UNC
CS Department

COMP 755– HW 1
Machine Learning

Instructor: Prof. Junier Oliva

2021/8/26

Name: Sharmin Aktar

This assignment contains 3 pages (including this cover page) and 1 multi-part questions. The total of points is 100. **The assignment is due September 1st, 2021 11:59PM via** https://docs.google.com/forms/d/e/1FAIpQLSekixyPflufkMEh3jXgBza22BRJwKP0cmICZnuVFG4jTXwbQQ/viewform?usp=pp_url. For any handwritten portions, please submit clear and legible handwritten or typefaced answers. **Do not try to search directly for the answers. You may speak to other students about the assignment at a high-level (e.g. sharing related references/slides). However, Sharing complete or partial answers is strictly prohibited.** Please see the UNC honor code: https://catalog.unc.edu/policies-procedures/honor-code/ for additional details on maintaining academic integrity.

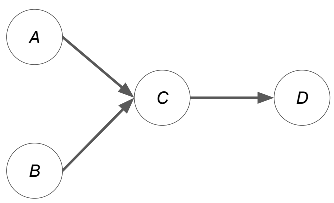
Distribution of Marks

Question	Points	Score
1	100	
Total:	100	

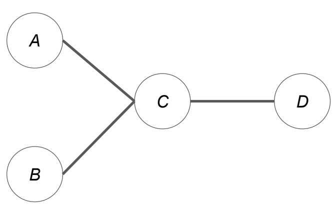
1. **Graphical Models**

   (a) (25 points) Suppose that we have 4 features A, B, C, D. What are all the possible conditional (and unconditional) independence statements to make between the features? (E.g. $C \perp\!\!\!\perp D, \ldots$)

   (b) (25 points) Which of all the possible independence statements (in *(a)*) are guaranteed to hold for the Bayesian network below?



   (c) (25 points) Which of all the possible independence statements (in *(a)*) are guaranteed to hold for the undirected graphical model below? Why is this not a valid graphical model for distributions satisfying the Bayesian network above?



   (d) (25 points) What is the smallest edit to the undirected graphical model above, such that it is a valid graphical model for distributions satisfying the Bayesian network (in *(b)*). What are the independence statements are guaranteed to hold for the edited undirected graphical model?

1(a) $A \perp C \mid B$, if B is observed and ($A \leftarrow B \rightarrow C$ or $A \rightarrow B \rightarrow C$ or $C \rightarrow B \rightarrow A$) [cascade or common parent situation]

$A \not\perp C$ if B is unobserved

another case, if $C \rightarrow B \leftarrow A$ and B is unobserved, then $A \perp C$

~~$A \not\perp C$ if~~ $A \not\perp C \mid B$ if B is observed

Similarly,

$A \perp D \mid B$ if B is observed, where B is common parent of A, D or A, B, D form cascade situation

$A \not\perp D$ if B is unobserved

for V-Structure

$$A \rightarrow B \leftarrow D$$

$A \perp D$ if B is unobserved

$A \not\perp D \mid B$ if B is observed

Similarly for the rest of the Conditional Independence we can write;

$A \perp B \mid C$ if C is observed
$A \not\perp B$ if C is unobserved
[Cascade or Common Parent]

$A \perp B$ if C is unobserved
$A \not\perp B \mid C$ if C is observed
[V-Structure]

$A \perp D \mid C$ if C is observed
$A \not\perp D$ if C is unobserved
[Cascade or Common Parent]

$A \perp D$ if C is unobserved
$A \not\perp D \mid C$ if C is observed
[V-Structure]

$A \perp B \mid D$ if D is observed
$A \not\perp B$ if D is unobserved
[cascade or Common Parent]

$A \perp B$ if D is unobserved
$A \not\perp B \mid D$ if D is observed
[V-Structure]

$A \perp C \mid D$, if D is observed
$A \not\perp C$, if D is unobserved
[cascade or Common Parent]

$A \perp C$, if D is unobserved

$A \not\perp C | D$ if D is observed

V-Structure

for Cascade or common parent situation the following Conditional Independence Statements hold:

$B \perp C | A$, if A is observed

$B \perp D | A$, " " " "

$B \perp A | C$, if C is observed

$B \perp D | C$, if C is observed

$B \perp A | D$, if D is observed

$B \perp C | D$, if D is observed

$C \perp B | A$, if A is observed

$C \perp D | A$, if A is observed

$C \perp D | B$, if B is observed

$C \perp A | B$, if B is observed

$C \perp A | D$, if D is observed

$C \perp B | D$, if D is observed

$D \perp B \mid A$, if A is observed

$D \perp C \mid A$, if A is observed

$D \perp A \mid B$, if B is observed

$D \perp C \mid B$, if B is observed

$D \perp A \mid C$, if C is observed

$D \perp B \mid C$, if C is observed

The following Conditional Independence statements hold for the V-Structure case:

$B \perp A$, if C is unobserved

$B \not\perp A \mid C$, if C is observed

$B \perp D$, if C is unobserved

$B \not\perp D \mid C$, if C is observed

$B \perp C$, if A is unobserved

$B \not\perp C \mid A$, if A is observed

$B \perp D$, if A is unobserved

$B \not\perp D \mid A$, if A is observed

$B \perp A$, if D is unobserved
$B \not\perp A \mid D$, if D is observed

$B \perp C$, if D is unobserved
$B \not\perp C \mid D$, if D is observed

$C \perp B$ if A is unobserved
$C \not\perp B \mid A$, if A is observed

$C \perp D$ if A is unobserved
$C \not\perp D \mid A$, if A is observed

$C \perp D$, if B is unobserved
$C \not\perp D \mid B$, if B is observed

$C \perp A$, if B is unobserved
$C \not\perp A \mid B$, if B is observed

$C \perp A$, if D is unobserved
$C \not\perp A \mid D$, if D is observed

$C \perp B$, if D is unobserved
$C \not\perp B \mid D$, if D is observed

$D \perp B$, if A is unobserved
$D \not\perp B | A$, if A is observed

$D \perp C$, if A is unobserved
$D \not\perp C | A$, if A is observed

$D \perp A$, if B is unobserved
$D \not\perp A | B$, if B is observed

$D \perp C$, if B is unobserved
$D \not\perp C | B$ if B is observed

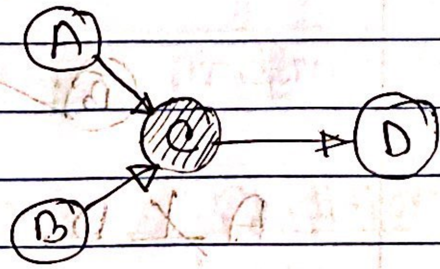
$D \perp A$, if C is unobserved
$D \not\perp A | C$ if C is observed

$D \perp B$, if C is unobserved
$D \not\perp B | C$ if C is observed

Scanned with CamScanner

1(b) if C is observed, then we can say that

$A \perp D \mid C$



$\because$ C holds all the information, that is needed to determine the outcome of D.

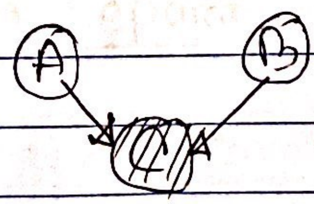
Knowing A does not make any change in the outcome ab D.

Similarly, we can say,

$B \perp D \mid C$, ib C is observed

Again, if C is observed, we can see

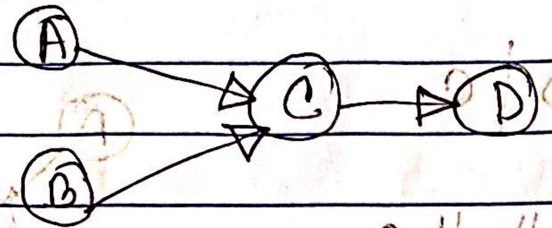
$A \not\perp B \mid C$,



Here, knowing C couples A and B

V-structure (explaining away)

If C is not observed, then we see



$A \not\perp D$, if C is not observed

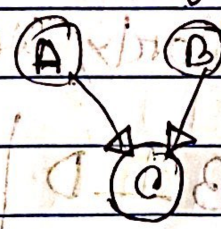
$B \not\perp D$, if C is not observed

Both situations are examples of Cascade

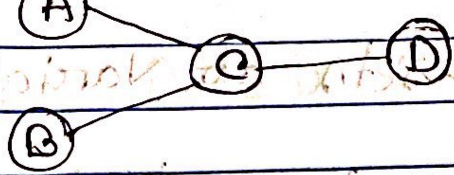
But in case of A, B, C

$A \perp B$, if C is unobserved

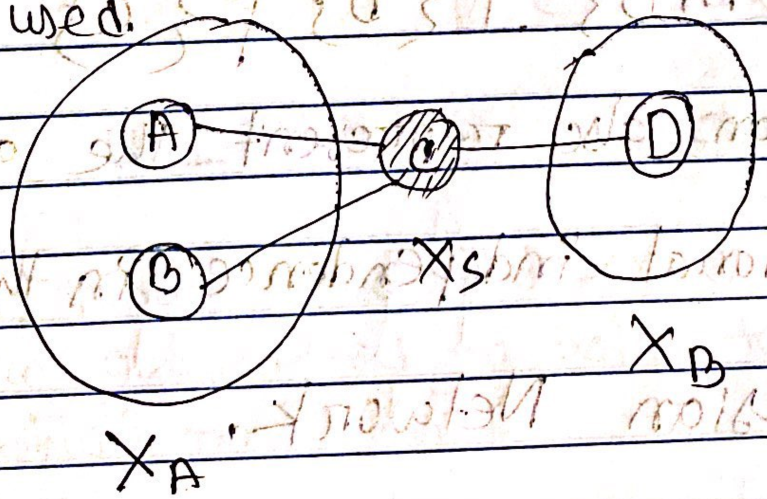
this is an example



of V-structure.



The above one is an undirected graphical model. To capture conditional independence is in such an undirected graphical model "Markov Random Fields" is used.



Here $X_S = \{C\}$, X_S is a separating subset

$X_A = \{A, B\}$, Subset X_A containing A and B

$X_B = \{D\}$, Subset X_B containing only D

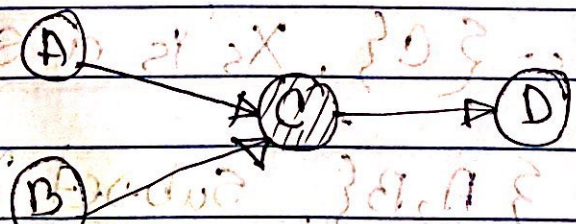
According to Global Markov property, any two subsets of variables are conditionally independent given a separating subset.

So, we can say;

$$X_A \perp X_B \mid X_S$$

$$\{A, B\} \perp \{D\} \mid \{C\}$$

We can also represent the above conditional independence in the following Bayesian Network.



If C is observed, then $A \rightarrow C \rightarrow D$

form a Cascade pattern, So, we can say, $A \perp D \mid C$, if C is observed

Again, $B \rightarrow C \rightarrow D$ forms a cascade pattern, So, we can say,

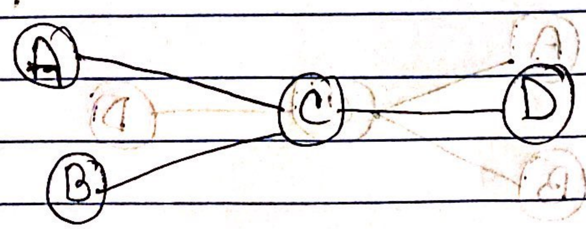
$B \perp D \mid C$, if C is observed.

Now, we know that, the following two conditional independence statements hold for the UGM.

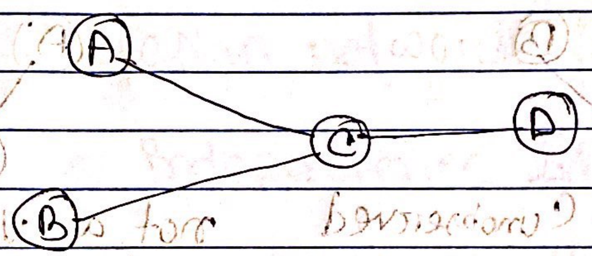
$A \perp D \mid C$, if C is observed

$B \perp D \mid C$, if C is observed

The above two independence statements are guaranteed to hold for the undirected graphical model below.

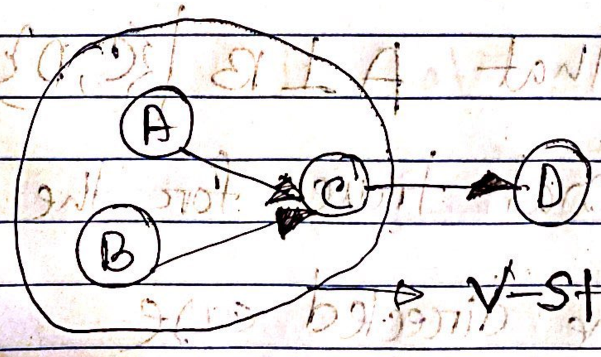


1(c) Question: (Part 2 of 1(c)) Why is this not a valid graphical model for distributions satisfying the Bayesian network above?

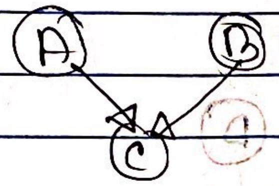


Answer:

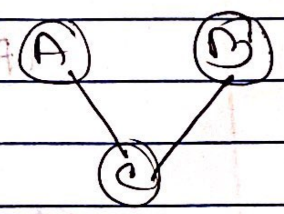
The above bayesian network has a V-Structure Component marked below



We know that for a directed V-structure case, there ~~ets~~ is no undirected model that can describe the independence assumption $A \perp B$



$A \perp B$ if C unobserved



not a valid undirected representation for directed $A \rightarrow C \leftarrow B$ ~~structu~~ structure to ~~Comp~~ capture $A \perp B$

In the given undirected model, we can show that $A \perp B \mid \{C, D\}$ which is not true for the directed case. In the directed case $A \perp B$ only if C is unobserved.

1(d) Question: What is the smallest edit to the undirected graphical model above, such that it is a valid graphical model for distributions satisfying the Bayesian network (in (b)). What are the independence statements are guaranteed to hold for the edited undirected graphical model?

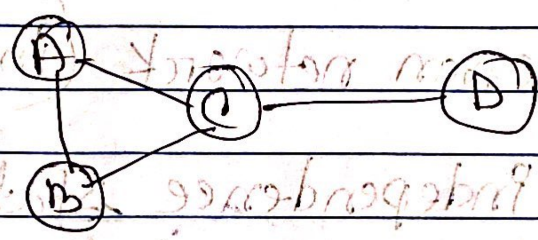
Answer:

One of the possible solutions that I can think of is that adding an edge between node A and node B.

That way, we can eliminate the independence

~~assu~~ assumption $A \perp B \mid \{C, D\}$ [using Pairwise Markov property]

So, the resulting undirected graphical model is:



there is no way we can show that $A \perp B$ holds for any specific scenarios.

The only guaranteed independence statements are

$$\{A, B\} \perp \{D\} \mid \{C\}$$ [using global Markov property]

$A \perp D \mid \{B, C\}$

$B \perp D \mid \{A, C\}$ [using Pairwise Markov Property]

So, the I-map at the edited undirected graphical ~~moded~~ model is the subset of the given Bayesian network

$I(G') \subset I(G_{BN})$

This page is intentionally left blank to accommodate work that wouldn't fit elsewhere and/or scratch work.