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BIT-MANIPULATION OPERATORS :

1. AND

a	b	a&b
0	0	0
0	1	0
1	0	0
1	1	1

NOTE : When we AND 1 with any number the number remains the same (and not vice versa & AND with same number gives the same)

Eg:
```
     1001010101101
(&)  1111111111111
     1001010101101
```

```
     10          10
(&)  11     (&)  10
     10          10
```

2. OR

a	b	a|b
0	0	0
0	1	1
1	0	1
1	1	1

NOTE : OR '0' with any number gives the same.

3. XOR (^) ⇒ if and only if

a	b	a^b
0	0	0
0	1	1
1	0	1
1	1	0

[only one should be true]

NOTE: $a \wedge 1 = \bar{a}$

$a \wedge 0 = a$

$a \wedge a = 0$

4. Complement (~) (NOT)

a	~a	
0	1	
1	0	

5. Left Shift Operator (<<)

Internally, computer first converts the decimal number to binary number, then perform shift operations.

* $(10)_{10} = (1010)_2$

* 1010 << 1

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* 10100 = 20 [Replacing extra no with 0]

General form

$$\boxed{a << b = 2^b * a}$$

6. Right Shift Operator (>>)

Q : 0011001 >> 1

001100 = 1100 (LSB is ignored)
= $(12)_{10}$

General form :-
$$a >> b \Rightarrow \frac{a}{2^b}$$

NUMBER SYSTEM :

1. Decimal (10) → 0,1,2,3,4,5,6,7,8,9.
2. Binary (2) → 0,1
3. Octal (8) → 0,1,2,3,4,5,6,7.
4. Hexadecimal (16) → 0-9, A-F

DECIMAL :	0	1	2	3	4	5	6	7	8	9	10	11	12
OCTAL :	0	1	2	3	4	5	6	7	10	11	12	13	14

13	14	15	16	17	18	19	20
15	16	17	20	21	22	23	24

$(9)_{10} = (11)_8$

CONVERSIONS:

1. Decimal – Base b
2. Base b – Decimal

Q i) $(17)_{10}$ to Base 2.

Keep dividing by base, take remainders, write it opposite

$$\begin{array}{r|l}
2 & 17 \\
2 & 8 - 1 \\
2 & 4 - 0 \\
2 & 2 - 0 \\
 & 1 - 0
\end{array}$$

$(17)_{10} = (10001)_2$

ii) $(17)_{10}$ to Base - 8

$$\begin{array}{r|l}
8 & 17 \\
 & 2 - 1
\end{array}$$

$(17)_{10} = (21)_8$

Q i) $(10001)_2$ – Base 10

Multiply & add the power of base with digits.

=

1 0 0 0 1

$2^0 \times 1 = 1 \times 1 = 1$

$2^1 \times 0 = 2 \times 0 = 0$

$2^2 \times 0 = 4 \times 0 = 0$

$2^3 \times 0 = 8 \times 0 = 0$

$2^4 \times 1 = 16 \times 1 = 16$

17

$(10001)_2 = (17)_{10}$

ii) $(21)_8$ - Base-10

2 1

$8^0 \times 1 = 1 \times 1 = 1$

$8^1 \times 2 = 8 \times 2 = 16$

17

$(21)_8 = (17)_{10}$

Q: Base-10 to Base-8

1. Base-10 to Base-2
2. Base-2 to Base-8

Q i) $(23.81252)_{10}$ to base-2

$\Rightarrow (28 + 0.81252)_{10}$

2	23
2	11 - 1
2	5 - 1
2	2 - 1
	1 - 0

$(10111)_2$

$0.81252 \times 2 = 1.62504 - 1$

$0.62504 \times 2 = 1.25008 - 1$

$0.25008 \times 2 = 0.50016 - 0$

$0.50016 \times 2 = 1.00032 - 1$

$0.00032 \times 2 = 0.00064 - 0$

$0.00064 \times 2 = 0.00128 - 0$

$0.00128 \times 2 = 0.00256 - 0$

$= (10111.1101000)_2$

ii) $(28.125)_{10}$ to Base-2

$$
\begin{array}{r|l}
2 & 28 \\
2 & 14 - 0 \\
2 & 7 - 0 \\
2 & 3 - 1 \\
2 & 1 - \cancel{0}1 \\
 & 0 - 1
\end{array}
$$

$(28)_{10} = (11\cancel{0}100)_2$

$= (11100)_2$

$0.125 \times 2 = 0.250 - 0$

$0.250\cancel{0} \times 2 = 0.500 - 0$

$0.500 \times 2 = 1.000 - 1$

$(28.125)_{10} = (11100.001)_2$

$$
\begin{array}{ccccccccc}
1 & 1 & 1 & 0 & 0 & . & 0 & 0 & 1 \\
\times & \times & \times & \times & \times & & \times & \times & \times \\
2^4 & 2^3 & 2^2 & 2^1 & 2^0 & . & 2^{-1} & 2^{-2} & 2^{-3} \\
16+ & 8+ & 4+ & 0+ & 0 & . & 0 & 0 & 0.125
\end{array}
$$

$$(28.125)_{10}$$

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Q 1) Given a number, find if it is odd/even

* Every number is calculated in binary form internally.

* Eg: 12 + 7 = 19

```
  1100
(+)0111
 10011
```

$(1)_2+(1)_2=(10)_2$

* ~~In~~ binary, for eg., 1001$\boxed{1}$, ~~this~~ leaving this LSB, every other is a power of 2. This LSB; makes a number even or odd.

i) 1001$\boxed{1}$

$1 \times 2^0 = 1$

∴ It is odd

ii) 110$\boxed{0}$

$0 \times 2^0 = 0$

∴ It is even.

$\boxed{\therefore\ a \,\&\, 1 \text{ gives } a}$ → odd else even.

2) arr [2,3,4,1,2,1,3,6,4]

Given an array, find the number that is not repeated [appears once].

* a ^ a = 0
* 2 ^ 3 ^ 2 = 3 = 3 ^ 2 ^ 2
*

3) arr [-2, -3, -4, -1, 2, 1, 3, 6, 4]

* sum of (+1, -1) = 0
* Order doesn't matter.

4) Find i^{th} bit of a number.

1	0	1	$\boxed{1}$	0	1	1	0
8	7	6	$\boxed{5}$	4	3	2	1
			?				

& any number with 1 will give that number

$$\begin{array}{cccccccc} 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{array}$$

→ This is called masking

→ To have 1 in the 5^{th} place

left shift 1 with 4.

General form:

num & (1 << n−1) → ith bit

[Bit Masking with &]

Q: 5) Set the ith bit:

Set → Turn it to 1 ⇒ if 0, → 1 and if 1, → 1.

$$\begin{array}{ccccccc} 1 & 0 & 1 & \boxed{0} & 1 & 1 & 0 \\ 7 & 6 & 5 & 4 & 3 & 2 & 1 \end{array}$$

set 4^{th} bit

OR any number with 1, gives 1

$$\begin{array}{ccccccc} 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 1 & 0 & 1 & 1 & 1 & 1 & 0 \end{array}$$

General form: num | (1 << n −1) → ith bit

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6) Reset ~~Toggle~~ i^{th} bit :

↳ 0→0 / 1→0

$$\begin{array}{rcccccccc} & 1 & 0 & \boxed{1} & 0 & 1 & 1 & 0 \\ \& & 1 & 1 & \boxed{0} & 1 & 1 & 1 & 1 \\ \hline & 1 & 0 & \boxed{0} & 0 & 1 & 1 & 0 \end{array}$$

General form :

$\boxed{num \,\&\, \overline{(1 << n-1)}}$ → $num \& !(1<<n-1)$

This complement $\overline{(1<<n-1)}$ gives 0 at i^{th} place.

7) Find the position of right most set bit.

Eg: 1 0 1 1 0 1 $\boxed{1}$ 0 0 Ans : 4

N = a 1 b   a = 1 0 1 1 0 1 ; b = 0 0

Except 1, we need to complement a, then

$= \overline{a}\ 1\ b$

since, b = 0 0 nothing to do with b

$-N = \overline{a}\ 1\ b$

Ans : N & (−N)

−N ⇒ * Complement

* Add 1

$$\begin{array}{rl} -N = & 0\,1\,0\,0\,1\,0\,0\,1\,1 \\ & \qquad\qquad\quad +\ 1 \\ \hline & 0\,1\,0\,0\,1\,0\,1\,0\,0 \end{array}$$

$-N = \underbrace{010010}_{\overline{a}}\ \underbrace{1}_{1}\ \underbrace{00}_{b}$

$N \& (-N) = 1\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 0$

$(\&)\ 0\ 1\ 0\ 0\ 1\ 0\ 1\ 0\ 0$

$0\ 0\ 0\ 0\ 0\ 0\ \boxed{1}\ 0\ 0$

4

Right most set bit.

NEGATIVE OF A NUMBER: IN BINARY

1 Byte = 8 Bits

8	7	6	5	4	3	2	1
0	0	0	0	1	0	1	0

10 ⇒ 00001010

MSB → Tells us whether the number is positive or negative. (Reserved bit)

LSB → Represents the value of a number

−10 ⇒

1 → (−)ve

0 → (+)ve

For integer, the size is 4 bytes i.e., 32 bits

To find negative of a number:

1. complement of a number
2. Add 1.

also known as 2's Complement method

For example,

$(10)_{10} = (00001010)_2$

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① Take complement

1 1 1 1 0 1 0 1

② Add 1

$$\begin{array}{r} 11110101 \\ +\quad 1 \\ \hline 11110110 \end{array}$$

$\Rightarrow$ −10 in binary.

Why does 2's complement give negative of a number?

⇒ Greater than 8 bits, the bits other than 8 is ignored, when it's stored in 8 bit

For eg; $\boxed{1}$ 0 1 1 0 1 0 0 1 0 ⇒ Storing this in 8 bits.

Ignored

⇒ Whenever we subtract 0 from a number ~~from 0~~, it'll give (−)ve of a number.

(0 − 10)

00000000 − 00001010.

$$\begin{array}{r} 100000000 \\ -\ 00001010 \\ \hline \end{array} \longrightarrow ①$$

→ This extra bit is ignored as it can store only 8 bits. So, it may not cause any change.

1000 → 8 → 7+1 → 0111 + 1

$$\begin{array}{r} 0111 \\ +\quad 1 \\ \hline 1000 \end{array}$$

That is the power of 2

So, ① can be written as

11111111+1 − 00001010

$$\underbrace{11111111 - 00001010}_{\text{This Gives the complement}} \; \underbrace{+1}_{\text{represents step-2}}$$

$$\begin{array}{r} 11111111 \\ -\,00001010 \\ \hline 11110101 \end{array} \rightarrow \text{Complement}$$

$$\begin{array}{r} +1 \\ \hline 11110110 \end{array} \longrightarrow -10$$

RANGE OF A NUMBER:

* How many numbers can be stored in 1 Byte ?

Sign ← first bit; Actual Value → remaining bits

1 Byte

0/1	0/1	0/1	0/1	0/1	0/1	0/1	0/1
2 ways	~~1~~ 2	2	2	2	2	2	2

Each bit can have either '0' or '1'.

Total = 2^8 = 256 numbers can be made

Actual NO is stored in bits = n − 1

In 1 Byte = 7 bits

Total I can make from 7 bits = $2^7 = 128$
i.e., 128 in positive & 128 in negative

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$-128 \xleftarrow{+} 0 \xrightarrow{+} 127 = 256$

Not 128 coz negative of 0 is not negative

Negative of 0.

① 0 0 0 0 0 0 0 0

② 1 1 1 1 1 1 1 1

③ + 1

1 0 0 0 0 0 0 0 0

9th bit is discarded

$\therefore \boxed{-0 = 0}$

RANGE FORMULA FOR N BITS :

$$-2^{n-1} \text{ to } 2^{n-1} - 1$$

Q 8) arr = [2, 2, 3, 2, 7, 7, 8, 7, 8, 8]

In an array, the numbers appeare thrice, find the no which occurs once.

* In the array, every number appears three times, then their set bits also appear three times

1 0 - 2

1 0 - 2

$\boxed{1\ 1 - 3}$

1 0 - 2

1 1 1 - 7

1 1 1 - 7

1 0 0 0 - 8

1 1 1 - 7

1 0 0 0 - 8

1 0 0 0 - 8

* If the array is storing a total no. of bits, then

3	3	7	4

* If in this array (arr), every no occurs thrice, then the total no. of bits are exactly the multiple of 3 (In this case, | 3 | 3 | 6 | 3 |

* Then, this will be divisible by 3.

* Taking modulo of | 3 | 3 | 7 | 4 | will that extra number the occurs once.

* | 3 | 3 | 7 | 4 | % 3 = | 0 | 0 | 1 | 1 | $\Rightarrow (3)_{10}$

<u>General</u>: In an array, if numbers appear even times except one, then taking XOR will give the answer.

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And if the numbers appear odd times, then taking modulo will give the ans.

Q: 9) Find the n^{th} Magic Number. (Amazon)

Magic Number

$1 = 0 \times 5^3 + 0 \times 5^2 + 1 \times 5^1 = 5$

$2 = 0 \times 5^3 + 1 \times 5^2 + 0 \times 5^1 = 25$

$3 = 0 \times 5^3 + 1 \times 5^2 + 1 \times 5^1 = 30$

$4 = 1 \times 5^3 + 0 \times 5^2 + 0 \times 5^1 = 125$

⋮

Eg: $n = 6$ 1 1 0

Looping
① n & 1 → Give last digit
② n >> 1 → omits the last digit, getting remaining digits

$1 \times 5^3 + 1 \times 5^2 + 0 \times 5^3 = 150$

$\boxed{\text{Complexity} = \log(n)}$

10) Find no. of digits in binary.

$(6)_{10} \rightarrow 1$ digit

$(6)_{10} = (110)_2 \Rightarrow 3$ digits

$\Rightarrow \log_b a = x$

$\Rightarrow a = b^x$

For Eg:

$\log_2 6 = x$

$6 = 2^x$

1) $\log_2 6 = 2.58 \Rightarrow 2+1$

$6 = 2^{2.58}$

No. of digits in $6 = 110 = 3$

so, No. of digits of 6 of base $2 = 2+1 = 3$

2) $\log_2 10 = 3.32$ ($10 = 1010_2 = 4$ digits)

No. of digits $= 3+1 = 4$

Formula:

No. of digits of base 'b' of Number 'a' is $\text{int}(\log_b a) + 1$.

$$\log_b a = \frac{\log_x a}{\log_x b}.$$

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11) Pascal's Triangle : ${}^1C_0 = \binom{1}{0}$

```
        1
      1   1
    1   2   1
  1   3   3   1
1   4   6   4   1
```

$$\binom{0}{0}$$
$$\binom{1}{0} \quad \binom{1}{1}$$
$$\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$$
$$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$$
$$\cdots$$

Sum of each row

$= nC_0 + nC_1 + nC_2 + \cdots + nC_n = 2^n$

For n^{th}, the sum is 2^{n-1}

Ans : $1 << (n-1) = 1 \times 2^{n-1}$

NOTE :

$1 << 0 \Rightarrow 1 - 2^0$

$1 << 1 \Rightarrow 10 = 2^1$

$1 << 2 \Rightarrow 100 = 2^2$

$1 << 3 \Rightarrow 1000 = 2^3$

12) Find whether the number is power of 2 or not

Note : If the number is power of 2, there will only one 1's and the rest will be 0's. For example, $0100 \rightarrow$ power of 2 (2^2)

$0110 \rightarrow$ Not ($2^2 + 2^1 = 6$)

For Example

'100000' can be written as

$11111 + 1$

* Taking '&' The number with (11111) gives the ans.

* If ans == 0, then the no is power of 2 otherwise, not.

$$\begin{array}{r} 100000 \\ \& 011111 \\ \hline 000000 \end{array} \Rightarrow 0$$

Then $(100000)_2 = (32)_{10}$ is power of 2.

$$\begin{array}{r} 10010 \\ \& 01111 \\ \hline 00010 \end{array} \neq 0$$

Then $(10010)_{2} = (17)_{10}$ is not power of 2.

$\boxed{\text{General form: } n \,\&\, (n-1) == 0}$

Exception if n == 0 then it will be false

Q: 13) Calculate a^b

$3^6 \Rightarrow 3*3*3*3*3*3$ (complexity $O(b)$)

Using bitwise

$3^6 = 3^{\overset{420}{110}}$

ans = 1

base = 3

n = 110

n & 1 = 0

n = 110

n >> 1 ⇒ 11 & 1 = 1

n = 11

n >> 1 ⇒ 1 & 1 = 1

n = 1

n >> 1 ⇒ 0 & 1 = 0

$\boxed{\text{ans} *= \text{base}}$

base *= base

$\boxed{3^{\overset{420}{110}} \Rightarrow 3^4 * 1 + 3^2 * 1 + 3^0 * 0}$ $O(\log(b))$

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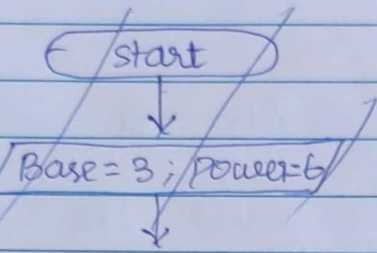

0. Start
1. Take power and base.
2. Initialize ans = 1
3. If the last digit of power is 1, then multiply the ans ~~by~~ with base
4. Then increment the base as $(base)^2$ &
~~5~~ right shift the power to skip the last digit each iteration
5. End.

Example - $3^6 = 3^{110}$

ans = 1

① 6 > 0 (base = 3 power = 6)

110 & 1 ≠ 1

base = 3 * 3 = 9

power = 011

② 011 > 0

011 & 1 == 1

ans = 1 * 9

ans = 9

base = 9 * 9 = 81

power = 001

③ 001 > 0

001 & 1 == 1

ans = 9 * 81 = 729

base = 81 * 81 = 6561

power = 000

④ 000 ≯ 0

Terminate

$\boxed{ans = 729}$

14) Given a number, find the no. of set bits

$n = 9$

$n = 1001 \Rightarrow Ans = 2$

$\boxed{n \& (-n)}$ ⇒ gives the right most set bit

$n - [n \& (-n)]$ ⇒ gives the right most set bit and neglecting it each time

① n=9 1 0 0 1
0 1 1 0
1
−n = 0 1 1 1

n & (−n) = 1 0 0 1
0 1 1 1
0 0 0 1

n − (n & (−n)) 1 0 0 1
1 0 0 0

count = +1 ;

② n = 1 0 0 0
0 1 1 1
−n = + 1
1 0 0 0

n & (−n) : 1 0 0 0
& 1 0 0 0
1 0 0 0

n − (n & (−n)) − 1 0 0 0
0 0 0 0

count = 2

① n = 15 = 1111
−n ⇒ 0 0 0 0
1
−n 0 0 0 1
n & −n = 1 1 1 1
0 0 0 1

n − (n & (−n))
= 0 0 0 1
1 1 1 1
1 1 1 0

count = 1

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② $n = 1110$

$-n = 0001$
$+ \quad 1$
0010

$n\&(-n)$ 1110
0010

$n-(n\&(-n)) = 0010$
1110
1100

count = 2

③ $n = 1100$

0011
$+1$

$-n$ 0100

$n\&(-n) = 1100$
0100

$n-(n\&(-n)) = 0100$
1100
1000

count = ③

④ $n = 1000$

$= 0111$
$+1$

$-n$ 1000

$n\&(-n)$ 1000
1000

$n-(n\&(-n)) = 1000$
1000
0

count = 4

$\boxed{\text{Ans} = 4}$

Q :15 Find the XOR of numbers from 0 to a.

a	XOR from 0 - a.
0	0
1	0^1 = 1
2	0^1^2 = 3
3	0^1^2^3 = 0
4	0^1^...3^4 = 4
5	0^1^..4^5 = 1
6	0^..5^6^ = 7
7	0^..6^7 = 0
8	0^..7^8 = 8
9	0^.....8^9 = 10

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$a \% 4 == 0 \Rightarrow a$
$a \% 4 == 1 \Rightarrow 1$
$a \% 4 == 2 \Rightarrow a+1$
$a \% 4 == 3 \Rightarrow 0$

16) XOR of all numbers between range (a,b)

$a = 3$ $b = 9$

$3 \wedge 4 \wedge 5 \wedge 6 \wedge 7 \wedge 8 \wedge 9$

we know $0 \wedge 1 \wedge 2 \wedge 3 \wedge 4 \wedge 5 \wedge 6 \wedge 7 \wedge 8 \wedge 9$

Subtracting (0 - 2) from (0 - 9) will give the required ans.

$\therefore$ XOR(3, 9) = XOR(0,9) ^ XOR(0,2)

xORing the duplicates will give 0

17) Flip the image horizontally

1) Reverse [0,1,1] → [1,1,0]
2) Invert [1, 1,0] → [0,0,1]

	Reverse	Invert
1 1 0	0 1 1	1 0 0
1 0 1 ⇒	1 0 1 ⇒	0 1 0
0 0 0	0 0 0	1 1 1

Image is a 2D array of 0's & 1's