Model Solutions to Tutorial 2

Problem 1

- $C_1 = \dfrac{(1-X_1)P_1}{Q_{13}} = \dfrac{(1-X_1)\,10^9}{10^7} = 100(1-X_1)$

$C_1 \leq 20$ is the constraint

$\Rightarrow \quad 100(1-X_1) \leq 20 \quad \Rightarrow (1-X_1) \leq 0.2$

$\Rightarrow \boxed{-X_1 \leq -0.8}$ ← Eq. 1

- $C_2 = \dfrac{(1-X_2)P_2}{Q_{23}} = \dfrac{(1-X_2)\,2\times10^9}{5\times10^7} = 40(1-X_2)$

$C_2 \leq 20 \Rightarrow$

$\Rightarrow \quad 40(1-X_2) \leq 20 \quad \Rightarrow (1-X_2) \leq 0.5$

$\Rightarrow \boxed{-X_2 \leq -0.5}$ ← Eq. 2

- $C_3 = \dfrac{R_{13}Q_{13}C_1 + R_{23}Q_{23}C_2 + (1-X_3)P_3}{Q_{34}}$

$= \dfrac{0.5\times10^7\times C_1 + 0.35\times5\times10^7\times C_2 + (1-X_3)\,4\times10^9}{11\times10^7}$

$= \dfrac{0.5}{11}C_1 + \dfrac{1.75}{11}C_2 + \dfrac{400}{11}(1-X_3)$

$= \dfrac{50}{11}(1-X_1) + \dfrac{70}{11}(1-X_2) + \dfrac{400}{11}(1-X_3)$

$= \dfrac{520}{11} - \dfrac{50}{11}X_1 - \dfrac{70}{11}X_2 - \dfrac{400}{11}X_3$

$C_3 \leq 20$ is the constraint

$$\Rightarrow -\frac{50}{11}x_1 - \frac{70}{11}x_2 - \frac{400}{11}x_3 \leq 20 - \frac{520}{11}$$

$$\Rightarrow \boxed{-\frac{50}{11}x_1 - \frac{70}{11}x_2 - \frac{400}{11}x_3 \leq -\frac{300}{11}} \leftarrow \text{Eq. 3}$$

- $C_4 = \dfrac{R_{34} Q_{34} C_3 + (1-X_4) P_4}{Q_{45}}$

$$= \frac{0.6 \times 11 \times 10^7 C_3}{25 \times 10^7} + \frac{(1-X_4)\, 2.5 \times 10^9}{25 \times 10^7}$$

$$= \frac{6.6}{25} C_3 + 10(1-X_4)$$

$$= \frac{6.6}{25}\left[\frac{520}{11} - \frac{50}{11}x_1 - \frac{70}{11}x_2 - \frac{400}{11}x_3\right] + 10(1-X_4)$$

$$= \frac{312}{25} - \frac{30}{25}x_1 - \frac{42}{25}x_2 - \frac{240}{25}x_3 + 10 - 10X_4$$

$$= \frac{562}{25} - \frac{30}{25}x_1 - \frac{42}{25}x_2 - \frac{240}{25}x_3 - 10X_4$$

$C_4 \leq 20$ is the constraint

$$-\frac{30}{25}x_1 - \frac{42}{25}x_2 - \frac{240}{25}x_3 - 10X_4 \leq 20 - \frac{562}{25}$$

$$\Rightarrow \boxed{-\frac{30}{25}x_1 - \frac{42}{25}x_2 - \frac{240}{25}x_3 - 10X_4 \leq -\frac{62}{25}}$$

↑ Eq. 4

The cost (objective function) to be minimized is:

$$Z_1 = d_1 P_1 X_1 + d_2 P_2 X_2 + d_3 P_3 X_3 + d_4 P_4 X_4$$

$$= (2\times10^{-6} \times 1\times10^{9}) X_1 + (2\times10^{-6} \times 2\times10^{9}) X_2 + (4\times10^{-6} \times 4\times10^{9}) X_3 + (4\times10^{-6} \times 2.5\times10^{9}) X_4$$

i.e. $$Z = 2000 X_1 + 4000 X_2 + 16000 X_3 + 10000 X_4$$

"fmincon" is used to minimize Z subject to the constraints given in equations 1 through 4.

- The four linear inequality constraints can be written as

$$\underbrace{\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ -\frac{50}{11} & -\frac{70}{11} & -\frac{400}{11} & 0 \\ -\frac{30}{25} & -\frac{42}{25} & -\frac{240}{25} & -10 \end{bmatrix}}_{A} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} \leq \underbrace{\begin{bmatrix} -0.8 \\ -0.5 \\ -300/11 \\ -62/25 \end{bmatrix}}_{B}$$

$$\Rightarrow AX \leq B$$

- There are no equality constraints
- lower bound for $X_1, X_2, X_3, X_4 = 0 \Rightarrow lb = [0\ 0\ 0\ 0]$
- upper bound for $X_1, X_2, X_3, X_4 = 1 \Rightarrow ub = [1\ 1\ 1\ 1]$

Please see file question1.m (mainfile) & gn1.m (objective function to be minimized.)

lowest cost = \$12,600 and $X_{1,opt} = 0.8$, $X_{2,opt} = 0.5$, $X_{3,opt} = 0.5625$, $X_{4,opt} = 0$