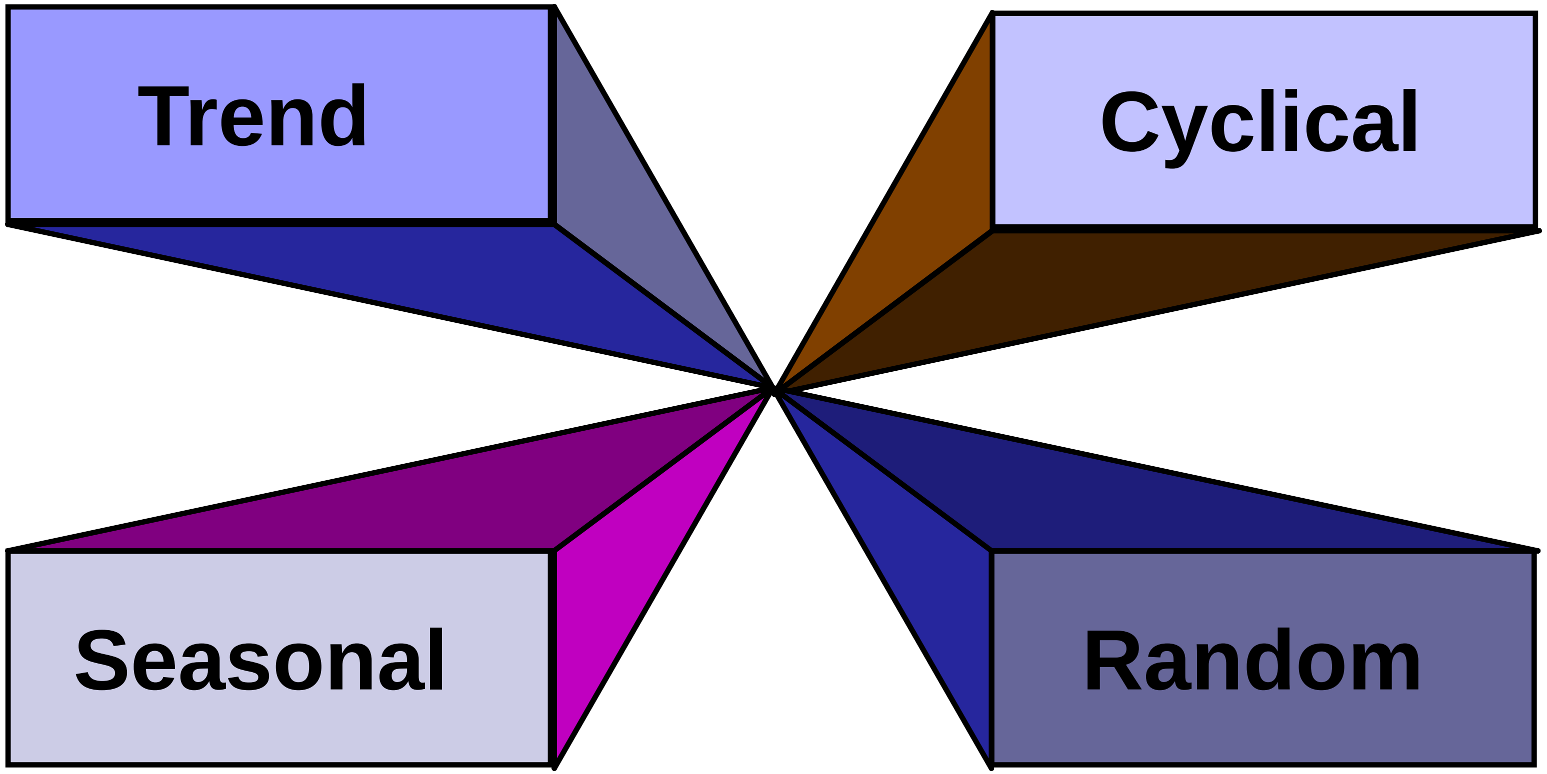
Stationary Time Series



Non-Stationary Time Series



Time Series Components



Components of Time Series Analysis

Trend

- Gradual upward or downward movement of data over time
- Trends reflect changes in population levels, technology, and living standards
- Long term movement

Cyclical

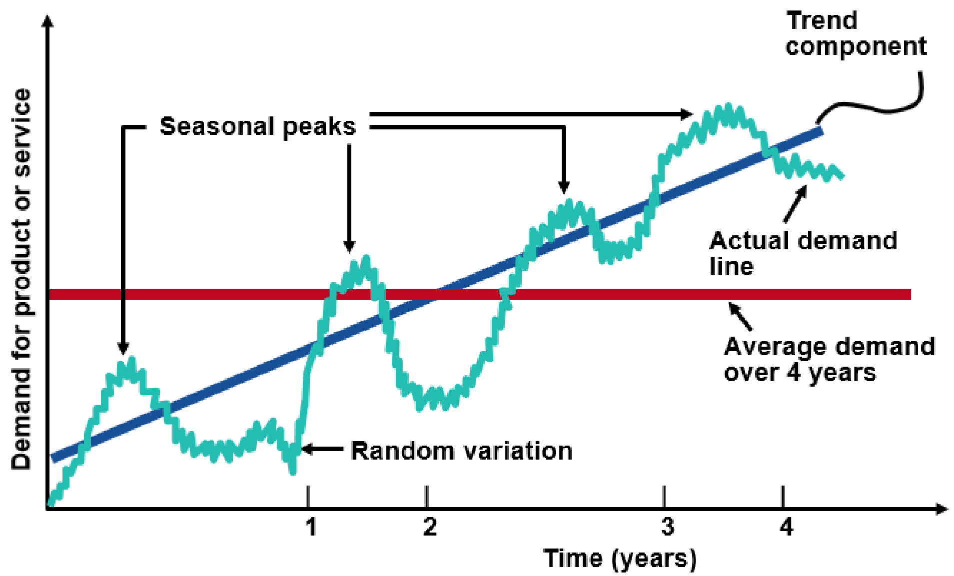
- Has a duration of at least one year and it varies from cycle to cycle
- Long term & require large historical data to determine repetitiveness/ unusual circumstance

Seasonality

- Variation repeat after fixed intervals
- Long as year/short as few hours
- Season / Holidays/ Special Events

Random

- Variation not explained by Trends/ Seasonality/Cyclicality
- Caused by chance



Components of Time Series Analysis

- Understanding the components of time series is crucial for analyzing and modeling the data effectively.

- The main components of time series are :Trend, Cyclical, Seasonality, Random variation.

- Analysis of these components can give insights to underlying patterns and relationships in time series data making informed decisions and improving forecasting accuracy.

- Time Frame (How far can we predict?)
  - Short-term (1 - 2 periods)
  - Medium-term (5 - 10 periods)
  - Long-term (12+ periods)
  - No line of demarcation

Model Evaluation: Measures of Forecast Accuracy

- Model evaluation in forecasting is the process of assessing the accuracy and reliability of a forecasting model.
- This is done by comparing the model's predictions to actual observations.
- This will help to know the error in forecasting.
- Also, this will hep to compare different forecasting techniques for a given dataset.
- Four common methods used for measuring the accuracy of forecast are shown below.

Mean Absolute Deviation

$$MAD = \sum_{t=1}^{n} \frac{|Y_t - \hat{Y}_t|}{n}$$

Mean Absolute Percentage Error

$$MAPE = \frac{100}{n} \sum_{t=1}^{n} \frac{|Y_t - \hat{Y}_t|}{Y_t}$$

Mean Square Error

$$MSE = \sum_{t=1}^{n} \frac{(Y_t - \hat{Y}_t)^2}{n}$$

Root Mean Square Error

$$RMSE = \sqrt{\sum_{t=1}^{n} \frac{(Y_t - \hat{Y}_t)^2}{n}}$$

Mostly Preferred

Time Series Models : Naïve Model

- Naive forecasting is one of the simplest forecasting techniques used to predict future values based solely on historical data.
- It assumes that the future will be the same as the past, without considering any other factors or variables that may affect the outcome.
- The forecasted value for the next period is simply the actual value of the last observed period.
- Uses recent past as the best indicator of the future

$$Y_{t+1} = Y_t$$

- The error associated with this model is computed as:

$$e_t = Y_t - \hat{Y}_t$$

Example for the Naïve Model

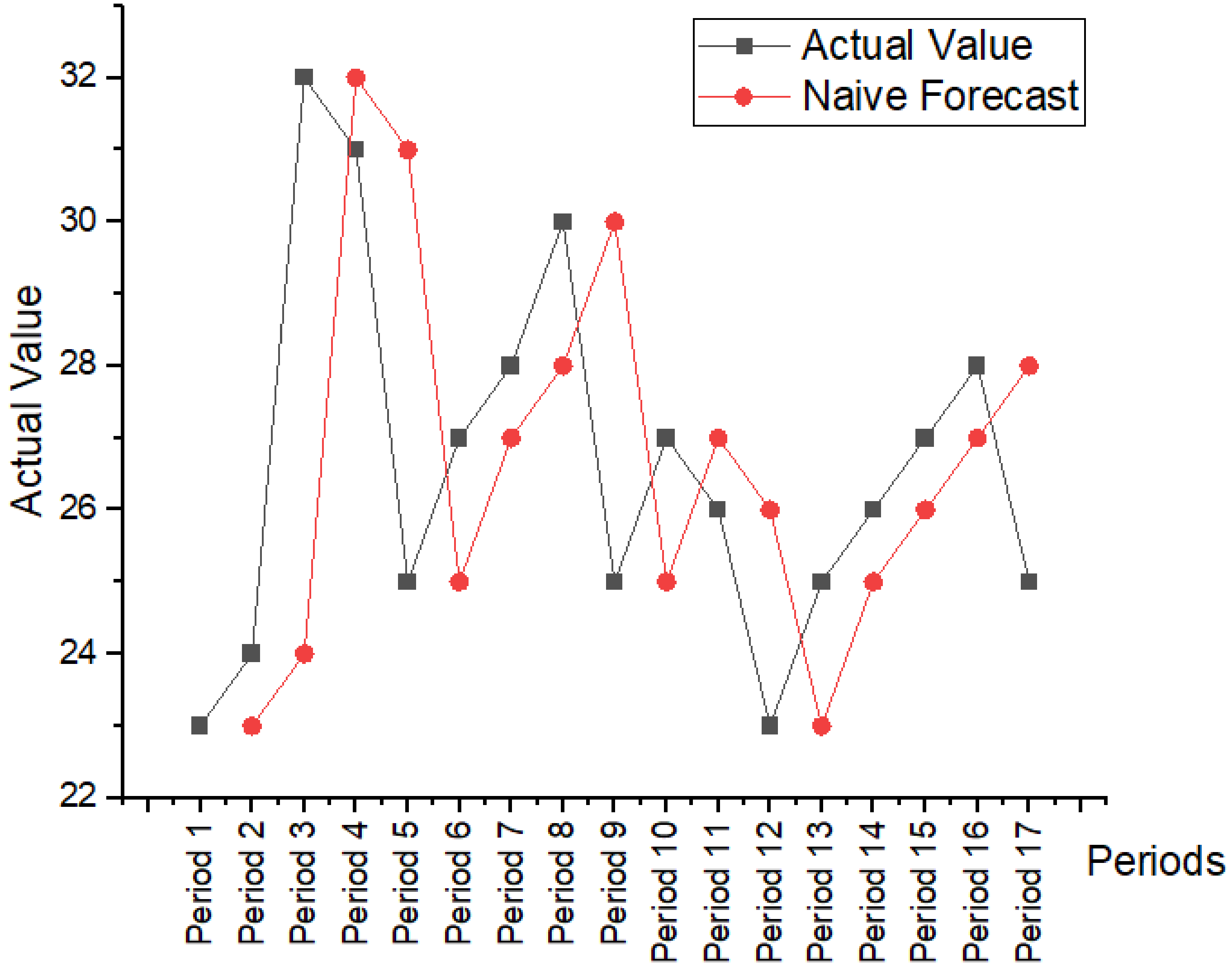
Week	Sales	Forecast
1	23	-
2	24	23
3	32	24
4	31	32
5	25	31
6	27	25
7	28	27
8	?	28

Naïve Model

Periods	Actual Value	Naïve Forecast	Error	Absolute Error	Error %	Squared Error
Period 1	23					
Period 2	24	23	1	1	4.17%	1
Period 3	32	24	8	8	25.00%	64
Period 4	31	32	-1	1	3.23%	1
Period 5	25	31	-6	6	24.00%	36
Period 6	27	25	2	2	7.41%	4
Period 7	28	27	1	1	3.57%	1
Period 8	30	28	2	2	6.67%	4
Period 9	25	30	-5	5	20.00%	25
Period 10	27	25	2	2	7.41%	4
Period 11	26	27	-1	1	3.85%	1
Period 12	23	26	-3	3	13.04%	9
Period 13	25	23	2	2	8.00%	4
Period 14	26	25	1	1	3.85%	1
Period 15	27	26	1	1	3.70%	1
Period 16	28	27	1	1	3.57%	1
Period 17	25	28	-3	3	12.00%	9
				2.5	9.34%	10.375
				MAD	MAPE	MSE

RMSE = 3.22

Naïve Model



Actual Value
Naive Forecast
Actual Value
32
30
28
26
24
22
Period 1
Period 2
Period 3
Period 4
Period 5
Period 6
Period 7
Period 8
Period 9
Period 10
Period 11
Period 12
Period 13
Period 14
Period 15
Period 16
Period 17
Periods

Time Series Models : Moving Average Methods

- Moving average methods are a class of forecasting techniques that calculate the average of a set of historical values to predict future values.
- They are one of the simplest and most widely used forecasting methods, as they are easy to understand and implement.
- Moving average methods are best suited for forecasting short-term trends in data that is relatively stable and free of outliers. They are also useful for smoothing out seasonal variations in data.
- There are three types of moving averages:
  - **Simple Moving Average (SMA):** Equally weights all data points within the selected time period
  - **Weighted Moving Average (WMA):** Assigns different weights to each data points based on its position in the series
  - **Exponential Moving Average (EMA):** Places more weights on recent data points, giving greater importance to current trends.

Simple Moving Average

- A simple moving average (SMA) is a calculation used to analyze data points over a certain period of time.
- The SMA is calculated by taking the average of a specified number of periods of a given data set.
- These periods can be daily, weekly, monthly, or any other time frame, depending on preference.
- SMA helps to identify trends by smoothing out short-term fluctuations.

How to calculate SMA?

- Choose a specific time period (e.g. 5 months) for calculating SMA.
- Add up the demands/prices for that period.
- Divide the sum by the number of days in the period to obtain the SMA.
- Repeat the process for each subsequent period to create a moving average.

$$k\ period\ moving\ average = \sum_{k=1}^{k}(Actual\ Value\ in\ previous\ k\ periods)/k$$

Simple Moving Average

Periods	Actual Value
Period 1	23
Period 2	24
Period 3	32
Period 4	31
Period 5	25
Period 6	27
Period 7	28
Period 8	30
Period 9	25
Period 10	27
Period 11	26
Period 12	23

Three-Month Moving Average

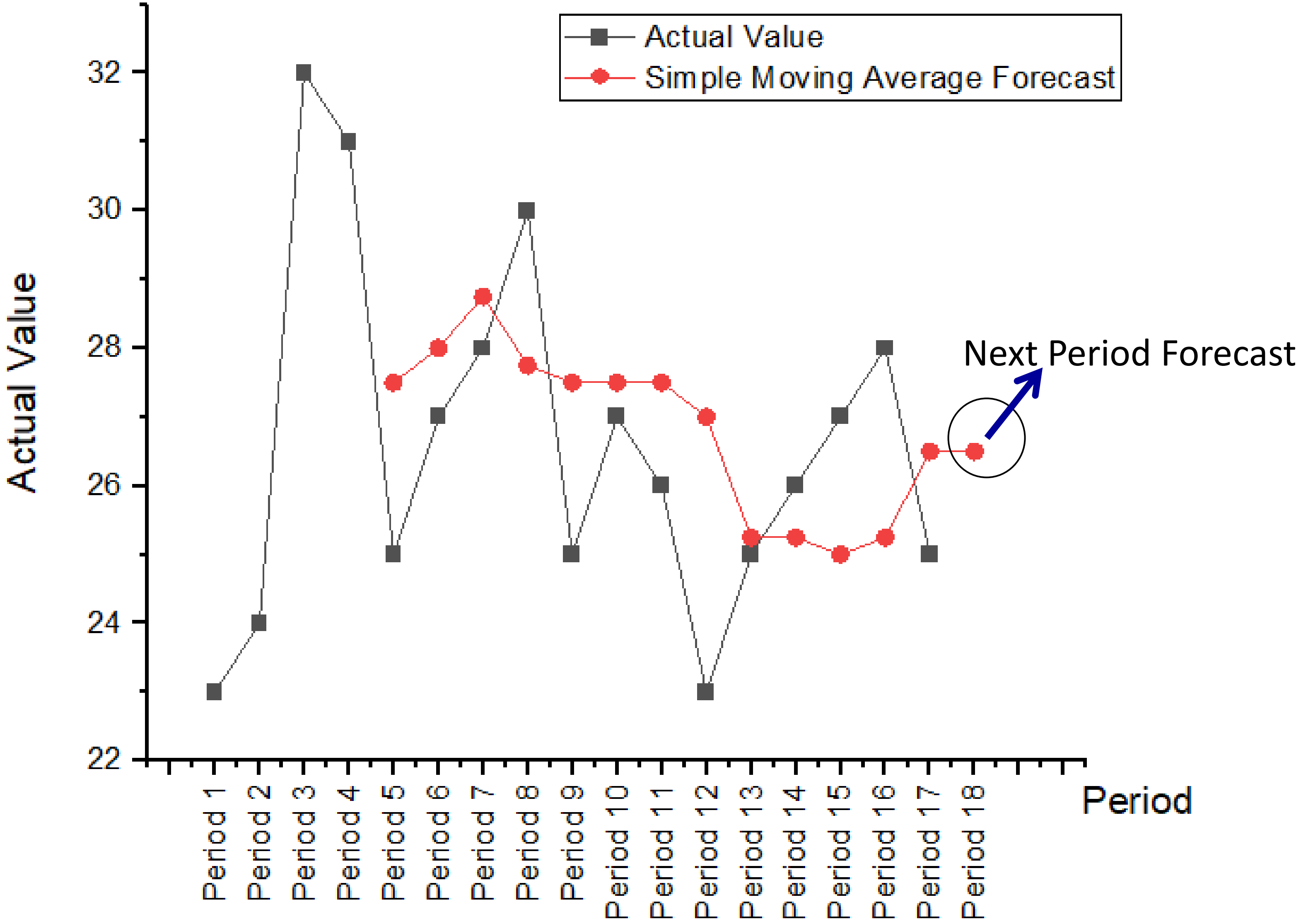
23+ 24 +32 / 3 =26.33
24 + 32 +31 / 3 = 29
32+31+25 /3 = 29.33
31+25+27/3 = 27.67
25+27+28 /3 = 26.67
27+28+30 /3 = 28.33
28+30+25 /3 =27.67
30+25+27 = 27.33
25+27+26 = 26

Simple Moving Average Model

Periods	Actual Value	Moving Average Forecast	Error	Absolute Error	Error %	Squared Error
Period 1	23	4 Periods average				
Period 2	24					
Period 3	32					
Period 4	31					
Period 5	25	27.5	-2.5	2.5	10.00%	6.25
Period 6	27	28	-1	1	3.70%	1
Period 7	28	28.75	-0.75	0.75	2.68%	0.5625
Period 8	30	27.75	2.25	2.25	7.50%	5.0625
Period 9	25	27.5	-2.5	2.5	10.00%	6.25
Period 10	27	27.5	-0.5	0.5	1.85%	0.25
Period 11	26	27.5	-1.5	1.5	5.77%	2.25
Period 12	23	27	-4	4	17.39%	16
Period 13	25	25.25	-0.25	0.25	1.00%	0.0625
Period 14	26	25.25	0.75	0.75	2.88%	0.5625
Period 15	27	25	2	2	7.41%	4
Period 16	28	25.25	2.75	2.75	9.82%	7.5625
Period 17	25	26.5	-1.5	1.5	6.00%	2.25
Period 18		26.5		1.71	6.61%	4.00
				MAD	**MAPE**	**MSE**

RMSE = 2.00

Simple Moving Average Model



Weighted Moving Average

- A Weighted moving average (WMA) is similar to SMA, only difference is that it uses a weighted average to give more importance to recent data.
- Weights determine the impact of each data point in the calculation.
- Recent data points typically have higher weights for responsiveness.
- Weights selection is crucial for accurate forecasting.
- Weights can be assigned based on domain knowledge or statistical methods
- Right weights depends on the characteristics of data and desired responsiveness.
- Higher weights lead to quicker adaptation to changes in the data.

How to calculate WMA?

- Multiply each data point by its weight.
- Sum up the weighted data points
- Divide the sum by the total weights.
- Repeat the process for each data point.

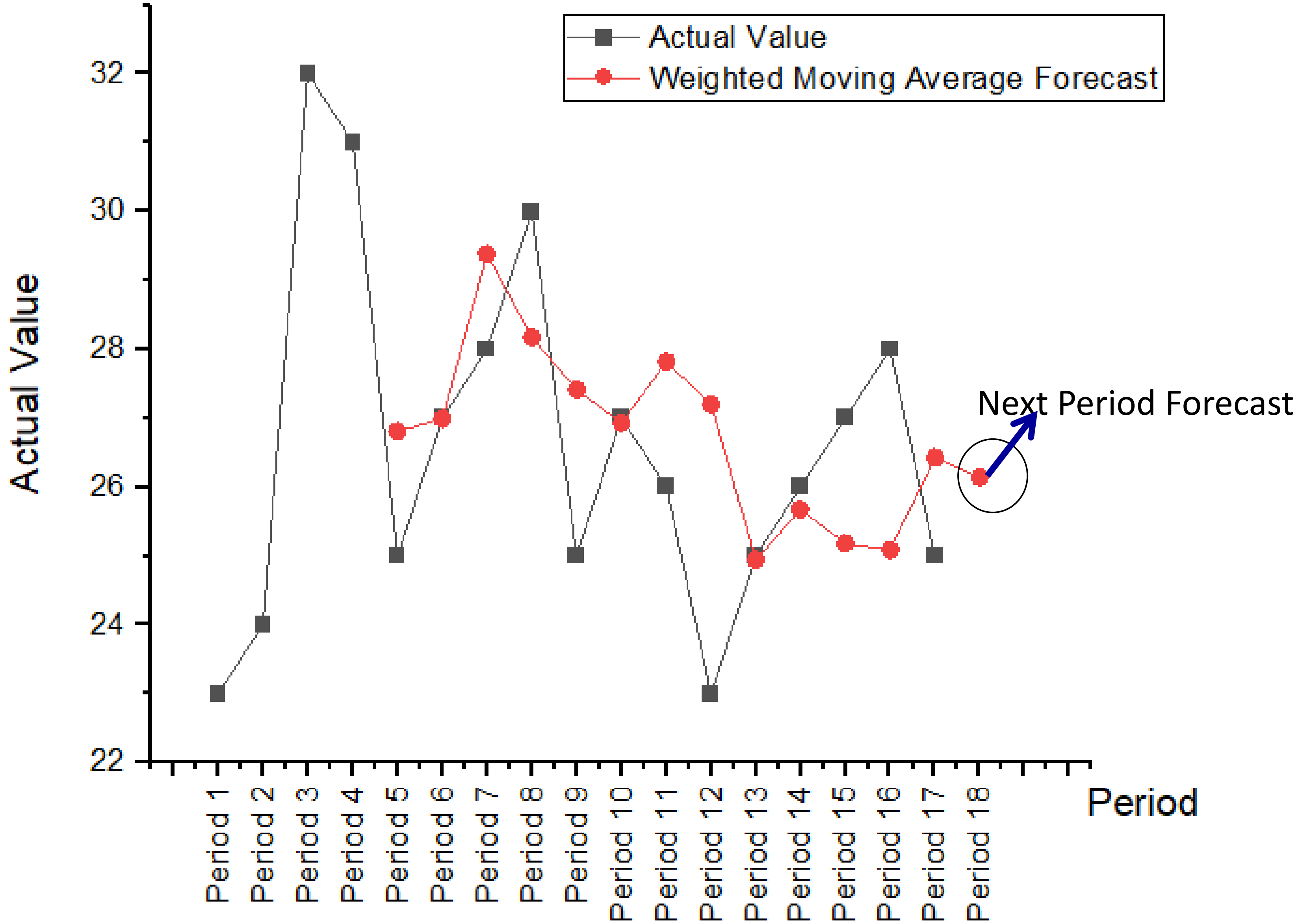
$$k\ period\ moving\ average = \sum_{i=1}^{k} \frac{(Weight\ for\ each\ period\ i)(Actual\ Value\ in\ previous\ k\ periods)}{weights}$$

Weighted Moving Average Model

Periods	Actual Value	Moving Average Forecast	Weights	Error	Absolute Error	Error %	Squared Error
Period 1	23	4 Periods average	0.3312				
Period 2	24		0.2366				
Period 3	32		0.1114				
Period 4	31		0.3209				
Period 5	25	26.81		-1.81	1.81	7.24%	3.2761
Period 6	27	26.99		0.01	0.01	0.04%	0.0001
Period 7	28	29.38		-1.38	1.38	4.93%	1.9044
Period 8	30	28.17		1.83	1.83	6.10%	3.3489
Period 9	25	27.41		-2.41	2.41	9.64%	5.8081
Period 10	27	26.93		0.07	0.07	0.26%	0.0049
Period 11	26	27.82		-1.82	1.82	7.00%	3.3124
Period 12	23	27.2		-4.2	4.2	18.26%	17.64
Period 13	25	24.94		0.06	0.06	0.24%	0.0036
Period 14	26	25.68		0.32	0.32	1.23%	0.1024
Period 15	27	25.18		1.82	1.82	6.74%	3.3124
Period 16	28	25.09		2.91	2.91	10.39%	8.4681
Period 17	25	26.42		-1.42	1.42	5.68%	2.0164
Period 18	?	26.14			1.54	5.98%	3.78
					MAD	MAPE	MSE

RMSE = 1.94

Weighted Moving Average Model



Exponential Moving Average

- A Exponential moving average (EMA) assigns more weights to recent data, providing better insights into short-term trends.
- EMA is calculated using a formula that applies a decay factor to previous data points.
- EMA reacts faster to new data compared to simple moving average
- What is Decay factor?
  - The decay factor determines the weightage of previous data points.
  - Decay factor is calculated using a smoothening constant or alpha value.
  - Higher alpha values give more weightage to recent data points.
  - Lower alpha values include more historical data in the calculation.

How to calculate EMA

- Choose a time period and select an alpha value.
- Calculate the initial EMA using a simple moving average.
- Apply the smoothing formula to compute subsequent EMAs.
- Repeat the process for each data point in the time series.

$$\boldsymbol{EMA} = (\boldsymbol{D} \ * \ \boldsymbol{\alpha}) + (\mathbf{Previous\ EMA} \ * (\mathbf{1} - \boldsymbol{\alpha}))$$

D = Current demand

α = Smoothing Factor = $\frac{2}{1+N}$

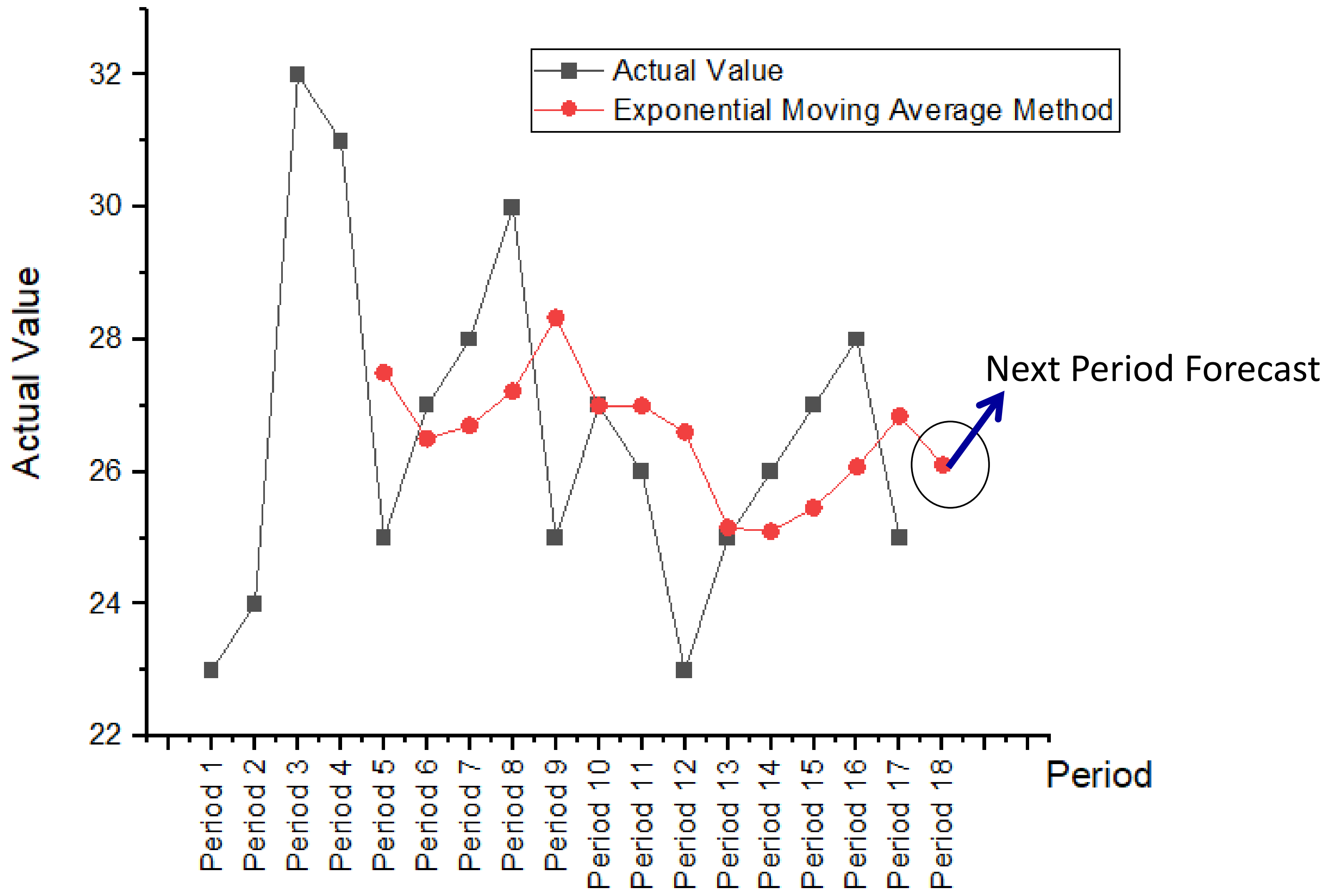
N = Number of Time Periods

Exponential Moving Average Model

Periods	Actual Value	Moving Average Forecast	Weights	Error	Absolute Error	Error %	Squared Error
Period 1	23	4 Periods average	0.4				
Period 2	24						
Period 3	32						
Period 4	31						
Period 5	25	27.50		-2.50	2.50	10.00%	6.25
Period 6	27	26.50		0.50	0.50	1.85%	0.25
Period 7	28	26.70		1.30	1.30	4.64%	1.69
Period 8	30	27.22		2.78	2.78	9.27%	7.73
Period 9	25	28.33		-3.33	3.33	13.33%	11.10
Period 10	27	27.00		0.00	0.00	0.00%	0.00
Period 11	26	27.00		-1.00	1.00	3.84%	1.00
Period 12	23	26.60		-3.60	3.60	15.65%	12.96
Period 13	25	25.16		-0.16	0.16	0.64%	0.03
Period 14	26	25.10		0.90	0.90	3.48%	0.82
Period 15	27	25.46		1.54	1.54	5.71%	2.38
Period 16	28	26.07		1.93	1.93	6.88%	3.71
Period 17	25	26.84		-1.84	1.84	7.38%	3.40
Period 18	?	26.11			1.65	6.36%	3.95
					MAD	MAPE	MSE

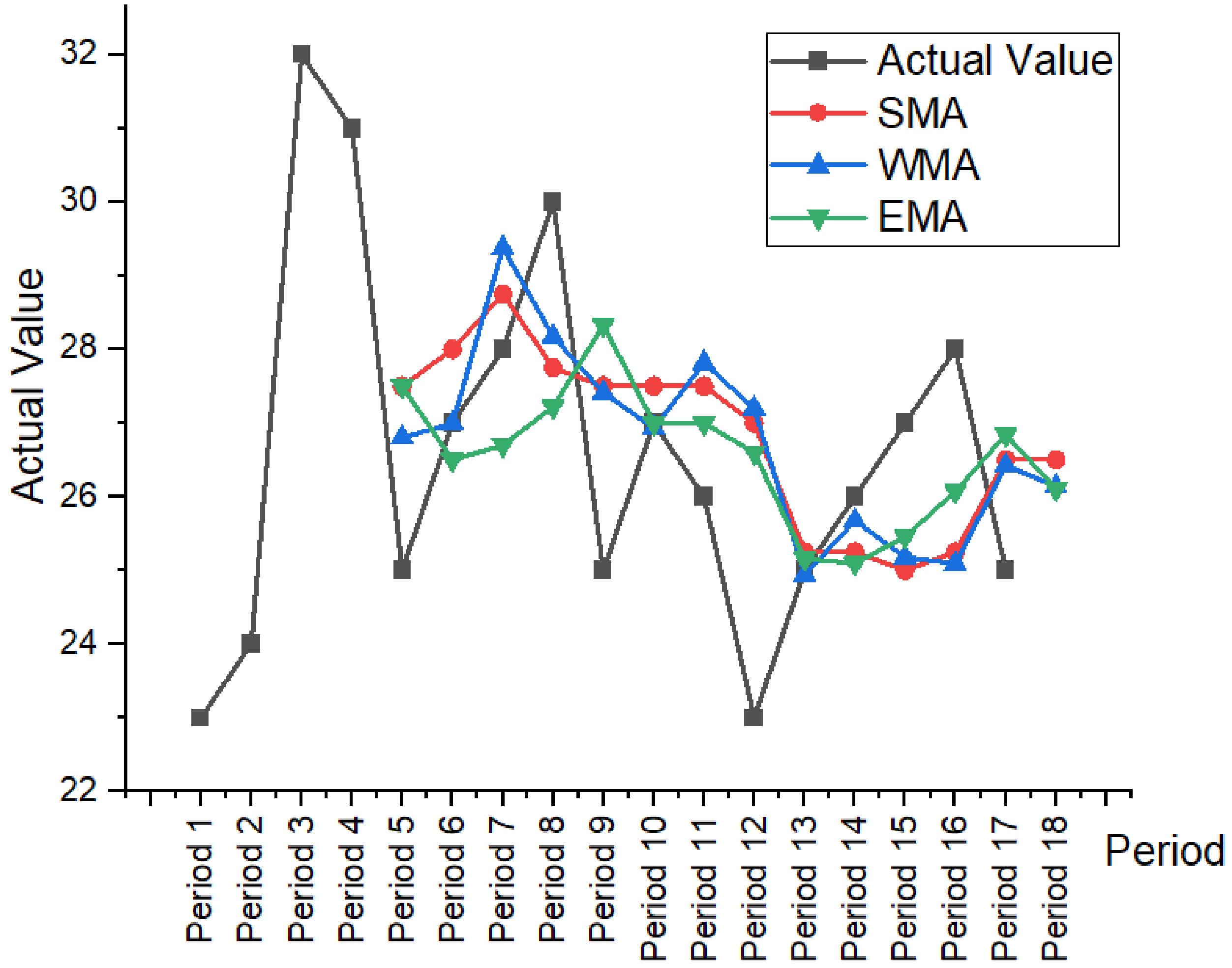
RMSE = 1.98

Exponential Moving Average Model



Comparison of Moving Average Models

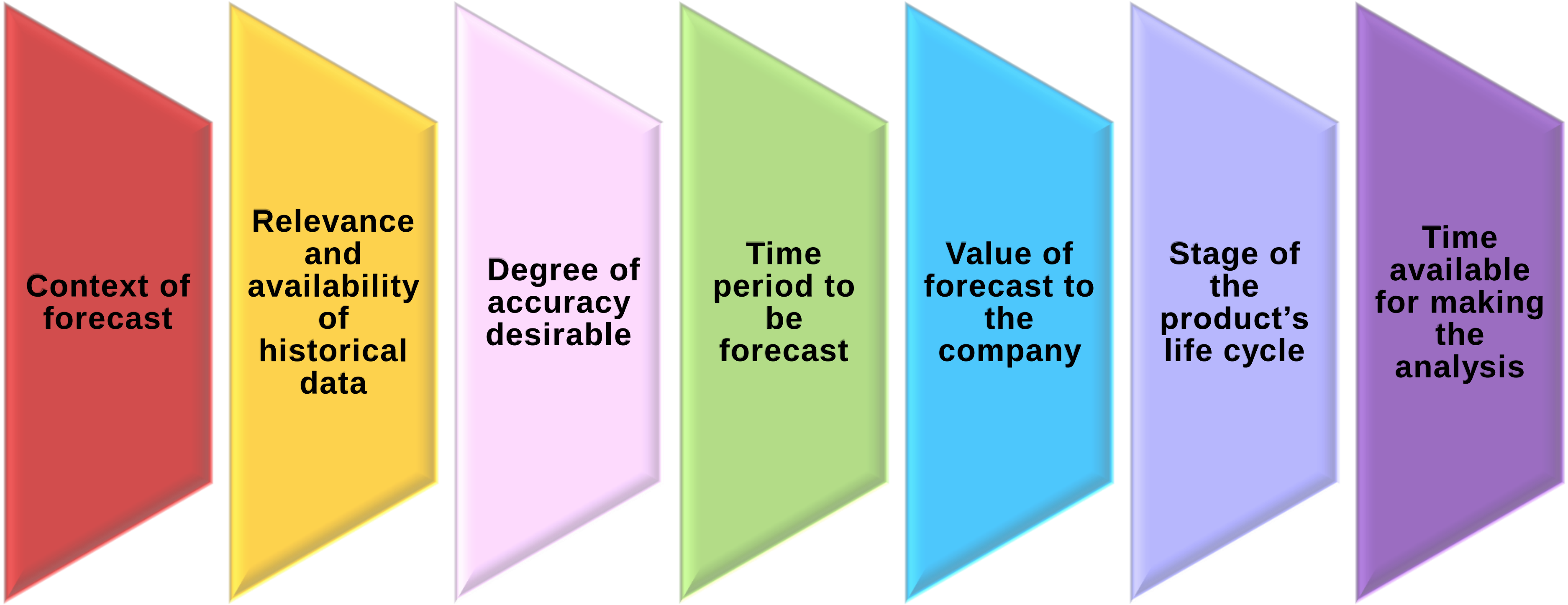
	MAD	MAPE	MSE	RMSE
SMA	1.71	6.61%	4	2
WMA	1.54	5.98%	3.78	1.94
EMA	1.65	6.36%	3.95	1.98



We are getting lowest error with WMA compared to SMA and EMA, Therefore Best Model for this dataset is Weighted Moving Average

Selection of a Method

The selection of a method depends on many factors:



Time Series Analysis: Exponential Smoothing

- Initialize the forecast by setting the initial forecast equal to the average of first few observations.
- The effectiveness of exponential smoothing heavily relies on the selection of smoothing parameter α.
- Calculate α by taking the weighted average of the current observation and the previous smoothed value. The weight given to the current observation is called the smoothing factor.
- Smoothing factor is decided based on optimization method.
  - A higher smoothing factor will give more weight to recent observations and will be more responsive to changes in the data.
  - A lower smoothing factor will give more weight to older observations and will be less responsive to changes in the data.
- Forecast the next value.

$$\hat{Y}_{t+1} = \alpha\, Y_t + \alpha\, (1-\alpha) Y_{t-1} + \alpha\,(1-\alpha)^2\, Y_{t-2} + \alpha\,(1-\alpha)^3\, Y_{t-3} + \cdots$$

$$\hat{Y}_{t+1} = \hat{Y}_t + \alpha\,(\,Y_t - \hat{Y}_t)$$

How to Forecast using Exponential Smoothing?

$$\hat{Y}_{t+1} = \alpha\, Y_t + \alpha\ (1-\alpha) Y_{t-1} + \alpha\, (1-\alpha)^2\ Y_{t-2} + \alpha\, (1-\alpha)^3\ Y_{t-3} + \cdots$$

$$\hat{Y}_{t+1} = \hat{Y}_t + \alpha\, (Y_t - \hat{Y}_t) = \alpha Y_t + (1-\alpha)\hat{Y}_t$$

- The model depends on three pieces of data:
  - Most recent actual
  - Most recent forecast
  - Smoothing constant.
- The value of alpha assigned as a smoothing constant is critical to the forecast.
- The best alpha should be chosen on the basis of minimal sum of error squared, i.e.; MSE.

How to Forecast using Exponential Smoothing?

Let us illustrate the case

- ❑ for α =.8 to the immediate period
- ❑ for α =.2 to the immediate period

$$\hat{Y}_{t+1} = \alpha\, Y_t + \alpha\ (1-\alpha)Y_{t-1} + \alpha\,(1-\alpha)^2\ Y_{t-2} + \alpha\,(1-\alpha)^3\ Y_{t-3} + \cdots$$

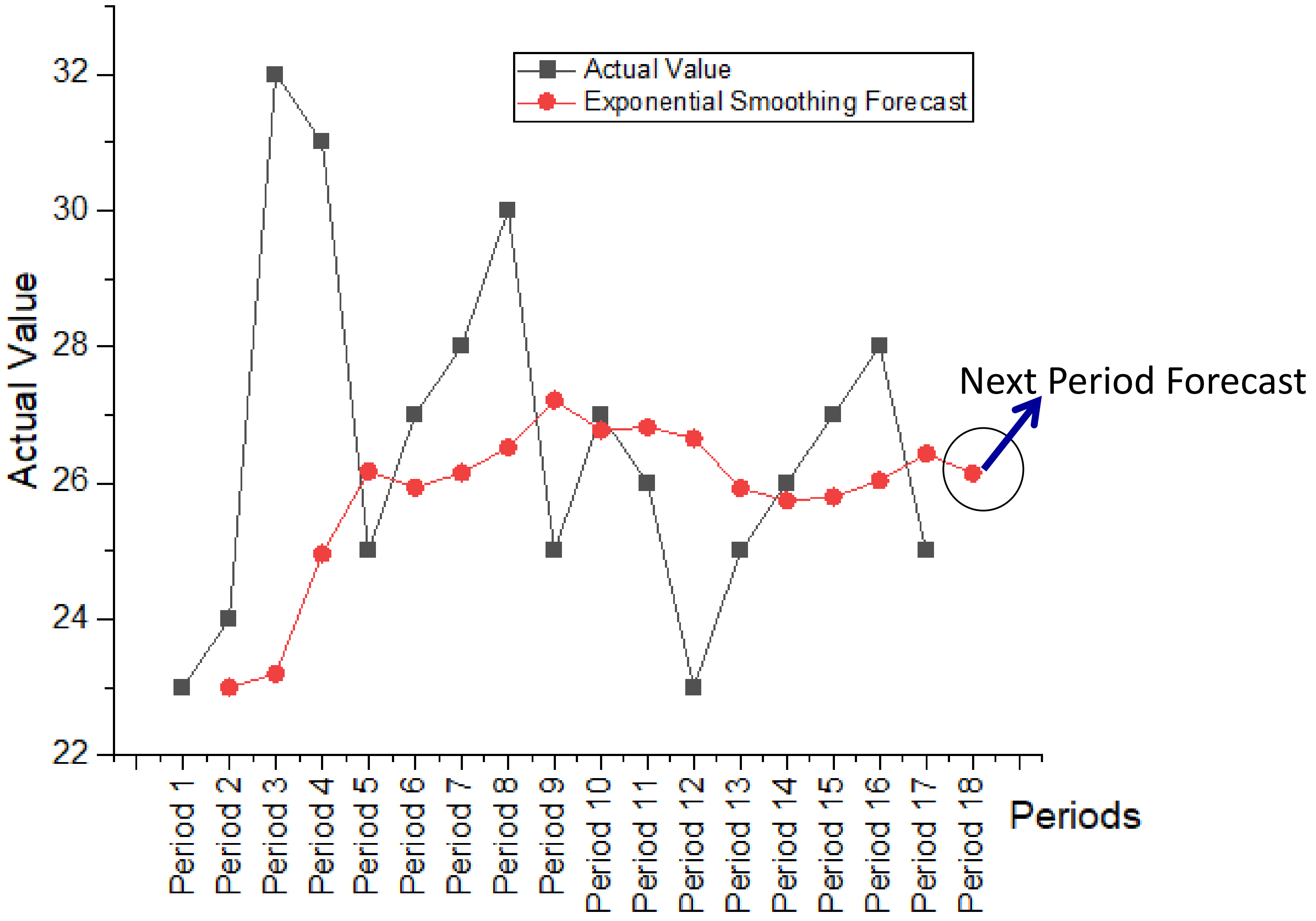
Exponential Smoothing Model (Excel Data)

Periods	Actual Value	Exponential Smoothing Forecast	Error	Absolute Error	Error %	Squared Error
Period 1	23					
Period 2	24	23.00	1.00	1.00	4.17%	1.00
Period 3	32	23.20	8.80	8.80	27.50%	77.44
Period 4	31	24.96	6.04	6.04	19.48%	36.48
Period 5	25	26.17	-1.17	1.17	4.67%	1.36
Period 6	27	25.93	1.07	1.07	3.95%	1.14
Period 7	28	26.15	1.85	1.85	6.62%	3.43
Period 8	30	26.52	3.48	3.48	11.61%	12.12
Period 9	25	27.21	-2.21	2.21	8.86%	4.90
Period 10	27	26.77	0.23	0.23	0.85%	0.05
Period 11	26	26.82	-0.82	0.82	3.14%	0.67
Period 12	23	26.65	-3.65	3.65	15.89%	13.35
Period 13	25	25.92	-0.92	0.92	3.69%	0.85
Period 14	26	25.74	0.26	0.26	1.01%	0.07
Period 15	27	25.79	1.21	1.21	4.48%	1.46
Period 16	28	26.03	1.97	1.97	7.03%	3.87
Period 17	25	26.43	-1.43	1.43	5.70%	2.03
Period 18	?	26.14		2.26	8.04%	10.01
				MAD	**MAPE**	**MSE**

$\alpha = 0.2$

RMSE = 3.16

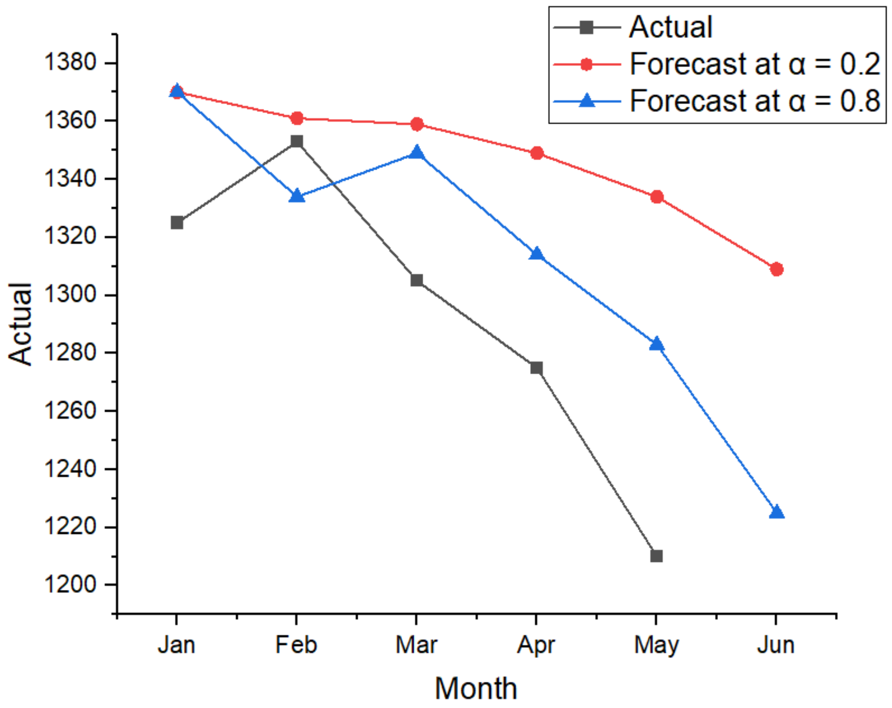
Exponential Smoothing Model



Impact of α in Downward Trend Data

- Using a low α value in simple exponential smoothing for recent observations in a downward trend data series can result in forecasts that are less responsive to recent changes
- It may not capture the declining trend as effectively as higher α would
- Choose α that balances the trade-off between responsiveness to recent data and smoothing out noise in the time series, based on the specific characteristics of your data and forecasting goals.

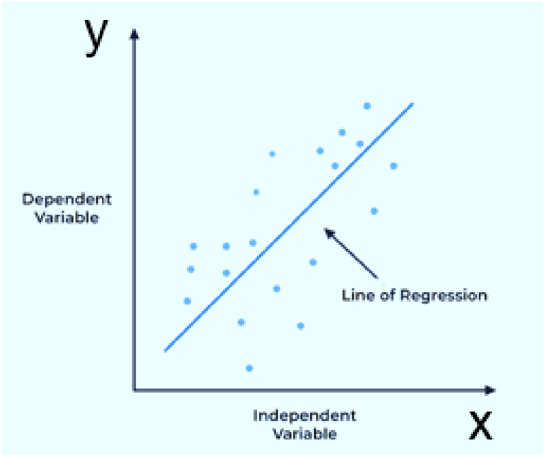
Month	Actual	Forecast at α = 0.2	Forecast at α = 0.8
Jan	1,325	1,370	1,370
Feb	1,353	1,361	1,334
Mar	1,305	1,359	1,349
Apr	1,275	1,349	1,314
May	1,210	1,334	1,283
Jun	?	1,309	1,225



Trend Projections

- Trend projections aims to fit a trend line to historical data points to identify underlying trend in the data and then use that trend to predict future values.
- Linear trends can be found using the least squares technique. The trend line can be used to predict future values of demand/ sales by extending the line into the future.

$y = a + bx$



y= computed value of the variable to be predicted (dependent variable)
a= y- axis intercept
b= slope of regression line
x= Independent variable

Time Series Models : Holt Model

- The simple exponential smoothing and moving average methods fail to respond to the presence of trends, if any, since they are constant level models.
- They are more applicable when the demand is constant and fluctuates within a range.
- Trend is a long-term direction of the data, either increasing or decreasing. When a time series exhibits a trend, it means that the future values of the series are likely to be different from the current values.
- Holt exponential smoothing/ Holt model is specifically designed to predict future values of time series that exhibits a trend. The model is versatile than exponential smoothing model by capturing the variations in the data over time.
- The model considers the level of the series at time t and the trend (up/ down) at time t-1 to produce a forecast for time t+1. Hence, it is also called 'Double Exponential Smoothing' model.
- α and β are the smoothing parameters for the level and trend respectively

How to Forecast using Holt Model?

- The effectiveness of holt exponential smoothing heavily relies on the selection of smoothing parameter α and β. The steps for forecasting using Holt model are:
- Initialize the α and β values (you can state any values in range 0 to 1).
- Calculate the level and trend of the series using equations (1) and (2).
- Calculate the error of the forecast.
- Experiment with different α and β to minimize the error (Optimization method).
- Calculate level and trend again for best α and β values.
- Forecast the next period value using equation (3).

Level	$L_t = \alpha Y_t + (1-\alpha) * (L_{t-1} + T_{t-1})$	(1)
Trend	$T_t = \beta * (L_t - L_{t-1}) + (1-\beta) * T_{t-1}$	(2)
Forecast	$\hat{Y}_{t+1} = L_t + T_t$	(3)

Where,

L_t= Level at Time t

T_t = Trend at time t

α = Smoothing parameter for the level

β = Smoothing parameter for trend

Y_t = Actual value at time t

L_{t-1} & T_{t-1}= Previous level and trend at time t-1

L_t = Current level, and $\hat{Y}_{t+1}$ = Forecast of next period

Exponential Smoothing with Trend Adjustment

Step 1: Forecast level for month 2

$$F_1 = \alpha A_1 + (1 - \alpha)(F_0 + T_0)$$
$$F_1 = (0.2)(12) + (1 - 0.2)(11 + 2)$$
$$= 2.4 + 10.4 = 12.8$$

Period	Actual value (A)	Level (F) α = 0.2	Trend (T) β = 0.4	Forecast Yt+1	Error
		11	2		
1	12	**12.8**		13	
2	17				
3	20				
4	19				
5	24				
6	21				
7	31				
8	28				
9	36				
10	?				

Level $L_t = \alpha\, Y_t + (1-\alpha) * (L_{t-1} + T_{t-1})$ (1)

Trend $T_t = \beta * (L_t - L_{t-1}) + (1-\beta) * T_{t-1}$ (2)

Forecast $\hat{Y}_{t+1} = L_t + T_t$ (3)

Exponential Smoothing with Trend Adjustment

Step 2: Forecast Trend for month 2

$T_1 = \beta(F_1 - F_0) + (1 - \beta)T_0$

$T_1 = (0.4)(12.8 - 11) + (1 - 0.4)(2)$

$= 0.72 + 1.2 = 1.92$ units

Period	Actual value (A)	Level (F) α = 0.2	Trend (T) β = 0.4	Forecast Yt+1 (FIT)	Error
		11	2		
1	12	**12.8**	**1.92**	13	
2	17				
3	20				
4	19				
5	24				
6	21				
7	31				
8	28				
9	36				
10	?				

Level $L_t = \alpha\, Y_t + (1-\alpha) * (L_{t-1} + T_{t-1})$ (1)

Trend $T_t = \beta * (L_t - L_{t-1}) + (1-\beta) * T_{t-1}$ (2)

Forecast $\hat{Y}_{t+1} = L_t + T_t$ (3)

Exponential Smoothing with Trend Adjustment

Step 3: Forecast value for month 2

$FIT_2 \quad = F_1 + T_1$

$FIT_2 \quad = 12.8 + 1.92$

$\quad\quad\quad = 14.72$

Period	Actual value (A)	Level (F) α = 0.2	Trend (T) β = 0.4	Forecast Yt+1 (FIT)	Error
		11	2		
1	12	**12.8**	**1.92**	13	
2	17			**14.72**	
3	20				
4	19				
5	24				
6	21				
7	31				
8	28				
9	36				
10	?				

Level $\quad L_t = \alpha\, Y_t + (1-\alpha) * (L_{t-1} + T_{t-1})$ (1)

Trend $\quad T_t = \beta * (L_t - L_{t-1}) + (1-\beta) * T_{t-1}$ (2)

Forecast $\quad \hat{Y}_{t+1} = L_t + T_t$ (3)

Exponential Smoothing with Trend Adjustment

Step 1: Forecast level for month 3

$$F_2 = \alpha A_2 + (1 - \alpha)(F_1 + T_1)$$
$$F_2 = (0.2)(17) + (1 - 0.2)(12.8 + 1.92)$$
$$= 3.4 + 11.78 = 15.18$$

Period	Actual value (A)	Level (F) $\alpha = 0.2$	Trend (T) $\beta = 0.4$	Forecast Yt+1	Error
		11	2		
1	12	**12.8**	**1.92**	13	
2	17	**15.18**		**14.72**	
3	20				
4	19				
5	24				
6	21				
7	31				
8	28				
9	36				
10	?				

Exponential Smoothing with Trend Adjustment

Period	Actual value (A)	Level (F) $\alpha = 0.2$	Trend (T) $\beta = 0.4$	Forecast Yt+1 (FIT)	Error
		11	2		
1	12	**12.8**	**1.92**	13	
2	17	**15.18**	**2.10**	**14.72**	
3	20				
4	19				
5	24				
6	21				
7	31				
8	28				
9	36				
10	?				

Step 2: Forecast Trend for month 3

$$T_2 = \beta(F_2 - F_1) + (1 - \beta)T_1$$

$$T_2 = (0.4)(15.18 - 12.8) + (1 - 0.4)(1.92)$$

$$= 0.95 + 1.15 = 2.10 \text{ units}$$

Exponential Smoothing with Trend Adjustment

Step 3: Forecast value for month 3

$$FIT_3 = F_2 + T_2$$
$$FIT_3 = 15.18 + 2.10$$
$$= 17.28$$

Period	Actual value (A)	Level (F) α = 0.2	Trend (T) β = 0.4	Forecast Yt+1 (FIT)	Error
		11	2		
1	12	**12.8**	**1.92**	13	
2	17	**15.18**	**2.10**	**14.72**	
3	20			**17.28**	
4	19				
5	24				
6	21				
7	31				
8	28				
9	36				
10	?				

Exponential Smoothing with Trend Adjustment

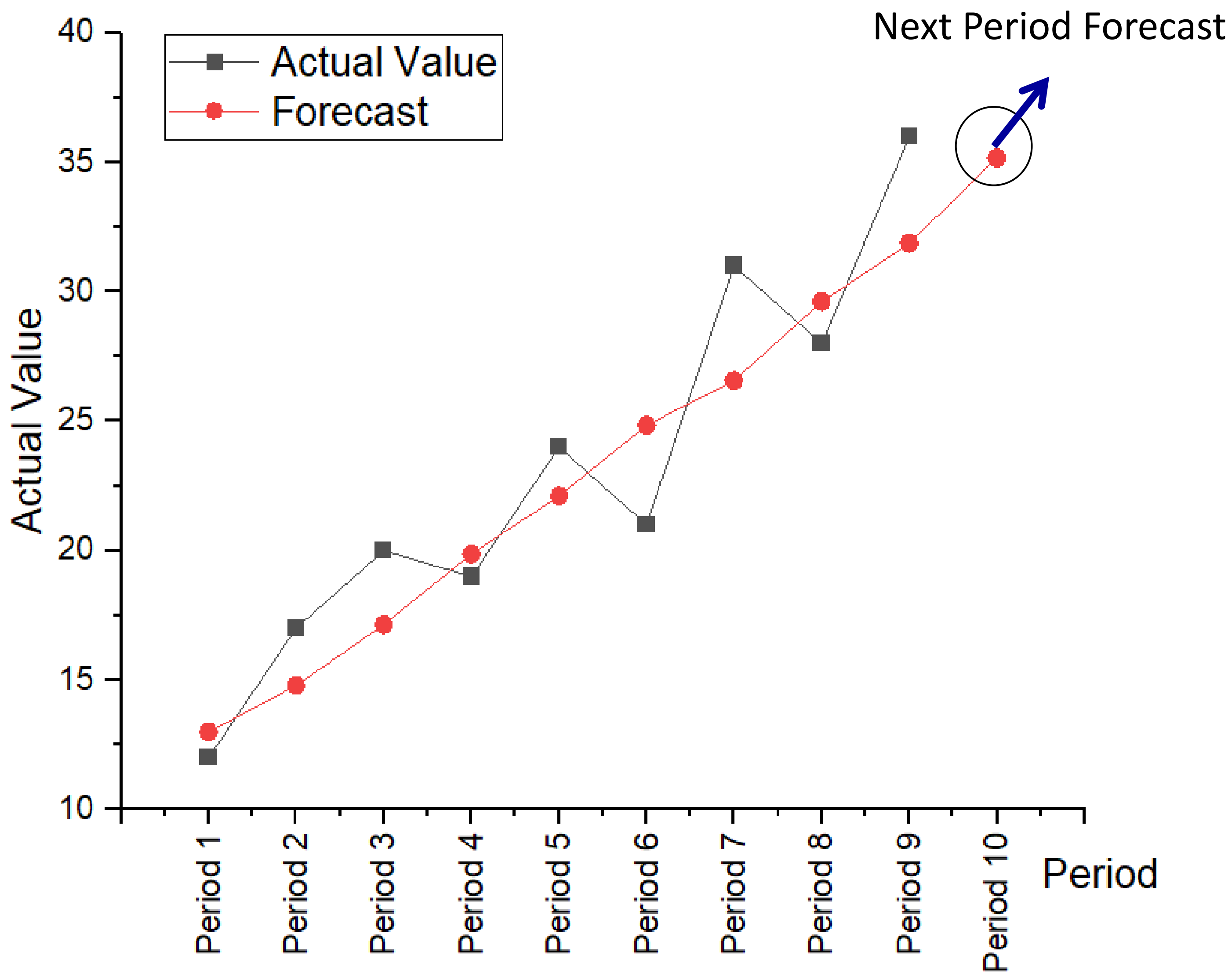
Period	Actual value (A)	Level (F) $\alpha = 0.2$	Trend (T) $\beta = 0.4$	Forecast Yt+1 (FIT)	Error	Absolute Error	Squared Error
		11.00	2.00				
Period 1	12	12.80	1.92	13.00	-1.00	1.00	1.00
Period 2	17	15.18	2.10	14.72	2.28	2.28	5.20
Period 3	20	17.82	2.32	17.28	2.72	2.72	7.41
Period 4	19	19.91	2.23	20.14	-1.14	1.14	1.31
Period 5	24	22.51	2.38	22.14	1.86	1.86	3.45
Period 6	21	24.11	2.07	24.89	-3.89	3.89	15.14
Period 7	31	27.14	2.45	26.18	4.82	4.82	23.24
Period 8	28	29.28	2.32	29.59	-1.59	1.59	2.54
Period 9	36	32.48	2.68	31.60	4.40	4.40	19.36
Period 10	?			35.16		2.63	8.74
						MAD	MSE

$\alpha = 0.2$

$\beta = 0.4$

RMSE = 2.95

Exponential Smoothing with Trend Adjustment

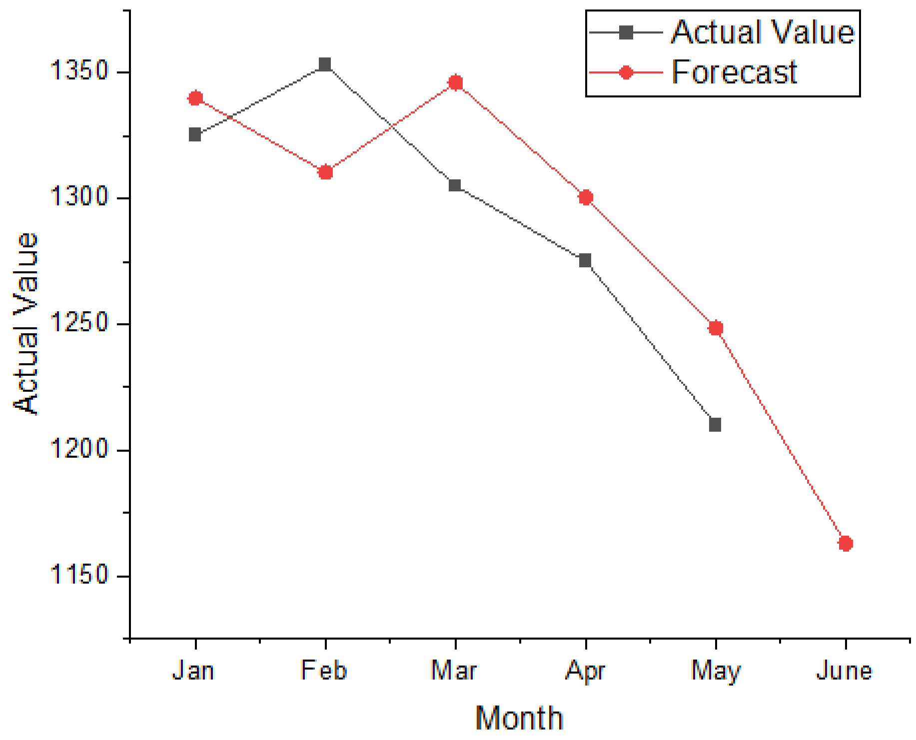


Holt Model in Down Trend Data

$\alpha = 0.65$

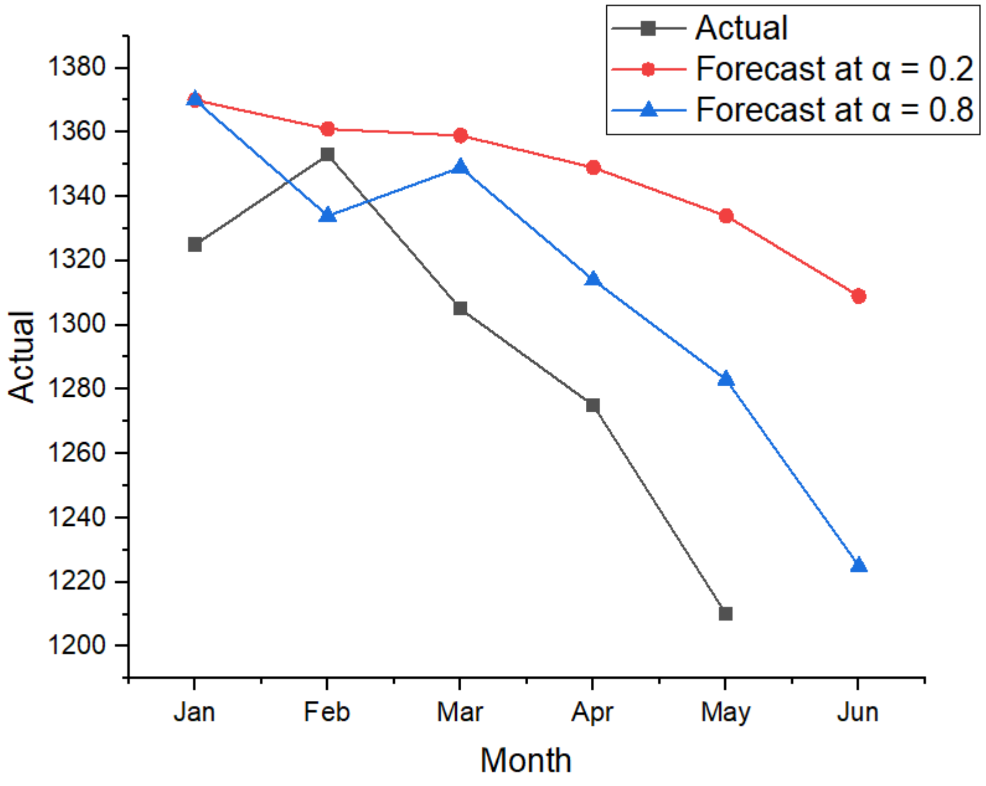
$\beta = 1.0$

Month	Actual Value	Level	Trend	Forecast
		1350	-10	
Jan	1325	1330.23	-19.77	1340.00
Feb	1353	1338.17	7.94	1310.46
Mar	1305	1319.33	-18.84	1346.11
Apr	1275	1283.89	-35.45	1300.49
May	1210	1223.40	-60.49	1248.44
Jun				1162.91

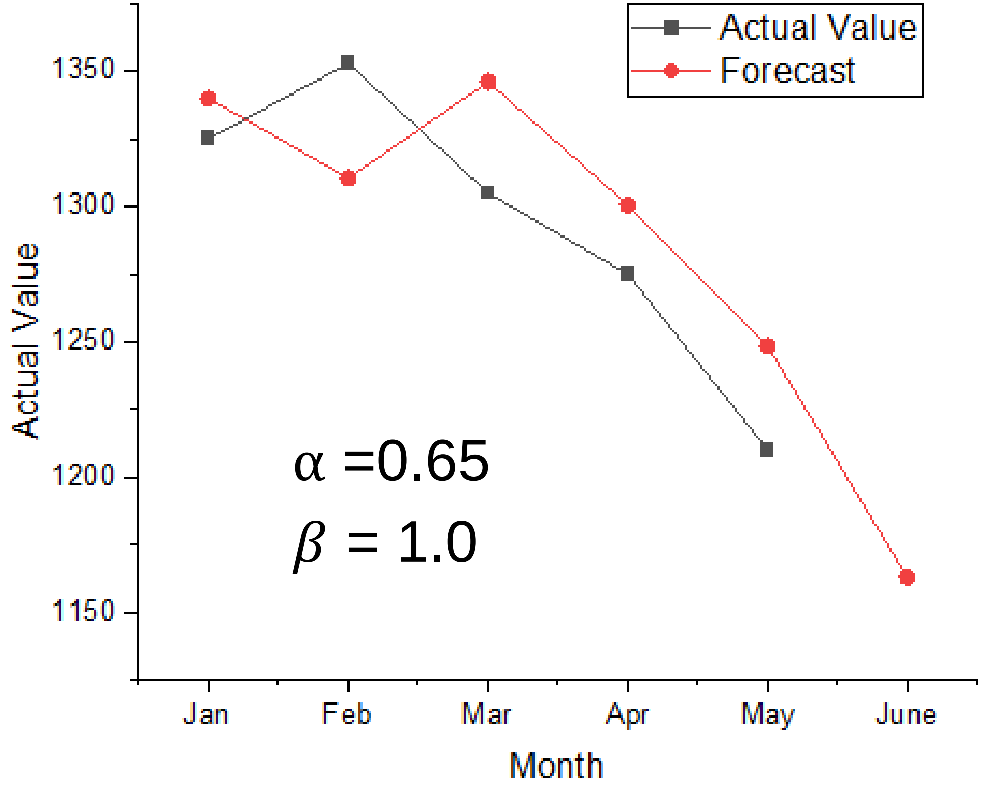


Holt vs. Simple Exponential Smoothing for Down Trend Data

Simple Exponential Smoothing



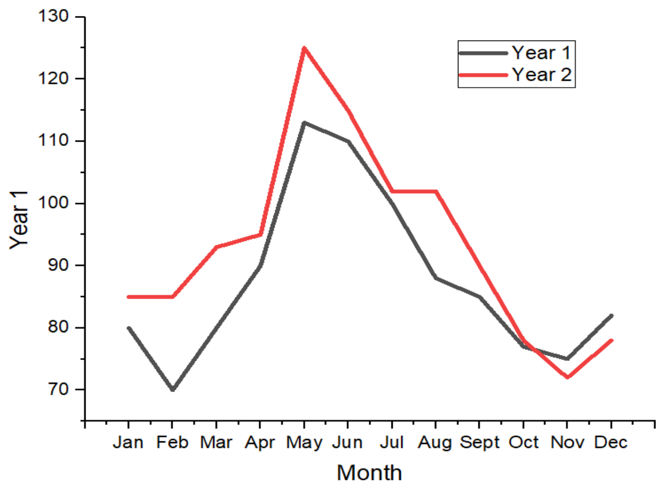
Holt Model



What is Seasonality in Forecasting?

- Seasonality refers to a regular & predictable patterns or fluctuations at specific interval in a time series
- Can be Quarterly, Monthly, Weekly or Daily
- It helps businesses anticipate and plan for changes in demand or other relevant factors
- From seasonal patterns, firms can plan their production, inventory, and marketing strategies.
- Often experienced in industries such as retail, tourism, agriculture, and weather forecasting
- Seasonality can be used to help analyze stocks and economic trends, and retail sales

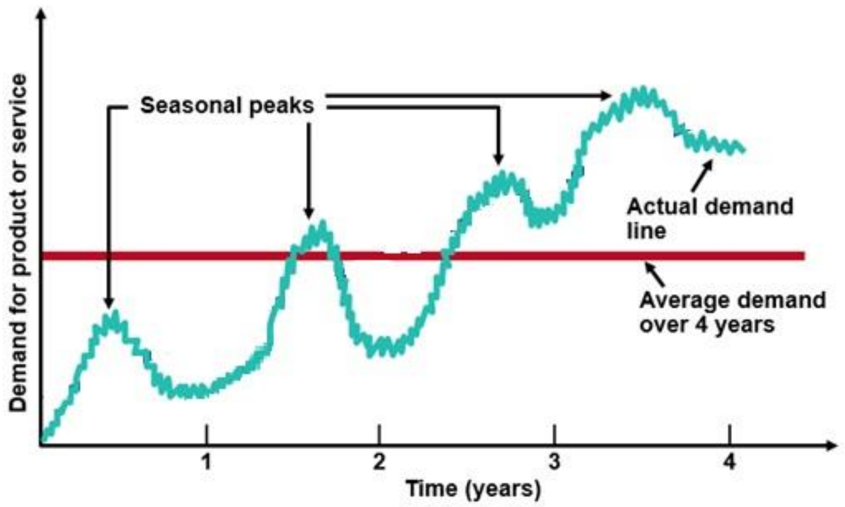
Seasons of India	Months
Summer Season	March to May
Monsoon Season	June to September
Autumn Season	October to November
Winter Season	December to February



Examples of Seasonality in Forecasting

- Retail sales: Retail sales typically peak during the holiday/festival seasons, from November to December.
- Airline traffic: Airline traffic typically peaks during the summer months, when people are on vacation or say during the festival season.
- Electricity consumption: Electricity consumption typically peaks during the summer months, when people are using their air conditioners.
- Ice cream sales: Ice cream sales typically peak during the summer months.
- E-commerce sales: E-commerce sales typically peak during high offer days like big billion days sale of Flipkart.

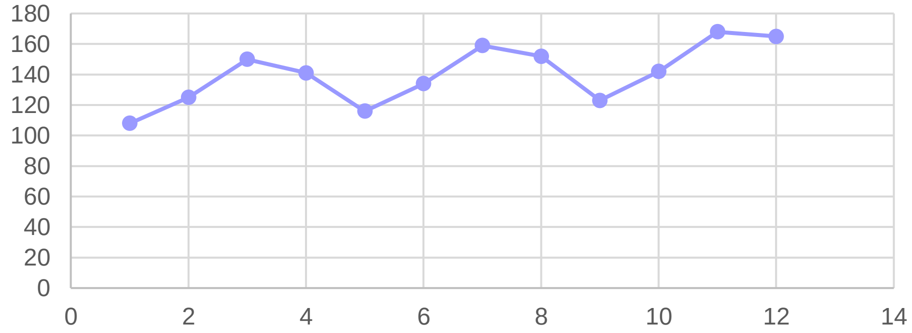
Simple Average Method



Steps in the forecasting process:

- Find the average historical demand for each season
- Compute the average demand over all seasons
- Compute a seasonal index for each season
- Estimate next year's total demand
- Divide this estimate of total demand by the number of seasons; then multiply it by the seasonal index for that season

Seasonal data



Seasonal Index: Calculate the Monthly forecast for 2013

Month	Demand		
	2010	2011	2012
Jan	438	444	450
Feb	420	425	438
Mar	414	423	434
Apr	318	331	338
May	306	318	331
Jun	240	245	254
Jul	240	255	264
Aug	216	223	231
Sept	198	210	224
Oct	225	233	243
Nov	270	278	289
Dec	315	322	335

Seasonal Index Example

$$Seasonal\ Index = \frac{Average\ 2010-2012\ Monthly\ Demand}{Average\ Monthly\ Demand}$$

$=444/309 = 1.435$

Month	Demand			Average	Average monthly	Seasonal Index
	2010	2011	2012	(2010-2012)		
Jan	438	444	450	444	309	1.435
Feb	420	425	438	428	309	1.382
Mar	414	423	434	424	309	1.369
Apr	318	331	338	329	309	1.063
May	306	318	331	318	309	1.029
Jun	240	245	254	246	309	0.796
Jul	240	255	264	253	309	0.818
Aug	216	223	231	223	309	0.722
Sept	198	210	224	211	309	0.681
Oct	225	233	243	234	309	0.755
Nov	270	278	289	279	309	0.902
Dec	315	322	335	324	309	1.047

Seasonal Index Example

y	x
3600	1
3707	2
3831	3

$$y = 115.5\,x + 3481.667$$

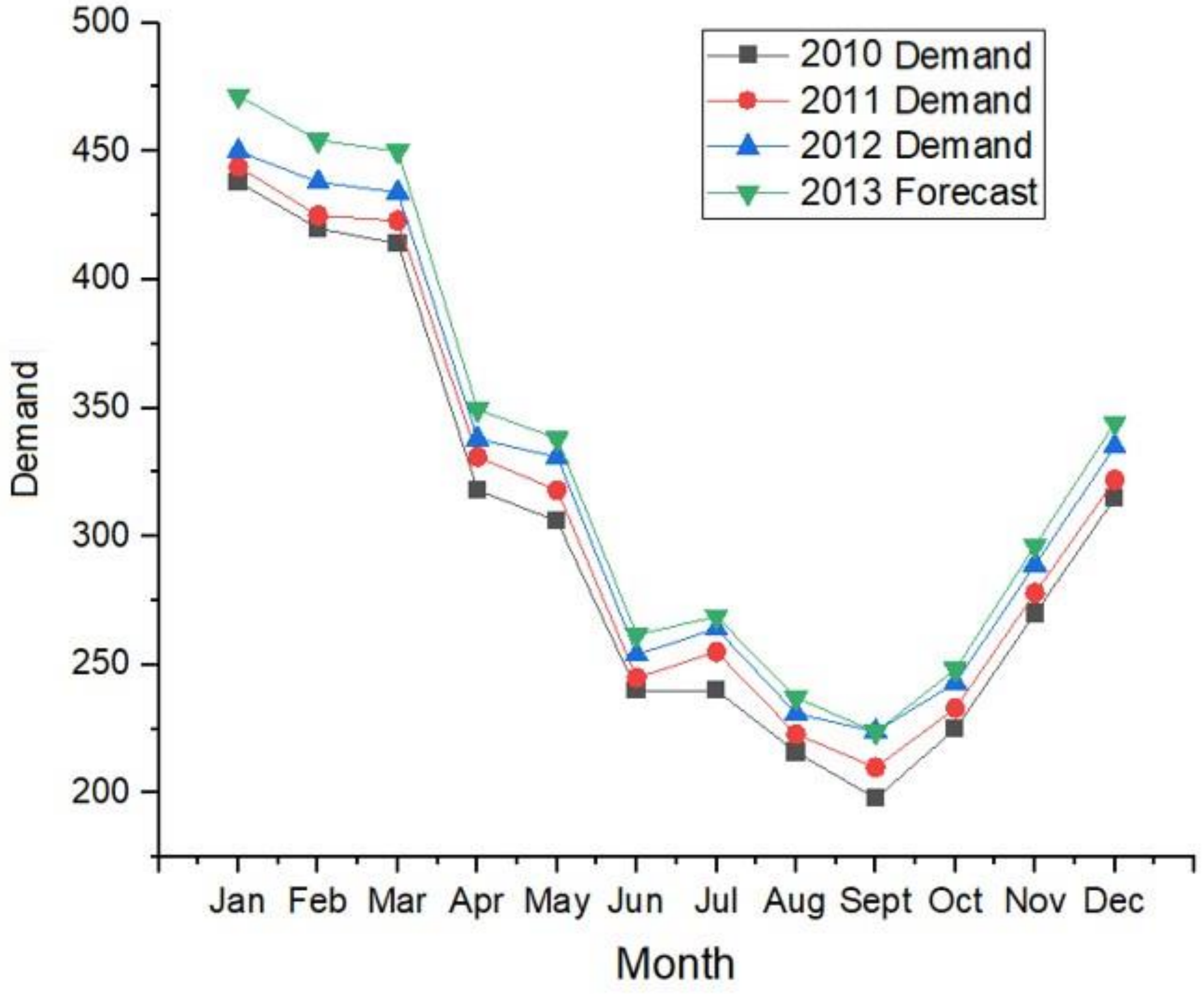
Expected annual demand = 3944 (From Regression)

$$\text{Jan} = \frac{3944}{12} \text{x } 1.435 = 472$$

$$\text{Feb} = \frac{3944}{12} \text{x } 1.382 = 454$$

Month	Demand			Average	Average monthly	Seasonal Index
	2010	2011	2012	(2010-2012)		
Jan	438	444	450	444	309	1.435
Feb	420	425	438	428	309	1.382
Mar	414	423	434	424	309	1.369
Apr	318	331	338	329	309	1.063
May	306	318	331	318	309	1.029
Jun	240	245	254	246	309	0.796
Jul	240	255	264	253	309	0.818
Aug	216	223	231	223	309	0.722
Sept	198	210	224	211	309	0.681
Oct	225	233	243	234	309	0.755
Nov	270	278	289	279	309	0.902
Dec	315	322	335	324	309	1.047
Total	3600	3707	3931			

Seasonal Index Example



Forecasting Seasonality for Quarters?

Year	Q1	Q2	Q3	Q4	Total
1999	18	27	23	14	82
2000	17	30	20	13	80
2001	21	26	23	20	90
2002	21	29	26	16	92
2003	19	28	22	14	83
2004	22	30	26	16	94
2005	24	32	24	19	99
2006	21	29	29	13	92
2007? 2008?					

What would be the seasonal forecast for the year 2007 (98) and 2008 (100)?

Seasonal Indices? (Use Simple/Quarterly Average Method)

Forecasting Seasonality for Quarters?

Year	Q1	Q2	Q3	Q4	Total
1999	18	27	23	14	82
2000	17	30	20	13	80
2001	21	26	23	20	90
2002	21	29	26	16	92
2003	19	28	22	14	83
2004	22	30	26	16	94
2005	24	32	24	19	99
2006	21	29	29	13	92
Quarterly Average	20.37 Index-.915	28.87 Index-1.29	24.15 Index-1.08	15.62 Index-.70	Aveg-22.25

Forecasting Seasonality for Quarters?

	Q1	Q2	Q3	Q4			
1999	18	27	23	14	82		
2000	17	30	20	13	80		
2001	21	26	23	20	90		
2002	21	29	26	16	92		
2003	19	28	22	14	83		
2004	22	30	26	16	94		
2005	24	32	24	19	99		
2006	21	29	29	13	92		
	163	231	193	125			
	20.375	28.875	24.125	15.625	22.25	22.25	
	0.91573	1.29775	1.08427	0.70225			4
2007	24.5	24.5	24.5	24.5	98	say, using regression	
2007?	22.4354	31.7949	26.5646	17.2051			
2008	22.8933	32.4438	27.1067	17.5562	100		

Trend + Seasonal Components

- **Multiplicative/decomposition Model**
  - Centered moving average
  - Calculate the Seasonal Indexes
  - **Deseasonalizing the Time Series**
  - Use deseanalized Time Series to Identify Trend
  - Make Seasonal Adjustments for Final Forecast

Introduction to ARIMA –Time Series Forecasting

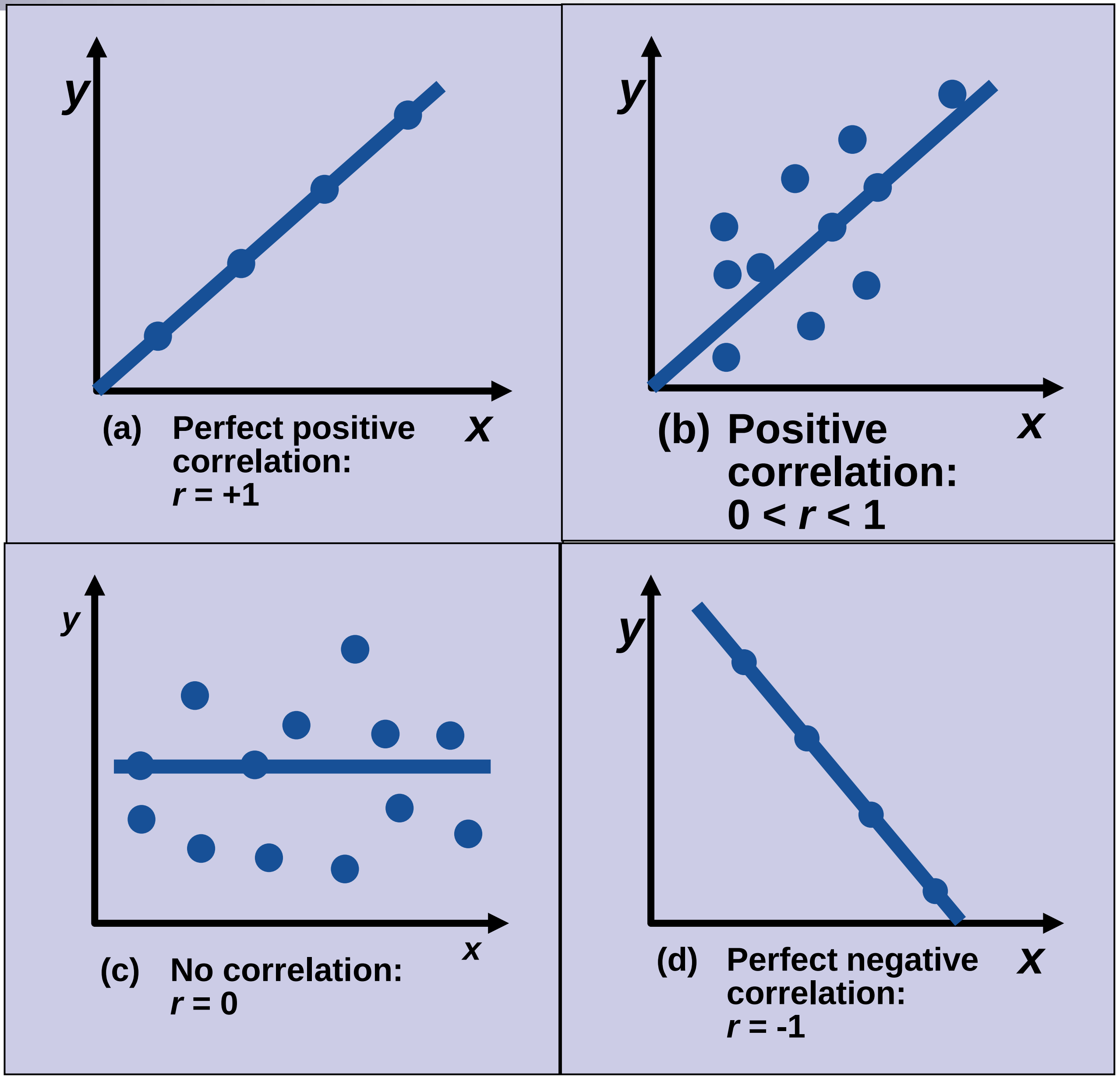
- Introduction-concepts
- Types of ARIMA processes
  - Auto-Regressive (AR)
  - Moving Average (MA)
  - Auto-Regressive Moving Average (ARMA)
- Differencing and ARIMA
- Role of ACF and PACF in ARMA models
- Excel Illustration of ARIMA

Correlation

- Correlation asses that How strong is the linear relationship between the variables?
- Correlation may be Positive, Negative and Zero
- Correlation does not necessarily imply causality!
- To quantify the correlation between the variables, we use Coefficient of correlation (r)

Values range from -1 to +1

$$r = \frac{n \sum xy - \sum x \sum y}{\sqrt{[n \sum x^2 - (\sum x)^2][n \sum y^2 - (\sum y)^2]}}$$



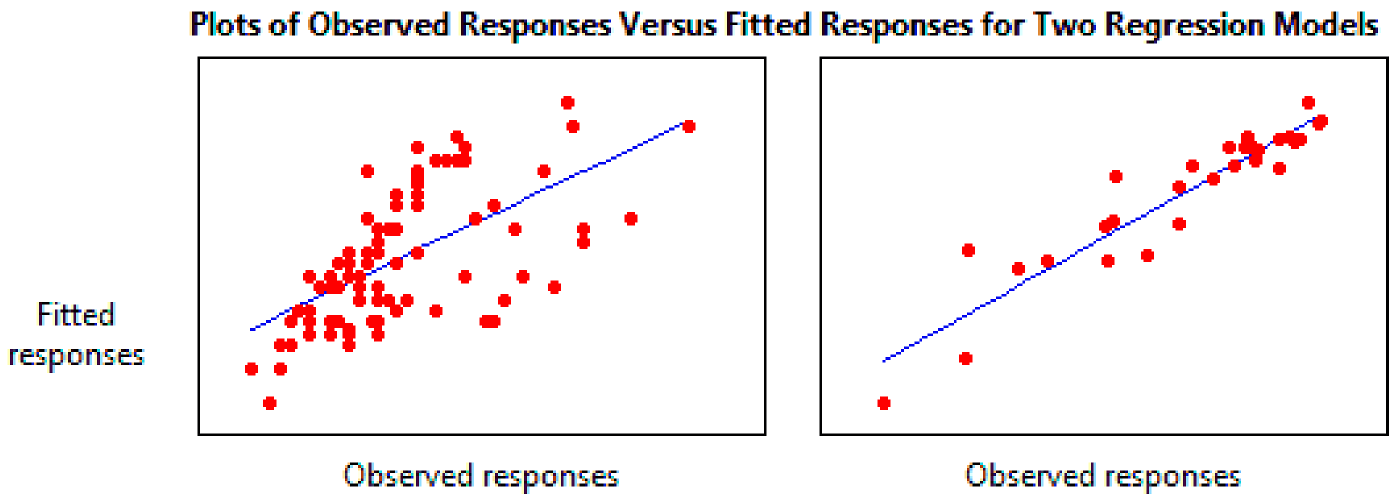
Correlation Coefficient

- Coefficient of Determination, r^2, measures the percent of change in y predicted by the change in x
  - Values range from 0 to 1
  - Easy to interpret

- For the Model Construction (say) example:

$r = .901$

$r^2 = .81$



Multiple Regression Analysis

If more than one independent variable is to be used in the model, linear regression can be extended to multiple regression to accommodate several independent variables

$$\hat{y} = a + b_1x_1 + b_2x_2 \dots$$

Computationally, this is quite complex and generally done on the computer

Example:

One Month's Sales (units)	Aptitude Test Score	Age (years)	Anxiety Test Score	Experience (years)	High School GPA
44	10	22.1	4.9	0	2.4
47	19	22.5	3	1	2.6
60	27	23.1	1.5	0	2.8
71	31	24	0.6	3	2.7
61	64	22.6	1.8	2	2
60	81	21.7	3.3	1	2.5
58	42	23.8	3.2	0	2.5
56	67	22	2.1	0	2.3
66	48	22.4	6	1	2.8
61	64	22.6	1.8	1	3.4
51	57	21.1	3.8	0	3
47	10	22.5	4.5	1	2.7
53	48	22.2	4.5	0	2.8
74	96	24.8	0.1	3	3.8
65	75	22.6	0.9	0	3.7
33	12	20.5	4.8	0	2.1
54	47	21.9	2.3	1	1.8
39	20	20.5	3	2	1.5
52	73	20.8	0.3	2	1.9
30	4	20	2.7	0	2.2
58	9	23.3	4.4	1	2.8
59	98	21.3	3.9	1	2.9
52	27	22.9	1.4	2	3.2
56	59	22.3	2.7	1	2.7
49	23	22.6	2.7	1	2.4
63	90	22.4	2.2	2	2.6
61	34	23.8	0.7	1	3.4
39	16	20.6	3.1	1	2.3
62	32	24.4	0.6	3	4
78	94	25	4.6	5	3.6

Correlation Coefficient Vs Auto-correlation Coefficient

ACF in ARMA

- The key statistic in time series analysis is the autocorrelation coefficient (the correlation of the time series with itself, lagged 1, 2, or more periods.)
- Recall the autocorrelation formula:

$$r_k = \frac{\sum_{t=k+1}^{n} (y_t - \bar{y})(y_{t-k} - \bar{y})}{\sum_{t=1}^{n} (y_t - \bar{y})^2}$$

Time Series Forecasting: ARIMA Models

- ARIMA models are a class of linear models that is capable of representing stationary as well as non-stationary time series.
- ARIMA models do not involve independent variables in their construction. They make use of the information in the series itself to generate forecasts.
- ARIMA models rely heavily on autocorrelation patterns in the data.
- ARIMA methodology of forecasting is different from most methods because it does not assume any particular pattern in the historical data of the series to be forecast.
- It uses an interactive approach of identifying a possible model from a general class of models. The chosen model is then checked against the historical data to see if it accurately describe the series

ARIMA Models

- **ARIMA** is a **highly refined curve-fitting device** that uses **current** and **past values** of the **dependent variable** to produce often accurate short-term forecasts of that variable
  - Examples of such forecasts are stock market price predictions created by brokerage analysts
  - Weather conditions in specific regions
  - Electricity consumptions in specific household/organization
  - Total sales in a store
  - Heart rate monitoring
  - Crude oil price predictions, etc.

- If ARIMA models thus essentially **ignores economic theory** (by ignoring "traditional" explanatory variables), why use them?
- The use of ARIMA is appropriate when:
  - little or nothing is known about the dependent variable being forecasted,
  - the independent variables known to be important cannot be forecasted effectively
  - all that is needed is a one or two-period forecast

Important Terminologies of ARIMA

- AR: To describe a model in which a variable's current value is dependent on its past values.
- MA: The moving average (MA) is calculated by taking the average of a specified number of periods of a given data set, and periods can be daily, weekly, monthly, or any other time frame, depending on preference.
- ARIMA: Auto Regressive Integrated Moving Average (ARIMA) model that combines AR and MA to model and predict future values.
  - It can involve more complex data, more advanced diagnostics, and the consideration of various ARIMA model orders.
  - It's often helpful to use statistical software to perform these calculations and analysis effectively
- ACF: To measure the correlation between time series and its own past values at different lags
- PACF: to measure the partial correlation between a time series and its own past values at different lags

Steps of ARIMA Models

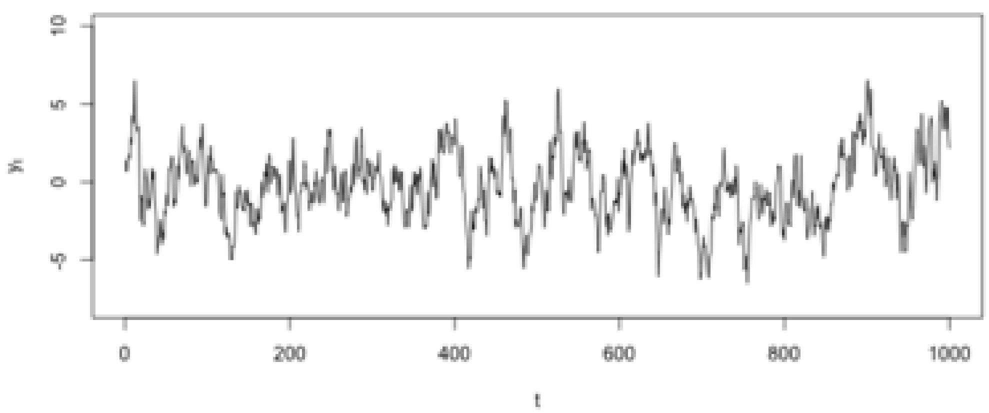
- Visualize the Time Series Data
- Make the Time Series data stationary, if not
  - Use differencing
  - If the data follows seasonality, then use 12 points differencing (i.e., scale down the data by taking differences of not immediate period, rather the next seasonal period/(January to January/Q1 to Q1).
- Plot the autocorrelation (ACF) and Partial autocorrelation (PACF) charts
- Select the lags (p, d, q) for the AR, differencing, and MA model as per the ACF and PCF charts
- Construct the ARIMA models or seasonal AIRMA models based on the stationary data
- Use the model to make future predictions

Stationary Vs Non-Stationary Time Series Data

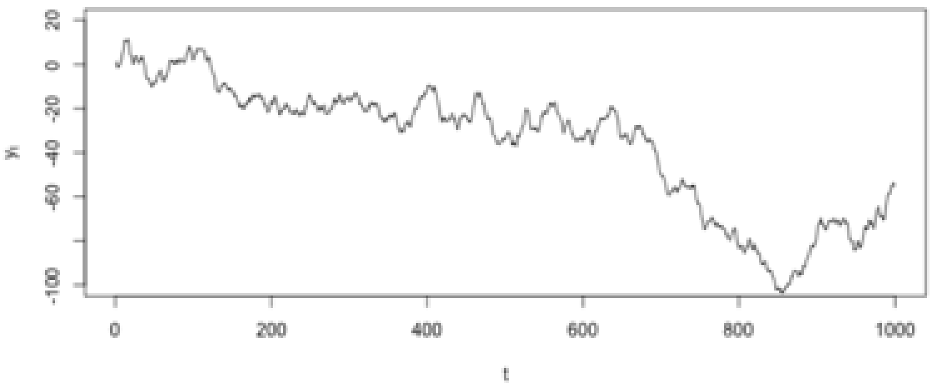
- How to check if the data is stationary or not?

- Plot the data and check whether the mean and variance remain constant (same distributions) for the whole series no matter where you chose a period.
- If it satisfies the above condition then the time series is said to be stationary.

Stationary Time Series



Non-Stationary Time Series



ARIMA Models

The ARIMA approach **combines two different specifications** (called processes) into one equation:

- An **autoregressive** process **(AR) or (p):**
  expresses a dependent variable as a function of past values of the dependent variable.

$$Y_t = a + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \cdots + \beta_n y_{t-n}$$

- A **moving average** process **(MA) or (q):**
  expresses a **dependent variable** as a function of **past values** of the **error term**

  Such a function is a moving average of past error term observations that can be added to the mean of Y to obtain a moving average of past values of Y

$$\mathrm{Y_t = Y_{Mean} + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \cdots + \beta_n \varepsilon_{t-n}}$$

ARIMA terminology

- A non Seasonal ARIMA model can be summarize in three numbers:

p= the number of autoregressive terms
d= the number of non-seasonal differences
q= the number of moving average terms

- The model can be called as “ARIMA (p,d,q)”.

- The model may also include a constant term (or not)

Autoregressive Model (AR)

An Autoregressive model is when a value from time series is regressed on the past values of the variable from the same time series

- Thus the autoregressive model can be written as:

$$y_t = \alpha + \beta_1 y_{t-1} + \cdots + \beta_p y_{t-p}$$

Where ε_t is white noise and AR(p) model is Autoregressive model of order *p*

- AR model are used for handling a wide range of different time series patterns.
- The value of *p* is called the order. For example $\mathrm{AR}(1)$ is first order autoregressive process which is currently based on the immediately preceding value;

while $\mathrm{AR}(2)$process is one in which value is based on the past two values.

- $\mathrm{AR}(0)$process is used for white noise and has no dependence between the terms.

AR (1) $\boldsymbol{y_t = \alpha + \beta 1 \ * y_{t-1}}$
AR (2) $\boldsymbol{y_t = \alpha + \beta 1 \ * y_{t-1} + \beta 2 \ * y_{t-2}}$
AR (3) $\boldsymbol{y_t = \alpha + \beta 1 \ * y_{t-1} + \beta 2 \ * y_{t-2} + \beta 3 \ * y_{t-3}}$

Moving Average Model (MA)

- A moving average model uses past forecast error value in regression like model.

$$Y_t = Y_{Mean} + w_1\varepsilon_{t-1} + \cdots + w_q\varepsilon_{t-q} + \varepsilon_t$$

where ε_t is white noise and MA(q) model is moving average model for order q.

- A moving average model is used for forecasting future values.

MA (1) $\boldsymbol{y_t = Y_{Mean} + w1 * \varepsilon_{t-1}}$
MA (2) $\boldsymbol{y_t = Y_{Mean} + w1 * \varepsilon_{t-1} + w2 * \varepsilon_{t-2}}$
MA (3) $\boldsymbol{y_t = Y_{Mean} + w1 * \varepsilon_{t-1} + w2 * \varepsilon_{t-2} + w3 * \varepsilon_{t-3}}$

Autoregressive and Moving Average Model (ARMA)

The autoregressive moving average (ARMA) is the basic model for analysing a linear stationary time series.

- ARMA is simply the merger between $\mathrm{AR}(p)$ and $\mathrm{MA}(q)$ models
- An Autoregressive model $\mathrm{AR}(p)$ is when a value from time series is regressed on the past values of the variable from the same time series
- The moving average model $\mathrm{MA}(q)$ makes predictions using the series mean and previous errors

Autoregressive and Moving Average Model (ARMA)

- A simple autoregressive moving average model would like this:

$$y_t = a + \varepsilon_t + \gamma_1 y_{t-1} + \beta_1 \varepsilon_{t-1}$$

- y_t and y_{t-1}represent the values in current and one period ago
- ε_t and $\varepsilon_{t-1}$are the error terms for two same periods. The error term from the last period is used to help us correct our predictions.
- a stands for constant factor
- The γ_1 and β_1 The former, γ_1, expresses on average what part of the value last period (y_{t-1}) is relevant in explaining the current one. Similarly, the latter, β_1 , represents the same for the past error term (ε_{t-1}).

ARMA Model (in general of order p and q)

Three basic ARIMA models for a stationary time series y_t

(1)Autoregressive model of order *p* (AR(*p*))

$$y_t = \alpha + \varepsilon_t + \beta_1 y_{t-1} + \cdots + \beta_p y_{t-p}$$

i.e., y_t depends on its *p* previous values

(2) Moving Average model of order *q* (MA(*q*))

$$y_t = a + \varepsilon_t - \beta_1 \varepsilon_{t-1} - \cdots - \beta_q \varepsilon_{t-q}$$

i.e., y_t depends on *q* previous random error terms

(3) Autoregressive-moving average model of order *p* and *q* (ARMA(*p,q*))

$$y_t = \alpha + \varepsilon_t + \beta_1 y_{t-1} + \cdots + \beta_p y_{t-p} + \varepsilon_t + \beta_1 \varepsilon_{t-1} - \cdots - \beta_q \varepsilon_{t-q}$$

i.e., y_t depends on its *p* previous values and *q* previous random error terms

ARMA Example Model

AR MA Example Models

ARIMA (0,0,1)=MA(1) $Y_t = 10 + e_t - 0.7e_{t-1}$

ARIMA (1,0,0)= AR(1) $Y_t = 3 + 0.7Y_{t-1} + e_t$

ARIMA (0,0,2)= MA(2) $Y_t = e_t - 0.4e_{t-1} + 0.9e_{t-2}$

ARIMA (2,0,0)= AR(2) $Y_t = 6 + 1.2Y_{t-1} - 0.7Y_{t-2} + e_t$

ARIMA(1,01)=ARMA(1,1) $Y_t = 10 + 0.3Y_{t-1} + e_t - 0.7e_{t-1}$

Differencing (I) in ARIMA Models

- Before this equation can be applied to a time series, however, it must be ensured that the time series is stationary, as defined earlier.
- For example, a **non-stationary** series can often be **converted** into a **stationary** one by taking the **first difference**:
- $y*_t = \Delta y_t = y_t - y_{t-1}$
- If the first differences do not produce a stationary series, then first differences of this first-differenced series can be taken—i.e. a **second-difference transformation:**
- $y*_t* = (\Delta y*_t) = y*_t - y*_{t-1} = \Delta y_t - \Delta y_{t-1}$

- **Example:** Transforming nonstationary data into stationary data:

X = [5,4,6,7,9,12];

After one lag differencing, we get X* = [-1, 2,1, 2, 3]

ARIMA Models

- For example, if d = 1 , then d is the number of differences taken to make Y stationary.
- This conversion process is **similar to integration** in mathematics, so the “I” in ARIMA stands for **“integrated”**
- **ARIMA** thus stands for **Auto-Regressive Integrated Moving Average**
  - An ARIMA model with p, d, and q specified is usually denoted as **ARIMA (p,d,q)** with the specific integers chosen inserted for p, d, and q
  - If the original series is **stationary** and d therefore equals 0, this is sometimes shortened to **ARMA**

Y_t → AR filter → Integration filter → MA filter → ε_t

(long term) (stochastic trend) (short term) (white noise error)

ARIMA (2,0,1) $y_t = a_1 y_{t-1} + a_2 y_{t-2} + b_1 \varepsilon_{t-1}$

ARIMA (3,0,1) $y_t = a_1 y_{t-1} + a_2 y_{t-2} + a_3 y_{t-3} + b_1 \varepsilon_{t-1}$

ARIMA (1,1,0) $\Delta y_t = a_1 \Delta y_{t-1} + \varepsilon_t$, where $\Delta y_t = y_t - y_{t-1}$

ARIMA (2,1,0) $\Delta yt = a_1 \Delta y_{t-1} + a_2 \Delta y_{t-2} + \varepsilon_t$ where $\Delta yt = yt - yt\text{-}1$

To build a time series model issuing ARIMA, we need to study the time series and identify p,d,q

Identification

- What does it take to make the time series stationary?
- In the stationary model AR, MA, ARMA
  - If AR(p) how big is p?
  - If MA(q) how big is q?
  - If ARMA(p,q) what are p and q?

AR (p) Model

- The ACF will show exponential decay
- The first p terms of the PACF will be significantly different from zero (outside the parallel lines)

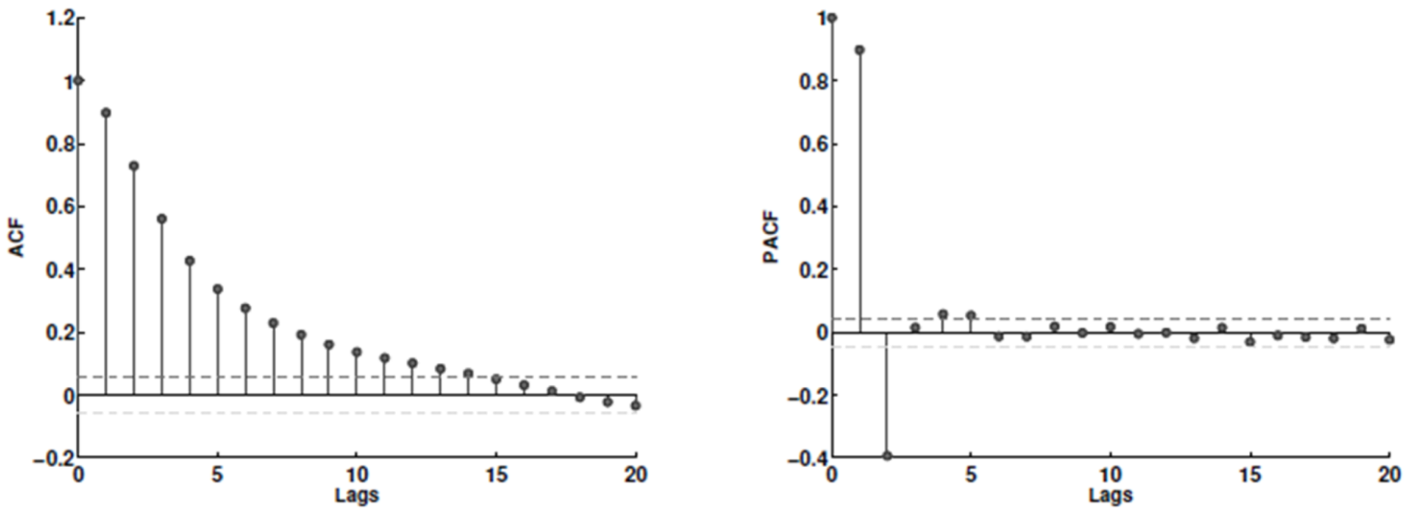
MA(q) models

- The first q terms of the ACF will be significantly different from zero
- The PACF will decay exponentially towards zero

PACF and ARMA Model

- PACF results as an alternative measure to ACF
- PACF of an AR(2) process indicates the order of the AR process
- PACF of an MA(1) process displays exponentials decay

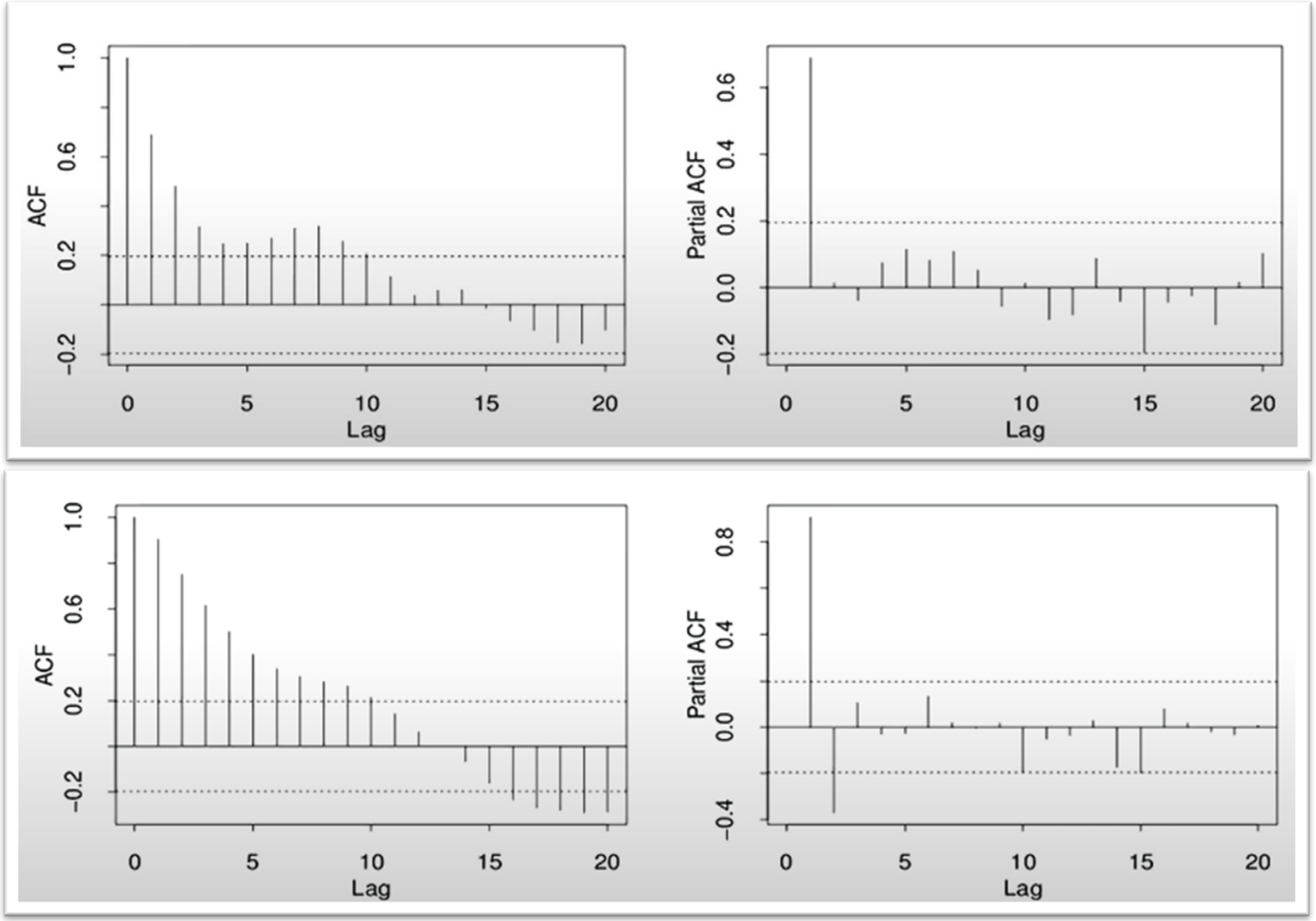
Plots of ACF and PACF of AR(1) model



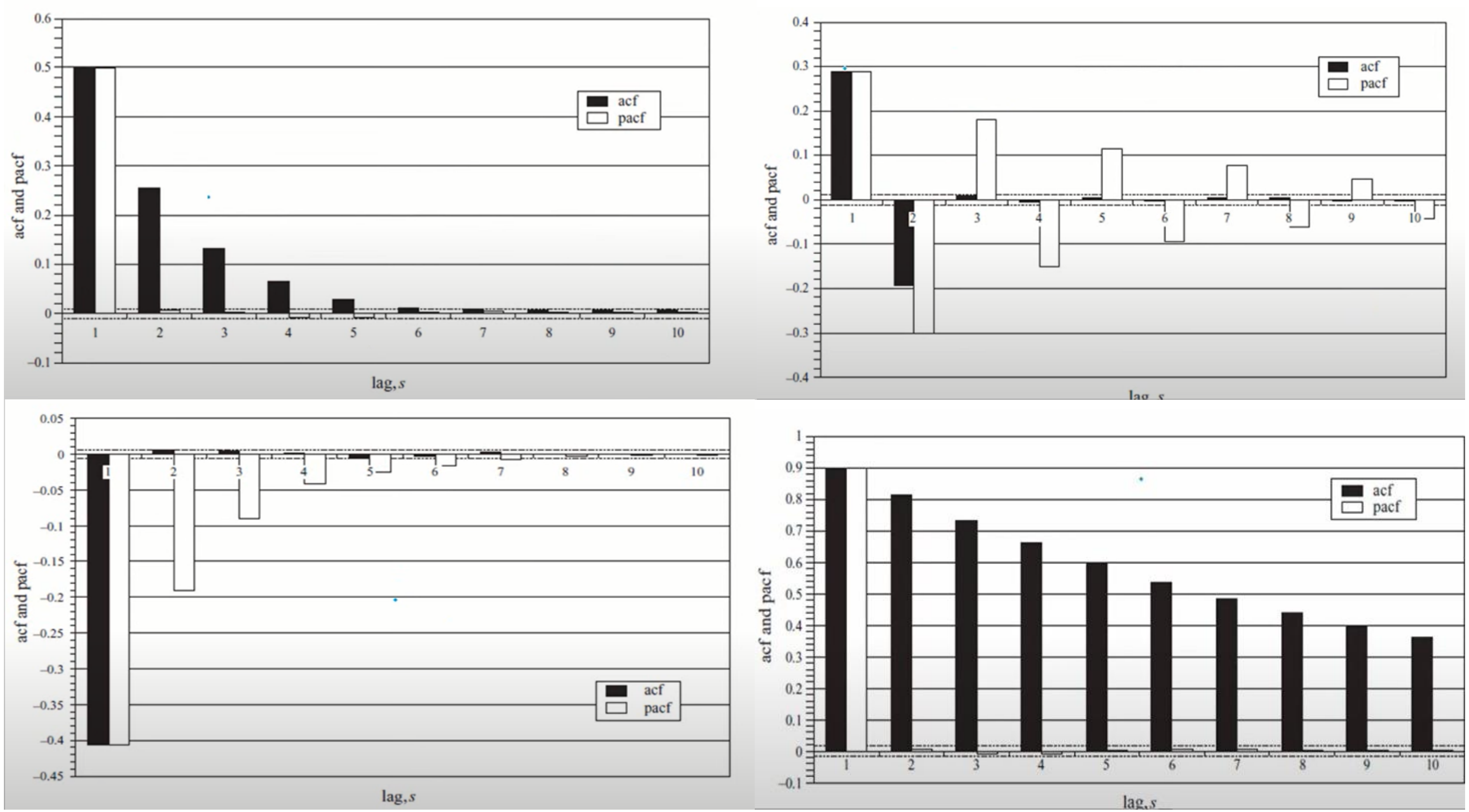
(a) ACF of the series

(b) PACF of the series

ACF and PACF graph



ACF and PACF graph



ARMA Models

ACFs	PACFs	Model
Decay to zero with exponential pattern	Cuts off after lag p	AR(p)
Cuts off after lag q	Decay to zero with exponential pattern	MA(q)
Decay to zero with exponential pattern	Decay to zero with exponential pattern	ARMA(p, q)

- That is,

	AR(p)	MA(q)	ARMA(p,q)
ACF	Tails off	Cuts off after lag q	Tails off
PACF	Cuts off after lag p	Tails off	Tails off

- If you can’t easily tell if the model is an AR or a MA, assume it is an ARMA model.

ARIMA Models

Data Preparation:

1) Plot data and examine for stationarity
2) Examine ACF for stationarity
3) If not stationary, take first differences
4) If variance appears non-constant, take logarithm before first differencing
5) Examine the ACF after these transformations to determine if the series is now stationary

Model Identification and Estimation:

1) Examine the ACF and PACF's of your (now) stationary series to get some ideas about what ARIMA(p,d,q) models to estimate.
2) Estimate these models
3) Examine the parameter estimates, the statistic and test of white noise for the residuals.

Forecasting:

1) Use the best model to construct forecasts
2) Graph your forecasts against actual values
3) Calculate the Mean Squared Error for the forecasts

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