**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans:

Given Mean = 45

Standard Deviation = 8

We have to find the probability of P(X>50) where X denotes the amount of time taken to complete the work on the car.

We know that P(X>50) = 1 - P(X≤50)

Since it is a normal distribution, we can consider Z = (X-*μ*)/*σ*

P(X≤50) = P(Z ≤ (50-45)/8)

P(X≤50) = P(Z ≤ 0.625) = 73.24%

Hence, the probability that the service manager cannot meet his commitment is

P(X>50) = 1 - 73.24 = 26.76% = 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: See Set\_2.ipynb

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans:

Given Y1 = 2X1 and Y2 = X1 + X2

Now Mean[Y1] = 2Mean[X1] = 2μ and Mean[Y2] = Mean[X1] + Mean[X2] = 2μ

But Var[Y1] = 22Var[X1] = 4σ2 and Var[Y2] = Var[X1] +Var[X2] = 2σ2

We can clearly see that Y1 and Y2 both follow normal distribution with same mean but their variances are different. Y1 has a higher variance compared to Y2

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: See Set\_2.ipynb

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans:

See Set\_2.ipynb