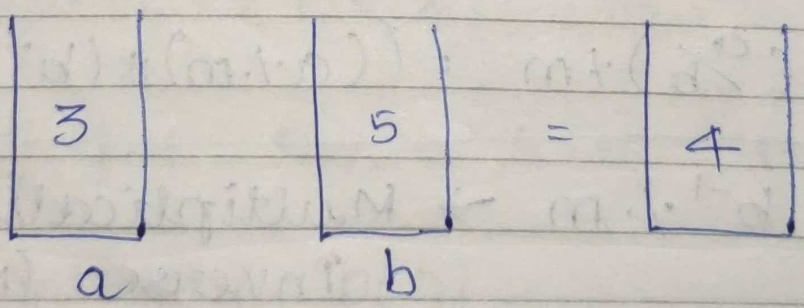
DIE - HARD - Example

To dis-harm a bomb, you have to exactly measure 4 gallons of water. But you are only given 3 & 5 gallons of jug



Don't fill water with any random assumptions, you've to measure exact 4 gallon.

Pass	a, b	pour		
① →	(0, 0)	⟶	(3, 0)	→ (0, 3)
② →	(0, 3)	⟶	(3, 3) → (1, 5)	
			↓	
			(0, 1) ← (1, 0)	
③ →	(0, 1)	→	(3, 1)	⟶ (0, 4) ⇒ Ans

Jug - a was filled s^1 times
Jug - b was emptied s^2 times
remainder $= as^1 - bs^2 = as^1 + (-bs^2)$
a - volume of 1st jug
b - volume of 2nd jug
as^1 - amt of water taking
bs^2 - amt of water removing

$\Rightarrow r = as^1 - bs^2$

$r = as^1 + (-bs^2)$

Add & sub $t'b$

$r = as^1 + t'b - t'b - bs^2$

let $L = s'a + t'b$

$s'a = L - t'b$

$r = L - (t' + u)b$

If $t' + u \neq 0 \Rightarrow r < 0$ (or) $r > b$
which is not true

The remainder water can't be negative

$t' + u = 0 \Rightarrow u = -t'$

$r = s'a + t'b = L$

Buckets we were given is converted to a linear equation

We are given 3 gallon & 5 gallon of jug to make 4 gallons of water This in a linear equation will be like $3x + 5y = 4$

① Pat x & y, a integer, then what is the minimum value (+ve), the equation can produce

For example.

$x = -3 \quad y = 2$

$= 3(-3) + 5(2) = 1 \rightarrow$ This is the min. (+)ve value

NOTE: This is called HCF (a, b)

Min. (+)ve value of eqn $ax+by$ where, $x, y \Rightarrow \mathbb{Z}^{rs}$

Example:

HCF (3, 9) = 3

1, (3) ↙ ↘ 1, (3), 9 $\Rightarrow$ 3 - HCF

Min $(3x + 9y) = 3$

$= 3(x + 3y)$

$x = -2 \quad y = 1$

$= 3(-2 + 3(1))$

$= 3(1)$

$\boxed{= 3}$

How does this note relate to our problem.

$$ax + by = L$$

For eg; $2x + 4y = 5$ lit

NOTE: What ever HCF you will get that will come out as common, & not a decimal (coz decimal value can't be common factor)

$3x + by = 9$

$3(x + 2y) = 9$

$3 \backslash 9$ ∴ true can form a lit

$3x + 5y = 17$

$1(3x + 5y) = 17$

$1 \backslash 17$ ∴ true